Transactions of the Canadian Society for Mechanical Engineering

Reliability modeling for competing failure systems with instant-shift hard failure threshold

Journal:	<i>Transactions of the Canadian Society for Mechanical Engineering</i>
Manuscript ID	TCSME-2017-0130.R1
Manuscript Type:	Article
Date Submitted by the Author:	27-Feb-2018
Complete List of Authors:	Liu, Jingyi; Northwestern Polytechnical University, Zhang, Yugang; Northwestern Polytechnical University Song, Bifeng; Northwestern Polytechnical University
Keywords:	Reliability model, Multiple dependent competing failure processes, Instant-shift hard threshold
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

SCHOLARONE™
Manuscripts

Article Type:

Original Article

Corresponding Author:

Jingyi Liu,
Northwestern Polytechnical University, 127 West Youyi Road, Xi'an, Shaanxi, 710072,
P.R.China
Email: liujy@mail.nwpu.edu.cn

Article Title:

Reliability modeling for competing failure systems with instant-shift hard failure threshold

Author lists:**Jingyi Liu^{1*}**

liujy@mail.nwpu.edu.cn

Yugang Zhang¹

zhangyugang@nwpu.edu.cn

Bifeng Song¹

songbif@foxmail.com

¹ Northwestern Polytechnical University, 127 West Youyi Road, Xi'an, Shaanxi, 710072, P.R.China

Reliability modeling for competing failure systems with instant-shift hard failure threshold**Abstract**

There are many researches about the modeling for system under multiple dependent competing failure processes (MDCFP) in recent years. Typically speaking, those failure processes are composed by degradation process (soft failure) and random shock process (hard failure). The threshold of hard failure is a fixed value in previous papers which is not compliance with the engineering practices. Threshold means the ability to resist external random shocks which is also shifting with time due to the usage of system. Thus, this paper establishes a model for MDCFP with instant-shift hard threshold. The hard failure threshold changes with time instantaneously and it is also influenced by external shocks. Afterwards, system reliability model is built. The effectiveness of presented model is demonstrated by the reliability analysis of the micro-engine of Sandia National Laboratories. In addition, sensitive analysis is performed for specific parameters.

Key words: Reliability model; Multiple dependent competing failure processes; Instant-shift hard failure threshold

Draft

1 Introduction

Mechanical systems are always under multiple failure processes including catastrophic failure and degradation failure, due to external shocks and degradation. The two failure processes are dependent and competing with each other. Dependence means one process may have influences on others and competing indicates that occurring of any failures will make the system fail.

1.1 Brief literature review

There have been many significant and meaningful researches related to competing failure modeling. We established new model for competing failure by renewing and expanding previous method and conduct new result.

Literatures about shock model mainly focus on the shock process (shock time and shock size) and its influences on system. Finkelstein *et al.* (2001) considered two types of non-cumulative damage shock models. One is based on the shock level while the other is based on the time interval between two consecutive shocks. System would fail when the damage caused by one shock exceeds a threshold or the time interval is too small. Mallor *et al.* (2003) extended shock model based on it. The damages caused by shocks to system are cumulative. Even one shock did not lead to the failure of system, the influence caused by it could be kept due to cumulativeness. Gut *et al.* (2005) expanded the shock model further. The threshold is a fix value in previous paper, but Gut considered that some shocks may harm the system and lead to the changing of threshold. Accordingly, the failure rate to time would also change. Bai *et al.* (2006) put forward to a generalized framework for shock models based on cluster point process with cluster marks. Durante (2017) introduced a new class Copula functions to shock models and the failure distributions are studied with dependence and asymmetric linkages considered.

Models for degradation can be classified into two types, the disperse process and the continuous process. Xue *et al.* (1995) considered systems with multi states. Systems degrade from one state to another state with different parameters and reliabilities. The system reliabilities are calculated at various states. Delia *et al.* (2006) studied a two-unit cold standby system whose unit goes through a finite number of stages during degradation. For continuous degradation model, there are three ways to describe the process, degradation path model, stochastic process model and statistical model Robinson *et al.* (2000) built the degradation curve to establish the reliability of units. The parameters are estimated by Bayesian method with little data. Bae *et al.* (2007) used degradation path to simulate the degradation process material fatigue. Lifetime distribution was well calculated by randomness considered. Zuo *et al.* (1999) used the random process to build reliability model for continuous state devices. Comparing with the general path model, the random process model has advantages in calculation and analyzing. Kharoufeh (2003) (2005) considered the reliabilities of a single-unit system with wear. The system failure when total wear exceeds a threshold. Markov additive process is introduced to model the wear process and an example demonstrates this method. Hsieh *et al.* (2009) used a non-homogeneous compound Poisson model to describe the degradation. The degradation process is measured at discrete schedule based on which Poisson model is established. Yang (2000) used inspected lifetime data to model the degradation process based on which life-time character was analyzed. Huang (2005) estimated the parameters in a new-presented degradation model based on degradation statistics

