Fuzzy Analytical Hierarchy Process Approach for Multicriteria Decision-Making with an Application to developing an ‘Urban Greenness Index’

by

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A thesis submitted in conformity with the requirements for the degree of Master of Science
Department of Public Health Sciences
University of Toronto

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Abstract

Urban greenness is a multifaceted concept that describes greenness in a built environment. Current approaches studying health outcomes quantify greenness one-dimensionally, leading to the necessity of developing an index that accurately reflects greenness aspects in an urban setting.

This thesis integrates Analytical Hierarchy Process with fuzzy logic (fuzzy-AHP) to develop an index based on opinions about urban greenness and on objective attribute data. Index attribute weights are obtained by triangular, trapezoidal and Gaussian membership functions; geometric mean method and fuzzy extent analysis are applied to obtain fuzzy weights. Defuzzification is performed by modal value dominancy and alpha-cuts. A numerical application is shown and resulting indices are compared.

The developed ‘Urban Greenness Index’ represents a more accurate reflection of the true exposure by incorporating multiple aspects of the urban environment. After obtaining experts input on index attributes, this index can be used to explore associations to different health outcomes.
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Chapter 1
Introduction

A growing body of empirical evidence has demonstrated associations between exposure to greenness, such as trees, grass, wetlands, or other types of vegetation, and various health outcomes ranging from obesity, asthma, and allergies, to mental health and perceived quality of life (James, Banay, Hart, & Laden, 2015). To study those associations, researchers commonly quantify greenness by qualitative and quantitative methods. However, limited studies have focused on studying the effect on such outcomes considering the interactions of greenness with specific characteristics of residential urban areas.

Qualitative methods for measuring greenness rely on evaluations by trained raters or on residents’ visual perception, which imply subjective assessment since the perception of different groups of people may be different (Y. Liu et al., 2016). More often than not, self-reported methods are replaced by quantitative methods involving satellite-derived measures, such as the normalized difference vegetation index (NDVI), as they provide repeatability and larger coverage area. By utilizing satellite images, researchers quantify greenness as a percentage of green cover per area or per inhabitant, or as proximity to green spaces. Those measures cannot comprehensively reflect the role of greenness in cities, as they do not account for the spatial arrangement of green spaces or the vertical dimensions, building structure, and density of urban built-up areas (ThiLoi, Anh Tuan, & Gupta, 2015). In addition, inconsistencies between greenness exposure and health conditions have been reported in the literature (James et al., 2015). Apart from differences in demographical characteristics this may reflect differences in how urban greenness is defined and measured.

1.1 Urban Greenness

Urban greenness refers to areas covered by vegetation in an urban environment. The term encompasses various factors such as urban forests, grassland, parks, gardens, recreational venues, planted trees along streets, and other permeable surfaces (Li et al., 2016).
It has been speculated that exposure to green areas in urban environments may influence the development of physiological and psychological conditions in the general population, as green areas absorb pollutants and release oxygen, reduce noise, concentrations of air pollution, and stress, as well as balance the natural urban environment (Crouse et al., 2017; Gupta, Kumar, Pathan, & Sharma, 2012; Y. Liu et al., 2016). The World Health Organization (WHO) states:

“Green spaces such as parks and sports fields as well as woods and natural meadows, wetlands or other ecosystems, represent a fundamental component of any urban ecosystem. Green urban areas facilitate physical activity and relaxation, and form a refuge from noise”.

It is necessary to integrate multiple factors of greenness exposure bearing in mind the built environment of cities, as building characteristics are believed to play a role in the impact of urban greenness in health. Existent works have addressed quantification of urban greenness by developing urban greenness indices that incorporate additional factors not identified as vegetation, such as building sparsity and building height. Studies that developed urban greenness indices are listed below, the different greenness and urban-related attributes that are considered in such studies are explained.

Li et al (2016) developed a ‘3D Building Proximity to Greenery Index’ in Székesfehérvár, Hungary, using ratios of green area and total buffer area around a building, accounting for building height and vegetation type. W. Wang et al. (2018) performed a similar study in Beijing, China, using ratios of visual green space area from buildings and total buffer area around buildings, rather than proximity. Indices were used to evaluate proximity of a building to its nearby urban greenery and green space viewed from buildings. Additional greenness-related characteristics of the surrounding environment were not included.

Gupta et al. (2012) developed an ‘Urban Neighborhood Greenness Index’ for Delhi, India, using an Analytical Hierarchy Process (AHP) method to incorporate pairwise comparison matrices and determine attributes (criteria) weights. Attributes included were percentage of green cover, height of buildings, proximity to green spaces, and density built-up. Using the Delphi method, Gupta et al. (2012) approached urban planners, educators, and research workers for input on their preferences of one attribute over the other. Similarly, Y. Liu et al. (2016) used AHP and the Delphi method to develop a ‘Building Neighborhood Green Index’ for Székesfehérvár, Hungary,
using percentage of green cover, proximity to green spaces, building sparsity, and high-rise sparsity as attributes. ThiLoi et al. (2015) developed a ‘Weighted Urban Green Space Index’ for Chandigarh, India, incorporating percentage of green cover, type of vegetation, and proximity to green spaces using the same methodology as Gupta et al. (2012) and Liu et al. (2016). Schöpfer, Lang, & Blaschke (2004) incorporated percentage of green cover, percentage of multi-story buildings, and distance between buildings in a cognition network to create a ‘Green Index’ on a grid for Salzburg, Austria.

In the studies mentioned above, AHP was used for index development. AHP has the advantage of incorporating more than two attributes to measure the multifaceted aspects of urban greenness exposure. However, AHP also requires human input in the form of pairwise comparison matrices, thus it is often criticized for its inability to handle the variability and uncertainty associated with multiple experts’ opinions (Vahidnia, et al., 2008). This criticism can be addressed by integrating fuzzy logic, which has proven to be an effective way for formulating decision problems where the information is subjective (Zimmermann, 1996).

1.2 Objectives

The primary objective is to develop a robust ‘Urban Greenness Index’ using an integrative Fuzzy Analytical Hierarchy Process (fuzzy-AHP) approach. The developed ‘Urban Greenness Index’ can then be used to evaluate the impact of urban greenness on multiple aspects of health.

In chapter one of this thesis, the definition and an overview of methods currently used to quantify urban greenness are reviewed. Chapter two presents a literature review of different applications of fuzzy-AHP for ranking of alternatives. Chapter three introduces the theoretical formulations of various integrated fuzzy-AHP methods and membership functions, and Chapter four presents the spatial scan statistic for detection of spatial clusters on attribute data. Chapter five presents a numerical example of the application of fuzzy-AHP to calculate an ‘Urban Greenness Index’. Finally, a discussion on limitations and future steps of the study, as well as a conclusion are presented in Chapter six.
Chapter 2
Previous applications of AHP and fuzzy logic

In this chapter, the basis of Multiple Criteria Decision-Making (MCDM) and AHP are explained, followed by a literature review on the integration of AHP with other techniques. The benefits of integrating AHP with fuzzy logic for developing an index are described, as well as a literature review on the different fuzzy-AHP methodologies applied on various areas of research.

2.1 Multiple Criteria Decision-Making Method

Decision-making comes from the act of choosing between two or more courses of action. Within that context, MCDM is a more complex methodology that structures and solves both decision and planning issues involving consideration of multiple criteria.

In real life applications, a unique optimal solution for a MCDM problem rarely exists (Guitouni & Martel, 1998; Majumder, 2015). In most scenarios, the most satisfactory decision, rather than the optimal one, is chosen; hence the need for methodology to help decision-makers choose the best outcome in complex situations involving multiple criteria, goals, or objectives of conflicting nature (Kahraman, et al., 2016). All approaches of MCDM depend on different assumptions, but they all have in common the use of experts or decision-makers (DM) input and analytical/algorithmic models to calculate relative weights for the attributes involved.

Majumder M. (2015) defines the steps of decision making as follows:

1. Identify the objective/goal of the decision-making process
2. Select the set of potential alternatives
3. Select the Criteria/Parameters/Factors
4. Select weighing methods
5. Define the method of Aggregation
6. Decision-making/recommendations based on the aggregation results
Following the points above, MCDM starts with establishing a goal, which is decomposed into a finite set of attributes or criteria to be weighted/ranked; this ranking leads to decision-making of the best alternative or solution among all previously defined (Figure 2.1).

Numerous methodologies have been proposed to solve MCDM problems, but none of them is perfect or can be considered applicable to all scenarios. The MCDM method that is chosen to be used should be decided upon based on the type of problem, the design type, and other contextual considerations (Guitouni & Martel, 1998; Majumder, 2015). Guitouni (1998) articulated seven general tentative guidelines to follow when deciding which MCDM method to use. The most relevant to the process of an index development are outlined below:

1. Determine the number of decision-makers of the decision process. This helps deciding whether to use group decision-making methods.
2. Consider the most appropriate way to compare attributes, i.e. by pairwise comparisons.
3. Choose an aggregation procedure that can handle properly the decision-makers inputs.
4. The fundamental hypothesis of the method should be met, otherwise one should choose another method.

### 2.2 Analytical Hierarchy Process

MCDM methods are divided between compensatory and outranking methods. The most commonly used compensatory method is the Analytical Hierarchy Process, initially proposed by R.W. Saaty (1987). AHP is used to derive relative priorities for a given set of alternatives by pairwise comparisons of attributes in order to arrive at a final solution. Pairwise comparisons matrices are obtained from decision-makers’ preferences over said set of attributes. In other words, AHP allows the use of subjective measures, such as human perceptions, as input in its calculations. The principal eigenvalue is used to derive and synthetize relative scales provided
that the decision-makers comparison matrices are positive, reciprocal, and consistent (Delgado-Galván, et al., 2010; R. W. Saaty, 1987; Vaidya & Kumar, 2006).

One of the many advantages of AHP is its ability for being easily integrated with different techniques, such as the Delphi method, fuzzy logic, neural networks, and Bayesian statistics, as well as other MCDM methods. In addition, AHP applications extend to numerous fields, including planning, manufacturing, environmental assessment, resource allocations, education, and government (Delgado-Galván et al., 2010; Vaidya & Kumar, 2006). Al-Harbi (2001) applied AHP to select the best contractor for a project due to its computational simplicity and the fact that it provides a structured yet relatively simple solution to the decision problem. Contractors were compared pairwise based on experience, financial stability, quality performance, manpower resources, equipment resources, and current workload. Ranking among the different attributes was performed to find the overall priority of each contactor and the best one was selected.

Mimović et al. (2015) used an integrated approach of AHP and Bayesian analysis to measure the relative probability of an alternative considering the factors relevant to the intentions of a new corporation over those of an existing corporation. The relative likelihood of outcomes was estimated with respect to each of the criteria, and the most likely intention of the new corporation was identified. Bayes theorem was used to derive a posteriori criteria weights based on a priori weights that were subjectively determined. These were later used as input in the AHP process. Mimović et al. (2015) took advantage of having environmental information to incorporate Bayesian analysis into the study. Alternatively, in a study aimed at computing performance measures for different products, Fogliatto & Albin (2001) created weights to evaluate the best product based on quantitative expert opinion, and sensory panel data using an extension of AHP. Fogliatto & Albin (2001) recognize the disadvantage of data gathering costs and the need to adjust for uncertainties in panel data for future studies.

The aforementioned publications are examples of a large variety of research papers integrating AHP with other techniques. All of these publications are not applicable to an index creation as their ultimate goal is not to calculate weights, but to identify the most viable goal and/or alternative, to use information from other sources (rather than experts’ input), to minimize time and cost, or to use simpler methods that offer computational and interpretational ease. A literature review of the broad areas of AHP application and its integration with different techniques can be found in the publication of Vaidya & Kumar (2006).
2.3 Integration of Fuzzy Logic and Analytical Hierarchy Process

As reviewed, AHP allows the integration of both quantitative and qualitative aspects of decision-making. However, its input only consists of subjective opinions from experts, which means that the importance of one attribute over the other is based solely on human expertise, which implies:

1. Variation between experts’ opinions
2. Subjective and ambiguous aspects within each expert opinion

The within and between variability of opinions can be addressed by fuzzy logic, which provides mathematical strength to incorporate the uncertainty and imprecision of such opinions into the calculations, by adding uncertainty to the preferences using linguistic terms, i.e. “more than”, “as high as”, “less than”, and “equally as”. Fuzzy logic also helps to conciliate conflicting observations due to human expertise (Ertuğrul & Karakaşoğlu, 2008; Kahraman et al., 2016; Mourhir et al., 2014). Modeling using fuzzy logic has proven to be an effective way for formulating decision problems where the information is subjective (Zimmermann, 1996), such as when using linguistic variables. It is important to emphasize that fuzzy theory addresses vagueness and uncertainty, rather than lack of knowledge about the value of an attribute. Therefore, deciding upon the group of experts that will provide input on the matter at hand is as important as in a traditional AHP approach.

