Framework to assess the pseudo-static approach for the seismic stability of clayey slopes

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Framework to assess the pseudo-static approach for the seismic stability of clayey slopes

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Abstract
Approaches commonly used to assess the seismic stability of slopes range from the relatively simple pseudo-static method to more complicated nonlinear numerical methods, e.g. finite element (FE) and finite difference (FD). The pseudo-static method, in particular, is widely used in practice as it is inexpensive and substantially less time consuming compared to the much more rigorous numerical methods. However, the pseudo-static method is widely criticized since it ignores the effects of the earthquake on the shear strength of the slope material and the seismic response of the slope. Hence, some researchers recommend its use only in slopes composed of cohesive materials that do not develop significant pore pressures or that lose less than about 15% of their peak shear strength during earthquake shaking. However, the use of the pseudo-static method in these soils is also problematic as clayey slopes generally fail in pseudo-static stability analyses (i.e., factors of safety is less than 1) and the failure surface is completely predominated by the thickness of the clayey layer in the slope or foundation. The reliability of the pseudo-static method in natural clayey slopes is examined here based on rigorous numerical simulations with FLAC. The numerical results are compared and verified using available static and dynamic 1-g laboratory tests. This article then addresses some of the crude assumptions of the pseudo-static method and provides practical suggestions to be applied to refine the outcomes of pseudo-static analyses not only in terms of the computed safety factors but also in the prediction of the failure surface through the consideration of additional aspects of the dynamic responses of the clayey slopes.

Keywords: pseudo-static; clay slopes; factor of safety; finite differences; failure surface.

Résumé
Les approches couramment utilisées pour évaluer la stabilité sismique des pentes vont de la méthode pseudo-statique, relativement simple, à des méthodes numériques non linéaires plus compliquées, par ex. élément fini (FE) et différence finie (FD). La méthode pseudo-statique, en particulier, est largement utilisée dans la pratique car elle est simple et demande beaucoup moins de temps de calcul que les méthodes numériques beaucoup plus rigoureuses. Cependant, la méthode pseudo-statique est largement critiquée car elle ignore les effets du séisme sur la résistance au cisaillement du matériau de la pente ainsi que sa réponse dynamique. Par conséquent, certains chercheurs recommandent son utilisation uniquement dans les pentes composées de matériaux cohésifs qui ne développent pas de pressions interstitielles significatives ou qui perdent moins qu'environ 15% de leur résistance au cisaillement lors d'un tremblement de terre. Cependant, l'utilisation de la méthode pseudo-statique dans ces sols est également problématique car les pentes argileuses ruptures généralement dans les analyses de stabilité pseudo-statique (ie, les facteurs de sécurité sont inférieurs à 1) et la surface de rupture est complètement dominée par l'épaisseur des couches argileuse dans la pente ou la fondation. La fiabilité de la méthode pseudo-statique dans les pentes argileuses naturelles est étudiée dans cet article à l'aide de simulations numériques rigoureuses avec FLAC. Les résultats numériques sont comparés et vérifiés à l'aide d'essais de laboratoire statiques et dynamiques en 1 g disponibles dans la littérature. Cet article aborde ensuite certaines hypothèses grossières de la méthode pseudo-statique et fournit des suggestions pratiques à appliquer pour raffiner les résultats des analyses pseudo-statiques non seulement en termes de facteurs de sécurité calculés, mais aussi dans la prédiction de la surface de rupture à travers la prise en compte d'aspects supplémentaires des réponses dynamiques des pentes argileuses.

Keywords: pseudo-statique; pentes argileuses; coefficient de sécurité; différences finies; surface de rupture.
Introduction

The states of natural and engineered slopes vary from marginally stable to very stable with respect to failure depending on their material, geometric, geologic characteristics as well as the geotechnical conditions. Earthquake-induced ground shaking creates alternating inertia forces within the slope and, possibly, significant reduction of shear strength and stiffness of the slope material (Bray 2007; Kramer 1996). Depending on the predominance of these effects, seismic slope instabilities can be categorized as “inertial”, “weakening” or combined “inertial-weakening” instabilities (Kramer 1996; Biondi et al. 2002). In fact, a reliable evaluation of the seismic stability of slopes, considering these effects, is very important since several slope stability failures have caused by moderate to large magnitude earthquakes. For example, the 1920 Haiyuan, the 1964 Alaska, the 1988 Saguenay, and the 2011 Tohoku Pacific earthquakes caused slope failures that resulted in extensive damage to lifeline systems and transportation networks (Youd 1978; Tiwari et al. 2013).

Procedures commonly used to assess the stability of slopes during earthquakes range from the relatively simple pseudo-static method (Terzaghi 1950) to much more sophisticated nonlinear numerical methods, such as finite element (FE) or finite differences (FD) simulations (Mizuno and Chen 1982; Daddaziao et al. 1987). Since the mid-1960s, most slopes have been analyzed using the pseudo-static method, an extension of static slope stability analysis in which the effects of an earthquake are approximated by a spatially constant horizontal acceleration applied to the geometric model. The stability of the slope is then evaluated using the method of slices and limit equilibrium to determine a critical failure surface and a factor of security. The assumption of a constant horizontal acceleration within the model is a crude approximation especially in slopes with relatively great heights as both field and laboratory observations have demonstrated that the seismic response (i.e. the
applied horizontal accelerations) varies with height (Duncan et al. 2014; Hynes-Giffin and Franklin 1984). The method also assumes that the shear strength of the soil within a slope remains essentially constant and the slope deformations are caused only by the inertial forces induced by earthquake shaking (it does not account for the potential reduction of shear strength and stiffness of the slope material due to the development of shear strains during earthquake shaking). This would be relatively true in cases where there is no significant shear strength loss during seismic shaking, such as in the case of cohesive soils that do not develop appreciable excess pore pressures or that lose less than about 15% of their strength during earthquake shaking. Duncan et al. (2014) indicated that a 15% reduction in clay strength during shaking can be compensated for by the expected increase in strength (between 20 to 50%) during undrained loading at the rates imposed by earthquakes. However, the use of the pseudo-static method in these situations is also problematic as the method typically indicates failure for clayey slopes (i.e. produces factors of safety less than 1) regardless of the actual stability of the slope and the resulting failure surface is determined by the thickness of the clayey layer within the slope of foundation, which may not be the case. Moreover, the results of the pseudo-static analyses are highly dependent on the selected value of the pseudo-seismic coefficient, $k_h$, which is used to apply a constant horizontal acceleration to the model. As noted, the representation of earthquake effects by constant acceleration is a crude approximation and conservative: it assumes that the resulting earthquake-induced forces are constant and act only in the direction of instability (Jibson 2011).

