CHARACTERIZATION OF QUANTUM STATES GENERATED ON CHIP

by

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Quantum protocols, which exploit non-classical correlations produced by quantum light sources, have demonstrated their advantages over their classical counterparts. Being able to engineer and characterize on-chip quantum light sources used to implement these quantum protocols is becoming crucial, as practical quantum-enhanced applications evolve.

In this thesis, we first demonstrate that the photon statistics of our on-chip quantum source can be measured using pulsed pumping and single-photon detectors. We show that non-classical correlations $g^{(2)}$ and noise reduction factor are degraded in a noisy and lossy environment. We also demonstrate that the bi-photon joint spectrum amplitude can be tuned by engineering the waveguide structure.

The ability to engineer and characterize the on-chip quantum source paves the way to practical quantum applications, such as quantum illumination (QI) protocol. We also show that the performance of QI can be enhanced by increasing the number of frequency modes through waveguide engineering.
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Chapter 1

Introduction

1.1 Motivation for Characterization of Quantum States Generated on Chip

In recent years, due to developments in quantum information [38], quantum state engineering [45] and quantum metrology [40, 42], more and more practical quantum protocols have been proposed and implemented using different quantum sources [46]. Among those quantum sources, compact, robust, scalable and tunable integrated quantum sources have shown their unique advantages [32]. The ability to engineer and characterize quantum light sources on chip become essential. On the other hand, quantum applications suffer from quantum decoherence caused by environment noise and loss. Hence, investigating quantum decoherence effects on performance of quantum source is also crucial for practical quantum applications.

Quantum applications utilize peculiar properties of quantum sources to achieve better performance that classical protocols cannot achieve. In particular, quantum correlations, such as path entanglement [37], sub-Poissonian statistics [53], phase squeezing [50], and time correlations [56], are crucial requirements for most quantum protocols. For example, quantum super phase resolution utilizes the path entanglement of a N00N state source to reach the Heisenberg limit $1/N$ and beat the ’standard quantum limit’ $1/\sqrt{N}$ [33–36]. Sub-shot-noise imaging can be achieved through correlated photon number fluctuations [46]. Quadrature-squeezed quantum light has smaller phase uncertainty, which is used in gravitational wave detection to reduce measurement noise [50, 52]. Time-correlated and frequency-uncorrelated photon pairs are the ideal resources for high-purity heralded single photons [56, 58].

Among those quantum sources, on chip sources are attracting more and more atten-
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In recent years, most quantum experiments generated quantum light from the nonlinear bulk crystal. However, in the real world, most of the practical applications require the system to be compact, mobile, alignment-free and robust, which is difficult for bulk crystal. Opposite to bulk crystal, the integrated chip can satisfy those requirements perfectly. Not only the size of the integrated chip is small but also the pump source and detection system can all be integrated onto the same chip, which makes on chip quantum system to be high efficient. The modes in waveguides usually have better confinement compared to modes in bulk crystal, and hence the on chip source has much higher conversion efficiency [30,31]. On the other hand, on chip quantum state engineering provides this system more tunability. By engineering waveguides, quantum states with different properties can be generated on chip [32].

Quantum source needs to be characterized before they are used in the real quantum applications. For example, the heralding efficiency of the system and bi-photon frequency correlation are crucial for heralded single photons quantum applications [45,56]. For applications based on quantum correlation, the non-classical correlation of quantum source is usually characterized using cross-correlation method [98]. Self correlation value can be used to identify light source type and mode number [56]. For sub shot noise application, noise reduction factor (NRF) is an important parameter to measure the quantum source non-classical correlation [29].

In real quantum applications, the environment noise and loss can break the quantum entanglement easily. A 10dB squeezing state can be degraded to 1dB in the presence of 6dB noise. The performance of entanglement of NOON state can easily be influenced by noise. The resolution of phase sensing decreases from $1/\sqrt{N}$ quadratic improvement to a constant improvement as entangled photon number $N \to \infty$ in noise environment [27,28]. The performance of quantum applications can be estimated from characteristics of their quantum source, therefore analyzing the noise and loss effects to those characteristics becomes the important step towards the practical quantum applications.

Low photon number is a crucial requirement for lots of fragile quantum sensing system, such as biological measurement [4], trapped atom interaction [6] and single molecules measurement [7,8]. To avoid sample damage, the interaction between probe photon and sample must be as weak as possible. As a result, the quantum source must be operated at low power region. However, many quantum protocols realized in lab is operated in nW ($10^8$ photon/s) power range [18,20,29,50]. In this thesis, we characterize and implement the quantum protocols between fW and pW ($10^4$ photon/s) power region, which makes a significant contribution towards the practical low photon quantum imaging applications.
1.2 On-Chip Quantum Sources

Spontaneous parametric down conversion (SPDC) is one of the major methods used to generate entangled photons. In early experiments, SPDC light was generated from a radiative atomic cascade of calcium [60], which was difficult to implement and the conversion efficiency was only about $1 \times 10^{10}$ pump photon per generated pair.

Later $\chi^{(2)}$ nonlinear material waveguide was used to generate the SPDC photon pairs. Good mode confinement inside the waveguide makes chip quantum source has much higher efficiency nonlinear process compared to bulk crystal source. Among those $\chi^{(2)}$ nonlinear waveguides, periodically poled lithium niobate (PPLN) and periodically poled potassium titanyl phosphate (PPKTP) waveguides are commonly used to produce photon pairs [61]. To satisfy phase matching condition, PPLN uses a quasi-phase-matching (QPM) technique (Equation 1.2.0.1), where the sign of $\chi^{(2)}$ coefficient is periodically changing by periodically poling (Figure 1.2.1).

$$k_p = k_s + k_i + \frac{2\pi}{\Lambda}$$  \hspace{1cm} (1.2.0.1)

Where $\Lambda$ is the poling period, $k_j (j = p, s, i)$ is k-vector of pump, signal and idler photon respectively. Similar to PPLN, QPM condition also can be satisfied using periodically poled KTP (PPKTP) crystal with a different grating period [64, 65]. Unlike PPLN requires high temperature during manufacturing, PPKTP waveguides can be manufactured under room temperature.

Photon pairs also can be generated in the fiber. Because fiber does not exhibit second order $\chi^{(2)}$ nonlinearity, instead of using SPDC, photon pairs are generated via spontaneous four-wave mixing (SFWM). The photon pair generation efficiency of fiber
is much lower than SPDC device due to the weak $\chi^{(3)}$ nonlinearity. But the output power of SFWM device is proportional to input power square. Compare to linear power relationship of SPDC device, SFWM device can generate very high power output if it is pumped by high power pump [66].

In recent years, ring resonator structure is implemented on chip to generate high power photon pairs [67] [68]. In [69], it demonstrates CMOS-compatible micron-size silicon nitride oscillator can generate high squeezing photon pairs via SFWM process (Figure 1.2.2). In our lab, we generate photon pairs from GaAs/AlGaAs Bragg reflection waveguides (BRWs). Different from QPM SPDC and SFWM methods, BRW structure utilizes modal phase matching (MPM) to generate quantum photon pairs via SPDC [70]. III-V semiconductor BRW has much larger $\chi^{(2)}$ nonlinearity and can be integrated with pump laser directly. BRW device can generate Type-I and Type-II SPDC on the same chip by pumping different polarization pump.

1.3 Characterization of Quantum Source

To characterize quantum sources, quantum correlations are measured using different methods, such as heralding efficiency, squeezing parameter, quantum correlation $g^{(2)}$, noise reduction factor (NRF) and bi-photon joint spectrum amplitude (JSA).

High heralding efficiency is an essential requirement for heralded single photon sources [56] [58]. Time and frequency correlated signal/idler photon pairs can be generated via
Figure 1.2.3: SPDC generation on BRWs. W: ridge width. D: etch depth. It has two Bragg reflector layers. Pump photon propagates in Bragg mode, down-converted signal and idler photon pairs are propagated in TIR mode. [71]

SPDC process. The detection of idler photon can herald the arrival of signal photon. The raw heralding efficiency is Klyshko efficiency, which equals to the ratio between the detected photon and generated photon pairs on waveguide [59]. The raw efficiency contains three major parts $\eta_E \eta_P \eta_D$, where $\eta_E$ is the extraction efficiency of the waveguide, $\eta_P$ is the path efficiency, $\eta_D$ is the detection efficiency of the detector. In the experiment, we can estimate the quality of single photon source from its heralding efficiency.

Photon correlation are usually measured using $g^{(2)}$ methods. Self-correlation $g^{(2)}_{j,j}(j = 1, 2)$ measures the photon statistics of the source, which can be used to identify the light source type. Thermal light has $g^{(2)}_{1,1} = 2$ and coherent light has $g^{(2)}_{1,1} = 1$. In [55], it measures self-correlation of signal and idler photon, where $g^{(2)}_{j,j} = 2(j = s, i)$ tells the signal ($j = s$) and idler ($j = i$) photon are single mode thermal state. Cross-correlation $g^{(2)}_{s,i}$ can be used to measure the quantum correlation of photon pairs. $g^{(2)}_{s,i}$ is proportional to coincidence to accidental ratio (CAR) in experiment. For two modes optical parametric amplifiers (OPAs), non-classical correlations $g^{(2)}_{s,i}$ value can tell the parametric gain of the device [54]. In heralded single photon experiment, besides the heralding efficiency, conditional self-correlation $g^{(2)}_c$ is another important figure of merit [55]. To measure $g^{(2)}_c$, one photon herald the self-correlation measurement of the other photon. For prefect single photon source, it has $g^{(2)}_c = 0$. In experiment these three types of $g^{(2)}$ can be measured using three single photon detectors (Figure 1.3.1).

Quadrature squeezed light is commonly in quantum metrology applications to improve SNR of phase measurement. One of the well-known applications is gravitational
CHAPTER 1. INTRODUCTION

Figure 1.3.1: $g^{(2)}$ measurement setup. SNSPDs: niobium nitride (NbN) superconducting nanowire single-photon detector with a quantum efficiency of 7%. InGaAs SPD: indium gallium arsenide single-photon detector with a quantum efficiency of 15% a) cross-correlation $g^{(2)}_{s,i}$ b) self-correlation of signal photon $g^{(2)}_{s,s}$ c) self-correlation of idler photon $g^{(2)}_{i,i}$ d) conditional correlation of signal photon $g^{(2)}_c$

wave detection [50–52]. The enhancement gained from less uncertainty of phase can be demonstrated using Homodyne type of detection system (Figure 1.3.2 (i)), where the variance of two detectors current difference is smaller than the shot noise (Figure 1.3.2 (ii)) [49] [48].

Noise reduction factor (NRF) is another way to analyze the quantum correlation of twin photon sources [53]. NRF is a good figure of merits for sub-shot noise experiment. Sub-shot noise experiment is very similar to squeezing detection, but sub-shot noise experiment detects photon number squeezing directly without using local oscillator and Michelson interferometer [47]. Sub-shot noise protocol utilizes the photon number correlation between signal and idler to reduce the photon number uncertainty (shot noise) of light. It has many applications in quantum imaging field [46].

Frequency correlated photon pairs can be described using joint spectrum amplitude (JSA). It not only contains the spectrum of individual signal and idler photon but also contains the bi-photon frequency correlation information. In heralded single photon experiment, the frequency correlated photon pair will degrade the purity of single photon. So a uncorrelated photon pairs source that has a round shape JSA is crucial. However, most quantum source JSA is not a round shape, hence the ability to tailor JSA shape becomes essential for the generation of high purity heralded single photons. Passing
Figure 1.3.2: Quadrature squeezing detection using Homodyne detection scheme. \[48\] i) outline of Homodyne detection. ii) (a) squeezing detection by scanning the phase of local oscillator (b) electronic shot noise \[50\].

photon pairs through a bandpass filter is a commonly used method to get round shape JSA. But this post selection methods introduce additional loss, which degrades the purity of photon pairs \[43\] \[44\]. Instead of using post-selection to tailor JSA, quantum source JSA can be controlled using waveguide engineering technique. In In \[45\] it proposes a JSA shaping method without post-selection. It can customize JSA shape by engineering the poling of PPLN waveguide.

1.4 Scope and Outline of Thesis

The objective of this research is to characterize SPDC quantum light generated on BRW ridge device and investigate the noise and loss effects on SPDC quantum source and quantum applications.

In chapter 2, the background of SPDC quantum source is reviewed. The basic photon statistics of the different light source is compared. The model of some quantum protocols is introduced. In chapter 3, the quantum source is characterized using the photon statistics method. The theory model of $g_{s,i}^{(2)}$ and noise reduction factor (NRF) is built. The noise and loss effects on SPDC quantum source are investigated. In chapter 4, our BRW SPDC source is characterized using quantum correlation methods in the experiment. The noise and loss effects on Quantum correlation $g^{(2)}$ and noise reduction factor (NRF) are confirmed in the lab. In chapter 5, the joint spectrum amplitude (JSA) of on-chip SPDC source is simulated and measured. We demonstrate the JSA can be controlled by engineering the waveguide structure. JSA of the straight and tapered waveguide are
measured in the lab. In chapter 6, theory model of quantum illumination (QI) protocol in high loss and noise is developed. The effects of JSA and frequency degree of freedom to quantum illumination is investigated. We also implement the QI protocol using our on-chip quantum source in the lab.

In this work, we successfully characterized the performance of an integrated, practical silicon-based quantum light source in the presence of noise and loss, and modeled the chip engineering and its effects on the performance of the source in low power region. We also fully characterized the JSA spectral properties and their relations to the source architecture. With this knowledge, we used this source in the first on-chip quantum illumination protocol. And we also demonstrate the quantum illumination protocol can be improved by engineering our waveguide structure.
Chapter 2

Background

2.1 Spontaneous Parametric Down-Conversion (SPDC)

Spontaneous parametric down conversion (SPDC) process involves a pump photon at a given frequency $w_p$ being down converted into two photons of low frequency at $w_s$ and $w_i$. In the generated photon pair, the photon with higher frequency ($w_s$) is called signal photon, and the other photon with lower frequency ($w_i$) is called idler photon. SPDC is a widely used method to generate quantum photon pairs. SPDC Photon pairs are used in many quantum applications such as quantum imaging, quantum key distribution, quantum computation, quantum cryptography, quantum ellipsometry and quantum communication [72]. In this chapter, the basic phase matching condition of SPDC is reviewed. The quantum properties of SPDC photon pair is discussed.

Spontaneous Parametric Down Conversion (SPDC) process must satisfy the energy conservation and phase matching requirements [79]:

$$\omega_p = \omega_s + \omega_i$$
$$\omega_j = \omega_j^0 + \Omega_j, (j = p, s, i)$$
$$\Delta k = k_p - k_s - k_i = 0$$ (2.1.0.1)

with $\omega_j(j = p, s, i)$ are the frequency of pump, signal and idler photon respectively. $\omega_j^0(j = p, s, i)$ and $\Omega_j(j = p, s, i)$ are the central frequency and frequency deviation. At degenerate point, signal and idler has the same frequency, $\omega^0 = \omega_s^0 = \omega_i^0 = 2\omega_p^0$.