A review of the literature showed that the ranking of alternatives obtained from fuzzy-AHP and from other fuzzy-MCDM methods is often the same (Ertuğrul & Karakaşoğlu, 2008; Hefny, Elsayed, & Aly, 2013; Kaya & Kahraman, 2011; Krejčí, Pavlačka, & Talašová, 2017; Mondal & Ghosh, 2016; Vahidnia et al., 2008). To select the optimal facility location, Ertuğrul & Karakaşoğlu (2008) apply fuzzy-AHP and fuzzy-TOPSIS methods. Within fuzzy-AHP, a triangular membership function and extent analysis were used, and fuzzy weights were normalized using the arithmetic mean procedure, which is explained by D.-Y. Chang (1996). Results show that both techniques yield the same order of alternatives. Similarly, Efe (2016) used fuzzy extent analysis and a triangular membership function to calculate criterion weights in a software selection problem. After fuzzy-AHP, the MCDM problem is solved using fuzzy-TOPSIS, which defines the most appropriate alternative with respect to a pre-defined goal. Krejčí, Pavlačka, & Talašová (2017) used triangular fuzzy numbers and a geometric mean method with a constrained fuzzy arithmetic concept to calculate attribute weights. The center of
gravity method was used to defuzzify and rank alternative weights, and the overall fuzzy weights were ranked with respect to their centers of area. Kaya & Kahraman (2011) proposed an environmental impact assessment methodology based on an integrated Fuzzy AHP-ELECTRE approach in the context of urban industrial planning. Using a trapezoidal membership function, criteria weights were calculated by fuzzy-AHP using geometric means and defuzzification was performed using modal value dominancy technique. Fuzzy ELECTRE was then used to assess the environmental impact generated by six different alternatives. Hefny, Elsayed, & Aly (2013) derived a priority vector of attribute weights by fuzzy extent analysis and a Gaussian membership function. Gaussian functions were transformed to triangular functions for extent analysis and retransformed to Gaussian for weight calculation. Weights were then input in an Analytical Network Process (ANP) model to rank electrical power generation alternatives. Mondal & Ghosh (2016) followed a similar approach to calculate attribute weights using a Gaussian function. Resulting weights were used to calculate the Spearman rank correlation coefficient to achieve evaluation of teachers’ performance. Vahidnia et al. (2008) used two fuzzy-AHP methods to analyze Geographical Information System (GIS) data: fuzzy extent analysis and an α-cut based method, both with triangular fuzzy numbers. Vahidnia et al. (2008) indicate that the advantage of using the α-cut based method is that the conclusion is less controversial, and the uncertainty and different attitude of decision-makers can be considered, while fuzzy extent analysis is easier to compute.

To our knowledge, there are no published studies that use fuzzy-AHP methodology to assess urban greenness exposure. Within the context of environmental quality, Sheikhian, Pahlavani, & Sabzevari (2015) used fuzzy-AHP to map air pollution severity in Tehran. Attributes the researchers used included: tropospheric ozone (O3), nitrogen monoxide (NO), carbon monoxide (CO), and particulate matter (PM10 and PM2.5). Such pollutants were discretized using the air quality index (AQI), which transforms the pollutant values to their relevant health concern degree. For AHP, opinions from 30 experts were included to catalogue the effect of existing AQI degrees in the overall health impact of each pollutant, and the AHP linguistic scale was used to map such opinions to a numeric scale. Attribute weights were calculated by fuzzy logic using the extent analysis method and a triangular membership function. The resulting attribute weights were used to create vulnerability maps for each concerned attribute and to obtain the final air pollution susceptibility map. Using experts’ opinions, Sheikhian et al. (2015) identified an area
of Tehran with crucial air pollution that was exceedingly threatening the health of the city’s residents. Sheikhian et al. (2015) also proved that particulate matters have greater influence on overall air pollution than gaseous pollutants. Additionally, the results provide evidence of industrial areas as the main sources of pollution, and green spaces as a controlling element. The application of fuzzy-AHP in this study allowed to map air pollution severity without inclusion of morbidity and/or mortality data, which in some cases may be difficult to obtain, as well as to provide evidence for decision-making on the necessity of air quality enhancement. In studies of this type, where it is difficult to quantify tangible boundaries of acceptable attribute levels, experts’ opinions are highly valued, and fuzzy-AHP may be more effective than data driven approaches, such as machine learning techniques.

As shown, different methods and membership functions for attribute weight calculation are available within fuzzy-AHP, however it is unclear if they would yield significantly different weights, as studies comparing them were not found in the literature. Given that attribute weights are the base of an index development, it would be important to understand the extent of weight differences obtained by the various fuzzy-AHP methods to find a balance between ease of computation and accuracy of results.
Chapter 3
Methods for index development

In this section the steps to apply Analytical Hierarchy Process are presented, followed by fuzzy set theorems and fuzzy membership functions. The theoretical formulation of fuzzy-AHP methodology and different approaches for fuzzy weight calculation and weight defuzzification are introduced.

3.1 Fuzzy set theory

To facilitate manipulation of fuzzy sets, the operators of classical probability set theory are adapted to fuzzy numbers\(^1\), such operators are presented in theorems one to six below. Addition, subtraction, multiplication and division operators of fuzzy numbers can be found in Appendix A.

**Theorem 1.**

A classical or crisp set is a collection of countable elements in which \(x \in X\), each single element can either belong or not belong to a set \(A\), for \(A \subseteq X\). In a fuzzy set, an element from the collection of countable elements has a continuum of grades of membership. Each set is characterized by a membership function which allows an element to be partly true and partly false at the same time by assigning it a grade of membership ranging between zero and one.

Let \(X\) be the complete universe and its elements denoted by \(x\). Then a fuzzy set \(\tilde{A}\) in the universe \(X\) is a set of ordered pairs:

\[
\tilde{A} = \{ x, \mu_A(x) \mid x \in X \}
\]

where \(\mu_A(x)\) = membership function of \(x\) in \(\tilde{A}\) that maps \(X\) to the membership space \(M\), and \(\mu_A: X \to [0, 1]\).

\(^1\) Throughout this document, a crisp number represented by \(A\), will have its fuzzy number represented by \(\tilde{A}\).
A membership degree of zero means that the value is not in the set, a degree of one means that the value is completely representative of the set, and a degree confined between zero and one means the value is partially in the set.

**Theorem 2.**

The support of a fuzzy set \( \tilde{A} \), \( S(\tilde{A}) \), is the crisp set of all \( x \in X \) such that \( \mu_{\tilde{A}}(x) > 0 \).

**Theorem 3.**

The crisp set of elements that belong to the fuzzy set \( \tilde{A} \) at least to the degree \( \alpha \) is called the \( \alpha \)-level cut set (Figure 3.1):

\[
A_{\alpha} = \{ \ x \in X \ | \ \mu_{\tilde{A}}(x) \geq \alpha \}
\]

Figure 3.1 Graphical representation of crisp set of elements for various alpha-level cuts

**Theorem 4.**

The membership function \( \mu_{\tilde{A}}(x) \) of the union of two fuzzy sets \( \tilde{A} = \tilde{B} \cup \tilde{C} \) over the same set \( X \) is defined as:

\[
\mu_{\tilde{A}}(x) = \mu_{\tilde{B} \cup \tilde{C}}(x) = \max \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \ | \ x \in X \}
\]

Where \( \mu_{\tilde{B}} \) and \( \mu_{\tilde{C}} \) are the membership functions for \( \tilde{B} \) and \( \tilde{C} \) respectively.

**Theorem 5.**

The membership function \( \mu_{\tilde{A}}(x) \) of the intersection of two fuzzy sets \( \tilde{A} = \tilde{B} \cap \tilde{C} \) over the same set \( X \) is defined as:
\[
\mu_{\tilde{A}}(x) = \mu_{\tilde{B} \cap \tilde{C}}(x) = \min \{ \mu_{\tilde{B}}(x), \mu_{\tilde{C}}(x) \mid x \in X \}
\]

**Theorem 6.**

The membership function of the complement of a normalized fuzzy set \(\tilde{A}\), \(\mu_{\tilde{A}}^c(x)\) is defined by:

\[
\mu_{\tilde{A}}^c(x) = 1 - \mu_{\tilde{A}}(x), x \in X
\]

### 3.2 Fuzzy membership functions

Within fuzzy-AHP theory, various membership functions can be utilized to map crisp pairwise comparison matrices into fuzzy numbers. Triangular membership functions are most widely used due to their simplicity and ease of computation, followed by trapezoidal and Gaussian membership functions, however these are mostly used in Fuzzy inference systems (FIS) where two or more fuzzy functions are incorporated as input to obtain an output signal.

**Triangular membership function (TMF)**

A triangular fuzzy number \(\tilde{A}\) is a fuzzy number whose membership function is determined by a triplet of real numbers such that \(l \leq m \leq u\) as shown in Figure 3.2 and is expressed as follows:

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{(x-l)}{(m-l)}, & l \leq x \leq m \\
\frac{(m-x)}{(u-x)}, & m \leq x \leq u \\
0, & \text{otherwise}
\end{cases}
\]

The real numbers \(l\) and \(u\) are called lower and upper limits, respectively. The real number \(m\) represents the modal point. The interval \([l, u]\) expresses the closure of the support of \(\tilde{A}\) and the modal point represents the core, the values of \(\mu_{\tilde{A}}(X)\) belong to the interval \([0, 1]\).
Trapezoidal membership function (TrMF)

A flat fuzzy number \( \bar{B} \) is a fuzzy number represented by a lower limit \( l \), an upper limit \( u \), a lower support \( m \) and an upper support \( n \), where \( l \leq m \leq n \leq u \). As depicted in Figure 3.3, a trapezoidal membership function is a case of flat fuzzy membership function where \( m \) and \( n \) represent the lower and upper endpoints respectively of the modal interval of \( \bar{B} \). A trapezoidal function where \( m = n \) simplifies to a triangular function.

The membership function of a trapezoidal fuzzy number is expressed as follows:
\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{(x - l)}{(m - l)}, & l \leq x \leq m \\
1, & m \leq x \leq n \\
\frac{(u - x)}{(u - n)}, & n \leq x \leq u \\
0, & \text{otherwise}
\end{cases}
\]

**Gaussian membership function (GMF)**

Similar to the measures of centrality and spread of a normal distribution, Gaussian membership functions are defined by a central value \( m (\mu) \) and a standard deviation \( \sigma > 0 \), and are expressed by:

\[
\mu_{\tilde{C}}(x) = \exp \left[ -\frac{1}{2} \left( \frac{x - m}{\sigma} \right)^2 \right], \quad x \in \mathbb{R}
\]

Figure 3.4 Graphical representation of a Gaussian fuzzy membership function

The advantage of using Gaussian membership function is that they are determined by only two parameters \( \mu \) and \( \sigma \). While the centers (\( \mu \)'s) must have the same value as the crisp numbers from the preference matrix, the spread (\( \sigma \)) is decided upon the amount of uncertainty present.
### 3.3 Classical Analytical Hierarchy Process

Analytical Hierarchy Process uses pairwise comparisons of a knowledgeable group of experts to determine the importance of criteria in a decision. The steps of AHP exclusively used for weight calculation described by R. W. Saaty (1987) are shown below.

1. Decompose the decision problem or goal into its decision elements. Note that setting of alternatives is omitted in this study.

![Figure 3.5 AHP decomposition of decision problem/goal into criteria and alternatives](image)

2. Determine a preference scale to make pairwise comparisons. The fundamental scale used in this study is shown in Table 3.1 and was proposed by R. W. Saaty (1987).

**Table 3.1 Numeric to Linguistic Preference Scale suggested by R. W. Saaty (1987).**

<table>
<thead>
<tr>
<th>Preference in absolute scale</th>
<th>Preference in linguistic scale</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two attributes contribute equally to the goal</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
<td>Experience and judgment moderately favor one attribute over the other</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
<td>Experience and judgment strongly favor one attribute over the other</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
<td>An attribute is strongly favor and its dominance demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
<td>The evidence favoring one attribute over another is of the highest possible order</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values between adjacent scale values</td>
<td>When compromise is needed</td>
</tr>
<tr>
<td>Reciprocals</td>
<td>For $a_{ij}$, it’s reciprocal is $1/a_{ij}$</td>
<td>Represented in the lower part of a matrix</td>
</tr>
</tbody>
</table>
3. For all attributes $i$ of the hierarchy structure, obtain a $(n \times n)$ reciprocal pairwise comparison matrix from each expert $k$. Each expert should provide a set of $m = n(n - 1)/2$ comparison judgments in the matrix. It is not recommended to build comparison matrices greater than 10x10.

$$A_k = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} \\ a_{21} & a_{22} & \cdots & a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1j} \\ 1/a_{12} & 1 & \cdots & a_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{ij} & 1/a_{2j} & \cdots & 1 \end{bmatrix}$$

4. Aggregate the $k$ comparison matrices by calculating geometric means on each cell.

5. Verify matrix consistency ratio (CR) which compares the consistency index (CI) of the matrix in question to the consistency index of a random-like matrix (RI):

$$CR = \frac{\text{Consistency Index (CI)}}{\text{Random Index (RI)}}$$

Where RI is obtained from Table 3.2, and:

$$CI = \frac{\lambda_{\text{max}} - n}{n - 1}$$

where $\lambda_{\text{max}}$ is the principal eigenvalue of the aggregated matrix obtained on step 4.