There may be competing between failures for systems under both catastrophic failure and degradation failure. The system reliabilities are distinct when competing failure considered. Coit *et al.* have made plenty of researches about the competing failure models with various factors considered. Peng and Coit (2011) built the basic competing failure model. The hard failure (catastrophic failure) and soft failure (degradation failure) are considered. Hard failure occurs when the size of external shock exceeds its

threshold. Degradation is composed by pure degradation and instantaneous increase lead by shocks. Soft failure happens if the degradation exceeds it threshold. The two failures are competing with each other based on which the system reliabilities are analyzed. Many extended papers are published by Coit based on the basic model in reference (Peng and Coit 2011). Jiang and Coit (2012) introduced the shift failure thresholds to competing failure model based on the basic one. Soft thresholds may shift if the shock size is larger than a pre-fixed value. Similarly, Rafiee and Coit (2013) considered the changing degradation rates which may change due to the larger shock size. Besides, Jiang and Coit (2015) classified the shock into three zones, the safe zone, damage zone and failure zone, according their sizes. Shocks in different zones have different influences on system reliability. Song and Coit (2014) (Song 2014) built the competing failure models for the multi-unit systems. In reference (Rafiee 2017), Coit built the models with mixed factor considered together that make the model complicated and challenging. There are also many other researchers studied the competing failure models. Fan *et al.* (2017) built a competing failure model in which the degradation process may influence the sizes and rate of external shocks. A sliding spool is chosen as an example to demonstrate the model. Liu *et al.* (2017) took the self-healing into consideration to build the model. The degradation may have the ability to healing itself which is the highlight in the model. An *et al.* (2017) built a model for systems under a shock failure and multi degradation failure and all the failures are dependent and competing with each other. Copula functions are used to establish the dependence based on which reliabilities are calculated.

1.2 Motivation

The hard failure is caused by external shock which means as soon as the shock size exceeds a threshold the hard failure occurs. The threshold for hard failure can be regarded as the ability to resist the external shocks which should be degraded due to usage. However, few literatures consider this during modeling. In reference (Jiang and Coit 2012) the shifting threshold is caused by specific shock. In practice, the shifting of hard failure is composed by two parts, the instant shifting with time and the shifting caused by specific shocks. There are also engineering practice to demonstrate this. The MEMS (Micro-Electro-Mechanical-Systems), which is chosen as a case in many references, ((Peng and Coit 2011); (Jiang and Coit 2012); (Rafiee and Coit 2013); (Jiang and Coit 2015); (Liu *et al.* 2017)) is a typical system under competing failure risks. The MEMS experiences the soft failures due to wear degradation and debris from external shocks and hard failures due to pin fracture (Jiang and Coit 2012). The wear may cause rub face that leads to soft failures. What's more, the wear will thin the pin so the strength will decrease with the wear process. In other words, the threshold of hard failure reduces with time continually.

There are also other examples to demonstrate the continually decreasing hard failure threshold. The landing gear system of aircraft subjects two risks, soft failure due to crack increasing (fatigue) and the hard failure due to external shocks (landing loads). Increasing crack not only leads to soft failure, but also decrease the ability to resist landing load. In other words, the hard failure threshold decreases.

The model will be complicated and challenging after the continually decreasing hard failure threshold introduced. Supposing there are three shocks at T_1 , T_2 and T_3 , the probability for fixed hard threshold is expressed by

$$R = \left\{ \begin{array}{l} Shock_1 < Hard \text{ threshold}, \\ Shock_2 < Hard \text{ threshold}, \\ Shock_3 < Hard \text{ threshold} \end{array} \right\}$$

which is determined just by shock sizes.

If the threshold is continually decreasing, the probability becomes

$$R = \begin{cases} Shock_1 < Threshold_{T_1}, \\ Shock_2 < Threshold_{T_2}, \\ Shock_3 < Threshold_{T_3} \end{cases}$$

which is influenced by shock sizes and shock time T (T_1 , T_2 and T_3). Comparing with the fixed threshold, the probabilities of decreasing threshold changes with T . Different shock time leads to different probabilities. Generally, shock time is random, making it challenging and hard to conduct the reliability expression. Figure 1 shows the shocks with different thresholds.

