Finite Element Analysis of Intermediate Crack Debonding in FRP Strengthened RC Beams

<table>
<thead>
<tr>
<th>Journal:</th>
<th>Canadian Journal of Civil Engineering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript ID</td>
<td>cjce-2017-0439.R2</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Article</td>
</tr>
<tr>
<td>Date Submitted by the Author:</td>
<td>09-Apr-2018</td>
</tr>
<tr>
<td>Complete List of Authors:</td>
<td>Cohen, Michael; University of Waterloo, Civil &amp; Environmental Engineering Monteleone, Agostino; AMTEC Engineering Ltd Potapenko, Stanislav; University of Waterloo, Civil &amp; Environmental Engineering</td>
</tr>
<tr>
<td>Keyword:</td>
<td>Debonding, Fracture, Interface, Finite Element Analysis, ABAQUS</td>
</tr>
<tr>
<td>Is the invited manuscript for consideration in a Special Issue? :</td>
<td>Not Applicable (Regular Submission)</td>
</tr>
</tbody>
</table>

https://mc06.manuscriptcentral.com/cjce-pubs
Finite Element Analysis of Intermediate Crack Debonding in FRP Strengthened RC Beams

Michael Cohen¹, Agostino Monteleone², Stanislav Potapenko³

¹ PhD Candidate, Dept. of Civil and Environmental Engineering, University Of Waterloo, Waterloo, ON, Canada (corresponding author). E-mail: mcohen@uwaterloo.ca
² AMTEC Engineering Ltd., Etobicoke, Ontario, Canada. E-mail: amontele@gmail.com
³ Dept. of Civil and Environmental Engineering, University Of Waterloo, Waterloo, ON, Canada. E-mail: spotapenko@uwaterloo.ca

Word Count: 7474

Abstract

The performance of the fibre reinforced polymer (FRP) to reinforced concrete (RC) interface is vital to ensure desired design capacity. Without proper understanding of the interfacial behaviour it is impossible to develop an effective, efficient and rational bonding technique. This paper presents the results of a comprehensive numerical investigation aimed to assess and better understand the debonding behaviour caused by different types of intermediate flexural crack distributions in FRP-RC strengthened beams. The model is based on damage mechanics modelling of concrete, a bilinear bond-slip relationship with softening to represent the interface, and a discrete crack approach to simulate crack propagation. The model also highlights how crack propagation and debonding is affected by the rate of change of moment. It is shown that the variation of crack spacing and rate of change of moment can significantly affect debonding crack propagation and strain development in the internal and external reinforcement, which directly influences debonding load.

Key Words: Debonding, Fracture, Interface, Finite Element Analysis
1 Introduction

In recent years, there has been an increased need for the strengthening or rehabilitation of reinforced concrete (RC) structures due to the aging of infrastructure, demand for higher vehicle loads, updates in design codes or inadequate original design. Fibre-reinforced polymers (FRP) are now routinely considered as an effective method for such applications in the structural engineering community due to their light weight and minimal labor and equipment requirements, as compared to tradition methods. Furthermore, the initial high material costs of FRP can be offset by the low installation and long-term maintenance costs. Numerous studies have been conducted to prove the efficiency of utilizing FRP composites in structural elements (Nanni 1995; Arduini et al. 1997; Grace et al. 1999; Ross et al. 1999; Brena et al. 2003). In spite of this, industrial practitioners are still concerned about premature debonding of the plates before reaching the desired strength or ductility. Unlike the failure modes of concrete crushing, shear failure and FRP rupture, such debonding failure cannot be predicted by conventional RC theory. Premature debonding initiates from the ends of the plate or from intermediate cracks (IC) in the concrete. In practice, FRP debonding from the end of an IC sometimes is unavoidable and more dominant despite careful surface preparation and good bond between FRP composites and concrete. Currently our basic understanding of the mechanics of the bond and failure between the FRP and concrete initiating from an IC is rather limited.

The presence of FRP bonded to the tension face of a RC beam will restrict but not prevent the opening of an IC. Empirical studies have shown that FRP debonding may initiate from the bottom of an IC near the middle of the span, where the bending moment and force in the FRP are high. With an increase in load, the debonded zone grows and propagates towards the free-end of the plate, leading to ultimate debonding failure of the strengthened structure. As of late,
debonding failure and the presence of multiple cracks has started to attract attention in the research community. In fact, FRP debonding failure under the presence of multiple cracks is now being considered in design recommendations (*fib* 2001). However, the proposed model has not yet been verified experimentally.

One of the advantages of finite-element models is their ability to capture quantities that could be difficult to quantify experimentally, such as stress/slip concentrations and distributions along the FRP-concrete interface. In addition, they provide insight into the effects of micro- and macro-cracking on the interfacial behaviour and they allow us to better obtain results which may vary significantly from researcher to researcher, such as FRP strain (Yao et al. 2004).