Table 3.2 Random Index levels for each sample size

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0.0</td>
<td>0.0</td>
<td>0.52</td>
<td>0.89</td>
<td>1.11</td>
<td>1.25</td>
<td>1.35</td>
<td>1.40</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

A consistency ratio <0.1 indicates that judgment is consistent enough while a CR > 0.1 indicates that judgment is untrustworthy. If the CR is larger than desired, T. L. Saaty & Tran (2007) recommend to identify the most inconsistent judgment in the matrix, determine the range of values to which that judgment can be changed corresponding to which the inconsistency would be improved and ask the decision maker to consider, if possible, to change his judgment to a

---

2 Table 3.2 is based on the average CI of 500 randomly generated matrices, details of its calculation are explained by T. L. Saaty & Tran., (2007).
plausible value in that range. If inconsistency is encountered, the final step is to re-calculate criteria weights after adjusting inconsistencies.

6. Calculate criteria weights \((w_i)\) of the aggregated matrix by the eigenvalue method:

\[ w_i = \lambda_{max} \cdot w_{ij} \]

where \(w_{ij}\) is the geometric mean of the row elements of the aggregated matrix:

\[ w_{ij} = \sqrt[n]{\prod_{j=1}^{n} a_{ij}} \]

and \(a_{ij}\) corresponds to position \(ij\) of the aggregated pairwise comparison matrix.

### 3.4 Fuzzy Analytical Hierarchy Process

As explained in Chapter 2, attributes weights can be obtained by integrating fuzzy logic and Analytical Hierarchy Process. The general steps to calculate attribute weights by the different fuzzy-AHP methods used in this thesis are shown in Figure 3.6. Two common methods for fuzzy weight calculation are introduced in this section: geometric means and extent analysis. After fuzzy-AHP calculation, fuzzy numbers need to be either “defuzzified” (transformed back to crisp numbers) or normalized. In this thesis the modal value dominancy technique is used for defuzzification, while for normalization the arithmetic mean and the alpha-cut methods are used.

The first step of fuzzy-AHP is to compare attributes by obtaining \(k\) pairwise comparison matrices \(A_k\) (one for each expert) using the crisp preference scale values from Table 3.1.

\[
A_k = \begin{bmatrix}
    a_{11} & a_{12} & \ldots & a_{1j} \\
    a_{21} & a_{22} & \ldots & a_{2j} \\
    \vdots  & \vdots  & \ddots & \vdots  \\
    a_{i1} & a_{i2} & \ldots & a_{ij}
\end{bmatrix} = \begin{bmatrix}
    1 & a_{12} & \ldots & a_{1j} \\
    1/a_{12} & 1 & \ldots & a_{2j} \\
    \vdots  & \vdots  & \ddots & \vdots  \\
    1/a_{1j} & 1/a_{2j} & \ldots & 1
\end{bmatrix}
\]
Figure 3.6 General steps to compute attribute weights by Fuzzy-AHP
All $A_k$ matrices are then converted to fuzzy matrices and their reciprocals are obtained. The conversion to fuzzy numbers shown in the matrix below corresponds to a triangular membership function with $l$, $m$, and $u$. The method can be generalized to trapezoidal or Gaussian functions by modifying the equivalences of the preference matrix.

$$
\tilde{A}_k = \begin{bmatrix}
(1, 1, 1) & (l_{12}, m_{12}, u_{12}) & \cdots & (l_{1i}, m_{1i}, u_{1i}) \\
(\frac{1}{u_{12}} \cdot \frac{1}{m_{12}} \cdot \frac{1}{l_{12}}) & (1, 1, 1) & \cdots & (l_{2i}, m_{2i}, u_{2i}) \\
\vdots & \vdots & \ddots & \vdots \\
(\frac{1}{u_{1i}} \cdot \frac{1}{m_{1i}} \cdot \frac{1}{l_{1i}}) & (\frac{1}{u_{2i}} \cdot \frac{1}{m_{2i}} \cdot \frac{1}{l_{2i}}) & \cdots & (1, 1, 1)
\end{bmatrix}
$$

**Matrix aggregation**

After transforming to $\tilde{A}_k$ pairwise comparison matrices, aggregation of expert’s opinions is performed. R. W. Saaty (1987) proposed to aggregate judgements by calculating geometric means of the lower limits, modal point, and upper limits of all triangular fuzzy sets. The resulting lower, modal, and upper points can be used as the representative preference judgement of the entire group on condition that the dispersion of the judgments is not usually large, however, using fuzzy numbers with a lower and upper bound for each expert, a wide spread of preference scores can be observed, and under scenarios of large dispersion this method does not reflect discrepancies between experts’ preferences (Jaskowski, Biruk, & Bucon, 2010).

A work around the issue of large judgment dispersion is proposed by C. W. Chang, Wu, & Lin (2009), which by the following equations preserves the limit values of the fuzzy numbers:

$$l_{ij} = \min(a_{ijk}) \quad (3.1)$$

$$m_{ij} = \sqrt[k]{\prod_{l=1}^{k} a_{ijk}} \quad (3.2)$$

$$u_{ij} = \max(a_{ijk}) \quad (3.3)$$

Equations 3.1, 3.2, and 3.3 are applied to the set of $l$, $m$, $n$ and $u$ bounds respectively to obtain an aggregated matrix of expert opinions for triangular and trapezoidal functions. In Gaussian
membership functions, only the modal values \( m(\mu) \) are used for aggregation, so the resulting aggregated matrix is identical as the one obtained from a classical AHP method.

### 3.4.1 Geometric means for weight calculation

After aggregation of preference matrices, weights calculation by geometric means is performed by applying Equation 3.2 to the elements of the fuzzy sets \((l, m, n, u)\), the results are denominated fuzzy weights. Although weights from Gaussian functions prior to defuzzification can be very similar to those obtained by triangular functions, slight differences are added due to the variation represented by \( \sigma \).

**Defuzzification**

In triangular and Gaussian membership functions the resulting matrix of weights is defuzzified for \( i \) attributes using the modal value dominancy method:

\[
 w_i = \frac{l_i + 4m_i + u_i}{6} \text{ for } i = 1, 2, \ldots, n 
\]  

(3.4)

where \( w_i \) is the defuzzified (crisp) weight for each attribute.

In trapezoidal membership functions, defuzzification of interval weights is calculated by a variation of equation 3.4 (Cheng and Hwang, 1992):

\[
 w_i = \frac{l_i + 2(m_i + n_i) + u_i}{6} \text{ for } i = 1, 2, \ldots, n 
\]  

(3.5)

The defuzzified weights are then composed by a unique crisp number that is later used for index calculation.

### 3.4.2 Fuzzy extent analysis for weight calculation

The extent analysis method is an alternative method for weight calculation that takes into consideration the areas of overlap between all membership functions when calculating weights.

In most MCDM scenarios, there is an attribute set \( X = \{x_1, x_2, \ldots, x_n\} \) and a goal set \( U = \{u_1, u_2, \ldots, u_m\} \). According to the method of extent analysis, an extent analysis must be performed on each object for each goal respectively, hence a maximum of \( m \) extent analysis values for each object can be obtained. Since this study does not include goals or alternatives to
choose from, the extent analysis is applied exclusively on the pairwise comparison matrices to calculate attribute weights and a maximum of $n$ extent analysis values can be obtained.

**Extent analysis on triangular and trapezoidal functions**

Once the pairwise comparison matrices of all decision makers have been fuzzified and aggregated, the value of fuzzy synthetic extent with respect to each attribute is defined as:

\[
\tilde{S}_i = \sum_{j=1}^n \tilde{a}_{ij} \otimes \left[ \sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1}
\]

where \( (\tilde{a}_{ij}) = [\tilde{a}_{ij}]^{-1} = (l_{ij}, m_{ij}, u_{ij})^{-1} = \left( \frac{1}{l_{ij}}, \frac{1}{m_{ij}}, \frac{1}{l_{ij}} \right) \),

and \( \left[ \sum_{i=1}^n \sum_{j=1}^n \tilde{a}_{ij} \right]^{-1} = \left( \frac{1}{\sum_{i=1}^n \sum_{j=1}^n u_{ij}}, \frac{1}{\sum_{i=1}^n \sum_{j=1}^n m_{ij}}, \frac{1}{\sum_{i=1}^n \sum_{j=1}^n l_{ij}} \right) \).

And we have a value of fuzzy synthetic extent \( \tilde{S}_i \) for each attribute. Next, the degree of possibility that \( x \in \mathbb{R} \) fuzzily restricted to belong to \( \tilde{S} \), to be greater than \( y \in \mathbb{R} \) fuzzily restricted to belong to \( \tilde{S} \) is calculated. For two triangular membership functions \( \tilde{S}_1 \) and \( \tilde{S}_2 \), the degree of possibility of \( \tilde{S}_1 \geq \tilde{S}_2 \) is defined as:

\[
V(\tilde{S}_1 \geq \tilde{S}_2) = \sup_{x_1 \geq x_2} \left[ \min(\mu_{S_1}(x), \mu_{S_2}(y)) \right]
\]

Where the supremum “sup” is the least element that is greater than or equal to all elements of \( \mu_{S_1}(x_1), \mu_{S_2}(x_2) \).

For every existent pair of attributes \( (x_1, x_2) \) such that \( x_1 \geq x_2 \) and \( \mu_{S_1}(x_1) = \mu_{S_2}(x_1) = 1 \) as depicted in Figure 3.1, then:

\[
V(\tilde{S}_1 \geq \tilde{S}_2) = 1 \text{ if } \tilde{S}_1 \geq \tilde{S}_2, \text{ and } V(\tilde{S}_1 \leq \tilde{S}_2) = \max(\tilde{S}_1 \cap \tilde{S}_2) = \mu_{S_1}(w)
\]

where \( w \) is the ordinate of the highest intersection point \( W \) between \( \mu_{S_1} \) and \( \mu_{S_2} \).

For two triangular membership functions, the ordinate of \( W \) is given by:
\[ V(\tilde{S}_1 \leq \tilde{S}_2) = \max(\tilde{S}_1 \cap \tilde{S}_2) = \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} \]  

(3.6)

And for two trapezoidal membership functions, assuming that they are symmetric, the core element \( n \) is added as an additional element on Equation 3.6, so the ordinate of \( W \) is given by:

\[ (\tilde{S}_1 \leq \tilde{S}_2) = \max(\tilde{S}_1 \cap \tilde{S}_2) = \frac{l_1 - u_2}{(m'_2 - u_2) - (m'_1 - l_1)} \]  

(3.7)

with \( m'_1 = \frac{m_1 + n_1}{2} \) and \( m'_2 = \frac{m_2 + n_2}{2} \).

To compare \( \tilde{S}_1 \) and \( \tilde{S}_2 \), the values of \( V(\tilde{S}_1 \geq \tilde{S}_2) \) as well as \( V(\tilde{S}_1 \leq \tilde{S}_2) \) are needed.

Consequently, the degree of possibility for the convex fuzzy number \( \tilde{S}_1 \) to be greater than all the other \( n \) convex fuzzy numbers \( \tilde{S}_i \) for \( i = 1, 2, \ldots, n \) is defined by:

\[
V(\tilde{S} \geq \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n) = V(\tilde{S} \geq \tilde{S}_1) \land V(\tilde{S} \geq \tilde{S}_2) \land \cdots \land V(\tilde{S} \geq \tilde{S}_n)
\]

\[ = \min V(\tilde{S} \geq \tilde{S}_i), \quad \text{for } i = 1, 2, \ldots, n \]  

(3.8)

To calculate the fuzzy weights of the fuzzy comparison matrix, it is assumed that
\[ \tilde{w}(A_i) = \min V(S_i \geq S_n), \text{ for } n = 1, 2, \ldots, l; n \neq i. \quad (3.9) \]

Then the weight vector is given by

\[ \tilde{W} = (\tilde{w}(A_1), \tilde{w}(A_2), \ldots, \tilde{w}(A_n))^T \quad (3.10) \]

where \( A_i \) \((i = 1, 2, \ldots, n)\) are the attributes.