Rigorous FE or FD numerical analyses using advanced constitutive soil models can provide direct and salient evaluations of seismic slope stability since they can account for the interrelated effects of both inertial forces and weakening during shaking. However, they are much more complex and require significant computational resources, and consequently are rarely conducted in engineering practice. As a result, force-based (pseudo-static) and
displacement-based (Newmark-type or sliding block; Newmark 1965) methods, despite their considerable deficiencies, remain in widespread use in engineering practice and research. Nevertheless, the pseudo-static method could be transformed into a reliable tool for the evaluation of the seismic stability of clayey slopes provided it can be properly calibrated using adequate case histories and physical models and can be shown to be in agreement with rigorous numerical simulations for a wide range of earthquake characteristics and ground conditions. In structural engineering, properly calibrated simplified procedures are an essential feature of seismic structural design and are the basis of many code regulations. However, in geotechnical engineering, simplified procedures such as the pseudo-static method have not been refined to a level such that they can be applied to clayey slopes with adequate confidence. They, therefore, should be continually evaluated, validated, and improved with case histories, small to large-scale physical models and parametric numerical simulations. The attractive features of the pseudo-static method, in particular its basis in the limit equilibrium method, routinely used in geotechnical engineering, make it worth the effort necessary to improve it. In an attempt of doing so, this article closely examines the reliability of the pseudo-static method in natural clayey slopes and addresses some of the crude assumptions in its implementation based on a rigorous finite difference modeling using the two-dimensional computer code FLAC (Itasca 2007). This article then provides some useful recommendations for the implementation of the pseudo-static method in a manner that considers the seismic response of the slope in the determination of the critical factor of safety and location of the corresponding failure surface. Special concern has been devoted in this article to validate the adopted numerical model using available static and dynamic 1-g laboratory physical models.

**Numerical simulations**
The two-dimensional explicit finite difference program FLAC (Itasca 2007) was employed to evaluate the static and dynamic performance of two homogeneous, 10-m-high clay slopes with inclinations of 1.75H:1V and 3H:1V, underlain by 20-meter-thick homogeneous clay foundation and then by a rigid bedrock. The slopes were analyzed statically, pseudo-statically, and dynamically. Figure 1a shows the basic characteristics (i.e., dimension, boundaries, and meshing) of the slopes. Quiet lateral boundaries were set away from the region of interest (near the slope) so that reflected artificial waves were sufficiently damped and their influence in the slope response was minimized. To determine an adequate width for the model, preliminary seismic analyses using Synthetic 1 and 2 input motions (Atkinson 2009) were carried out on slope models with widths ranging from 70 m to 960 m. The results of these analyses are portrayed in Figs. 1b and 1c. In particular, Fig. 1b shows the variation of the difference between a reference displacement and the maximum horizontal displacement of the slope toe versus the model width normalized by the slope height (10 m). The reference displacement is defined here as the maximum horizontal displacement of the slope toe in the widest model considered (i.e., mesh width equal to 960 m). The analyses were conducted for different undrained shear strengths (reduction factors, RF, ranging from 1.0 to 1.4 were considered). The corresponding maximum horizontal displacements of the slope toe are plotted against the RF in Fig. 1c for different model widths. The displacement-RF curve from static slope stability is plotted as a reference. Fig. 1b indicates that the change in the horizontal displacement of the slope toe becomes insignificant a model width of 320 m, and Fig. 1c confirms that adopting a width of 320 m produces a displacement-RF curve identical to that of 960-m wide model. Quadrilateral soil elements were used since they are less prone to strain concentration and the mesh extended a horizontal distance of 150 m from both the toe and the crest of the slope as shown in Fig. 1a.
The simulations were performed in two stages. In the first stage the slope and foundation soils were represented by the elastic constitutive model implemented in FLAC and in situ stresses were developed in the model due to gravity. Following this phase, the strains and displacements within the model were reset to zero. In the second stage, the soils were represented using the Mohr-Coulomb model as implemented in FLAC. In this stage, the model was charged by external static or dynamic (an earthquake ground motion) loads. For the first phase and the static second phase simulations, the mechanical boundary conditions consisted of horizontal fixity on both sides of the model and horizontal and vertical fixity on the bottom of the model. In the second phase dynamic simulations, quiet boundary conditions were applied on the sides to avoid spurious wave reflection and thus simulate the effect of an infinite elastic medium. The height of the elements affects the transmission of high frequency shear waves. For this reason, the height of the elements was limited to a maximum height, \( h \), calculated with (Matthees and Magiera 1982):

\[
    h \leq \frac{1}{5} \left( \frac{V_s'}{f_{\text{max}}} \right)
\]

where \( f_{\text{max}} \) is the highest excitation frequency used in the analyses, \( f_{\text{max}} \) of 10 Hz and \( V_s' \) is the shear wave velocity expected after the degradation of the shear modulus, \( G \), due to the developed shear strain and can be related to the initial shear wave velocity \( V_s \) by:

\[
    \left( \frac{G}{G_{\text{max}}} \right) = \left( \frac{V_s'}{V_s} \right)^2 = \frac{1}{10}
\]

where 1/10 is a typical reduction factor of the shear modulus in a range of 1–10% on the strain dependent shear modulus curve.