A commonly accepted method of studying IC debonding by researchers and industrial practitioners is to apply a direct shear test on the composite beam (Chen and Teng 2001). This test involves pulling a FRP plate bonded to a concrete prism along the direction of its length to determine the bond capacity of the strengthened specimen. Due to the shear lag phenomenon (i.e: the uneven distribution of strains along the FRP-Concrete interface relative to the distance from the loaded end), the bond capacity approaches a plateau value with increasing bond length. By performing the direct shear test on members with different bond lengths, it is possible to determine the effective bond length necessary to develop sufficient bond capacity between FRP reinforcements and concrete members. While this test may be frequently employed, both an empirical and analytical study performed by Teng et al. (2002) and Teng et al. (2005) prove otherwise. The results from those studies indicate that debonding models with parameters derived from the direct shear test significantly underestimate the maximum FRP strain in the strengthened beam. These findings may be attributed to the presence of multiple secondary cracks along the beam as the opening of cracks along the beam act to reduce relative sliding
between concrete and FRP, and may lessen the rate of softening along the interface. Moreover, the portion of FRP plate situated between two adjacent cracks is subjected to tension at both cracks. In contrast to loading setting observed in conventional shear test where one end of the FRP plate is loaded and the other end remains intact. The discrepancy between commonly accepted practices and empirical evidence is just one example of why further investigations into the mechanism that trigger FRP debonding from IC are needed in an attempt to improve the efficiency of rehabilitation projects in civil engineering applications.

In this study, the results of a comprehensive numerical investigation are presented to illustrate the complex interaction between the debonding strain and the flexural crack distribution. The model is based on damage mechanics modelling of concrete and a bilinear bond-slip relationship with softening behaviour to represent the FRP-concrete interfacial properties. A discrete crack approach was adopted to simulate crack propagation through a nonlinear fracture mechanics based finite element analysis. The model also highlights how crack propagation and debonding is affected by moment gradient. The numerical simulations were validated against experimental results and are capable of predicting behaviour observed during such tests. The results provide an insight on the behaviour of a repair system that is gaining widespread use and will be of interest to researchers and design engineers looking to successfully apply FRP products in civil engineering applications.

2 Numerical Modelling

2.1 Geometric Discontinuities

Generally, there are two approaches to simulate fracture process in finite element modeling: a continuum approach and a discrete approach. The continuum approach, commonly referred to as...
the “smeared crack approach”, treats fracture as the end process of localization and accumulation of damage in continuum without creating a real discontinuity in the material. It is unable to trace individual macro-cracks because it tends to spread crack motion over a region of the structure rather than at localized points unless the characteristic dimension of the finite elements are chosen small enough from the beginning of the analysis to accurately resolve the evolving damage zone, as demonstrated by Lu et al. (2005). However, for real-life structures the computation costs become excessive and impractical. Alternatively, the discrete crack approach models a crack discretely as a geometric entity and was selected to simulate discontinuities due to opening of dominant cracks, slipping of rebar, and debonding of the FRP sheet. This approach has been modelled using zero-thickness interface elements for the aforementioned discontinuities and their constitutive relationships are discussed in subsequent sections.

2.2 Concrete Modelling

The concrete damage plasticity model provided in ABAQUS (2007) is used to simulate the nonlinear behaviour of concrete. To ensure that major cracking and crack growth do not occur in locations other than where the predefined cracks are set, the compressive and tensile material models were defined without limitation of strain capacity, as shown in Figure 1. The plasticity of concrete material was modelled using the concrete damage plasticity model. In this plasticity model, the damage of concrete was represented by a continuum (smeared) approach; meaning that the model does not physically generate macro-cracks in concrete, instead the cracks are indirectly accounted for by the way their presence affects the stress and material stiffness. The crack process in concrete is not a sudden onset of new free surfaces but a continuous forming and connecting of macro-cracks. Macro-cracking and crushing in concrete are represented by increasing values of the hardening variables and these variables control the degradation of the...
elastic stiffness and the evolution of the yield surface. They are also closely related to the dissipated fracture energy required to generate macro-cracks. Typically, the formation of macro-cracks is represented macroscopically as softening behavior of the material, which causes the localization and redistribution of strain in a structure. One way to remove the evolution of cracking is to modify the compressive hardening and tensile softening of the material model, within the original smeared model. Cracking leads to softening behaviour of the material so if any post-yield softening is removed from the model, the growth of cracking could be avoided in the material thereby only permitting major cracking to occur within the predefined locations.