**Extent analysis on Gaussian functions**

Fuzzy extent analysis on Gaussian membership function requires the function to be approximated by a triangular membership function. This is achieved by setting a small enough \( \alpha \) – level (the smaller the \( \alpha \) – level, the better the approximation to a triangular function). Such approximation is used to obtain values of fuzzy synthetic extent \( \tilde{S}_i \), which are then converted back to Gaussian numbers for weight calculation. The \( \alpha \) – level is obtained as:

\[ \alpha = \exp \left[ -\frac{(x-\mu)^2}{\sigma^2} \right] \]

At any \( \alpha \) – level, the approximation to the triangular boundaries \( l_i' \) and \( u_i' \) are calculated as:

\[ l_i' = \mu - \sigma \left( \sqrt{-\ln(\alpha)} \right) \]
\[ u_i' = \mu + \sigma \left( \sqrt{-\ln(\alpha)} \right) \]

Resulting triangular approximations can be found in Appendix D. The fuzzy synthetic extent values of the triangular function are then calculated using Equation 3.6 and converted back to Gaussian fuzzy numbers by:

\[ \sigma_{\tilde{S}_i}^l' = \frac{m_{\tilde{S}_i} - x_{\tilde{S}_i}^l}{\sqrt{-\ln(\alpha)}} \]
\[ \sigma_{\tilde{S}_i}^u' = \frac{x_{\tilde{S}_i}^u - m_{\tilde{S}_i}}{\sqrt{-\ln(\alpha)}} \]

Where \( m_{\tilde{S}_i} \) is the modal value of a Gaussian function. Note that the Gaussian function becomes asymmetric as:
\[
\mu_{S_i}(x) = \begin{cases} 
\exp \left[ -\left( \frac{x - m_{S_i}}{\sigma_{S_i}^l} \right)^2 \right] & \text{if } x \leq m_{S_i} \\
\exp \left[ -\left( \frac{x - m_{S_i}}{\sigma_{S_i}^u} \right)^2 \right] & \text{if } x > m_{S_i}
\end{cases}
\]

And the intersection points between two Gaussian functions \(m_{S_1}\) and \(m_{S_2}\) is given by:

\[
v = \begin{cases} 
\exp \left[ -\left( \frac{m_{S_2} - m_{S_1}}{\sigma_{S_1}^l + \sigma_{S_2}^l} \right)^2 \right] & \text{if } m_{S_1} > m_{S_2} \\
\exp \left[ -\left( \frac{m_{S_2} - m_{S_1}}{\sigma_{S_1}^u + \sigma_{S_2}^u} \right)^2 \right] & \text{if } m_{S_1} \leq m_{S_2}
\end{cases}
\]

Using the same two Gaussian functions \(m_{S_1}\) and \(m_{S_2}\), the degree of possibility of \(\bar{S}_2 = \mu_{S_2}(x) \geq \bar{S}_1 = \mu_{S_1}(x)\) is defined as:

\[
V(\bar{S}_2 \geq \bar{S}_1) = \max (\bar{S}_1 \cap \bar{S}_2) = \mu_{S_2}(d)
\]

\[
V(\bar{S}_2 \geq \bar{S}_1) = \begin{cases} 
1 & \text{if } \mu_{S_2} \geq \mu_{S_1} \\
\exp \left[ -\left( \frac{m_{S_2} - m_{S_1}}{\sigma_{S_2}^u + \sigma_{S_1}^u} \right)^2 \right] & \text{if } \mu_{S_2} < \mu_{S_1}
\end{cases}
\]

Consequently, and as with triangular functions, the degree of possibility for the convex fuzzy number \(\bar{S}_1\) to be greater than all the other \(k\) convex fuzzy numbers \(\bar{S}_i\) for \(i = 1, 2, ..., n\) is calculated using Equations 3.8 and 3.9, and the weight vector is given by equation 3.10.

**Normalization**

Since fuzzy AHP is used to model uncertainty and vagueness, to generate a unique weight vector from an interval, the fuzzy weight vector has to be normalized to assure unbiasedness and ease of interpretation; otherwise there will be an infinite number of weight vectors that can be derived from a fuzzy pairwise comparison matrix based on different membership values (Y.-M. Wang & Elhag, 2006).

The conventional and most common normalization method proposed by D.-Y. Chang (1996) is based on interval arithmetic operations using the arithmetic mean. For a triangular membership function, normalization is achieved by the following equation:
\[ W = (w(A_1), w(A_2), ..., w(A_n))^T = \left( \frac{\tilde{w}(A_1)}{\sum_{i=1}^{n} \tilde{w}(A_i)}, \frac{\tilde{w}(A_2)}{\sum_{i=1}^{n} \tilde{w}(A_i)}, ..., \frac{\tilde{w}(A_n)}{\sum_{i=1}^{n} \tilde{w}(A_i)} \right) \]

where \( W \) is a nonfuzzy number and \( \sum_{i=1}^{n} w(A_i) = 1 \).

Y.-M. Wang & Elhag (2006) propose a different method to normalize weights by using \( \alpha \)-cuts and the extension principle. The \( \alpha \)-cut is used to account for uncertainty in the fuzzy range chosen, ranging from 0 to 1 from the least confidence to the most confidence. Normalized fuzzy weights can be generated very precisely by setting different \( \alpha \) levels \( n \).

In this method, a set of L-R fuzzy weight intervals \( \tilde{w}_i = (w_i^l, w_i^m, w_i^u)_{LR} \) with \( 0 \leq w_i^l \leq w_i^m \leq w_i^u, i = 1, ..., n \), is used, whose membership function for triangular fuzzy numbers is defined as:

\[
\mu_{\tilde{w}}(x) = \begin{cases} 
L \left( \frac{(w_i^m - x)}{(w_i^m - w_i^l)} \right), & w_i^l \leq x \leq w_i^m \\
1, & w_i^m < x < w_i^u \\
R \left( \frac{(x - w_i^m)}{(w_i^u - w_i^m)} \right), & w_i^m \leq x \leq w_i^u 
\end{cases} \tag{3.11}
\]

Normalized fuzzy weights of trapezoidal membership functions follow the same principle as triangular ones, by including the core element \( w_i^n \) to the L-R fuzzy weight intervals as \( \tilde{w}_i = (w_i^l, w_i^m, w_i^n, w_i^u)_{LR} \) with \( 0 \leq w_i^l \leq w_i^m \leq w_i^n \leq w_i^u, i = 1, ..., n \), hence the membership function from Equation 3.11 becomes:

\[
\mu_{\tilde{w}}(x) = \begin{cases} 
\left( \frac{(w_i^m - x)}{(w_i^m - w_i^l)} \right), & w_i^l \leq x \leq w_i^m \\
1, & w_i^m < x < w_i^n \\
\left( \frac{(x - w_i^n)}{(w_i^u - w_i^n)} \right), & w_i^n \leq x \leq w_i^u 
\end{cases} \tag{3.12}
\]

In Equations 3.11 and 3.12, L and R are the left and right shape functions, which are continuous and non-increasing mapping from [0,1] to [0,1] such that \( L(0) = R(0) = 1 \) and \( L(1) = R(1) = 0 \).

By the extension principle, the alpha cuts of the fuzzy weights can be expressed as:

\[ (w_i)_{\alpha} = \{ x | \mu_{\tilde{w}_i}(x) \geq \alpha \} = [(w_i)^l_{\alpha}, (w_i)^u_{\alpha}] = \inf \{ x | \mu_{\tilde{w}_i}(x) \geq \alpha \}, \sup \{ x | \mu_{\tilde{w}_i}(x) \geq \alpha \} \].
\[ \text{for } i = 1, \ldots, n. \]

If \( \alpha = 0 \) then:

\[
(w_i)_0 = [(w_i)_0^l, (w_i)_0^u] = [w_i^l, w_i^u], \quad i = 1, \ldots, n \tag{3.1}
\]

If \( \alpha = 1 \) the result is the case presented on the right of Figure 3.1 and:

\[
(w_i)_1 = [(w_i)_1^l, (w_i)_1^u] = [w_i^m, w_i^n], \quad i = 1, \ldots, n \tag{3.2}
\]

Which represent, respectively, the support and cores of the L-R fuzzy weights. When the fuzzy weights are dependent, and the set of fuzzy weights has to be summed to one, the normalized weights \( l \) and \( u \) for each \( \alpha - \text{cut} \) are obtained by the equations:

\[
(\hat{w}_i)_\alpha^l = \max \left\{ (w_i)_\alpha^l, 1 - \sum_{j \neq i} (w_j)_\alpha^u \right\}
\]

\[
= \max \left\{ \inf \left\{ x \mid \mu_{\hat{w}_i}(x) \geq \alpha \right\}, 1 - \sum_{j \neq i} \sup \left\{ x \mid \mu_{\hat{w}_j}(x) \geq \alpha \right\} \right\}, \quad i = 1, \ldots, n
\]

\[
(\hat{w}_i)_\alpha^u = \min \left\{ (w_i)_\alpha^u, 1 - \sum_{j \neq i} (w_j)_\alpha^l \right\}
\]

\[
= \min \left\{ \sup \left\{ x \mid \mu_{\hat{w}_i}(x) \geq \alpha \right\}, 1 - \sum_{j \neq i} \inf \left\{ x \mid \mu_{\hat{w}_j}(x) \geq \alpha \right\} \right\}, \quad i = 1, \ldots, n
\]

The normalized weights are the resulting attributes weight that are used for ranking of attributes.

### 3.5 Index calculation

After calculation of attributes weights through fuzzy-AHP, calculation of attribute data by region of interest (i.e. census tracts, neighborhoods, grids, etc.), and detection of spatial clusters where required, index values for each geographic region are calculated as the product of the attribute value and its assigned weight.
\[ Index_{l} = \sum_{i=1, h=1}^{i=5, h=2707} w_i \cdot A_{ih} \]

Where \( w_i \) is the relative weight for the \( i \)th attribute obtained from fuzzy-AHP, and \( A_{il} \) refers to the value of the \( i \)th attribute in the \( l \)th geographic region where the attribute was quantified.
Apart from expert opinions, index calculation requires objective attribute data that fluctuates based on spatial, time or spatial-time characteristics. Attribute data used in green index calculations are generally continuous and spatially-dependent, measured in geographic areas such as census tracts, census metropolitan areas, or neighborhoods, where geographical limits are not necessarily based on the attribute of interest, hence attribute values for neighboring regions are generally different. Tobler (1970) claims in his first law of geography, that:

“Everything is related to everything else, but near things are more related than distant things.”

Tobler suggests a spatial correlation of attributes between close geographical regions. For instance, subjects living at the border of a census tract might share similar characteristics and life dynamics to those of neighboring regions rather than of those living on the center of their own census tract; therefore, assuming that a population shares the same characteristics just because they live in the same delimited geographic area is not always correct. In addition, detecting clusters of attributes is an easier way to visualize how they are distributed across a map. Understanding the way attribute values are spatially distributed helps discern the amount of information that each attribute brings to the index prior to deciding to include it.

In the case of continuous attribute data, the implications of the first law of geography can be addressed by a Spatial Scan Statistic based on the normal model. The spatial scan statistic method evaluates areas with significantly increased or reduced values by scanning a large geographic area for possible clusters using circular windows. Attribute data may be either aggregated at the census tract or other geographical level, or there may be unique coordinates for each observation. Once the approximate area and location of the clusters is identified, a significance test is performed on each cluster to analyze if the values observed inside the cluster are statistically different from the values outside it. The spatial scan statistic method allows to identify if fluctuations in attribute values at a geographic area are random or whether they represent statistically significant deviations from randomness over space, over time or over space.
and time, and it can also adjust for any number of categorical covariates, as well as for temporal trends (Kulldorff, Feuer, Miller, & Freedman, 1997; Sheehan et al., 2004).

