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Effect of Aspect Ratio on Developing and Developed Narrow Open Channel Flow with Rough Bed

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Abstract
This study attempts to unravel the effect of aspect ratio on the turbulence characteristics in developing and fully developed narrow open channel flows. In this regard, experiments were conducted in a rough bed open channel flow. Instantaneous 3-D velocities were acquired using an acoustic Doppler velocimeter at various locations along the centerline of the flume. The variables of interest include the mean components of the flow velocity, turbulence intensity, wall normal Reynolds Shear Stress, correlation coefficient, turbulence kinetic energy and anisotropy. A new correlation between the equivalent roughness and velocity shift from the smooth wall logarithmic equation as a function of aspect ratio is proposed. Aspect ratio was found to influence the velocity characteristics throughout the depth in the developing flow region, while the effects are confined to the outer layer for fully developed flows. New equations to describe the variation of turbulence intensities and turbulent kinetic energy are proposed for narrow open channel flows. Reynolds stress anisotropy analysis reveals that level of anisotropy in narrow open channel flow is less than in wide open channel flows. Finally, a linear regression model is proposed to predict flow development length in narrow open channel flows with a rough bed.

Keywords: turbulence, narrow open channel, aspect ratio, developing flow, fully develop flow, flow development length
1. Introduction

It is essential to understand the flow characteristics in developing and developed open channel flows to assess fluid-sediment interaction and sediment erosion, which are directly linked to the hydrodynamic characteristics such as the vertical distribution of time-averaged velocities, turbulence intensities, Reynolds shear stresses and turbulent kinetic energy. However, most open channel flows are non-uniform because of ever changing boundary conditions resulting from changes in discharge, cross-section, bed roughness, the presence of hydraulic structures, etc. Flow becomes disturbed whenever there is a change in boundary condition because of imbalance of viscous, gravitational and inertial forces. Subsequently, the flow tries to attain a fully developed flow condition in the downstream direction. The developing turbulent flow in the open channel is a complex three-dimensional flow, which is influenced by aspect ratio, bed roughness, and other parameters. A significant number of studies have focussed on the flow in smooth, wide open channels. A study of turbulent characteristics of developing narrow open channel flow on rough and transitionally rough surfaces which is often encountered in hydraulic engineering is of importance.

Literature indicates that the classical log-law is applicable for the vertical distribution of streamwise velocity within 20% of the flow depth above the channel bed (Cardoso et al. 1989; Nezu and Rodi 1986; Xinyu et al. 1995). Investigators have reported that above 20% of the flow depth, the log-law deviates from the experimental data and the most reasonable expression is the wake law (e.g., Coles 1956). Absi (2011) has stated that the log-wake law is not valid for flow in a narrow open channel where the value of the aspect ratio, defined by the ratio of width of the open channel to depth of flow is less than five. Moreover, for an aspect ratio greater than 10, which corresponds to a wide open channel, the log-wake law is not valid closer to the side-walls (Vanoni 1941). The complexity of the narrow open channel turbulent flow is enhanced due to its susceptibility to disturbances from the side-walls in addition to the bed roughness and free surface (Guo 2014). This phenomenon is similar to shallow open channel flow where disturbances from the bed and free surface lead to turbulent flow with coherent motions (Roussinova et al. 2010; Balachandar and Patel 2005). In shallow open channel flows, the depth is the controlling length scale, whereas the distance between the sidewalls is expected to play an important role in the generation of coherent eddies in narrow open channel flows.
A century ago, researchers (Francis 1878 and Stearns 1883) have reported that in fully
developed narrow open channel flow, the maximum velocity occurs below the free surface,
which is commonly referred to as the dip-phenomenon. In the last few decades, several
experimental studies have discussed the dip-phenomenon in open channel flows (Kirkgoz and
studies have also predicted a velocity distribution with a dip-phenomenon (Absi 2011 and Yang
et al. 2004). It is well known that to perform an experimental or theoretical study in narrow
open channel flow, it is very important to know the characteristic length by which the flow
would acquire a fully developed condition. This study aims to investigate the turbulence
characteristics in developing and fully developed narrow open channel flow at different aspect
ratios over a fixed rough bed. In addition, the effects of aspect ratio on equivalent roughness
and the velocity shift from the log-law are studied. The investigation is focused to include
hydraulically rough and transition conditions to develop an equation to predict flow
development length as a function of aspect ratio and equivalent sand-grain roughness.

The paper mainly investigates the effects of aspect ratio on: (i) vertical distribution of
streamwise, lateral and vertical velocities, turbulence intensities and Reynolds shear stress to
explore the physics behind their behavior and (ii) vertical distribution of correlation coefficient,
turbulent kinetic energy (TKE) and Reynolds stress anisotropy. New equations to predict
turbulence intensities and TKE for narrow open channel flow are developed. Finally, the paper
proposes a linear regression model as a function of aspect ratio to compute the length required
for attaining fully developed flow conditions in a narrow open channel.

2. Experimental methodology

Experiments were conducted in a 7.0 m long, 0.6 m wide, and 0.7 m deep glass-walled
rectangular flume in the Hydraulic and Water Resources Engineering Laboratory at Indian
Institute of Technology, Kharagpur, India. The bed slope of the flume was 0.002. Fig. 1 shows
a schematic diagram of the experimental facility. By looking into the flow direction, the glass
wall on the right-hand side is referred as RHS wall and similarly, the left-hand side wall is
referred as LHS wall. At the inlet to the flume, a screen is provided to reduce the turbulence
levels in the flow. The bed is made of concrete with a smooth finish and cladded with uniform
sand particles of median diameter $d_{50} = 2.5$ mm to make it rough. Two centrifugal pumps
were used to recirculate the water between the underground sump and the flume. The desired
flow depth in the flume is achieved by adjusting the tailgate and the flowrate is controlled by

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a valve located upstream of the inlet chamber. For the coordinate system, \( x \)-axis is denoted as streamwise direction, \( y \)-axis is denoted as lateral direction from the RHS wall to the LHS wall and \( z \)-axis is denoted as vertical direction upwards from the channel bed. The origin of the coordinate system is fixed at the intersecting point between channel bottom and the RHS wall at the inlet. The flow depth was measured with the help of a point gauge equipped with a Vernier scale. The test section is 3.0 m long and starts 2.15 m downstream of the inlet.