The reliability model with continually decreasing hard threshold, which correspond with engineering practice, is established in this paper.

1.3 Overview

In this paper, we build a new reliability model for systems under competing failures. During modeling process, the continually decreasing hard threshold is considered.

The main contributions of the present study are:

- 1 Expanding competing failure model for system under multiple dependent competing failure processes (MDCFP) with instant-shift hard failure threshold considered.
- 2 Providing analytical reliability formulas for expanded model.
- 3 Assessing the validity of present model by comparing the failure rate curves.
- 4 Applying presented model for an engineering practice (a micro-electromechanical systems).

The remainder of this paper is organized as follows. Section 2 is the notation list and system description. Failure principles and failure processes are introduced. Reliability models are established in section 3, including hard failure modeling, soft failure modeling and system reliability modeling. In section 4, an application example is presented to demonstrate the model. Sensitivity analysis is also studied. Section 5 draws the conclusion and summaries this paper.

2 Notation list and system description

The notations used in this paper are listed in Table 1 below for a better illustration.

Systems are subjected to soft failure and hard failure which are dependent and competing with each other. The degradation process, which causes the soft failure, is a continuous random process if there is no external shock just like Figure 2 (a). If the system suffers random external shocks, as shown in Figure 2 (c), instantaneous increase in each shock is added to the degradation process like Figure 2 (b). Besides, if the hard failure threshold is continues decreasing, the shock process will change into Figure 2 (d). Figure 2 (b) and (d) are two processes studied in this paper.

Error! Reference source not found. (b) shows the degradation of soft failure consists of pure degradation $X(t)$ and increments Y_i caused by each shock. If the total degradation exceeds pre-fixed value H , soft failure occurs. The pre-fixed value H is defined in advance so it is usually a constant and won't change.

Figure 2 (d) demonstrates the hard failure process with shifting threshold. W_i is the size of each shock. Once the size is larger than the threshold at time t , the hard failure occurs. Hard threshold means the ability to stand the shocks, so it is decreasing with time. What's more, the external may reduce the ability suddenly. Each shock leads to a threshold reduce Z_i .

3 Reliability modeling

3.1 Reliability analysis for degradation process

Just as Figure 2 (b) shows, total degradation $X_s(t)$ is composed by two parts, the pure degradation $X(t)$ and instantaneous increases Y_i s. The linear degradation path is used to model the pure degradation.

Linear is expressed by $X(t) = \varphi + \gamma t$ where φ is the initial degradation and γ represent the degradation rate. Figure 2 (a) shows the pure degradation process when rate γ is a random variable.

Due to randomness of coming time of external shocks, Poisson process is used to model the shocks process. Supposing a homogeneous Poisson process $\{N(t), t \geq 0\}$ with intensity λ , the probability that n shocks occurs is

$$P(N(t) = n) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}, n = 1, 2, \dots \quad (1)$$

The cumulative increments caused by random shocks is

$S(t) = \sum_{i=1}^{N(t)} Y_i$ where $N(t)$ is the total number that shock occurs until time t . Thus, the total degradation of soft failure process is expressed by

$$X_s(t) = X(t) + S(t) = \varphi + \gamma t + \sum_{i=1}^{N(t)} Y_i \quad (2)$$

The cumulative distribution of the total degradation is expressed by

$$\begin{aligned} R_X(t) &= P(X_s(t) < x) \\ &= \sum_{i=0}^{\infty} P(X(t) + S(t) < x \mid N(t) = i) P(N(t) = i) \end{aligned} \quad (3)$$

By a convolution integral, the distribution could be derived as Equ. (4) (Peng and Coit 2011).

$$R_X(t) = \sum_{i=0}^{\infty} \left(\int_0^x G(x-u, t) f_Y^{<i>} du \right) \frac{\exp(-\lambda t) (\lambda t)^i}{i!} \quad (4)$$

where $G(x, t)$ is the CDF of $X(t)$, f_Y is the PDF of Y_i and $f_Y^{<i>}$ is the PDF of the sum of i i.i.d. Y_i variables.