4.1 Likelihood calculations

A spatial scan statistic based on the normal probability model aims to identify spatial clusters of individuals or locations with high or low values of some continuous data attribute. Under the null hypothesis, all observations come from the same distribution across all locations in a map and the maximum likelihood estimates (MLE) of the mean and variance are:

\[ \mu = \frac{X}{N}, \text{ and } \sigma^2 = \frac{(x_i - \mu)^2}{N} \]

respectively. The likelihood under the null hypothesis is calculated as:

\[ L_0 = \prod_{1}^{s} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} \]

The log likelihood is then obtained as:

\[ \ln(L_0) = -(N \ln(\sqrt{2\pi} \cdot \sigma) - \sum_{1}^{s} \frac{(x_i - \mu)^2}{2\sigma^2} \]

\[ \ln(L_0) = -(N \ln(\sqrt{2\pi})) - (N \ln(\sigma)) - \sum_{1}^{s} \frac{(x_i - \mu)^2}{2\sigma^2} \]

Under the alternative hypothesis there are one or more clusters that have a higher mean compared to the mean outside the cluster. The radius of each circle varies from zero to a chosen upper bound that restricts the percentage of the population included in the cylinder (Kleinman, Abrams, Kulldorff, & Platt, 2005). The spatial scan clustering imposes a large number \( z \) of overlapping circles of different location and size in the map (Klassen, Kulldorff, & Curriero, 2005). The MLE of each circle \( z \) is calculated first as:

mean inside the circle: \( \mu_z = \frac{x_z}{n_z} \)

mean outside the circle: \( \lambda_z = \frac{(X - x_z)}{(N - n_z)} \)
The MLE for the common variance is:

$$\sigma_z^2 = \frac{1}{N} \left( \sum_{i \in z} (x_i^2 - 2x_z \mu_z + n_z \mu_z^2) + \sum_{i \notin z} (x_i^2 - 2(x - x_z) \lambda_z + (N - n_z) \lambda_z^2) \right)$$

The log likelihood for each circle $z$ is:

$$\ln(L_z) = -\left( N \ln(\sqrt{2\pi}) \right) - (N \ln(\sigma_z)) - \frac{1}{2\sigma_z^2} \left( \sum_{i \in z} (x_i^2 - 2x_z \mu_z + n_z \mu_z^2) + \sum_{i \notin z} (x_i^2 - 2(x - x_z) \lambda_z + (N - n_z) \lambda_z^2) \right)$$

$$\ln(L_z) = -\left( N \ln(\sqrt{2\pi}) \right) - (N \ln(\sigma_z)) - \frac{N}{2}$$

To only search for cluster with low values of population density, an indicator function is added to the likelihood calculation as:

$$\ln(L_z) = \left( -\left( N \ln(\sqrt{2\pi}) \right) - (N \ln(\sigma_z)) - \frac{N}{2} \right) \mathbb{I}(\mu_z > \lambda_z)$$

The maximum log likelihood ratio $\max_z \left( \frac{\ln L_z}{\ln L_0} \right)$ is used as the statistic:

$$\max_z \left( \frac{\ln L_z}{\ln L_0} \right) = \max_z \left( -\left( N \ln(\sqrt{2\pi}) \right) - (N \ln(\sigma_z)) - \frac{N}{2} \right) - \left( -\left( N \ln(\sqrt{2\pi}) \right) - (N \ln(\sigma)) - \sum_{i=1}^{s} \frac{(x_i - \mu)^2}{2\sigma^2} \right)$$

$$= \max_z \left( -\left( N \ln(\sigma_z) \right) - \frac{N}{2} + (N \ln(\sigma)) + \sum_{i=1}^{s} \frac{(x_i - \mu)^2}{2\sigma^2} \right)$$

for only those cases where $\mu_z > \lambda_z$.

Since only the term $N \ln(\sigma_z)$ depends on $z$ and it corresponds to the variance under the alternative hypothesis, it is shown that the most likely cluster selected is the one that minimizes the variance between the inside and outside of the circle.

### 4.2 Statistical significance

The statistical significance of the most likely cluster is evaluated by Monte Carlo hypothesis testing. For the analysis, the observations values are randomly re-distributed over space by generating 999 data sets from a random permutation of the observations $x_i$ and their locations $s$. 
By randomizing this way, the correct alpha level is maintained even if the observations don’t come from a normal distribution.

The log likelihood $lnL(z)$ is calculated for each circle and the most likely cluster is found under each of the randomly Monte Carlo replicates using the equations described above. If the likelihood ratio from the original dataset is found within the 5% highest of all datasets (original and randomly permutated datasets), it is concluded that the cluster from the original dataset is significant at the 0.05 alpha level.

For the 999 random datasets plus 1 original dataset, the p-value of the most likely cluster is calculated as the proportion of the scan statistics that are larger than the one obtained from the original dataset:

$$p\text{-value} = \frac{R}{M + 1}$$

Where $R$ is the place or rank in which the cluster from the original dataset is located with respect to the rest of the datasets. Calculating significance using this method allows to adjust for the multiple testing that originates from the many cylinder sizes and locations evaluated (Dwass, 1957; Kleinman et al., 2005).

When identification of attributes spatial clusters is obtained by applying regular window bandwidth, the identified clusters are always of circular shape. Chapter 5 shows that most attribute clusters are used for visualization purposes of their spatial distribution, however, for index calculation, one attribute is included in the form of cluster categorization rather than on its continuous value, and since real world situations do not necessarily present clusters in perfect circles, a given geographic area could possibly be described as part of a cluster based on the shape limitations of the scan statistic, affecting the final cluster values. Results from the spatial scan statistic of all attributes are shown in Chapter 5.
Chapter 5
An application to Toronto greenness data

In this section, a summary of real attribute data from the city of Toronto and the motivation behind including it is presented. Results from spatial cluster analysis on each attribute and an overview of the results is also included. The application of AHP and Fuzzy-AHP using simulated pairwise comparison matrices and real attribute data is shown. Attributes weights obtained by the various AHP methods and the resulting ‘Urban Greenness Indices’ for the different methods are compared.

5.1 Attribute data

In order to develop an ‘Urban Greenness Index’, grids of 500m x 500m were generated across a map of Toronto. Inclusion of the attributes is based on previous studies that generated urban greenness indices, and studies that measured the effects of greenness in different health outcomes. Such attributes include: population density, building height, proximity to green spaces, overall vegetation obtained by Normalized Difference Vegetation Index (NDVI), and basal area of public street pollen generating trees. A brief description of their relevance with respect to urban greenness is presented in Table 5.1. All attribute data used in this study was publicly available.

Table 5.1 Attributes relevance to greenness and health outcomes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Impact on greenness</th>
<th>Impact on health outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population density</td>
<td>- Surrogate measure of building sparsity and population dynamics.</td>
<td>- Mental Health*</td>
</tr>
<tr>
<td></td>
<td>- Systematic differences on built environment between high and low populated areas</td>
<td>- Respiratory Health</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Cardiovascular Health</td>
</tr>
<tr>
<td>Building height</td>
<td>- More effective in approximating population density as it is not specified by geographic region.</td>
<td>- Mental Health*</td>
</tr>
<tr>
<td></td>
<td>- Surrogate measure of different spatial arrangements of vegetation</td>
<td>- Respiratory Health</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Cardiovascular Health</td>
</tr>
</tbody>
</table>
Overall vegetation (NDVI)

- Measure of overall greenness around a geographical area, indiscriminate of greenness type.
- Differentiates sparsely vegetated from abundantly or scarcely vegetated areas.

Proximity to green spaces

- Surrogate of time a person is likely to spend outside for leisure (i.e. indirect exposure to greenness)
- Exposure to pesticides, herbicides, etc.

Pollen generating trees basal area

- Measure of exposure to pollen generator trees

<table>
<thead>
<tr>
<th>Health Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Respiratory Health</td>
</tr>
<tr>
<td>Mental Health*</td>
</tr>
<tr>
<td>Quality of life</td>
</tr>
<tr>
<td>Birth &amp; Developmental Outcomes</td>
</tr>
<tr>
<td>Cardiovascular Health</td>
</tr>
<tr>
<td>Respiratory Health</td>
</tr>
<tr>
<td>Mental Health*</td>
</tr>
<tr>
<td>Obesity/Overweight</td>
</tr>
<tr>
<td>Screen time</td>
</tr>
<tr>
<td>Hyperactivity</td>
</tr>
<tr>
<td>Allergies</td>
</tr>
</tbody>
</table>

5.1.1 Network buffers

A buffer is defined as a zone around a map feature measured in units of distance or time. An individual’s environment is frequently measured by radial buffers, which are circular spatial units around the individual’s location. The use of radial buffers implies measuring distance in a straight line, which may cause errors pertaining to over- or underestimation of exposure measurements. A relatively new approach to address these limitations is to build street network-based buffers, which have variable sizes and shapes as a function of the characteristics of the surroundings, i.e. street connectivity (Frank et al., 2017; Oliver, Schuurman, & Hall, 2007).

Even though network buffers represent the areas individuals can access around their residences with greater precision, there are no published studies in Canada that use network buffers to measure urban greenness exposure.

In this study, network buffers based on street connectivity were built around each 500m x 500m grid centroid. A total of 2,707 buffers were calculated based on a 15-minute walking distance from each grid centroid at a walking speed of 1.2m/sec. All attribute data, with the exception of population density, were summarized as either the proximity, proportion, or median value of an attribute inside each buffer.
5.1.2 Population density

G. Liu, Wilson, Qi, & Ying (2007) state that systematic differences exist in the built environment, building sparsity, and landscape across areas of different population densities. It is generally believed that there is an inverse relationship between population density and vegetation cover of all types, hence the importance of incorporating a factor for population density when quantifying urban greenness.

Data on the population of Toronto was obtained from the 2016 census tract and downloaded using the CHASS census analyzer (Government of Canada, 2016). Population density was available at the census metropolitan area (CMA) level, which is defined as a geographic area consisting of one or more neighboring municipalities situated around a core (Statistics Canada, 2012). A CMA must have a total population of at least 10,000 people, of which 5,000 or more live in the core. CMAs abide by a set of rules defined by Statistics Canada, and are listed in the following order of priority:

1. A Census Metropolitan Area boundary must follow permanent and easily recognizable physical features. Street extensions, utility or transportation easements, property lines, and former municipal limits may be used as CMA boundaries if physical features are not in close proximity or do not exist.
2. For the 2016 Census, CMA boundaries must follow the boundaries of the Census subdivision types associated with 'on reserve' population.
3. The population of a CMA usually ranges between 2,500 and 10,000 people, with a preferred average of 5,000. CMAs on reserves, in central business districts, major commercial and industrial zones, or peripheral areas, can have populations outside this range.
4. CMAs should be as homogeneous as possible in terms of socioeconomic characteristics, such as similar economic status and social living conditions, at the time of its creation.
5. The shape of CMAs should be as compact as possible.
6. CMA boundaries respect census metropolitan area, census agglomeration, and provincial boundaries, but do not necessarily respect census subdivision (municipality) boundaries.
As shown above, CMA boundaries are somewhat arbitrarily defined in relation to the topic of interest of this study. As a result, assigning a fixed population density to each 500m x 500m grid within a CMA could result in misspecification of characteristics of the population under analysis. For example, if a house is located in a low populated CMA but surrounded by highly populated CMAs, the environmental effects (traffic, noise, air pollution, etc.) within that low populated area may be largely influenced by the dynamics of the highly populated areas surrounding it. On the other hand, a house located and surrounded by low populated CMAs will have very different dynamics. When assigning population density to a grid, the population density of the CMAs location should not only be considered, but also the density of the CMAs surrounding it.

Instead of using the raw population density shown in Figure 5.1 as the attribute value for each grid, a spatial cluster analysis was used to assign each grid to a low population density category, and all non-clustered grids were identified as highly populated. For index calculation, a value of 0.25 was assigned to low populated grids and 0.75 for high populated grids. Detected low population clusters are shown in section 5.3.
5.1.3 Building height

Inclusion of the height of buildings for assessing urban greenness has been suggested in the literature and it is believed to be more effective in approximating population density by indirectly denoting the number of people living in a particular area. In addition, since building height can act as a surrogate measure of different spatial arrangements of vegetation, it can also influence individual perception of greenness (Gupta et al., 2012; Schöpfer et al., 2004).

Average building height and median building height were calculated within each network buffer. Since height data within some buffers were highly skewed, only median building height was used for index calculation. All buildings with average height smaller than 1m and greater than 390m were removed.

Figure 5.2 shows the spatial distribution of building height, where darker colors, mostly located around the downtown area denote areas with higher buildings.

![Figure 5.2](image-url)
Building height data were extracted from a publicly available 3D Massing model in shapefile format last updated on November 2017. Data was downloaded from the City of Toronto open data catalogue and it contains information licensed under the Open Government Licence – Toronto (City of Toronto, 2013).

5.1.4 Normalized Difference Vegetation Index

The Normalized Difference Vegetation Index (NDVI) is a measure of vegetation density on an area of land. NDVI is highly used in the literature as it measures the overall vegetation of an area regardless of its type, in fact, most studies use a ‘Greenness Index’ based on the proportion of green cover as defined by NDVI over an area of land. NDVI ranges from -1 to 1, where zero indicates no vegetation and a value close to 1 indicates the highest possible density of green leaves (NASA, 2000). Sparse vegetation (shrubs and grassland) may result in moderate NDVI values from approximately 0.2 to 0.5, while free-standing water such as oceans, lakes and, rivers tend to present very low or even slightly negative values. NDVI is calculated by the equation below:

$$\text{NDVI} = \frac{\text{Near Infrared} - \text{Visible}}{\text{Near Infrared} + \text{Visible}}$$

To obtain higher spatial accuracy, the minimum available resolution, as of July 2017, was used by means of remote sensing images from LANDSAT 7 satellite. Satellite images were downloaded from Earth Explorer during summer months to assure vegetation levels were at their peak. Images were obtained for June, July, and August, for 2013, 2014, and 2015. Images that had more than 30% of land covered by clouds were removed, and NDVI was ultimately calculated using information of July 2013, July 2014, June 2015, and August 2015.