The slope and the foundation were sub-divided into 1-m-thick sub-layers with constant properties assigned to each sub-layer. The parameters of the Mohr-Coulomb model are the density, \( \rho \), the cohesion, \( c \), the friction angle, \( \phi \), the elastic modulus, \( E \), and the shear wave velocity, \( V_s' \).
Poisson's ratio, \( \nu \). A Poisson's ratio of 0.45 was used assuming undrained loading. Accordingly, a friction angle of 0 was used and the cohesion represented the undrained shear strength, \( S_u \). The \( S_u \) of the slopes was assumed to be 25 kPa at the surface (top of the slope) and to increase at a rate of 1.5 kPa/m with depth. This relatively low-rate increase of \( S_u \) with depth was expected to induce deeper failure surfaces and corresponds to an unfavorable situation. The maximum shear modulus, \( G_{\text{max}} \) of the soil were evaluated according to the value of the undrained shear strength, \( S_u \) (or \( C_u \)) following the correlations suggested by Locat and Beauséjour (1987):

\[
G_{\text{max}} = 0.379 C_u^{1.05} \tag{3}
\]

The shear wave velocity, \( V_s \) of the soil was determined based on \( G_{\text{max}} \) from the elastic relationship between the \( G_{\text{max}} \) and \( V_s \); \( G_{\text{max}} = \rho V_s^2 \), where \( \rho \) is the soil density. Input parameters of the Mohr-Coulomb model used in this study are listed in Table 1. The parameters of the elastic model that was used in the first phase of the simulations are the elastic modulus and the Poisson's ratio. The values used were the same as those of the second phase.

In the dynamic simulations, the shear modulus of the soils were degraded using the constitutive model SIG4 (Itasca 2007) capped by the Mohr-Coulomb failure criteria following the experimental curves suggested by Vucetic and Dobry (1991) for a plasticity index, PI, of 30. The fitted parameters of the SIG4 model are listed in Table 1. Additionally, a Rayleigh damping ratio of 0.0015 was used to ensure stability of the numerical solution process at low strain levels.

**Validation of the numerical method based on physical modelling**
Prior to the main part of this study, the validity of the numerical method was verified by simulation of the monotonic (static) and dynamic responses of clayey slopes from the 1-g physical model testing conducted by Ozkahriman (2009). The physical tests were performed in a $2.03 \times 1.22 \times 0.60$ m rigid Plexiglass box bolted to a shaking table. Three clayey slope models (static models S1, S2, and S3) were constructed and then tested to study the stability of slope under static loading conditions. Three other models (dynamic models D1, D2, and D3) were built and then shaken to investigate the stability of the slope under dynamic loading conditions. These models typically consisted of two soil zones: an upper soft clay layer underlain by a stiffer clay layer that was in contact with the front and rear walls of the box. The geometries of the slope models are given in Table 2. The largest models (S1 and D1) of the two sets of experiments were considered as the “prototype” earth structures. For the two smaller models, geometry, strength, low-strain properties (shear modulus), and frequency content of the input motions were adjusted by applying the laws of similitude (Iai, 1989) to reflect their reduced scales. The selected scaling factors ($\lambda$) for smallest and middle model were 2.5 and 1.43, respectively. Assuming a geometric scaling factor ($\lambda$) of 22, the models (S1 and D1) can be said to represent a $45^\circ$, 12 m height of soft clay embankment having a constraint undrained shear strength of 75 kPa and shear wave velocity of 45 m/sec. The Model S0 in Table 2 was a duplicate model of S1 to evaluate the repeatability of experimentations. Detailed geometries and the instrumentation of models S1 and D1 are respectively shown in Fig. 2a and Fig. 2b. The dynamic responses of the model were measured at different depths using capacitive spring mass-based miniature accelerometers (i.e. $A_1$ to $A_7$). Surface displacements were measured using either 50-mm or 100-mm-range linear motion potentiometers (i.e. $P_1$ to $P_7$) with a linearity of less than +/- 1% and a resolution that is reported as “essentially infinite”. Detailed descriptions of the instrumentation can be found in (Ozkahriman 2009).
The models were constructed using a kaolinite-bentonite “model clay” (kaolinite to bentonite ratio of 1:3) originally developed by Seed and Clough (1963) for use at the University of California, Berkeley in physical modeling to simulate the stress-strain behavior of the San Francisco Bay Mud characterized by a low rate of consolidation and a $V_s$ of 120-180 m/sec. Another important characteristic of the mixture is that its undrained shear strength is determined by its water content (Wartman and Riemer 2002), thus the undrained strength can be controlled. The clay has liquid (LL) and plastic (PL) limits of 120 and 25, respectively, and a plasticity index (PI) of 95 (Ozkahriman 2009).

Prior to model construction, the geometry of the slope and the instrumentation layout were marked on the Plexiglas walls to provide guidance. The stiff base layer was placed first, followed by the upper layer of soft clay, and the accelerometers and potentiometers were placed in the models at the predetermined locations shown in Fig. 2. Each handful of clay was carefully "worked" into the layer below to ensure uniformity and prevent formation of any construction-influenced preferential shear surfaces. Once model construction was completed, surface elevations were measured, and they were found to be essentially uniform across the width of the model slope typically varying by no more than several millimeters (Ozkahriman 2009).