A three-dimensional down looking ADV system (VectrinoPlus) was used to measure the instantaneous flow velocities. The velocity measurements were undertaken along the centerline at different distances from the inlet (\( x = 2.15, 3.65 \) and \( 5.15 \) m). Velocity measurement nearest to bed is achieved at 0.3 mm above the bed and the farthest measuring point is taken at 50 mm below the free surface. Near the bed, the velocity measurements are carried out at every 0.003 m interval, while away from the bed, the velocity measurements are obtained at every 0.01 m interval and the details of co-ordinates are: \( z = 0.003, 0.005, 0.007, 0.009, 0.015, 0.02, 0.025, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.1 \) (for flow depth 0.15 m), 0.11, 0.12, 0.13, 0.14 and 0.15 m (for flow depth 0.2 m) from the channel bed. The ADV measures instantaneous velocities at a location 50 mm below the probe emitter to minimize the influence of the probe on the measured data. The sampling rate was 100 Hz. The sampling cylinder dimensions were fixed at \( 6 \times 10^{-3} \) m in height and \( 6 \times 10^{-3} \) m in diameter. The SNR and correlation values were greater than 22 and 80, respectively. The sampling duration was set to 300 s. The low frequency noise in the Vectrino raw data was despiked by using the phase-space threshold method as proposed by Goring and Nikora (2002) and Wahl (2003). The removed spikes were replaced using a cubic interpolation method.

Six experimental runs were conducted with Reynolds number ranging between \( R_e = 2.8 \times 10^4 \) and \( 4.5 \times 10^4 \) and four aspect ratios, \( A_r = 2.5, 2.75, 3.0, \) and \( 4.1 \), where \( A_r \) = ratio of channel width to flow depth. These experiments are denoted as E1 to E6. The experiments E1, E3, E5 and E6 are at the same Reynolds number \( (R_e = 4.5 \times 10^4) \) with aspect ratios 3.0, 4.1, 2.5 and 2.75 respectively. Similarly, experiments E2 and E4 are for the second Reynolds number \( (R_e = 2.8 \times 10^4) \) with aspect ratios 3.0 and 4.1, respectively. Finally, an additional experiment (E7) at a \( R_e = 1.95 \times 10^4 \) with an aspect ratio of 3.3 was conducted at the University of Windsor to confirm the results obtained from the experiments E1 to E6. The aspect ratios in these experiments are less than five and the experimental regimes are classified as narrow open channel flow (Nezu et al. 1985). In experiments E1 to E4, instantaneous
velocity measurements are captured at two locations in the developing flow region \((x = 2.15 \text{ m and 3.65 m})\) and one location in the fully developed flow region \((x = 5.15 \text{ m})\). Velocity measurements were obtained only in the fully developed flow region for the experiments E5, E6 and E7.

In this study, \(u, v\) and \(w\) are the time averaged streamwise, lateral and vertical velocity components, respectively; \(u', v'\) and \(w'\) are the fluctuations of corresponding velocity components; similarly \(u^+ = u/u_* , v^+ = v/u_*\) and \(w^+ = w/u_*\) are the normalized time averaged velocities and \(z^+ = zu_*/\theta\) is defined as the normalized vertical height in inner scaling. Here, \(u_*\) is the shear velocity obtained from the profile of the Reynolds shear stress \((-u'w')\) extended to the channel boundary. The streamwise, lateral and vertical turbulence intensities are represented by \(u_{rms}, v_{rms}\) and \(w_{rms}\) respectively; similarly \(-u'w^+ = (-u'w')/u_*^2\) ; \(-u'v'^+ = (-u'v')/u_*^2\) and \(-v'w'^+ = (-v'w')/u_*^2\) are the normalized Reynolds shear stresses. Uncertainty errors in the inner layer in \(u\) and \(-u'w'\) are found to be less than 5% and 12.5%, respectively.

Complete experimental conditions are provided in Table 1, where \(k_{r+} = k_\delta u_*/\theta\) is the roughness Reynolds number and \(U\) is depth averaged flow velocity in the fully developed flow region, \(\Delta u^+\) is downward shift from the law of the wall for hydraulically smooth flow and \(\theta\) is the kinematic viscosity of water. In this study, following the classification of Nezu and Nakagawa (1993), the flow field is divided into four regions: (i) inner region \((z/h < 0.2)\); (ii) outer region \((z/h \geq 0.2)\); (iii) intermediate region \((0.2 \leq z/h \leq 0.6)\) and (iv) free surface region \((0.6 < z/h \leq 1)\). The classification of flow field helps to study the important trends in the turbulence characteristics in the in different regions.

3. Results and discussion

In this section, results of tests E1, E2, E3 and E4 are analysed to differentiate the effect of aspect ratio on turbulence characteristics in the developing flow region. Furthermore, results of E1, E3, E5, and E6 at the same Reynolds number are compared in the fully developed flow.

3.1. Time Mean Velocities

Experimental conditions of the present study are provided in Table 1. Shear velocities are estimated using the distribution of normalized Reynolds shear stress with the normalized vertical height \((z/\delta)\) as shown in Fig. 2(a), where \(\delta\) is the dip position at which the maximum
velocity occurs (see Table 1). The required values of the conventional zero-velocity shift and roughness height are computed using the log-law equation. Equivalent sand roughness values are obtained using the law of the wall equation iteratively since the constant of integration in the equation is also a function of equivalent sand roughness. The computed Nikuradse equivalent roughness height and a downward shift in velocity distributions in each experiment are given in Table 1. The hydraulically rough or transition state is classified based on the criteria presented in Nezu and Nakagawa (1993). The flow is classified as hydraulically smooth if $k_\tau^+$ is less than five; transitionally rough flow if $k_\tau^+$ is between five and 70; and hydraulically rough (fully rough flow) if $k_\tau^+$ is greater than 70. The experiments E1, E2, E3 and E4 belong to the hydraulically rough condition, whereas E5, E6 and E7 fall in the range of transitional flow. From Table 1, for the two available Reynolds numbers, an increase in the aspect ratio increases the shear velocity and hydraulic roughness of the flow.