For SPDC generated on chip, the phase mismatching can be calculated by solving the pump, signal and idler modes. In our case, we use BRW to generate photon pairs.
Figure 2.1.1: Type-II SPDC phase matching in 3um ridge width BRW device. a) Pump TE polarization with frequency $2\omega_0$ propagates at Bragg mode. b) Generated signal/idler in TE/TM polarization with frequency $2\omega_0$ propagates at TIR mode.

via SPDC process, detail layer structure about the BRW device used in this articles can be found in Appendix [A]. Unlike other phase matching techniques, such as birefringence phase-matching, quasi phase-matching (QPM) and artificial birefringence, BRW satisfies modal phase matching (MPM) [80]. BRW structure can support three modes: pump propagate in Bragg mode, generated signal and idler are propagate in TIR mode (Figure 2.1.1). The k-vector for each mode can be written in terms of effective index of mode inside the waveguides:

$$
\begin{align*}
    k_p &= \frac{2\pi n_{\text{Bragg,TE}}^{\text{eff}}}{\lambda_p} \\
    k_s &= \frac{2\pi n_{\text{TIR,TE}}^{\text{eff}}}{\lambda_p/2} \\
    k_i &= \frac{2\pi n_{\text{TIR,TM}}^{\text{eff}}}{\lambda_p/2}
\end{align*}
$$

(2.1.0.2)

where the effective index can be obtained by solving the mode of the waveguide. We can solve the mode using Matlab or Lumerical MODE FDTD. The phase matching at the degenerate point is given by:

$$
\Delta k = \frac{2\pi}{\lambda_p/2} \left(2n_{\text{Bragg,TE}}^{\text{eff}} - n_{\text{TIR,TE}}^{\text{eff}} - n_{\text{TIR,TM}}^{\text{eff}}\right) = 0
$$

(2.1.0.3)

To find the phase matching at other frequency, we can apply Taylor’s expansion to
\[ \Delta k = k_p^0 + \frac{d k_p}{d w_p} (w_p - w_p^0) + \frac{1}{2} \frac{d^2 k_p}{d w_p^2} (w_p - w_p^0)^2 + \ldots \text{ (higher order term)} \]

\[- k_s^0 - \frac{d k_s}{d \omega_s} (\omega_s - \omega_s^0) + \frac{1}{2} \frac{d^2 k_s}{d \omega_s^2} (\omega_s - \omega_s^0)^2 + \ldots \text{ (higher order term)} \]

\[- k_i^0 - \frac{d k_i}{d \omega_i} (\omega_i - \omega_i^0) + \frac{1}{2} \frac{d^2 k_i}{d \omega_i^2} (\omega_i - \omega_i^0)^2 + \ldots \text{ (higher order term)} \]

We can simplify the equation using:

\[ v_j = \frac{d \omega_j}{d k_j} \]

\[ D_j = -\frac{2\pi c d^2 k_j}{\lambda_q^2 \frac{d w_j^2}{d^2}} \]

\[ \Lambda_j = -\frac{\lambda_x^2 D_x}{4\pi c} = \frac{1}{2} \frac{d^2 k_j}{d w_j^2} \]

and we can obtain phase mismatching at any wavelength:

\[ \Delta k(\omega_p, \omega_s, \omega_i) = \frac{1}{v_p} (\omega_p - \omega_p^0) + \frac{1}{v_s} (\omega_s - \omega_s^0)^2 - \frac{1}{v_i} (\omega_i - \omega_i^0)^2 \]

The intensity of output down-converted field has the following relationship with phase mismatch:

\[ E(L) \propto \int_0^L e^{j \Delta k z} \, dz \propto \text{sinc}(\frac{\Delta k L}{2\pi}) L \]

\[ I \propto |E|^2 \]

Where \( L \) is interaction length of SPDC process. According the equation, SPDC only have power near the phase matching point \( \Delta k = 0 \).

The quantum state of photon pairs generated via SPDC can be expressed as:

\[ |\psi\rangle = \int \int d\omega_s d\omega_i f(\omega_s, \omega_i) \Gamma(\omega_s, \omega_i) \hat{a}_s^\dagger(\omega_s) \hat{a}_i^\dagger(\omega_i) |0\rangle |0\rangle \]

\[ = \int \int d\Omega_s d\Omega_i f(\Omega_s, \Omega_i) \Gamma(\Omega_s, \Omega_i) |w_s^0 + \Omega_s\rangle |w_i^0 - \Omega_i\rangle \]
where \( \Omega_j \) are the frequency deviation given by \( \omega_j = \omega_j^0 + \Omega_j \), \( j = p, s, i \). For simplicity, let’s assume the spatial overlap of the interaction mode \( \Gamma(\Omega_s, \Omega_i) \approx 1 \) for any frequencies deviation \( \Omega_s \) and \( \Omega_i \). The biphoton joint spectral amplitude (JSA) \( f(\omega_s, \omega_i) \) is equal to:

\[
f(\omega_s, \omega_i) = E_p(\omega_s + \omega_i)\phi(\omega_s, \omega_i) \tag{2.1.0.13}
\]

where \( E_p \) is the field of pump photon. Phase matching intensity function \( \phi(\omega_s, \omega_i) \) can be written in terms of phase mismatch \( \Delta k \):

\[
|\phi(\omega_s, \omega_i)|^2 = |\text{sinc}(\Delta kL/2)|^2 \tag{2.1.0.14}
\]

### 2.2 Photon Statistics of Different Light Sources

Quantum applications utilize quantum light to achieve high performance. The different physics between quantum and classical light source can be understood in terms of photon statistics. In this section we compare three different light source: coherent light, thermal light, and two modes squeezed light.

**Coherent Light**  Coherent light can be easily generated from a laser in the lab. For any coherent state \( |\alpha\rangle \), it can be written in terms of number states \( |n\rangle \):

\[
|\alpha\rangle = \sum_{n=0}^{\infty} C_n |n\rangle \tag{2.2.0.1}
\]

And coherent state \( |\alpha\rangle \) must satisfy the following equation [86]:

\[
\hat{a} |\alpha\rangle = \alpha |\alpha\rangle \tag{2.2.0.2}
\]

Where \( C_n \) can be solved by substituting equation 2.2.0.1 into 2.2.0.2:

\[
C_n = \frac{\alpha^n}{\sqrt{n!}} C_0 \tag{2.2.0.3}
\]

\[
C_0 = e^{-|\alpha|^2/2} \tag{2.2.0.4}
\]
Constant $C_0$ can be obtained using the normalization constrain, $\langle \alpha | \alpha \rangle = 1$. Substituting $C_n$ into equation 2.2.0.1, we get normalized coherent state:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

(2.2.0.5)

Given the coherent state equation 2.2.0.5 in terms of number state, average photon number per mode $\mu$ can be calculated using the photon number operator $\hat{n} = \hat{a}^\dagger \hat{a}$:

$$\mu = \langle n \rangle = \langle \alpha | \hat{n} | \alpha \rangle = |\alpha|^2$$

(2.2.0.6)

The variance of coherent state can be expressed as:

$$\langle \delta^2 n \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$$
$$= (|\alpha|^4 + |\alpha|^2) - (|\alpha|^2)^2$$
$$= \mu$$

(2.2.0.7)

In fact, the variance of photon number per mode is equal to average photon number $\mu$. Projecting the coherent state equation 2.2.0.5 onto number state, the photon number distribution can be obtained:

$$P_n = |\langle n | \alpha \rangle|^2 = e^{-\mu \frac{\mu^n}{n!}}$$

(2.2.0.8)

From equation 2.2.0.8 we can see the photon number distribution of coherent state is Poisson distribution.

**Thermal Light** Noise in most of quantum system is thermal light. Sunlight is a simple example of thermal light. To describe thermal light in terms of photon statistics, we can first decompose its density operator into number operator:

$$\hat{\rho} = \sum_{n=0}^{\infty} P_n |n\rangle \langle n|$$

(2.2.0.9)

Where $P_n$ is the photon number distribution of thermal light 86:

$$P_n = \frac{e^{-E_n/k_B T}}{\sum_{n=0}^{\infty} e^{-E_n/k_B T}} \frac{\mu^n}{(1 + \mu)^{n+1}}$$

(2.2.0.10)
Where \( E_n = \hbar \omega (n + 1/2) \), \( k_B \) is Boltzmann constant, \( T \) is temperature. \( \mu \) is the average photon number per mode:

\[
\mu = \langle n \rangle = \text{Tr}(\hat{n} \hat{\rho}) = \sum_{n=0}^{\infty} n P_n
\]
\[
= \frac{1}{e^{\hbar \omega / k_B T} - 1}
\]

(2.2.0.11)

The photon number variance of thermal light is then given by:

\[
\langle \delta^2 n \rangle = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2
\]
\[
= \text{Tr}(\hat{n}^2 \hat{\rho}) - \mu^2
\]
\[
= \mu + \mu^2
\]

(2.2.0.12)

Compared with coherent light variance \( \mu \), thermal light has higher fluctuation or noise.

**SPDC Light** From previous sections, we know type-II Spontaneous Parametric Down-Conversion (SPDC) can be used to generate two modes squeezing vacuum states. The squeeze operator of type-II SPDC process is defined as [86]:

\[
S(\xi) = e^{\frac{\xi^* a_s a_i - \xi a_s^* a_i^*}{2}}
\]

(2.2.0.13)

\[
\xi = r e^{i\theta}
\]

(2.2.0.14)

Where \( a_s^\dagger \) and \( a_i^\dagger \) are the creation operator for signal and idler photon respectively. Squeezing parameter \( r \) is a constant depending on the design of quantum source and pump [102]:

\[
r = \frac{\chi_{\text{eff}} \omega_p}{2 n_c} |\varepsilon_p| z
\]

\[
|\varepsilon_p| = \sqrt{\frac{I_p}{2 n \epsilon_0 c}}
\]

(2.2.0.15)

\[
I_p = \frac{P}{\pi W^2}
\]

Where \( \chi_{\text{eff}} \) is the effective nonlinear susceptibility. \( z \) is the length of waveguide. \( \omega_p \) is the frequency of pump. \( W \) is the pump focus radius. \( P \) is pump field power. The operator of signal and idler photon are given by:

\[
b_s = S(\xi) a_s S(\xi) = \cosh(r) a_s - e^{i\theta} \sinh(r) a_i^\dagger
\]

\[
b_i = S(\xi) a_i S(\xi) = \cosh(r) a_i - e^{-i\theta} \sinh(r) a_s^\dagger
\]

(2.2.0.16)
CHAPTER 2. BACKGROUND

The two mode squeezing modes generated from SPDC can be written in terms of $r$ [86]:

$$|\psi\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} (-1)^n e^{in\theta} (\tanh r)^n |n, n\rangle$$  \hspace{1cm} (2.2.0.17)

The average photon number per mode of this two mode squeezing modes sate is [2.2.0.16]:

$$\mu = \langle \hat{b}_j^\dagger \hat{b}_j \rangle = \langle n_j \rangle = \sinh^2 r, \text{ where } j = s, i$$  \hspace{1cm} (2.2.0.18)

The variance of signal or idler is given by [87]:

$$\langle \delta^2 n_x \rangle = \langle n_x^2 \rangle - \langle n_x \rangle^2$$

$$= (\mu + 2\mu^2) - \mu^2$$  \hspace{1cm} (2.2.0.19)

$$= \mu + \mu^2$$

We note the variance expression of the thermal state is the same as the variance of signal/idler photon. In fact, for type-II SPDC, signal and idler are thermal states with photon correlation $n_s = n_i$. The signal and idler have the same photon number distribution as the thermal state [2.2.0.10].

Given squeezing parameter $r$ and average photon number $\mu$ relation in [2.2.0.18] The photon statistics of SPDC light also can be written in terms of $r$ [86]:

$$P_j(n) = \frac{(\tanh r)^{2n}}{(\cosh r)^2}$$

$$\langle n_j \rangle = \sinh^2 r$$  \hspace{1cm} (2.2.0.20)

$$\langle \delta^2 n_j \rangle = \sinh^2 r \cosh^2 r = \frac{1}{4} \sinh^2 (2r)$$

$$\langle n_1 n_2 \rangle = \frac{1}{4} \sinh^2 (2r) + \sinh^4 r$$

2.3 Quantum Correlations and Sensing Protocols

Quantum sensing protocols utilize non-classical correlation of quantum source to enhance the sensing. In this section, we will review two important quantum sensing protocols: ghost imaging and sub-shot noise. The enhancement of both protocols is gained from photon number correlation.
Figure 2.3.1: Schematic setup for the ghost imaging. BBO bulk crystal is used for type-II SPDC; D1 and D2 detector are used to detect idler and signal respectively; D1 is bucket detector and doesn’t have the spatial resolution; D2 detector has the spatial resolution, it can be a CCD camera or a scanning detector.

2.3.1 Ghost Imaging

In ghost imaging experiment, bulk crystal is usually used to generate spatial intensity correlated photon pairs. Signal beam works as reference, idler beam hit an object in phase distortion environment. Figure 2.3.1 shows the ghost imaging system with type-II SPDC as the quantum source.

Because of the spatial correlation between signal and idler photon, the object spatial information still can be retrieved from a bucket detector by jointly measure the correlation of signal and idler beam. For D2 at \( x \) position, the measured correlation \( S(x) \) that contains signal and idler correlation information.

\[
S(x) = f (\langle n_1 \rangle, \langle n_1, n_2(x) \rangle, \langle n_1^2 \rangle, \langle n_2(x)^2 \rangle, \ldots) \tag{2.3.1.1}
\]

Where \( \langle X \rangle \) is the expectation value of \( \kappa \) number of repeat realizations.

\[
\langle X \rangle = \frac{1}{\kappa} \sum_{k=1}^{\kappa} X^{(k)} \tag{2.3.1.2}
\]

Depends on the different ghost imaging protocols, the correlation function \( S(x) \) is different. The following three protocols are used in most of ghost imaging articles. The first is Glauber intensity correlation as \( S \) function:

\[
S(x) = G^{(2)}(x) = \langle n_1n_2 \rangle \tag{2.3.1.3}
\]
CHAPTER 2. BACKGROUND

The second is quantum cross correlation $g^{(2)}$:

$$S(x) = g^{(2)}(x) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2(x) \rangle}$$  \hspace{1cm} (2.3.1.4)

The third is the covariance $\text{cov}(n_1, n_2(x))$:

$$S(x) = \text{cov}(n_1, n_2(x)) = \langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2(x) \rangle$$  \hspace{1cm} (2.3.1.5)

In order to compare the performance of different ghost imaging protocols, the signal to noise ratio (SNR) of ghost imaging is defined \[87\]:

$$\text{SNR} = \frac{|\langle S_i - S_o \rangle|}{\sqrt{\langle \delta^2 (S_i - S_o) \rangle}}$$  \hspace{1cm} (2.3.1.6)

Where $S_i$ and $S_o$ are two situations corresponding to object presence and object absence. This SNR definition reflects the contrast between inside ($S_i$) and outside ($S_o$) of object profile. In \[87\], it shows that the $g^{(2)}$ and covariance ghost imaging have much better performance than $G^{(2)}$.