If the sizes of shocks Y_i and the degradation rate γ are under normal distribution, $Y_i \sim N(\mu_Y, \sigma_Y^2)$,

$\gamma \sim N(\mu_\gamma, \sigma_\gamma^2)$ the CDF becomes Equ. (5). (Peng and Coit 2011)

$$R_X(x, t) = \sum_{i=0}^{\infty} \Phi \left(\frac{x - (\mu_\gamma t + \varphi + i\mu_Y)}{\sqrt{\sigma_\gamma^2 t^2 + i\sigma_Y^2}} \right) \frac{\exp(-\lambda t) (\lambda t)^i}{i!} \quad (5)$$

3.2 Reliability analysis for shock process

First we consider the fixed hard threshold situation. As shown in Figure 2 (c), the hard failure will occur when the magnitude of shock W_i exceed threshold H . Generally speaking, magnitude W_i are random variable. Supposing F_W denotes the CDF of W_i , the probability that no hard failure occurs at the i th random shock is

$$P(W_i < D_0) = F_W(D_0) \quad i = 1, 2, 3, \dots \quad (6)$$

When random variable W_i is under normal distribution $W_i \sim N(\mu_{W_i}, \sigma_{W_i}^2)$, the probability becomes

$$F_W(D_0) = \Phi \left(\frac{\mu_{D_0} - \mu_W}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2}} \right) \quad (7)$$

When the hard threshold is shifting, the instant threshold with time should be analyzed before reliability modeling. The time interval of two consecutive events of a Poisson process is subjected to exponential

distribution.

Poisson process with intensity λ is used to model the shocks process in this paper. Supposing the time interval between i th shock and $(i-1)$ th shock is t_i , then $t_i \sim \text{Exp}(\lambda)$ with PDF

$$f(t_i) = \begin{cases} \lambda e^{-t_i \lambda}, & t_i > 0 \\ 0, & \text{others} \end{cases} \quad (8)$$

If there is no external shock, the hard failure will not occur and the reliability equals to 1. The probability could be expressed as

$$R(t | N(t) = 0) = 1 \quad (9)$$

The final probability that shock failure doesn't occur is

$$R_0(t) = R(t | N(t) = 0) \times P(N(t) = 0) = e^{-\lambda t} \quad (10)$$

Further, if there is one shock happens until time t (Figure 3), the probability that shock failure doesn't occur is expressed as

$$\begin{aligned} R_1(t | N(t) = 1, t_1 = T_1) \\ = P(W_1 < D_0 - \beta T_1 | N(t) = 1, t_1 = T_1) \end{aligned} \quad (11)$$

R_1 is the reliability probability for shock process when there is just one external shock. t_1 is the time interval between the occurring time of shock and origin time with value T_1 . t_1 is under exponential distribution according to the theorem. W_1 is the size of the shock. $D_0 - \beta T_1$ is the hard threshold when the shock happens where β is the degradation rate with randomness. Due to the time randomness of Poisson process, the T_1 can be any value of $[0, t]$.

The Equ. (11) can be derived as Equ. (12) based on above-mentioned

$$\begin{aligned} R_1(t) &= \int_0^t P(W_1 < D_0 - \beta T_1) \times P(t_1 = T_1) P(N(t) = 1) dT_1 \\ &= \int_0^t P(W_1 + \beta T_1 < D_0) \times P(t_1 = T_1) P(N(t) = 1) dT_1 \\ &= \int_0^t \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta T_1)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2 T_1^2}} \right) \lambda e^{-T_1 \lambda} (-\lambda t) e^{-\lambda t} dT_1 \end{aligned} \quad (12)$$

Similarly, if there are two shocks happening until time t (Figure 4), the probability that shock failure doesn't occur is expressed as

$$\begin{aligned} R_2(t | N(t) = 2, t_1 = T_1, t_2 = T_2) &= P(W_1 < D_0 - \beta T_1, \\ W_2 < D_0 - Z_1 - \beta(T_1 + T_2) | N(t) = 2, t_1 = T_1, t_2 = T_2) \end{aligned} \quad (13)$$

where T_2 is the time interval between of shock 1 and shock 2 under exponential distribution.

The meaning of this expression is that each shock should be smaller than its hard threshold. For shock 1, its threshold is $D_0 - \beta T_1$. However, the threshold of shock 2 is $D_0 - Z_1 - \beta(T_1 + T_2)$ including the pure degradation $\beta(T_1 + T_2)$ and instantaneous decrement Z_1 caused by shock 1. The Z_1 is also random variable.

If Z_i is assumed to be normal distributed $Z_i \sim N(\mu_Z, \sigma_Z^2)$, the probability that shock failure doesn't occur is shown Equ. (14).

