MIDDLE SCHOOL TEACHERS' SUCCESSES AND CHALLENGES IN SUPPORTING STUDENTS' MATHEMATICAL COMMUNICATION USING MANIPULATIVES AND TECHNOLOGY IN A PROFESSIONAL DEVELOPMENT STUDY

by

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A thesis submitted in conformity with the requirements for the degree of Master of Arts Department of Curriculum, Teaching and Learning Ontario Institute for Studies in Education University of Toronto

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2018

Abstract

The case studies of four inner-city Grade 6 educators involved in a year-long Professional Development (PD) study were examined to determine: a) successes and challenges teachers face supporting students' mathematical communication, b) teachers' manipulatives and technology use to support communication, and c) the study's effects on teachers' attitudes and beliefs. The Ten Dimensions of Mathematics Education Framework (McDougall, 2004) was used to examine the findings. Teachers had success with learning environment, student tasks and teachers’ comfort with mathematics. Teachers faced many challenges, primarily in learning environment, communicating with parents, and teacher’s comfort with mathematics. Teachers provided varied integration of manipulative use, and need to continue to integrate technology, to support mathematics communication. It was determined that, while the PD study was greatly successful in providing support towards aligning teachers' attitudes and beliefs with current mathematical practices, PD alone is insufficient for improving teachers' attitudes.
Acknowledgements

The completion of my thesis, and my Master of Arts degree, has been a journey that was supported by many people, and I would like to take this opportunity to express my sincere gratitude to all who helped me with my accomplishments.

First, and foremost, I must thank my supervisor, Dr. Douglas McDougall. Your support and patience made writing this thesis much easier than I could have ever anticipated. Thank you for your constant encouragement and understanding, especially through unexpected life events. I have learned a tremendous amount from you, and it has truly been a pleasure to have been your student.

I must also express my gratitude to my second reader, Dr. James Hewitt. Thank you for taking the time to read my thesis, and for providing helpful feedback.

Thank you to my participating educators: Gloria, Alison, Betty, and Joanne. Without you this thesis would not be possible. Your thoughtful and candid responses provided tremendous insight into the world of Grade 6 mathematics.

I would like to thank the research team who participated in the larger mathematics study for support with the project. I wish you all the best in your future endeavors.

My studies were generously supported by the Social Studies and Humanities Research Council of Canada, making the journey of completing my Master's degree significantly less stressful. Thank you for choosing to support my project.

I am fortunate to have received much support aside from the academic realm, and must begin with thanking my husband, Jonathan Yau. Your unwavering support and encouragement throughout my entire degree - through both successes and challenges - means more than you will ever know. I feel extremely blessed to have you in my life.
I would also like to thank my daughter, Mary, for reminding me every day to question what I know about the world. Watching you interact with the world around you motivates me to be innovative in my approach to life. You inspire me every day to try new things, both in and out of the classroom. I dedicate this thesis to you.

A thank you to my parents, Emily and Tasso, and my sister, Georgia. Mom, Dad, thank you for raising me to be the person I am today, and for your constant encouragement. Your love and support throughout my life, and especially during the middle of my degree, means so much to me. Georgia, thank you for always being there for me. I am lucky to be able to call you my sister.

Finally, to my friends and colleagues, thank you for your support over the last few years. Thank you again to all of you for being a part of this journey.
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Chapter One: Introduction

1.1 Introduction

The purpose of this thesis is to identify how four inner-city Grade 6 mathematics educators utilize manipulatives and technology to support development in mathematics communication. In my experience, mathematics is a subject that many students struggle to grasp, and I am very interested in learning more about how teachers help students to grasp concepts.

Given our increasingly technology-driven world, I am curious regarding technology’s role in supporting mathematics achievement and improvement, particularly in the area of mathematics communication. This chapter describes the research context and questions addressed by the research, as well as the significance of the study and the plan of the thesis. My relationship with the study will also be discussed.

1.2 Research Context

Mathematics is one of the subjects considered to be a top priority in education by many stakeholders, including parents, educators, and policymakers (Boaler, 2015). While mathematics was once perceived as an elite subject that could only be mastered by few people in society, policymakers are pushing for a more inclusive approach to mathematics education believing that all students are capable of a high level of mathematics learning (NCTM, 2000; Ontario Ministry of Education, 2005). A strong foundation in mathematics and comfort with the material can set up students for success in their educational careers, as well as life in general (NCTM, 2000).

In Ontario, teachers must follow the expectations dictated by the Ministry of Education for each subject and grade that they teach, as laid out in the Curriculum documents. In addition, each Curriculum document includes information relevant to the particular subject. The Ontario Mathematics Curriculum, Grades 1 to 8 (Ontario Ministry of Education, 2005) outlines five
mathematics strands and seven mathematical processes. The five strands are: Number Sense and Numeration, Measurement, Geometry and Spatial Sense, Patterning and Algebra, and Data Management and Probability (Ontario Ministry of Education, 2005). The seven mathematical processes are: Problem Solving, Reasoning and Proving, Reflecting, Selecting Tools and Computational Strategies, Connecting, Representing, and Communicating (Ontario Ministry of Education, 2005). The Ontario Ministry of Education separates the strands and processes because it recognized that students needed to be applying the processes in all mathematics work and not solely in relation to certain strands (Ontario Ministry of Education, 2005). However, the two lists are separated only to make their content explicit as the processes and the acquisition of student knowledge and skills are interconnected (Ontario Ministry of Education, 2005).

The National Council of Teachers of Mathematics (NCTM), the largest organization in the world to focus on mathematics education, is a guiding force in shaping mathematics educational policy in both the United States and Canada. According to NCTM's mission statement, the organization "advocates for high-quality mathematics teaching and learning for each and every student" (NCTM, 1989). In 2000, NCTM published a list of Principles and Standards that they suggest should be used when creating and delivering an effective mathematics program. There are five Content Strands that lay out in which strands students need to gain knowledge. The strands are: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability (NCTM, 2000). In addition, there are five Process Strands that specify the ways through which students should apply content knowledge to achieve mastery in the content: Problem Solving, Reasoning and Proof, Communication, Connections, and Representations (NCTM, 2000).
For this thesis, I will focus on the Communication Process Standard. The Communication Standard stresses the importance of mathematical communication as the catalyst for reflection, clarification, and deeper understanding of concepts (NCTM, 2000). Through communication, students learn to form arguments, discuss different perspectives, learn how to listen effectively, and accurately use language to convey ideas and their thinking in oral and written form (NCTM, 2000).

The Ten Dimensions Framework (McDougall, 2004) also supports a focus on communication, through explicitly naming Students' Mathematical Communication (Dimension 8) as a necessary component of any successful mathematics program. According to this framework, providing students with communication opportunities in math allows students to have a better and deeper understanding of concepts and their own thinking (McDougall, 2004). Moreover, student communication in math helps make student thinking more apparent to teachers, which allows teachers to help students with any misconceptions (McDougall, 2004).

All of the skills listed above can then be transferred to other subjects and areas of students' lives, making them better prepared to face the challenges of the future (NCTM, 2000; Ontario Ministry of Education, 2005).

There are also six NCTM Principles: Equity, Curriculum, Teaching, Learning, Assessment, and Technology (NCTM, 2000). For this thesis, the Teaching Principle is of particular importance as it stresses the need for teachers to have a deep understanding of the mathematics they teach in order to effectively support student learning (NCTM, 2000). The Ten Dimensions Framework also supports having mathematics teachers with a high comfort level for mathematics, specifically with Dimension 10 - Teacher's Attitude and Comfort with Mathematics (McDougall, 2004). Teachers who are comfortable with math are better able to...
work with students to enhance student learning through deeper, real-world problems, and be flexible in the application of assessments based on student needs (McDougall, 2004; NCTM, 2000).

Teachers need to constantly engage in professional development (PD) activities. Participation in PD can support teachers to enhance their teaching practice, remain current, and gain knowledge on new and different pedagogy (McDougall, 2004; NCTM, 2000). Through ongoing professional development, teachers may also gain comfort with the content. Another significant aspect of the role of the teacher is to model a positive attitude towards mathematics, which may be easier when teachers are more comfortable with the relevant content and pedagogy (McDougall, 2004; NCTM, 2000).

The Ontario Ministry of Education (2005) has crafted policies to move away from traditional approaches and the belief that the goal of mathematics education is solely about mastering basic mathematics skills. The Ministry's vision now sees math as the avenue through which students learn "... a concise and powerful means of communication" (Ontario Ministry of Education, 2005, p. 3). Continuing on the idea of the role of communication in mathematics, the Mathematics Curriculum names communication as an "essential process" (Ontario Ministry of Education, 2005, p. 17) for mathematics learning. The current educational policy initiatives support teacher PD in mathematics as the preferred route to improving students' mathematical communication and in turn overall math understanding and learning. This focus on communication through professional development in mathematics teaching is crucial, and will be the topic explored in this thesis in the context of Grade 6 mathematics.
1.3 Research Questions

For my thesis, I explored how four Grade 6 educators support student learning in mathematics by focusing on mathematics communication, and how it is supported through the use of technology and manipulatives. My thesis will focus on the following research questions:

1. What successes and challenges do middle school teachers encounter in supporting students' mathematical communication?

2. How do middle school teachers utilize manipulatives and technology to support students' mathematical communication?

3. What effect does a year-long PD study have on teachers' attitudes and beliefs?

1.4 Significance of the Study

This study is significant because the case studies shed some light on key successes and challenges faced by mathematics teachers when supporting students' mathematical communication. Educators may gain insight into how to improve their students' mathematical communication, as well as potential challenges they can avoid through careful planning. Furthermore, administrators can utilize this information to support their mathematics team, and carefully consider the effect of any school policies on students' learning.

Given the large professional development component of the study, the findings regarding individual teacher differences, and their response to PD opportunities, may prove useful to administrators and school districts. PD is an area given great importance in the current educational climate, and administrators want to maximize the impact of workshops and training on their educators in order to maximize the effect on student learning through the participation of teachers in meaningful PD opportunities. This PD involvement could lead to higher engagement
with the material, on the part of the teacher, and achieve more gains in moving teachers towards adopting attitudes and practices that align with current, research-based mathematics instruction.

The study school, with its low achievement scores, low socioeconomic status, and high student needs, provides an opportunity to examine at-risk students' mathematics communication in the mathematics classroom. Students at the study school face many challenges, both academically and in the community, and the study findings may allow teachers to gain understanding, as well as pedagogical ideas, for how to support the most at-risk students in their own classrooms.

Finally, this study uses the Ten Dimensions of Mathematics Education Framework (McDougall, 2004) to examine successes and challenges faced by middle school teachers in supporting mathematics communication. There is only some research available on this framework, and this study will help by adding to the body of research on the Ten Dimensions of Mathematics Education Framework (McDougall, 2004) for both a successful professional development approach, as well as a mathematics communication focus.

1.5 Background of the Researcher

My interest in this topic stems from my own experiences as a mathematics teacher and continuously looking to improve my teaching practice to better engage my students and support their learning. My own early mathematics teachers focused solely on getting to the 'right answer', allowing only for a single approach. This singular approach not only discouraged creativity, but also any discussion about the process of problem-solving, relevant concepts, or misconceptions. In addition, student supports were nearly non-existent. Technology, at that time, was limited to calculators, and manipulatives were reserved only for remediation purposes of 'dumb' students.
I realized early in my teaching career that I wanted to move away from the traditional approaches I encountered in my own mathematics education, even though I received pushback as early as my first practicum, in a Grade 8 classroom, during my initial teacher training. My first Associate Teacher, in particular, thought I was naive for wanting to utilize manipulatives and include discussions that went beyond taking up homework or tests. However, this setting provided my first glimpse into the power of math discourse, especially when supported with manipulative and technology use. I saw an increase in achievement and confidence in my students in just a short time period.

My next math teaching experience came when I was teaching a summer school remedial mathematics course for Grade 9 students, who had already failed the course at least once. Sadly, many of my colleagues had a poor perception of my students' abilities, and had given up on them becoming successful. However, I found that, with the implementation of math discussions and collaborative activities, students became more engaged and showed a deeper understanding of concepts.

At this particular school, manipulatives and technology were limited. However, I had some success in creating my own manipulatives and utilizing diagrams to support students. In just over a month, I found students were excited for class and many were performing better than they had ever experienced in their mathematics career. One student told me he never thought he would enjoy a math class, but that he had in my classroom.

I am fortunate to have taught in a Grade 7 and 8 mathematics classroom. From the start, I placed a focus on mathematics communication, supporting students with math talk prompts, modeling, and think-aloud activities. At first, students saw math talk as meaningless, but soon realized we could delve deeper into concepts, and better understand them. High-level students
appreciated the challenge, and low-level students were able to identify their misconceptions and improve their learning.

With the Grade 7 and 8 students, I once again focused on manipulative use to support concept exploration, but faced significant resistance from my students. Students viewed manipulatives as only needed by those who struggled. With much effort and persistence, students eventually increased their manipulative use, although some quite reluctantly. On the other hand, students were much more perceptive to technology use and were enthusiastic during its use. Unfortunately, once again, I was at a school with limited technological resources.

During this appointment, I participated in a professional development (PD) series focused on building innovative practices in mathematics. I found the resources provided to be extremely useful, eliciting a high level of engagement from my students. Furthermore, the activity examples provided were rich tasks that encouraged collaboration, and I was able to focus on my mathematics communication goal. However, I found the PD information to be quite disjointed, and had difficulty with planning in a way that did not feel like I was jumping around from topic to topic. In addition, I found some of the sessions to not be useful as I already had a background in mathematics, and felt comfortable with the content.

With these experiences in mind, I am very interested in successes and challenges faced by teachers in using mathematics communication in their practice. I am also interested in how teachers utilize manipulatives and technology to support mathematical communication in their teaching. Given my experience as a PD participant, I am interested in best practices for effective mathematics PD.
1.6 Plan of the Thesis

This thesis is divided into five chapters to organize the details of my study and provide the reader with the necessary details. Chapter One introduces the background and significance of the study, and lays out the research questions.

Chapter Two contains a literature review of previous research and findings in the study area. Particular emphasis is placed on professional development in elementary education, mathematics education, mathematics communication, and the integration of technology and manipulatives. Through the lens of policy directives, mathematical communication, as well as technology and manipulatives implementation in mathematics, is examined. The Ten Dimensions of Mathematics Education (McDougall, 2004) is used as a framework for assisting educators in professional growth.

The methods used in this study are described in Chapter Three. Data collection and analysis are included, as are ethical considerations pertaining to the study. As the participants from this study are drawn from the larger School Improvement Study (McDougall et al., 2014), I describe how the larger study sets the tone for this study.

Chapter Four outlines the case studies of the four educators in the study. Through the examination of the four cases, I illustrate what successes and challenges three Grade 6 teachers and one principal face while supporting mathematics education through the use of mathematical communication. Participants' use of technology and manipulatives to support mathematics communication are also examined. In addition, I examine the effects of the PD study on teachers' attitudes and beliefs.
Finally, in Chapter Five I answer the research questions outlined in Chapter One using the findings from the cases in Chapter Four. In addition, I address areas of possible future research.
Chapter Two: Literature Review

2.1 Introduction

As in all areas of education, mathematics teachers are always striving to improve their practice in order to increase their students' learning. One area mathematics educators focus on to support their students' learning is the implementation of mathematics communication. While there are numerous benefits to the use of mathematics communication in the classroom, there are also challenges. Thus, teachers can learn more, and gain support in improving their practice, through participation in professional development opportunities.

As there are numerous approaches to professional development available, teachers may gain greater benefit from an existing framework, such as the Ten Dimensions of Mathematics Education (McDougall, 2004), to aid them in identifying areas of need and provide criteria for exceptional teaching in each component of mathematics. In this chapter, I will review all of these areas, and their impact on student learning, as well as concerns about mathematics education from stakeholders.

2.2 Mathematics Education - Ontario Context

Currently, there is an increasing focus on improving mathematics education. This focus is caused, in part, by concerns about student performance on large-scale assessments (Boaler, 2015). In many countries around the world, there is an increased value placed on scores from large-scale assessments, and a constant desire to improve students' performance (Boaler, 2015). While this phenomenon is very powerful in the United States of America (Boaler, 2015), with students being tested at least yearly during most of their elementary education (EQAO, date unknown), Ontario students are also subjected to large-scale assessments.
In Ontario, the Education Quality and Accountability Office (EQAO) administers province-wide large-scale assessment tests in mathematics, reading, and writing in Grade 3 (Primary) and 6 (Junior), as well as Grade 9 mathematics, and the Ontario Secondary School Literacy Test in Grade 10. The purpose of these assessments is to determine the number of students that meet or exceed the provincial standards, based specifically on the expectations outlined in the Ontario Curriculum. Between 2013 and 2017, both Primary and Junior mathematics scores have declined (EQAO, 2017). Primary scores dropped from 67% to 62% of students performing at or above the provincial standard (EQAO, 2017). Junior scores dropped from 57% to 50% of students performing at or above the provincial standard (EQAO, 2017).

Canadian and Ontario students also participate in international large-scale assessments. The latest Trends in International Mathematics and Science Study (TIMSS), administered in 2015, found that Canadian and Ontario students are performing above the international average in mathematics at both the Grade 4 and 8 level (Council of Ministers of Education, Canada, 2017). However, at the Grade 4 level, twenty-six countries scored higher than Canada, while four countries scored the same statistically, and eighteen countries scored lower than Canada (Council of Ministers of Education, Canada, 2017). At the Grade 8 level, Canadian students performed much better, with only six countries scoring higher than Canada, three countries at the same score statistically, and twenty-nine countries achieving a lower score (Council of Ministers of Education, Canada, 2017).

Another international assessment that Canadian and Ontario students participate in is the Programme for International Student Assessment (PISA). PISA is an international assessment that measures 15-year olds' achievement in reading, mathematics, and science (EQAO, 2016). PISA scores showed a decline in Ontario mathematics achievement scores from 2003 to 2012,
with a stabilization in 2015 (EQAO, 2016). Ontario mathematics scores were in the 25th percentile of participants, with only eleven jurisdictions scoring higher statistically (EQAO, 2016).

Historically, the focus of assessment was typically quantitative (Novak & Gowin, 1984). In Ontario, there is a shift away from assessment that is purely quantitative and for the purpose of evaluation. The Ministry of Education outlines, in its *Growing Success* document, three types of assessment categories: diagnostic, formative, and summative (Ontario Ministry of Education, 2010). Furthermore, the document supports the use of varied assessments for, as, and of learning (Ontario Ministry of Education, 2010). Assessment practices now require teachers to consider: conversations, observations, and products (Ontario Ministry of Education, 2010). Its stated goal for assessment is improving student learning, and supporting student development using strategies such as descriptive feedback (Ontario Ministry of Education, 2010). With the move away from purely quantitative approach to assessment in the classroom, mathematics education displays a disconnect with the emphasis on quantitative results from large-scale assessments.

Mathematics education is being placed in a priority spot in Ontario, as school boards, schools, and other stakeholders want to see improvement in mathematics achievement. For example, the Waterloo Region District School Board (WRDSB) has made mathematics an explicit focus since 2015, with the inclusion of Mathematics as a Target Area in its *Board Improvement Plan for Student Achievement and Well-being 2015 - 2016* (BIPSA) (WRDSB, 2015). In 2015, the WRDSB also published a statement, titled *Mathematics Beliefs*, supporting mathematics for all learners (WRDSB, 2015). Furthermore, the WRDSB places an emphasis on high-level mathematics that is centered around collaborative problem-solving using varied
assessment strategies to prepare students for success with mathematics in and out of the classroom (WRDSB, 2015).

In 2016, the WRDSB took steps to place an even greater focus on Mathematics as a board-wide goal, one of only three areas of focus for the board in their 2016 - 2019 Operational Goals (WRDSB, 2016). Specifically, their Mathematics goal is to "Increase the percentage of students achieving at provincial standard by 8 percent yearly on Grade 3, Grade 6 and Grade 9 Applied EQAO Mathematics Assessments for three years" (WRDSB, 2016).

Many supports have been put into place for educators, both in schools and at the system level. The teacher supports include: the creation of a comprehensive math strategy for the elementary grades, professional development opportunities, increased physical resources in the classroom, and increased home-school communication supports (WRDSB, 2016). To support school administrators, there is a focus on professional development opportunities surrounding the comprehensive mathematics strategy, further supported by the appointment of an Elementary Principal Instructional Leadership Coach with a Math Focus (WRDSB, 2016).

Aside from school board-specific supports and professional development, teachers in Ontario can access funding to improve their mathematics instruction in self-led professional development. Since 2014, the Ministry of Education, through the Ontario Teachers' Federation, has provided a subsidy for teachers seeking to enroll in Mathematics university courses or Additional Qualifications courses. The courses supported for subsidy will help teachers to move their practice towards current practices that are research-based.

Some parents and community members fear that the current approach to mathematics education, which is based on conceptual understanding and preparing students for real-world mathematics situations, is doing a disservice to students (Boaler, 2015). These fears arise, at least
in part, to a teaching approach that is very different from parents' educational experiences, 
resulting in parents feeling uncomfortable with the mathematics instruction (Boaler, 2015). 
Furthermore, some parents strongly believe that students need to have a greater focus on 'the 
basics' of math, contrary to research findings (Boaler, 2015).

The Ontario Ministry of Education also understands the importance of engaging parents 
in education. To support this aim in mathematics education, in 2014, the Ontario Ministry of 
Education published Doing Mathematics With Your Child: Kindergarten to Grade 6 (A Parent 
Guide) (Ontario Ministry of Education, 2014). This document attempts to engage parents in 
supporting math outside of classroom (Ontario Ministry of Education, 2014). Furthermore, it 
tries to help parents feel comfortable with the instructional move away from traditional math by 
providing explanations of benefits to the approaches used, in the short- and long-term for 
students (Ontario Ministry of Education, 2014). Finally, the document provides ideas to parents 
about activities they can engage in with their child to support their classroom learning, with links 
to the Ontario Curriculum (Ontario Ministry of Education, 2014). Activities are centered on 
mathematical processes, such as problem-solving and communication.

2.3 Mathematics Communication

Mathematics communication is a crucial component in mathematics learning (Boaler, 
2015; Ontario Ministry of Education, 2005). This section will examine benefits to implementing 
mathematics communication in a mathematics program, as well as challenges educators may 
face during implementation. Furthermore, this section will discuss the role language plays in 
mathematics communication, and how math anxiety affects student performance.
2.3.1 Benefits of Mathematical Communication

Traditional mathematics instruction focuses on the teacher as the one who knows the answers, and is not interested in understanding student thinking or their strategies (Cirillo, 2013). In this teaching approach, the goal is only to reach the correct answer (Cirillo, 2013). This approach does not leave much room for mathematics communication practice for the students. However, there are numerous benefits to students, and their mathematics understanding, if mathematics communication is incorporated into mathematics instruction (Boaler, 2015; Ontario Ministry of Education, 2005). Furthermore, mathematics discussions can shift the math authority from the teacher and/or textbook to the classroom community (Cirillo, 2013).