Ghost imaging is one of first real 'quantum' imaging protocols. It was first realized in the experiment using quantum entangled light \[88\]. But after a few years, people find the classical correlated thermal light also can be used for ghost imaging \[89\]-\[92\]. The correlated thermal light is generated by passing thermal light through a beam splitter. Because the discord of correlated thermal light is not zero, in \[93\] it argues that it is the discord that is the requirement for ghost imaging other than the entanglement. But one year later, in \[94\], it demonstrates that spatial modulation light without discord can also be used to realize ghost imaging. It seems ghost imaging is not a strict quantum application since it not really need a quantum source. But the concept of ghost imaging pave the path to lots of quantum imaging applications, such as ghost imaging ladar \[96\] and ghost imaging in turbid environment \[95\].

2.3.2 Sub-Shot Noise

Shot noise is caused by the fluctuation of electric current, which is one of the fundamental limitation of measurement. For photon detector, shot noise also can be understood in terms of photon statistics. The fluctuation of current is proportional to the uncertainty of the photon number. For example, shot noise of coherent light is caused by the Poisson distribution of photon number, hence the shot noise is equal to the standard deviation of Poisson distribution. From equation 2.2.0.7, we know the variance of coherent light is
Figure 2.3.2: Scheme of sub shot noise experiment. BBO nonlinear crystal is used to generate photon pairs. Signal and idler photon are detected by detector D2 and D1 respectively. The variance of photon current is analyzed using the computer.

\[ \mu, \] so the shot noise of coherent light is limited to \( \sqrt{\mu} \).

In [97], sub-shot noise protocol was proposed to break the limit of shot noise using two modes squeezing quantum light. Figure 2.3.2 shows the typical setup of sub-shot noise.

Because of the photon number correlation between signal and idler, \( n_s = n_i \), the photon number fluctuation is the same. So the fluctuation of the photon number difference should always be zero, \( \langle \delta^2(n_s - n_i) \rangle = 0 \). To measure sub-shot noise effect, noise reduction factor (NRF) is given by:

\[
NRF = \frac{\langle \delta^2(n_s - n_i) \rangle}{\langle n_s + n_i \rangle} = \frac{\langle \delta^2 n_s \rangle + \langle \delta^2 n_i \rangle - 2 \text{cov}(n_s, n_i)}{\langle n_s \rangle + \langle n_i \rangle} \tag{2.3.2.1}
\]

Given the photon statistics of the light source, the sub shot noise effect can be estimated using NRF equation 2.3.2.1. In an ideal situation, where the system has zero loss and noise, the NRF of coherent light can be written as:

\[
\langle n_s \rangle = \langle n_i \rangle = \mu
\]

\[
\langle \delta^2 n_s \rangle = \langle \delta^2 n_i \rangle = \mu
\]

\[
\text{cov}(n_s, n_i) = \langle n_s n_i \rangle - \langle n_s \rangle \langle n_i \rangle = 0
\]

\[
NRF_{\text{coherent}} = 1
\]
For type-II SPDC light:

\[
\langle n_s \rangle = \langle n_i \rangle = \mu \\
\langle \delta^2 n_s \rangle = \langle \delta^2 n_i \rangle = \mu + \mu^2 \\
cov(n_s, n_i) = \langle n_s n_i \rangle - \langle n_s \rangle \langle n_i \rangle = (\mu + 2\mu^2) - \mu^2 = \mu + \mu^2 \\
NRF_{quantum} = 0
\] (2.3.2.3)

In fact, NRF of all classical light is larger than 1, quantum light is always less than 1. Due to this feature, NRF can be used as to measure the non-classical correlation of quantum source \[98\]. The lower the NRF value, the stronger the non-classical correlation.

Sub-shot noise can break the fundamental shot noise limitation, which paves the path to noise-free imaging. In \[99\], it proposes an application that uses sub-shot noise to detect a weak absorbing object. One year later, the same method is applied in the experiment to achieve high-sensitivity imaging \[101\] \[100\].
Chapter 3

Photon Statistics of Quantum Source

3.1 Background

From previous sections, we know the photon statistics of classical light and quantum light is different. Some non-classical correlations of quantum light can be calculated from photon statistics, such as cross-correlation $g^{(2)}$, noise reduction factor (NRF). The ability to measure the photon statistics of the quantum source in the lab is an important step for real quantum applications. Unlike the ideal photon statistic model in the previous section, the equipment limitation and environment noise and loss must be taken into account in the practical quantum application. Motivated by practicality, a theory model that can simulate the loss and noise effects on the quantum correlation is built in this section.

3.2 Theory Model

Figure 3.2.1 shows the theory model of the photon pairs generation via type-II SPDC and detection.

Quantum source is generated by pumping Bragg reflection waveguides (BRWs) with 780nm TE light. Type-II spontaneous parametric down-conversion (SPDC) process inside the waveguide generates two orthogonal polarization photons: signal (TM) and idler (TE) photon. Combine equation 2.2.0.16 and 2.2.0.18, SPDC process can be described...
Figure 3.2.1: Theory model of quantum light generation and detection. H (idler) and V (signal) polarization photon pairs are generated by pumping 780nm TE pump onto a BRW chip. The loss of photon idler and signal are $\eta_1$ and $\eta_2$ respectively. Single photon detector D1 and D2 record the click timestamps of idler and signal photons.

using following two equations:

$$b_1 = \sqrt{1 + \mu a_1 + \sqrt{\mu} a_2^\dagger}$$
$$b_2 = \sqrt{1 + \mu a_2 + \sqrt{\mu} a_1^\dagger}$$

(3.2.0.1)

Where $b_j$ ($j = 1, 2$) are the output modes (idler and signal) of quantum source. $\mu$ is photon number per mode. The collinear signal and idler are separated by a polarization beam-splitter. $\eta_j$ is the total efficiency of the detect photon in mode $b_j$, which is equal to the ratio of detected photon and generated photon pairs. We can use a beam-splitter to model the linear loss of the mode:

$$b_3 = \sqrt{\eta_1 b_1 + i \sqrt{1 - \eta_1} a_{vac}}$$
$$b_4 = \sqrt{\eta_2 b_2 + i \sqrt{1 - \eta_2} a_{vac}}$$

(3.2.0.2)

(3.2.0.3)

When the photon pairs arrive at detectors, photon statistic of the quantum source can be measured:

$$\langle n_j \rangle = \langle b_j^\dagger b_j \rangle = \eta_j \mu$$
$$\langle n_j^2 \rangle = \langle (b_j^\dagger b_j)^2 \rangle = \eta_j \mu + 2\eta_j^2 \mu^2$$
$$\langle \delta^2 n_j \rangle = \langle n_j^2 \rangle - \langle n_j \rangle^2 = \eta_j \mu + \eta_j^2 \mu^2$$
$$\langle n_1 n_2 \rangle = \langle b_1^\dagger b_1 b_2^\dagger b_2 \rangle = \eta_1 \eta_2 (\mu + 2\mu^2)$$

(3.2.0.4)

To detect above photon statistics, we assume detector is photon number resolvable and
CHAPTER 3. PHOTON STATISTICS OF QUANTUM SOURCE

3.2.1 Pulsed Pump and Single-Photon Detectors

Based on theory model 3.2.1, a photon statistics measurement experiment with pulse pump and single photon detectors is designed in Figure 3.2.2.

Different from ideal theory model, the detector is not photon number resolving. To analysis this effect on photon statistics measurement, we can write average photon num-
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ber $\langle n_j \rangle$ in terms of photon number detection probability:

$$\langle n_j \rangle = Tr \{ \rho_{1,2} n_j \}$$
$$= \sum_{n_j=0}^{\infty} n_j P_j(n_j)$$
$$= 0P_j(0) + 1P_j(1) + 2P_j(2) + 3P_j(3) + ...$$

(3.2.1.1)

Where $\rho_{1,2}$ is the density matrix of two mode squeezing states. $n_j (j = 1, 2)$ is the photon number per mode of idler and signal. $P_j(n)$ is the probability of detect $n$ photon.

From equation (3.2.1.1) we can see it requires photon number resolving detectors to measure photon number $n_j > 1$. This requirement can be relaxed in low power condition. In spontaneous regime the photon number per mode $\mu \ll 1$. Given the photon number distribution of thermal state 2.2.0.10 we can assume the photon number probability $P(n > 1) \ll P(1)$. Under this low power assumption, we can estimate average photon number $\mu$ from single photon detector click probability, $P_j(1)$:

$$\langle n_j \rangle = M\eta_j \mu \approx P_j(1)$$

(3.2.1.2)

The single photon detector click probability for signal/idler with pulse pump is:

$$P_j(1) = \frac{N_j}{\kappa} = \frac{\sum_{k=1}^{\kappa} n_j[k]}{\kappa}$$

(3.2.1.3)

Where $N_j (j = 1, 2)$ is the signal/idler single counts. $\kappa$ is the total number of pulse. $n_j[k]$ is the photon number at k-th pulse. Pulse pump trigger two gated single photon avalanche diode (SPAD) detectors to record detector click at each pulse (Figure 3.2.2). If SPAD $j (j = 1, 2)$ click at k-th pulse, we record $n_j[k] = 1$, otherwise record $n_j[k] = 0$.

The same as average photon number, the average coincidence counts $\langle n_1 n_2 \rangle$ can be written as:

$$\langle n_1 n_2 \rangle = Tr \{ \rho_{1,2} n_1 n_2 \}$$
$$= \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} n_1 n_2 P(n_1, n_2)$$
$$= P(1, 1) + 2P(1, 2) + 2P(2, 1) + 4P(2, 2) + ...$$

(3.2.1.4)

Where $P(n_1, n_2)$ is the joint probability of detect mode 1 has $n_1$ photon and mode 2 has $n_2$ photon. $P_j(n_j)$ is the marginal probability of $P(n_1, n_2)$. Because our source is
twin photon, we always have $n_1 = n_2$, so $P(n_1, n_2) = 0$ for $n_1 \neq n_2$:

$$\langle n_1 n_2 \rangle = P(1, 1) + 4P(2, 2) + \ldots$$  \hspace{1cm} (3.2.1.5)

Under the low power assumption $\mu \ll 1$ and $P(1, 1) \gg P(2, 2) \gg P(n > 2, n > 2)$, which simplifies equation to:

$$\langle n_1 n_2 \rangle \approx P(1, 1)$$  \hspace{1cm} (3.2.1.6)

Where $P(1, 1)$ is the coincidence probability:

$$P(1, 1) = \frac{N_c}{\kappa} = \frac{\sum_{k=1}^{\kappa} n_1[k]n_2[k]}{\kappa}$$  \hspace{1cm} (3.2.1.7)

$N_c$ is the total coincidence counts over $\kappa$ number of pulse.

### 3.2.2 Mode Number and Loss Calculation

In real quantum applications, entanglement is fragile and can be easily broken by noise and loss. In our experiment, the loss is unavoidable and could come from waveguide propagation, lens, mirror, fiber coupling, and detector. Total loss $\eta_j$ contains all those loss, which is modeled as a beam-splitter in Figure 3.2.1. Here we will demonstrate the total loss can be estimated from photon statistics 3.2.0.5.

Based on setup shows in Figure 3.2.1, detector single counts per realization $\langle n_j \rangle$ and coincidence counts per realization $\langle n_1 n_2 \rangle$ can be measured using 2 single photon detectors.

$$\langle n_1 \rangle = M\eta_1\mu$$
$$\langle n_2 \rangle = M\eta_2\mu$$
$$\langle n_1 n_2 \rangle = M\eta_1\eta_2\mu(1 + \mu + M\mu)$$  \hspace{1cm} (3.2.2.1)

Where $M$ is number of spatial mode and temporal mode, $M = M_{\text{spatial}} \times M_{\text{temporal}}$. Because the signal/idler generated from waveguide is single mode, $M_{\text{spatial}}$ is assumed equal to 1 in our case. $M_{\text{temporal}}$ can be obtained using equations:

$$M_{\text{temporal}} = \frac{T_{\text{pump}}}{T_{\text{coh}}}$$
$$T_{\text{coh}} = \frac{\lambda_{\text{coh}}^2}{c\Delta\lambda_{\text{coh}}}$$  \hspace{1cm} (3.2.2.2)

Where $T_{\text{coh}}$ is photon coherence time and $T_{\text{pump}}$ is pump pulse duration. Given equation
we can write the $\eta_j$ and $\mu$ in terms of $\langle n_1 \rangle$, $\langle n_2 \rangle$ and $\langle n_1 n_2 \rangle$:

$$\eta_2 = \langle n_1 n_2 \rangle - \frac{1 + M}{M} \langle n_1 \rangle \langle n_2 \rangle$$

$$\mu = \frac{\langle n_2 \rangle}{M \eta_2}$$

$$\eta_1 = \frac{\langle n_1 \rangle}{M \mu} \quad (3.2.2.3)$$

Substituting 3.2.1.2 and 3.2.1.6 into equation 3.2.2.3, we are able to write it in terms of detection probability using equation:

$$\eta_2 = P(1, 1) - \frac{1 + M}{M} P_1(1) P_2(1)$$

$$\mu = \frac{P_2(1)}{M \eta_2}$$

$$\eta_1 = \frac{P_1(1)}{M \mu} \quad (3.2.2.4)$$

### 3.3 Quantum Correlations

#### 3.3.1 Quantum Cross Correlation $g_{1,2}^{(2)}$

Second-order coherence, $g^{(2)}$, is usually used to measure the quality of single photon source and non-classical correlations. The general definition of $g^{(2)}$ is:

$$g_{j,k}^{(2)}(\tau) = \frac{\langle a_j^+(t) a_k^+(t+\tau) a_k(t+\tau) a_j(t) \rangle}{\langle a_j^+(t) a_j(t) \rangle \langle a_k^+(t+\tau) a_k(t+\tau) \rangle} \quad (3.3.1.1)$$

Two modes are equal ($j = k$) for self-correlations measurements. Our quantum source is two mode squeezing, we will focus on cross-correlations, $j \neq k$. Cross-correlation $g_{1,2}^{(2)}$ can be described by normalized Glauber correlation function:

$$g_{1,2}^{(2)}(0) = \frac{\langle n_1 n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} \quad (3.3.1.2)$$

Where $n_j (j = 1, 2)$ is the photon number operator of the two modes.
In our experiment, we use pulse source, equation 3.3.1.1 need to be modified to:

\[ g_{1,2}^{(2)}(m) = \frac{\langle a_1^\dagger[k]a_2^\dagger[k+m]a_2[k+m]a_1[k] \rangle}{\langle a_1^\dagger[k]a_1[k] \rangle \langle a_2^\dagger[k+m]a_2[k+m] \rangle} \tag{3.3.1.3} \]

Where \( k \) is the pulse number, \( m \) is the number of pulse delay between mode 1 and 2. For zeros delay, \( m = 0 \), equation 3.3.1.3 can be simplified as:

\[ g_{1,2}^{(2)}[0] = \frac{\langle n_1[k]n_2[k] \rangle}{\langle n_1[k] \rangle \langle n_2[k] \rangle}, \text{for} \ k = 1, 2, 3, \ldots \tag{3.3.1.4} \]

Where \( \langle n_j \rangle (j = 1, 2) \) is average photon number of each pulse for signal and idler respectively. \( \langle n_1n_2 \rangle \) is the average coincidence number of each pulse.