Negative NDVI was classified as soil or built-up area and was removed from the study, positive values less than 0.2 were also removed. The calculated NDVI value was assigned to each pixel on a 30m x 30m resolution raster of the city of Toronto. A graphical representation of NDVI levels before and after removing non-vegetation data is shown in Figure 5.3. A Google maps aerial view of Toronto and the finalized NDVI map were overlapped to visually verify concordance of green areas.
Figure 5.3 Toronto NDVI values on a 30mx30m resolution. NDVI values between -1 and 0.20 represent no vegetation, and values between 0.21 and 1 represent low to high vegetation. Top: Positive NDVI values. Bottom: NDVI values restricted to greenness.
5.1.5 Pollen generating trees basal area

Data on pollen generating trees located on public streets is used in this study as an indirect measure of exposure to pollen. None of the studies that focused on generation of urban greenness indices incorporate individual tree data into their analysis. While the reasoning behind this is unknown, it may be due to lack of data availability, low impact of this attribute in health, or other reasons. Tree data was summarized by tree basal area, which is a useful measure of site occupancy, and refers to the cross-sectional area of a tree stem as measured at breast height (at 1.3m). Tree basal area was used in this study since it takes tree size into consideration rather than simply counting the number of trees within a buffer.

As pollen data is often summarized at the genus level, it was used to identify those predominant offenders known to produce allergic pollen in Toronto. Twenty-two species were identified and included in the study. Genus type and total count for those species is shown in Table 5.2.

Table 5.2 Genus types corresponding to allergic pollen producers.

<table>
<thead>
<tr>
<th>No.</th>
<th>Offender Genus</th>
<th>Total Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Abies</td>
<td>241</td>
</tr>
<tr>
<td>2</td>
<td>Acer</td>
<td>20793</td>
</tr>
<tr>
<td>3</td>
<td>Aesculus</td>
<td>639</td>
</tr>
<tr>
<td>4</td>
<td>Alnus</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>Betula</td>
<td>1641</td>
</tr>
<tr>
<td>6</td>
<td>Carya</td>
<td>16</td>
</tr>
<tr>
<td>7</td>
<td>Fagus</td>
<td>293</td>
</tr>
<tr>
<td>8</td>
<td>Fraxinus</td>
<td>2316</td>
</tr>
<tr>
<td>9</td>
<td>Juglans</td>
<td>199</td>
</tr>
<tr>
<td>10</td>
<td>Juniperus</td>
<td>365</td>
</tr>
<tr>
<td>11</td>
<td>morus</td>
<td>620</td>
</tr>
<tr>
<td>12</td>
<td>Picea</td>
<td>4514</td>
</tr>
<tr>
<td>13</td>
<td>Pinus</td>
<td>1810</td>
</tr>
<tr>
<td>14</td>
<td>Populus</td>
<td>173</td>
</tr>
<tr>
<td>15</td>
<td>Pseudotsuga</td>
<td>110</td>
</tr>
<tr>
<td>16</td>
<td>Quercus</td>
<td>2749</td>
</tr>
<tr>
<td>17</td>
<td>Salix</td>
<td>196</td>
</tr>
<tr>
<td>18</td>
<td>Taxus</td>
<td>190</td>
</tr>
<tr>
<td>19</td>
<td>Thuja</td>
<td>756</td>
</tr>
<tr>
<td>20</td>
<td>Tilia</td>
<td>4749</td>
</tr>
<tr>
<td>21</td>
<td>Tsuga</td>
<td>88</td>
</tr>
<tr>
<td>22</td>
<td>Ulmus</td>
<td>1846</td>
</tr>
</tbody>
</table>
Tree basal area was calculated as the mean basal value inside each network buffer. The distribution of the five most common trees on streets by genus type is presented in Figure 5.5 (note that a single point may represent more than one tree or a one-to-many relationship) and a heat map of tree basal area distribution is presented in Figure 5.4.

Preprocessed Toronto street tree data was obtained via the Canadian Urban Environmental Health Research Consortium (CANUE) from a publicly available and annually updated shapefile containing more than 500,000 trees located on Toronto public streets. The Toronto Street Tree data contains information licensed under the Open Government License – Toronto (City of Toronto, 2013).
5.1.6 Proximity to green spaces

Living in proximity to green spaces affects the purpose and frequency of visiting an outside green area and has effects on the microclimate and environmental quality (Jim and Chen 2009, Troy and Grove 2008). Spending time outside in open green spaces implies being exposed to environmental modifiable factors, such as traffic pollution, allergens produced by trees, pesticides, etc., but it is difficult to measure exactly how much time a person spends outside. A proxy variable has been used in the literature to approximate how much leisure time someone is likely to spend outside in proximity to public green spaces, such as parks, parkettes, playgrounds, and other green areas.

The variable proximity to public green spaces was calculated as the distance in meters from each grid centroid to the nearest green area, as defined by the City of Toronto. All green areas outside a network buffer were removed. Figure 5.6 displays grids that have a higher distance to the nearest green space in darker color.
Green spaces data contains boundaries and park names within the City of Toronto. The shapefile was last updated on April 2017 and downloaded from the City of Toronto open data catalogue and it contains information licensed under the Open Government Licence – Toronto (City of Toronto, 2013).

### 5.2 Attribute data summary

Data on NDVI levels, building height, population density, tree basal area, and green space locations were obtained for a total of 2,707 grids, drawn over the city of Toronto, Canada. All attribute data was calculated within the limits of each network buffer belonging to a grid.

Building height and tree basal area were normally distributed across the whole city, but once the city was broken into grids and attribute data was calculated for each, both variables became highly skewed. As a result, the median within each buffer, instead of the mean, was used as the representative value of each grid. Proximity to green spaces was calculated as a unique value in meters from the centroid of the grid to the closest green space. Population density of each grid was defined as an indicator variable of high or low populated area based on the cluster it belonged to. Details on resulting clusters are explained in the following section.
In order for the attributes to have a comparable scale, all were converted to their respective percentile values. Table 5.3 shows the mean and standard deviation by attribute before and after converting to percentiles for index calculation. Counts and percentages are shown for high and low population density clusters.

Table 5.3 Summary statistics of attribute data (mean and standard deviation (SD) of 500m x 500m grids)

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Mean (SD)</th>
<th>Mean percentiles (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population density</td>
<td>4506.73 (4538.95)</td>
<td>0.47 (0.23)</td>
</tr>
<tr>
<td>Building Height</td>
<td>5.73 (3.37)</td>
<td>0.47 (0.15)</td>
</tr>
<tr>
<td>NDVI</td>
<td>0.34 (0.04)</td>
<td>0.48 (0.28)</td>
</tr>
<tr>
<td>Proximity to green spaces</td>
<td>136.24 (152.71)</td>
<td>0.47 (0.27)</td>
</tr>
<tr>
<td>Pollen gen. tree basal area</td>
<td>0.05 (0.03)</td>
<td>0.49 (0.26)</td>
</tr>
</tbody>
</table>

Correlation was calculated among all attributes to assure absence of conflict during inclusion of the attributes in the index. The heat map in Figure 5.7 shows that most variables have a low correlation (between 0 and -0.41). Higher negative correlation is observed between NDVI and proximity to green spaces ($r = -0.41$, p-value < 0.05), followed by a positive correlation between population density and average height ($r = 0.39$, p-value < 0.05). Even though the correlation is statistically significant, based on the valuable added information of each attribute in the index and in the low correlation values, the attributes were maintained.

### 5.3 Spatial clusters of attributes

Using the spatial scan statistic explained in Chapter 4, spatial clustering for continuous data based on the normal probability model was used to identify those grids with higher or lower levels of each attribute.
With the centroid as a geographical location, attribute data from 2,707 grids were inputted in the SaTScan™ version 9.6 software for identification of clusters. A similar approach was taken by Kleinman et al. (2005), in which summary measures of observations were assigned to their census tract centroid. With respect to population density, eight overlapping clusters with statistically significant lower values were found. Information of each low population density cluster is shown in Table 5.4.

Table 5.4 Identified clusters of population density by spatial clustering analysis using SaTScan. Statistically significant clusters highlighted in gray

<table>
<thead>
<tr>
<th>Cluster</th>
<th>Grids on cluster</th>
<th>Population Density mean inside</th>
<th>Population density mean outside</th>
<th>Std. Deviation</th>
<th>Cluster Radius (km)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>366</td>
<td>2291.195</td>
<td>4765.717</td>
<td>3680.167</td>
<td>8.714</td>
<td>0.001</td>
</tr>
<tr>
<td>2</td>
<td>801</td>
<td>3228.912</td>
<td>4936.392</td>
<td>3694.889</td>
<td>14.268</td>
<td>0.001</td>
</tr>
<tr>
<td>3</td>
<td>60</td>
<td>377.710</td>
<td>4523.029</td>
<td>3726.565</td>
<td>3.969</td>
<td>0.001</td>
</tr>
<tr>
<td>4</td>
<td>752</td>
<td>3502.096</td>
<td>4788.514</td>
<td>3731.986</td>
<td>8.868</td>
<td>0.001</td>
</tr>
<tr>
<td>5</td>
<td>93</td>
<td>1779.785</td>
<td>4525.479</td>
<td>3742.948</td>
<td>2.688</td>
<td>0.001</td>
</tr>
<tr>
<td>6</td>
<td>106</td>
<td>2210.278</td>
<td>4521.658</td>
<td>3749.503</td>
<td>2.912</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Figure 5.7 Correlation levels between attributes of Urban Greenness Index
Spatial clustering analysis was performed in all remaining attributes. Resulting significant clusters are depicted by circles in Figure 5.8, where grids corresponding to low value clusters are contained in black striped circles for population density, building height, tree basal area, and NDVI. Grids with significantly higher proximity to green spaces are also shown in black striped circles, and in green for tree basal area.

In Figure 5.8, the map titled ‘Overall Vegetation’ shows a clear pattern of higher NDVI values for regions on the borders of the city, however, some of those zones present low street tree basal area and low proximity to green spaces. This means that NDVI may not be reflecting tree density, but other types of vegetations such as grass or wetlands. This is an example of the information that could be omitted when quantifying greenness by individual attributes rather than by quantifying them together.

Figure 5.8 Spatial distribution and spatial clusters detected on attribute data. Only significant clusters are shown.
Population density and building height appear to be more homogeneous throughout the maps with the exception of some areas in the center that are catalogued as low populated but have high buildings (Figure 5.8). Although clusters are calculated for all attributes, only population density clusters are used for index calculation, as it is the only attribute that cannot be summarized by grid. After calculation of attribute data per grid, the next step for developing an index is to obtain experts’ opinions on attribute importance and apply AHP to calculate attribute weights.

5.4 Attributes Weights calculation

To calculate attribute weights, three 5x5 pairwise comparison matrices \(A_k\) for \(k=1,2,3\) are randomly simulated from a uniform distribution with bounds at 1 and 9, this means that real input from experts was not used in this study. Bounds of the simulated matrices were based on R. W. Saaty (1987) preference scale from Table 5.5. The three simulated matrices are used in the following sections to calculate weights by triangular, trapezoidal and Gaussian membership functions, and can be found in Appendix B.

Table 5.5 Preference table and its equivalent crisp and fuzzified numbers

<table>
<thead>
<tr>
<th>Absolute scale</th>
<th>Linguistic scale</th>
<th>Triangular ((l, m, u))</th>
<th>Trapezoidal ((l, m, n, u))</th>
<th>Gaussian ((\mu, \sigma))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>(0.5,1,2)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
<td>(2.3,4)</td>
<td>(2, 2.5, 3.5, 4)</td>
<td>(3, 1)</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
<td>(4,5,6)</td>
<td>(4, 4.5, 5.5, 6)</td>
<td>(5, 1)</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
<td>(6,7,8)</td>
<td>(6, 6.5, 7.5, 8)</td>
<td>(7, 1)</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
<td>(8,9,9)</td>
<td>(8, 8.5, 9, 9)</td>
<td>(9, 1)</td>
</tr>
</tbody>
</table>

Application of classical AHP

Prior to initiating an AHP procedure, the three expert comparison matrices are aggregated by geometric means (Appendix C) and consistency is verified. Consistency ratios for matrix \(M_1\), \(M_2\), and \(M_3\) are 0.096, 0.167 and 0.126 respectively; \(M_2\) and \(M_3\) are consistent at borderline, considering the cut-off of \(CR<0.1\). Consistency ratio for the aggregated matrix is 0.08 which indicates that the resulting matrix is acceptable for further AHP calculations. Classical AHP yields the weights shown in Table 5.6 where NDVI is given the highest weight, followed by proximity, population density, tree basal area and building height.
Table 5.6 Resulting aggregated crisp matrix and weights from classical AHP

<table>
<thead>
<tr>
<th>Aggregated matrix</th>
<th>NDVI</th>
<th>Proximity</th>
<th>Pop. Density</th>
<th>Tree basal area</th>
<th>Bldg. Height</th>
<th>AHP Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDVI</td>
<td>1</td>
<td>2.714</td>
<td>5.241</td>
<td>5.646</td>
<td>8.277</td>
<td>0.480</td>
</tr>
<tr>
<td>Proximity</td>
<td>0.368</td>
<td>1</td>
<td>4</td>
<td>6.604</td>
<td>6.542</td>
<td>0.300</td>
</tr>
<tr>
<td>Pop. density</td>
<td>0.191</td>
<td>0.250</td>
<td>1</td>
<td>4.160</td>
<td>4.121</td>
<td>0.125</td>
</tr>
<tr>
<td>Pollen gen. tree basal area</td>
<td>0.177</td>
<td>0.151</td>
<td>0.240</td>
<td>1</td>
<td>2.410</td>
<td>0.056</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>0.121</td>
<td>0.153</td>
<td>0.243</td>
<td>0.415</td>
<td>1</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Application of fuzzy-AHP

Fuzzy-AHP is performed separately for triangular, trapezoidal and Gaussian membership functions. The same simulated pairwise comparison matrices used for classical AHP are used as fuzzy-AHP input. Prior to aggregation, the matrices are fuzzified using the preference table for conversion of linguistic variables (Table 5.5) and are then aggregated using Equations 3.1, 3.2, and 3.3. Resulting fuzzified matrices before and after aggregation are shown in Appendix C.