Teachers need to have many pedagogical strategies to support the learners in their classroom. Mathematical discussion is an effective tool for supporting mathematics learning (Cirillo, 2013). Mathematics discourse can help students consolidate their thinking (Marks Krpan, 2008). Furthermore, math communication can also support deeper understanding of content through metacognition as it helps students become aware of their own learning and provides opportunities for students to explain their thinking (Marks Krpan, 2008). A focus on mathematics communication can also help improve students' overall problem-solving skills in mathematics (Marks Krpan, 2008).

When students are provided opportunities for discussion, the result is a higher degree of understanding of the content (Boaler & Staples, 2008). Boaler and Staples (2008) found that, when teachers adopt a 'communicative' approach, students outperform those exposed to a traditional mathematics teaching approach. They suggest that 'communicative' teaching should focus on different ways for students to communicate mathematics, which can include: words, diagrams, tables, symbols, objects, and graphs (Boaler & Staples, 2008).
Aside from the academic benefits, mathematics communication also benefits students' wellbeing in the mathematics classroom. Math discussions can move students from being passive learners to active participants in their learning through an increase in motivation (Cirillo, 2013; Boaler & Staples, 2008). In addition, math discussion can increase student agency over their own learning (Cirillo, 2013). Mathematics discourse provides additional benefits to middle school students. Marks Krpan (2008) found that students in the Junior and Intermediate divisions possessed a negative view of mathematics prior to the inclusion of math discourse, but felt an increased personal connection after an increase of math discourse in their classes.

Mathematics discourse is a valuable tool for educators as well, as it can provide insight to educators into student learning (Marks Krpan, 2008). Cirillo (2013) states that successfully integrated math discussion is not focused on assessing student knowledge. Rather, the objective of math discussion should be for teachers to determine how they can help develop student understanding (Cirillo, 2013). Cirillo (2013) also points out that mathematics discussion can support teachers in understanding and assessing student thinking. In addition, math discussion supports teachers in determining if any misconceptions exist (Cirillo, 2013). Through identification of misconceptions, teachers can purposefully plan their instruction to address the misconceptions present (Peppers, Wan & Phillips, 2014). Teachers can then work with students to help build math knowledge (Cirillo, 2013).

For students to reap all the benefits that mathematics communication has to offer, teachers need to not only encourage participation of students in math discourse, but also help them understand that different types of contributions are valuable and help advance class understanding, including: asking questions, proposing alternate solutions, false starts, and making conjectures (Cirillo, 2013). Furthermore, if the focus for math discourse is on student
explanations for reasoning, then students experience a high level of success on standardized testing (Chapin, O'Connor & Anderson, 2003). This finding may be relevant to the Ontario large-scale assessments written by students, and administered by the Education Quality and Accountability Office (EQAO).

2.3.2 Challenges in Incorporating Mathematics Communication

Although there are many benefits for students and teachers when moving away from a traditional approach to teaching, transforming mathematics instruction is difficult because teachers are unsure of what their class should look like, or even how to get there (Hufferd-Ackles, Fuson & Sherin, 2004). There are many necessary components to successful mathematics communication in classroom instruction. Stein (2007) found that, to encourage discourse, teachers need to set up a classroom community that has a focus on understanding, not only finding the right answer.

In addition, to encourage discourse, teachers can engage in two different types of discourse - cognitive and motivational (Stein, 2007). The purpose of cognitive discourse is to: push students to determine the links between strategies and procedures used, highlight the importance of mistakes and learning from them, and to stress individual accountability to increase student engagement (Stein, 2007). The purpose of motivational discourse is to provide praise and supportive comments in order to encourage student participation in math discourse (Stein, 2007).

However, the many components of setting up mathematics discourse can be challenging for teachers. This is supported by research that shows that math discourse is not very frequent in mathematics classrooms (Marks Krpan, 2008). When math discourse does take place, it is mostly through whole-class activities (Marks Krpan, 2008). Furthermore, when math discourse occurs in
small-group settings, it includes sharing answers, not strategies, which does not support student learning (Marks Krpan, 2008). Educators report valuing math discourse in the classroom, but do not always have success in its meaningful integration, and have concerns about time restraints in their teaching practice (Marks Krpan, 2008). Marks Krpan (2008) also stressed the importance of students being taught metacognition and math talk strategies, but teachers do not always engage in this teaching.

Another challenge faced by teachers when integrating mathematics communication into their practice is the assessment component. As discourse is a dynamic process, it is difficult to assess (Stein, 2007). Stein (2007) suggests that, instead of trying to only assess the level of discourse, teachers must also assess individual students in the discourse process. Stein (2007) also warns that the language teachers use is very important.

2.3.3 Language

In general, language plays a unique role in learning (Novak & Gowin, 1984). Novak and Gowin (1984) stressed the importance of teachers explicitly conveying to learners that words convey and carry meaning on their own. They also pointed out that communication can sometimes be difficult because the same concept can be slightly different for each individual person as everyone must acquire their own meaning for concepts (Novak & Gowin, 1984).

The term 'mathematics communication' implies a strong language component. Language is an integral part of mathematics communication, in both oral and written form. Thus, language difficulties may have a stronger impact on student learning within the mathematics communication context as, "Learning the meaning of a piece of knowledge requires dialog, exchange, sharing, and sometimes compromise" (Kinchin & Hay, 2000, p. 20). When engaging in learning through language, there is also an emotional response on the part of the learner
(Novak & Gowin, 1984). Sometimes the emotional response is positive, but it can be negative or fearful if the learner recognizes how wrong or ignorant they were (Novak & Gowin, 1984).

2.3.4 Math Anxiety

A significant challenge faced by mathematics teachers is dealing with students' math anxiety (Ramirez, Gunderson, Levine & Bellock, 2013; Vukovic, Kieffer, Bailey & Harari, 2013). Math anxiety is so prevalent and widespread that it has been reported as early as the Primary grades (Ramirez et al., 2013; Vukovic et al., 2013). The negative impact on student achievement as a result of math anxiety can lead to a dislike, and avoidance, of mathematics (McCoy, 1992). Vukovic et al. (2013) suggest that there are two sub-categories of math anxiety - numerical anxiety and math test anxiety.

Working memory has been shown to be affected by math anxiety in learners. Vukovic et al. (2013) found that math anxiety is a unique source of individual differences in calculation skills and the ability to apply mathematical concepts. Furthermore, math anxiety is a barrier for learning mathematical applications, especially for those students with a high working memory capacity (Vukovic et al., 2013). Students with a higher working memory also see a drop in mathematical achievement with increased math anxiety, suggesting that the situation depletes working memory (Ramirez et al., 2013).

Teachers need to be aware of the effects of math anxiety and must consider the cognitive and affective aspects in assessments when dealing with students' math anxiety (Vukovic et al., 2013). Moreover, teachers must consider the emotions and anxiety felt by their students, not just focus on concepts and procedures, when teaching math (Ramirez et al., 2013). Miller (2010) supports a holistic approach to education, advocating for the consideration of the students’ body, mind, and soul during planning and instruction. These findings support individualized instruction
and moving away from a one-size-fits-all approach to education. Just because students appear to be 'stronger', or high-level, academically does not mean that teachers should not give them as much attention or support.

2.3.5 Student Identity

Student identity also plays a role in mathematics learning. There is a positive correlation between student attitude and student achievement (Bishop, 2012). In small-group activities, students begin to take on different identities, such as 'smart' and 'dumb' (Bishop, 2012). There is a significant change in student role during small-group problem-solving activities depending on whether students identify as 'smart' or 'dumb' (Bishop, 2012).

Bishop (2012) found that the 'smart' student directed and controlled the activities to solve an assigned problem, dominated the discussion, and overall exercised a great deal of power over the 'dumb' student (Bishop, 2012). In comparison, the 'dumb' student's alternate thinking was not considered, and when the 'dumb' student presented an idea found to be worthy of the 'smart' student, the 'dumb' student was not given credit for the idea (Bishop, 2012). The students also engaged in very harsh comments towards one another, particularly so from the 'smart' student.

As time progresses, with students remaining in the same role of 'smart' or 'dumb', the power imbalance accumulates (Bishop, 2012). This power imbalance leads to an accumulation of advantage and disadvantage for 'smart' and 'dumb' students, respectively (Bishop, 2012). Thus, teachers must constantly listen to, and monitor mathematics discussion, and where necessary intervene (Bishop, 2012).

2.4 Manipulatives and Technology

Education is always changing and evolving to reflect the needs of the students in the present, and to prepare learners for the future. Today's 21st Century learners are deeply
immersed in a technological world where they will need to have the skills necessary to solve novel problems. Thus, many stakeholders want students engaged in hands-on learning, with an increased integration of technology in students' learning environments.

2.4.1 Manipulatives

Manipulative use is increasingly being supported for mathematics education. NCTM (2000) states, “Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics” (p. 136). This statement supports the idea that students require concrete materials to learn new mathematical concepts (Weiss, 2005). Manipulatives are one type of concrete material that teachers can use to support students in their acquisition of new concepts in math. Manipulatives can be used by teachers to provide students with concrete experiences to acquire mathematical concepts, leading to a higher conceptual understanding (Weiss, 2005). In this study, the Ontario teachers delved deeper into concepts when utilizing manipulatives in their instruction (Weiss, 2005).

Manipulatives are "designed to represent explicitly and concretely mathematical ideas that are abstract" (Moyer, 2001, p.176). However, students must be taught how to properly use the manipulatives to explore a concept (Weiss, 2005). Student familiarity with manipulative use is imperative to the learning tool effectively supporting learning (Weiss, 2005). Lack of student knowledge surrounding proper use of manipulatives can turn these supports into distractions (Weiss, 2005). Furthermore, for manipulative use to positively impact student learning, students must be actively engaged in using the learning tools (Moyer, 2001).

Teachers must be aware that manipulatives are only one component of effective instruction, and not a 'cure-all' (Weiss, 2005). Teachers still need to employ effective teaching
strategies during manipulative use (Weiss, 2005). When utilizing manipulatives, teachers' focus should remain on student learning (Moyer, 2001).

Studies into the effectiveness of manipulative use show that experienced teachers display the most effective manipulative integration in their practice (Weiss, 2005). Teachers need to be familiar with different types of manipulatives, both virtual and concrete (Peppers et al., 2014). This familiarity will allow teachers to be aware of the advantages and disadvantages of different manipulatives (Peppers et al., 2014). This knowledge helps teachers to properly select the right manipulatives to support an activity or task (Peppers et al., 2014).

The use of manipulatives in mathematics instruction can help support students that are struggling (Weiss, 2005). Peppers, Wan and Phillips (2014) conducted an action research project in a Grade 8 classroom to support student learning and fill in learning gaps regarding fractions. The project focused on the use of manipulatives to provide students with concrete understanding of the underlying concepts (Peppers et al., 2014). Through the use of virtual and concrete manipulatives, students significantly increased their understanding of fractions (Peppers et al., 2014).

Manipulatives may also help to address learning style differences. Manipulative use by kinesthetic learners may decrease math anxiety in 10% of students (McCoy, 1992). Through the support of conceptual learning of fractions using manipulatives, Grade 8 students displayed a decrease in math anxiety, and also an increase in their reported enjoyment of mathematics (Peppers et al., 2014). Peppers et al. (2014) reported that some students benefited from the hands-on learning with the use of concrete manipulatives. Other students reported a higher level of enjoyment with the use of virtual manipulatives (Peppers et al., 2014). Students running an experimental probability simulation, through the use of a Plinko game, were also highly engaged,
even through a long, two-hour lesson (Naresh & Royce, 2013). These findings suggest that teachers can deliberately choose different manipulative types to differentiate their instruction and engage more of their students.

Some manipulatives skeptics believe that manipulatives are limited to concepts that are concrete (Weiss, 2005). However, manipulatives can be used to support learning of abstract concepts as well, such as algebra (Weiss, 2005). Manipulatives have also been shown to aid students in learning about experimental probabilities through a game of Plinko (Naresh & Royce, 2013). Through the Plinko game, students were given opportunities for real-life data collection (Naresh & Royce, 2013). Students also worked on important mathematical processes, including: performing simulations, making conjectures, and testing conjectures using collected experimental data (Naresh & Royce, 2013). Thus, manipulatives can support the development of mathematical processes.

Many mathematics experts view manipulative use in the mathematics classroom as a crucial component to effective instruction (Boaler, 2015; McDougall, 2004). However, there is also skepticism on the integration of manipulatives in the mathematics classroom. McNeil and Jarvin (2007) summarized what they call a 'manipulatives debate', with a primary concern that time spent on manipulative use meant less time for students to engage in the formal symbolic notation of mathematics. However, they also point out that true mathematicians do in fact utilize manipulatives (McNeil & Jarvin, 2007), thus working with manipulatives is representative of true math work.

McNeil and Jarvin (2007) criticized how researchers generalize their findings when presenting their research, and warn teachers that manipulative effects must be considered within the appropriate context. Just because a certain manipulative was beneficial for a particular
concept or grade level, does not mean that the manipulative in question will be effective with all students, or in all scenarios (McNeil & Jarvin, 2007).

McNeil and Jarvin (2007) present two theories why manipulatives may be ineffective. The first theory involves the teacher not being familiar with the proper teaching and implementation approaches necessary for a particular manipulative to support learning. Teachers may use a manipulative solely for engagement, thus limiting its educational purpose (McNeil & Jarvin, 2007; Moyer, 2001). However, when teachers use manipulatives in a constructivist setting, student learning increases (McNeil & Jarvin, 2007). Teachers need to explicitly link students' knowledge constructed through manipulative use with the formal symbolic mathematic notation (McNeil & Jarvin, 2007). These findings support professional development for teachers that aids them in increasing their knowledge of manipulative use and its benefits.

The second theory presented surrounds children's behaviour. McNeil and Jarvin (2007) posit that children will struggle with the nontransparent nature of dual representation in manipulative use, have limited cognitive resources from which to draw, and are naturally resistant to change. Thus, teachers need to carefully consider the type of manipulatives they integrate into their practice. They recommend that teachers avoid the use of manipulatives that are rich in perceptual detail, or familiar to students outside of school (McNeil & Jarvin, 2007). The use of simpler manipulatives may allow for easier use of the learning tool for deep exploration of a concept (McNeil & Jarvin, 2007).

As a result of the 'manipulatives debate', there can be a lack of ‘buy-in’ from teachers, students, and parents, particularly in the middle school grades (Martinie & Stramel, 2004). Middle school students are typically transitioning between two of Piaget's stages of cognitive development, from the concrete operational to the formal operational stage (Weiss, 2005;
Martinie & Stramel, 2004). Through this transition, middle school students still require concrete materials to support their acquisition of new concepts (Martinie & Stramel, 2004). While McNeil and Jarvin (2007) point out some potentially negative findings of manipulatives use, Weiss (2005) claims there is no evidence to support that manipulative use ever has a negative effect on student learning. Martinie and Stramel (2004) also point out that there are many benefits to manipulative use. These research findings support manipulative use in middle school mathematics instruction, but teachers need to carefully consider complementary teaching strategies and implementation.

2.4.2 Technology

The Ontario Curriculum states that technology should be considered because it can "reduce the time [students] might otherwise spend on purely mathematical activities" (Ontario Ministry of Education, 2005, p.14). The document clarifies that "students’ work with the software [should] be focused on the mathematics related to the data" (Ontario Ministry of Education, 2005, p.15). However, the amount of meaningful incorporation of technology in mathematics classrooms has been decreasing, even as funding is increasing (OECD, 2004). Given these conflicting findings, it is worthwhile to investigate how teachers utilize technology to support student learning.

Technology use in education, which is often touted as the solution to any student learning issues, is increasing. However, it is hard to generalize findings from specific studies as these studies are typically focused on specific grades, contexts, technologies, and/or mathematical concepts. As such, I will present findings from specific cases to highlight what successes and challenges the research has illuminated.
Teachers feel pressure to increase the amount of technology in their instruction, even though all teachers will have a different level of comfort, and experience, with technology. Fortunately, teachers do not need to be experts in educational technology use to meaningfully incorporate technology into their instruction (Karchmer-Klein, Mouza, Harlow Shinas, & Park, 2017). Teachers just need to use their imagination when considering technology integration into their practice (Karchmer-Klein et al., 2017). In a 1:1 iPad initiative in an all-boys private school, Karchmer-Klein et al. (2017) found that teachers' use of apps in education is highly varied. Moreover, all implementation approaches were beneficial to students and supporting their learning (Karchmer-Klein et al., 2017). The 1:1 iPad initiative provided an opportunity for a flexible teaching approach (Karchmer-Klein et al., 2017).

To optimize student learning, teachers need to use both physical and virtual spaces to complement digital communication with face-to-face communication (Karchmer-Klein et al., 2017). These findings provide some insight into the relationship between teachers' attitude towards technology and the implementation into their practice. Teachers who viewed the iPads are supplementary displayed full integration, while one teacher who viewed them as central to instruction had only partial implementation (Karchmer-Klein et al., 2017).

Karchmer-Klein et al. (2017) stress that teachers need to create a balanced approach to app use. Doing so, teachers will provide students with opportunities to engage in digital content as both consumers and active producers (Karchmer-Klein et al., 2017). Furthermore, teachers need to plan their instruction with the learning goal in mind, considering the strategies that will be most beneficial to students, incorporating both digital and non-digital elements (Karchmer-Klein et al., 2017). To effectively do so, teachers need to be aware of evidence-based pedagogy to make appropriate decisions. Thus, teachers need PD support that provides knowledge of
effective strategies as well as curriculum planning (Karchmer-Klein et al., 2017). This conclusion is supported by earlier research conducted by Demetriadis et al. (2003), who found that PD programs that introduce new technologies must provide constant support to teachers.

A study by Bottge, Grant, Stephens and Rueda (2010) into how technology teachers focused on math skills, through the use of computer-based instructional tools, found a great improvement in student math skills. Embedded teaching of fractions, compared to explicit teaching of fractions, yielded the best results, especially when compared to no mathematics instructions focus (Bottge et al, 2010). Math skills integration into other curricular areas can lead to increased achievement for students. Placing math skills instruction within a context of a rich task is beneficial to student learning and achievement, especially when paired with life-sized manipulatives (Bottge et al, 2010). When teaching practice incorporates both computation and problem-solving, it helps supports low achieving students (Bottge et al, 2010). Furthermore, it helps students understand that mathematics is useful and worthwhile (Bottge et al, 2010).

A group of New Brunswick teachers participated in PD centered around games of chance, using virtual interactive simulations, to help students better understand probabilities (Savard, Freiman, Theis, & Larose, 2013). These teachers were provided with flexible software that allowed for different scenarios, which helped deepen students' learning (Savard et al., 2013). Participating teachers showed concern that the technology used should support the aim of the curriculum (Savard et al., 2013). When teachers were provided with technology, and supports such as implementation strategies and scenarios, they appreciated the potential for the positive impact on student learning (Savard et al., 2013). They also saw the potential for cross-curricular and real-world connections, such as critical thinking, regarding games of chance and gambling (Savard et al., 2013).
Savard et al. (2013) reported that teachers who used the technology saw a transformation of their teaching practice, and moved towards a culturally responsive approach. In addition, students displayed a high level of engagement and motivation with the technology (Savard et al., 2013). As a result, teachers wanted to have access to resources and potential activity ideas to draw from, to make their instruction and implementation easier (Savard et al., 2013).

In another study, Grade 7 and 8 teachers experimented with the SimCalc software (Roschelle et al., 2010). This representational software was introduced to teachers, and further explored, over 3 to 5 sessions, depending on district funding and time. The PD presented to Grade 8 teachers was implemented through a train-the-trainer approach (Roschelle et al., 2010). This study placed a large focus on the tight interconnections between curriculum, technology, and teacher PD (Roschelle et al., 2010). Roschelle et al. (2010) found that representational software greatly supports student learning both for achieving curriculum expectations, as well as learning the necessary prerequisite algebra skills for success in later mathematics. These findings provide support for technology use in mathematics to go beyond teaching students 'the basics', as it can help teachers teach higher-level math work and concepts (Roschelle et al., 2010).

Technology can also facilitate collaborative activities, as found in higher education (O Broin, & Raftery, 2011). O Broin and Raftery (2011) investigated the use of a collaborative Google Docs project, and found that it allowed students to participate and work on the project without the difficulty of having to schedule a mutually agreed-upon time and place. Students could work at the same time or individually (O Broin, & Raftery, 2011). However, a third of the students reported that they would prefer not to use the Google Docs platform in the future, in part because of technical issues (O Broin, & Raftery, 2011). The authors suggested that students
would benefit from additional training with the software prior to the commencement of the project (O Broin, & Raftery, 2011).

The use of the Google Docs software also provided teachers with benefits. It allows the teacher to monitor the progress of the work (O Broin, & Raftery, 2011). Google Docs also allows the teacher to better assess individual student contributions, compared to traditional assessments, even though it had limitations (O Broin, & Raftery, 2011). We can extract from these successes, and apply them to K-12 education, modifying where necessary given students' differences in age and development.

While many studies have found positive findings of technology use, even with some limitations, others have found major difficulties to consider during technology implementation. Technology integration does not guarantee improvement in teaching practice, thus must be complemented with effective teaching strategies to have a positive impact (Bottge et al., 2010). Successful supports in mathematics, in addition to technology, include: graphics, verbal descriptions, and integration of different concepts and skills into an application setting (Bottge et al., 2010).

Technology integration must be carefully considered to determine if it meets the goals of the activity. For instance, the use of a computer lab may hinder student collaboration, as students may be more comfortable collaborating at a table or other physical space (Naresh & Royce, 2013). Furthermore, technology must be used appropriately to ensure the focus of instruction remains on the concept. If not, technology use can confuse and frustrate learners (Fital-Akelbek & Akelbek, 2012). These findings are summarized well in the Ministry's recommendations regarding technology, "[t]echnology can be beneficial when it is wisely integrated with effective pedagogy" (Ontario Ministry of Education, 2013, p. 16).
Overall, teachers must be skilled in technology implementation for it to be effective. Teachers need PD in how to effectively use a type of technology to support student learning (Savard et al., 2013). Furthermore, as educational leaders, principals need to support the meaningful use of technology in math (Bottge et al., 2010). Principals must also support teachers in accessing PD in order to be effective when implementing technology, especially in an applied mathematics setting (Bottge et al., 2010).

2.5 Professional Development

Professional development has become a major component in today's education system, as the belief that teacher education is not limited to the initial teaching degree becomes more accepted (Cochran-Smith, 2011; Shizu Kutaka et al., 2017). Weiss (2005) suggests that many teachers' initial education programs are contradictory to research-based best practices. Thus, teachers must participate in on-going professional development to remain current in their teaching practice, especially as mathematics education moves away from the traditional approach (Cochran-Smith, 2011; Won, 2017). Teachers must be supported in increasing their content knowledge (Won, 2017), as well as focusing on students' mathematical thinking (Jacobs, Franke, Carpenter, Levi, & Battey, 2007).