The discrete crack approach is adopted to account for “major” concrete cracking in the model by predefining possible flexural cracks in the beam. The predefined cracks are divided into two zones: traction-free and cohesive crack, as shown in Figure 2. A traction free crack physically represents a “notch” that would be set in the concrete beam at the time of casting to ensure that cracking initiates and evolves at a predefined location. No forces are transferred along this zone and crack surfaces are completely separated. The force in the cohesive crack surface follows a predefined response whereby a linear softening curve is employed to model tension softening of concrete. The cohesive crack, preceding the formation of a real crack, is assumed to initiate if tensile stresses reach the tensile strength of concrete, $f_t$, whereas the real macro-crack is formed when the energy required to create a unit area of crack is achieved. The area below the curve represents the model I interfacial fracture energy, $G_f$. Moreover, minor “hairline” cracks might still occur in locations outside the predefined discrete cracks when internal stresses exceed the tensile capacity of concrete, $f_t$. However, the development of these minor cracks is restricted, as opposed to the major discrete cracks defined earlier (Niu and Zhishen 2006).
Unloading and reloading behaviours are modelled by a secant path, which means following a straight line back to the origin upon unloading the stress. After cracking, no shear stress is assumed to transfer along the crack surface.

### 2.3 Reinforcing Steel

The stress-strain relationship for reinforcing steel was modelled with the use of the classical metal plasticity model, provided in ABAQUS (2007), using three linear segments. Three linear segments are employed to account for yielding and strain hardening of the reinforcing steel. Interface elements surround the reinforcing steel to account for slippage between rebar and concrete. Interfacial slippage was defined according to the CEB-FIB model code (1993).

### 2.4 Fiber Reinforced Polymer Composites

The FRP composites are assumed to be linear-elastic. The rupture point in the stress-strain relationship defines the ultimate stress and strain values for the FRP. These ultimate points were originally adopted from the experimental program conducted by Brena et al. (2003).

### 2.5 FRP-Concrete Interface

The FRP-concrete interface refers to a thin layer of the adhesive and adjacent concrete within which the relative deformation between FRP and concrete mainly happens, as revealed by experimental studies (Yuan et al. 2004). This deformation occurs mainly due to shearing stresses along the FRP-concrete interface (Mode II). In addition, interfacial normal (peel) stress also exits at intermediate crack locations. However, these type of stresses were not considered in this numerical study, because it is generally accepted that the debonding of the FRP resembles mode II fracture behaviour as the adhesive layer transfers shear stresses from the concrete to the FRP.
In strict sense (microscopic), any interface is naturally mixed-mode and the stress state within the interface is very complicated (Hutchinson and Suo 1992). Only the debonding behaviour within the adhesive layer may be like mode II fracture behaviour, whereas the general debonding within the adjacent concrete layer may be associated with a concrete mode I fracture and mode II shearing fracture behaviour. However, the latest research on this topic has shown that it is acceptable to ignore the mode I fracture behaviour since it has a negligible effect on the overall debonding process. The works that has led to this assumption are briefly outlined below:

- According to Rabinovitch and Frostig (2000), the concrete beam and FRP plate are in contact at the vicinity of the flexural crack. This suggests that the normal interface stress is compressive at this location, and therefore, doesn’t affect the debonding of the FRP-concrete interface if friction is neglected. This is different from the normal stress at the FRP plate end, which is tensile and plays a critical role in the plate-end debonding.

- Existing solutions proposed by Smith and Teng (2001) show that the normal stress has little effect on the derivation of shear stress.

- By using a displacement discontinuity model, Wu et al. (2002) found that there exists a linear correlation between mode I concrete and a mode II interfacial fracture energy values for a given shearing fracture energy introduced on the crack surface. This means that the overall debonding behaviour can be regarded as a mode II for intermediate cracks.

Thus, it is considered to be an acceptable assumption that the IC debonding is treated as a mode II fracture.
On the other hand, the interface is modelled using the non-linear fracture mechanics approach proposed by Lu et al. (2005) and shown in Figure 3. Its validity has been proven by numerous researchers (Baky et al. 2007; Ebead et al. 2007). Under external load, interfacial shear stresses, $\tau$, develops along the interface due to the opening of the predefined cracks. The opening of the predefined cracks is represented numerically as a finite slip, $s$, between the FRP plate and the concrete beam. Initially, when the applied load is small and the interfacial stress, $\tau$, is less than the maximum interfacial stress, $\tau_{\text{max}}$, the interface is considered to be in its elastic stage. The elastic stage ends when $\tau$ exceeds $\tau_{\text{max}}$, which signifies the onset of micro-debonding. As slip continues to increase, the interfacial shear stresses reduce to zero, and full debonding initiates and propagates along the interface, representing macro-debonding. Slip at the interface is given by:

$$s = u_p - u_c$$

(1)

Where $u_p$ is the horizontal displacement of the FRP element and $u_c$ is the horizontal displacement of the concrete element. The interface is formulated as:
\[ \tau = \begin{cases} 
\tau_{max} \left( \frac{s}{s_0} \right) & \text{if } s \leq s_o \\
\tau_{max} \exp[-\alpha(s/s_o - 1)] & \text{if } s > s_o
\end{cases} \]  
(2)

\[ s_o = 0.0195 \beta_w f_t ; \quad \tau_{max} = 1.5 \beta_w f_t \]  
(3)

\[ \alpha = \frac{1}{G_f/\tau_{max}s_o^{2/3}} ; \quad G_f = 0.308 \beta_w^2 \sqrt{f_t} \]  
(4)

\[ \beta_w = \sqrt{(2.25 - b_f/b_c)/(1.25 + b_f/b_c)} \]  
(5)

where \( \tau_{max} \) is the maximum interfacial shear stress; \( s_o \) is the slip at maximum interfacial shear stress; \( \alpha \) is a coefficient; \( G_f \) is the interfacial fracture energy; and \( \beta_w \) is FRP width factor.