As shown in Fig. 2a, the static models were subjected to a surface loads applied by means of a 2.5-cm-thick Plexiglas footing on the crest of the slope. A load of a maximum magnitude of 900 N was gradually applied using a hand crank at a constant rate of 360 N/min. The dynamic models were subjected to a suite of ground motions including synthetic (frequency sweeps and sine waves) and recorded earthquake motions with varied frequency content, duration, and amplitude. The frequency sweep motions consisted of a four-second ramping windows at the beginning and at the end of the full amplitude motion. The input frequency linearly increased from 5.4 Hz to 12.5 Hz during the 20-second duration of the motion. The
models were shaken with low amplitude (0.03g < PGA < 0.08g) sine sweeps to provide
information on the dynamic characteristics of the models before moderate-to-high amplitude
sine pulses and the recorded 1989 Loma Prieta earthquake motion at Redwood City (RWC)
seismograph station (details are tabulated Table 3) have been applied. These later motions
induced permanent deformation in the models (Ozkahriman 2009).

The responses of the S1 and D1 models were simulated numerically using the numerical
model presented in the previous section. The grid was built sequentially to model the actual
construction sequence of the physical models. The soil was modeled as an elastic material
during the construction process to prevent any localized construction-induced rupture zones.
After the simulation of staged of construction, the strains and displacements of the model
were reset to zero and the Mohr-Coulomb model was assigned to the soil. The sigmoidal
(SIG4) model was adopted to fit the shear modulus degradation curves of the clay during
dynamic loading based on the experimental curves suggested by Vucetic and Dobry (1991)
for a PI of 100%. The parameters used to numerically simulate the physical models are given
in Table 4. The boundary conditions (e.g. mechanical fixity) and static and dynamic load
application in the numerical simulations were the same as those used in the physical models.

The post-failure localized shear surfaces at the middle sections of physical models S0 and
S1 are plotted on the corresponding numerically simulated deformed mesh in Fig. 3a. In Fig.
3b, the failure surfaces of physical models S2 and S3 were respectively scaled by 1.43 and
2.5 times (i.e., geometric scaling factor, λ, in Table 2) relative to that of S1 model. Figs. 3a
and 3b show that the observed post-test profiles are generally similar for all tested slope
models (S0, S1, S2, and S3) though the behavior near the toe of S0 and S1 models differ
somewhat from that of the other models (S2 and S3) in which the rotational shear surfaces
pass through the toes of the slopes. Furthermore, Fig. 3a shows that the internal shear surface
of the model S0 is somewhat shallower than the model S1, which can be interpreted through
a close examination of the vertical deformation at the top of slope models S1 and S0 beneath the Plexiglas plate. As shown in Fig. 3a, the loaded plate in the first case S1 has a relatively non-uniform vertical deformation, as the inner vertical settlement is significantly greater than the outer one. It seems that the load control of the Plexiglas plate is improved when the test was repeated (the second model S0) resulting in a more uniform foundation settlement. In other words, the unbalanced vertical deformation of the loaded plate generated in the model S1 resulted in a deeper localized shear surface, rationalizing the differences in the internal shear surfaces presented in Fig. 3a. To cope with these uncertainties in the control of the loaded plate translation and rocking deformations, two cases of numerical simulation have been considered. In the first case (Case 1), the lateral displacement of the loaded plate was permitted and in the second case (Case 2), the lateral motion of the plate was prevented. The computed failure surface of the first case was found to fall within the upper and lower failure surfaces observed in S0 and S1 physical tests shown in Fig. 3a, while that of the second case coincides with the observed post-test profiles of slope models S2 and S3 presented in Fig. 3b.

Measured and computed load-deformation responses of the Plexiglas plate at P6 in model S1 are presented in Fig. 4a. The observed load-deformation curves of other tests (S2 and S3) are also presented in Fig. 4a for comparison. The curves presented in Fig. 4a reflect the real and the simulated strain-softening nature of the model clay. Although the current numerical model doesn’t completely simulate the gradual reduction in the clay strength after failure (i.e., after the peak resistance) as it utilized the elastic-perfectly plastic Mohr-Coulomb model, the computed pre-failure load-deformation responses are generally in good agreement with the measured curves. The measured and the computed horizontal displacements at potentiometers P3 and P5 are respectively plotted as functions of the vertical displacement at P6 in Figs 4b and 4c. The numerical prediction of the horizontal displacement when the plate is not allowed to displace horizontally (case 2) is generally greater than the experimental
responses at P3 (Fig. 4b) but it is very close to the response of the experimental model S1 (Fig. 4c). When the horizontal displacement of the plate is allowed (Case 1), the numerical results are significantly improved and become closer to the experimental results. Even if the numerical prediction generally overestimates the data at the lower vertical displacements and provides an underestimation at greater vertical displacements, they can be viewed as an average estimation of the experimental results.

It should be noted that the observed localization of the shear surface shown in Fig. 3a and the load-deformation curves in Fig. 4a are more similar to a bearing capacity problem (for a shallow foundation near a slope) than that to a pure slope stability problem reflecting the response of the experimental model to the type of the load applied. Simulating these experimental static results, however, constitutes one of the first and important stages of the validation process of the numerical procedure adopted in this study and serves to ensure the applicability of the numerical procedure to predict soil deformations under static loading before going further and examining its pertinence in modeling the more complicated dynamic slope stability problem.

The dynamic response of Model D1 during shaking table test subjected to the sine pulse shown in Table 3 was also simulated numerically. The recorded time history at an accelerometer attached to shaking table, the input motion used, was baseline corrected before using it in the numerical analysis. Figure 5a shows the spectral accelerations used in the physical modeling and numerical simulations. Figure 5b presents the measured and the computed responses of accelerometers A4 and A5 shown in Fig. 2b. Figure 5b shows that the FLAC simulation of the dynamic response of the slope model is in good agreement with the shaking table measurements, and the shape of the predicted spectral acceleration curves are very similar to the measured curves.