To optimize the effectiveness of PD, professional development should not be a lecture-style information session that is isolated in nature (Cochran-Smith, 2011). Instead, for professional development opportunities to be effective, they should be an on-going process, taking place over a significant period of time (Cochran-Smith, 2011; Shizu Kutaka et al., 2017), as well as be coherent and allow teachers opportunities for active learning (Shizu Kutaka et al., 2017).
The inclusion of active learning opportunities during PD can have a greater effect on the participating teacher's practice (Desimone, Porter, Garet, Yoon, & Birman, 2002). When teachers are explicitly taught specific instructional methods, teachers are more likely to include them into their own practice (Desimone et al., 2002). In addition, mathematics PD should support teachers in learning the necessary content knowledge for their teaching (Shizu Kutaka et al., 2017). Furthermore, increasing teachers' content knowledge should be explicitly linked with the introduction of new mathematical practices (Won, 2017).

Successful PD also needs to include opportunities for collaboration (Cochran-Smith, 2011; Shizu Kutaka et al., 2017). For concepts or practices that are new or unfamiliar to teachers, opportunities for discussion with peers are crucial (Won, 2017). Through discussions, teachers can develop a better understanding of their role in facilitating a particular process (Won, 2017). Furthermore, collaboration with peers of different grades can benefit teachers' understanding of how processes progress and develop through mathematics education (Won, 2017). Teacher collaboration is also important when learning new technological skills, and provides benefits for teacher learning during PD opportunities (Desimone et al., 2002). When teachers collaborated with other teachers within the same school, department, or grade level, there was an additional benefit to teacher learners (Desimone et al., 2002).

To examine different types of teacher learning and the focus of PD opportunities, Cochran-Smith and Lytle (1999) developed a Teacher Learning Framework consisting of three different conceptions: knowledge-for-practice, knowledge-in-practice, and knowledge-of-practice. The knowledge-for-practice category focuses on knowing as much content knowledge as possible, and typically refers to 'formal' knowledge created and obtained from an external source, such as university research (Cochran-Smith & Lytle, 1999). This conception also
includes knowledge regarding discipline, pedagogy, assessment, and other teacher administrative actions (Cochran-Smith & Lytle, 1999).

Knowledge-in-practice typically refers to strategies and practices that are 'tried-and-true', that have been developed by veteran, expert, and highly competent teachers through many years of experience (Cochran-Smith & Lytle, 1999). Finally, knowledge-of-practice relates to teachers conducting inquiries in their teaching practice to construct knowledge (Cochran-Smith & Lytle, 1999). These inquiries typically challenge current assumptions, are context-specific, and help to move the understanding of education forward (Cochran-Smith & Lytle, 1999).

Furthermore, the final conception can be engaged in by any teacher, novice or experienced, making it more useful to the practicing teacher (Cochran-Smith & Lytle, 1999). Cochran-Smith and Lytle's (1999) Teacher Learning Framework will be used to examine the content and structure of the PD series offered to participants of this study.

In a follow-up to her Teacher Learning Framework, Cochran-Smith (2011) outlined how teachers can move their practice forward through the constant examination of their practice, in the context of their current setting, to determine what would benefit their students. Teachers need to ask questions about their practice, and thoughtfully find answers to improve their teaching practice (Cochran-Smith, 2011). Teacher gains from an on-going quest for improvement are not limited to student achievement (Cochran-Smith, 2011). Through a continual focus on improving their teaching practice, teachers may experience an increase in: teacher learning, student learning, and student equity (Cochran-Smith, 2011). Cochran-Smith (2011) concluded that teacher learning does not, in her opinion, ever end. However, teacher attitudes may impact the degree and quality of that teacher learning.
Teacher attitudes have consequences on their students' learning. Lower teacher math anxiety has been linked with higher student mathematical achievement (Shizu Kutaka et al., 2017). In addition, teachers must possess a positive attitude for learning mathematics themselves to effectively teach the subject (Shizu Kutaka et al., 2017). In order to improve their teaching practice, teachers must be open-minded in their approach to programming (Clarke, Thomas & Vidakovic, 2009). Differences in teachers' attitudes can result in a different interpretation of the same learning goal, affecting whether they adopt a product- or process-oriented teaching approach (Won, 2017). As such, teacher attitudes play an integral part in whether the teacher will change their instruction to reflect research-informed pedagogy.

Targeted PD participation can result in a shift in teachers' attitudes (Shizu Kutaka et al., 2017). In particular, a four-year longitudinal PD study by Traci Shizu Kutaka et al. (2017) found that teachers felt less anxious, focused on student-centered mathematics, and enjoyed more growth in content knowledge compared to the control group. The same study found that students, whose teachers were involved in the PD study, demonstrated greater achievement levels than the control group teachers (Shizu Kutaka et al., 2017). Long-term involvement in the study greatly benefited students as they demonstrated the highest gains in the second- and third- study year (Shizu Kutaka et al., 2017).

Currently, with the push for professional development in mathematics, there are a number of PD programs that have been commercially created and available for purchase at the school- or board-level (Jacob, Hill & Corey, 2017). However, some programs have limited impact on improving teacher knowledge of mathematics, even when this is a stated focus and supposed outcome (Jacob et al., 2017). Some programs may even not show any impact on student outcomes, even where the PD program runs over several years (Jacob et al., 2017).
One issue with a pre-packaged approach to PD could be that it does not meet the needs of the teacher participants (Jacob et al., 2017). When teachers begin a mathematics PD program with a low level of mathematics understanding, they face difficulties in incorporating new instructional strategies into their teaching practice (Hill & Charalambous, 2012; Jacob et al., 2017). Furthermore, if the content knowledge presented is too complex for participants, and is not linked well to instructional practices, the benefit of the PD program to teachers can be limited, if any (Desimone et al., 2012; Jacob et al., 2017).

Another factor that must be considered when implementing professional development is the support of individuals in leadership positions, both at the school- and board-level. Administrator attitudes play a role in the potential success of a PD program. Research studies have found that principal support was critical in a PD program's implementation (Matsumara, Sartoris, Bickel, & Garnier, 2009; Wanless, Patton, Rimm-Kaufman, & Deutsch, 2012). Members of the senior leadership of a school district must also demonstrate support of the implemented PD program in order for their teachers and students to realize a benefit of the program (Jacob et al., 2017). The findings surrounding commercially available PD programs, support creating PD programs that are tailored to a specific set of teachers, either at the school- or board- level, and are supported by school and board leaders.

Inner-city schools present unique challenges to teachers and their delivery of content to students. Inner-city schools historically struggle with low scores on standardized achievement tests (Jacob, 2007), which, in Ontario, could relate to lower EQAO scores. Inner-city schools also face a higher percentage of students from minority and low socioeconomic backgrounds (Jacob, 2007). Inner-city teachers face challenging teaching conditions as their students often display learning gaps, poor classroom behaviour, and the effects of poverty (Jensen, 2009;
The 2015 Trends in International Mathematics and Science Study (TIMSS), an international achievement test in mathematics and science, found that Canadian students who attended schools that needed to provide breakfast scored much lower for both subjects, for both Grades 4 and 8 students (Council of Ministers of Education, Canada, 2017).

Furthermore, inner-city schools face teacher shortages (Jacob, 2007). Teachers in inner-city schools require support in order to be able to succeed in their practice. DuFour (2002) found that these supports must start at the administrative level, with principal support for teachers to work collaboratively in order to focus on maximizing student achievement. For teachers to experience success in an inner-city setting, they must demonstrate a commitment to their students, as well as have high expectations for their students (Won, 2017). In addition, new teachers within an inner-city school setting need to be provided with mentorship (Stewart Rose, Markus, & Kugler, 2009). Stewart Rose, Markus and Kugler (2009) found that mentorship of new teachers can encourage risk-taking in the classroom.

2.6 Ten Dimensions Framework

This thesis utilizes the Ten Dimensions of Mathematics Education Framework (McDougall, 2004) to examine the data. The Ten Dimensions Framework provides educators with the necessary planning and instruction components for a successful, and current, mathematics program. As the name implies, the framework outlines ten different dimensions for teachers to consider during the implementation of their mathematics instruction. The Ten Dimensions are: 1) Program Scope and Planning, 2) Meeting Individual Needs, 3) Learning Environment, 4) Student Tasks, 5) Constructing Knowledge, 6) Communicating with Parents, 7) Manipulatives and Technology, 8) Students' Mathematics Communication, 9) Assessment, and 10) Teacher Attitude and Comfort with Mathematics (McDougall, 2004).
The Ten Dimensions Framework is a useful tool to educators, as it provides a theoretical framework of an ideal and current mathematics program, and measures for self-assessment for teachers to identify areas that need improvement. Once those areas of need have been identified, the Ten Dimensions Framework then outlines specific components of each Dimension, and what needs to be achieved in each, for teachers to improve their mathematics practice to align with current mathematical beliefs.

The Ten Dimensions of Mathematics Education Framework provides teachers with a self-assessment tool in the form of a survey - Attitudes and Practices to Teaching Math Survey (APTMS) (McDougall, 2004, p. 87) (see Appendix A). The APTMS is a quick assessment tool, containing 20 questions, that teachers can use to determine how closely their current beliefs and teaching practice align with best practices in mathematics instruction. The APTMS will be discussed in further detail in Chapter 3, in the context of data collection.

Each Dimension is further broken down into different components (identified as criteria), to delve deeper into each category of mathematics education being considered. In addition to further assisting teachers in identifying the key components of successful mathematics education, and its implementation, the Ten Dimensions Framework provides a continuum for each Dimension and its criteria. This continuum provides a more in-depth and detailed self-assessment rubric with descriptions of teacher behaviour and characteristics from Level 1 to Level 4, where Level 4 describes the ideal classroom conditions. The Level 4 descriptions for each Dimension focus for this thesis, specifically Dimension 7 and Dimension 8, and their criteria are outlined below (adapted from McDougall, 2004).
Table 1

*Dimension 7 - Manipulatives and Technology*

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Manipulatives</strong></td>
<td>The teacher regularly uses a variety of manipulatives for all students to develop conceptual understanding; encourages students to explore mathematics through use of the manipulatives</td>
</tr>
<tr>
<td><strong>Technology</strong> (calculators and computers)</td>
<td>The teacher integrates technology as required by the curriculum; enhances instruction using technology in additional ways; ensures students use technology correctly and efficiently for maximum potential and student benefit</td>
</tr>
</tbody>
</table>

Table 2

*Dimension 8 - Students' Mathematical Communication*

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Oral Communication</strong></td>
<td>The teacher regularly assigns group tasks and asks questions that require students to communicate orally using mathematical language</td>
</tr>
<tr>
<td><strong>Written Communication</strong></td>
<td>The teacher regularly assigns tasks that require students to express their mathematical thinking in writing, in a variety of forms/types; provides regular instruction and feedback in writing for mathematics</td>
</tr>
</tbody>
</table>

The above descriptors provide an example of what proficient use of Dimension 7 and 8 would look like in the classroom. These descriptors are provided as a comparison for the study participants, to determine the similarity of their practice to the identified ideal mathematics program, at least for the two focus Dimensions.
2.7 Summary

Much emphasis is placed on improving mathematics education, at both the Ministry and school board level. Many stakeholders worry that today’s students are underperforming compared to their international peers, although the evidence for this claim is mixed. One approach that is being targeted to support mathematics learning is the use of mathematics communication. Mathematics communication has many benefits for students, but also presents teachers with challenges. Another learning support that many teachers are being encouraged to implement into their teaching practice is the use of manipulatives and technology. Again, these supports provide students with numerous benefits, but there are challenges to their effective implementation.

Teachers require support in implementing all of the changes recommended by the professional and academic literature surrounding successful mathematics communication in their classroom. They also need assistance in increasing effective integration of manipulatives and technology into their instruction for mathematics. One avenue for providing the needed support is through formal professional development opportunities for educators. As there are numerous PD frameworks presented in research studies, the selection of a specific one, namely the Ten Dimensions of Mathematics Education Framework (McDougall, 2004), can provide teachers, and PD facilitators, with a helpful starting point.
Chapter Three: Research Methodology

3.1 Introduction

In this thesis, I discuss the role of mathematics communication in shaping the teaching of Grade 6 teachers in a high needs, inner-city school setting, and will examine the usage of technology and manipulatives to support students' mathematical communication.

Throughout this thesis, the study school will be referred to as Bon Echo School, to maintain the privacy of the school, participating educators, and students. Teachers at Bon Echo School use a variety of teaching strategies to support student learning, which are outlined in this chapter along with challenges they face.

Furthermore, in this chapter, I discuss the research context, introduce the teacher participants, and the school setting. Data collection and analysis methods are also described, as are the ethical considerations for the study.

3.2 Research Context

Given the exploratory nature of this study, I utilize a qualitative methodology "...to explore a central phenomenon and engage in an emerging process of research" (Creswell, 2012, p. 137). To answer the study questions, one type of qualitative data were collected, specifically interview data. Interview data were collected through semi-structured interviews with participants, with more specific details to follow in the chapter.

This study will use a case study approach (Creswell, 2012). To allow for an in-depth examination of the study topic, four participants were chosen from a single school (Creswell, 2012). This approach is utilized because it allows for an in-depth analysis of a special case, a STEM school, thus making the focus an intrinsic case. Furthermore, this design is useful as the purpose of the study is to describe the attitudes and activities of the participants.
To allow for more defined start and end points of teacher attitudes and perceptions, and more clear interpretation, one type of quantitative data were collected, specifically measures of attitude (Creswell, 2012). Attitude measure data were collected through the administration of self-report surveys.

A crucial first step was determining the initial attitude of teachers towards mathematics communication, as well as the comfort level of teachers with technology and manipulatives in their mathematics instruction, and was achieved through the completion of the Attitudes and Practices to Teaching Math Survey (McDougall, 2004, p. 87). A second administration of the survey at the end of the PD sessions provides a valuable comparison, which is helpful for determining the effectiveness of the PD sessions in boosting teachers’ comfort and confidence on focusing their practice around mathematics communication and in using technology and manipulatives to support mathematics communication.

3.3 Participants

The particular school was chosen by the school board because of its STEM focus, as well as its low socioeconomic status, and the historically low EQAO performance of the student population. Participants from the school volunteered to be part of the study and were accepted as they demonstrated eagerness and willingness to learn and expand their professional knowledge.

Four educators were involved in this project - three teachers and one administrator. All are veteran teachers, with at least nine years of teaching experience. All three teachers taught at least one class of Grade 6 mathematics, and all four participants had been at the same school for at least one year. None of the participants had completed a degree in mathematics, or held Basic Qualifications in mathematics. One teacher had completed Additional Qualification courses, to the Specialist level, in Primary/Junior Mathematics.
3.4 Data Collection

Data for this study was collected from two sources: surveys and interviews. Data collection took place during the 2014 - 2015 school year. Bon Echo School educators participated in a larger School Improvement in Mathematics study (McDougall, 2014) that included five other school teams of mathematics educators. At four PD sessions, educators learned about evidence-based activities and teaching strategies they can implement and were introduced to the Ten Dimensions of Mathematics Education Framework (McDougall, 2004). All sessions placed an emphasis on teacher collaboration and facilitators used various strategies to encourage discussions both amongst and between school teams. This inter-mingling of educators promoted richer discussion and idea generation through exposure to more diverse teacher backgrounds and teaching philosophies.

The first PD session, which took place in December 2014, started by introducing the study and its objectives to participants. Educators then took part in workshops that focused on problem-solving in mathematics, followed by participants engaging in collaborative creation of open-ended tasks that suited the needs of their own classrooms. Teachers and principals where then introduced to the Ten Dimensions of Mathematics Education Framework (McDougall, 2004), and asked to complete the first administration of the Attitudes and Practices for Teaching Math Survey (APTMS).

The next session, which took place in January 2015, consisted of workshops on the Number Sense strand in the Curriculum, manipulative teaching strategies, and a constructivist approach to mathematics. The workshop leaders promoted the use of rich tasks and non-traditional assessments, within the context of a three-part lesson plan.
The third session, which took place in March 2015, began with a summary of the school visits. Educators then participated in workshops to increase their understanding of, and comfort with, the Number Sense strand of the Curriculum. Finally, teachers were introduced to one of two types of software, Microsoft Excel or Geometer's Sketchpad, depending on the grades they were teaching during the study year. Teachers were provided with ideas on how to implement the software, as well as time to independently explore the program.

The final session, which took place in April 2015, provided participants with workshops in the Patterning and Algebra strand of the Curriculum, with links to rich student tasks, assessment, and technology use. Participants were once again asked to complete the APTMS, and were provided an opportunity to compare their scores with the initial administration. Educators then had time to reflect, independently or collaboratively, on their scores, as well as the overall experience of the PD sessions and its effect on their perceptions of mathematics instruction.

Overall, the focus of the larger Grade 3 to 6 School Improvement Study was on the following four Dimensions: Student Tasks (Dimension 4), Manipulatives and Technology (Dimension 7), Students' Mathematical Communication (Dimension 8), and Assessment (Dimension 9). The data collected, described below, was used to determine what changes, if any, there were following participation in the PD sessions on the educators' perceptions.

**3.4.1 Attitudes and Practices for Teaching Math Survey**

The Ten Dimensions Framework outlines best practices for mathematics education, and each participant's adherence to the framework can be quantitatively assessed using the Attitudes and Practices for Teaching Math Survey (McDougall, 2004, p. 87-88) (Appendix A). The survey consists of 20, 6-point Likert-scale questions that are centered around the Ten Dimensions
Framework. The APTMS provides participants with a score for each Dimension, as well as an Overall Score, to help them determine how closely their attitudes and practices align with current understanding of best practices in mathematics education.

Once completed, teachers used the scoring guide to calculate their score for each of the Dimensions, as well as an Overall Score. Scores range from 1 to 6 for each question, where a score of 1 reflects a traditional approach to mathematics teaching, while a score of 6 reflects a teaching practice that aligns with current mathematics beliefs. To ensure accuracy of the calculations, I re-calculated and verified the scores.

For the Dimension Scores, a higher score indicated teacher attitudes that align well with current mathematics teaching instruction, whereas a lower score can suggest areas to focus on for personal growth. For the Overall Score, higher scores not only indicate an alignment of teacher attitudes with current mathematics teaching instruction, but also that the teacher is more likely to be willing to adapt and grow in his/her teaching practice. Survey data was collected at the first (December 2014) and final (April 2015) PD session, to compare pre- and post- differences in participants' responses. Any differences in response can indicate changes in participants' thinking regarding mathematics education, as well as changes in their teaching style.

The primary function of the APTMS scores was for teachers to self-reflect on their practice and determine the areas of mathematics education on which they wanted to focus on for their own professional development. Using the APTMS data, participants were asked to choose two Dimensions to focus on for personal growth, as well as two Dimensions to focus on as a school team. Through this identification, teachers could work on their specific Dimensions to improve on their teaching practice and help to align it with current educational practices and philosophies.
The second function of the APTMS scores was to help me identify what changes, if any, took place between the initial and final PD sessions in teachers' attitudes and practices regarding mathematics education. Even though there were only five months between the survey administrations, PD sessions provided to teachers were very rich and full of engaging material with practical ideas for their classrooms, as well as new considerations for teachers to push their teaching practice.

3.4.2 Teacher Interviews

Individual educator interviews were conducted on March 26, 2015. The interviews took place at Bon Echo School for the convenience of the educators. The semi-structured interviews took place after the first two PD sessions, typically lasted between 30 to 60 minutes, and included a total of 36 questions (Appendix B). Question categories were educator background (7 questions), versions of success (5 questions), challenging circumstances (5 questions), ideas about mathematics (6 questions), mathematics communication (6 questions), school support (4 questions), and overall questions (3 questions). To ensure preservation of the data, interviews were recorded, with the participants' consent, on my laptop. A secondary recording was made on my tablet in the event of technical difficulties with the laptop recording.

There are various definitions of 'mathematics communication', both in the education system and in the general population. For that reason, I did not start with a specific definition of what 'students' mathematical communication' is, but I asked each teacher for their own. The teacher's definition will help to add more depth into the view towards communication in mathematics, and could also have an impact on the use of mathematics communication and its intended purpose.
3.5 Data Analysis

3.5.1 Attitudes and Practices to Teaching Math Survey

Pre- and post-scores were compared for each of the Ten Dimensions, as well as the Overall Scores. Differences in the scores may indicate a change in attitudes towards mathematics education, or changes in their teaching practices.

All three teacher participants completed the Attitudes and Practices to Teaching Math Survey, at both the first and last PD sessions. Joanne, the administrator, chose to not complete the survey as she felt it was not relevant given her non-classroom role.

3.5.2 Educator Interviews

The interview recordings were transcribed and verified against the recording for accuracy. From previous studies, a codebook was created, with coding categories that included: professional development, each of the Ten Dimensions, and roles of stakeholders. Transcriptions were then divided up into smaller sections of data and coded under the appropriate, corresponding categories. Coding of transcribed interviews was completed using the program NVivo because it allows comparisons across nodes, automatically removes redundant coding, and I had access to the software through the Ontario Institute for Studies in Education.

Reading through transcripts several times helped me identify recurring themes and patterns in participant responses. While time-consuming, the creation of coding categories cannot be completed in advance, at least not completely, as unexpected themes may occur. Likely themes include: the Ten Dimensions, goals, challenges, parents, and student success.

The interview data was examined to discover the major themes in supporting students' mathematical communication and how it was supported by educators through technology and manipulative integration. A careful examination of these themes was used to guide my findings.
on effective technology and manipulative use in mathematics instruction to support mathematics communication, as well as how to deal with the struggles of current teachers. This thesis will also look at what support educators need to overcome their challenges.

3.6 Ethical Considerations

The ethics review for this thesis project was conducted under the larger McDougall (2014) project. To maintain confidentiality of the participants and school, pseudonyms were used in all sections of this thesis. To further ensure confidentiality, no identifying details of the school have been included.

Consent from participants was first obtained verbally, then formally with a signed consent form. Participants were informed that they could withdraw consent at any time in the study, and that they could choose to not participate in certain aspects of the study. Participants were made aware that pseudonyms would be used for participants and the school to protect their identity.
Chapter Four: Findings

4.1 Introduction

In this thesis, I explore how three individual Grade 6 teachers and one administrator approach elementary mathematics, both in their own classroom practices and collaboratively as a school team. For each case, I will present the data collected from the APTMS survey administrations and the interviews.

I begin by providing the data collected from the APTMS at the first and fourth PD sessions, and examining the data for each Dimension score, as well as the overall score, to determine if there was any change. Next, I explore the interview data from each educator. I start with the educator's definition of mathematics communication before diving into the successes and challenges they shared regarding their Grade 6 mathematics teaching at Bon Echo School.

Differences were found in individual educators' practices and attitudes towards mathematics, as is to be expected in a human population, and these are further explored below.

4.2 School Context

This study took place at Bon Echo School, a middle school (Grade 6 to 8) in an inner-city setting in Southern Ontario. During the study year, the school served approximately 550 students. The school population is diverse, and is composed of approximately 30% Caribbean descent, 30% Southeast Asian background, 15% South American heritage, and 25% representing the rest of the world's cultures.

The local school board conducts an equity survey on the population of each school. Factors such as familial income, familial make-up, and other external difficulties faced by students are considered. These factors were selected because of the added burden on students, and the potential for a negative impact on their academic success. Based on the findings of the
survey, Bon Echo School was one of the 20th neediest schools out of 474 elementary schools in 2014. In previous publications of the survey in 2011 and 2009, Bon Echo School had performed slightly better, with the school placing between the twentieth and fortieth neediest spots, respectively. These findings point towards the school's population facing increasingly more difficult life circumstances outside of school, which the interviews with the participating educators would confirm.