The velocity distributions in the format of the law of the wall are shown in Fig. 2b, which shows that the results of the present study follow the log-law. The smooth wall data is represented by $u^+ = \frac{1}{k} \ln(z^+) + 5.0$, where $u^+$ is the streamwise velocity in inner scaling and $z^+$ is the vertical co-ordinate in inner scaling above the bed. The plot shows the downward shift in the velocity distribution of the present experiments from the smooth wall data. The increasing downward shift as shown in Fig. 2b is consistent with increasing roughness. Figure 2c shows transitionally rough flows are clustered separately from fully rough flows. For transitionally rough flows, $\Delta u^+$ values range between 2.0 and 4.7 and, similarly $\Delta u^+$ values for fully rough flows are between 7.0 and 7.9. The velocity shift values are comparable to the values available in the literature; 9.7 to 11 (Balachandar and Patel, 2002) and 10 to 15.5 (Hanmaiahgari et al. 2016) and 10 to 14 (Krogstad et al. 1992). The velocity shift values in the present experiments are slightly less because the experiments barely cross the hydraulically rough flow threshold.

In this study, an important observation found is that $\Delta u^+$ and $k_\tau^+$ are changing with the aspect ratio in the narrow open channel flows. As noted earlier, the experiments E1, E3, E5, and E6 are at the same Reynolds number. The aspect ratios are in the decreasing order of $A_r(E3) > A_r(E1) > A_r(E6) > A_r(E5)$ and the corresponding $\Delta u^+$ are also in the decreasing order. For a given Reynolds number, the downward velocity shift from the smooth wall data increases with increasing aspect ratio. From Table 1, it is clear that $\Delta u^+$ is related to $k_\tau^+$ irrespective of the roughness regime. As observed in the present experiments and other
studies in literature, $\Delta u^+$ is a function of $k_s^+$. A few equations are available in the literature such as $\Delta u^+ = \frac{1}{k} \ln(k_s^+) - 3.2$ (Raupach et al. 1991) and $\Delta u^+ = \frac{1}{k} \ln(k_s^+) + 1.2$ (Krogstad et al. 1992). However, these equations are developed for wide open channel flow and are functions of von Karman constant $\kappa$. In this study another correlation between $\Delta u^+$ and $k_s^+$ is developed as given below which is a function of $A_r$.

$$\Delta u^+ = p_1 A_r \ln(p_2 k_s^+ A_r^{p_3}), \quad p_1 = 2.43, p_2 = 0.22, p_3 = 0.1$$

(1)

The proposed empirical equation is developed by using data from the present experiments, immobile bed experiments in Hanmaiahgari et al. (2016), Balachandar et al. (2002), and Balachandar and Blakely (2004). The coefficient of correlation ($R^2$) of the proposed equation is 0.97. The satisfactory agreement between the experimental measurements and computed values using Eq. (1) between mean velocity shift and equivalent roughness height has been presented in Fig. 2c.

Figs. 3(a-c) show the streamwise velocity in inner scaling along the normalized depth in the developing flow region. Marusic et al. (2010) stated that a flow is called fully developed if all the turbulence quantities and the flow quantities are independent of the streamwise distance along the flow development. By examining the velocity profiles measured along the centerline at every 10 cm, flow field was found to be fully developed when $x \geq 5.15$ m for the experimental runs (E1, E2, E3, and E4). This is consistent with the observation of other researchers (Kironoto and Graf 1994).

Velocity profiles presented in Fig. 3(a-c) show the upward shift in the location of local maximum velocity along the channel which indicates the increase in the thickness of the boundary layer along the developing zone (Kirkgoz and Ardiclioglu 1997 and Bonakdari et al. 2014). Slight inward bending of velocity profiles in the outer region near the free surface is observed in all the cases due to the secondary currents in the flow. Figs. 3(a-b) show the effect of aspect ratio in the developing flow region. The data at the same Reynolds number (tests E1 and E3) do not collapse on to each other. However, towards the end of developing flow region, the aspect ratio effect is diminished in the inner layer and its effect is seen only in the outer region (Fig. 3c). Fig. 3(c) also compares hydraulically rough and transitional flows at the same Reynolds number but different aspect ratios. It is clear from the figure that velocity profiles normalized with shear velocities are distinct for hydraulically rough and transitional flows. The hydraulically rough flows are approximately self-similar and similarly, the three transition
flows appear to collapse on to a single profile. In Fig. 3(c), the smooth wall results of Roussinova et al. (2008) is also plotted to compare the velocity profiles in the three regimes. Hydraulically rough flows have the lowest normalized velocities as compared to that of transition flows and smooth flows as indicated by the arrow (see Fig. 3d). Comparing the results of E1, E3, E5, and E6, it is clear that for a given Reynolds number, as the aspect ratio increases, the flow is becoming increasingly rough. Fig. 3(d) shows the vertical distribution of velocity defect for fully-developed flows with different aspect ratios. The hydraulically smooth flow has a higher velocity defect in the outer layer followed by the transition flows and the least velocity defect is found in hydraulically rough flows. The arrow direction in Fig. 3d shows that among the same Reynolds number flows (E1, E3, E5, and E6), the velocity defect is small for hydraulically rough flows in the inner layer.