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Figures 6a and 6b, respectively, present comparisons between the measured and the computed spectral accelerations of the accelerometers A4 and A5 during a ground motion equivalent to that recorded at the Redwood City location during the 1989 Loma Prieta earthquake. Two different numerical simulations with and without pre-shaking (using the low amplitude motions mentioned above) were performed and the results of both analyses are presented in Fig. 6. Figure 6 shows that the numerical spectral accelerations at both locations (A4 and A5) match well the measurements from the physical models, specifically, the shapes of the simulated spectral acceleration curves are in general in good agreement with the measured curves from physical modeling. The measured and the computed overall response during dynamic tests are further represented in Fig. 7 in terms of peak ground accelerations (PGAs) at the surface as functions of the maximum input accelerations. Figure 7 indicates that similar trends can be seen between computed and measured amplification ratios and thus numerical simulations produce PGAs that match well those measured in the shaking table tests. Also, the computed and the measured post-test profiles and failure surfaces due to the Redwood city ground motion are presented in Fig. 8, and these surfaces are generally similar. Figure 8 shows also that the deformations are distributed along the height of the model and localized along a single failure surface indicated by deep rotational and translational displacements passing through toe of the slopes which is well predicted in simulation by the localization of shear strain close to the base (toe) of soft clay layers and at the interface between the soft and stiff clay layers.

The comparative results presented in Figs. 3 to 8 indicates that the adopted numerical model could be used to simulate the static and dynamic responses of clayey slopes such as those presented here.
**Results of static, pseudo-static and dynamic analyses**

The basic condition for static or dynamic stability of slopes is that the mobilized shear resistance (capacity) must be greater than the applied shear force (demand). A slope could be brought to the brink of instability either by a reduction of the shear resistance of the soil on some potential failure surface or by an increase in the imposed static and dynamic loads. In this study, a unified framework has been adopted to access the stability of slopes under static and dynamic loadings. This approach is used in all the static, pseudo-static and dynamic analyses and consists of a determination of a reduction factor applied to the resistance ($S_u$) of the cohesive soil that leads to the slope failure (Karray et al. 2001). In other words, the main component of the framework is a systematic search for the value of the reduction factor (i.e., the factor of safety) that will cause the slope to fail. For a given static, pseudo-static, or dynamic analysis, the calculations were repeated several times with different values of soil shear strength and the relative horizontal displacement between two arbitrary points (typically, at the slope toe and the corresponding point at the bedrock) was noted and plotted as a function of the applied strength reduction factor. The equivalent factor of safety can be then determined from this plot as the reduction factor corresponding to the general plastification (i.e., the significant and sudden increase in the relative horizontal displacement). Moreover, the formation of the failure surface is also investigated during the numerical simulations for the shear strength reduction procedure, as the factor of safety is known to be directly linked to the development of the failure surface.

The static numerical simulations of the clayey slopes considered in this study were performed using the undrained shear strength parameters and the above procedure was used to obtain the factors of safety and the corresponding failure surfaces. There was no initial assumption concerning the shape and location of the failure surface. It is noted that the static stability of a slope doesn’t ensure its stability under dynamic conditions. However, the static
analyses were carried out in this study to: (i) assess the slope stability under static conditions; (ii) compare these results with the subsequent pseudo-static and dynamic analyses; and (iii) evaluate the procedure adopted for estimating the safety factors. The height-normalized relative displacement (an approximation of the soil shear strain, γ%)-reduction factor curves of the static analyses are presented in Fig. 9a, while the corresponding failure surfaces are plotted in Figs. 9b and 9c for the 1.75H:1V and 3H:1V slopes, respectively. Figure 9a shows that the safety factors obtained for the 1.75H:1V and 3H:1V slopes, respectively, are 1.48 and 1.76. These values agree well with those estimated using well-known limit equilibrium methods such as Bishop’s modified (Bishop 1955), Spencer (Spencer 1967), Morgenstern-Price (Morgenstern, and Price 1965), and Fredlund-Krahn (GLE) (Fredlund and Krahn 1977) as presented in Table 5. Figures 9b and 9c, respectively portray the corresponding yielded zones obtained from FLAC analyses for 1.75H:1V and 3H:1V slopes. The slip surfaces determined from the limit equilibrium analyses are also reported on Figs. 9b and 9c, and they generally fall within or close to the plastified zones determined by the current FD model.

The pseudo-static analyses were conducted using the same conditions as the static analyses and the corresponding reduction factors-normalized displacement curves are plotted in Fig. 10a. A pseudo-static coefficient of 0.15 was used to calculate the equivalent horizontal inertial forces. Because the prime objective of this work is to assess the seismic stability of clayey slopes in the Québec city region, the value of the pseudo-static coefficient (0.15) was selected following the recommendation of the Centre d’expertise hydrique du Québec (Centre d’expertise hydrique du Québec 2013). This value corresponds to about half of the maximum acceleration on rock for this region. Figure 10a shows that the safety factors obtained for the 1.75H:1V and 3H:1V slopes are almost the same of 0.95 and they are successfully compared to those estimated using typical limit equilibrium methods (Bishop 1955; Fredlund and Krahn 1977) as presented in Table 5. Figures 10b and 10c respectively portray the corresponding
yelled zones obtained from FLAC analyses for the 1.75H:1V and 3H:1V slopes. The slip surfaces determined from typical static limit equilibrium analyses are also reported in Figs. 10b and 10c. In both cases, the failure surface obtained from the pseudo-static analysis is very deep and passes close to bedrock. There is a significant difference between these surfaces and those estimated from the static limit equilibrium analyses.