All students in Ontario write large-scale assessment tests in Grades 3, 6, 9, and 10, administered by the Education Quality and Accountability Office (EQAO). For the purpose of this study, I will focus on Grade 6 Mathematics test scores. In 2011 to 2013, 34% of Grade 6 students performed at or above the provincial standard, with 29% reaching or exceeding the standard in 2012 to 2014, and 27% in 2013 to 2015. The decreasing percentages show a worsening performance in mathematics achievement.

One unique feature of Bon Echo School is its Science, Technology, Engineering, Mathematics (STEM) focus. STEM is at the core of all instruction, across all content areas. The STEM focus was appealing when selecting this school as the study setting, as there is a lot of excitement currently in education about STEM pedagogy, and this study will explore how mathematics communication and the use of manipulatives and technology supports Grade 6 mathematics in the context of a STEM school.

Following the second PD session, the principal investigator, Douglas McDougall, conducted school visits and found that schools in the overall study experienced both successes and challenges in their mathematics practice. Successes included: the availability and visibility of manipulatives in the classroom, use of technology, explicit success criteria, use of word walls, and proper implementation of the three-part lesson plan. Challenges included confusion.
regarding success criteria versus learning goals, and a preference for displaying teacher-created posters.

4.3 Case of Gloria

Gloria has a background in Educational Administration. She began her teaching career in Guyana before moving to Canada. At the time of her interview, Gloria had over twenty-five years experience, and had been at the study school for eight years. She has taught Grade 6 to Grade 12, and also had experience in adult education. At the time of the study, she was teaching Language, Mathematics, and Music to her Grade 6 homeroom, an additional Grade 6 Mathematics class, as well as Grade 6 Rotary Music. According to the Ontario College of Teachers' website, she completed Additional Qualifications courses in Primary and Junior Mathematics to the Specialist level.

For Gloria, mathematics communication was defined as:

the ability to clearly explain whatever mathematical thinking you have as it pertains to a specific problem. So, using correct language, that is what I would say is mathematics communication ... you have to be mathematically literate to some extent to be able to communicate your mathematical thinking. (Gloria, Interview, March 26, 2015)

4.3.1 Attitudes and Practices to Teaching Math Survey - Results

Gloria completed the APTMS at both the initial (December) and final (April) PD sessions, and her scores for both survey administrations, as well as any changes between the two occurrences, are summarized in the table below.

Gloria experienced a positive gain in score across nine of the Ten Dimensions, and her Overall score. She also showed no change in one Dimension (7 - Manipulatives & Technology), but she was already at the highest possible score during the first survey administration, thus no further increase was possible.
### Table 3

*Initial and Final Attitudes and Practices to Teaching Math Survey Scores for Gloria*

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Day 1 Score</th>
<th>Day 4 Score</th>
<th>Change in Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Program Scope &amp; Planning</td>
<td>5.33</td>
<td>5.67</td>
<td>Increase</td>
</tr>
<tr>
<td>2) Meeting Individual Needs</td>
<td>4.6</td>
<td>5.8</td>
<td>Increase</td>
</tr>
<tr>
<td>3) Learning Environment</td>
<td>5.67</td>
<td>6</td>
<td>Increase</td>
</tr>
<tr>
<td>4) Student Tasks</td>
<td>4.8</td>
<td>5.8</td>
<td>Increase</td>
</tr>
<tr>
<td>5) Constructing Knowledge</td>
<td>4.6</td>
<td>5.8</td>
<td>Increase</td>
</tr>
<tr>
<td>6) Communicating with Parents</td>
<td>4</td>
<td>6</td>
<td>Increase</td>
</tr>
<tr>
<td>7) Manipulatives&amp; Technology</td>
<td>6</td>
<td>6</td>
<td>No Change</td>
</tr>
<tr>
<td>8) Students' Mathematics Communication</td>
<td>5.75</td>
<td>6</td>
<td>Increase</td>
</tr>
<tr>
<td>9) Assessment</td>
<td>4.75</td>
<td>6</td>
<td>Increase</td>
</tr>
<tr>
<td>10) Teacher Attitude &amp; Comfort with Mathematics</td>
<td>5.2</td>
<td>5.8</td>
<td>Increase</td>
</tr>
<tr>
<td>Overall</td>
<td>5.03</td>
<td>5.87</td>
<td>Increase</td>
</tr>
</tbody>
</table>

#### 4.3.2 Successes in Mathematics Teaching

Gloria identified that her teaching philosophy is based on the belief that, "every student can learn" (Gloria, Interview, March 26, 2015), and that, "learning is a lifelong process" (Gloria, Interview, March 26, 2015). Gloria explained that she feels she has succeeded:

> when ... students as they move on from Grade 6 ... to Grade 7, they can come to say, "Oh, Miss you taught us this, or Miss thank you, ... for this." Or even when they ... go off to high school, and they come to visit ... and they can recall some of the experiences they had at [Bon Echo School] ... It is awesome! (Gloria, Interview, March 26, 2015)

In addition, her "mathematical philosophy is that as mathematical learners these students should be able to link real life experiences with what they are doing in the classroom, and vice versa" (Gloria, Interview, March 26, 2015).

To engage students, Gloria uses "lots of real life experiences and situations ... [in] the classroom" (Gloria, Interview, March 26, 2015). For her, success in mathematics is achieved
"when a student can apply what they have learned to real life experiences" (Gloria, Interview, March 26, 2015) and when "they can make connections" (Gloria, Interview, March 26, 2015).

Gloria also encourages big picture thinking, wanting:

Students ... making connections to what they have learned in the classroom, and then when given a similar problem they are able to apply what they have learned to this new problem ... and ... the problem might not even be similar, but then cross-strand they are able to go back to that other strand. Like ... we do fractions, but in fractions you have to think about multiples and factors. (Gloria, Interview, March 26, 2015)

Gloria provides this example to illustrate that:

when [students] are able to see the relationships in numbers ... that is a success. Because now they are not seeing math by strand ... they are seeing this is mathematics. I am learning math, I am not ... doing Measurement, I am not doing Geometry." (Gloria, Interview, March 26, 2015)

Gloria's goal is to have students understand "I am doing math" (Gloria, Interview, March 26, 2015). Moreover, Gloria stated, "that is one of my goals, getting the students to understand the math. Not just do the math, but to understand the math. Understand what they are doing" (Gloria, Interview, March 26, 2015).

Gloria presents herself as a role model with a high understanding of math, for students, through her teaching style. She stated, "I do not teach textbook. I teach problem-solving" (Gloria, Interview, March 26, 2015). She explained that, "you have to know content to be able to teach in this non-conventional manner. [Because] you cannot be having to battle with content and be experimenting ... with stuff." (Gloria, Interview, March 26, 2015). Furthermore, this style of teaching involves, "taking a risk of not [makes motions as if flipping through pages] Chapter 1, Chapter 2, Chapter 3, Chapter 4, Chapter 5 ... I would just pull out a question out of the textbook and try to modify it - and to have the kids work on it" (Gloria, Interview, March 26, 2015).
To ensure comfort with the mathematics content and non-traditional teaching approach, Gloria is continuously seeking out opportunities to learn and improve her teaching practice. She reported, "I am always investing in resources, for my personal self, I am always buying books" (Gloria, Interview, March 26, 2015). In addition, "I will attend ... the ... OAME [Ontario Association for Mathematics Education] ... I normally would attend their annual conference" (Gloria, Interview, March 26, 2015). Gloria engages in this self-led PD "because if I am going to be very effective in continuing to be as effective as I want to be in the classroom, I have to keep abreast" (Gloria, Interview, March 26, 2015). During the study year, aside from her self-led PD, Gloria had also attended "the EQAO workshop" (Gloria, Interview, March 26, 2015). These "workshops are held at the Board" (Gloria, Interview, March 26, 2015), and "they teach ... the model school cluster" (Gloria, Interview, March 26, 2015). Also in the study year, "there is the OISE program that ... Grade 6 teachers are a part of" (Gloria, Interview, March 26, 2015).

Due to her non-traditional teaching practice, coupled with her high content knowledge and on-going professional learning, Gloria is also a role model to her colleagues, in both an official and unofficial capacity. She is "a part of the model school [cluster] ... [as a] Learning Teaching Coach" (Gloria, Interview, March 26, 2015). As "part of that cluster ... we have monthly meetings, so what I do is try to share lots of the stuff with my colleagues" (Gloria, Interview, March 26, 2015). If Gloria "found something to be successful I share ... with my colleagues" (Gloria, Interview, March 26, 2015).

Finding that formal meeting times are limited, Gloria stated, "basically I have lots of [informal] contact with my colleagues ... and communication. And that helps us to ... create an environment because we share information not at the meeting setting, because we do not have too much time" (Gloria, Interview, March 26, 2015). Gloria observed "some of [my colleagues]
are buying into it" (Gloria, Interview, March 26, 2015), referring to a non-traditional teaching approach in mathematics.

She provides further support to colleagues through classroom visits, having many requests of, "Miss, I would like to come into your [class] to come and see one of your math lessons" (Gloria, Interview, March 26, 2015), and she states, "I visit my colleagues, they visit me" (Gloria, Interview, March 26, 2015). As a result, Gloria and her colleagues "are always in discussion" (Gloria, Interview, March 26, 2015), and "talk about what [they are] doing" (Gloria, Interview, March 26, 2015).

In addition, Gloria takes on a role of supporting mathematics initiatives even at the system level as a Learning Teaching Coach ...

Gloria went on to explain her perception of the purpose of her role, "being a part of that Learning Teaching Coach ... that model school cluster, all roads should be to come back to the school, and ... work with teachers ... to ... realize greater student success" (Gloria, Interview, March 26, 2015). While she seemed to enjoy the role, Gloria mentioned some drawbacks, "I mean it has its challenges, because ... of preparation you have to do. And you have to know content" (Gloria, Interview, March 26, 2015).

Gloria perceived the role of mathematics communication within her mathematics program as "very important, because it enables the... learner to explain and to justify and to prove what he or she is thinking mathematically" (Gloria, Interview, March 26, 2015). Furthermore, mathematics communication "allows the teacher to be able to be knowledgeable of
the student's thinking" (Gloria, Interview, March 26, 2015). For Gloria, "using correct math language, that is what I would say is ... mathematics communication" (Gloria, Interview, March 26, 2015), because "you have to be mathematically literate to some extent to be able to communicate your mathematical thinking" (Gloria, Interview, March 26, 2015).

To determine a particular lesson's success, "it depends on [the] learning goal and success criteria for the day. So, what is the learning goal? What are the success criteria? That would be what I would ... want to see how [the students] ... have been successful" (Gloria, Interview, March 26, 2015). One example Gloria provides, relates to students' mathematical communication:

So, we are looking at ... comparing fractions greater than a whole on a number line. So are you able to show me 5/3? and 2 1/4? To tell me which is greater, which is least? So that would be the communication that would come out ... but that is being able to use mathematical language ... To communicate. (Gloria, Interview, March 26, 2015)

With an understanding of student thinking, and the use of learning goals and success criteria, "assessment for learning can be facilitated there because I would know ... what is my next step, and I could communicate to the child what would be their next step for improvement" (Gloria, Interview, March 26, 2015). Furthermore, assessment "can be very triangular too, with ... teacher-student, student-student" (Gloria, Interview, March 26, 2015).

To support students in developing their mathematics communication, Gloria engaged students in using math talk. In her classroom, "students would share their thinking ... sharing what they are doing ... there is time for student talk" (Gloria, Interview, March 26, 2015). Gloria observed that "[students] they like it, this talking. They like talk" (Gloria, Interview, March 26, 2015). Gloria used many different strategies to support mathematics communication during her
lessons. First off, "students would share their thinking ... through gallery walks, or bansho, or math congress" (Gloria, Interview, March 26, 2015).

At other times, students were instructed to "talk with your partner, now talk with your group, now talk with the class" (Gloria, Interview, March 26, 2015). Gloria indicated that, "the ones that really do like the math talk you have to really ... guide them through it. “Okay, let us use these prompts [points to a poster of math talk prompts] or let us use these two [points to the same poster]" " (Gloria, Interview, March 26, 2015).

To support her mathematics program, Gloria stated, "I use a lot of manipulatives ... I really let the kids touch the stuff" (Gloria, Interview, March 26, 2015). One example that was current at the time of her interview:

we are looking at using ... the number line to compare and order fractions greater than a whole ... I drew out number lines for them, so that they actually know [to] begin ... I prepare this for them, I photocopy this, and then I walk them through [it]. I have mine there [points to front blackboard] so I am modeling for them also. "It is a number line. Okay, identify the wholes. If I am at this point [points to number line] where am I?" ... See you really have to be explicit with them. (Gloria, Interview, March 26, 2015)

Gloria has enjoyed high levels of parental engagement, which she attributes to her mathematical philosophy:

Parents I think they do understand my vision for their kids, in terms of success, and that every child can learn. So they ...have bought into this. And I see that because of their ... attendance at parent-teacher conferences ... twenty-eight out of twenty-nine parents would ... be present ... and we ... do have conversations, ongoing conversations (Gloria, Interview, March 26, 2015)

She added that, "the parents [pauses] they want ... their child, or children to be successful at school, so they are very cooperative" (Gloria, Interview, March 26, 2015).
4.3.3 Challenges in Mathematics Teaching

During her interview, Gloria identified several challenges in implementing her mathematics program, and many of them centered around the students at Bon Echo School, and their community. The first thing she identified as a challenge in her teaching was:

the neighbourhood in which we are living in ... this is an inner-city school ... it is a challenging neighbourhood, and students come from various socioeconomic ... backgrounds, and issues in life that cannot be separated from ... their school life. So they are coming hungry, they are coming tired, and all the rest. (Gloria, Interview, March 26, 2015)

As a result, "behaviour is a very ... challenging thing for us here, at least for me" (Gloria, Interview, March 26, 2015). She observed a perceived link between behaviour and academics, stating that she needs to focus on "behaviour management, we have a large number of our students on the IEP. Not working at grade level" (Gloria, Interview, March 26, 2015). Gloria stated that, "what I found in terms of ... [meeting] those provincial goals, you have to re-teach. This is Grade 6, [but] I am having to re-teach Grade 4" (Gloria, Interview, March 26, 2015), "so those are challenging, having to be able to ... help the child to be successful and having to deal with all these things" (Gloria, Interview, March 26, 2015).

Gloria felt that Bon Echo School was more challenging than other schools, "because some of my colleagues [at other schools] they do not have to deal with the behaviour management. They do not have ... a large number of students on IEP, so academics is more of the focus than having to deal with all these other issues" (Gloria, Interview, March 26, 2015).

Gloria also struggled with students' math perceptions:

The first challenge is - from the kids. That is the first challenge. They see math as paper and pencil work [pauses], and they have this hatred towards math, they say "I do not like math" ... They ... hate it. Yeah! So that is my greatest challenge. (Gloria, Interview, March 26, 2015)
Gloria observed:

I do not create [an] opportunity for them with a ... worksheet and say, "Do one" and they just copy from their friends ... they do not really like that ... because ... they think on their own, they think with a partner, but they still got to think. And they do not want to think. (Gloria, Interview, March 26, 2015)

To combat students' 'hatred' of math, Gloria wants "to have that growth mindset take place" (Gloria, Interview, March 26, 2015), as "growth mindset is a whole shift they have to go through in my math class" (Gloria, Interview, March 26, 2015). However, "that is what we are still working on, probably will continue to work on it until [the last day of] June" (Gloria, Interview, March 26, 2015).

Another challenge related to students and academics was the curriculum. Gloria used the curriculum to meet the province's math goals, "because that is my guideline. I cannot use anything else" (Gloria, Interview, March 26, 2015). However, she had some concerns about the document. She stated, "I think it is important that we see the whole, in relation to the parts" (Gloria, Interview, March 26, 2015). Currently:

we have the curriculum that is broken down from ... Grade 1 to Grade 8 ... So we have that, we have it in its entirety ... If we can see that whole 6, 7, 8 whole [pauses] in relation to the parts, that I think would be a great accomplishment. Even elementary school, because we ... are receiving from the elementary schools. (Gloria, Interview, March 26, 2015)

She also felt that the curriculum made incorrect assumptions:

when you are finding out when the kids come ... into Grade 6 they are not at a Grade 5 level ... They are not at the Grade 5 level. We assume because they are coming into Grade 6, I should start the Grade 6 curriculum. No! (Gloria, Interview, March 26, 2015)

To determine student content knowledge:

we have the ON app ... and we use that to do our diagnostic ... it is an Ontario math program. So we use the Grade 5 one, as a diagnostic for our Grade 6, and it is unbelievable. So that proves to us that when they come into Grade 6, they are not at a Grade 5 level. (Gloria, Interview, March 26, 2015)
Because of all her observations, she stated, "I think that is why the ... province is having this breakdown in mathematics. It is not working!" (Gloria, Interview, March 26, 2015). To remedy this breakdown:

what I would recommend is that we revisit and really understand. When I get a new group of students, what is the level they are at mathematically? ... Do not just take it for granted, "Oh, so I start my new curriculum. Bang!" (Gloria, Interview, March 26, 2015)

Although Gloria has experienced success in developing students' mathematics communication, as mentioned earlier, she also shared some challenges, first of which was "getting them to use math language" (Gloria, Interview, March 26, 2015). She provides support, for example, the math talk prompts poster, but she still finds it difficult to "[get] them to use these math prompts" (Gloria, Interview, March 26, 2015). When she can get students to communicate using the proper language, "they are always more willing to ... express themselves orally, but not to write it" (Gloria, Interview, March 26, 2015).

When students are working on concepts, they have trouble "understanding word problems" (Gloria, Interview, March 26, 2015). Gloria used "lots of manipulatives" (Gloria, Interview, March 26, 2015) to support student learning and stated, "I prepare [the manipulatives] for them. I photocopy this and I walk them through it" (Gloria, Interview, March 26, 2015). She prepares manipulatives because, "If I tell them draw a number line and divide it, they are going to be looking at me like, "What are you talking about?" " (Gloria, Interview, March 26, 2015).

To further help students with word problems:

we have worked on a process thinking ... so they know okay you have to work through, but then many of them still forget to come back after they have shown their process thinking. They leave just the process thinking. (Gloria, Interview, March 26, 2015)

Gloria shared an example that was current at the time of her interview:
we looked at a problem during the week,... it said, "This girl said she ate 2/4 of a cake, and the other girl said she ate 8/16 of a cake. Did they eat the same amount?" So they did the process thinking, they showed the equivalent fraction, and then they forgot now to say, "Yes, they ate the same amount, because both fractions are equal." (Gloria, Interview, March 26, 2015)

Gloria expressed some frustration at students not being able to tie it all together.

Even though Gloria has a strong desire for professional learning, and engages in self-led PD, she has found that middle school is an underserved area in mathematics. She explained that "lots of the programs I am finding they are... for either elementary school or high school, [but nothing for middle school] ... That is what I found last year" (Gloria, Interview, March 26, 2015).

She found that neither the elementary or secondary option fit for her needs, for example:

I have gone to the elementary program, now the high school's [program]. I cannot really flip it, because the last time I went there is the flipped classroom. I am still to try it here. I cannot flip ... my classroom. Because it is not like high school where everybody got a phone, or you know, Internet. No ... it is just a few kids in this class who have a phone. It is not a big thing in Grade 6. (Gloria, Interview, March 26, 2015)

She concluded she cannot flip her class because if she did "I am not being fair to the kids" (Gloria, Interview, March 26, 2015).

While Gloria did identify successes in dealing with her students' parents, she also shared challenges she had encountered. She found that parents "want their ... children to be successful at school ... but they are not as involved as they should be" (Gloria, Interview, March 26, 2015).

She added:

they are not supporting the child as they should in terms of homework completion, you know ... ensuring that ... they are taken to the library, to read. Lots of students need ... extra academic support outside of school. Parents are not providing that. (Gloria, Interview, March 26, 2015)

Gloria attributed this low parental support to "the absence of the hours away from the home" (Gloria, Interview, March 26, 2015). In addition, regarding parental knowledge of her
mathematics approach, she responded, "I do not know if the parents know what is going on. [laughs] ... They just know their kid's doing math" (Gloria, Interview, March 26, 2015).

4.4 Case of Alison

Alison has a background in Equity Studies, Caribbean Studies, and History. She had spent her entire teaching career of nine years within the same school board and had taught from Grade 4 to Grade 8, at the time of her interview. The study school year was her second at Bon Echo School. At the time of her interview, she was teaching Language and Mathematics to her Grade 6 homeroom, as well as Grade 6 Social Studies, Drama and Media Literacy on Rotary.

Alison explained that, when considering communication in mathematics, she begins by "introducing the vocabulary [for] whatever unit or whatever strand that we are going to be talking about" (Alison, Interview, March 26, 2015). She provided an example, "first, talking about what is math. What is geometry? What do you mean by a vertex? What do you mean by ray or line? ... math language is important" (Alison, Interview, March 26, 2015). She discussed manipulatives and technology in her definition, expressing her desire for:

[students] having the opportunity to see it. So that is where the technology comes in. There are so many virtual manipulative sites where they actually can move lines and create angles. And having the actual manipulatives in your classroom that we are talking about. Actual materials in your classroom. So ... that is how I would communicate math. (Alison, Interview, March 26, 2015)

4.4.1 Attitudes and Practices to Teaching Math Survey - Results

Alison completed the APTMS at both the first (December) and fourth (April) PD sessions. Her Dimension scores and Overall score is summarized below for both sessions, as well as any changes between the two administrations. All ten of Alison's Dimension scores as well as her Overall score showed an increase between the first and final administration of the APTMS.
Table 4

*Initial and Final Attitudes and Practices to Teaching Math Survey Scores for Alison*

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Day 1 Score</th>
<th>Day 4 Score</th>
<th>Change in Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Program Scope &amp; Planning</td>
<td>4.33</td>
<td>5.33</td>
<td>Increase</td>
</tr>
<tr>
<td>2) Meeting Individual Needs</td>
<td>3.8</td>
<td>4.6</td>
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</tr>
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<td>3) Learning Environment</td>
<td>4.33</td>
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<td>Increase</td>
</tr>
<tr>
<td>4) Student Tasks</td>
<td>3.8</td>
<td>4.6</td>
<td>Increase</td>
</tr>
<tr>
<td>5) Constructing Knowledge</td>
<td>3.6</td>
<td>4.6</td>
<td>Increase</td>
</tr>
<tr>
<td>6) Communicating with Parents</td>
<td>4.5</td>
<td>5</td>
<td>Increase</td>
</tr>
<tr>
<td>7) Manipulatives &amp; Technology</td>
<td>5</td>
<td>6</td>
<td>Increase</td>
</tr>
<tr>
<td>8) Students' Mathematics Communication</td>
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<td>5.75</td>
<td>Increase</td>
</tr>
<tr>
<td>9) Assessment</td>
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<td>5.75</td>
<td>Increase</td>
</tr>
<tr>
<td>10) Teacher Attitude &amp; Comfort with Mathematics</td>
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<td>5.4</td>
<td>Increase</td>
</tr>
<tr>
<td>Overall</td>
<td>4.18</td>
<td>5.21</td>
<td>Increase</td>
</tr>
</tbody>
</table>

**4.4.2 Successes in Mathematics Teaching**

Alison experienced some successes while delivering her mathematics program. She explained that her students can have difficult lives, that "some of them are coming in here with so many other things that for them to just be here is a celebration" (Alison, Interview, March 26, 2015), and "I think the success is them wanting to be here, them being excited about being at school, and ... relationship building" (Alison, Interview, March 26, 2015). To support her students, she says she "wants them to be engaged" (Alison, Interview, March 26, 2015). She linked engagement to success, "with me, when it comes to success ... it would be engagement [with] whatever is being taught in the classroom" (Alison, Interview, March 26, 2015).