The distribution of lateral and vertical velocities of the flow represents the secondary currents. Conventionally, secondary currents have been found to be strong in narrow open channels and their strength reportedly increases with a decrease in aspect ratio. Fig. 4(a) shows the vertical distribution of time-averaged lateral velocity along the centerline of the developing flow. Near the inlet, at $x = 2.15$ m, it is observed that for a higher Reynolds number flow, irrespective of aspect ratios (E1 and E3), $v^+$ profiles show an increasing trend with increasing vertical height, whereas for a lesser Reynolds number flow with different aspect ratios (E2 and E4), $v^+$ increases in the inner layer and decreases in the intermediate and outer layers. At $x = 3.65$ m, the aspect ratio effect is clearly visible at the same Reynolds number, i.e., E3 which has higher aspect ratio and higher Reynolds number flow, shows a trend similar to the fully developed flow. It is an indication that higher aspect ratio and higher Reynolds number flows develop fully in a shorter distance. In all the experiments, the flow is fully developed at $x = 5.15$ m, where $v^+$ increases throughout the flow depth. It can be also observed that near the inlet, the range of $v$ is $-0.5u_*$ to $0.5u_*$, whereas in the fully developed section that range is narrowed to 0 to $0.5u_*$. In Figs. 4a and 4b, comparison between E1 and E3 and between E2 and E4 shows the effect of aspect ratio throughout the depth in the developing flow. Fig. 4(c) presents the effect of aspect ratio on lateral velocities in fully developed narrow open channel flow. Lateral velocity distributions are different between hydraulically rough and transition flows. Lateral velocities in the transition flows are found to be negative and they are positive in hydraulically rough flows. Comparing E1, E3, E5 and E6, the arrow direction shows that as the aspect ratio increases, normalized lateral velocities change from negative to positive with flow becoming hydraulically rough from the transitional flow regime. Depending on rotational direction of the
cellular secondary currents near the walls, the direction of lateral velocities at the centreline is different for flow is either hydraulically rough or hydraulically transition.

Figure 5 depicts the variation of the normalized time-averaged vertical velocity $w^+$ along the centerline of the developing flow with normalized vertical distance. It is observed that $w^+$ decreases with an increase in vertical height from the channel bed up to 0.4$h$ and further upwards, $w^+$ show an increasing trend along the flow development for hydraulically rough flows. In the inner region, $z/h < 0.2$, at all the streamwise stations, $w^+$ profiles show the evidence of self-similarity, which shows that there is no significant effect of aspect ratio on the time averaged vertical velocity in the inner region of both the developing flow and fully developed flow, provided that the roughness regime should be the same based on $k_r^+$ criteria of Nezu and Nakagawa (1993). The maximum value of $w^+$ in fully developed flow (Fig. 5c) occurs near the channel bed and decreases in the upward direction. In addition, $w^+$ is greater in the fully developed flow region, i.e., $x = 5.15$ m than the other stations in the developing flow. Fig. 5c also shows that the vertical velocity distributions in the fully developed flow are different in hydraulically rough and transition flows. Vertical velocities in the hydraulically rough flows are in the downward direction which shows higher resistance to the flow which is consistent with formation of downflows over the rough bed as stated by Nezu and Nakagawa (1993). In contrast, vertical velocities are in the upward direction in the hydraulically transitional flow again which is consistent with upflows occur over smooth beds (Nezu and Nakagawa 1993). At the same Reynolds number, as the aspect ratio increases, flow is becoming hydraulically rough from hydraulically transition which is shown by the arrow direction in Fig. 5c. The maximum lateral velocities occurring close to the channel boundary are strongly and locally correlated with the large negative vertical velocities directing fluid inwards towards the boundary as shown in the Figs. 4 and 5. It is concluded that with respect to a developing flow in a narrow open channel, the strength of secondary current decreases with the increasing streamwise distance. This phenomenon indicates that over an erodible sediment bed, the erosion ability of developing flow in a narrow open channel decreases with the increasing streamwise distance. Finally, it is observed that the non-uniformity of the normalized time averaged streamwise, lateral and vertical velocities in the developing flow region is due to secondary circulation, lateral mixing, free surface flow effects and strong three dimensionality of flow (Raju et al. 2000).
3.2. Turbulence Intensities and Turbulence Kinetic Energy

The variation of vertical distribution of the three components of turbulence intensities, $u_{\text{rms}}^+ = u_{\text{rms}}/u_*$, $v_{\text{rms}}^+ = v_{\text{rms}}/u_*$, and $w_{\text{rms}}^+ = w_{\text{rms}}/u_*$, along streamwise, transverse and vertical directions are shown in Figs. 6, 7 and 8, respectively. In the developing flow region (Figs. 6a and 6b), both Reynolds number and aspect ratio are influencing the distribution of $u_{\text{rms}}^+$. In the Fig. 6a, the results of tests E1 and E3 are clustered together while the results of E2 and E4 are closer together, which shows the effect of Reynolds number. Further, at the same Reynolds number, the results of tests E1 and E3 are diverging in the outer layer due to different aspect ratios. Similarly, at the second Reynolds number, the results of E2 and E4 show the effect of aspect ratio. As the flow develops and further downstream at $x = 3.65$ m (Fig. 6b), the effect Reynolds number is diminished. However, it can be observed that the normalised values of $u_{\text{rms}}^+$ for E2 are greater than that of E4 in the outer layer. A similar observation can be made while comparing tests E1 and E3. This indicates that streamwise turbulence intensity ($u_{\text{rms}}^+$) of flow with low aspect ratio is greater than the flow at the higher aspect ratio for the same Reynolds number. In the fully developed flow region, i.e., at $x = 5.15$ m, the Reynolds number effects are negligible, but when E1, E3, E5 and E6 are compared, aspect ratio effect is not completely diminished in the outer layer for example E1 and E3 are of same Reynolds number but they are not collapsing in outer layer for $v_{\text{rms}}^+$ and $w_{\text{rms}}^+$ as shown in Figs. 7c and 8c. As indicated by the results of Roussinova et al. (2008), smooth flow in a wide open channel has higher and lower turbulence intensities in the inner and outer regions, respectively, as compared to that of the narrow open channel flow. Therefore, it can be inferred that the aspect ratio influences the distribution of streamwise turbulence intensity above the inner layer. Fig. 6c also shows the comparison of experimental streamwise turbulence intensities in narrow open channel flows with the universal equation provided by Nezu and Nakagawa (1993). There are differences between the equation proposed by Nezu (1977) and $u_{\text{rms}}^+$ values in the outer layer of the fully developed narrow open channel flow.