### 2.6 Structural Model

Due to symmetry, only half of beam was modeled in two-dimensions. The structural member is broken down into finite elements to model the composite beam. Since more than one type of materials and interfaces are considered in the analysis, different types of finite elements are required to discretize the structure. Concrete is modelled using continuum elements; reinforcing steel and FRP are modelled using one-dimensional beam elements; and predefined flexural cracks, rebar-concrete interface and FRP-concrete interface are modelled using cohesive/interface elements. Linear reduced-integration continuum elements are employed throughout the analysis with a fine mesh for their ability to withstand severe distortion in plasticity and crack propagation applications. Reinforcing steel is modeled as one-dimensional beam elements discretely defined and superimposed onto the ‘host’ concrete element through the use of beam (stringer) elements. The FRP composite is modelled as singly defined beam elements attached to the concrete through interface elements.
The interface elements are attached to the concrete substrate through the use of “tie constraints”. At each end of the interface element, the interaction between the two nodes is represented by two springs: shear spring with stiffness, $k^s_{int}$, and normal spring with stiffness, $k^n_{int}$. Load was applied in the form of an imposed displacement at the top face of the specimen and a pin-support was used to restrain the beam in the vertical direction. Plane strain condition was assumed throughout the analysis, since during debonding failure the conditions at the crack tip are “neither plane stress nor plane strain, but three-dimensional” (Anderson 1994; Coronado and Lopez 2007). IC debonding is a common failure mode in composites beams that exhibit flexural behaviour when the shear span-to depth (a/d) ratio is approximately 2.5 or greater (Rosenboom and Rizkalla 2008). Since the model has a shear span of 3.05, IC debonding behaviour is expected.

2.7 Model Verification

The results of a comprehensive experimental study reported by Brena et al. (2003) have been used to validate the original finite element model. To ensure the accuracy and effectiveness of the numerical modelling approach followed in this study, the calibration of results was conducted on a global (structural beam) level and a local (constitutive materials) level. Initially, the numerical beam was calibrated against three different experimental specimens. At first, a control finite element model (without CFRP) was validated against a corresponding experimental specimen to the same author. Then, CFRP was added to the numerical model, but with the assumption of perfect bond between the concrete substrate and the adjacent CFRP. Similarly, the simulation of this model was calibrated with the tested beam, in order to emphasize the influence of accounting for the appropriate interfacial mechanisms. Finally, the response of the main ABAQUS model in this study (CFRP with cohesive zone approach) was compared to the real beam, and was found
to replicate the original data very closely. Figure 4(a) through (c) illustrate the comparison between each model and the experimental beam, in term of load-midspan deflection curve.

On the other hand, the validation of the FE model on local level was achieved by obtaining the load-strain relationships of concrete, steel rebar, and CFRP laminate, and compared them with experimental results (see Figure 5(a) through (c)). Good correlations between the numerical and experimental results were found suggesting that the model has been adequately calibrated. The numerical investigations have been designed to expand over previous experimental studies and better understand the structural behaviour of crack induced debonding. The length of the FRP plate was increased from 1322 to 2200 mm from the calibrated model to the model used for this study, respectively. The modification was made to allow for more flexural cracks to span across the entire length of the beam and study how the structural behaviour and debonding mechanisms are influenced by multiple intermediate cracks, which develop under external loading, and are virtually impossible to measure experimentally. The geometric properties of the model used in this study is shown in Figure 6(a).

2.8 Mesh Convergence

A general check on the mesh density was investigated prior to the start of the analysis. Mesh refinement was investigated for two models: a single discrete crack located under the load point and multiple cracks distributed along the interface at 50 mm apart. The latter represents a case in which convergence problems are expected to be most severe due to the complicated debonding behaviour for such closely spaced cracks. Figure 7 shows the results of the load-deflection response using five levels of mesh refinement for the case of a single discrete crack located under
the load point. It can be seen that the structural performance is overestimated with the use of the coarse mesh, while refining the mesh leads to convergence of response.

While it appears that convergence is achieved by employing a fine structured mesh of element size 5 mm by 5 mm, a closer inspection of the results in terms of interfacial shear stress versus midspan deflection proves otherwise. Figure 7 demonstrates that the fine mesh employing element sizes of 5 x 5 mm overestimates the interfacial shear stress response of the specimen (at location immediately under the loading point), whereas mesh convergence is ultimately obtained with the use of a finer mesh size of 3.5 x 3.5 mm. Overlooking the effect of interfacial shear stress when selecting a mesh for the analysis may lead to inaccurate results along the interface as the coarser mesh appears to be incapable of converging prior to macro-debonding. The finer mesh seems to be able to accurately capture the complicated fracture behaviour involving concrete cracking and interfacial debonding. This mesh was found to be suitable for the model employing cracks spaced at 50 mm apart.