The probability expression is derived due to the same reason when there are n shocks until time t .

$$\begin{aligned}
R_2(t) &= \int_0^t \int_{T_1}^t P(W_1 < D_0 - \beta T_1) P(W_2 < D_0 - Z_1 - \beta(T_1 + T_2)) \times P(t_1 = T_1, t_2 = T_2) \times P(N(t) = 2) dT_2 dT_1 \\
&= \int_0^t \int_{T_1}^t P(W_1 + \beta T_1 < D_0) P(W_2 + Z_1 + \beta(T_1 + T_2) < D_0) \times P(t_1 = T_1, t_2 = T_2) \times P(N(t) = 2) dT_2 dT_1 \\
&= \int_0^t \int_{T_1}^t \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta T_1)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2 T_1^2}} \right) \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta(T_1 + T_2) + \mu_Z)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2(T_1 + T_2)^2 + \sigma_Z^2}} \right) \times \\
&\quad \lambda e^{-T_1 \lambda} \lambda e^{-T_2 \lambda} \frac{(-\lambda t)^2}{2!} e^{-\lambda t} dT_2 dT_1
\end{aligned} \tag{14}$$

$$\begin{aligned}
R_n(t) &= \int_0^t \int_{T_1}^t \cdots \int_{T_{n-1}}^t P(W_1 < D_0 - \beta T_1) P(W_2 < D_0 - Y_1 - \beta(T_1 + T_2)) \\
&\quad \cdots P(W_n < D_0 - (Z_1 + Z_2 + \cdots + Z_{n-1}) - \beta(T_1 + T_2 + \cdots + T_n)) \\
&\quad \times P(t_1 = T_1, t_2 = T_2, \dots, t_n = T_n) P(N(t) = n) dT_n \cdots dT_2 dT_1 \\
&= \int_0^t \int_{T_1}^t \cdots \int_{T_{n-1}}^t P(W_1 + \beta T_1 < D_0) P(W_2 + Y_1 + \beta(T_1 + T_2) < D_0) \\
&\quad \cdots P(W_n + (Z_1 + Z_2 + \cdots + Z_{n-1}) + \beta(T_1 + T_2 + \cdots + T_n) < D_0) \\
&\quad \times P(t_1 = T_1, t_2 = T_2, \dots, t_n = T_n) P(N(t) = n) dT_n \cdots dT_2 dT_1 \\
&= \int_0^t \int_{T_1}^t \cdots \int_{T_n}^t \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta T_1)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2 T_1^2}} \right) \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta(T_1 + T_2) + \mu_Z)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2(T_1 + T_2)^2 + \sigma_Z^2}} \right) \\
&\quad \times \cdots \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta(T_1 + T_2 + \cdots + T_n) + (n-1)\mu_Z)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2(T_1 + T_2 + \cdots + T_n)^2 + (n-1)\sigma_Z^2}} \right) \\
&\quad \times \lambda e^{-T_1 \lambda} \lambda e^{-T_2 \lambda} \cdots \lambda e^{-T_n \lambda} \frac{(-\lambda t)^n}{n!} e^{-\lambda t} dT_n \cdots dT_2 dT_1
\end{aligned} \tag{15}$$

$$\begin{aligned}
R_n(t) &= \frac{(-\lambda t)^0}{0!} e^{-\lambda t} + \\
&\quad \sum_{n=1}^{\infty} \int_0^t \int_{T_1}^t \cdots \int_{T_{n-1}}^t \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta T_1)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2 T_1^2}} \right) \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta(T_1 + T_2) + \mu_Z)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2(T_1 + T_2)^2 + \sigma_Z^2}} \right) \\
&\quad \times \cdots \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta(T_1 + T_2 + \cdots + T_n) + (n-1)\mu_Z)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2(T_1 + T_2 + \cdots + T_n)^2 + (n-1)\sigma_Z^2}} \right) \\
&\quad \times \lambda e^{-T_1 \lambda} \lambda e^{-T_2 \lambda} \cdots \lambda e^{-T_n \lambda} \frac{(-\lambda t)^n}{n!} e^{-\lambda t} dT_n \cdots dT_2 dT_1
\end{aligned} \tag{16}$$