Comparison of lateral turbulence intensities ($v_{\text{rms}}^+$) is presented in Fig. 7. The vertical distribution of $v_{\text{rms}}^+$ shows a decreasing trend up to $0.3h$ and then increases with vertical distance in the developing flow region (Figs. 7a and 7b). In this region, i.e., $x = 2.15$ m and $3.65$ m, Reynolds number and aspect ratio effects are present throughout the depth. Flows with low aspect ratio for a given Reynolds number, have higher lateral intensities in the developing flow region. These effects are diminished in the fully developed flow region at $x = 5.15$ m.
However, the effect of aspect ratio on $v_{rms}$ can be found in the outer layer by comparing E1, E3, E5 and E6 (Fig. 7c). Fig. 7c also shows the comparison of experimental lateral turbulence intensities in narrow open channel flows with the universal equation proposed by Nezu (1977). The experimental $v_{rms}$ values in the fully developed narrow open channel flow are larger than the values provided by the universal equation in the outer layer.

Vertical profiles of normalized vertical turbulence intensities ($w_{rms}^+$) are presented in Fig. 8. The trend of $w_{rms}^+$ has an increasing tendency with $z/h$ but in the free surface region it is suppressed by the free surface. The suppression of vertical turbulence intensities in the outer region explains the characteristic pattern of secondary currents in the open channel flow (Tominaga et al. 1989). In the developing flow region at $x = 2.15$ m and $3.65$ m, the effects of aspect ratio and Reynolds number on the distribution of $w_{rms}^+$ are visible in the outer layer, similar to $u_{rms}^+$ and $v_{rms}^+$ profiles. In the fully developed flow region ($x = 5.15$ m), Reynolds number effect is diminished but the aspect ratio effect on $w_{rms}^+$ is found in the outer layer. In the developing as well as fully developed flow, for a given Reynolds number, lower aspect ratio flow has a higher vertical turbulence intensity (Fig. 8). Further, the hydraulically smooth results (Roussinova et al. 2008) in wide open channel flow has lower vertical turbulence intensities in the outer region as compared to that of narrow open channel flow (Fig. 8c). Therefore, it can be inferred that the aspect ratio influences the distribution of vertical turbulence intensity above the inner layer. Finally, from Figs. 8a, b and c, it is inferred that vertical turbulence intensities near the bed are not influenced by the aspect ratio as the wall controls the flow features, but in the outer layer, low aspect ratio flows have higher vertical turbulence intensities.

The effect of aspect ratio on turbulence characteristics is further examined by investigating the comparison of the turbulent kinetic energy $[k_{3D} = 0.5(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})]$ in the developing flow and fully developed flow section as shown in Fig. 9. In this plot, TKE is normalized with $u^*_2$. Figs. 9a and 9b show that for the same Reynolds number but different aspect ratios, the profiles do not collapse on to each other throughout the depth. Similarly, for the same aspect ratio, but different Reynolds number flows, the results also do not collapse on to each other. In the developing region, for a given Reynolds number, the lower aspect ratio flow has a higher TKE ($k_{3D}^{E1} > k_{3D}^{E3}, k_{3D}^{E2} > k_{3D}^{E4}$, where E1 & E3 are of same $R_e$ and similarly E2 & E4 are of same $R_e$) as shown in Figs. 9a and 9b. The profiles of TKE in the fully developed flow are plotted in Fig. 9c, which shows higher values of TKE near the wall region because of higher
streamwise and lateral turbulence intensities in this zone. Similar to the wide open channel flows, the TKE profile in narrow open channel flows record the maximum value near the bed and decreases rapidly in the outer layer. In the fully developed flow region (Fig. 9c), the effect of Reynolds number is diminished. However, the aspect ratio effect is found to be still significant in the outer layer of the fully developed flow. The TKE vertical distribution for a given Reynolds number is higher for lower aspect ratio flows as compared to the higher aspect ratio flows. Fig. 9c also shows the comparison between the universal equation given by Nezu (1977) and the present data in the developed flow. The prediction of turbulence kinetic energy by the universal equation deviates from the narrow open channel flow results.

An attempt was made to modify the universal equations suggested for wide channels for use in narrow channels. The format of the semi empirical relationships proposed by Nezu and Azuma (2004) and Nakagawa (1993) are followed in this paper to predict 3D turbulence intensities and turbulence kinetic energy as given below:

\[
\frac{u_{rms}}{u_*} = \alpha_1 \exp\left(-\beta_1 \frac{z}{h}\right) \tag{2}
\]

\[
\frac{v_{rms}}{u_*} = \alpha_2 \exp\left(-\beta_2 \frac{z}{h}\right) \tag{3}
\]

\[
\frac{w_{rms}}{u_*} = \alpha_3 \exp\left(-\beta_3 \frac{z}{h}\right) \tag{4}
\]

\[
\frac{k}{u_*^2} = \alpha_4 \exp\left(-\beta_4 \frac{2z}{h}\right) \tag{5}
\]

In the above equations, constants \(\alpha_1, \alpha_2, \alpha_3,\) and \(\alpha_4\) are obtained from fitting the measured data of the present narrow open channel flow experiments. For simplicity and ease of use, the exponential coefficients \(\beta_1, \beta_2, \beta_3\) and \(\beta_4\) are considered as unity similar to Nezu and Azuma (2004) and Nakagawa (1993). To compute the constants \(\alpha_1, \alpha_2, \alpha_3,\) and \(\alpha_4\) in universal equations, a regression analysis is carried out using the results in the intermediate region. The present study indicates a change in the constant values from that provided by Nezu (1977). The constants for universal equations given by the present study and other researchers are presented in Table 2. The derived new equations are plotted in Figs. 6(c), 7(c), 8(c), and 9(c). Nezu and Nakagawa (1993), and Papanicolaou and Hilldale (2002) reported that the constant \((\alpha_2)\) of the lateral turbulent intensity equation is larger than the constant of vertical turbulence intensity \((\alpha_3)\) i.e., \(\alpha_2 > \alpha_3\) and the same is also observed in the present study (see Table 2). Secondary currents will facilitate the increase in the lateral turbulence intensities, while the free surface
will tend to reduce the vertical turbulence intensities. The comparison between the constants in the present study and literature values of smooth bed flow indicates that the constants for narrow open channel flows are on the higher side, which shows the turbulence intensity distribution in narrow open channel flow is different from that of wide open channel flow. The derived Eqs. 2, 3, 4 and 5 are satisfactorily fitting the experimental data as shown in Figs. 6(c), 7(c), 8(c), and 9(c).