3 Numerical Analyses and Discussions

In this study, no attempt was made to consider the application of FRP composites to a pre-damaged RC structure. While in practice many cracks will have already formed with some spacing before the application of FRP composites, a clear explanation on how these cracks may affect the bond characteristics between concrete and FRP and ultimately influence the mode of failure is currently unavailable. Without established empirical evidence on this topic, any finite element model created to carry out such an analysis can potentially be contradicted. The cracking
spacing, $x_c$, is varied throughout the model to observe the how the spacing between cracks affects the debonding mechanisms and efficiency of internal and external reinforcement. A single localized crack model and five (5) crack spaced models ($x_c = 280, 125, 100, 75$ and 50 mm) were considered as these values closely represent the real pattern in the tested specimens. The following parameters are used in this study: $b = 1000$ mm, $k_{s}^{\text{int}} = 160$ MPa/mm (taken from the experimental program), $k_{R}^{\text{int}} = 0$ MPa/mm, $\tau_{b} = 4.5$ MPa, $G_{f}^{\text{int}} = 0.5$ N/mm, the width factor, $\beta_w$ was found to be 0.75, the $s_o$ value was 0.0516, and the value of the parameter $\alpha$ was 1.1015, $E_{\text{frp}} = 230,000$ MPa, $f_{l} = 3.5$ MPa, $E_{c} = 26,500$ MPa, $f'_{c} = 35$ MPa, $A_{sb} = 200$ mm$^2$ ($\varnothing 16$ mm), $A_{st} = 71$ mm$^2$ ($\varnothing 10$ mm), $A_{\text{stirrup}} = 51$ mm$^2$ ($\varnothing 8$ mm), Reinforcement ratio, $\rho$, is 0.0063. A schematic representation of the structural model is shown in Figure 6(b). It must be noted that despite the presence of very few crack openings generated outside the constant moment region of the beam, the cracks within the shear span were actually accounted for and modelled as inclined cracks. The schematic representation of the FE model in Figure 6(b) is intended to show the location of each crack, not the orientation of cracks.

### 3.1 Global Response

The effect of crack spacing is first analyzed based on the load-deflection behaviour of the FRP-strengthened beam. Figure 8 demonstrates that the stiffness and ultimate capacity of the model decreases with the change in crack spacing. This can be attributed to the existence of more closely spaced cracks reducing the rigidity of the structure as loading progresses. Single localized and large crack space models produce similar stiffness and ultimate load, which are both greater than the smaller crack space models. Table 1 lists key data throughout the parametric analysis and is

https://mc06.manuscriptcentral.com/cjce-pubs
referenced throughout Section 3. Once macro-debonding initiates, the debonding crack propagates towards the end of the FRP sheet and the load would remain relatively constant until final debonding failure (ultimate capacity). Subsequent to the initiation of macro-debonding, debonding failure was reached earlier in models with more closely spaced cracks as opposed to the larger crack spaced models by inspection of the deflection values for each model in Table 1. The single localized crack model was able to deflect 18.9 mm after macro-debonding in comparison to only 9.5 mm for the model with cracks spaced at 50 mm. This may be attributed to the existence of more closely spaced cracks quickening the debonding propagation.

3.2 Interfacial Shear Stress Response

The above results can be explained by studying the interfacial behaviour of the analyses models. For the case of the single localized crack predefined beneath the load, it was found that prior to the initiation of flexural cracking there is no slip and therefore no shear stress at the FRP-concrete interface. With further loading, interfacial shear stresses develop along the interface until micro-debonding initiates at a midspan deflection of 2.4 mm. At this point, micro-debonding occurs in the weaker concrete layer of the interface and high bond stresses develop near the toe of the crack creating sliding between the concrete and FRP plate. The strain in the plate is no longer equal to the strain in the adjacent concrete and the difference is defined as slip strain. In order to accommodate the stress development, the FRP plate would require infinite strains across the crack, which is not possible, and thus results in micro-cracking. This point is illustrated in Figure 9 where the stress concentration at the toe of the crack reaches its maximum value of 4.5 MPa. As loading progresses, the maximum interfacial shear stress shifts along the beam in two directions: towards the support and midspan. This shift represents that the shear capacity of the interface is reached and the development of the debonding crack, which is
propagating along the soffit in two directions. To demonstrate the debonding propagation along
the interface Figure 9 captures the interfacial shear stress distribution at six (6) stages during the
analysis. It should be noted that the effective shear transfer length, $L_{\text{eff}}$, required to attain the
ultimate load- carrying capacity may be regarded as 140 mm, which compares well with the
analytical solution proposed by Chen and Teng (2001). However, this parameter, $L_{\text{eff}}$, is obtained
schematically from the strain distribution of FRP reinforcement, and is calculated as the distance
required to achieve full FRP bond capacity along the concrete interface.