$$\begin{aligned}
R_{sys}(t) &= P(X_S(t) < H) \times \int_0^t \int_{T_1}^t \cdots \int_{T_{n-1}}^t P(W_1 < D_0 - \beta T_1) P(W_2 < D_0 - Y_1 - \beta(T_1 + T_2)) \\
&\quad \cdots P(W_n < D_0 - (Z_1 + Z_2 + \cdots + Z_{n-1}) - \beta(T_1 + T_2 + \cdots + T_n)) \\
&\quad \times P(t_1 = T_1, t_2 = T_2, \dots, t_n = T_n) P(N(t) = n) dT_n \cdots dT_2 dT_1 \\
&= \frac{(-\lambda t)^0}{0!} e^{-\lambda t} \times \Phi \left(\frac{H - (\mu_\gamma t + \varphi)}{\sqrt{\sigma_\gamma^2 t^2}} \right) + \sum_{n=1}^{\infty} \left(\int_0^t \int_{T_1}^t \cdots \int_{T_{n-1}}^t \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta T_1)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2 T_1^2}} \right) \right. \\
&\quad \times \Phi \left(\frac{\mu_{D_0} - (\mu_W + \mu_\beta(T_1 + T_2 + \cdots + T_n) + (n-1)\mu_Z)}{\sqrt{\sigma_{D_0}^2 + \sigma_W^2 + \sigma_\beta^2(T_1 + T_2 + \cdots + T_n)^2 + (n-1)\sigma_Z^2}} \right) \\
&\quad \times \lambda e^{-T_1 \lambda} \lambda e^{-T_2 \lambda} \cdots \lambda e^{-T_n \lambda} \frac{(-\lambda t)^n}{n!} e^{-\lambda t} dT_n \cdots dT_2 dT_1 \times \Phi \left(\frac{H - (\mu_\gamma t + \varphi + n\mu_\gamma)}{\sqrt{\sigma_\gamma^2 t^2 + n\sigma_\gamma^2}} \right) \Big)
\end{aligned} \tag{17}$$

With all possible situation (form 0 shock to infinite shocks) considered, the probability that no hard

failure occurs is demonstrated in Equ. (16).

3.3 System reliability modeling

As we said above, the system is under two failures, the hard failure and soft failure. Neither the hard failure nor the soft failure occurs if the system wants to keep normal. The system reliability is derived as Equ. (17).

Failure rate is defined as the proportion of failure during a unit interval. The typical failure rate curve is the famous *tub curve*. The expression of failure rate and reliability is demonstrated as

$$FR(t) = -\frac{1}{R(t)} \frac{dR(t)}{dt} \quad (18)$$

4 Case: an MEMS application

Consider a micro-electromechanical system (MEMS) (Peng and Coit 2011) (An *et al.* 2017). The MEMS is subjected to two failures. The first is the visible wear on rubbing surfaces between the gear and the pin joint which may lead to inaccuracy transmission. The wear volume is caused by the aging degradation process and debris from external shocks. The other failure is that the external shocks may cause the catastrophic fracture of the pin if loads led by external shocks are too large. Thus, the MEMS are under soft failure caused by aging and debris from shocks and hard failure caused by fracture of pin.

The parameters for Equ. (17) to do reliability analysis are in Table 2. Threshold H of wear degradation, hard failure threshold D_0 , initial degradation value φ and degradation rate γ are obtained from Sandia's experimental results. (Tanner *et al.* 2003) The distribution of external shock sizes is supposed to be normal distributed according to Jiang L. as well as Y_i . (Jiang and Coit 2015) For a better illustration, the shifting rate β of hard threshold and instantaneous increases Z_i are assumed to normal distribution with its parameters.

According to Equ. (17), the system reliability is calculated which is demonstrated in Figure 5 with parameters in Table 2. From Figure 5 (a) we can obtain that the reliability for system changes with different speeds at different time t . Figure 5 (b) shows the failure rate which seems like the "*tub curve*". Comparing the failure rate curve, showed in Figure 6 calculated by a former model (Peng and Coit 2011), the tub curve is much closer to engineering practice which demonstrated the advantage of presented model in this paper.

Figure 7 shows the threshold H has obvious influences on system reliability and failure rate especially when t is larger than 100000. A smaller threshold H will have a smaller system reliability and higher failure rate than a larger H , after 100000 times. It is also indicated that the smaller the threshold H is, the more likely failure occurs.

Figure 8 indicates that both system reliability and failure rate are sensitive to the mean value of D_0 at any time stage. A larger initial mean value D_0 corresponds to a higher reliability. Besides, a larger mean value D_0 accompanies a smaller failure rate. Figure 9 is the sensitive analysis for the standard deviation of D_0

which indicate that system reliability and failure rate are also sensitive to σ_{D_0} .

Obviously, system reliability and failure rate are much more sensitive to external shocks rate λ . When rate λ increases from 1.5×10^{-5} to 3.5×10^{-5} , the system reliability curve shifts to left which indicates a decreasing reliability. Meanwhile, the failure rate varies with rate λ just as Figure 10 (b) demonstrates.