3.3. Reynolds Shear Stresses

Figure 10 depicts the vertical distribution of normalized Reynolds shear stress $\tau_{uw} = -\frac{u'w'}{u^*}$ along the developing and fully developed flow at different stations. For all the measurements in the developing flow, it is observed that the vertical distribution of $\tau_{uw}$ attains a peak value in the wall region and decreases towards the bed, which has been also reported by other investigators (Grass 1971; Nikora and Goring 2000; Bigillon et al. 2006). In developing flow region, $\tau_{uw}^+$ profile is found to deviate from the theoretical line. Shear stress $\tau_{uw}^+$ near the bed in developing flow is greater than that in the fully developed flow. The change in the sign of $\tau_{uw}^+$ of E1 from positive to negative occurs at $z/h \geq 0.6$ at $x = 2.15$ m and 3.65 m. These negative values of $\tau_{uw}^+$ in the outer layer are caused by the velocity retardation in upper flow region (Nezu and Nakagawa 1993). In fully developed flow region, $\tau_{uw}^+$ profile deviates from the theoretical line in the outer layer and takes the shape of a concave curve due to the effect of aspect ratio and secondary currents. Yang et al. (2004) explained that the existence of dip phenomena in open channel flow results in negative Reynolds shear stress values in the outer layer. However, measurements could not be carried out in the top five centimetres in the present study.

3.4. Fluctuating Velocities

Quadrant analysis was developed by Lu and Willmarth (1973) to investigate the existence of coherent eddies and calculate their contribution to the turbulent shear stresses. In the present study, fluctuating velocities in four quadrants were studied in the inner region in both the developing and fully developed flows to study the relative contribution of turbulent events to the corresponding Reynolds shear stress. In the quadrant analysis, streamwise $u'$ and vertical $w'$ velocity fluctuations are divided into four quadrants on a $u' - w'$ plane. Each quadrant denotes one type of turbulent event which takes place in the flow region. Outward interactions
(\(u' > 0, w' > 0\)) which define the outward motion of high speed fluid parcel are represented by the first quadrant. Ejections events (\(u' < 0, w' > 0\)) characterize pushing away of low speed fluid parcel from the bed and are represented by the 2nd quadrant. The third quadrant (\(u' < 0, w' < 0\)) represents inward interactions, which are characterized by low speed fluid parcel moving towards the bed. Sweep events (\(u' > 0, w' < 0\)) are characterized by high speed fluid parcel moving towards the bed, are in fourth quadrant. For the quadrant analysis, the location \(z = 0.035h\) in the inner layer at each streamwise location along the centerline of the developing flow region was chosen as a monitoring point. Figure 11(a) depicts the scatter plots of flow velocity fluctuations \(u'\) and \(w'\) along the developing flow. It is observed that the strength of the turbulent events decrease with increasing streamwise distance for all the experimental runs (E1-E4). However the decrease is more significant in the lower Reynolds number flows (E2 and E4). From all the plots in Fig. 11a, it is observed that in the inner layer, more number of events occurring in the 2nd and 4th quadrants characterizing the ejections and sweeps dominate the flow (Bomminayuni and Stoesser 2011). By comparing the scatter plots, it may be concluded that the strength of the turbulent events is stronger with increasing aspect ratio. The major axis of ellipse enclosing \(u' - w'\) scatter plot at \(z = 0.035h\) along the developing flow at \(x = 2.15, 3.65\) and \(5.15\) m respectively for each experiment from E1 to E4 is presented in Fig. 11b. It is clear from Fig. 11b that comparing E1 and E3, fluctuations are decreasing more in E1. Similarly comparing E2 and E4, fluctuations are decreasing more in E2. It is therefore inferred that attenuation of turbulent fluctuations in the developing flow is less for high aspect ratios.

Figs. 11(c) and 11(d) show streamwise and vertical velocity fluctuations without normalization for hole size \(H = 0\) (which denotes that at \(z = 0.035h\) all \(u'\) and \(w'\) are included) in the inner and outer layers, respectively, for fully developed flow. Figs. 11(c) and 11(d) show sweeps and ejections are dominant in the inner and outer layers for the higher aspect ratio flows, whereas outward and inward events are equally significant in addition to the sweeps and ejections throughout the depth in the lower aspect ratio flows (E1, E5 and E6) This characteristic is demonstrated by the increased angle between the major axis of the ellipse and horizontal. In the lower aspect ratio flows, the scatter area of turbulence fluctuations in the outer layer is approximately circular which states that turbulence is nearly isotropic in the outer layer of lower aspect ratio flows. The strength of the fluctuations without normalization increases with increasing aspect ratio as seen in the Figs. 11c and 11d. From Figs. 11b, 11c and
11d, for a given Reynolds number, the highest aspect ratio has the largest fluctuations and flow with the lowest aspect ratio has the smallest fluctuations.

### 3.5 Correlation Coefficient

Figure 12 shows the vertical distribution of the correlation coefficient \([-\overline{u'u'}/u_{rms}^2 w_{rms}\] in the fully developed flow region in hydraulically rough, narrow open channel flows at approximately the same Reynolds number with different aspect ratios.