The effects of crack spacing will be discussed by considering the 280 and 100 mm crack spacing
simulations. The case with cracks spaced at 280 mm represents a case in which flexural cracks
are well spaced along a beam (large crack spacing). It can be seen from Figure 10 that the
occurrence of a new crack adjacent to Crack 2 affects the interfacial shear stress distribution
since there is a change in slip direction. Now there exists a negative slip between neighbouring
cracks resulting in a change in direction of interfacial shear stress in order to maintain
equilibrium. This was not the case for the single localized crack. The development of negative
slip can be explained by examining the behaviour along the interface between Cracks 1 and 2.
When the slip at Cracks 1 and 2 are small, i.e. prior to the initiation of macro-debonding, shear
stresses along the interface develop at the toe of the flexural cracks, but in opposite directions.
The stresses at the right of Crack 1 develop towards the support and stresses to the left of Crack 2
develop towards the applied load. As load increases and macro-debonding initiates, the
debonding crack propagating towards the midpoint from Crack 2 restricts the debonding crack
that intends to propagate towards the support from Crack 1. The point at where these two
debonding cracks meet is referred to as the point of zero slip (Liu et al. 2007). From Figure 10, it
can be seen that the point of zero slip between Cracks 3 and 4 shifts towards the subsequent
crack, Crack 4, as the slip at Crack 3 increases. This suggests that the uncracked concrete soffit is gradually slipping towards Crack 4 and hence reducing the slip on the left of Crack 4. The debonding crack that was developing from Crack 4 towards the applied load cannot propagate any further and is restricted by the debonding crack propagating towards the support from Crack 3, indicating that the debonding crack propagating from the left of Crack 4 has closed up. Now the debonding crack propagates in only one direction: towards the support. Since there are no cracks predefined between Crack 4 and the plate-end, the debonding crack propagates rapidly from Crack 4 to the plate-end once macro-debonding initiates causing debonding failure to occur.

Similar behaviour was observed between Cracks 2 and 3 but not between Cracks 1 and 2. This may be attributed to the fact that the region between Cracks 1 and 2 lie in a constant moment region, whereas the regions between Cracks 2 and 3 and Cracks 3 and 4 are located in a varying moment region. As shown in Figure 10, to maintain equilibrium the point of zero slip in this constant moment region was found to occur at the midpoint between Cracks 1 and 2, as opposed to the varying moment region where the point of zero slip moves towards the next crack as slip increases. It was found that the debonding crack in the constant moment regions will propagate in both directions and will not close up like those in varying moment regions prior to debonding failure. The effective shear transfer length was found to be 145 mm.

The case of cracks spaced at 100 mm along the interface is used to exemplify a case in which cracks are closely situated along the soffit (i.e. small crack spacing). The debonding propagation does not appear very smooth like that observed for the case of the single localized crack and large crack spaced model \((x_c = 280 \text{ mm})\). This may be attributed to the existence of many cracks along the interface spaced less than the effective shear transfer length. The effective shear transfer length for the single localized crack was found to be 140 mm and predefining the cracks
less than $Leff$ appears to be complicating the debonding behaviour. The debonding crack encounters resistance from the opposite direction near the adjacent cracks resulting in an increased $Leff$ of 215 mm as shown in Figure 11. $Leff$ can be more easily determined by referring to the FRP strain distribution along the interface, since the existence of very closely spaced cracks complicates the debonding propagation. Similar trends were found for other small crack spaced models of 125, 75, and 50 mm with $Leff$ values of 190, 245, and 330 mm, respectively. It appears as if more energy is required for the debonding crack to propagate through the flexural cracks in small crack spaced models, which contribute to the increase in $Leff$. Inspecting the deflection values in Table 1 suggests that the existence of multiple cracks spaced less than $Leff$ helps prolong the initiation of micro-debonding, rebar yielding, and macro-debonding.

The existence of a secondary crack and the spacing between adjacent cracks appears to have an effect on the initial of debonding and rebar yielding, which occurred at earlier stages in models with larger crack spacing (see Table 1). This may be attributed to the abrasion effect along the interface, where additional work is required for debonding to propagate beyond secondary cracks (Leung and Yang 2006). When a flexural crack opens under loading, longitudinal displacements at the bottom of the beam increase. Due to the abrasion effect, the residual shear stress at any point along the debonded zone decreases with interfacial relative sliding. As a result, the relative displacement in the debonded zone is reduced and the interfacial shear stress will increase. In another word, the presence of cracks was found to reduce the initiation of micro-debonding and rate of interfacial softening. Consequently, the maximum force in the FRP was found to increase with a decreased crack spacing, demonstrating the effectiveness of FRP rehabilitation in delaying debonding and crack propagation.
However, while some researchers have reported increases in ultimate load when crack spacing is reduced in their finite element models (Niu and Wu 2005), such a phenomenon was not found to occur in this study as the existence of more closely spaced cracks in the model greatly reduces the rigidity of the structure. Subsequent to the initiation of macro-debonding, the rate of debonding propagation was found to be increased in models with smaller crack spacing, evidenced by the lower deflection values obtained before debonding failure in Table 1.