Alison's teaching practice was centered around, "rich assessments ... good assessments, better assessments, so we can see where our kids are" (Alison, Interview, March 26, 2015),
because "assessments need to be useful. Sometimes ... they are not useful, and do not give you as much ... information on the child in terms of their ability" (Alison, Interview, March 26, 2015).

In her mathematics class, "the kids really have to work in groups or by themselves [on] a rich problem" (Alison, Interview, March 26, 2015). Students then share their solutions after "they do the problem, and you can do a bansho" (Alison, Interview, March 26, 2015), which sometimes can result in Alison thinking, "Oh, I never thought about that." But it is okay. My comfort level's having that I can learn from the kids and ... that I will be exposing them to models that they had not done before" (Alison, Interview, March 26, 2015).

Alison also uses her experience to support students "because I have been teaching Grade 6 for the past years, we see the areas that students had difficulty in EQAO ... we are being more deliberate and giving more time with those strands" (Alison, Interview, March 26, 2015). When discussing mathematics planning, she stated, "our focus was Number Sense, and just an understanding that everything - that every other strand has Number Sense in it" (Alison, Interview, March 26, 2015).

For students to succeed in mathematics, Alison felt that "it is important to have that communication in my program because whatever it is that they are learning it is ... the foundation" (Alison, Interview, March 26, 2015), because "the math language is important" (Alison, Interview, March 26, 2015). In her class, Alison begins with a focus on vocabulary. To build:

the vocabulary they have their own dictionary in the back of their notebooks. So whatever word I want them to know, they put the word in the back of their notebook and they will draw a picture. That is something that they are responsible for doing. (Alison, Interview, March 26, 2015)
Furthermore, "it is important for them to know the language and then they can connect it with things that they can see or things they have experienced" (Alison, Interview, March 26, 2015). Alison provides opportunities for students to connect math language to things they 'see' by "having the words around them, having different objects around them that they can see. They can see the math, feel the math, be a part of the math" (Alison, Interview, March 26, 2015). She uses this approach because she feels "that would be the best way ... To have them involved, or to be a part of their learning, their space" (Alison, Interview, March 26, 2015).

An example of how Alison connects math language to students' experiences was, "saying the word "parallel", and then knowing that, "Okay, well the streets Jane, and Keele, and Dufferin are parallel" " (Alison, Interview, March 26, 2015). She also observed, "even in conversation ... if we are taking something up ... [Student:] "It is not beside it." [Teacher:] "What do you mean by beside it? Do you mean parallel?" ... having them re-enforcing the language" (Alison, Interview, March 26, 2015) is important.

Alison includes technology and manipulatives in her mathematics program to increase students' mathematical communication. Technology supports students through "them having the opportunity to see it, so that is where the technology comes in. There are so many virtual manipulative sites where they actually can move lines and create angles" (Alison, Interview, March 26, 2015). Alison felt it was important "having actual manipulatives in your classroom ... Actual materials in your classroom" (Alison, Interview, March 26, 2015). She shared this example of how students use manipulatives:

they help themselves to tracing paper because they know that it will assist them, they are able to see how a shape will move ... moving it 90 degrees, just so that they are comfortable with math ... The math resources that are there, they are able to use and not having to remind them, "Okay go and use them". They just use them ...it will help them with their math learning. (Alison, Interview, March 26, 2015)
She stated she knew the resources were working when "you hear them use the words" (Alison, Interview, March 26, 2015).

Alison reflected on how her teaching practice includes the use of technology and manipulatives to support students. She felt that, "I think we are doing, pretty well ... with the technology piece" (Alison, Interview, March 26, 2015), and the Grade 6 teachers, "we have been looking at ... the different resources that are available ... especially us trying to incorporate more technology in our classrooms" (Alison, Interview, March 26, 2015). Regarding technology, Alison's "personal ... goals ... [are to] have more technology in my classroom, be more comfortable with it, and I can use it on a day-to-day basis" (Alison, Interview, March 26, 2015). Alison also wanted technology, "more ... incorporated in my ... units, and ... assessments" (Alison, Interview, March 26, 2015).

At the time of her interview, "we have been using our SMARTBoards, we are using our laptops, so I think that has been successful" (Alison, Interview, March 26, 2015). She expressed some frustration about the past because, "I did not have a SMARTBoard last year. [Students would ask:] "Are we getting a SMARTBoard?""(Alison, Interview, March 26, 2015). She felt technology use:

now ... has been an expectation, because our family of schools, even ... the elementary schools ... they also have a lot of technology in the classroom. So, when they come here ... it is almost an expectation, which is good. It is pretty great because there are a lot of other communities within the [school board, where]it is an expectation for those things to be available. (Alison, Interview, March 26, 2015)

Aside from technology and manipulatives, Alison mentioned that last year:

we had someone from the Ministry come in to help us do the rich tasks ... our focus was on Number Sense, and just an understanding that everything - that every other strand has Number Sense in it, so if they are not getting Number Sense [they will struggle].(Alison, Interview, March 26, 2015)
She added that "we have not really had much of him this year." (Alison, Interview, March 26, 2015).

In dealing with parents, Alison has experienced some success. She observed that, during "parent-teacher interviews, math is always the one that the parents" (Alison, Interview, March 26, 2015) focus on as important. She also found that for her students:

you can always tell the kids that their parents are involved because they have a sense of advocacy for themselves, "Oh, [Teacher] are you available after school? Can I come to you at lunch?" ... But that has to be told ... They think the lesson's over, the day is over, and they cannot ask questions. (Alison, Interview, March 26, 2015)

But, "I ... see a small number of those parents" (Alison, Interview, March 26, 2015).

4.4.3 Challenges in Mathematics Teaching

Alison faces challenges in delivering her mathematics program, and those challenges begin on the personal level:

So my own goals in mathematics is to become comfortable with teaching all strands of mathematics. I think for me it is my comfort level in teaching math. I mean ... we have the school plan we go by, but ... I guess it is easier to save those strands that you are not really comfortable with [laughs] a bit towards the end ... But ... for me it is comfort. Comfort level. (Alison, Interview, March 26, 2015)

She revealed that she has been "experiencing some anxieties when it comes to math" (Alison, Interview, March 26, 2015).

She also faces challenges related to Bon Echo School and its students. At the time of her interview, she had taught at five different schools, and described Bon Echo School as "my most challenging school" (Alison, Interview, March 26, 2015). The community itself is a challenge for students because "some of these kids are coming with a sense of fear ...walking to school with that sense of fear" (Alison, Interview, March 26, 2015), resulting in some students "not even coming to school" (Alison, Interview, March 26, 2015). Alison shared an example of what is
causing this fear, "last year when I started ... a couple weeks before school started two or three people were shot on [this street], and some kids here knew those guys" (Alison, Interview, March 26, 2015). She added:

some of them... do not have ... the most [pauses] positive perception of police, because of the things that they have heard or the things that they have seen. [And also from] family members - because some of these kids, their parents also grew up in this community. (Alison, Interview, March 26, 2015)

Even though "[my last school] is only four stoplights south of here ... it is different ... this school [pauses] it has me working [laughs]" (Alison, Interview, March 26, 2015). Furthermore, community challenges center around "the socioeconomic situation with a lot of our parents" (Alison, Interview, March 26, 2015), and as a result at the school, "we are feeding them, or we are trying to feed them" (Alison, Interview, March 26, 2015).

Alison observed that her students also have "social issues with each other, in the community" (Alison, Interview, March 26, 2015). Even with the teachers "I find that a lot of our kids here if we do not have that relationship piece that they do not feel somewhat connected with you ... it is more challenging to do a unit and have them connect" (Alison, Interview, March 26, 2015). She attributed this challenge to "they are so disconnected with what is going on" (Alison, Interview, March 26, 2015), even though as a teacher, "I want them to be engaged" (Alison, Interview, March 26, 2015).

Another major challenge for Alison is "there are lots of students who have gaps in their learning" (Alison, Interview, March 26, 2015). She observed, "a lot of them are not getting ... whatever strand we are doing" (Alison, Interview, March 26, 2015) because of "those gaps that are there" (Alison, Interview, March 26, 2015). She made a connection between engagement and learning, "whatever it is ... that is causing these gaps ... I would say a lot of it is engagement ..."
that they are not getting it. They are not able to connect with whatever it is" (Alison, Interview, March 26, 2015). In addition, "sometimes [students] they have a phobia of math. "I cannot do it." " (Alison, Interview, March 26, 2015). Alison provides students with tasks where the "expectation with the kids in giving them these math problems [is] them feeling comfortable" (Alison, Interview, March 26, 2015).

Alison was concerned with class sizes, having "a class of 27" (Alison, Interview, March 26, 2015), especially because "they are at varying abilities ... I have quite a few ... maybe 40% who are maybe Level 2 and below" (Alison, Interview, March 26, 2015). This wide range of student abilities "that is the most challenging piece for myself because I do not have an assistant in my classroom, it is just myself" (Alison, Interview, March 26, 2015).

The challenge, for Alison, lies with the lack of resources, because:

when we talk about ... how we are going to accommodate for students, and finding the resources that we are needing to accommodate ... that is very challenging. Because you are teaching a class that there are so many different ... abilities, and you want to obviously make sure that ... everybody is reaching their potential. Whatever their potential is ... whatever level they are at. But, ... being one person in the classroom, makes it quite difficult when it comes to planning and when it comes to delivery. (Alison, Interview, March 26, 2015)

She felt that a challenge "I would say ... finding the resources, finding enough resources" (Alison, Interview, March 26, 2015). She stated that with her school board "the push right now is math" (Alison, Interview, March 26, 2015), but questioned "how do you support us as teachers?" (Alison, Interview, March 26, 2015).

Alison felt that "the provincial Ministry's vision of mathematics ... It is a focus right now... because of the [EQAO] scores" (Alison, Interview, March 26, 2015). She considered EQAO a challenge regarding time:
Especially teaching grade 6, you have the EQAO and it is important that we are on top of things and we get done what we need to get done because we are supposed to be finished the curriculum by May and that is when the EQAO happens, whereas you still have a whole month of school left. I think it is time, it is always that time crunch, and getting everything done. That would be the most challenging. (Alison, Interview, March 26, 2015)

In addition to the time challenge related to EQAO:

I guess we are always sort of on a time crunch ... when it comes to strands, you know that you have to cover this strand, and then you have to cover that strand. Sometimes I would love to have the opportunity to spend more time on a strand, especially if the kids are not quite getting it or they need more support. Sometimes you [have] to move on to the next strand, and you know they did not get it ... (Alison, Interview, March 26, 2015)

Furthermore, with Bon Echo School being a middle school, teachers and students have even less time available "because there is so much going on at your school ... you are planning your unit on another subject, you have class trips, you have kids running sports teams, so there are so many other things" (Alison, Interview, March 26, 2015).

Alison thought that the push for EQAO stemmed from "the students are not faring as well as they would like" (Alison, Interview, March 26, 2015). She felt this 'push' "it is great ... in terms of ... PD, it is fine to say this is where we are going ... but how am I being supported as a teacher ... for this vision that you have ... to bear fruit?" (Alison, Interview, March 26, 2015).

Alison felt that:

teachers would love to hear about things that they can do to help their students be successful. But it is not only hearing, it is also having somebody come in and help you do that. Help you to help your students, because as I said, I am not 100% comfortable teaching math. It is something I have to do but I do it. I think that sometimes the PD that we receive is kind of like, "do this session, do this session", and you go out there and you should be better. (Alison, Interview, March 26, 2015)

To help with implementing the PD, she suggested collaboration:

I think what is very useful for us teachers is when somebody comes in and you are able to collaborate with them, and they do the lesson with you, see how the lesson is done, or just do the lesson with you. (Alison, Interview, March 26, 2015)
She reflected on a past experience:

I think it would be nice if we had ... someone who is more attached to the teacher, like it becomes a journey. For the year, this person is going to come into your school. I did have one when I was at [another school] ... [she] was called the Teaching Coach and she'd work with me for math. She came in every other week ... she was spread over like 10 different schools ... so sometimes I wouldn't see her for a whole month. Whenever she was around, whenever we get a chance at that, it was amazing! Because [math] was her forte. She was good at math and she had been teaching it for awhile and she had resources, like online things that I could do. She would come in at the end of the week, and she'd do lessons by herself and we co-taught a lesson together. But it was really ... useful that I had somebody who knew math. She was there to help me with the math, and I felt like, when she left, I felt a little bit better, a little more comfortable with the math in whatever area. (Alison, Interview, March 26, 2015)

Another challenge Alison faced when planning for her mathematics instruction was related to the curriculum document, and she stated, "some of the curriculum is great, but I think there needs to be more work around student engagement" (Alison, Interview, March 26, 2015).

The final challenge that Alison faced in her mathematics program was her students' parents. During her tenure at Bon Echo School, "I cannot say I have had much parent involvement" (Alison, Interview, March 26, 2015). She attributes some of the lack of parental involvement to:

sometimes ... it can be seen as the parent does not want to be involved, or is not interested ... And parents are working night shifts, or two shifts, and ... it makes it difficult ... Especially if it is a single parent home. (Alison, Interview, March 26, 2015)

Regardless of the parental situation, Alison struggles as a teacher when there is a lack of support:

you have a student in your classroom, and ... he or she is not doing well, and... is not ... getting the homework done, and not getting their assignments done on time, and you want to meet the parent, and the parent is not even available. (Alison, Interview, March 26, 2015)

Even when parents are involved:

the parents that I have, that are supportive, they are as supportive as they can be, but it is more about the marks ... "How does my child do better?" Not, "What are you guys
teaching? How are you teaching?" It is just more about the marks. (Alison, Interview, March 26, 2015)

Although parents focus on math "most of the time I have parent-teacher interviews... parents, are like "Oh, I want him to do better ... Can you help?" (Alison, Interview, March 26, 2015).

However, parents "they will say that with the math, homework will come home, and they are like "I have not done this in so long. Ask your teacher." (Alison, Interview, March 26, 2015). Alison believed that parents "are just ... as challenged with some of the math as we [teachers] are" (Alison, Interview, March 26, 2015), as she has had "parents who have said to me, "[Teacher] can you help me? Can you help me help him or help her?" In whatever it is that they need help in, because they understand that education is important" (Alison, Interview, March 26, 2015).

She stated, "I have experienced a little of the ... parents who are involved, and they know [the math]" (Alison, Interview, March 26, 2015).

Another parental issue for Alison are immigrant families, "some of them are new - English is not their first language, so they have that language barrier" (Alison, Interview, March 26, 2015). Furthermore, "a lot of them ... have not been educated here, so they will not know the system so they really depend on us as teachers here to ... really help their kids to be successful" (Alison, Interview, March 26, 2015). Overall, Alison struggles with such low parental involvement that when asked how she perceived parental acceptance of her goals in mathematics, she stated, "I really cannot speak to parents" (Alison, Interview, March 26, 2015).

4.5 Case of Betty

Betty has a background in Sociology, and has spent her entire teaching career within the same school board. At the time of her interview, Betty had been teaching for nine years, eight of which were at Bon Echo School. Betty had experience teaching Grade 6 and Grade 8. At the
time of the study, she was teaching Language, Mathematics, and Science to her Grade 6 homeroom, as well as Rotary Science to all the Grade 6 students in the school.

Betty's definition of mathematics communication was, "using common language in our classroom and as a school" (Betty, Interview, March 26, 2015).

### 4.5.1 Attitudes and Practices to Teaching Math Survey - Results

Betty completed the APTMS at both the introductory and concluding PD sessions. Her scores, and any changes in score from the first (December) and final (April) PD sessions, are summarized in the table below.

**Table 5**

<table>
<thead>
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<th>Dimension</th>
<th>Day 1 Score</th>
<th>Day 4 Score</th>
<th>Change in Score</th>
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<td>1) Program Scope &amp; Planning</td>
<td>5.33</td>
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<td>2) Meeting Individual Needs</td>
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</tr>
<tr>
<td>Overall</td>
<td>4.95</td>
<td>4.74</td>
<td>Decrease</td>
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The majority of Betty's Dimension scores (seven out of ten), as well as her Overall score display a negative change between the initial and final administration of the APTMS. Two of the Dimensions scores (2- Meeting Individual Needs, and 7- Manipulatives & Technology) showed
no change in score between administration of the APTMS, indicating the teacher did not perceive any changes in her teaching practice in those areas. Only one Dimension score (4-Student Tasks) demonstrates an increase between the first and fourth PD session.

4.5.2 Successes in Mathematics Teaching

In her teaching practice, Betty focused on individual students in order to determine success. From the start, and throughout her interview, she repeated that success looks different for different students, and that as a teacher she defines success as "academically ... students achieving their potential" (Betty, Interview, March 26, 2015). Betty stressed that educators need to start by understanding their students, "because for every student it is not always Level 3" (Betty, Interview, March 26, 2015), and that the ability to "move any student forward ... is success" (Betty, Interview, March 26, 2015). Overall, she stated very passionately that, "you have to look at ... individual students and what success means to them" (Betty, Interview, March 26, 2015). Even if success is measured by something other than academic success, it still must be focused on the individual student, specifically, "for students who do not want to come to school getting them to want to come to school would be success" (Betty, Interview, March 26, 2015).

Starting with this individualized approach in mind, Betty has mobilized as many resources as she can to support her students in mathematics. To begin with, she has in-class support, "two days a week I have a teacher come in and help me in the math classroom" (Betty, Interview, March 26, 2015). She also arranged for students to receive support outside the classroom, "one day a week I can send five or six students to another teacher for resource support" (Betty, Interview, March 26, 2015). Betty was grateful for the extra staff available to her students, and stated, "having the extra support has been ... a huge support for me" (Betty, Interview, March 26, 2015). She also provides students with mathematics support outside of
Betty was very excited to share successes she had experienced in her classroom:

When I see a student get a concept that they have struggled with ... it blows my mind ... it makes it all worth it. When you see a student who all of a sudden gets something, you are like, "Okay, it ... makes it worthwhile." (Betty, Interview, March 26, 2015)

Betty makes mathematical communication a focus in her classroom, stating, "I'll do rich tasks that ask [students] to use the terminology" (Betty, Interview, March 26, 2015). Betty has had some success in implementing rich tasks to support students' mathematical communication, and shared the following example:

I have done back-to-back where the students have to ... explain to another student and [have their partner] draw or complete the task using just language, and it is really neat when it works ... because you start to hear and see the words they are using ... and when the other student understands the ... proper language and the proper terminology ... it is good. I love it when they get it. (Betty, Interview, March 26, 2015)

She went on to give a recent successful experience of the back-to-back activity in her current unit of teaching to support students' mathematical communication:

for example, ... if you are doing Geometry, or Measurement, ... "translate, start with a shape at" and they give all the ordered pairs. So the person that is on the back, and they are doing the shape or whatever, and they draw the ordered pairs, and then they translate it five units left, and then they actually, move it. So one student gives the instruction, the other has to match what the first person said. And then they are using the language. (Betty, Interview, March 26, 2015)

Betty enjoys this kind of activity, because "[the students] come up with [the language], and then I have the other person write down ... the key words that were used" (Betty, Interview, March 26, 2015). Betty finds these type of student tasks useful because she does not "have to sit down with each group individually" (Betty, Interview, March 26, 2015), which would not be
possible to do with all groups simultaneously, and she can still "see what language was used" (Betty, Interview, March 26, 2015).

Betty's students also "have math journals" (Betty, Interview, March 26, 2015) "to be able to communicate using math language" (Betty, Interview, March 26, 2015). In addition, the use of proper terminology is an expectation in Betty's classroom, "and it is always part of my rubric" (Betty, Interview, March 26, 2015).

While Betty felt pressed for time when delivering her mathematics program, she had some success in combating this issue. Her strategy was, "trying to combine as much into each unit as possible, like pulling from different units all at once" (Betty, Interview, March 26, 2015). Her example, "Patterning works with ... Geometry and Measurement" (Betty, Interview, March 26, 2015). When planning she would consider, "how do [the topics] work together, instead of doing ten different units we'll meld them together as much as we can" (Betty, Interview, March 26, 2015).

4.5.3 Challenges in Mathematics Teaching

During her interview, Betty discussed the numerous challenges she faced in her daily teaching practice. She felt that many of the challenges were specific to Bon Echo School and its students, and that her "number one [challenge] is [the] behaviour of students" (Betty, Interview, March 26, 2015), especially "after working in another school where students were not as challenging" (Betty, Interview, March 26, 2015). The unique challenges faced by students were easily attributed to "location" (Betty, Interview, March 26, 2015) by Betty, who then elaborated that a major issue faced by the students is "the responsibilities that are placed on the students outside of school ... is a lot ... They are at home babysitting, they are at home cleaning, on top of their daily activities as a student" (Betty, Interview, March 26, 2015).
Another observation that Betty made was about the differences in behaviour between students in different years, describing "cycles of students" (Betty, Interview, March 26, 2015) where one year "you get a really good cohort and then the next year, the cohort is a little more challenging" (Betty, Interview, March 26, 2015). She mentioned how even her colleagues:

talk about in our grade 8s actually that this cohort is a really good group. But they had a rough group last year and looking at our grade 7s this year, they are going to be a rough group next year in grade 8. (Betty, Interview, March 26, 2015)

She went on to tie behaviour and academic ability, by describing the current 'good' grade 8 group as "stronger academically, which I think lessens the behaviour" (Betty, Interview, March 26, 2015).

Students' academic ability is another area in which Betty feels challenged when implementing her mathematics program, and stated, "this year math has been [pauses] challenging ... my students have been low from the start"(Betty, Interview, March 26, 2015).

Another academic challenge is specific to students' mathematical communication:

I believe it is important that the kids have the proper terminology and that they are using it regularly and throughout every class and throughout every grade ... however, sometimes I feel that it almost takes a back burner when you just want them to get the concept. (Betty, Interview, March 26, 2015)

Furthermore, when mathematical communication is made the focus of the lesson, students use "the language that they have learned in the past" (Betty, Interview, March 26, 2015), even though Betty tries "getting them to move to that new language" (Betty, Interview, March 26, 2015). She provided an example, "I am doing ... Geometry right now where we are talking about transformations, and I have "slide, flip, turn" [from the students] and I am like "No, it is rotation, translation" " (Betty, Interview, March 26, 2015), later reflecting "it has been my biggest challenge just getting them to use the proper terminology" (Betty, Interview, March 26,
2015). She then needs to backtrack in her lesson, with a phrase such as, "Okay, let us go back to what we had and ... what was our new word?" (Betty, Interview, March 26, 2015).