Figure 12 shows that the correlation coefficient of E3 is larger than E1, E5, and E6 throughout the depth of the flow. The figure also demonstrates that the correlation coefficient at any depth is related to the aspect ratio of the flow. The variation of correlation coefficient along the depth is also a function of aspect ratio. The correlation coefficient of E3 decreases very slowly near the wall and becomes constant at \(z/h = 0.4-0.5\) in the intermediate region and then again decreases near the free surface similar to observation of Kironoto and Graf (1994) in the hydraulically rough wide channel. However, the trend of the correlation coefficient for E1, E5 and E6 decreases strongly in the intermediate region and slightly increases just after the dip positions because of the damping of vertical fluctuations near the free surface. Another observation from Fig. 12 is that occurrence of negative correlation coefficient in the vicinity of free surface \((z > 0.6)\) which is because of flow retardation caused by the existence of secondary currents in narrow channels. For the same Reynolds number, the correlation coefficient increases with the aspect ratio. The effect of aspect ratio is clearly noticeable even within the hydraulically rough flows, the correlation coefficient of E3 > E5 as shown in Fig. 12. Since flow regime is a function of the aspect ratio, the correlation coefficient increases with increasing flow roughness as shown by the direction of arrow in the Fig. 12.

### 3.6 Reynolds Stress Anisotropy Tensor

Reynolds stress anisotropy tensor \(b_{ij}\) is defined as the ratio of Reynolds stress term to the turbulence kinetic energy minus its isotropic equivalent quantity. Reynolds stress anisotropy is a measure of the departure from isotropic turbulence \((b_{ij} = 0)\). Here, \(b_{ij}\) is computed as \(b_{ij} = \left(\overline{u'_iu'_j}/2k_3\right) - \left(\delta_{ij}/3\right)\) where \(\delta_{ij}\) is the Kronecker Delta function. Anisotropy components \(b_{11}, b_{13}\) and \(b_{33}\) are plotted in Fig. 13. Experimental data of smooth bed wide channel (Roussinova et al. 2009) and rough bed wide channel (Mazouz et al. 1998) have also been included. Comparison of \(b_{11}\) at three locations along the developing flow for different Reynolds numbers and aspect ratios, reveal that anisotropy is high in flow with higher Reynolds
number and higher aspect ratio. Among $b_{11}$, $b_{13}$ and $b_{33}$, the variable $b_{11}$ is found to be more sensitive to the flow conditions such as channel aspect ratio, smooth or rough bed etc. Analysis of $b_{11}$ demonstrates that rough wide channel has higher anisotropy as compared to smooth channel. Present experiments, i.e., narrow and rough channels have the least anisotropy, which implies, anisotropy is sensitive to aspect ratio.

Reynolds shear stress anisotropy tensor components $b_{22}$, $b_{12}$ and $b_{23}$ are plotted in Fig. 14. Importantly, the trend of $b_{22}$ is found to be different from that of $b_{11}$, which means higher anisotropy has lower $b_{22}$. The experiment E3 which has the maximum $b_{11}$ has the minimum $b_{22}$. Similarly $b_{22}$ for wide rough open channel flow is less than the narrow rough open channel, which demonstrates that higher anisotropy is present in wide rough open channel flow. Shafi and Antonia (1995) found that $b_{22}$ is not a function of bed roughness, but a function of aspect ratio and the same is being observed in the present experiments. It can be concluded that anisotropy in narrow open channel flow is less than in wide-open channel flow. The particular characteristic may be attributed to better mixing of flow in the narrow open channel due to secondary currents resulting in lesser anisotropy.

Anisotropy components are also plotted for same Reynolds number, but different aspect ratios as shown in Fig. 15. The components $b_{11}$, $b_{13}$, $b_{22}$ and $b_{12}$, in the outer layer seem to be a function of aspect ratio. Especially, the values of $b_{11}$ and $b_{22}$ in the narrow open channel flow are different for aspect ratios less than and greater than four. For aspect ratio equal to four, $b_{11}$ and $b_{22}$ have values close to that of wide open channel flow with either hydraulically smooth or rough open channel conditions.

3.7. Flow Development Length

The flow development length ($L_e$) has been investigated in many studies (Kirkgoz and Ardiclioglu, 1997). However, most studies focused on flow development length in fully developed turbulent wide open channel flow. In this study, to explore the dependency of the flow development length ($L_e$) on the flow characteristics and bed roughness, nonlinear regression analysis of the experimental data of the present study and data collected from the literature (Sharma, 2015) was performed. The median size of bed roughness ($d_{50}$) used in the present analysis varies from 2.25 mm to 14 mm. The roughness Reynolds number varied between 70 and 4080. The data consists of different non-dimensional numbers such as scaled bed roughness, Reynolds number, Froude number, aspect ratio, particle Reynolds number and
flow development length. Various models were formulated by trial and error and studied for their accurate prediction ability. Finally, the best model was selected based on the highest regression coefficient ($R^2$) value, the lowest standard error of the estimate and lower Akaike’s information criterion (AIC) scores. The proposed model for the prediction of flow development is given as follows:

$$\frac{L_e}{h} = 38.8 - 66.71 \left(\frac{d_{50}}{h}\right) - 2.147 \log\left(\frac{Re}{Fr}\right) + 5.061 A_r$$  \hspace{1cm} (6)

The regression coefficient of the above model is 0.895 and the mean absolute percent error is 5%. Coefficients of all independent variables (non-dimensional parameters such as scaled equivalent sand roughness, $R_e$, $F_r$ and $A_r$) were estimated using the bootstrapping method. The same experimental data which was used to develop a regression model has also been used for validation purpose. Figure 16 shows the comparison plot of observed and predicted values of $L_e$. Predicted flow development length falls within 11% envelope of perfect agreement line, which demonstrates the efficiency of the proposed model. The proposed model can be used to predict the flow development length in hydraulically rough channels with varying aspect ratio and flow conditions.