In the same manner, the dynamic analyses were conducted using Synthetic 1 and 2 input motions (Atkinson 2009) compatible with the seismicity of the region of Quebec city, which are respectively shown in Fig. 11b and 11c. They were selected because the resulting response spectra (Fig. 11a) are compatible with that given by the CNBC 2005, 2010, and 2015 for Quebec and Montreal for site class A sites (hard rock). Before discussing the results of the seismic analyses, recall that the seismic analysis of slopes is rather complex and requires consideration of the effect of the dynamic stresses induced by the earthquake (inertial effects) as well as the potential for strength loss within the soil (weakening effects). In the former, the shear strength remains relatively constant, but strains develop due to temporary exceedance of the capacity. In the latter, the earthquake may induce soil strength loss, reducing the capacity, and strains develop due to exceedance of the reduced capacity. In the current analyses, strength loss was considered. However, the analyses were conducted considering typical degradation of the shear modulus and an increase in damping with increasing shear strain. This assumption is probably acceptable in clayey slopes that do not develop significant dynamic pore pressures or lose more than about 15% of their peak shear strength during earthquake shaking as reported by Kramer (1996) or can be inferred from the results presented by Matasovic and Vucetic (1995); and Vucetic and Dobry (1988). This assumption is compatible with the pseudo-static method implemented using limit equilibrium.
The computed normalized displacement-reduction factor curves of both 1.75H:1V and 3H:1V slopes under Synthetic 1 and 2 input motions (Atkinson, 20) are plotted in Fig. 12a. The computed factors of safety of the 1.75H:1V under Synthetic 1 and 2 input motions are 1.17 and 1.25, respectively. The corresponding factors of safety of the 3H:1V are 1.35 and 1.45. It can be noticed from Fig. 12a that the change in the displacement-reduction factor curve is relatively smooth compared to the static and the pseudo-static curves. Thus, the factor of safety is determined in this case by constructing two tangents: one to the first straight segment of the curve and the other one to the lower straight segment. A bisector is then drawn intersecting the curve at a point that corresponds to where the failure would occur. It should be also noticed that these safety factors correspond to a normalized relative displacement (average shear strain) of 0.2. Figures 12b and 12c, respectively, portray the corresponding yielded zones obtained from FLAC analyses for the 1.75H:1V and 3H:1V slopes. The slip surfaces determined from the static limit equilibrium analyses are also shown on Figs. 12b and 12c for reference. Figures 12b and 12c indicate there are two potential failure surfaces due to the dynamic loading. The shallower one rather resembles the slip circle obtained from the static analysis, and it is very different from the surfaces predicted by the pseudo-static approach. The deeper failure surface is generated as expected at the interface between clay and bedrock. Similar failure surfaces have been observed in the physical test results as presented in Fig. 8. In the case of the 3H:1V slope, the dynamic failure surface is a bit wider than in the 1.75H:1V slope.

It should be mentioned here that both the clayey slopes considered have been shown to be stable in static and the dynamic analyses (factors of safety greater than 1) but failed in the pseudo-static analyses that produced, moreover, unrealistic slip surfaces.

The dynamic numerical analyses conducted in this study could lead to some valuable information with respect to the fundamental behavior of clayey slopes during earthquakes and
would enrich the current understanding of common assumptions used in the evaluation of such slopes by the pseudo-static method. For example, Figs. 13a-d depict the shear stress-shear strain response of the soil at different locations within the slope during the dynamic analyses of both cases considered. Specifically, Figs. 13a and 13b respectively show the soil stress-strain loops of a point close to the computed yield zone (Fig. 12b) at a 13-m depth and a 10-m distance from the slope toe plotted for a reduction factor of 1.0 (i.e., immediately prior to failure) and 1.2 (i.e., instantly after the failure). Similarly, Figs. 13c and 13d respectively show the soil stress-strain loops of a point close to the computed yield zone (Fig. 12c) plotted for reduction factors of 1.0 and 1.45. Figs. 13a and 13c (plotted immediately before the slope failure) show the degradation of the clay stiffness with shear strain. Unlike the cohesionless soil response to seismic excitation that exhibit a substantial reduction in its stiffness manifested by a significant rotation of the stress-strain loop towards the horizontal axis, the clay experiences limited rotation of the stress-strain loop and consequently less relative reduction in its shear modulus. Immediately after the failure, Figs. 13b and 13d indicate that a decrease of the soil peak shear resistance (42 kPa in Fig. 13a) to a residual strength (36 kPa) has been occurred and the difference between the peak and the residual strength of the soil at the point in question is transferred to the surrounding soil. These redistributions of stresses may cause the peak strengths in the surrounding soil to be reached resulting in a progressive growth of the failure zone (i.e., propagation of the failure zone) until the entire slope become unstable.

Figure 14 shows the spectrum corresponding to the input motion used in the dynamic analysis versus the design spectrum of Quebec and Montreal provided in the national building code, CNBC 2015. The spectral accelerations of different points within the slope model considered are also plotted in Fig. 14. These points are located: (1) on the top of the slope, 70 m from the crest (upstream); (2) at mid-slope; and (3) at the bottom of the slope, 50
m from the toe. Figure 14 demonstrates that the fundamental period varies from 1.19 sec. at the top of the slope (1) to 0.72 sec at the bottom of the slope (3) and that the second-mode periods of the slope converge close to the maximum design spectral acceleration. These effects are because of geometry and the distribution of the soil properties. Figure 14 demonstrates also that there a substantial amplification of the seismic response around the fundamental mode of vibration (i.e., period = 1.19 sec) at the surface both at the top and base of the slope with an average amplification ratio of 3 with respect to the bedrock motion. With this average amplification, the spectral acceleration at the fundamental mode approaches the design spectrum. On the other hand, the amplification ratio at the second mode of vibration can be neglected. The results presented in Fig. 14 could, in part, justify the use of the pseudo-static method if the design acceleration value is consistent with the fundamental period. In other words, the earthquake effect could be replaced by an acceleration that acts in the direction of instability, such as the fundamental mode of vibration. These results also call in to question the use of a constant acceleration over the entire soil slope as there a substantial variation of the seismic response at different locations of the slope at the fundamental mode of vibration.