Another challenge that Betty faces when delivering her mathematics program is time constraints. She explained, "our biggest challenge here I think is time" (Betty, Interview, March 26, 2015). Specifically, Betty had issues with the curriculum. While she felt that regarding the provincial mathematics vision, "what they ask students to do is fair" (Betty, Interview, March 26, 2015), she was critical that "they have overstuffed what students should know at each grade level. There is a lot of specific expectations ... there is too much to do in a year" (Betty, Interview, March 26, 2015). Her suggestion was to "[focus] more on the big ideas in math instead of [pauses] all the specific expectations that we need to cover" (Betty, Interview, March 26, 2015).

Another contributing factor to the time challenge is scheduling, because "there is a lot going on" (Betty, Interview, March 26, 2015) within the school. Betty explained that the previous month had been particularly busy, "March was a complete disaster for us as a Grade 6 team between March Break, three trips ... and then an assembly here, an assembly there, dance afternoon" (Betty, Interview, March 26, 2015). She summarized by saying, "yes, it is scheduled into our schedule, but it does not mean we hit math every day, because of all the interruptions" (Betty, Interview, March 26, 2015).

Betty struggled with her class sizes, and when asked how schools can increase their effectiveness in supporting mathematics, she answered, "class size I think plays a huge role" (Betty, Interview, March 26, 2015). She explained that, "I have a class of twenty-eight, I started out with ... thirty at the beginning of the year ... one teacher with thirty students - it is sometimes
hard to get around to all of them" (Betty, Interview, March 26, 2015). To explain her frustration with large class sizes, she provided the following example:

You want to do all these activities with ... manipulatives. You want to ... be able to foster a love for mathematics, but it is really hard to do that when you have a really large class, ... and not enough support ... When I have the extra teachers in my room, I find those are the easiest days to do activities with the manipulatives. (Betty, Interview, March 26, 2015)

EQAO was another challenging aspect of Betty's mathematics teaching. In Ontario, Grade 6 "is an EQAO year so you feel like there is so much pressure to do well and get them through the math" (Betty, Interview, March 26, 2015). With this focus on EQAO, "there have been a lot more math initiatives because ... EQAO was so low last year" (Betty, Interview, March 26, 2015). The high volume of professional development offered to Betty made her feel like "you need to do this activity, and this activity, and this activity" (Betty, Interview, March 26, 2015), resulting in her "[feeling] like I am jumping [around] and the kids feel like we are jumping" (Betty, Interview, March 26, 2015). She added that she felt, in terms of professional development, that "nothing really blends or melds" (Betty, Interview, March 26, 2015).

Betty expressed frustration with the parents of her students, and when discussing challenging circumstances, she described how "[other] barriers would be parental. I want to say the amount they care" (Betty, Interview, March 26, 2015). Betty's wish was for "parents wanting to be involved" (Betty, Interview, March 26, 2015). On a slightly hopeful note, "you have a few [parents] that are really, really engaged and ... a lot that are not" (Betty, Interview, March 26, 2015). Furthermore, "the kids that ... I see their parents on parent-teacher interviews are ... the ones that ... I would say I least need to see ... The ones I need to see, you do not see"(Betty, Interview, March 26, 2015). When describing the parents of her students, and wishing to engage the parent to support a student, Betty remarked:
I could give you an example. I met with a parent yesterday who this morning their child was supposed to be at my door at 8:15 [a.m.] for extra help, and they did not show up. And the meeting we had was yesterday and when I phoned them they are like, "Oh, that was today?" (Betty, Interview, March 26, 2015)

Parental engagement is so low that, when I asked Betty to describe the views of the parents, she replied, "I honestly do not know" (Betty, Interview, March 26, 2015).

4.6 Case of Joanne

Joanne has a background in Sociology, with a teaching career of over fifteen years. She has taught Grade 1 Reading Recovery, Grade 2 to Grade 4, Grade 8, and Physical Education. At the time of her interview, she had been at Bon Echo School for three years - two as Vice Principal and one as Principal.

Joanne is the school principal, and as such her interview was focused on what happens at her school as a whole rather than in a specific classroom. She also shed some light on mathematics learning and school-wide goals from an administrator's position. She shared her actions on how she stayed connected with the mathematics team and how those actions helped to inform the school improvement plan. Mathematics communication for Joanne:

I would define it as the ability to ... express ideas. That to me is that simple. The ability to express ideas, so when you have a particular concept, and you want to be able to express it, and they express it in a variety of ways to be able to explain. So, sometimes, there is vocabulary associated with that, and that is key ... a key part of your explaining. (Joanne, Interview, March, 26, 2015)

As mentioned earlier, Joanne declined to complete the APTMS because she did not feel it was relevant given her administrative role.

4.6.1 Successes in Mathematics Teaching

When considering success, Joanne's "number one is the students’ sense of wellbeing, and safety" (Joanne, Interview, March 26, 2015). She elaborated, "I consider us successful if the
students are having a good experience. If they want to come to school, they want to be here, if they feel positive about being here" (Joanne, Interview, March 26, 2015). In the school:

I think this is a universal goal. I think that everybody that is working at [Bon Echo School] has the students' best interests at heart ... and student wellness is certainly a goal of all the teachers on the staff I would say. (Joanne, Interview, March 26, 2015)

To achieve this goal, "there is the caring and safe aspect of our school improvement plan, which incorporates the mental health and wellness piece" (Joanne, Interview, March 26, 2015).

Once the wellness piece is considered, "then obviously the next step after that is ... academic success" (Joanne, Interview, March 26, 2015). Her approach is individualized, aiming for "students who are performing to the best of their ability, and we are constantly helping them to improve their performance. In that instance, we are successful" (Joanne, Interview, March 26, 2015). In the school improvement plan:

we have our literacy and numeracy goals, and that is one of the focus of the academics. Our expectation is that this is going to filter into social studies, science, all the other subject areas. That once students have a strong sense of literacy, and a strong sense of capability in mathematics, then their chances of achieving well in the other subject areas obviously increase. (Joanne, Interview, March 26, 2015)

Regarding student wellness and academic success, Joanne believed, "the two of them ... are very much combined, and very much linked. They go together" (Joanne, Interview, March 26, 2015).

Joanne enjoyed "hearing success stories from the students and from the teachers ... when students talk about being excited about a particular subject, being excited about a particular exercise or activity ... that is a real feeling of success" (Joanne, Interview, March 26, 2015).

Joanne felt that, "when kids are enthusiastic and feeling good and excited about what they are learning, and excited about what they are doing, that is very gratifying" (Joanne, Interview, March 26, 2015). For example, " when they have a sense of achievement where a student will
bring me a writing piece that they did saying, "Look [Principal], so and so... and [I] got a little sticker on it" (Joanne, Interview, March 26, 2015).

Joanne also "had kids that have come and shared their work ... when they ... used to have trouble with [the subject]" (Joanne, Interview, March 26, 2015). She takes that opportunity to "talk about ... growth mindset, with the students and we talk about the idea that you can get better at whatever it is you want to get better at. Anything that you practice, and try hard at, you can improve" (Joanne, Interview, March 26, 2015). She found that through this approach "kids are encouraged to come and share their story, and say "I was not able to do this, but now I can." " (Joanne, Interview, March 26, 2015). These discussions allow her to:

- talk about [growth mindset] with academics, and we especially talk about it with behaviour. Because the transformation over three years can be quite staggering, and we talk to the student in grade 8 and, "Remember when I used to do this [Principal]?" "Yes. I remember" [laughs] "I do not do that anymore." "Good for you!" (Joanne, Interview, March 26, 2015)

Pointing out the changes when discussing with students is:

- a great combination because ... the kids are feeling successful about their social-emotional development, and their coping skills, and their sense of confidence, and wellbeing, their attitude. They feel better about all those things, and certainly when they feel better about their academics. So, I am really, really pleased, and I think that is really ... a big sign of success when we see that. (Joanne, Interview, March 26, 2015)

Rather than describe how she plans for students in the classroom, Joanne described how the school improvement plan is formulated for the year. Although Joanne is the head of the school:

- We do it as a committee. So we meet usually the early part of the year, and then we meet at different term points ... in the year. So we meet in the fall, and ... it usually consists of the school chairs, and the Principal, the Vice-Principal, and anybody else who wants to participate. (Joanne, Interview, March 26, 2015)

To determine the direction of the school improvement plan for mathematics:
we take a look at our [EQAO] test scores, we take a look at last year's school improvement plan, see whether or not we made gains in the areas that we needed to, and then we start again with the process. So you are not really creating a brand new plan every year ... you are kind of looking over two years, or over three years. You are going to implement something over a space of time. (Joanne, Interview, March 26, 2015)

For example, "if we are at year two on our school improvement plan, we say "Okay, what do we need to do this year to move things forward?" So, that is the process" (Joanne, Interview, March 26, 2015). To ensure that the school improvement plan remains current and relevant:

we are meeting again once or twice ... through the year ... it is always there and always referred to, but as a committee, we'll meet one or two more times throughout the year, just to check in and see how we are doing. (Joanne, Interview, March 26, 2015)

In addition, Joanne worked closely with the school chairs, who are also on the school improvement committee:

We work with our grade team chairs ... because you are only meeting with the staff once a month, and ... there is a lot to check in with at a monthly staff meeting ... So, in between the staff meetings, you are looking to meet with your grade team chairs, your leaders in the subject areas as much as possible ... So, you are meeting with them, say "How are we doing? What else do we need to do? These are the challenges I am facing, what are you guys experiencing?" (Joanne, Interview, March 26, 2015)

For Joanne, it was important that "you have that communication, so I meet with the chairs once every 2 weeks ... and we just talk about whatever we need to do school-wide in regards to math, or any other subject area" (Joanne, Interview, March 26, 2015).

In her schools' mathematics instruction, Joanne wanted to "see some fundamental skill building, and we are going beyond just ... being able to memorize multiplication tables" (Joanne, Interview, March 26, 2015). Continuing with the multiplication example, "we are talking about understanding the fundamental concept of multiplication. Right? It is fine and dandy to say that "5 times 4 is 20" but what does that mean? ... What does that look like?" (Joanne, Interview, March 26, 2015). The reason for this focus is "we are trying to get some fundamentals into the
kids' understanding and into their minds. A certain comfort level with numbers, relational understanding of numbers" (Joanne, Interview, March 26, 2015).

Joanne clarified "not fundamentals at a basic level, but fundamentals at a deeper level" (Joanne, Interview, March 26, 2015). In her opinion, this focus was one that the Ministry shares, and she felt that "the shift is a positive one because it is a shift towards depth versus breadth" (Joanne, Interview, March 26, 2015).

The focus on 'fundamentals' is also explicit in the school improvement plan, "in our school improvement plan it says basically that our focus of the five strands would be Number Sense and Numeration" (Joanne, Interview, March 26, 2015), because "again, that being the foundation for a lot of stuff that you are going to do in Measurement, and the foundation for a lot of stuff that you are going to do in ... Data Management, and those sorts of things" (Joanne, Interview, March 26, 2015). Furthermore, choosing a strand to focus on allowed math instruction:

"to have a sense of consistency, because [if] we are trying to do too many things at one time, you do not make a lot of progress ... it is funny ...it is not the area where you want to multi-task [laughs], because then things are not getting done ... as well. (Joanne, Interview, March 26, 2015)

Joanne believed this approach was better than "hopping all over the place and saying, "Okay, we are going to do something in Patterning, and we are just going to bounce over to Measurement next week" " (Joanne, Interview, March 26, 2015). To support gains in mathematics, "we try to keep it kind of consistent, and keep our goals reasonable" (Joanne, Interview, March 26, 2015).

Having addressed the need for 'fundamentals', "from there we are looking at the problem solving, and just wanting the kids to do a better job in terms of applications" (Joanne, Interview, March 26, 2015), because "after we spend all this [time] getting ... the concepts down, you still
have to be able to apply them ... And that is usually linked to problem-solving" (Joanne, Interview, March 26, 2015). Reflecting once again on the Ministry's vision of mathematics, Joanne stated:

now the Ministry is emphasizing this idea of depth of knowledge and depth of understanding. And let us take that one problem and get more out of it. In terms of knowledge and understanding in application and communication. Let us get more out of the time that we are spending and the things that we are doing, the tasks we are asking the kids to do, the problems, everything that we do in mathematics. So I think that that shift on the part of the Ministry is a positive one because I think that you'll do better covering the curriculum actually if you do spend more time on one, so called, "rich tasks" or one "sophisticated problem". I think that that is a better approach. (Joanne, Interview, March 26, 2015)

Joanne believed that "mathematics communication has to do with development of thinking skills" (Joanne, Interview, March 26, 2015). For her, mathematics communication is an important component of a mathematics program, "and that is why we focus on that area..." (Joanne, Interview, March 26, 2015). In the school's mathematics program, "one of the things that we ... worked and thought about was the questioning, and having students thinking about certain questions, thinking about how they explain" (Joanne, Interview, March 26, 2015).

To support students' mathematical communication development:

We have put up charts, in the classroom, every classroom that has a math class going on in it, that has a suggested list of questions and answers that kids can use to help them to explain, to help them to describe, to help them think about, and to help them communicate with each other.(Joanne, Interview, March 26, 2015)

These "created posters" (Joanne, Interview, March 26, 2015) provide students with language support, and Joanne felt this approach was successful:

when you see kids raising their hands, saying, "I see this another way." Actually, using the language, and using it correctly in the appropriate context, that is very encouraging because that is what we are trying to get at, that is what we are trying to do. (Joanne, Interview, March 26, 2015)

Joanne reflected on a specific lesson she had observed:
From what I have seen in a particular class that I remember visiting the other day, I think that the comfort level of the students and the attitude, and the potential for learning is a real indicator of our success. (Joanne, Interview, March 26, 2015)

Working within an environment that students felt comfortable, Bon Echo School teachers were "encouraging the risk-taking in mathematics, and the students were responding to that ... I would count that as a success" (Joanne, Interview, March 26, 2015). In her school, Joanne also wanted to "encourage the teachers who work with the students ... to display their work ... and to explain as well" (Joanne, Interview, March 26, 2015). Joanne felt that as a result, students also developed:

The ability to say, "I do not understand" - huge. Huge, because we really need to give kids the language, and also encourage them to say I do not understand, and not feel badly about it. That sort of conversation piece is huge. (Joanne, Interview, March 26, 2015)

Tying together the language component of mathematics communication with the thinking process, Joanne believed that "once you get the language going and the questioning, I think that helps to foster the communication" (Joanne, Interview, March 26, 2015).

Joanne felt that "if you are able to think in mathematics, then that is going to spill over into your thinking and problem-solving in science, it is going to spill over into your thinking and problem-solving in health" (Joanne, Interview, March 26, 2015). To assist students in their academics:

we want to develop the ability to think and problem-solve. It is really not limited to the mathematics program, that is where it starts and that is where it develops a foundation, but this is something that we expect to see influencing the other subjects. So the students' ability to do this well should impact their learning overall. (Joanne, Interview, March 26, 2015)

The focus on mathematics is also reflected in the school timetable "by having that math block in the morning ... We give it a good amount of time" (Joanne, Interview, March 26, 2015). As an administrator Joanne felt that she needed to "[create] an environment that [prioritizes] the
subject, putting it in the morning when the kids are best able to focus" (Joanne, Interview, March 26, 2015). According to Joanne, "these are some of the decisions that I think ... help support mathematics success" (Joanne, Interview, March 26, 2015).

At Bon Echo School, Joanne has put a great deal of emphasis on increasing the resources that teachers have access to, particularly in terms of technology, and stated, "we have gotten much better in terms of resources ... the technology that has come in" (Joanne, Interview, March 26, 2015). Joanne went on to discuss "some big changes in terms of technology" (Joanne, Interview, March 26, 2015), and explained "that we have gotten our drops in ... we have purchased more mobile laptops and iPads ... the school's gone wireless over the past couple of years" (Joanne, Interview, March 26, 2015). In her opinion, "that has really, really made a huge difference ... in terms of what we can offer, what we are able to do with the kids, the technology that we are bringing forth and utilizing in classrooms" (Joanne, Interview, March 26, 2015).

To further support teachers and student learning, the school has placed a focus on manipulatives, and as such "we are very well-keeled in terms of manipulatives" (Joanne, Interview, March 26, 2015). When describing the access to manipulatives, Joanne explained that teachers "have a lot of things in the room" (Joanne, Interview, March 26, 2015).

Joanne also offered support to her teachers in their teaching practice, mostly through allowing and encouraging teachers' access to PD opportunities, because "PD opportunities for teachers is key" (Joanne, Interview, March 26, 2015). Joanne understood that different types of PD were needed based on the situation, and supported different opportunities, "so that those who need support on content can get it, and those who need support in terms of instructional programming and how to deliver can get that as well" (Joanne, Interview, March 26, 2015).
At Bon Echo School, "we work hard in terms of our teachers, in terms of professional development around mathematics" (Joanne, Interview, March 26, 2015). Joanne's approach to PD was "kind of having that open flow" (Joanne, Interview, March 26, 2015) meaning "that, when we need to bring in somebody external we bring them in, if we need to just kind of work amongst ourselves, we do" (Joanne, Interview, March 26, 2015). Joanne's focus on PD was based on her beliefs:

that the appropriate content knowledge, probably more so for math than any other subject, is key. You can have great pedagogy, wonderful teaching style, but if the content knowledge is not there you can kind of come up against a wall. (Joanne, Interview, March 26, 2015)

Joanne provided an example of providing PD to her teachers in response to a need, "we have had our coach in, back in the fall, specifically to do a workshop on manipulative use" (Joanne, Interview, March 26, 2015).

Joanne spoke frequently about the teaching community at Bon Echo School, explaining that there were high levels of teacher collaboration in mathematics centered around "sharing the PD ... sharing the information" (Joanne, Interview, March 26, 2015). The collaboration:

Sometimes it is just teachers having conversations ... sometimes you go a lot further with a couple of teachers getting together on their prep or after school and talking math, sometimes that is more impactful than bringing everybody out for some PD during the school day. (Joanne, Interview, March 26, 2015)

For these informal meetings, "mentoring is a huge piece" (Joanne, Interview, March 26, 2015). In the school, "we have teachers who have ... AQ Specialists in math" (Joanne, Interview, March 26, 2015), "so, mentoring their colleagues, saying "Hey, I have this resource. Let us read this." That sort of piece is key" (Joanne, Interview, March 26, 2015).

Joanne also provided more formal support in-school through bringing "the math teachers together, for some PD, for some opportunities, ... and I think that that team building really does
help to support teachers' professional development into mathematics, and to encourage them to try new things" (Joanne, Interview, March 26, 2015).

At Bon Echo School, they work "as a staff" (Joanne, Interview, March 26, 2015) where the goal is "to do some planning as a team" (Joanne, Interview, March 26, 2015) and together they were "looking at the tasks, looking at the problems, planning has been a huge part of what we have been trying to do" (Joanne, Interview, March 26, 2015). Joanne provided some historical context for the focus on team planning:

So, one of the biggest barriers that we had in the building years ago was that this person over here was doing their own thing ... and somebody over here was doing something different. And, then when you said, "Okay, let us come together and let us look at this task, or go back to our classrooms and try something," it was very difficult. "I cannot do that now because I am doing such and such." That sort of thing, right? So the idea of working to get everybody on the same page, at least around the same area. It is not that there is no discretion on the part of the teachers to do what they need to do in any given time. They certainly can, but we kind of want to be in the same area as we go through the year, as you go through planning your weeks, your months, your blocks. So, I think we have gotten sort of a little bit more on board in terms of planning, so that was the reason. If you have things planned, that enables you to ... sit down and mark a piece of student work, to look at student work, and we are all looking at this. To go back to classrooms and try different things ... to just have that feedback piece amongst each other. So planning has been a focus for that reason. (Joanne, Interview, March 26, 2015)

Joanne reflected that, "the planning piece has been huge ... It takes a lot more time to do something together if we are not all on the same page" (Joanne, Interview, March 26, 2015).

After considering the team planning and collaboration, Joanne felt that "now we want to move and do some reflection pieces" (Joanne, Interview, March 26, 2015). Joanne felt that:

the math project with OISE has been beneficial ... because we get exposed to the rich tasks, ... we get ideas, an opportunity to talk and discuss ... that really, really helps. It has been incredibly helpful, so we continue to go forward and to work to plan and see what we can do next year. (Joanne, Interview, March 26, 2015)

Joanne briefly touched on the school setting and explained "it is a middle school so that is a big difference from being a K-8 school" (Joanne, Interview, March 26, 2015). However, at Bon
Echo School, they still "have mentorship" (Joanne, Interview, March 26, 2015), because they "foster it, and build it" (Joanne, Interview, March 26, 2015).

When considering the parents of her students, Joanne described them as "very enthusiastic and very caring" (Joanne, Interview, March 26, 2015). She clarified that "what I mean by enthusiastic is they are ... very passionate about ... their children's education" (Joanne, Interview, March 26, 2015). Her description was "about the parents with whom I have regular interaction, through things like Parent Council, through things like volunteering, and things like that" (Joanne, Interview, March 26, 2015). In her role, she felt that "the ones we do see are certainly very enthusiastic, and as I said, very passionate" (Joanne, Interview, March 26, 2015).

Joanne also described success in gaining support for students from the community, and "found that that has really been increasing and improving over the past few years" (Joanne, Interview, March 26, 2015). The school has people "who are involved in our nutrition program, people who do the parent conferences..." (Joanne, Interview, March 26, 2015). The school has also benefited from stronger ties with "the neighbourhood and the community" (Joanne, Interview, March 26, 2015), and "our relationship with police has changed over the years as well" (Joanne, Interview, March 26, 2015). Joanne attributed these positive changes, at least in part, to "policy, because of school initiatives" (Joanne, Interview, March 26, 2015).

### 4.6.2 Challenges in Mathematics Teaching

Joanne felt there were many "challenges of the community" (Joanne, Interview, March 26, 2015) at Bon Echo School, because "there is a fair degree of poverty and needs in the community" (Joanne, Interview, March 26, 2015). To support students:

we have our nutrition program. To make sure that kids are ... receiving the nutrition that they require... We have assisted kids with lunch meals, we have assisted kids with
getting certain clothing ... In winter time, winter jackets and things like that. So that creates ... a challenge in and of itself. (Joanne, Interview, March 26, 2015)

There is "a legacy piece as well, in terms of incidents that have happened in the community historically" (Joanne, Interview, March 26, 2015). For example:

there was a shooting ... about four years ago, on the school grounds ... it was not a student at this school ... it was a drive-by type of thing ... the student was not targeted it was an accidental - got hit with a stray bullet type of thing. (Joanne, Interview, March 26, 2015)

In addition, there were instances where "guns ... are found on the property" (Joanne, Interview, March 26, 2015)."So those sorts of things change ... how ... the students perceive the school ... This is the reputation of the building" (Joanne, Interview, March 26, 2015). Joanne felt that, as a result, "those sorts of things, it creates a whole other way to ... how people perceive the building" (Joanne, Interview, March 26, 2015).