In the following section we compare several different experiment measurement methods of cross-correlation \( g_{1,2}^{(2)} \). Fundamentally the three methods are the same. However in real experiment, depending on certain experiment setup, \( g^{(2)} \) is easier to measure from some methods.

**Method 1: Photon Number Probabilities**  The first method is photon number probabilities. If we write equation 3.3.1.4 in terms of number operator and density matrix:

\[ g_{1,2}^{(2)}[0] = \frac{Tr\{\rho_{1,2}n_1n_2\}}{[Tr\{\rho_{1,2}n_1\}][Tr\{\rho_{1,2}n_2\}]} = \frac{\sum_n \sum_n n_1n_2P(n_1, n_2)}{\sum_n n_1P_1(n_1)[\sum_n n_2P_2(n_2)]} \tag{3.3.1.5} \]

In low pump power regime, it can be simplified using equation 3.2.1.2 and 3.2.1.6:

\[ g_{1,2}^{(2)}[0] \simeq \frac{P(1, 1)}{P_1(1)P_2(1)} \tag{3.3.1.6} \]

Where \( P(1, 1) \) is the coincidence probability of signal and idler. \( P_1(1) \) and \( P_2(1) \) are the click probabilities of signal and idler mode respectively. Substituting equation 3.2.1.3 and 3.2.1.7 into it, we can get:

\[ P(1, 1) = N_c/\kappa \]
\[ P_j(1) = N_j/\kappa \]
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Figure 3.3.1: Coincidence counts example, green: coincidence counts at $m = 0$, red: accidental counts at $m \neq 0$, black: dark noise

Where $\kappa$ is number of pulse, $N_j (j = 1, 2)$ is the single counts for idler and signal. We can see from equation 3.3.1.6, $g_{1,2}^{(2)}$ can be simply calculated from SPAD clicks probabilities, $P_{j}(1)(j = 1, 2)$ and $P(1, 1)$.

**Method 2: Coincidence to Accidental Ratio (CAR)** Coincidence to accidental ratio (CAR) is commonly used to detect non-classical correlation of SPDC.

$$ CAR = \frac{N_c/\kappa}{N_{acc}/\kappa} = \frac{N_c[0]}{N_c[m]} = \frac{\sum_{k=1}^{\kappa} n_1[k]n_2[k]}{\sum_{k=1}^{\kappa} n_1[k]n_2[k + m]}, (m \neq 0) $$

(3.3.1.8)

Where $N_c$ is the coincidence counts at $m = 0$, $N_{acc}$ is the accidental counts. Accidental counts can be obtained by counting coincidence at $m \neq 0$. Figure 3.3.1 shows an example of coincident and accidental counts in experiment.
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In spontaneous regime, we can approximate \( P(\text{accidental}) \approx P_1(1)P_2(1) \):

\[
\text{CAR} \approx \frac{P(1,1)}{P_1(1)P_2(1)} \approx g_{1,2}^{(2)}[0] \tag{3.3.1.9}
\]

From equation \(3.3.1.9\), we can see the cross correlation \( g_{1,2}^{(2)} \) can be estimated from CAR ratio. In the experiment, the dead time of SPAD detector can be corrected by using the single counts:

\[
\text{CAR}_{\text{correct}} = \frac{\text{CAR}}{(1 - N_1 t_1)(1 - N_2 t_2)} \tag{3.3.1.10}
\]

Where \( N_j \) and \( t_j \) are the single counts rate and dead time for SPAD detector \( j \).

**Method 3: Average Photon Number** \( g^{(2)} \) also can be calculated from average photon number \( \mu \). Substituting the photon statistic equation of bi-photon \(3.2.2.3\) into \(3.3.1.2\) we can get:

\[
g_{1,2}^{(2)} = \frac{1 + \mu + M \mu}{M \mu} \tag{3.3.1.11}
\]

For \( M = 1 \):

\[
g_{1,2}^{(2)} = \frac{1}{\mu} + 2 \tag{3.3.1.12}
\]

Where \( \mu \) is the average photon number per pulse:

\[
\mu = \sum_{n_j=0}^{\infty} n_j P_j(n_j) \tag{3.3.1.13}
\]

As with other methods, we assume system is in spontaneous regime, \( P_j(1) \gg P_j(n > 1), (j = 1, 2) \):

\[
\mu \simeq P_j(1) \simeq \frac{N_j}{\kappa} \tag{3.3.1.14}
\]

Substitute into \(3.3.1.12\)

\[
g_{1,2}^{(2)} \simeq \frac{\kappa}{N_j} + 2 \tag{3.3.1.15}
\]

In the experiment, if we assume signal and idler have the same loss, the cross-correlation \( g_{1,2}^{(2)} \) can be calculated by simply measuring the single counts.
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Figure 3.3.2: $g^{(2)}_{1,2}$ versus average photon number per mode $\mu$ with $M = 1$ (left). $g^{(2)}_{1,2}$ versus mode number per pulse $M$ with $\mu = 1 \times 10^{-5}$ (right). Blue line: twin photon quantum light $g^{(2)}_{1,2} > 1$. Red line: coherent light $g^{(2)}_{1,2} = 1$.

Simulation To investigate the different parameters (loss $\eta$, power $\mu$ and mode number $M$) effects on $g^{(2)}_{1,2}$, we simulate $g^{(2)}$ based on equation 3.3.1.11:

$$g^{(2)}_{1,2} = \frac{1}{M\mu} + \frac{1}{M} + 1 \quad (3.3.1.16)$$

From equation 3.3.1.16 we can see $g^{(2)}_{1,2}$ is independent of loss $\eta_j (j = 1, 2)$.

Figure 3.3.2 shows quantum cross correlation $g^{(2)}$ is approaching to classical coherent light $g^{(2)} = 1$ as average photon number $\mu$ and the mode number per realization $M$ increasing. Based on this results, $g^{(2)}$ method requires pump power is low ($\mu \ll 1$) and the detector only detect small number of modes per realization ($M \approx 1$).

3.3.2 Noise Reduction Factor $\sigma$

In sub-shot noise protocol, noise reduction factor (NRF) is an important figure of merit. The smaller the value of NRF the better performance of the system. NRF also can be used to measure the correlation of non-classical light. From equation 2.3.2.2 and 2.3.2.3 in the loss-free environment, the NRF of coherent light should be 1, and the NRF of two modes squeezing quantum light should be 0. But in the real experiment, due to the loss and noise in the system NRF of quantum light will be a value between 0 and 1.

To investigate the loss and mode number effect on NRF, we substitute equation 3.2.0.5
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Figure 3.3.3: Noise Reduction Factor (NRF) of two modes squeezing quantum light and classical coherent light.

into NRF equation 2.3.2.1

\[
NRF_{\text{quantum}} = \frac{\langle \delta^2 n_1 \rangle + \langle \delta^2 n_2 \rangle - 2 \text{cov}(n_1, n_2)}{\langle n_1 \rangle + \langle n_2 \rangle}
= \frac{\eta_1 + \eta_2 - 2\eta_1 \eta_2 + \eta_1^2 \mu - 2\eta_1 \eta_2 \mu + \eta_2^2 \mu}{\eta_1 + \eta_2}
\]

\[
(3.3.2.1)
\]

\(n_1\) and \(n_2\) are independent for classical coherent light, so \(\text{cov}(n_1, n_2)\) of coherent light is always equal to 0:

\[
NRF_{\text{coherent}} = \frac{\langle \delta^2 n_1 \rangle + \langle \delta^2 n_2 \rangle - 0}{\langle n_1 \rangle + \langle n_2 \rangle}
= \frac{M\eta_1 \mu + M\eta_2 \mu}{M\eta_1 \mu + M\eta_2 \mu}
= 1
\]

\[
(3.3.2.2)
\]

Figure 3.3.3 shows NRF comparison between quantum light and classical light. Both NRF of quantum and classical light are independent of mode number per pulse \(M\) and average mode number \(\mu\). However, as the loss increasing NRF of quantum light will approaching to classical limit 1.

Both \(g^{(2)}\) and NRF can detect the non-classical correlation of quantum light. \(g^{(2)}\) method is insensitive to loss, but \(g^{(2)}\) decrease to classical if the average photon is too high or mode per pulse is too larger. This limit \(g^{(2)}\) method only can be used in low power regime. In opposite to \(g^{(2)}\) method, NRF method is independent of power and
Figure 3.4.1: Theory model of quantum light generation and detection with loss and noise. The loss of photon idler and signal are $\eta_1$ and $\eta_2$ respectively. Noise $\langle N_b \rangle$ is added to signal path through a 50:50 beam splitter.

mode number. But NRF is very sensitive to loss. Each method has its advantages, depends on the situation different method should be chosen.

### 3.4 Noise Effect

In the real quantum application and experiment, environment noise effects the SNR and results. To simulate the noise effects, we add a beam splitter at the signal path to mix signal with noise $\hat{a}_b$. The model in 3.2.1 is changed to:

Because adding noise to one path is enough to show the effect of noise. So to simplify the process, we only add noise to signal path. In Figure 3.4.1, noise $\langle N_b \rangle$ and signal is mixed by a 50:50 beam splitter. It causes signal has additional 50% loss and contains $N_b$ noise photon per pulse.

$$b_5 = \frac{1}{\sqrt{2}} b_4 + i \frac{1}{\sqrt{2}} a_b$$  \hspace{1cm} (3.4.0.1)

Substituting it into photon statistics equation 3.2.0.5 and we can obtain:

$$\langle n_1 \rangle = M \mu \eta_1$$

$$\langle n_2 \rangle = \langle \hat{b}_5^\dagger \hat{b}_5 \rangle = \frac{1}{2} M (N_b + \mu \eta_2)$$

$$\langle n_1 n_2 \rangle = \frac{1}{2} M \mu \eta_1 (N_b M + \eta_2 (1 + \mu + M \mu))$$

$$\langle n_1 n_1 \rangle = M \mu \eta_1 (1 + (1 + M) \mu \eta_1)$$

$$\langle n_2 n_2 \rangle = \frac{1}{4} M (N_b + \mu \eta_2) (2 + N_b + N_b M + (1 + M) \mu \eta_2)$$

(3.4.0.2)
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Figure 3.4.2: Noise effects on $g^{(2)}$. Left: $g^{(2)}$ versus noise photon $N_b$, average photon number per mode $\mu = 1 \times 10^{-4}$, mode per pulse $M = 1$, mode 2 efficiency $\eta_2 = 1$. Right: $g^{(2)}$ versus signal loss, $\mu = 1 \times 10^{-4}$, noise $N_b = \mu$, mode per pulse $M = 1$. Red curve is classical coherent light, $g^{(2)} = 1$.

Substituting above equation into $g^{(2)}$ equation 3.3.1.2:

$$g^{(2)}_{1,2} = \frac{N_b M + (1 + \mu + M \mu) \eta_2}{M(N_b + \mu \eta_2)}$$  \hspace{1cm} (3.4.0.3)

The same for NRF, from equation 3.3.2.1 we can write:

$$NRF_{quantum} = \frac{N_b(2 + N_b) + 4\mu^2 \eta_1^2 + 2(1 + N_b)\mu \eta_2 + \mu^2 \eta_2^2 - 4\mu \eta_1(-1 + (1 + \mu) \eta_2)}{2(N_b + 2\mu \eta_1 + \mu \eta_2)}$$  \hspace{1cm} (3.4.0.4)

The simulation plot of $g^{(2)}$ equation 3.4.0.3 is shown in Figure 3.4.2. From the $g^{(2)}$ plot, we can see the quantum cross-correlation $g^{(2)}$ is fragile to noise. The $g^{(2)}$ value degrade by half if the noise level is equal to signal level, $N_b = \mu$. In the presence of noise, the loss tolerance $g^{(2)}$ start to become sensitive to loss. As shown in Figure 3.4.2, the $g^{(2)}$ value will drop sharply as the efficiency of mode 2 decreasing.

The noise effect to NRF is shown in Figure 3.4.3. Because of the additional 3dB loss of beam splitter on the signal path, the NRF is not zero at low noise condition. The same as $g^{(2)}$, NRF value is degraded in the presence of noise. Due to the noise, NRF is smaller at high power. To explore full parameter space, we sweep both noise and loss value. Figure 3.4.4 shows the simulation results. From the Figure 3.4.4, we can see $g^{(2)}_{1,2}$ has good performance at low loss and low noise region. NRF is also sensitive to loss and noise, the unbalance of the signal, and idler path loss will degrade NRF. In addition to
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3.4.3: Noise effects on NRF. Left: NRF versus noise photon $N_b$, average photon number per mode $\mu = 1 \times 10^{-4}$, mode per pulse $M = 1$, efficiency $\eta_1 = 1, \eta_2 = 1$. Right: NRF versus average photon number per mode $\mu$, noise $N_b = \mu$, mode per pulse $M = 1$

- Red curve is classical coherent light, NRF=1.

the unbalanced loss, the environment noise will cause NRF increasing.

3.5 Conclusions

In the section, the experiment setup we used to characterize quantum source is introduced. We demonstrate how to use pulse laser and two single photon detectors to measure photon statistics of the quantum source. Our quantum source operates at the spontaneous regime, which makes it possible for us to use two single photon detectors to measure photon number. We show that the total loss of signal and idler can be estimated using photon statistics. The theory model of quantum cross-correlation $g^{(2)}$ and noise reduction factor (NRF) in the lossy noisy environment is built. We demonstrate that in ideal condition $g^{(2)}$ is insensitive to loss and has better performance at low power. NRF is sensitive to loss and degraded if signal and idler detection efficiency is unbalance. However, in noisy and lossy condition, the performance of both $g^{(2)}$ and NRF is degraded due to quantum decoherence.
Figure 3.4.4: Noise effects on $g_{1,2}^{(2)}$ and NRF. Left: noise and loss effects on $g_{1,2}^{(2)}$, higher $g^{(2)}$ value is better. Right: noise and loss effects on NRF, the lower NRF value the better.
Chapter 4

On-Chip Quantum Correlation

Quantum applications utilize quantum correlation to enhance their performance. From the last section, we show cross-correlation $g^{(2)}_{1,2}$ and noise reduction factor (NRF) are two important figure of merits for quantum correlation. In this section, we implement the $g^{(2)}_{1,2}$ and NRF measurement methods in the lab and investigate the noise effects on quantum correlation.