**Recommendations to refine the pseudo-static procedure outcomes**

Based on the comparative results of the dynamic and pseudo-static analyses discussed in the previous section and the literature pertaining to the topic, one of the main simplifying approximations of the pseudo-static method is that it uses a constant seismic coefficient over the entire height of the slope. In this study, the authors postulated that using either linear or segmental variation of the pseudo-static coefficient over the height of the slope in the analysis of clayey slopes would be a significant improvement. Several numerical analyses with different variations of the pseudo-static coefficient with depth were conducted and the
authors found that a variation of the seismic coefficient in the hyperbolic form (Eq. 4) would lead to a significant improvement of the method:

\[ k_{hz} = k_{ho}[1 + 2(z / H_t)^2] \]  (4)

where \( k_{hz} \) and \( k_{ho} \) are the seismic coefficients at any distance \( z \) measured from the bedrock level and at the bedrock (initial value), respectively; \( H_t \) is the total height of both the slope and the deposit. The modified hyperbolic profile of the seismic coefficient could be then incorporated into numerical slope stability analysis by multiplying the mass of each row of soil elements in the slope model at a certain depth by the value of the seismic coefficient at the same elevation, thus obtaining a non-uniform distributed seismic force along the entire depth of the slope. Figure 15 presents an example of the obtained results. Figure 15a shows the normalized relative displacement-reduction factor curves obtained from the suggested pseudo-static analysis at for the 1.75H:1V and 3H:1V slopes under Synthetic 1 and 2 input motions. For the 1.75H:1V slope, \( k_{ho} \) values of 0.035 and 0.045 are selected for the pseudo-static analyses under Synthetic 1 and 2 input motions, respectively. These values lead to seismic coefficient at the ground surface of 0.105 and 0.135 (these values are relatively close to the assumed value of the seismic coefficient in conventional pseudo-static procedure (0.15)). As shown in Fig. 15a, these selected values produce a factor of safety identical to that obtained from the numerical simulation at similar shear strain of 0.2. On the other hand, the best safety factors for the 3H:1V slope were been reached at values of seismic coefficients at the ground surface of 0.081 and 0.105 for Synthetic 1 and 2 motions, respectively. The corresponding failure surfaces are also improved when the suggested profile of the seismic coefficient is included in the pseudo-static analysis. Figures 15b and 15c show the slip surfaces obtained from modified pseudo-static analysis over which the slip surfaces of the dynamic analysis is plotted in dashed line for the 1.75H:1V and 3H:1V slopes, respectively. It is observed that the yield surfaces are very similar. Moreover, the deeper failure surface
previously observed in the dynamic analysis of the 3H:1V slope has been appeared in the suggested pseudo-static analysis, but it is somewhat wider than that in detected in the numerical analysis.

**Conclusions**

A close examination of the simplistic assumptions of the conventional pseudo-static method was conducted through a rigorous framework based on static and dynamic finite difference simulations conducted using the Mohr-Coulomb constitutive model and the SIG4 shear modulus reduction and damping model. The numerical modeling was validated using published data from static and dynamic 1-g physical modeling. Two homogeneous clayey slopes: 1.75H:1V and 3H:1V overlying a 20-meter thick homogeneous clay foundation layer underlain by bedrock were analyzed statically, pseudo-statically and dynamically. The results of these analyses were compared in terms of the estimated factor of safety and the developed failure surfaces. Based on the comparative results presented in this study, the following conclusions were made:

1. Generally, the conventional pseudo-static procedure significantly under-estimates the factor of safety of clayey slopes.

2. The failure surfaces obtained from the pseudo-static analysis are typically very deep and there are significant differences between those surfaces and the failure surfaces estimated from numerical simulations.

3. Careful examination of the spectral acceleration ratios of different points within the slopes based on dynamic numerical simulations show that the assumption of replacing the earthquake effect by a unidirectional acceleration could be acceptable. However, the use of a constant acceleration over the entire soil slope mass is a very simple approximation and overly conservative.
4. The use of a hyperbolic variation of the seismic coefficient with depth would produce factors of safety and failure surfaces very similar to that obtained from the numerical simulations and can be readily accomplished in numerical analysis.

Acknowledgements

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References


**Figure captions**

**Fig. 1**: Basic characteristics of the slope under investigation, meshing, and associated boundary conditions.

**Fig. 2**: Geometry and instrumentation of slope models reproduced from Ozkahriman (2009): (a) S1 and (b) D1.

**Fig. 3**: Static loading: localized shear surfaces and their corresponding deformed meshes of numerical models: (a) S0, S1 and (b) S2, S3.

**Fig. 4**: Static loading: real and the simulated load-deformation behavior of the slopes.

**Fig. 5**: Dynamic loading (sine sweep): measured and computed spectral accelerations in slope model D1: (a) base input motion and (b) accelerometer A4, A5.

**Fig. 6**: Dynamic loading (Redwood motion): measured and computed spectral accelerations at: (a) A4 and (b) A5.

**Fig. 7**: Dynamic loading: measured and computed PGAs.

**Fig. 8**: Dynamic loading (Redwood motion): measured and computed post-test profiles and failure surfaces.

**Fig. 9**: Static analysis: (a) relative displacement-factor of safety curves; failure surface of slopes: (b) 1.75H:1V and (c) 3H:1V.