3.3 Internal Reinforcement Response

Figure 12 compares the rebar strain distribution along the beam for the single localized crack model, and two (2) crack spacing of 125 and 50 mm at the initiation of macro-debonding and at debonding failure. The results suggest that prior to macro-debonding the decrease in crack spacing may be helpful to utilize the full strengthening effect of the internal reinforcing steel. However, this may not be the case for prolonged loading as the results suggest that the internal reinforcement becomes less effective in smaller crack spaced models subsequent to macro-debonding, as shown in Figure 12. This may be attributed to the fact that as the debonding crack propagates along the interface, multiple flexural cracks open under loading creating longitudinal displacements at the FRP-concrete interface. As the displacements increase, the flexural cracks migrate upward and reduce the bond action between the concrete and rebar, creating slippage at the concrete-rebar interface. In order to accommodate the load demand, the system now relies more on the FRP reinforcement. Consequently, the maximum force in the rebar was found to decrease with a decrease in crack spacing. This may also help to explain why lower ultimate load was found in models with smaller crack spaced models than for the single localized crack or large crack spaced models.
3.4 External (FRP) Reinforcement Response

Once yielding occurs in the rebar, FRP strain increases at a much higher rate until interfacial debonding occurs. For the case of the single localized crack and large crack space models, debonding propagation along the interface occurs easily and remains constant once macro-debonding initiates as shown in Figure 13. This may be explained by considering cracks spaced greater than $L_{eff}$ are less susceptible to the abrasion effect, as previously explained in Section 3.2, suggesting that the FRP sheet will produce similar strengthening results in cases where $x_c > L_{eff}$. The single localized crack and large crack spaced model ($x_c = 280$ mm) produce similar results following macro-debonding. However, with the case of small crack spacing, FRP strain was found to continue to increase (at a comparatively lower rate) following macro-debonding, suggesting that small crack spacing may be helpful to further utilize the strengthening effect of the FRP sheet. This phenomenon is shown in Figure 13 for the $x_c = 100$ mm model and may be attributed to the abrasion effect, which is more prominent in the models with crack spacing less than the $L_{eff}$. In these models the FRP sheet continues to contribute to the load-carrying capacity despite debonding at a particular location so long it is located within the effective shear transfer length of a nearby flexural crack. Inspecting the FRP strain values listed in Table 1 supports this observation as strain values are higher in smaller crack spaced models than larger spaced models at debonding failure. It is interesting to note how the optimum crack spacing to achieve the maximum plate strain is dependent on the crack spacing within the beam. As listed in Table 1 the FRP strain prior to failure is the greatest in the $x_c = 100$ mm model, followed by the $x_c = 75$, 125, and 50 mm models, respectively. Intuitively, one would expect the $x_c = 50$ mm to have the maximum FRP strain value at failure since it has the largest $L_{eff}$ of the group. However, the existence of very closely spaced cracks in the model quickens the debonding propagation and
fails the structure earlier, thus not allowing the FRP sheet to be fully utilized for the \( x_c = 50 \) mm model. Moreover, Table 1 lists that the \( x_c = 50 \) mm model was only able to deflect an additional 8.5 mm after macro-debonding initiates prior to debonding failure, in comparison to 13.7 mm and 18.9 mm for the \( x_c = 100 \) mm and single crack spaced model, respectively. This suggests that crack spacing and the amount of cracks along the beam influence the local stress and strain in a beam. Furthermore, the \( x_c = 100 \) mm model had the largest FRP strain increase after the initiation of macro-debonding followed by the \( x_c = 75 \), 50 and 125 mm models as listed in Table 1.

### 4 Conclusion

In this study, a detailed finite element model was developed to carry out a comprehensive finite element investigation that provides useful information related to the mechanics of flexural IC debonding in the FRP strengthened RC members. Numerical results presented in this paper indicate that:

- Cracks spaced greater than the \( L_{eff} \) produce similar ultimate load and FRP strain results to the single localized crack model.

- Subsequent to the initiation of macro-debonding, the interface cracks spread towards the end of the FRP more rapidly in models with smaller crack spacing. Ultimate capacity, therefore, was reached earlier in models with more closely spaced cracks.

- The shear stress is both additive and subtractive on either side of each flexural crack due to equilibrium of forces. The value will be a maximum in high moment regions where the crack opening displacement has the greatest magnitude.
• To maintain equilibrium in the constant moment region, a point of zero slip always occurs at the mid-distance between two cracks and the debonding crack propagates in both directions and will not close up easily like those in varying moment regions.