Joanne emphasized "achieving a sense of wellbeing ... and student wellness" (Joanne, Interview, March 26, 2015). For Joanne, "you cannot make any gains academically when the kids are constantly distracted by fear, or discomfort, or those sorts of things" (Joanne, Interview, March 26, 2015).

Another challenge Joanne faced as an administrator was the make-up of the student population, specifically the middle school setting, "because you have so many students of a particular age group in the same building" (Joanne, Interview, March 26, 2015). Joanne believed that:

in K-8 schools, usually, ... the primary tends to soften the feel of the school. You have the little, little ones going around, and there is that big brother, big sister nurturing type of ... feeling ... You do things like book buddies, and it changes the entire environment. (Joanne, Interview, March 26, 2015)
However, "with middle schools, there is not such a big gap, between the 6s and 8s, therefore you do not necessarily have that" (Joanne, Interview, March 26, 2015). While the school does try to encourage mentorship:

   it is not quite the same as when you have a real big age range. And so suddenly you have over 100 kids that are 11 and 12 years old, and 150 kids who are 13 years old ... it really, really changes the dynamic. (Joanne, Interview, March 26, 2015)

Joanne also pointed out that students at her school "can be lacking in some areas" (Joanne, Interview, March 26, 2015). Joanne clarified, "we talk about the education gap, or the achievement gap, ... that is very prevalent ... in this building" (Joanne, Interview, March 26, 2015). As a school, "we want to close that gap. We want to ... have students achieving at the provincial standard or above provincial standard" (Joanne, Interview, March 26, 2015). When considering students transitioning to secondary school, "we want them to be prepared when they get there, not feeling as if they have some huge gulf to leap in order to catch up. We want them to go in, feel ready, feel confident, and ... obviously achieve" (Joanne, Interview, March 26, 2015).

   Joanne also mentioned academic issues specific to mathematics, and dealing with "the perception piece ... dealing with math phobia" (Joanne, Interview, March 26, 2015). In her opinion, "it is a real phenomenon in people in terms of not necessarily having the comfort level with mathematics content and concept" (Joanne, Interview, March 26, 2015), which "pertains to students, parents, teachers, principals [laughs] everybody!" (Joanne, Interview, March 26, 2015). Regarding her teachers, she acknowledged that they "are fantastic teachers, they are just not confident" (Joanne, Interview, March 26, 2015). Joanne felt that low teacher confidence can impact teaching, and shared how it affected manipulative use:
sometimes these things can be underutilized. You'll have a lot of things in the room, but [nobody] knows what to do with them [laughs] or they may sit, you know, in a room and collect dust because people just do not have the ideas and the facility to use them. (Joanne, Interview, March 26, 2015)

To combat the math anxiety, Joanne focused on "growth mindset ... "We can learn this. We can do this. We can make progress" ... we cannot just accept that math is hard or accept that some people are just good" (Joanne, Interview, March 26, 2015). Through this approach, Joanne wanted to "change the outlook, and the viewpoints, and the attitudes towards mathematics" (Joanne, Interview, March 26, 2015). At the time of her interview, she was "working with students and staff to change mindsets" (Joanne, Interview, March 26, 2015).

Another challenge in the mathematics program, specifically with mathematics communication, was "getting that reflection piece" (Joanne, Interview, March 26, 2015). She felt that teachers needed to be "looking back and asking ourselves, "What kind of questioning are we doing?" "As teachers, are we doing the best questioning?" "(Joanne, Interview, March 26, 2015). Joanne also wanted teachers to consider " "Are we asking the right questions? Are we encouraging the conversation?" " (Joanne, Interview, March 26, 2015). In her opinion:

that is the biggest thing, because you can put it out there and say, "Oh, that is a great question," and "That is a great way to word it," but you have to see whether or not it is making an impact. Whether or not it is having a positive effect on student learning. (Joanne, Interview, March 26, 2015)

Joanne felt "if you do not take the time to ... look back and say, "Is this working?" ... If you do not have that reflection piece it is difficult to know the impact you are having, if any" (Joanne, Interview, March 26, 2015).

One major challenge that Joanne felt she faced was time. As mentioned in the previous section, Joanne was very supportive of providing her teachers with PD opportunities. However, "it is always a case ... of time. You always ... want to offer PD, but there is so many hours in a
day" (Joanne, Interview, March 26, 2015). Joanne's concern was that each "time you pull teachers out for PD, that is time that they are not with the kids, so you need to have that balance ... having them miss a period of math to [laughs] plan is kind of counterproductive" (Joanne, Interview, March 26, 2015).

Joanne tried to "find the right time and right balance. How many times you are pulling teachers out" (Joanne, Interview, March 26, 2015). Joanne understood that "there is lots that needs to be done, and you do need to get to that assessment ... stage at some point" (Joanne, Interview, March 26, 2015), because eventually teachers "have report cards to do" (Joanne, Interview, March 26, 2015).

In her role as principal, Joanne felt "the biggest challenge I would say is ... time" (Joanne, Interview, March 26, 2015). She explained, "you do not have enough time [laughs] to do everything you want to do" (Joanne, Interview, March 26, 2015). Even checking in with staff was a time challenge:

because you are only meeting with the staff once a month, and ... you can check in with staff around your mathematics program, but there is a lot to check in with at a monthly staff meeting, so you do not get much time. (Joanne, Interview, March 26, 2015)

As mentioned earlier, Joanne met with the grade team chairs. However, even those meetings are affected by her perceived time crunch, since "we used to have weekly meetings, but we cannot always do weekly" (Joanne, Interview, March 26, 2015). In addition, while these meetings keep Joanne updated, they also contribute to another challenge of the principal role:

It is very, very important that for example principals are in classrooms, and observing the teaching and observing the learning that is going on, and contributing, and being part of those conversations. But, at the same time you have ... safety issues to deal with, and ... school climate issues to deal with, parents that ... require your time, and so it is a very difficult sort of balance to, ... meet with the parents, deal with safety concerns, issues, bullying, do investigations, that sort of thing, ... attend your meetings, get your paperwork done, and do all those things, but at the same time do one of the most
important things, which is interacting with the students and being in the classroom. So ... for me that is probably the biggest challenge of the job. (Joanne, Interview, March 26, 2015)

Joanne reflected, "at the end of the day, there are still only so many minutes in the day" (Joanne, Interview, March 26, 2015).

Finally, Joanne shared some challenges she faced with the parents of her students, particularly how "there are some parents that you do not see much of, because they work a lot of jobs, or their hours, or their shift work, or for whatever reason you do not see them" (Joanne, Interview, March 26, 2015).

Another parental challenge was "how parents perceive the school" (Joanne, Interview, March 26, 2015). "There are some that have experienced difficulties in the school system, difficulties relating to either teachers or administrators, and they experience frustration" (Joanne, Interview, March 26, 2015). However, "those are a small ... number, but they nevertheless need to be counted and acknowledged" (Joanne, Interview, March 26, 2015).

When considering the mathematics program in her school, overall, Joanne stated, "we are still resolving issues. [laughs] ... we continue to try different things, and try different ways of reaching out in terms of having the students perform adequately in math, or perform well in math" (Joanne, Interview, March 26, 2015). She summarized that "to meet your school goals it takes a lot of different steps, and there are a lot of pieces" (Joanne, Interview, March 26, 2015).
Chapter Five: Discussion and Interpretation of Findings

5.1 Introduction

During the educator interviews, the participants discussed many of the same themes and issues. While participants had differences in their definitions of mathematics communication, discussions were focused on two major areas: success in their mathematics practice, and the many challenges they faced in implementing their mathematics program. These topics will be examined in further detail in this chapter, in particular how they relate to the chosen two Dimensions of focus for this thesis, namely Dimension 7 - Manipulatives and Technology, and Dimension 8 - Students' Mathematical Communication.

Three of the participating educators completed the Attitudes and Practices to Teaching Math Survey. Gloria and Alison saw an increase in their Overall scores. Betty saw a decrease in her Overall score. The differences in scores, and their possible implications, will be discussed further in this chapter.

The research questions for this thesis, posed in Chapter One, were:

1. What successes and challenges do middle school teachers encounter in supporting students' mathematical communication?

2. How do middle school teachers utilize manipulatives and technology to support students' mathematical communication?

3. What effect does a year-long professional development study have on teachers' attitudes and beliefs?

I will answer these questions in-depth in the next section by examining the findings from the four case studies of: Gloria, Alison, Betty, and Joanne.
5.2 What successes and challenges do middle school teachers encounter in supporting students' mathematical communication?

This study began with no set definition of mathematical communication, in order to allow educators to share their own individual interpretations. There were differences in the definitions, and their complexities. One common element of the definitions shared was a focus on language. They believed that ensuring that learners are using language correctly, and focusing explicitly on the meaning of words helps learners advance their understanding of concepts (Novak & Gowin, 1984). Furthermore, most definitions, except the one provided by Betty, also included a need for student explanations, reflecting teachers' understanding of the importance of a communicative focus in mathematics to help students understand their own thinking (Boaler & Staples, 2008; Marks Krpan, 2008). Alison's definition also included using technology to support students' mathematical communication.

Educators at Bon Echo School shared many successes they encountered when supporting their students' mathematical communication. The most commonly referred to were Meeting Individual Needs (Dimension 2), Student Tasks (Dimension 4), Manipulatives and Technology (Dimension 7) and Teacher Attitude and Comfort with Mathematics (Dimension 10). This section will exclude Manipulative and Technology (Dimension 7) as it will be addressed in detail to answer Research Question 2.

I will summarize teachers' APTMS scores, for easy reference, for the Dimensions included in this section. Gloria started with scores of 4.6, 4.8, and 5.2, for Dimensions 2, 4, and 10, respectively. Her final scores were 5.8, 5.8, and 5.8, respectively. Alison started with scores of 3.8, 3.8, and 4.6, respectively. Her final scores were 4.6, 4.6, and 5.4, respectively. Betty
started with scores of 4.8, 4.2, and 5.4, respectively. Her final scores were 4.8, 4.8, and 5.2, respectively.

5.2.1 Successes With Meeting Individual Needs (Dimension 2)

The participating educators all placed a great focus on the individual student, and having an individual approach for defining student success. All four expressed that it is important to consider individual progress. Although the province's standard for achievement is a Level 3 (Ontario Ministry of Education, 2005), all teachers focused on improving the level of understanding of their students and helping students moving forward. Not all Bon Echo School students had a goal of achieving Level 3 or above.

The school as a whole worked hard to address students’ well-being, and it was explicit in their school improvement plan. Bon Echo School educators showed a strong understanding that success was not limited to the academic sphere, and that students wanting to be at school and displaying excitement is important and needs to be celebrated. Through the consideration of students’ well-being, Bon Echo School is beginning to incorporate the students’ bodies, minds, and souls in education, as supported by Miller (2010). Miller (2007) calls for a more balanced approach to education, with an equal focus of yin and yang activities, which would further support students at Bon Echo School.

A growth mindset was an area of focus at the school. Using Carol Dweck's (2008) work on mindset as a guide, the school works with students to move from a fixed to a growth mindset. Growth mindset (Dweck, 2008) is applied in both the academic and well-being context. Joanne happily shared successes that students had shared with her regarding possession of a growth mindset in academics as well as behaviour.
Student confidence in mathematics is also related to their achievement. Teachers provided examples of student tasks that included a strong communication component, such as Betty's back-to-back activity, and reflected on how these activities helped increase student confidence as well as understanding. Joanne also shared that, in a recent classroom observation, she had seen a high level of confidence and engagement from students. These findings are supported by Marks Krpan (2008), who found that Junior students felt a higher personal connection to mathematics when discourse was included in instruction, and Cirillo (2013), who states that math discussions can lead to an increase in student agency, as students may become more motivated when they are actively learning (Boaler & Staples, 2008; Cirillo, 2013).

5.2.2 Successes With Student Tasks (Dimension 4)

Teachers reported using a wide range of activities to support student learning. These activities required students to think about mathematical concepts and, through the use of math communication, provided support for student knowledge consolidation (Marks Krpan, 2008). In addition, student tasks were focused on having students communicating, with each other as well as the teacher. The shift away from a traditional teacher-led communication approach can help move the authority to the classroom community, and further support an increase in student agency (Cirillo, 2013). Tasks often included collaboration among students, with varied structures including: pairs, small group, and whole class.

Gloria was adamant that, in her classroom, students needed to think through the tasks provided. Furthermore, Gloria's students never get simple fill-in-the-blank worksheets or low-level textbook work. Betty provided the example of the back-to-back activity she had used in her classroom. This activity had a really strong communication component, as one student had to describe to the other what they were seeing.
5.2.3 Successes With Teacher Attitude and Comfort With Mathematics (Dimension 10)

Participants demonstrated an eagerness for improving their mathematics practice through their on-going participation in professional development opportunities. This participation demonstrates that teachers understand the need for continual improvement of their teaching practice to support students (Cochran-Smith, 2011). Teachers reported that, in addition to the OISE study, they were involved in professional development at the Board level as well.

Gloria talked at length about the importance of PD to remain effective in her practice. This desire to be effective was her reasoning for ongoing self-led PD. She participated annually in the OAME conference, and was continuously purchasing new resources for her personal use.

There was a great culture of collaboration within the school and Gloria, as a veteran teacher with Mathematics Specialist qualifications, was sought-after for advice and ideas. Alison and Joanne also touched on the collaborative spirit of the Grade 6 mathematics team, and the informal PD that constantly took place. This high level of collaboration within the Grade 6 mathematics team, and the school, is highly supported by the research as it is a great help for teachers who are unsure about content, teaching strategies, or their role in the classroom (Cochran-Smith, 2011; Shizu Kutaka et al., 2017; Won, 2017). Betty was the only study participant who did not mention any collaboration with peers outside of formal PD.

The culture of ongoing professional development was greatly supported by the principal, particularly in the field of mathematics. Principal support is necessary for PD success, as supported by studies from Wanless et al. (2012) and Matsumara et al. (2009).
5.2.4 Challenges in Supporting Mathematics Communication (Dimension 8)

Participants at Bon Echo School identified numerous challenges they faced when attempting to support their students' mathematical communication abilities. Discussions were centered around: meeting individual needs (Dimension 2), communicating with parents (Dimension 6), manipulatives and technology (Dimension 7), and teacher attitude and comfort with mathematics (Dimension 10).

I will summarize the APTMS scores, for easy reference, for Dimension 6. Gloria's pre- and post- scores were 4 and 6, respectively. Alison's pre- and post- scores were 4.5 and 5, respectively. Betty's pre- and post- scores were 4 and 3.5, respectively (For summary of scores on Dimensions 2 and 10, see Successes section above.) Once again, Dimension 7 will be excluded from this section, and will be used to answer Research Question 2.

5.2.5 Challenges With Meeting Individual Student Needs (Dimension 2)

Meeting students' needs was a great challenge for Bon Echo School teachers. All participants mentioned the difficulties of the community in which the school was placed. Students' personal lives required overcoming issues of poverty and hunger, which the school attempted to improve, through nutrition and clothing programs. The students also had to take on additional roles at home, to support parents, placing a heavy burden on their young lives.

A major challenge of the community was the historic violence, and the fear students faced even coming to school. Unlike other schools, Bon Echo School had to work to get their students through the doors. The challenges of inner-city schools are well documented in the literature, and Bon Echo School sadly aligns well with previous findings (Jacob, 2007; Jensen, 2009; Predmore, 2004).
With the high absenteeism comes another challenge - learning gaps, which other studies have also reported (Jacob, 2007; Jensen, 2009; Predmore, 2004). Many students lack the fundamental knowledge required to receive the Grade 6 curriculum, and so teachers need to take time to try to close the gaps before beginning the grade-appropriate curriculum. With an already tight schedule to complete the mathematics curriculum in one year, teachers struggle to find the time to adequately support student learning. Betty specifically expressed frustration about feeling the need to move to another unit, even though not all students 'got it' and needed more time.

Another academic issue teachers faced was a high number of students with an Individualized Education Program (IEP). Many students were performing below the provincial standard, with Alison estimating at least 40% of her students fall in this category. Gloria and Betty also expressed frustration with the high number of students with IEPs. Betty explained that the IEP situation once again presented a time challenge as it made planning and implementing instruction more challenging for meeting the needs of all her students.

Alison and Betty both mentioned the challenge of large class sizes and little to no in-class support, resulting in teachers struggling to address all students. The time constraints affected Betty's ability to focus on mathematical communication, with it being put on the 'back burner', as sometimes she shifted her focus to conceptual understanding so that she could move on to the next topic.

The students at Bon Echo School also exhibited very difficult behaviour, making classroom management another issue for teachers. Betty linked poor behaviour with poor academic ability. Another factor surrounding poor student behaviour was the students' perceptions of math. When students felt that the math content was not relevant to their lives, they...
were disengaged, and, according to Alison, the behaviour worsened. This finding is supported by previous studies (Jacob, 2007; Jensen, 2009; Predmore, 2004).

Students' feeling towards the subject of math also matters. Gloria discussed how students come into math class with a hatred for the subject. She felt she first had to change their attitudes before she could see an improvement. Her practice is supported by Ramirez et al. (2013) as she considers students' anxiety and emotions in math in addition to the concepts she attempting to teach, as well as Miller (2010) since she is beginning to implement instructional supports for students’ minds and souls.

Joanne also discussed perceptions of mathematics, but from the lens of mathematics anxiety. She felt that this was an issue in the school-wide population that needed to be addressed. Joanne's concerns align with those of McCoy (1992), who found that math anxiety can lead to dislike, and subsequent avoidance, of math.

Finally, Joanne expressed concerns about the middle school setting. She felt middle school required a more time-intensive fostering of community to achieve any type of mentorship amongst students.

All of these challenges faced by Bon Echo School educators take time away from working to support students' needs. In the case of Betty, she was explicit that it has an impact on how she can support her students' mathematical communication.

5.2.6 Challenges Communicating With Parents (Dimension 6)

All educators experienced challenges in communicating with the parents of their students. They described most parents as disengaged, not involved, or that they are not often seen or heard from by teachers. While some parents are not highly visible because they are working to support their families with two jobs or night shifts, other parents have had bad experiences themselves
with the education system and avoid contact. Some parents had a negative perception of Bon Echo School because of the history of the school. The last group of challenging parents to reach are new immigrants. As they are English Language Learners themselves, language was a barrier to communication.

Regardless of the type of challenging parent, or reason for lack of communication with parents, teachers felt a lack of support from parents. Betty provided an example where a parent failed to support their child's learning by not following up on a teacher-parent plan for the student. Gloria expressed frustration that parents are not providing opportunities for student learning outside of school. All three teachers felt that parents are unsure of what is happening in the math classroom. Furthermore, teachers were unaware of parental perceptions regarding the mathematics instruction of their students.

Interestingly, Gloria reported having a good relationship with her students' parents. She had high parental attendance for parent-teacher conferences, with twenty-eight out of twenty-nine families coming to discuss their child's progress with her. Reasons for this unique relationship with parents could be related to her very open personality, or her belief that every one of her students had the capacity to learn and do well in mathematics.

5.2.7 Challenges With Teacher Attitude & Comfort With Mathematics (Dimension 10)

In their interviews, Alison and Joanne were very open about their math anxiety, and desire to work on becoming more comfortable with mathematics. Joanne was able to identify that math anxiety, low teacher confidence in teaching mathematics, and low math content knowledge was very prevalent within the school. Alison revealed that she is one of the teachers in the school that falls into the category of low confidence with mathematics. She also lacks
content knowledge, and was eager to engage in PD to improve her teaching, in order to benefit her students. Won (2017) supports teachers' involvement in PD for the purposes of increasing content knowledge.

However, Alison expressed a great deal of frustration at much of the PD she had experienced. She felt the PD was disjointed, and was not easy to implement into her teaching practice. For PD to be effective, it must be coherent (Shizu Kutaka et al., 2017). Alison also felt frustrated about her perceived lack of support to improve her teaching practice. She expressed a desire to have in-class support to implement the PD strategies. Betty also would have liked additional in-class support.

Both teachers lacked the confidence to make the changes without support. Many Junior teachers, at least in Ontario, need to teach math, whereas many Intermediate teachers choose to teach math. One possible reason for the resistance to implementing the PD learning is the lack of an open-minded approach to instruction, which is necessary for teachers to improve their teaching practice (Clarke, Thomas, & Vidakovic, 2009).

Finally, a challenge relevant to Dimension 10 was Betty's attitude during her interview. While other participants showed a great deal of excitement during their interviews, Betty did not. She was disengaged for most of her interview, provided the shortest answers overall, and was looking at the clock frequently. Teacher attitudes play a role in many aspects of teaching, including: their ability to teach the subject (Shizu Kutaka, et al., 2017), if they change their instruction to align with research-based pedagogy (Won, 2017; Clarke, Thomas, & Vidakovic, 2009), how they implement technology in the classroom (Karchmer-Klein et al., 2017), and their views on manipulatives use (Martinie & Stramel, 2004). However, Betty did reply enthusiastically a couple of times when describing students successes she had encountered.
5.3 How do middle school teachers utilize manipulatives and technology to support students' mathematical communication?

As this question deals with Dimension 7, teachers' APTMS scores will be summarized here as a reference. Gloria started at the highest possible score of 6, at the start of this study, and maintained her score. Alison started with a high score of 5 at the start of the study, and increased her score to the highest possible score, 6, by the conclusion of the study. Betty started the PD study with a high score of 5.5, and demonstrated no change in attitude. These APTMS scores demonstrate that teacher’s attitudes and beliefs towards Manipulatives and Technology greatly align with current mathematics practice.

5.3.1 Manipulatives

Manipulative use was a focus of instruction at Bon Echo School, and one that was strongly supported by the principal. Joanne ensured that all teachers had easy access to manipulatives by equipping all classrooms with the learning tools, understanding that middle school students still require concrete tools to support their learning (Martinie & Stramel, 2004). All teachers reported using manipulatives in their practice. Furthermore, teachers reported that the implementation of manipulatives helped increase students' mathematical communication. Students needed to effectively communicate to succeed in the collaborative, hands-on tasks they were given.

Teachers expressed a highly positive attitude towards using manipulatives. Alison reported always having manipulatives available in her classroom for all students to use. She found that students would take initiative and independently choose manipulatives to use to support their learning. Clearly, her students had the appropriate knowledge for manipulative use, and familiarity, to effectively support their learning without becoming distracted as suggested by
Weiss (2005). She found that this student-led manipulative selection also assisted students in increasing their communication abilities with their peers.

Gloria reported always using manipulatives in her teaching, which she found facilitated math communication for students. Gloria is the most experienced mathematics teacher in this study, and her effective use with manipulatives is supported by Weiss (2005). She described also using manipulatives as a support in class, and ensuring students know how to use them properly and effectively by first modeling their use. This approach, which helps students better understand the link between the learning tool and the mathematics concept, is supported by the literature as a successful practice (McNeil & Jarvin, 2007). She provided the use of number lines to explore fractions as an example.

Betty used manipulatives to support students' mathematical communication through careful selection of rich student tasks. She provided the example of the back-to-back activity, where a major component for success was the continued, and proper, use of math communication between students. Many studies stress the importance of manipulative use in complement with appropriate teaching strategies (McNeil & Jarvin, 2007; Weiss, 2005), which Betty demonstrates in this example. To make mathematical communication an explicit part of her expectations for tasks, Betty always included math communication in her assessment, through the use rubrics.