**Fig. 10**: Pseudo-static analysis: (a) relative displacement-factor of safety curves; failure surface of slopes: (b) 1.75H:1V and (c) 3H:1V.

**Fig. 11**: Saguenay Earthquake used in the dynamic analysis: (a) acceleration spectra; (b) accelerogram.

**Fig. 12**: Dynamic analysis: (a) relative displacement-factor of safety curves; failure surface of slopes: (b) 1.75H:1V and (c) 3H:1V.

**Fig. 13**: Dynamic analysis: shear stress-shear strain curves estimated at factor of safety of: (a) 1.0 in the 1.75H:1V slope, (b) 1.3 in the 1.75H:1V slope, (c) 1.0 in the 3H:1V slope, and (d) 1.45 in the 3H:1V slope.

**Fig. 14**: Dynamic analysis: acceleration spectra for the used input motion and the spectral accelerations at different locations within the slope.

**Fig. 15**: Modified pseudo-static analysis: (a) relative displacement-factor of safety curves; failure surface of slopes: (b) 1.75H:1V and (c) 3H:1V.
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Fig. 2: Geometry and instrumentation of slope models reproduced from Ozkahrman (2009): (a) S1 and (b) D1.
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Δcu = 1.5 kPa/m
Su₁ = 25 kPa

F.S. = 1.48
F.S. = 1.76

H₁ = 10 m
H₂ = 20 m

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Fig. 10: Pseudo-static analysis: (a) relative displacement-factor of safety curves; failure surface of slopes: (b) 1.75H: 1V and (c) 3H: 1V.
Fig. 11: Earthquake signals compatible with Québec and Montréal: (a) acceleration spectra; (b) and (c) accelerogram.
Fig. 12: Dynamic analysis: (a) relative displacement-factor of safety curves; failure surface of slopes: (b) 1.75H: 1V and (c) 3H: 1V.
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### Table 1: Parameters of Mohr-Coulomb and SIG4 models.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Depth (m)</th>
<th>Cohesion, (kPa)</th>
<th>Density (t/m³)</th>
<th>Shear modulus, $G_{max}$ (MPa)</th>
<th>Poisson's ratio</th>
<th>Shear wave velocity, $V_s$ (m/s)</th>
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<th>$b$</th>
<th>$X_0$</th>
<th>$Y_0$</th>
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<td>0.95</td>
<td>-0.6</td>
<td>-0.9</td>
<td>0.06</td>
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### Table 2: Summary of slope model geometries (Ozkahriman, 2009).

<table>
<thead>
<tr>
<th>Slope model</th>
<th>Geometric scaling factor ($\lambda$)</th>
<th>Slope height (m)</th>
<th>Slope height (°)</th>
<th>Model width (m)</th>
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<tr>
<td>S0</td>
<td>1.00</td>
<td>0.533</td>
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<td>0.508</td>
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<tr>
<td>S1</td>
<td>1.00</td>
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<td>S2</td>
<td>1.43</td>
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<tr>
<td>S3</td>
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<td>0.508</td>
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<tr>
<td>D2</td>
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<td>45</td>
<td>0.711</td>
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<td>D3</td>
<td>2.50</td>
<td>0.216</td>
<td>45</td>
<td>0.508</td>
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</table>

### Table 3: Details of Input Motions in “Model” Scale (Ozkahriman, 2009).

<table>
<thead>
<tr>
<th>Model</th>
<th>Motion type</th>
<th>PGA (g)</th>
<th>Significant duration (sec)</th>
<th>Mean frequency (Hz)</th>
<th>Mean period (sec)</th>
<th>Arias intensity (m/sec)</th>
<th>Predominant frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>Sine pulse</td>
<td>0.502</td>
<td>5.855</td>
<td>8.175</td>
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<td>RWC</td>
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<td>D2</td>
<td>Sine pulse</td>
<td>0.472</td>
<td>4.909</td>
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<td></td>
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<td>3.176</td>
<td>7.00</td>
<td>0.211</td>
<td>1.545</td>
<td>5.00</td>
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<td>D3</td>
<td>Sine pulse</td>
<td>0.449</td>
<td>3.352</td>
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<td>8.545</td>
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Table 4: Parameters of Mohr-Coulomb and SIG4 models used in the experimental validation.

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<tr>
<th>Soil</th>
<th>Cohesion, $c$ (kPa)</th>
<th>Density, $\rho$ (t/m³)</th>
<th>Bulk modulus, $K$ (MPa)</th>
<th>Shear modulus, $G_{max}$ (MPa)</th>
<th>Shear velocity, $V_s$ (m/s)</th>
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<th>$b$</th>
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<td>0.2</td>
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<td>-0.4</td>
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<tr>
<td>Stiff layer</td>
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<td>1.0</td>
<td>25.7</td>
<td>1</td>
<td>-0.5</td>
<td>-0.4</td>
<td>0.35</td>
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Table 5: Safety factors calculated using limit equilibrium methods and strength reduction using FLAC.

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<tbody>
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<td>Static analysis</td>
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<tr>
<td>1.75H :1V</td>
<td>1.348</td>
<td>1.493</td>
<td>1.485</td>
<td>1.485</td>
<td>1.480</td>
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<tr>
<td>3H :1V</td>
<td>1.683</td>
<td>1.827</td>
<td>1.822</td>
<td>1.822</td>
<td>1.760</td>
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<tr>
<td>Pseudo-static analysis $k_h = 0.15$</td>
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<tr>
<td>1.75H :1V</td>
<td>1.038</td>
<td>1.038</td>
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<td>3H :1V</td>
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