• The existence of the multiple cracks prolongs the initiation of micro-debonding, rebar yielding, and macro-debonding. This may be attributed to the abrasion effect along the interface.

• The internal reinforcement (rebar) becomes less effective in smaller crack spaced models subsequent to the initiation of macro-debonding and the beam relies more on external FRP reinforcement to contribute to its load carrying capacity.
References


List of Figure Captions

Fig 1. Strain hardening behaviour of concrete: (a) compression; (b) tension.

Fig. 2. Stress-relative displacement relationship of discrete cracking.

Fig. 3. Bond-behaviour of FRP-concrete interface.

Fig. 4: Global calibration of numerical model: (a) control beam; (b) CFRP Beam (Perfect Bond); (c) CFRP Beam (CZM).

Fig. 5: Local calibration of numerical model: (a) concrete compressive response; (b) axial strain in steel rebar; (c) axial strain in CFRP laminate.

Fig. 6: Schematic representation: (a) analysis model; (b) finite element model.

Fig. 7: Effect of mesh refinement for a single localized crack in terms of: (a) load versus deflection; (b) interfacial shear stress under the loading point.

Fig. 8. Load versus deflection for single and large crack spacing.

Fig. 9. Interfacial stress distribution for the case of a single localized crack.

Fig. 10. Interfacial stress distribution for the crack spacing $x_c = 280$mm.

Fig. 11. Interfacial stress distribution for the crack spacing $x_c = 100$mm.

Fig. 12. Comparison of rebar strain distribution: (a) initiation of macro-debonding; (b) debonding failure.

Fig. 13. Comparison of FRP reinforcement strain distribution: (a) single localized crack; (b) $x_c = 100$ mm.
565  **List of Table Captions**

566

567  Table 1: Summary of effect of crack spacing analysis.
Fig. 1: Strain hardening behaviour of concrete: (a) compression; (b) tension
Fig. 2: Stress-relative displacement relationship of discrete cracking
Fig. 3: Bond-behaviour of FRP-concrete interface
Fig. 4: Global calibration of numerical model: (a) control beam; (b) CFRP Beam (Perfect Bond); (c) CFRP Beam (CZM)
(a) Concrete response under compression (cylinder test)

(b) Axial strain in steel rebar

(c) Axial strain in CFRP laminate

**Fig. 5:** Local calibration of numerical model: (a) concrete compressive response; (b) axial strain in steel rebar; (c) axial strain in CFRP laminate
Fig. 6: Schematic representation: (a) analysis model; (b) finite element model
Fig. 7: Effect of mesh refinement for a single localized crack in terms of: (a) load versus deflection; (b) interfacial shear stress under the loading point.
Fig. 8: Load versus deflection for single and large crack spacing.
Fig. 9: Interfacial stress distribution for the case of a single localized crack.
Fig. 10: Interfacial stress distribution for the crack spacing $x_c = 280$ mm.
Fig. 11: Interfacial stress distribution for the crack spacing $x_c = 100$ mm.
**Fig. 12:** Comparison of rebar strain distribution: (a) initiation of macro-debonding; (b) debonding failure.
Fig. 13: Comparison of FRP reinforcement strain distribution: (a) single localized crack; (b) $x_c = 100$ mm.
Table 1: Summary of effect of crack spacing analysis.

<table>
<thead>
<tr>
<th>$x_c$</th>
<th>Init. of Micro-debonding</th>
<th>Yield load</th>
<th>Init. of Macro-debonding</th>
<th>Ultimate load</th>
<th>FRP Strain (Micro strain)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta$ mm</td>
<td>$P$ kN</td>
<td>$\Delta$ mm</td>
<td>$P$ kN</td>
<td>$\Delta$ mm</td>
</tr>
<tr>
<td>mm</td>
<td></td>
<td></td>
<td>mm</td>
<td>kN</td>
<td>mm</td>
</tr>
<tr>
<td>SC</td>
<td>0.4</td>
<td>2.4</td>
<td>176.7</td>
<td>6.9</td>
<td>184.9</td>
</tr>
<tr>
<td>280</td>
<td>0.8</td>
<td>2.7</td>
<td>165.7</td>
<td>7.1</td>
<td>178.9</td>
</tr>
<tr>
<td>125</td>
<td>4.0</td>
<td>3.4</td>
<td>163.8</td>
<td>7.8</td>
<td>178</td>
</tr>
<tr>
<td>100</td>
<td>10.7</td>
<td>4.5</td>
<td>168.5</td>
<td>8.1</td>
<td>179.8</td>
</tr>
<tr>
<td>75</td>
<td>126.8</td>
<td>5.7</td>
<td>167</td>
<td>8.5</td>
<td>178.2</td>
</tr>
<tr>
<td>50</td>
<td>121.7</td>
<td>6.1</td>
<td>164.2</td>
<td>8.9</td>
<td>178.2</td>
</tr>
</tbody>
</table>

*Percent difference between macro and ultimate strain.