4.1 Quantum Correlation Characterization

4.1.1 Experiment Setup

Based on the photon statistics measurement model (Figure 3.2.1), a experiment setup is build. Figure 4.1.1 shows the simplified experiment setup. In the experiment, an 80MHz pulse laser with center wavelength 780nm, 100 fs duration is used to pump our BRW nonlinear waveguide. The pump power and polarization are controlled by a half wave plate (HWP) and polarizer beam splitter (PBS). The TE polarization, the 1mW pump is focused onto the waveguide through a 100x lens. Type-II SPDC generates two orthogonal polarization photon pairs. The output signal and idler photon pairs are collected by the 40x lens. CCD camera and 780nm detector are used for alignment purpose. The 780nm pump is removed by a 1450nm long pass filter. The collinear signal (V polarization) and idler (H polarization) photon are separated using a PBS. The idler photon is collected by a single mode fiber (SMF) and detected by ID210 gated single photon detector. ID210 is externally triggered by the 80MHz pulse laser signal with gated width 2ns, dead time 20us and detection efficiency 25%. Signal photon is detected by free-running mode ID220 single photon detector. The dead time of ID220 is set to 20us and detection efficiency is 20%. Two detectors click timestamps are recorded by ID800 time to digital converter.
Figure 4.1.1: Experiment setup used to measure photon statistics of type-II SPDC quantum source. 780nm 100fs pulse pump is coupled onto BRW waveguide by 100x lens; the generated SPDC photon pairs are collected by 40x lens; 1450nm long pass filter removes 780nm pump; signal and idler photon are detected by ID220 and ID210 single photon detector; single photon detectors click timestamps are recorded by ID800 time to digital converter (TDC). The minimum time resolution of ID800 is 81ps. The single photon detector click for each pulse, \( n_j[k] \) \( (j = 1, 2; k = 1, 2, ... \) ), can be retrieved from the ID800 timestamps. Coincidence for each pulse can be calculated from \( n_c[k] = n_1[k]n_2[k] \). ID800 also able to calculate and plot the coincidence, which makes calculation process of CAR value very easy (Figure 3.3.1). The following quantities are calculated from experiment data:

i) The click of detector at each pulse, \( n_j[k] \) \( (j = 1, 2) \).

ii) The coincidence click at each pulse, \( n_1[k]n_2[k] \).

We record \( \kappa \) number of pulse is equal to repeat the experiment \( \kappa \) times (\( \kappa \) realizations). With the average measurement equation 3.2.0.6, we can obtain the average photon number \( \langle n_j \rangle \) and coincidence \( \langle n_1n_2 \rangle \).

In order to estimate the loss of this system, we need to calculate the temporal mode number per realization \( M \) first. \( M = M_{\text{spatial}} \times M_{\text{temporal}} \), where \( M_{\text{spatial}} = 1 \) because our signal and idler is a single TIR mode. Our pump pulse duration \( T_{\text{pump}} = 100fs \), signal/idler photon center wavelength \( \lambda_{\text{coh}} = 1560nm \). Twin photon bandwidth is about 600nm (left figure 4.1.2), but due to 1450nm long pass filter and detector detection upper limit 1600nm (right figure 4.1.2), the real detected bandwidth is \( \Delta \lambda_{\text{coh}} \approx 100nm \). Substituting those values into equation 3.2.2.2, we can get the temporal mode number per realization \( M \approx 1 \).

Substituting \( M = 1 \) into equation 3.2.2.3, we can obtain efficiency for signal (H) and
idler (V) are $\eta_1 \approx 0.71\%$ and $\eta_2 \approx 0.93\%$ respectively. The SMF coupling efficiency is about $\eta_{SMF} \approx 30\%$ and the output objective lens efficiency is about $\eta_{Lens} \approx 50\%$. The total efficiency is $\eta_{total} = \eta_{waveguide} \times \eta_{lens} \times \eta_{SMF} \times \eta_{detector}$, we can estimate the efficiency in waveguide is about $\eta_{waveguide} \approx 30\%$.

### 4.1.2 Quantum Cross Correlation $g_{1,2}^{(2)}$

The three different $g_{1,2}^{(2)}$ measurement methods are implemented in the lab, and their results are compared. Figure **4.1.3** shows the experiment results.

Theory prediction is calculated from equation 3.4.0.3 with weak background noise ($N_b = 4 \times 10^{-6}$). Method 1 and method three results match our expectation very well. However $g^{(2)}$ value calculated from coincidence to accidental ratio (CAR) method is much lower than the other two methods. This divergence is caused by noise in the experiment setup. Due to the limitation of equipment, we use one free-running detector instead of the gated detector in the experiment. In method 1 and 3, we post select valid clicks of the free-running detector, which reduce the noise of free-running detector. But in method 2, the CAR is directly measured from ID800 coincidence count module. Hence the noise is much higher. In next section, we will show noise can degrade the $g^{(2)}$ value.

### 4.1.3 Noise Reduction Factor (NRF)

We measured NRF of classical and quantum light in the experiment. Quantum light is the twin photon generated via type-II SPDC. Classical light is the correlated thermal light generated by passing thermal light through a beam splitter. Figure [4.1.4](#) shows NRF
Figure 4.1.3: $g_{1,2}^{(2)}$ experiment results of different methods: photon number probability, coincidence to accidental ratio (CAR), average photon number. The dashed line is the theory prediction. Temporal mode number $M=1$. The total efficiency of signal and idler are 0.7% and 0.9% respectively. Noise photon per mode is $N_b = 4 \times 10^{-6}$.

Experiment results. In this experiment, the idler and signal loss are calculated using the equation 3.2.2.3, $\eta_1 = 0.71\%$, $\eta_2 = 0.93\%$.

From the plot, we can see the quantum experiment results matches the theory simulation (equation 3.3.2.1) very well. In the ideal situation, NRF of classical light should always larger than 1. However, in our results, NRF of classical light is decreasing as the power increasing. This can be explained by the two reasons: high photon number violates low power assumptions ($\mu \ll 1$) and saturation of the detector.

In zeros loss and noise environment, NRF of twin photon pairs should be 0. But due to the unavoidable loss in our experiment setup, we only get $NRF_{quantum} = 0.9945$.

4.2 Noise Effect on Quantum Correlation

In real quantum applications, quantum correlation can easily be degraded by environment noise. Motivated by practicality, the noise effects on quantum correlation $g_{1,2}^{(2)}$ and NRF
are investigated in the experiment.

### 4.2.1 Experiment Setup

Based on theory model in Figure 3.4.1, we add noise and loss model into the system. Figure 4.2.1 shows the modified experiment setup:
Figure 4.2.1: Experiment setup used to measure photon statistics of type-II SPDC quantum source. 780nm 100fs pulse pump is coupled onto BRW waveguide by 100x lens; the generated SPDC photon pairs are collected by 40x lens; 1450nm long pass filter removes 780nm pump; signal and idler photon are detected by ID220 and ID210 single photon detector; loss of signal (V) path is tuned by HWP and PBS pair; signal photon mix with amplified spontaneous emission (ASE) noise from semiconductor optical amplifiers (SOA) through 50:50 fiber beam splitter; single photon detectors click timestamps are recorded by ID800 time to digital converter (TDC).

The flat spectrum white noise is generated from amplified spontaneous emission (ASE) of semiconductor optical amplifiers (SOA). Noise level can be tuned using a digital tunable attenuator. The noise is mixed with signal photon using a 50:50 fiber beam splitter. The beam splitter adds the additional 3dB loss to the signal path. Signal photon loss is controlled by tuning the connection of fiber.

To compare the quantum light with classical light, we also measure the quantum correlation of correlated thermal light in the experiment. The correlated thermal light is generated by passing the idler photon through a 50:50 fiber beam splitter. In fact, the $g^{(2)}$ of correlated thermal light is the self-correlation of idler photon. The loss is controlled by tuning the connection of fiber.
Figure 4.2.2: Experiment setup used to measure photon statistics of correlated thermal light. SPDC generate orthogonal polarization photon pair. Signal (V) is blocked. Idler thermal light is separated using a 50:50 fiber beam splitter to generate the correlated thermal state. One arm is detected directly by ID210 detector, the other arm is passed through loss and noise module and detected by ID220. Loss can be adjusted by tuning the coupling of the fiber connector. Noise is generated using ASE from SOA.

To make the classical and quantum results comparable, the source brightness need to be the same. We tuned the pump power to make sure the single photon number of the classical and quantum experiment are equal. As a result, the correlated thermal light and quantum light have the same photon flux $\langle n_j \rangle$ ($j = 1, 2$) at the detector.

### 4.2.2 Quantum Cross Correlation $g^{(2)}_{1,2}$

We measure quantum correlation $g^{(2)}$ of twin photon source and correlated thermal light in the experiment (Figure 4.2.3). As we can see from the results, the self-correlation of thermal light is always 2. The cross-correlation of twin photon source is degraded to classical limit if the noise is too high.
Figure 4.2.3: $g_{1,2}^{(2)}$ experiment results of twin photon quantum source (blue circle) and thermal light (red triangle). The blue solid line is the original simulation. Blue dashed line is the $g_{1,2}^{(2)}$ simulation with noise correction. Temporal mode number $M = 1$, idler and signal path efficiency are $\eta_1 = 0.56\%$ and $\eta_2 = 0.39\%$ respectively. Idler and signal average photon number are $\langle n_1 \rangle = 1.4 \times 10^{-4}$ and $\langle n_2 \rangle = 5.1 \times 10^{-5}$.

In our experiment, the noise is generated from SOA, which is a continuous wave (CW) source. But our quantum source is fs duration pulse with 12.5ns repetition rate (80MHz). Our gated detector has gate width 2ns, so only $2/12.5 \approx 1/6$ of CW noise is in the experiment realization. To include this effect in the simulation, we need to reduce the noise photon number per realization $N_b$ to $N_b/6$ in $g^{(2)}$ equation 3.4.0.3. The dashed blue curve in Figure 4.2.3 is the theory prediction after the noise correction, which matches the experiment results very well.

To investigate the loss effect on $g^{(2)}$ in the experiment, we add additional loss into the signal path. The experiment results (Figure 4.2.4) confirm with our simulation results, where correlation $g^{(2)}$ is decreasing as the loss increasing. The background noise of system is estimated to be $N_b = 4 \times 10^{-6}$. 
Figure 4.2.4: $g^{(2)}_{1,2}$ versus signal loss experiment for twin photon quantum source (blue circle) and thermal light (red triangle). Blue dashed line is the $g^{(2)}_{1,2}$ simulation with noise $N_b = 4 \times 10^{-6}$. Temporal mode number $M = 1$, idler and signal path efficiency are $\eta_1 = 0.47\%$ and $\eta_2 = 0.47\%$ respectively. Idler average photon number are $\langle n_1 \rangle = 1.5 \times 10^{-4}$.

4.2.3 Noise Reduction Factor (NRF)

We use the same setup as $g^{(2)}$ to measure the NRF value. Figure 4.2.5 shows the how NRF changing with noise. Theory model matches the experiment results at noise below $1 \times 10^{-3}$. The decreasing of both quantum and classical light at high noise level is due to the saturation of single photon detectors.
Figure 4.2.5: NRF of twin photon quantum source (blue circle) and thermal light (red triangle). The solid blue line is the original simulation. Blue dashed line is the NRF theory prediction with noise correction. Temporal mode number $M = 1$, idler and signal path efficiency are $\eta_1 = 0.56\%$ and $\eta_2 = 0.39\%$ respectively. Idler and signal average photon number are $\langle n_1 \rangle = 1.4 \times 10^{-4}$ and $\langle n_2 \rangle = 5.1 \times 10^{-5}$.

The loss has the similar effects as the noise. In high loss environment, NRF of quantum light approaches the classical NRF limit 1 (Figure 4.2.6).
Figure 4.2.6: $g^{(2)}_{1,2}$ versus signal loss experiment for twin photon quantum source (blue circle) and thermal light (red triangle). Blue dashed line is the $g^{(2)}_{1,2}$ simulation with noise $N_b = 4 \times 10^{-6}$. Mode number $M = 1$, idler and signal path efficiency are $\eta_1 = 0.47\%$ and $\eta_2 = 0.47\%$ respectively. Idler average photon number are $\langle n_1 \rangle = 1.5 \times 10^{-4}$.

4.3 Conclusions

We characterized our BRW SH3 waveguides using two quantum correlation: quantum cross-correlation $g^{(2)}$ and NRF.

In the first part, we calculate the temporal mode number per pulse is calculated to be around 1. The experiment results of three different $g^{(2)}$ measurement methods are compared. We demonstrate photon number probability method and average photon number methods give very similar $g^{(2)}$ value. But CAR method has much lower $g^{(2)}$ value due to the system noise. We also conduct a sub-shot noise experiment in the lab. The noise reduction factor (NRF) of our quantum source is measured to be 0.9945. The high NRF value is due to the high loss in the signal ($\eta_2 = 0.71\%$) and idler ($\eta_1 = 0.93\%$). We also show the NRF of classical light is about 1.

In second part, we repeat the $g^{(2)}$ and NRF measurement under high noise and loss. Both $g^{(2)}$ and NRF are degraded in the presence of noise and loss, which matches our
expectation.

As we may notice, in our experiment the NRF of quantum light is very close to 1. The performance of sub-shot noise experiment can be improved by reducing the system loss. For our experiment, we can use MMF to increase fiber coupling from 30% to 75%. We also can improve the detection efficiency by replacing the free running detector with triggered detectors.
Chapter 5

Tapered Waveguide JSA Engineering

5.1 Background

Joint spectrum amplitude (JSA) of the biphoton pairs generated from SPDC plays an important role in many quantum applications, such as high-purity heralded single photon [43], clock synchronization [24], correlated photons [25], high-dimensional Hilbert space [26]. Different application has different requirements of JSA, hence the ability to engineer and characterize JSA is important.

For twin photon generated via SPDC, we can see JSA contains all frequency information signal and idler photon. Equation 2.1.0.13 can be written into two parts:

\[
f(\omega_s, \omega_i) = E_p(\omega_s + \omega_i)\phi(\omega_s, \omega_i)
\]  

(5.1.0.1)

The first part \(E_p\) is pump envelope intensity (PEI). If the pump is Gaussian with bandwidth \(\delta_p\), we can write it as:

\[
|E_p(\omega_s + \omega_i)|^2 = \exp\left[-\left(\frac{\omega_s + \omega_i - \omega_p}{\delta_p}\right)^2\right]
\]  

(5.1.0.2)

The second part \(\phi(\omega_s, \omega_i)\) is phase matching intensity (PMI):

\[
|\phi(\omega_s, \omega_i)|^2 = |\text{sinc}(\Delta k L/2)|^2
\]  

(5.1.0.3)

Figure 5.1.1 shows an JSA example. As we can see from the plot, the shape of JSA is defined by PEI and PMI. The width of PEI is proportional to the bandwidth of pump.
Figure 5.1.1: Example of JSA plot. The left figure shows pump envelope intensity (PEI) of a Gaussian pulse. Center figure shows the phase matching intensity (PMI) of a device with -85.3 degree slope. The right figure shows the joint spectrum amplitude (JSA) by combine PEI and PMI.