Fuzzy-AHP by geometric means

After aggregation, weights are calculated using fuzzy geometric means and fuzzy extent analysis on each membership function. The fuzzy weights obtained from geometric means prior to defuzzification are shown in Table 5.7. The bounds are also shown graphically for triangular (top), trapezoidal (middle) and Gaussian approximated by triangular (bottom) on Figure 5.9.

Table 5.7 Fuzzy weights l, m, n, and u bounds for each membership function using geometric means method

<table>
<thead>
<tr>
<th>Fuzzy Weights by geometric means method</th>
<th>Triangular</th>
<th>Trapezoidal</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Modal</td>
<td>Max</td>
</tr>
<tr>
<td>NDVI</td>
<td>0.194</td>
<td>0.480</td>
<td>0.646</td>
</tr>
<tr>
<td>Proximity</td>
<td>0.145</td>
<td>0.300</td>
<td>0.573</td>
</tr>
<tr>
<td>Popl. density</td>
<td>0.052</td>
<td>0.126</td>
<td>0.338</td>
</tr>
<tr>
<td>Pollen gen. tree basal area</td>
<td>0.026</td>
<td>0.057</td>
<td>0.130</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>0.019</td>
<td>0.037</td>
<td>0.104</td>
</tr>
</tbody>
</table>
Figure 5.9 shows that although the modal values for triangular and Gaussian functions are identical and that the mean value for trapezoidal is similar, the upper and lower bounds of the three functions vary considerably; the triangular function yields the lowest spread, followed by Gaussian and trapezoidal. It is also shown that in all cases the functions become asymmetrical with a higher spread towards the upper bound, with exception of NDVI fuzzy weights using a
triangular membership function, in which case slightly higher spread is observed towards the lower bound. The defuzzified weights are shown in Table 5.8, it is the differences on spread of the three membership functions that make weight values differ between functions after defuzzification by equations 3.4 and 3.5.

Table 5.8 Attributes weights by geometric means and defuzzification by modal value dominancy

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Triangular</th>
<th>Trapezoidal</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDVI</td>
<td>0.46</td>
<td>0.38</td>
<td>0.46</td>
</tr>
<tr>
<td>Proximity</td>
<td>0.32</td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td>Popl. density</td>
<td>0.15</td>
<td>0.16</td>
<td>0.15</td>
</tr>
<tr>
<td>Pollen gen. tree basal area</td>
<td>0.06</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>0.05</td>
<td>0.08</td>
<td>0.07</td>
</tr>
</tbody>
</table>

**Fuzzy-AHP by extent analysis**

Attributes weights were also calculated using fuzzy extent analysis, resulting weights were normalized using the arithmetic mean method proposed by P.-T. Chang & Lee (1995) and the $\alpha$-cut method introduced by Y.-M. Wang & Elhag (2006). The weights after normalization are presented in Table 5.9.

Fuzzy weights obtained by extent analysis lack of an upper and lower bound due to the calculation of degrees of possibility, hence they cannot be represented graphically when normalizing by geometric means. On the other hand, normalizing by $\alpha$-cut allows a graphical representation of weight bounds by showing weights at each possible $\alpha$-level, as in Figure 5.10. Compared to fuzzy weights from geometric means (Figure 5.9, top), the modal value decreases for NDVI and increases for all other attributes, and the spread increases considerably in all.
Table 5.9 Attributes weights by extent analysis (after normalization)

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Fuzzy Extent Analysis</th>
<th></th>
<th>Wang’s Normalization</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Chang's Normalization</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Triangular</td>
<td>Gaussian</td>
<td>Triangular</td>
<td>Gaussian</td>
</tr>
<tr>
<td>NDVI</td>
<td>0.34</td>
<td>0.35</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Proximity</td>
<td>0.29</td>
<td>0.33</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Popl. Density</td>
<td>0.21</td>
<td>0.19</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Pollen gen. tree basal area</td>
<td>0.12</td>
<td>0.07</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Bldg. Height</td>
<td>0.02</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Ranking of attributes by all methods follow the same order: higher weight for NDVI, followed by proximity to green spaces, population density, tree basal area and building height. Overall, the weights obtained with all fuzzy-AHP methods are similar; considerably higher weight for NDVI and one of the lowest weights for building height are obtained using classical AHP. When using extent analysis, triangular functions yield the same weights as trapezoidal functions, hence results from the latter are omitted. Weights assigned to tree basal area and building height are close to zero, which in the case of this simulation is convenient due to their correlation level, however this might not be the case on a real scenario. In regard to population density, tree basal area and building height weights, fuzzy geometric mean method and fuzzy extent analysis using α-cut normalization render almost identical results though fuzzy geometric mean yields slightly higher values for all attributes.
Results from fuzzy extent analysis using normalization by arithmetic means, although following the same ranking order as the other methods, present a different distribution of weight values: using this method, weights assigned to NDVI and proximity are more similar within them and smaller in magnitude compared to the other methods; also, higher weights are given to population density and tree basal area. This method yields the highest weight for population density (weight = 0.21 compared to weights ranging from 0.15 to 0.17 by other methods).

Overall, weights obtained by all fuzzy weighting methods are reasonably similar, higher consistency is observed using fuzzy geometric mean method and fuzzy extent analysis using α-level normalization, although it is difficult to understand the extent to which the differences in weights affect the calculated index.

### 5.5 Development and Comparison of Urban Greenness Indices

In this section, the resulting ‘Urban Greenness Indices’ of Toronto obtained by AHP and Fuzzy-AHP methods are shown. An example of the calculation using triangular – fuzzy geometric mean weights is:

\[
UGI = 0.44 \,( \text{NDVI})_i + 0.34 \,( \text{Proximity})_i + 0.17 \,( \text{Population density})_i + 0.07 \,( \text{Basal Area})_i + 0.05 \,( \text{Building height})_i
\]

The ‘Urban Greenness Indices’ obtained with the fuzzy-AHP methods used are shown in heat maps in Figure 5.11. Main differences of index values are observed in the Toronto downtown area, where fuzzy geometric means yields index values greater than 0.6 for a larger land area, similar to those obtained using classical AHP. Within extent analysis, weight values differ by normalization method and by membership function, however the resulting indices are quite similar. With respect to the south-west and north-west regions of the city, a notable difference on values is observed on the eight maps. These differences are not specific to a certain fuzzy-AHP method or membership function. In regard to the built environment, certain characteristics of the areas can be reviewed to understand the accuracy of the methods. One characteristic that can be reviewed is that the south-west area belongs to a highly industrial area of Toronto located in proximity to an international airport. This area is classified as low populated and has low count of trees compared to the rest of the city. It also has low NDVI levels and sparse road networks, which may indicate that the index levels assigned, using fuzzy extent analysis and alpha levels normalization, are more accurate. The north-west area that is closer to the city limits consists of a
combination of residential and industrial areas, as well as a rail yard. A large area in this part of
the city has a high contrast of very low NDVI levels and low count of trees, compared to higher
values of such attributes in the surrounding grids. Fuzzy extent analysis with $\alpha$-cut levels
normalization is capable of distinguishing that contrast by providing different index values.
Fuzzy geometric mean using a Gaussian function yields similar results even though the
differentiation is not as distinguishable when a triangular function is used.

Figure 5.11 Comparison of indices obtained by the different fuzzy AHP methods and
membership functions
Chapter 6
Discussion and Conclusion

6.1 Discussion

The association between health outcomes and urban greenness is a relatively new area of interest that has had inconsistent findings to date. Though many of these inconsistencies may be due to lifestyle differences between geographic regions, heterogeneity of results may also be due to the use of individual measures of greenness, that fail to reflect accurately the exposure to greenness in an urban environment. For example, some studies define urban greenness as the percentage of green cover around a household, while other studies define it as the proximity to all green spaces. Each of these individual measures reflect different aspects of urban greenness exposure, thus incorporating them into an index is necessary.

In this thesis, fuzzy-AHP is used to develop an ‘Urban Greenness Index’ that incorporates various aspects of urban greenness, rather than analyzing them separately. The methodology of this thesis considers simulated input from experts, and objective measures of overall vegetation (NDVI), proximity to green spaces, tree basal area, building height, and population density. This approach differs from previous studies generating urban greenness indices as it integrates fuzzy logic, multiple criteria decision-making, and advance statistical methods for quantification of urban greenness. It is the first study to analyze the variation between weights obtained by different fuzzy-AHP methods and to incorporate the five attributes stated above measured using network buffers.

Important considerations on obtaining experts input

In every study focused on developing an index, two major features should be carefully decided on to assure the index reflects a truthful approximation of urban greenness exposure. First a group of health experts would need to provide input as to which attributes are relevant for the health outcome of interest. Another group of experts then provides pairwise comparison matrices on the attributes chosen without being informed of the health outcomes of interest to the study. Ideally these two groups of experts would work independently in order to avoid bias in the ranking of attributes.
Another approach to obtain input for AHP is the Delphi method. Previous research on urban greenness has used this method to obtain attributes relative importance from educators and researchers in the urban planning field (Gupta et al., 2012; Y. Liu et al., 2016). The Delphi method is based on a group of experts arriving to an acceptable degree of consensus regarding the attributes of interest. The consensus is reached by iteratively administering, and subsequently applying data from questionnaires. This iterative process involves identifying areas of agreement and disagreement and then making modifications based on previous questionnaire responses. Using results from the Delphi method could be beneficial, as it would assure consistency of the input pairwise comparison matrix used in fuzzy-AHP. Subject selection, time frames for conducting and completing the study, possibility of low response rates, and unintentionally guiding feedback from the respondent group, are areas which should be considered when designing a Delphi method (Hsu & Sandford, 2007). Unfortunately, studies published using the Delphi method to develop greenness indices fail to specify on the factors mentioned above. The extent to which the Delphi method is able to represent a variability in experts’ opinions compared to fuzzy methodology is unknown.

An option not explored in this study is to have experts grade themselves on their ability to provide input, that is, to weight experts’ input based on their self-perceived expertise. This approach helps in acknowledging that input in AHP is received from experts with different backgrounds and different levels of confidence with respect to the attributes involved, and it will be considered for future steps of this study.

Mapping spatial distribution

Published studies focused on urban greenness quantification map the spatial distribution of attributes at different resolutions. Some studies focus on exposure and distribution of greenness at lower scales, quantifying attributes at grid cell sizes of 20m, 100m, and 200m. Others are focused mainly on urban planning and community development, using larger areas of study, such as neighborhoods (Gupta et al., 2012; Schöpfer et al., 2004; ThiLoi et al., 2015).

Although 500m x 500m grids were used in this study, results were replicated using Census Metropolitan Areas as the region of interest. Using CMAs, the individual mean attribute values were similar to those values obtained using grids, but once attributes were incorporated into the
index, they differed considerably, and more variations were observed between the different fuzzy-AHP methods (Appendix E).

This study minimized computational times by using 500m x 500m grids, resulting index values using this resolution can still be directly transferable at the individual level by assigning each subject to the index value of the grid where they are located. In future research, higher resolutions may be applied to evaluate the effect they have on the index values using the same attribute datasets. Values for all attributes could be computed within network buffers of each individual building or household of Toronto and then the weights determined from fuzzy-AHP could be applied. One of the main challenges of this approach is the high computational times generated from calculating network buffers, cluster detection, and attribute quantification on such increased number of locations.

**Comparison of defuzzification methods**

Another feature not evaluated in this thesis is the variation originating from the defuzzification method. The modal value dominancy method was used to defuzzify all weights obtained by fuzzy geometric means. Other methods, such as defuzzification based on center of gravity (COG), or on the beta distribution, are explored to analyze the variability of the results of each method.