5 Summaries and future research

The modeling for MDCFP is established with instant-shift hard failure threshold considered. Hard threshold means the ability to resist external shocks which should be degenerate with time. However, it is

set as fixed value in previous researches. In the case section, the reliability and failure rate are calculated based on the equation in presented model. The obtained failure rate curve is closed to the typical *tub curve*, which indicates that the presented model tallies with engineering practice. Sensitive analyses for specific parameters are studied. The results show that system reliabilities and failure rates change a lot with different parameter value.

For future research, the instant-shift of hard failure threshold can be modeled by nonlinear expression and test of threshold trend can also been studied. The maintenance policies could also be studied based on presented reliability model.

Acknowledgment

The research in this paper is supported by “the Fundamental Research Funds for the Central Universities” of China. (Grant No.3102015 BJ (II)JL01)

Reference

- An Z., Sun D. 2017. Reliability modeling for systems subject to multiple dependent competing failure processes with shock loads above a certain level. Reliability engineering and system safety. 157:129-138. Available from: <http://dx.doi.org/10.1016/j.ress.2016.08.025>
- Bae S., Kuo W. 2007. Degradation models and implied lifetime distributions. Reliability engineering and system safety. 92:601-608. doi:10.1016/j.ress.2006.02.002
- Bai J., Li Z., Kong X. 2006. Generalized shock models based on a cluster point process. IEEE Transactions on reliability. 55(3):542-550. doi: 10.1109/TR.2006.879661
- Delia M., Rafael P. 2006. A deteriorating two-system with two repair modes and sojourn times phase-type distributed. Reliability engineering and system safety. 91:1-9. doi:10.1016/j.ress.2003.12.013
- Durante F. 2017. Shock models with dependence and asymmetric linkages. Fuzzy sets and systems. 323: 152-168. Available from: <http://dx.doi.org/10.1016/j.fss.2016.09.006>
- Fan M., Zeng Z., Zio E. 2017. Modeling dependent competing failure processes with degradation-shock dependence. Reliability engineering and system safety. 165:422-430. Available from: <http://dx.doi.org/10.1016/j.ress.2017.05.004>
- Finkelstein M., Zarudnij V. 2001. A shock process with a non-cumulative damage. Reliability engineering and system safety. 71:103-107. PII: S0951-8320(00)00065-X
- Gut A. 2005. Realistic variation of shock models. Statistics and probability letters. 75:187-204. doi:10.1016/j.spl.2005.04.043
- Hsieh M., Jeng S., Shen P. 2009. Assessing device reliability based on scheduled discrete degradation measurements. Probabilistic Engineering Mechanics. 24:151-158. doi:10.1016/j.probengmech.2008.04.003
- Huang W., Dietrich D. 2005. An Alternative Degradation Reliability Modeling Approach Using Maximum Likelihood Estimation. IEEE Transactions on reliability. 54(2):310-317. doi: 10.1109/TR.2005.845965
- Jiang L., Feng Q., Coit D. 2012. Reliability and Maintenance Modeling for Dependent Competing Failure Processes With Shifting Failure Thresholds. IEEE Transactions on Reliability. 61 (4): 932-948. doi: 10.1109/TR.2012.2221016
- Jiang L., Feng Q., Coit D. 2015. Modeling zoned shock effects on stochastic degradation in dependent failure processes. IIE Transactions. 47(5): 460-470. DOI:10.1080/0740817X.2014.955152