The width of PMI is controlled by the phase matching (PM) bandwidth of the waveguide device, which is defined by the waveguide structure. The slope of PEI is always -45 degree. The slope of PMI can be calculated from the gradient of $\Delta k$ \[85\]:

$$
\text{grad}\Delta k = \left( \frac{\partial \Delta k}{\partial \omega_s}, \frac{\partial \Delta k}{\partial \omega_i} \right) \quad (5.1.0.4)
$$

$$
= \left( \frac{1}{v_p} - \frac{1}{v_s}, \frac{1}{v_p} - \frac{1}{v_i} \right) \quad (5.1.0.5)
$$

Where $v_j (j = s, i, p)$ are the group velocity of signal, idler and pump photon. The PMI tilting angle can be expressed as:

$$
\theta = \arctan\left( -\frac{1}{v_p} - \frac{1}{v_s} \right) \quad (5.1.0.6)
$$

The spectrum of signal and idler photon can be calculated by projecting the JSA onto x or y two axis.

$$
f(\omega_s) = \int_0^{+\infty} d\omega_i f(\omega_s, \omega_i) \quad (5.1.0.7)
$$

$$
f(\omega_i) = \int_0^{+\infty} d\omega_s f(\omega_s, \omega_i)
$$

Figure 5.1.2 shows the spectrum of signal and idler photon pair in Figure 5.1.1.
Figure 5.1.2: Signal and idler photon spectrum of Figure 5.1.1.

Figure 5.2.1: Top view of BRW SH3 ridge waveguide. $2\omega$ Bragg mode pump generates $\omega$ signal and idler in TIR mode through SPDC on the chip. a) straight waveguide with ridge width 2um. b) tapered waveguide with ridge width tapered from 2um to 3um.

5.2 Theory

From the previous section, we know JSA contains pump envelope intensity (PEI) and matching intensity (PMI) two parts. PEI function is defined by the external pump. PMI is dependent on the structure of waveguide. In our experiment, we use BRW SH3 design to generate SPDC. The cross-section structure detail of BRW waveguide can be found in Appendix A. Based on standard straight BRW structure, a tapered ridge BRW structure is designed to enhance the control of JSA. (Figure 5.2.1 b).

To calculate the PMI of the waveguide, we need to find phase mismatching $\Delta k(\omega_1, \omega_2)$. The relationship between the intensity of SPDC photon pair and phase mismatching can
be described as 2.1.0.9

\[ E(L, \omega_1, \omega_2) = \int_0^L \exp[-i \Delta k(\omega_1, \omega_2)z]dz \] (5.2.0.1)

For tapered waveguide, because the structure is changing along the \( z \) direction, phase mismatching of taper structure is dependent on \( z \). To model this taper structure, we can write equation 5.2.0.1 as:

\[ E(L, \omega_1, \omega_2) = \int_0^L \exp[-i \int_0^z \Delta k(z', \omega_1, \omega_2)dz']dz \] (5.2.0.2)

The phase mismatching \( \Delta k(z, \omega_1, \omega_2) \) can be obtained by solving modes along the \( z \) direction. PMI of the \( L \) length waveguide is proportional to output down-converted field \( E \):

\[ \text{PMI}(\omega_1, \omega_2) = |E(L, \omega_1, \omega_2)|^2 \] (5.2.0.3)

From equation 5.1.0.2, pump envelope intensity (PEI) is:

\[ \text{PEI}(\omega_1, \omega_2) = |E_p(\omega_1, \omega_2)|^2 = \exp[-(\frac{\omega_1 + \omega_2 - \omega_0^p}{\delta \omega_p})^2] \] (5.2.0.4)

Where \( \omega_1 \) and \( \omega_2 \) are the signal and idler photon respectively. \( \omega_0^p = 2\pi c/\lambda_0^p \) is the central frequency of the pump. \( \delta \omega_p \) is the bandwidth of pump. We can assume the pulse pump is Gaussian:

\[ \delta \omega_p = \frac{2\sqrt{2\log 2}}{\Delta T_{\text{pump}}} \] (5.2.0.5)

\( \Delta T_{\text{pump}} \) is the pulse duration. Once we get PMI and PEI, we can calculate the JSA of the single and idler photon pairs:

\[ \text{JSA}(\omega_1, \omega_2) \propto \text{PEI}(\omega_1, \omega_2) \times \text{PMI}(\omega_1, \omega_2) \] (5.2.0.6)

Figure 5.2.2 and 5.2.3 show the pulse pump JSA simulation of straight and tapered waveguide respectively. The spectrum of signal and idler can be calculated from JSA using equation 5.1.0.7. The normalized spectrum of straight and tapered waveguide under 100fs pump is shown in Figure 5.2.4. Comparing the JSA of the tapered and straight waveguide, they have similar JSA length, but JSA width of the tapered waveguide is
Figure 5.2.2: Joint spectrum amplitude (JSA) of 2um ridge width straight waveguide. Pump pulse duration $T_{\text{pump}} = 100\text{fs}$, pump center wavelength $\lambda_0^p = 780.96\text{nm}$. JSA length: 600nm. JSA width: 2nm.

Figure 5.2.3: Joint spectrum amplitude (JSA) of 2um-3um tapered ridge width waveguide. Pump pulse duration $T_{\text{pump}} = 100\text{fs}$, pump center wavelength $\lambda_0^p = 791.04\text{nm}$. JSA length: 600nm. JSA width: 25nm.

around 25nm, the width of straight waveguide only has 2nm. The photon bandwidth of both straight and tapered waveguide is around 600nm.

To investigate the PEI effect on the JSA shape, we also simulate JSA under CW pump (Figure 5.2.5 and 5.2.6). A 10ps pulse is used to simulate semi-CW pump. Comparing CW pump JSA with pulse pump JSA, pulse pump has wider PEI, and the JSA is wider and longer. Under the CW pump, the tapered waveguide has slightly longer JSA than the straight waveguide. The width of the straight and tapered waveguide is very narrow. The discontinued JSA line in the plot is due to the computational artifacts. We can reduce this artifact by increasing the simulation points.
5.3 Experimental Setups and Results

The joint spectrum of straight and tapered BRW SH3 waveguides are measured in the experiment (5.3.1). Our waveguide is pumped by an 80MHz 100fs pulse pump to generate orthogonal polarization photon pairs. The collinear photon pairs are coupled into a 2km long fiber. The 100fs signal and idler pulse are stretched to around 10ns and then separated by a fiber polarization beam splitter. To reduce ID210 noise, an 80MHz trigger signal from fs laser is used to gate the ID210 detector with 10ns gate width. We also need to record the pulse time reference, but the 80MHz frequency is too high, and it will saturate the ID800 time to digital converter (TDC). So we use the other 40MHz signal from fs laser to trigger a function generator to produce an 8.7KHz signal.

The detection output of ID210 and ID220 click is connected to CH2 and CH3 of ID800 TDC. 8kHz laser trigger signal is connected to CH1. The path difference between signal and idler can be obtained from coincidence peak delay. Figure 5.3.2 shows the JSA calculation process. In the calculation, we can retrieve the 80MHz laser pulse trigger from 8kHz CH1. The arrival time of CH2 and CH3 photon can be expressed in terms of ID800 bin number. ID800 has timing resolution 81ps per bin size. Hence it has $12.5\text{ns}/81\text{ps} \approx 154$ bins per pulse duration. Due to the dispersion, frequency domain correlation between photon pairs become time correlation in the temporal domain. So the spectrum shape of signal and idler can be retrieved from Figure 5.3.2 a) and b) pulse temporal shape. The full JSA can be obtained from coincidence counts at $d1$ and $d2$ time delay.

The long fiber used in the experiment is about 2km, which is not long enough to
stretch the pulse to maximum 12.5ns duration. From Figure 5.3.2 we can see the pulse is only about 5ns, 40% of full 12.5ns. Due to the limitation of detector range and long-pass filter, the bandwidth of detected signal and idler photon is only about 100nm. The bin number difference can be written in terms of wavelength: \( \frac{\Delta d}{\Delta \lambda} = \frac{N}{(100 \text{nm})/0.4} = 0.616 \text{bin/nm} \), where \( N = 12.5\text{ns}/81\text{ps} \approx 154 \) is total bin number in 12.5ns.

JSA of a straight waveguide (Figure 5.3.3) and a 2-3um tapered waveguide (Figure 5.3.4) are measured in experiment.

Length and width of the straight waveguide are around 100nm and 22 nm. The tapered waveguide has the same 100nm length and 22nm width. Tapered waveguide and straight waveguide have the same JSA length, which matches our expectation. However, compare to JSA simulation in the previous section, the JSA width of the straight waveguide is much wider. This expanded width is due to the timing resolution limit of equipment. The timing resolution of ID210 and ID220 detector is 200ps and 250ps respectively. The laser trigger also has a fluctuation 80MHz \( \pm 1MHz \), which is about 300ps, four bins. Due to those timing resolution limit, CH2 and CH3 data have about > 4 bins uncertainty on each channel. To confirm that, we also measured JSA without long fiber in Figure 5.3.5. In the ideal situation, the JSA width of fs pulse should be one bin, but due to the timing uncertainty measured JSA width expand to about 12 bins, 20nm.
5.4 Conclusions

In this section, we show the tapered BRW structure can shape JSA. We first build JSA theory model. JSA of straight and tapered waveguides for CW and pulse cases are simulated. We demonstrate straight and tapered waveguide has the same JSA length, 600nm. But JSA of the tapered waveguide has a larger width (25nm) compare to straight waveguide (2nm) in pulse pump case.

In experiments, we measured the JSA of a 3um ridge width straight waveguide and a 2-3um tapered waveguide using the 80MHz pulsed pump. For straight waveguide, the width and length of JSA are 22nm and 100nm. For tapered waveguide, JSA width and length are 22nm and 100nm. Compared with simulation, the length difference is due to detector bandwidth limitation and long-pass filter. The difference in width is due to the timing uncertainty of equipment.

In the current setup, the stretched pulse duration is around 5ns, which do not reach the limit 12.5ns. Resolution of JSA can be improved by increasing the length of the long fiber. JSA width difference between straight and tapered waveguides will be more significant if the width of JSA is larger than the timing uncertainty 12 bins.
Figure 5.3.1: Experiment setup used to JSA. 780nm 100fs pulse pump is coupled onto BRW waveguide by 100x lens; the generated SPDC photon pairs are collected by 40x lens; 1450nm long pass filter removes 780nm pump; signal and idler photon are detected collect by a single mode fiber (SMF) and pass through 2km long fiber; signal and idler polarization is control by a fiber polarization controller (FPC); signal and idler photon are separated by a fiber polarization beam splitter (PBS); signal (V) photon is detected by ID220 free running; idler is detected by gated mode ID210 with 10ns gate width; 80MHz laser trigger is used to gate ID210, 40MHz trigger is down sample to 8.7KHz and used as time reference; 8.7KHz time reference and single photon detectors click timestamps are recorded by ID800 time to digital converter (TDC).
Figure 5.3.2: JSA calculation process. a,b) CH2(CH3) histogram, d1(d2) is the bin number difference between detection click and CH1 pulse trigger. c) JSA(d1,d2) is the coincidence counts of CH2 photon at time delay d1 and CH3 photon at time delay d2.

Figure 5.3.3: JSA of 3um ridge width straight waveguide with 180s integration time and 2mw pump.
Figure 5.3.4: JSA of 2-3μm ridge width tapered waveguide with 180s integration time and 2mw pump.

Figure 5.3.5: JSA of 2-3μm ridge width tapered waveguide without long fiber.
Chapter 6

Quantum Illumination (QI) On-Chip

6.1 Background

In recent years, quantum imaging and quantum metrology have attracted lots of attention [40, 41]. Due to the non-classical correlation of quantum source, quantum imaging protocols show better performance than classical protocols. Among those quantum protocols, some of them can beat the classical limit and show great potential for real practical applications.

Shot noise is caused by the uncertainty of photon number. The fluctuation of the photon number adds additional noise to electric current of the detector, which limits the detection SNR. For classical light, the coherent state has the minimum photon number variance. The coherent state with $N$ average photon number per mode has photon number variance $N$ due to the Poissonian statistics [86]. Sub shot noise quantum imaging protocol is proposed to use photon pairs generated via SPDC to resolve this problem. Signal and idler photon number has the same fluctuation, as a result, the photon number difference between signal and idler has less uncertainty. This protocol is very useful in detecting low absorption object in low flux illumination regime.

Ghost imaging was thought as one of the few early real applications of quantum imaging. As with most quantum imaging protocols, it uses photon pairs generated from SPDC process as the light source. Signal photon passes through the object and is collected by a bucket detector. The idler photon is collected directly by a spatial resolving detector (usually CCD camera). Ghost imaging protocol utilizes the spatial correlation between signal and idler to retrieve the spatial information of an object from a bucket detector. This feature makes ghost imaging protocol very useful in high spatial distortion environment. For example, an object is in the highly scattering environment or object itself is highly scattering and diffusive (biomedical tissues).
The resolution of classical light is limited by standard quantum limit, $\frac{1}{\sqrt{N}}$. In [23], it demonstrates quantum imaging using entangled photons can beat the diffraction limit and achieve the Heisenberg bound, $\frac{1}{N}$. Where $N$ is the number of the entangled photon. To achieve higher resolution, a large number of entangled photons is crucial. In [21][22], high N00N states with 2-5 photons are generated.

Quantum illumination is a very promising protocol [18][20]. Unlike the other protocols introduced above, which are very sensitive to noise and loss [27][28]. The entanglement can be easily broken by environment noise and loss, which makes the quantum imaging are hard to realize for practical application. In [39], it shows the quantum illumination (QI) can still have better performance than classical illumination (CI) even in the noisy and lossy environment. Quantum illumination protocol attracts lots of attention due to this noise resilience feature. The low-brightness quantum radar was proposed in [16], it shows quantum radar is stealthier, more resilient to jamming and higher SNR for low reflection object. In [20] and [14], it shows in high noise and loss environment quantum illumination protocol has better bit error rate (BER) and SNR than classical light in communication.

Quantum illumination sends a probe photon to detect an object in high noise and loss environment. If the object is there, probe photon will hit the object and reflect back to the detector. Otherwise, in case of the object is not there, detector only detect background noise.

**Classical Light** The untangled classical light in state $\rho_c$ is considered first. In case there is not object, the detected state can be expressed as:

$$\rho_{c0} = \rho_{th}$$  \hspace{1cm} (6.1.0.1)

where $\rho_{th}$ is the background thermal light, which can be written in terms of Fock state:

$$\rho_{th}(b) = \sum_{n=0}^{\infty} \frac{b^n}{(b+1)^{n+1}} |n\rangle_k \langle n|$$  \hspace{1cm} (6.1.0.2)

with $b$ is the average number of noise photon in mode $k$. Under the low photon assumption $b \ll 1$, we can ignore the high order of $b$ and simplify (6.1.0.1) to:

$$\rho_{c0} = (1 - d_1 b) |vac\rangle \langle vac| + b \sum_{k=1}^{d_1} |k\rangle \langle k| = (1 - d_1 b) |vac\rangle \langle vac| + b I_c$$  \hspace{1cm} (6.1.0.3)
where \( I_c \) is the identity operator on the single photon, \( d_1 \) is the number of distinguishable modes per realization. In the second case, the object is there, part of probe photon is reflected back and detected by the detector. We can write the detected state as:

\[
\rho_{c1} = (1 - \eta)\rho_{c0} + \eta \rho_c
\]  

(6.1.0.4)

Where \( \eta \) is reflectivity of the object, \( \rho_c \) is the density operator of classical probe light.