Alison was the only teacher to report using virtual manipulatives in her instruction. Virtual manipulatives allow teachers to combine manipulatives and technology to support student learning. Teacher familiarity with virtual as well as concrete manipulatives may better support student learning (Peppers et al., 2014).
Overall, when using manipulatives, all teachers maintained their focus on student learning. As encouraged by previous research, manipulatives were seen as a tool to support learning (Moyer, 2001), used in combination with effective instruction (Weiss, 2005).

Participants described challenges they faced when using manipulatives in their teaching practice. Joanne shared that many teachers at her school did not have the knowledge regarding how to use the manipulatives, and so they sat around 'collecting dust'. Peppers et al. (2014) stress the importance of teacher knowledge regarding appropriate use and selection of manipulatives, which Joanne attempted to support through planning an in-service for manipulative use. Betty experienced a similar challenge, sharing that she found it hard to use manipulatives, particularly because she had a large class size and limited classroom support. This view could be linked to her attitude and comfort, as mentioned earlier.

Gloria described a high integration of manipulatives in her class. However, her implementation was teacher-directed and, where applicable, teacher-created. She seemed to lack faith in her students' abilities to create their own supports, or appropriately select their own manipulatives. Moreover, she seemed to lack confidence in her students' abilities to select their own approach to using a manipulative even when it was chosen for them by her.

5.3.2 Technology

Educators in the study did not place as much of an emphasis on technology during their interviews, except Joanne. Joanne was excited that Bon Echo School had significantly increased its available technology in the last few years prior to her interview. The school had invested heavily in technology, and was able to purchase more laptop carts as well as provide wireless Internet throughout the school. While Joanne was supportive in increasing the amount of
technology, Bottge et al. (2010) suggests that administrators increase teachers' access to PD surrounding technology use in mathematics.

In addition to the increase in technology access comes a need for ongoing teacher support in these new technologies through PD (Demetriadis et al, 2003). Teachers can learn how to effectively use technology to support student learning, and pair it with effective instructional approaches that maintain focus on concept exploration (Fital-Akelbek & Akelbek, 2012; Karchmer-Klein et al., 2017; Savard et al., 2013).

Although Betty did not mention the laptop carts, there was one present in her classroom at the time of her interview. Her failing to mention the use of technology, when she had recently used it, could be a result of her disengagement with the study, her lack of focus on technology use in mathematics, or simple forgetfulness.

Alison was the only teacher to mention technology use in her interview. She expressed excitement that she had a SMARTBoard installed in her classroom during the study. She added that it was something the students had been calling for, and reacted with excitement when it became a classroom addition. Alison was also the only teacher to mention the use of virtual manipulatives to support students' mathematical communication.

Similarly to manipulatives, Bon Echo School educators faced challenges when attempting to incorporate technology into their mathematics instruction. Alison expressed a desire to use technology on a daily basis, but, at the time of her interview, she felt that she could not achieve this goal. This view may be related to a lack of confidence in technology use. However, Karchmer-Klein et al. (2017) found that teachers do not need to be expert technology users, but just need to be creative in how to incorporate technology. PD on technology use may help move this attitude towards a more current teaching approach.
Even with the great strides achieved in increasing the available technology in the school, the students did not possess a great deal of personal technology, such as smartphones. Gloria found this to be an issue when she considered adopting a flipped classroom teaching approach. Although the flipped classroom is gaining popularity, she did not feel it would be fair to most students, and so abandoned the idea, and the innovation that would have brought to her teaching.

5.4 What effect does a year-long PD study have on teachers' attitudes and beliefs?

The PD sessions presented to participants were focused on helping teachers move towards practices that align with current educational policy and philosophy, thus the two survey administrations provide insight into the change in teacher's attitudes and practices in response to the PD. Of the three teachers from Bon Echo School who completed the APTMS, two of them exhibited an increase in scores between the initial and final administration of the survey.

In this section, I will address changes in teachers' attitudes and beliefs, using their pre- and post-APTMS scores. I will also examine how well teachers' self-reported practices align with the Ten Dimensions Framework presented in Chapter Two for Manipulatives and Technology (Dimension 7) and Students' Mathematical Communication (Dimension 8). There are limitations to how well I am able to compare teachers’ practices with the continuum, as I am relying solely on teachers’ self-reported practices, specifically what they chose to share in their interviews, which may not represent the full story.

5.4.1 Case of Gloria

As the most veteran teacher, Gloria goes against the stereotype of older teachers being stuck in their ways, and showing a preference for more traditional approaches. On the contrary, she is a leader in bringing about positive change in the Grade 6 mathematics program at Bon
Echo School, and many of her colleagues look to her for advice and new ideas to engage students in the mathematics classroom.

Gloria’s teaching does well when compared with the Continuum for Dimension 7. She always uses manipulatives, and incorporates them into activities for all students. However, the encouragement of math exploration through manipulatives is limited as she leads students in a particular method of manipulative use. I am not able to determine how Gloria utilizes technology in mathematics as she did not mention any technology use.

Gloria’s alignment with the Oral Communication component of Dimension 8 is excellent, and the best one of the teachers in this study. She uses group tasks on a regular basis that require students to think and communicate with each other orally. However, Gloria reported that she struggled with the Written Communication component. Beyond that, no additional information was given, and thus no further conclusions can be drawn. To enhance her practice in areas in which she is lacking, for both Dimensions, Gloria can refer to the Continuum and use it as a guide for her own professional development.

5.4.2 Case of Alison

An increase of APTMS scores across the board shows that Alison is working towards aligning her attitudes and practices in the teaching practice with those of the current mathematical educational philosophy. Alison's growth in her mathematical teaching, and the corresponding alignment with current teaching approaches, reflects the goals of the PD sessions. Another finding in Alison's scores is her movement to the highest possible score in Dimension 3 (Learning Environment) and Dimension 7 (Manipulatives & Technology). Alison has come a long way in just a few short months to achieve the highest score for two of the Ten Dimensions.
Alison demonstrated the largest improvement towards aligning her practice with the Ten Dimensions of Mathematics Education Framework (McDougall, 2004). Her alignment for Dimension 7 is the best of the three teachers in the study. She incorporates manipulatives on a regular basis, uses them to support student learning, and encourages students to use manipulatives when they need them. Alison also allows students to self-select tools and strategies. She was the most enthusiastic teacher regarding Technology, and attempts to use technology, displaying a desire to incorporate more of it into her instruction. However, she reported that she knew she could incorporate more technology, and this would be an area of improvement for her in Dimension 7. The effectiveness with which she incorporates technology cannot be evaluated.

For Dimension 8, the Written Communication component cannot be evaluated as there was no mention of it in Alison’s interview. However, she did seem to have some success with Oral Communication. Students in Alison’s class worked on group activities that encouraged student communication and discussion. As the frequency of these tasks was not shared, this aspect cannot be evaluated.

5.4.3 Case of Betty

Betty's APTMS scores show an interesting finding - of the eleven scores calculated (ten Dimension scores and the Overall Score), ten show either no change, or a decrease in Betty's attitude and perception of her practice regarding mathematics education. Only one score increased between the initial and final administration of the APTMS. This overall negative outcome runs counter to the goal of PD to help teachers improve their teaching practice. I will present two possible explanations for this outcome.
One potential explanation for Betty's negative trend could be a reflection of her personality. All individuals have different learning styles, and thus different learning needs (Gardner, 1983). Just as the Ten Dimensions for Mathematics Education Framework (McDougall, 2004) calls for teachers to consider Meeting Individual Needs (Dimension 2), PD sessions must also consider the individual needs of participating teachers. The study did attempt to address individual needs through the use of exit cards asking for feedback and areas of interest. However, the feedback could be provided anonymously, thus not allowing for facilitators' understanding of individual teacher needs. Furthermore, as there was no information available for facilitators on teachers' needs prior to the start of the PD series, facilitators could not tailor the first session to teachers' needs.

One potential remedy for these issues could be a teacher package of surveys to be completed before teachers commence the PD sessions. The package should include the APTMS, as well as teachers’ intended Dimensions of focus. In addition, the teacher package should include surveys that determine teachers': learning styles, personality, and readiness to change their teaching practice.

Although this package would require more work on the part of the teacher, it could lead to a richer experience as the facilitators would be better able to meet teachers' needs. Furthermore, through better addressing teachers' needs, the facilitators may observe higher teacher engagement with the material. Higher engagement may lead to increased teacher gains.

Through the package information, facilitators can also tailor sessions to suit teachers' needs based on learning style and personality. Facilitators can purposeful plan for, and potentially group together, teachers that prefer more individual reflection rather than
collaboration. In addition, they can determine for which teachers they need to focus on preparing them to change their practice if they are not yet ready.

The second possible explanation for Betty's drop in Dimension and Overall scores could be that the PD sessions were indeed a success. It is possible that, at the start of the study, Betty reported higher scores because she was overconfident in how well her mathematics program aligned with current practices. During the sessions, through rich tasks, Betty may have increased her understanding of what current mathematics practice should look like. At the final administration of the APTMS, she may have reported lower scores as a result of her increased knowledge and understanding that she still needed to improve her teaching to better align with research-based mathematics instruction.

Based on her interview responses, Betty experienced the greatest struggle with aligning her practice with the Continuum for both Manipulatives and Technology (Dimensions 7) and Student Communication (Dimension 8). She mentioned that, while she appreciates the value of manipulatives as a learning tool, she feels more comfortable incorporating their use when she has additional support in her classroom. Thus, students have limited access to manipulatives in her classroom, losing out on rich learning potential. Technology use was not reported during her interview, and thus cannot be evaluated.

Betty fared a bit better with Student Communication (Dimension 8), explaining that students were given collaborative group tasks that placed an emphasis on communication. However, the frequency or richness of tasks cannot be evaluated. Regarding the Written Communication component, Betty provided the most information of all teachers as she described the written component of the back-to-back activity that required students to record all language used during the activity. Of course, this level of written communication is very shallow, and can
be improved through the use of the Continuum. Betty has many potential areas for improvement, and the Continuum can be a useful guide for her as she works to improve her practice to better align with the Ten Dimensions of Mathematics Education Framework (McDougall, 2004) to support her students.

**5.5 Review of the Professional Development Initiative**

The Professional Development Initiative generated some success with two teachers’ progress in aligning their teaching practice to reflect evidence-based instruction, namely Gloria and Alison. This finding is supported by Shizu Kutaka et al. (2017), who support PD partnerships with universities as these partnerships have been shown to have positive outcomes on mathematics achievement.

The third teacher, Betty, showed a decrease in working towards aligning her attitudes and beliefs with current mathematics practice. The major differences in the effect on teachers' attitudes and beliefs requires further examination.

This PD initiative was designed in a fashion that is greatly supported by the existing literature. Workshops provided teachers with coherent information that was easy to understand and implement into their instruction (Cochran-Smith, 2011; Shizu Kutaka et al., 2017). Furthermore, as the workshops were spread out over the school year, they helped provide more effective support to teacher development (Cochran-Smith, 2011; Shizu Kutaka et al., 2017).

The PD sessions had a large focus on collaboration opportunities, as supported by numerous studies (Cochran-Smith, 2011; Shizu Kutaka et al., 2017; Won, 2017). During workshops, participants were exposed to activities that helped increase their content knowledge, which is a key element to effective PD (Shizu Kutaka et al., 2017; Won, 2017). In addition, workshops were focused on increasing teachers’ pedagogical knowledge through the inclusion of
sample student tasks and the appropriate teaching strategies, which has been shown to increase the likelihood of implementation into teacher practice (Desimone et al., 2002).

Finally, the PD sessions included a technology component. To support teachers in learning about technology and effective strategies for implementation, workshops included time for technology (Demetriadis et al., 2003; Desimone et al., 2002). All activities, technology and non-digital, were approached through the lens of active learning, to further support teacher development (Desimone et al., 2002; Shizu Kutaka et al., 2017).

This PD series is largely composed of a combination of two of the three conceptions from Cochran-Smith & Lytle's (1999) Framework. Workshops and other supports provided to teachers had an emphasis on building teacher content knowledge in math. Educators were presented with formal knowledge for different strands in the Ontario Curriculum. This emphasis aligns with Cochran-Smith & Lytle's (1999) knowledge-for-practice conception.

Workshops also provided teachers with learning opportunities to increase their knowledge of pedagogical strategies for mathematics. During workshops, participants were provided with sample activities, modeling possible implementation, and using evidence-based practices. Through introducing numerous sample tasks, this PD study helped to build teachers' pedagogical strategies in math, aligning with the knowledge-in-practice conception (Cochran-Smith & Lytle, 1999).

Workshops seamlessly integrated the two conceptions so that teachers had the opportunity to learn math concepts through learning about best practices. This approach provided teachers opportunities to improve their teaching practice in a meaningful way, with all participants finding something of use. Furthermore, this approach helped to address Alison's critique of many PD opportunities feeling disjointed. This PD study provided teachers with
activities that were already ready to integrate into their classrooms, thus reducing the time cost of planning for teachers, a concern shared by all participants.

The incorporation of Cochran-Smith and Lytle's (1999) third conception, knowledge-of-practice, into this PD study was limited. This PD study touches on the third conception by providing teachers with opportunities to discuss with peers. Furthermore, teachers were given opportunities, while collaborating with peers, to come together and raise questions. However, one critical component of the knowledge-of-practice conception was missing, specifically teacher inquiry.

5.6 Major Findings

The major findings of this thesis are:

1. A school focus on Students' Mathematical Communication is beneficial to supporting teachers and teacher collaboration, which in turn benefits students.

2. Professional development alone is not enough to move these teacher's beliefs forward in aligning with the current mathematics education approach.

3. These teachers draw from several of the Ten Dimensions (McDougall, 2004) to support students' mathematical communication.

4. These middle school mathematics teachers face many challenges, particularly in the inner-city setting. These teachers require more support, particularly in-class, to be effective in their instruction.

5. These mathematics teachers, particularly those that teach other subjects, have many time constraints. Professional development experiences should consider these time constraints, and provide information and activities that are easily integrated into teaching practice.
5.7 Future Research

A longer study, which included an on-going PD portion, would shed light on the effects of continuing support for teachers in their mathematical practice. In lieu of that, a follow-up study should be conducted to determine the lasting effects, and how, or if, teachers seek to continue to work on aligning their teaching practice with the Ten Dimensions Framework. This follow-up would be beneficial for understanding teachers' long-term responses to the PD session, as well as teachers' need for professional growth.

The increase in the EQAO scores suggests that there may be some other positive impacts of this PD initiative. The percentage of students scoring at the provincial standard or higher has increased at this school, based on available data, before and after teachers' PD participation. Between 2014 and 2016, only 26% of students achieved the provincial standard, while between 2015 and 2017 that number increased to 34%. While, this increase is a positive sign of change, more investigation is required to determine if it was caused, even in part, by this PD study.

This study sheds light on the differences in efficacy of professional development on individual teachers. I suggest that differences in personality may be a factor in the effectiveness of PD. Future studies could incorporate a personality survey, to be completed by the teacher, and administered in conjunction with the APTMS. The personality survey could identify a participant's readiness to change, learning style, personality, and preference for individual or group PD activities. This additional information can help determine the effectiveness of PD sessions based on personality, and can help PD facilitators tailor the sessions to the group of teachers involved. Considering the attitude of the teacher is an important component to successful PD (Won, 2017).
One potential route to increasing teacher engagement with PD is the inclusion of an inquiry component. Cochran-Smith & Lytle's (1999) third conception relies heavily on inquiry to move teaching practice forward. Perhaps, the pursuit of inquiry could lead to higher engagement with the material for teachers if they are able to pursue answers to their own questions. To ensure a lack of confidence in mathematics does not negatively impact their ability, or desire, to form a meaningful inquiry question, teachers need to be provided with support. Assistance should be provided to teachers to form a good inquiry question given: the time period of the study, the interests of individual teachers, the needs of students, and the assessment tools that teachers can utilize for inquiry purposes.

Another potential challenge with implementing an inquiry component would be time constraints. While adding an inquiry component to the PD would add another layer to teachers' learning, which may lead to a richer experience, it would also require a greater time commitment from teachers. Participants in this study all named time as a major challenge. To prevent teachers from additional time management stress, the inquiry component could be optional. On the other hand, the structure of the study could be changed so that teachers could voluntarily apply to the PD program from across the school board, instead of superintendents pre-selecting participating schools, to ensure the appropriate teacher commitment and readiness.
References


Education Quality and Accountability Office. PROVINCIAL ASSESSMENT RESULTS 2017 Primary and Junior Divisions: Five-year trend: Grade 3 and Grade 6 students who met the provincial standard.


Education Quality and Accountability Office. (December 2016). Programme for International Student Assessment (PISA), 2015: Highlights of Ontario Student Results.


Appendix A - Attitudes and Practices to Teaching Math Survey (McDougall, 2004, pp. 87-88)

Instructions:
Circle the extent to which you agree with each statement, according to the A to F scale below.
Then, use the charts at the top of the next page to complete the Score column for each statement.

A  Strongly Disagree  B  Disagree  C  Mildly Disagree  D  Mildly Agree  E  Agree  F  Strongly Agree

<table>
<thead>
<tr>
<th>Statement</th>
<th>Extent of agreement</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>A B C D E F</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>A B C D E F</td>
<td></td>
</tr>
</tbody>
</table>
Attitudes and Practices to Teaching Math Survey Scoring Chart

For statements 1–5, 7–10, 12–14, and 17, score each statement using these scores:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

For statements 6, 11, 15, 16, 18, 19, and 20, score each statement using these scores:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To complete this chart, see instructions below:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Related Statements</th>
<th>Statement Scores</th>
<th>Sum of the Scores</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program Scope and Planning</td>
<td>4, 8, 13</td>
<td>6, 4, 5</td>
<td>15</td>
<td>+ 3 = 5</td>
</tr>
<tr>
<td>2. Meeting Individual Needs</td>
<td>2, 6, 7, 15, 16</td>
<td></td>
<td></td>
<td>+ 5</td>
</tr>
<tr>
<td>3. Learning Environment</td>
<td>3, 5, 6</td>
<td></td>
<td></td>
<td>+ 3</td>
</tr>
<tr>
<td>4. Student Tasks</td>
<td>1, 2, 11, 15, 16</td>
<td></td>
<td></td>
<td>+ 5</td>
</tr>
<tr>
<td>5. Constructing Knowledge</td>
<td>5, 11, 14, 15, 16</td>
<td></td>
<td></td>
<td>+ 5</td>
</tr>
<tr>
<td>6. Communicating With Parents</td>
<td>19, 9</td>
<td></td>
<td></td>
<td>+ 2</td>
</tr>
<tr>
<td>7. Manipulatives and Technology</td>
<td>10, 18</td>
<td></td>
<td></td>
<td>+ 2</td>
</tr>
<tr>
<td>8. Students’ Mathematical Communication</td>
<td>3, 6, 10, 17</td>
<td></td>
<td></td>
<td>+ 4</td>
</tr>
<tr>
<td>9. Assessment</td>
<td>8, 11, 12, 19</td>
<td></td>
<td></td>
<td>+ 4</td>
</tr>
<tr>
<td>10. Teacher’s Attitude and Comfort with Mathematics</td>
<td>4, 7, 13, 15, 20</td>
<td></td>
<td></td>
<td>+ 5</td>
</tr>
</tbody>
</table>

Total Score (All 10 dimensions) 152
Overall Score (Total Score ÷ 38) 4

**Step 1** Calculate the **Average Score** for each dimension:
1. Record the score for each **Related Statement** in the third column.
2. Calculate the **Sum of the Scores** in the fourth column.
3. Calculate the **Average Score** and record it in the last column.

For example:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Related Statements</th>
<th>Statement Scores</th>
<th>Sum of the Scores</th>
<th>Average Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Program Scope and Planning</td>
<td>4, 8, 13</td>
<td>6, 4, 5</td>
<td>15</td>
<td>+ 3 = 5</td>
</tr>
</tbody>
</table>

**Step 2** Calculate the **Overall Score**:
1. Calculate the **Total Score** of the sums for all 10 dimensions in the fourth column.
2. Calculate the **Overall Score** by dividing the Total Score by 38.

For example:

<table>
<thead>
<tr>
<th>Total Score (All 10 dimensions)</th>
<th>152</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Score (Total Score ÷ 38)</td>
<td>4</td>
</tr>
</tbody>
</table>

**Step 3** Interpret the results:

<table>
<thead>
<tr>
<th>Average Score for Each Dimension</th>
<th>Overall Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average scores will range from 1 to 6. The higher the average score, the more consistent the teacher’s attitude and teaching practices are with current mathematics education thinking, with respect to the dimension. A low score indicates a dimension that a teacher might focus on for personal growth and professional development.</td>
<td>The overall score will range from 1 to 6. The higher the overall score, the more consistent the teacher’s attitude and teaching practices are with current mathematics education thinking and the more receptive that teacher will likely be to further changes in his or her practice.</td>
</tr>
</tbody>
</table>
Appendix B - School and District Improvement in Elementary Mathematics Questions

Background questions

What is your name?
Where did you go to university? What is your degree of specialization?
Why did you become a teacher?
How many years have you been teaching?
Where did you teach before and what grades have you taught?
How long have you been here at this school?
What subjects and grades do you teach or what is your role in the school?

1. Versions of success

For you, what counts as success for students in this school?
What are your goals in education?
How widely accepted are your goals with other teachers in the school? Among parents?
How does your school improvement plan incorporate your goals for students?
How is the school improvement plan created in this school (principal)?

2. Challenging circumstances

What are the most challenging things (the barriers) for you as you go about your work in this school?
What are the most successful things for you as you go about your work in this school?
Do you think this school is different from other schools in its challenges?
How would you describe the community of parents with whom you work?
How has the school context changed over the past few years, and what changes are going on now?

3. Mathematics

How would you describe your goals in mathematics?
How widely accepted are these views in the school? Among the parents?
How would you describe the provincial ministry's vision of mathematics?
How do you meet the mathematics goals of the province?
Which of the Ten Dimensions have you selected for your personal growth? Why did you select those dimensions?
Which of the Ten Dimensions have you selected for your school improvement plan? Why did you select those dimensions?

4. Fostering Mathematics Communication

How would you define mathematics communication?
How do you perceive the role of mathematics communication in your mathematics program?
What mathematics communication goals do you have for your class?
How do you create a classroom environment that fosters students' mathematics communication?
What are some of the challenges you have encountered when attempting to develop students' mathematics communication?
What are some of the successes you have encountered when attempting to develop students' mathematics communication?
5. School support
   How do you create an environment, which supports success in mathematics?
   What challenges (barriers) have you faced in trying to create a culture that supports student achievement in mathematics?
   How do you work with staff and administration to develop the goals/vision of the school? To develop mathematics improvement?
   How were the issues resolved?

6. Overall
   What are the programs that support success in mathematics outside of the classroom?
   What do you think we should say in our report about how schools can be more effective in supporting mathematics improvement?
   Do you have a mathematics implementation team? If so, what is their role and what do they do?