- Kharoufeh J. 2003. Explicit results for wear processes in a Markovian environment. *Operations research letters*. 31:237-244. PII: S0167-6377(02)00229-8
- Kharoufeh J. Cox S. 2005. Stochastic models for degradation-based reliability. *IIE Transactions*. 37(6): 533-542. DOI: 10.1080/07408170590929009
- Liu H., Yeh R., Cai B. 2017. Reliability modeling for dependent competing failure processes of damage self-healing systems. *Computers and industrial engineering*. 105:55-62. Available from: <http://dx.doi.org/10.1016/j.cie.2016.12.035>
- Mallor F., Santos J. 2003. Reliability of systems subject to shocks with a stochastic dependence for the damages. *Sociedad de Estadística e Investigación Operativa*. 12(2):427-444.
- Peng H., Feng Q., Coit D. 2011. Reliability and maintenance modeling for systems subject to multiple dependent competing failure processes, *IIE Transactions*. 43 (1): 12-22. DOI: 10.1080/0740817X.2010.491502
- Rafiee K., Feng Q., Coit D. 2013. Reliability modeling for dependent competing failure processed with changing degradation rate, *IIE Transactions*. 46 (5): 483-496. DOI: 10.1080/0740817X.2013.812270
- Rafiee K., Feng Q., Coit D. 2017. Reliability assessment of competing risks with generalized mixed shock models. *Reliability engineering and system safety*. 159:1-11. Available from: <http://dx.doi.org/10.1016/j.ress.2016.10.006>
- Robinson M. Crowder M. 2000. Bayesian Methods for a Growth-Curve Degradation Model with Repeated Measures. *Lifetime date analysis*.: 357-374.
- Song S., Coit D., Feng Q. 2014. Reliability for systems of degrading components with distinct component shock sets. *Reliability engineering and system safety*. 132: 115-124. <http://dx.doi.org/10.1016/j.ress.2014.06.020>
- Song S., Coit D., Feng Q. 2014. Reliability Analysis for Multi-Component Systems Subject to Multiple Dependent Competing Failure Processes. *IEEE Transactions on Reliability*. 63 (1) : 331-3450. doi: 10.1109/TR.2014.2299693
- Tanner D., Dugger M. 2003. Wear mechanisms in a reliability methodology. In: *Proceedings of IEEE SPIE-reliability, testing, and characterization of MEMS/MOEMSII*, San Jose, CA. pp. 22-40.
- Xue J., Yang K. 1995. Dynamic reliability analysis of coherent multistate systems. *IEEE Transactions on reliability*. 44(4):683-688.
- Yang Y., Klutke G. 2000. Lifetime-Characteristics and Inspection-Schemes for Lévy Degradation Processes. *IEEE Transactions on reliability*. 49(4):377-382. PII: S 0018-9529(00)11784-1
- Zuo M., Jiang R. 1993. Approaches for reliability modeling of continuous-state devices. *IEEE Transactions on reliability*. 48(1):9-18.

Tables

Table 1 Notations

Notations	Meaning
$X(t)$	Pure degradation value
$X_S(t)$	Total degradation value
T_i	Coming moment of the i th shock
Y_i	Instantaneous increment of degradation value due to the i th shock
D_0	Fixed hard failure threshold value
D	Shifting hard failure threshold value
H	Soft failure threshold
Z_i	Decreasing value of hard failure due to the i th shock
W_i	Size of the i th shock
$R_i(t)$	Reliability value when there are i shocks
R_{sys}	System reliability

Table 2 Parameters for case analysis

Parameters	Values	Sources
H	0.00125	(Tanner 2003)
D_0	$\square N(\mu_{D_0}, \sigma_{D_0}^2)$ $\mu_{D_0} = 1.3$ $\sigma_{D_0} = 0.3$	(Tanner 2003)
φ	0	(Tanner 2003)
γ	$\square N(\mu_\beta, \sigma_\beta^2)$ $\mu_\beta = 8.4823 \times 10^{-9} \mu m^3$ $\sigma_\beta = 6.0016 \times 10^{-10} \mu m^3$	(Tanner 2003)
W_i	$\square N(\mu_w, \sigma_w^2)$ $\mu_w = 1.2 Gpa$ $\sigma_w = 0.2 Gpa$	(Jiang and Coit 2012)
λ	2.5×10^{-5} /revolutions	(Jiang and Coit 2012)
Y_i	$\square N(\mu_y, \sigma_y^2)$ $\mu_y = 2 \times 10^{-8} \mu m^3$ $\sigma_y = 1 \times 10^{-9} \mu m^3$	(Jiang and Coit 2012)
Z_i	$\square N(\mu_c, \sigma_c^2)$ $\mu_c = 1 \times 10^{-4} \mu m^3$ $\sigma_c = 2 \times 10^{-5} \mu m^3$	Assumption
β	$\square N(\mu_\beta, \sigma_\beta^2)$ $\mu_\beta = 6 \times 10^{-7} \mu m^3$ $\sigma_\beta = 3 \times 10^{-8} \mu m^3$	Assumption

Figure Captions list

Figure 1 Hard failure threshold

Figure 2 Competing failure process: (a) Continuous degradation (b) Total degradation (c) Shock process with fixed threshold (d) Shock process with shifting threshold

Figure 3 Shifting hard threshold with one shock

Figure 4 Shifting hard threshold with two shock

Figure 5 Reliability result

Figure 6 Reliability result from former model

Figure 7 Sensitivity analysis for H

Figure 8 Sensitivity analysis for μ_{D_0}

Figure 9 Sensitivity analysis for σ_{D_0}

Figure 10 Sensitivity analysis for λ

Draft

<https://mc06.manuscriptcentral.com/tcsme-pubs>

(a) System Reliability

(b) Failure rate