Object detection process is to distinguish the two quantum state \( \rho_{c0} \) and \( \rho_{c1} \). The upper bound of minimal error probability to distinguish \( \rho_{c0} \) and \( \rho_{c1} \) can be described using Quantum Chernoff Bound (QCB) method. More details about QCB can be found in Appendix B. The error probability is given by \( p_{err}(\kappa) \approx \frac{1}{2} Q^\kappa \), where \( \kappa \) is number of realization. In [39], it shows the \( Q \) parameter of classical illumination is:

\[
Q_c = \min_{0 \leq r \leq 1} (1 - \eta r + b(1 + (1 + \eta b) r)) + O(b^2, \eta b) 
\]  

(6.1.0.5)

**Quantum Light**  
The quantum state of photon pairs generated via type-II SPDC process is given by:

\[
|\psi\rangle_{si} = \frac{1}{\sqrt{d_1}} \sum_k |k\rangle_s |k\rangle_i 
\]  

(6.1.0.6)

The object absence case can be written as:

\[
\rho_{q0} = \rho_{th} \otimes \frac{I_i}{d_1} 
\]  

(6.1.0.7)

\[
= [(1 - d_1 b) |\text{vac}\rangle_s \langle \text{vac}| + b I_s] \otimes \frac{I_i}{d_1} + O(b^2) 
\]

where \( I_s \) and \( I_i \) are the identity operators on the single photon Hilbert spaces for the signal and idler photon respectively. In the other case, the signal photon has probability \( \eta \) to be reflected by the object and detected. The received state is given by:

\[
\rho_{q1} = (1 - \eta)\rho_{q0} + \eta \rho_q
\]  

(6.1.0.8)

where \( \rho_q = |\psi\rangle_{si} \langle \psi| \) is the density operator of quantum light. The QCB \( Q \) parameter of quantum illumination with SPDC light is given by [39]:

\[
Q_q = \min_{0 \leq r \leq 1} (1 - \eta r + \frac{b}{d_1}(-1 + (1 + \eta d_1 b) r)) + O(b^2, \eta b) 
\]  

(6.1.0.9)

Comparing \( Q \) parameter equation (6.1.0.5) and (6.1.0.9) we can clearly see the effective noise reduction from \( b \) to \( b/d_1 \). This also explicitly highlight the importance of number
of distinguishable mode per realization $d_1$ in quantum illumination protocols. If $d_1$ is polarization degree of freedom, then $d_1 = 2^m$, where $m$ is number of bits of entanglement, $m = 1$ for SPDC twin photon pair. The enhancement is increasing exponential with the number of entangled photon.

6.2 Theory

6.2.1 Quantum Illumination Experimental Model

In the quantum illumination theory proposed a few years ago [39], two Gaussian state quantum illumination receiver: OPA receiver and phase-conjugate receiver are proposed in [3]. In recent years, quantum illumination protocol is implemented in experiment using OPA receiver [14, 15] and coincidence counting receiver [18]. However, none of those receivers is the ideal receiver, and they only utilize parts of quantum correlation. In this thesis, we use coincidence counting receiver due to its practicality.

Quantum Illumination (QI) The theory model of coincidence counting quantum illumination receiver is very similar to the quantum correlation theory model in the previous chapter (Figure 3.4.1). In the QI scheme, shown in 6.2.1, the mode operators can be expressed as follows:

$$
\begin{align*}
    b_1 &= \sqrt{1 + \mu a_1} + \sqrt{\mu a_2^\dagger} \\
    b_2 &= \sqrt{1 + \mu a_2} + \sqrt{\mu a_1^\dagger} \\
    b_3 &= \sqrt{\eta_1} b_1 + i \sqrt{1 - \eta_1} a_{\text{vac}} \\
    b_4 &= \sqrt{\eta_2} b_2 + i \sqrt{1 - \eta_2} a_{\text{vac}} \\
    b_{\text{in}} &= \sqrt{1/2} (b_4 + ia_b) \\
    b_{\text{out}} &= \sqrt{1/2} (b_{\text{vac}} + ia_b)
\end{align*}
$$

(6.2.1.1)

with $b_1$ and $b_2$ being the reference and probe modes before losses, $b_3$ and $b_4$ the reference and probe modes after losses, and $b_{\text{in}}$ and $b_{\text{out}}$ the probe mode at detector D2 after the noise injection ($a_b$), when the target object is present (subscript "in") or absent (subscript "out") respectively. We can express the average number of photons at detectors D1 and
Figure 6.2.1: Theory model of quantum illumination protocol. Two orthogonal polarization photon pairs are generated through type-II SPDC process. One photon is used as the reference; the other photon is used as the probe to detect the object. A 50:50 beam splitter is used as the object. If the object is absence, detector only detects thermal noise. If object presence, the detector will detect noise and probe mix light. The loss of reference and probe path are $\eta_1$ and $\eta_2$ respectively. Thermal noise with average $N_b$ photon per realization is added to reference path.

D2 respectively as:

$$
\langle n_1 \rangle = \langle b_3^\dagger b_3 \rangle = M\eta_1\mu \\
\langle n_2 \rangle = \langle b_{in}^\dagger b_{in} \rangle = \frac{1}{2}M(N_b + \eta_2\mu) \\
\langle n'_2 \rangle = \langle b_{out}^\dagger b_{out} \rangle = \frac{N_b}{2}
$$

where $M$ is the number of temporal mode per realization in our case, $N_b$ is the number of noise photons per realization. From this we can derive the correlation function between the two detectors, in the presence or absence of the target:

$$
\langle S_{in} \rangle = \langle \delta n_1 \delta n_2 \rangle = \frac{1}{2}M\mu(1 + \mu)\eta_1\eta_2 \\
\langle S_{out} \rangle = \langle \delta n_1 \delta n'_2 \rangle = 0
$$
Figure 6.2.2: Theory model of classical illumination protocol. Two orthogonal polarization photon pairs are generated through type-II SPDC process. Vertical polarization photon is blocked, and horizontal polarization photon is passed through a 50:50 beam splitter. One arm is used as the reference; the other arm is used as the probe to detect the object. A 50:50 beam splitter is used as the object. If the object is absence, detector only detects thermal noise. If object presence, the detector will detect noise and probe mix light. The loss of reference and probe path are $\eta_1$ and $\eta_2$ respectively. Thermal noise with average $N_b$ photon per realization is added to reference path.

With the assumption of a noisy, lossy environment, the SNR for the QI scheme becomes:

$$ SNR_{QI} = \frac{\langle S_{in} - S_{out} \rangle}{\sqrt{\langle \delta^2 S_{in} \rangle + \langle \delta^2 S_{out} \rangle}} $$

$$ \langle S_{in} - S_{out} \rangle = \frac{1}{2} \eta_1 \eta_2 \mu (\mu + 1) M $$

$$ \langle \delta^2 S_{in} \rangle + \langle \delta^2 S_{out} \rangle = M (\eta_1 (2 \eta_2 \mu^2 (B (2 M^2 - M + 2) + M + 2) + 2 \mu (B (M^2 - M + 1) + 1) - 2 B \sqrt{\mu + 1} \sqrt{\mu (\mu + 1) \sqrt{\mu} (M - 1) M} + B (B + 2) \mu M + \eta_2^2 \mu (6 \mu^2 + 8 \mu + (4 \mu^2 + 4 \mu - 4 \sqrt{\mu + 1} \sqrt{\mu + 1} \sqrt{\mu} (M + 2)) + 2 (B + 1) \eta_2 (\mu + \mu M + 1) + B (B + 2) M + \eta_2^2 \mu (M + 2) + 2) } ) $$

(6.2.1.4)

Classical Illumination (CI) We use correlated thermal light as the light source for CI. In CI scheme (Figure 6.2.2), we pass horizontal polarization photon generated from SPDC through a 50:50 beam splitter to generate correlated thermal light. One arm is used as the reference; the other arm is used as the probe. The mode operators for the
CI scheme (Figure 6.2.2) are expressed as follows:

\[
\begin{align*}
    b_1 &= \sqrt{1 + \mu a_1 + \sqrt{\mu} a_2^\dagger}, \\
    e_1 &= \sqrt{1/2}(b_1 + ia_{\text{vac}}), \\
    e_2 &= \sqrt{1/2}(ib_1 + a_{\text{vac}}), \\
    e_3 &= \sqrt{\eta_1} e_1 + i \sqrt{1 - \eta_1} a_{\text{vac}}, \\
    e_4 &= \sqrt{\eta_2} e_2 + i \sqrt{1 - \eta_2} a_{\text{vac}}, \\
    e_{\text{in}} &= \sqrt{1/2}(e_4 + ia_b), \\
    e_{\text{out}} &= \sqrt{1/2}(a_{\text{vac}} + ia_b)
\end{align*}
\]

with \(e_1\) and \(e_2\) indicating the reference and probe modes before losses, \(e_3\) and \(e_4\) the reference and probe modes after losses, and \(e_{\text{in}}\) and \(e_{\text{out}}\) the probe mode at detector D2 after the noise injection, when the target object is present or absent respectively. It follows that:

\[
\begin{align*}
    \langle n_1 \rangle &= \langle e_3^\dagger e_3 \rangle = \frac{1}{2} M \eta_1 \mu, \\
    \langle n_2 \rangle &= \langle e_{\text{in}}^\dagger e_{\text{in}} \rangle = \frac{1}{4} M (N_b + \eta_2 \mu), \\
    \langle n_2' \rangle &= \langle e_{\text{out}}^\dagger e_{\text{out}} \rangle = \frac{B}{2}, \\
    \langle S_{\text{in}} \rangle &= \langle \delta n_1 \delta n_2 \rangle = \frac{1}{8} M \eta_1 \eta_2 \mu^2, \\
    \langle S_{\text{out}} \rangle &= \langle \delta n_1 \delta n_2' \rangle = 0
\end{align*}
\]

The correlation function between the two detectors, in the presence or absence of the target, is:

\[
\begin{align*}
    \text{SNR}_{CI} &= \frac{\langle S_{\text{in}} - S_{\text{out}} \rangle}{\sqrt{\langle \delta^2 S_{\text{in}} \rangle + \langle \delta^2 S_{\text{out}} \rangle}} \\
    \langle S_{\text{in}} - S_{\text{out}} \rangle &= \frac{1}{8} \eta_1 \eta_2 \mu^2 M \\
    \langle \delta^2 S_{\text{in}} \rangle + \langle \delta^2 S_{\text{out}} \rangle &= \frac{1}{32} \eta_1 \mu M (\eta_1 (\eta_2 (B(4\mu^2 + 4\mu^2 + 4\mu - 4\sqrt{\mu + 1/\sqrt{\mu}(\mu + 1)}\sqrt{\mu})M^2 \\
    &+ (-2\mu^2 - 4\mu + 4\sqrt{\mu + 1}\sqrt{\mu + 1/\mu(\mu + 1)}\sqrt{\mu})M) + 2\mu^2(M + 2)) \\
    &+ 2B(B + 2)\mu M + \eta_2^2 (3\mu^3 + (-2\sqrt{\mu + 1}\sqrt{\mu + 1/\mu(\mu + 1)}\mu^{3/2} + 2\mu^3 + 2\mu^2)M^2 \\
    &+ (2\sqrt{\mu + 1}\sqrt{\mu + 1/\mu(\mu + 1)}\mu^{3/2} - \mu^3 - 2\mu^2)M)) + 4(B + 1)\eta_2 \mu (M + 1) \\
    &+ 4B(B + 2)M + \eta_2^2 \mu^2 (M + 2)) + \frac{1}{16} B \eta_1 \mu M (\eta_1 \mu + 2)
\end{align*}
\]
Figure 6.2.3: QI CI SNR versus average photon number per mode \( \mu \) (Left) and temporal mode number per realization \( M \) (Right). Reference and probe path total efficiency are \( \eta_1 \) and \( \eta_2 \) respectively. QI uses photon pairs generated from SPDC, average photon number per mode is \( \mu \). CI uses correlated thermal light by splitting SPDC idler photon, average photon number per mode generated from SPDC is \( 2\mu \).

**QI and CI simulation** We next investigate noise \( N_b \), loss \( \eta \), power \( \mu \) and temporal mode number per realization \( M \) effects on SNR of QI and CI. To balance the QI and CI light source brightness, we use \( \mu \) in QI SNR 6.2.1.4 and \( 2\mu \) in CI SNR 6.2.1.7 in all simulations.

Figure 6.2.3 shows the how SNR of QI and CI changing with source brightness and detected mode in each realization. As the power and temporal mode number per realization increasing, the SNR of QI and CI increasing. Figure 6.2.4 shows the SNR ratio between QI and CI. As we can see from the plot, the SNR ratio is high at low photon number region. QI has better performance than CI in low brightness conditions.

The noise and loss effects on QI and CI are shown in Figure 6.2.5. SNR of both QI and CI will drop as the noise and loss increasing. Compared to CI protocol, QI protocol has higher SNR and less sensitive to noise. From QI/CI SNR ratio plot, Figure 6.2.6, QI protocol has more advantage over CI protocol at the high noise and loss region. Unlike other quantum applications are very fragile to noise and loss, QI protocol has better performance at higher noise and loss.

**6.2.2 Tapered Waveguide Improvement on QI SNR**

**QI with Ancillary Degrees of Freedom (DOF)** In [103], it proposes a new theory that utilizes frequency degree of freedom (DOF) to improve the QI protocol. To demon-
Figure 6.2.4: QI and CI SNR ratio. Left: SNR ratio versus average photon number, to get the same photon flux $\mu_{CI} = 2\mu_{QI}$. Right: SNR ratio versus detected temporal mode number per realization, $\mu_{CI} = 2\mu_{QI}$.

To demonstrate the model of ancillary DOF QI, we replace original SPDC quantum state (6.1.0.6) with two DOF:

$$\ket{\psi}_{si} = \frac{1}{\sqrt{d_1}} \frac{1}{\sqrt{d_2}} \sum_{k_1=1}^{d_1} \sum_{k_2=1}^{d_2} \ket{k_1, k_2}_s \ket{k_1, k_2}_i$$ (6.2.2.1)

where $k_1$ and $k_2$ are the two different DOF, $d_1$ and $d_2$ are the number of modes in that two DOF respectively. For object absent DOF, the received state can be expressed as:

$$\rho_{q0} = \rho_{th} \otimes \frac{I_i}{d_1 d_2}$$

$$= [(1 - d_1 d_2 b) \ket{vac}_s \bra{vac} + b I_s] \otimes \frac{I_i}{d_1 d_2} + O(b^2)$$ (6.2.2.2)

In the other case, the received state is:

$$\rho_{q1} = (1 - \eta)\rho_{q0} + \eta\rho_q$$ (6.2.2.3)

where $\rho_q = \ket{\psi}_{si} \bra{\psi}$ is the density operator of quantum light. The $Q$ parameter of QCB can be obtained by substituting $\rho_{q0}$ and $\rho_{q1}$ into $Q = \min_{0 \leq r \leq 1} \text{Tr}[\rho_{q0}^{-r} \rho_{q1}^r]$. After calculating the QCB bound (see Appendix B for details), we can get (103):

$$Q_q' = \min_{0 \leq r \leq 1} (1 - \eta r + \frac{b}{d_1 d_2}(-1 + (1 + \frac{\eta d_1 d_2}{b} r)^r)) + O(b^2, \eta b)$$ (6.2.2.4)
Figure 6.2.5: Noise and loss effects on QI CI SNR. $\mu$ is the average number of photon per mode. $N_b$ is the number of noise photon per realization. $M$ is the number of mode per realization. Probe path total efficiency is $\eta_2$. Left: QI SNR; Right: CI SNR

Comparison with one DOF QCB bound 6.1.0.9 shows effective noise per mode reduce from $b/d_1$ to $b/(d_1d_2)$, hence ancillary degrees of freedom can improve the QI protocol performance. If $d_1 = 2$ is polarization DOF and $d_2 = d_{freq}$ is frequency DOF, the total enhancement in terms of $m$ e-bits of entanglement is $d = (2d_{freq})^m$.