4. Conclusions

The effects of aspect ratio on the turbulence characteristics in developing and fully developed narrow open channel flow were presented in this paper. In the range of aspect ratio studied herein, for a given Reynolds number, the shear velocity was found to increase with increasing aspect ratio. For a given Reynolds number, the aspect ratio is found to influence the classification of flow regime to be either fully rough or transitionally rough. For a given Reynolds number, the downward velocity shift from smooth wall log-law increases with increasing aspect ratio. The correlation between equivalent roughness and the downward velocity shift is found to be a function of the aspect ratio. The aspect ratio was found to influence the velocity characteristics throughout the depth in the developing flow region, while the effects are confined to only to the outer layer for fully developed flows. For a given Reynolds number, the streamwise, lateral and vertical velocity profiles normalized with inner layer scales in fully developed flow are distinct for high aspect ratio (hydraulically rough) and low aspect ratio (transition) flows. In fully developed flow, the normalized streamwise and
lateral velocity profiles of low aspect ratio flows are greater than those of high aspect ratio flows. Vertical velocities in the high aspect ratio (hydraulically rough) flows are in the downward direction, whereas vertical velocities in the low aspect ratio (hydraulically transitional) flows are in the upward direction. Lower aspect ratio flows have higher normalized streamwise, transverse and vertical turbulence intensities in the outer layer. Turbulence intensities and TKE of higher aspect ratio flow correlated better with available universal equations compared to the lower aspect ratio flows. New exponential equations with slightly higher constants were proposed for turbulence intensities and TKE in narrow open channel flows. For a given Reynolds number, the non-dimensional TKE vertical distribution is higher for lower aspect ratio flow as compared to higher aspect ratio flows. In the fully developed flow, aspect ratio effect on the non-dimensional Reynolds shear stress profile is observed in the outer layer.

The analysis using quadrant decomposition demonstrates that the strength of instantaneous turbulence fluctuations without normalization increases with increasing aspect ratio for a given Reynolds number. In addition, the attenuation of instantaneous velocity fluctuations along the developing flow is decreasing with increasing aspect ratio. For the same Reynolds number, the correlation coefficient increases with increasing aspect ratio. Reynolds stress anisotropy tensor analysis reveals that level of anisotropy in narrow open channel flow is less than wide open channels irrespective of the roughness regime. Finally, a nonlinear regression equation has been proposed to estimate the flow development length of hydraulically rough and transition open channel flow.
References


Francis, J.B. 1878. On the cause of the maximum velocity of water flowing in open channels being below the surface. *Transactions of the American Society of Civil Engineers*, 7(1), 109–113.


Table captions

Table 1 Hydraulic Parameters Calculated for the Test Runs

Table 2 Review of empirical constants for exponential equations of turbulence intensities and TKE
Table 1 Hydraulic Parameters Calculated for the Test Runs

<table>
<thead>
<tr>
<th>Exp. Run No</th>
<th>h (m)</th>
<th>$A_r$</th>
<th>U (m/s)</th>
<th>$u_*$ (m/s)</th>
<th>$R_e$ ($\times 10^{-4}$)</th>
<th>$k_s^+$</th>
<th>$\Delta u^+$</th>
<th>$\delta$ (m)</th>
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*Dip position could not be found since velocity attained maximum at 5 cm below the water surface and further measurements were not available

**Experiment was conducted at University of Windsor; flume details are: length = 10.5 m, width = 0.4 m and sand bed with median sediment size ($d_{50}$) = 0.77 mm

Table 2 Review of empirical constants for exponential equations of turbulence intensities and TKE

<table>
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<tr>
<th>Literature</th>
<th>$\alpha_1$</th>
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<td>*</td>
<td>0.67</td>
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<td>Hydraulically Smooth and Rough</td>
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**Figure captions**

Fig. 1 A schematic diagram of experimental flume (Not to scale)

Fig. 2a Vertical distribution of Reynolds shear stress

Fig. 2b Inner scaling of time averaged streamwise velocity distribution in fully developed flow

Fig. 2c Plot of mean velocity shift vs. equivalent roughness height

Fig. 3(a-c) Vertical distribution of normalized streamwise velocity in developing and developed flow and (d) velocity defect in fully developed flow

Fig. 4 Vertical distribution of normalized lateral velocity

Fig. 5 Vertical distribution of normalized vertical velocity

Fig. 6 Vertical distribution of normalized streamwise turbulence intensity

Fig. 7 Vertical distribution of normalized lateral turbulence intensity

Fig. 8 Vertical distribution of normalized vertical turbulence intensity

Fig. 9 Vertical distribution of TKE in developing and fully developed flow

Fig. 10 Vertical distribution of normalized Reynolds shear stress for E1, E2, E3, and E4

Fig. 11(a) $u' - w'$ scatter plots with hole size = 0 at selected point 0.035$h$ in the developing flow region for E1, E2, E3 and E4

Fig. 11(b) Major axis of ellipse enclosing $u' - w'$ scatter at $z = 0.035h$ in developing flow

Fig. 11(c) $u' - w'$ scatter plots with hole size = 0 at selected point 0.035$h$ in fully developed flow for E1, E3, E5 and E6

Fig. 11(d) $u' - w'$ scatter plots with hole size = 0 at selected point 0.6$h$ in fully developed flow for E1, E3, E5 and E6

Fig. 12 Correlation Coefficient in fully developed flow

Fig. 13 Components of Reynolds stress anisotropy tensor ($b_{11}$, $b_{13}$ and $b_{33}$) along the flow development in a narrow rough channel

Fig. 14 Components of Reynolds stress anisotropy tensor ($b_{12}$, $b_{22}$ and $b_{23}$) along the flow development in a narrow rough channel

Fig. 15 Reynolds stress anisotropy tensor ($b_{12}$, $b_{22}$ and $b_{23}$) at the fully developed flow section for E1, E3, E5 and E6.

Fig. 16 Scatter plot of measured flow development length vs predicted using the proposed model
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Fig. 11(c) \( u' - w' \) scatter plots with hole size = 0 at selected point \( 0.035h \) in fully developed flow for E1, E3, E5 and E6

Fig. 11(d) \( u' - w' \) scatter plots with hole size = 0 at selected point \( 0.6h \) in fully developed flow for E1, E3, E5 and E6

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