Coincidence Counting Receiver QI with Ancillary DOF Instead of using QCB with the ideal receiver to demonstrate QI improvement, here we show QI protocol improvement from practical coincidence receiver and SNR aspects. For an experiment with two entangled photons generated via SPDC ($m = 1$), $\kappa$ realization, $M$ temporal mode per realization, $N_b$ noise photon per realization, $\mu$ photon per signal mode, the SNR is:

$$SNR = f(\mu, M, N_b)$$

Where function $f$ is equation 6.2.1.4 for QI, equation 6.2.1.7 for CI. SNR is the signal to noise ratio per realization. If we have another degree of freedom, e.g., frequency, with $d_{freq}$ number of the distinguishable mode in that DOF, we can write the SNR equation as:

$$SNR = \sqrt{d_{freq}} \times f(\mu/d_{freq}, N_b/d_{freq})$$

We can think the $d_{freq}$ number of distinguishable modes as additional realizations. The original realization is split into $d_{freq}$ sub-realization. If the pump power keeps the same, the average photon number per mode is $\mu/d_{freq}$ due to the additional mode. The noise per realization also decrease to $N_b/d_{freq}$ due to the additional realization. The SNR per
Figure 6.2.6: QI CI SNR Ratio. $\mu$ is the average number of photon per mode. $N_b$ is the number of noise photon per realization. $M$ is the number of mode per realization.

Probe path total efficiency is $\eta_2$.

sub-realization will be $f(\mu/d_{freq}, N_b/d_{freq})$, so the total SNR per realization is $\sqrt{d_{freq} \times f(\mu/d_{freq}, N_b/d_{freq})}$.

Figure 6.2.7 shows the SNR comparison between QI with DOF and without DOF. From the simulation, we can see the QI with additional DOF can improve SNR a lot, especially at high noise and loss.

Tapered Waveguide Improvement on QI SNR. The last section shows QI SNR can be improved to the additional degree of freedom. In previous QI protocol discussion, we only considered the polarization DOF of the quantum source. The frequency DOF information of quantum source can be obtained from bi-photon joint spectrum amplitude (JSA). In chapter 5, we show JSA can be shaped through waveguide engineering, hence the frequency DOF can be controlled through the tapered waveguide.

Figure 6.2.8 shows the JSA of a 3um ridge width straight waveguide and a 2-3um tapered waveguide. JSA length of both straight and tapered waveguides are around 600nm. The JSA width of straight and tapered waveguides are 2nm and 25nm respectively. The number of distinguishable frequency mode depends on the frequency resolution of the detector. In our experiment, we use long fiber to stretch pulse. The frequency correlated
 photon pairs become time correlated. The number of distinguishable frequency mode is the number of temporal modes the detector can distinguish. The ID210 time resolution is around 0.2ns; the pulse duration time is 12.5ns (80MHz). So the maximum number of the distinguishable temporal mode is $12.5/0.2 \approx 62$. Because of the detectors wavelength limitation and long-pass filter, our detection system only has the 150nm bandwidth. The bandwidth per temporal mode can be estimated as $150/62 \approx 2.4 \text{ nm/mode}$. JSA length of both straight and tapered waveguide is larger than 150nm, so the maximum number of the temporal mode is 62 if we only consider JSA length. The width of JSA makes JSA contains more frequency correlated information. The number of mode along width for straight and tapered waveguide is $2/2.4 \approx 1$ and $25/2.4 \approx 10$ respectively. The total distinguishable correlated frequency modes are $1 \times 62 = 62$ and $10 \times 62 = 620$ for straight and tapered waveguide.

Figure 6.2.9 shows the SNR ratio of straight and tapered waveguide. As we can see from the plot, tapered structure can improve the QI SNR at least three times and has better performance than straight waveguide at high loss and noise environment.
Figure 6.2.8: JSA of straight (a) and tapered (b) BRW SH3 waveguides with 100fs pulse pump. $w_1$: JSA length, $w_2$: JSA width. a) straight waveguide with width 3um. $w_1 \approx 600$nm, $w_2 \approx 2$nm b) tapered waveguide with ridge width 2-3um. $w_1 \approx 600$nm, $w_2 \approx 25$nm

6.3 Experimental Setups and Results

6.3.1 QI/CI Comparison

QI protocol measures the contrast between object presence and absence two situations. The experiment setup for QI with an object is the same as setup used for photon statistics measurement Figure 4.1.1. We block the probe (V) photon path to simulate the object absence situation. The same for CI experiment setup, the object presence setup is the same as Figure 4.2.2. For CI object absence situation, we disconnect the fiber in loss module to block probe photon.

To balance the QI and CI source brightness, we set the pump power of CI to about twice of QI pump power. Figure 6.3.1, 6.3.2 and 6.3.3 show the SNR per pulse for QI and CI in noise and loss environment. From the experiment results, we can see QI always has better performance than CI in noise and loss environment.

6.4 Conclusions

In this section, we first build the theory model of quantum illumination (QI) and classical illumination (CI). The effects of noise, loss, power and temporal mode number are simulated. We demonstrate the QI outperforms CI protocol at high noise and loss region. These results are confirmed in the experiment using our BRW quantum source. The experiment results show QI has about 10dB SNR improvement compared to CI.
We also demonstrate QI SNR can be improved at least 8dB through frequency degree of freedom (DOF). By engineering the JSA shape through the tapered waveguide, we can achieve additional 6dB SNR improvement. As the noise and loss increasing, the improvement due to frequency DOF and tapered structure also increasing.
CHAPTER 6. QUANTUM ILLUMINATION (QI) ON-CHIP

Figure 6.3.1: QI and CI SNR versus average photon number $\langle n_1 \rangle$. Left: SNR without any additional loss and noise. Right: SNR in high noise ($N_b = 1 \times 10^{-3}$) and loss (18dB loss in prob path) environment.

Figure 6.3.2: QI and CI SNR versus probe path loss. Left: SNR in zero noise environment. Right: SNR in high noise ($N_b = 1 \times 10^{-3}$) environment.
Figure 6.3.3: QI and CI SNR versus probe path noise. Left: SNR without any additional loss. Right: SNR in high loss (18dB loss in prob path) environment.
Chapter 7

Conclusions and Future Roadmap

7.1 Summary of Contributions

This thesis discussed how to characterize and engineer spontaneous parametric down conversion (SPDC) quantum source on chip. The environment noise and loss effects on the quantum source were also investigated. This study paved the way to practical quantum applications.

We first build the theory model of photon statistics measurement using two single photon detectors and pulse pump. The loss and temporal mode number are measured in the experiment. Then we compared two non-classical quantum correlation characterization methods, cross-correlation $g^{(2)}_{1,2}$ and noise reduction factor (NRF). In zero noise environment, NRF is independent of power, but $g^{(2)}_{1,2}$ value is degraded as pump power increasing. In the noisy environment, both $g^{(2)}_{1,2}$ and NRF are degraded at the high noise and loss region. We also measured the $g^{(2)}$ and NFR in the experiment using BRW device to confirm our theory model.

The joint spectrum amplitude (JSA) of BRW device was simulated and measured. We demonstrate that JSA shape can be modified by engineering the BRW structure. JSA length of tapered and straight waveguide is similar (600nm). But under the pulse pump, tapered waveguide has much wider JSA (25nm) compared to straight waveguide (2nm). We also measured the JSA of straight and tapered in the experiment with pulse pump and long fiber. Due to the limitation of equipment, JSA of straight waveguide is much wider than simulation. The JSA resolution can be improved using longer fiber.

The ability to engineer the on chip quantum source and characterize its performance in the noisy and lossy environment makes the practical quantum applications possible. We demonstrate the SNR of quantum illumination (QI) protocol can be improved using the frequency degree of freedom (DOF). By engineering the JSA, we can achieve about
14dB improvement compared to original QI protocol. The comparison between original quantum illumination (QI) and classical illumination (CI) are also implemented in the lab using on chip SPDC source. We show original QI has 10dB SNR improvement over CI, which matches our theory model.

7.2 Future Roadmap

We measure JSA of straight and tapered waveguides in the lab. However, the fiber we use is not long enough to stretch the pulse to 12.5ns, which reduces the resolution of JSA measurement. It can be improved by adding a long fiber. In theory simulation, we demonstrate QI SNR can be improved through tapered waveguide. But this improvement needs to be observed and confirmed in the lab. In the experiment, the noise level is limited by the detector saturation. We could push the QI operation bound to higher noise level if high saturation detector is used. In the experiment, the total efficiency $\eta_1$ and $\eta_2$ can be improved by replacing SMF with MMF and using higher efficiency detectors.

This thesis demonstrates our ability to characterize quantum source performance in the noisy environment and engineer quantum source on chip. Based on current work, there are several potential research topics and directions.

**Quantum Imaging & Sensing** In this study, we demonstrate the one of quantum sensing protocols, quantum illumination. One direction is to continue the quantum imaging/sensing research. Based on requirements of different imaging/sensing system, we can engineer our on chip quantum source to provide unique advantages over the conventional quantum source.

**Non-Gaussian State Quantum Illumination** In current work, all quantum illumination is implemented using Gaussian state quantum source \[18, 20\]. In recent years, non-Gaussian states are reported in the lab \[9, 10, 12, 13\]. In \[11\], it demonstrates the non-Gaussian state can carry more information than the conventional Gaussian quantum state. Quantum imaging/sensing protocols, such as Quantum illumination (QI), may benefit from the non-Gaussian source.
Appendix A

BRW SH3 Structure

The layer structure of BRW SH3 device is shown in Figure A.0.1. The device is made using AlGaAs material; the substrate is GaAs. The growth configuration is [001] direction, which is commonly used in Al$_x$Ga$_{1-x}$As material waveguide manufacture. The core layer thickness is about 500nm and the refractive index around 0.61. The core layer is surrounded by two matching layers to enhance the conversion efficiency. Two matching layers are used to increase the modal overlap between Bragg mode and output TIR mode. Two Bragg mirrors consist repeat layers are symmetric to the waveguide core. The thickness and material of the BRW layers are shown in the Table A.1.

To get the ridge structure, we etch the waveguide through the core; the etch depth is about 5um. The ridge width of the device is about 3um.
Figure A.0.1: BRWs layer structure with 5μm etches depth and 3μm ridge width. The core layer is surrounded with matching layer and two symmetric Bragg mirrors.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Core</th>
<th>Matching</th>
<th>Repeat 1</th>
<th>Repeat 2</th>
<th>Substrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness [nm]</td>
<td>500</td>
<td>374.9</td>
<td>461.5</td>
<td>129.2</td>
<td>700</td>
</tr>
<tr>
<td>x</td>
<td>0.61</td>
<td>0.2</td>
<td>0.7</td>
<td>0.25</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A.1: Thickness and refractive index of different layers of BRW device. x is the aluminum fraction in Al$_x$Ga$_{1-x}$As. Repeat 1,2 are two repeat layer in Bragg mirror.
Appendix B

Quantum Chernoff Bound (QCB)

In sensing and communication protocols, multiple signals are sent to the receiver. The error probability bound of the system can be described by Quantum Chernoff Bound (QCB) \[1,2\]. In quantum illumination protocol, the probability of error is the probability to distinguish $\rho_0$ and $\rho_1$ two detected states. For $N$ repeatable measurement, we can write the minimal error probability as:

$$P_{err}(N) = \frac{1}{2}(1 - \frac{1}{2}||\rho_1^{\otimes N} - \rho_0^{\otimes N}||)$$ (B.0.0.1)

where $||R|| = Tr[R^\dagger R]$. The calculation of this error probability is very difficult, but the upper bound of equation [B.0.0.1] can be obtained from QCB:

$$P_{err}(N) \leq P_{QCB}(N) = \frac{1}{2}Q^N$$ (B.0.0.2)

If $N$ is large, we can approximate the minimal error probability as $P_{err}(N) \approx \frac{1}{2}Q^N$. The parameter $Q$ is given by:

$$Q = \min_{0 \leq r \leq 1} Tr[\rho_0^{1-r}\rho_1^r]$$ (B.0.0.3)

In $b \ll 1$ region, we can write the two case $\rho_0$ and $\rho_1$ for classical illumination with single photon as:

$$\rho_{c0} = (1 - d_1b) |vac\rangle \langle vac| + b \sum_{k=1}^{d_1} |k\rangle \langle k|$$ (B.0.0.4)

$$\rho_{c1} = (1 - \eta)\rho_{c0} + \eta\rho_c$$

We have:

$$\rho_{c0}^{1-r} = (1 - d_1b)^{1-r} |vac\rangle \langle vac| + b^{1-r}I_c$$

$$\rho_{c1}^r = (1 - bd - \eta)^r |vac\rangle \langle vac| + b^r(I_c - \rho_c) + (b + \eta)^r\rho_c$$ (B.0.0.5)
Thus we can get the $Q$ parameter in QCB:

$$Q_c = \min_{0 \leq r \leq 1} (1 - \eta r + b(-1 + (1 + \frac{\eta}{b}))^r) + O(b^2, \eta b) \quad (B.0.0.6)$$

For quantum illumination, the density matrix operator of the two cases are:

$$\rho_{q0} = [(1 - d_1 b) |vac\rangle_s \langle vac| + b I_s] \otimes \frac{I_i}{d_1} + O(b^2) \rho_{q1} = (1 - \eta) \rho_{q0} + \eta \rho_q \quad (B.0.0.7)$$

So we have:

$$\rho_{q0}^{1-r} = [(1 - d_1 b)^{1-r} |vac\rangle_s \langle vac| + b^{1-r} I_s] \otimes \frac{I_i}{d_1} + O(b^2)$$

$$\rho_{q1}^r = (1 - bd - \eta)^r |vac\rangle_s \langle vac| \otimes \frac{I_i}{d}$$

$$+ (1 - \eta)^r (\frac{b}{d})^r (I_s \otimes I_i - \rho_q) + ((1 - \eta)^r \frac{b}{d} + \eta)^r \rho_q \quad (B.0.0.8)$$

The $Q$ parameter can be obtained:

$$Q = \min_{0 \leq r \leq 1} (1 - \eta)^r (1 + \frac{b}{d}(-1 + (1 + \frac{\eta d}{b(1 - \eta)})^r))$$

$$= \min_{0 \leq r \leq 1} (1 - \eta r + \frac{b}{d_1}(-1 + (1 + \frac{\eta d_1}{b})^r)) + O(b^2, \eta b) \quad (B.0.0.9)$$
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