The COG technique is the most prevalent of all the defuzzification methods (Voskoglou, 2016). For all membership functions, the COG is given by the equation:

\[
A = \frac{\int \mu_A(x) \cdot x \, dx}{\int \mu_A(x) \, dx}
\]

The COG technique is widely used when combining multiple membership functions on fuzzy inference systems due to its capability to find the centroid of unbalanced functions. On the other hand, novel defuzzifying techniques that have not been widely used also exist in the literature. An example is defuzzification using the statistical Beta distribution, since it is the only distribution bounded to (0,1) and is zero outside this interval.
The equations for defuzzification by Beta distribution for triangular and trapezoidal functions respectively, are shown below. Defuzzification by beta distribution for Gaussian membership functions has not been developed.

\[
A = \frac{l + m + n}{3}
\]

\[
A = \frac{2l + 7m + 7n + 2u}{18}
\]

Proof of the equations shown above, are demonstrated by Rahmani, Hosseinzadeh Lotfi, Rostamy-Malkhalifeh, & Allahviranloo (2016).

Using the fuzzy weights obtained by geometric means (Table 5.7), weights were defuzzified by the COG technique and by Beta distribution. Resulting weights are compared to those obtained by modal value dominancy (Table 6.1). All COG calculations were performed in Matlab (Version R2018a).

Table 6.1 Comparison of defuzzified weights by different defuzzification techniques

<table>
<thead>
<tr>
<th>Membership Function</th>
<th>Triangular</th>
<th>Trapezoidal</th>
<th>Gaussian</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Defuzzification Technique</td>
<td>Modal value Dominancy</td>
<td>COG</td>
</tr>
<tr>
<td>NDVI</td>
<td>Modal value Dominancy</td>
<td>0.46</td>
<td>0.44</td>
</tr>
<tr>
<td>Proximity</td>
<td>Modal value Dominancy</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>Popl. density</td>
<td>Modal value Dominancy</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>Modal value Dominancy</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>Modal value Dominancy</td>
<td>0.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Using a triangular membership function, weight values calculated by each defuzzification technique do not differ considerably. When resulting fuzzy membership functions are highly skewed to the right with the lower bound at zero, weight values using the COG technique are lower, i.e. weights for population density, tree basal area, and building height on trapezoidal functions (Figure 5.9). Conversely, using a Gaussian function that also resulted on unbalanced, highly spread fuzzy weights, all COG weights are higher than those obtained by modal value dominancy. It could be argued that the COG technique appears to be more sensitive to skewness
and to the spread of fuzzy weights, while Beta defuzzification and Modal Value dominancy provide similar crisp weights mostly impacted by the modal value. Repetitive simulations are needed to confirm this hypothesis.

**Limitations and Considerations**

An ‘Urban Greenness Index’ should be developed using input from experts in forestry, urban planning, research, and other professions, and possibly considering assigning weights to their input based on their level of expertise. Although this study does not include input from experts, future research could obtain it by completing pairwise comparison matrices or by using the Delphi method.

In regard to the index attributes, this thesis only considered objective data related to urban vegetation, building height, and population density characteristics. In contrast, previous studies mention that urban environmental quality is also affected by subjective aspects of the surrounding environment (Y. Liu et al., 2016; ThiLoi et al., 2015). Future studies may benefit from incorporating human factors, such as perceived quality of life or different questionnaires that evaluate individual perception of surrounding vegetation.

A challenge that occurred when developing the index was the identification of the most robust Fuzzy-AHP method. This was mainly due to the close similarity between the results of each approach. Identifying the most robust method can be achieved by evaluating which method is least sensitive to changes on the input matrices. This is achieved by continuously modifying the simulated input matrices and measuring deviations from the mean weight of each attribute. An example of this is presented in Appendix F, where it is shown that weights obtained by geometric means present lower standard deviations than those obtained by extent analysis. Such variations were only explored using a triangular function, analysis of variation between membership functions is outside the scope of this thesis but will be analyzed in the future.

Lastly, it is important to measure the appropriateness of the index, which can be achieved by validating which resulting index is most similar to a real observed scenario based on a thorough cross-inspection between index levels and CMA characteristics, i.e. overlapping a map of all attributes to the index map. It is to be noted that this approach would be only based on subjective information. Another way of measuring appropriateness is by analyzing spatial associations
between a health outcome and the urban greenness index. Results would then be compared to those obtained from analysis of individual attributes. This approach could also provide a sense of the information that is lost when measuring greenness one-dimensionally, i.e. solely by proximity to green spaces, versus using an index. Even though this index omits inclusion of environmental time-modifiable factors such as seasonality, relative humidity, temperature, and air pollution, (among others), the fluctuation of such variables through time makes it difficult to summarize them as a fixed value. It would be beneficial to include these factors as covariates in a secondary analysis, rather than to incorporate them as part of the index.

6.2 Conclusion

This thesis explores and demonstrates the usefulness of an integrated Fuzzy-AHP approach for developing an index. Ultimately, numerical examples are used to calculate an ‘Urban Greenness Index’ for Toronto to display the performance of the different Fuzzy-AHP methods.

It is complex to discern which fuzzy-AHP method yields a more realistic index. Simulated matrices provide an insight into the advantages and disadvantages of each fuzzy-AHP method. Triangular and Gaussian functions yield more consistent weights, with the former having the benefits of ease of computation and straightforward applications that do not require transformations. Modal value dominancy proves to be an acceptable defuzzification method when there is minimal imbalance in the fuzzy weights, otherwise COG should be used. Results obtained from the simulated preference matrices indicate that Fuzzy geometric means using a triangular membership function, is an acceptable approach, as it reflects the fluctuation of attributes on the different geographic locations with less variation to changes on the input. A limitation of the study is the lack of input from real experts on the areas of forestry, city planning, and urban development, among other disciplines. Obtaining such input enables the researcher to develop an appropriate index using the methodology suggested in this study.

Despite the limitations and areas for future study that are mentioned above, this study improves upon previous studies of the same nature in that it incorporates fuzzy logic to account for the variability of opinions. This study also accounts for walkability and street connectivity around the grid centroids by using network buffers, as well as attributes not only related to vegetation, but also to the urban built environment.
Studies analyzing the multifaceted aspects of greenness and their association with health outcomes help develop a further understanding of the pathways through which greenness affects health. Once input from real experts is obtained, the ‘Urban Greenness Index’ developed in this study can be used to study associations between urban greenness and different health outcomes, as it represents exposure more accurately by incorporating various aspects of the urban environment.
References


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Appendix

Appendix A. Basic fuzzy operators

Consider two fuzzy numbers \( \tilde{A}_1 \) and \( \tilde{A}_2 \) belonging to a triangular membership function, such that \( \tilde{A}_1 = (l_1, m_1, u_1) \) and \( \tilde{A}_2 = (l_2, m_2, u_2) \), their arithmetic operations are as follows:

1. \( (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \)

2. \( (l_1, m_1, u_1) \ominus (l_2, m_2, u_2) = (l_1l_2, m_1m_2, u_1u_2) \)

3. \( (l_1, m_1, u_1) \otimes (l_2, m_2, u_2) \approx (l_1l_2, m_1m_2, u_1u_2) \)

4. \( (\lambda, \lambda, \lambda) \otimes (l_2, m_2, u_2) \approx (\lambda l_2, \lambda m_2, \lambda u_2) \), \( 0 < \lambda < \infty \)

5. \( (l_1, m_1, u_1)^{-1} \approx \left( \frac{1}{l_1}, \frac{1}{m_1}, \frac{1}{u_1} \right) \)

Trapezoidal and Gaussian fuzzy numbers arithmetic operations are approximated in a similar manner. Note that the solutions to fuzzy addition and subtraction are of closed form and the definitions provided are exact. Multiplication changes the shape of the fuzzy number because of the non-linear effect of multiplication, hence fuzzy multiplication and division are not closed and the definitions are standard approximations to the actual result; a comparison between standard approximation to the product and the actual result is explained in Giachetti & Young (1997).
Appendix B. Simulated pairwise comparison matrices

To calculate attribute weights, three 5x5 pairwise comparison matrices $A_k$ for $k=1,2,3$ are randomly simulated from a uniform distribution with bounds at 1 and 9 based on R. W. Saaty (1987) preference scale. The simulated matrices are shown in their reciprocal form in table B.1

Table B. 1 Three simulated pairwise comparison matrices for AHP and fuzzy-AHP

<table>
<thead>
<tr>
<th></th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Simulated Expert k=1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>1</td>
<td>1</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Proximity</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Popl. density</td>
<td>1/9</td>
<td>1/4</td>
<td>1</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>1/9</td>
<td>1/8</td>
<td>1/6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>1/9</td>
<td>1/5</td>
<td>1/5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td><strong>Simulated Expert k=2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>NDVI</td>
<td>1</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Proximity</td>
<td>1/5</td>
<td>1</td>
<td>8</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>Popl. density</td>
<td>1/8</td>
<td>1/8</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>1/4</td>
<td>1/6</td>
<td>1/3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>1/7</td>
<td>1/7</td>
<td>1/2</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td><strong>Simulated Expert k=3</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>Proximity</td>
<td>1/4</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Popl. density</td>
<td>1/2</td>
<td>1/2</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>1/5</td>
<td>1/6</td>
<td>1/4</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>1/9</td>
<td>1/8</td>
<td>1/7</td>
<td>1/7</td>
<td>1</td>
</tr>
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Appendix C. Fuzzification and aggregation of pairwise comparison matrices

Simulated matrices fuzzified using equivalences of Table 5.5. Resulting fuzzified matrices before and after aggregation are shown below.

Table C. 1 Simulated pairwise comparison matrices converted to triangular fuzzy numbers

<table>
<thead>
<tr>
<th></th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Simulated Expert k=1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>(1;1;1)</td>
<td>(1/2;1;2)</td>
<td>(8;9;9)</td>
<td>(8;9;9)</td>
<td>(8;9;9)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1/2;1;2)</td>
<td>(1;1;1)</td>
<td>(3;4;5)</td>
<td>(7;8;9)</td>
<td>(4;5;6)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(1/9;1/9;1/8)</td>
<td>(1/5;1/4;1/3)</td>
<td>(1;1;1)</td>
<td>(5;6;7)</td>
<td>(4;5;6)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(1/9;1/9;1/8)</td>
<td>(1/9;1/8;1/7)</td>
<td>(1/7;1/6;1/5)</td>
<td>(1;1;1)</td>
<td>(1/2;1;2)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(1/9;1/9;1/8)</td>
<td>(1/6;1/5;1/4)</td>
<td>(1/6;1/5;1/4)</td>
<td>(1/2;1;2)</td>
<td>(1;1;1)</td>
</tr>
<tr>
<td><strong>Simulated Expert k=2</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>(1;1;1)</td>
<td>(4;5;6)</td>
<td>(7;8;9)</td>
<td>(3;4;5)</td>
<td>(6;7;8)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1/6;1/5;1/4)</td>
<td>(1;1;1)</td>
<td>(7;8;9)</td>
<td>(5;6;7)</td>
<td>(6;7;8)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(1/9;1/8;1/7)</td>
<td>(1/9;1/8;1/7)</td>
<td>(1;1;1)</td>
<td>(2;3;4)</td>
<td>(1;2;3)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(1/5;1/4;1/3)</td>
<td>(1/7;1/6;1/5)</td>
<td>(1/4;1/3;1/2)</td>
<td>(1;1;1)</td>
<td>(1;2;3)</td>
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<tr>
<td>Bldg. height</td>
<td>(1/8;1/7;1/6)</td>
<td>(1/8;1/7;1/6)</td>
<td>(1/3;1/2;1)</td>
<td>(1/3;1/2;1)</td>
<td>(1;1;1)</td>
</tr>
<tr>
<td><strong>Simulated Expert k=3</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>(1;1;1)</td>
<td>(3;4;5)</td>
<td>(1;2;3)</td>
<td>(4;5;6)</td>
<td>(8;9;9)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1/5;1/4;1/3)</td>
<td>(1;1;1)</td>
<td>(1;2;3)</td>
<td>(5;6;7)</td>
<td>(7;8;9)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(1/3;1/2;1)</td>
<td>(1/3;1/2;1)</td>
<td>(1;1;1)</td>
<td>(3;4;5)</td>
<td>(6;7;8)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(1/6;1/5;1/4)</td>
<td>(1/7;1/6;1/5)</td>
<td>(1/5;1/4;1/3)</td>
<td>(1;1;1)</td>
<td>(6;7;8)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(1/9;1/9;1/8)</td>
<td>(1/9;1/8;1/7)</td>
<td>(1/8;1/7;1/6)</td>
<td>(1/8;1/7;1/6)</td>
<td>(1;1;1)</td>
</tr>
</tbody>
</table>
Table C. 2 Simulated pairwise comparison matrices converted to trapezoidal fuzzy numbers

<table>
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<tr>
<th></th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Simulated Expert k=1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>(1, 1, 1.5, 2)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(8, 8.5, 9, 9)</td>
<td>(8, 8.5, 9, 9)</td>
<td>(8, 8.5, 9, 9)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1, 1, 1.5, 2)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(3, 3.5, 4.5, 5)</td>
<td>(7, 7.5, 8.5, 9)</td>
<td>(4, 4.5, 5.5, 6)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(-0.88, -0.38, 0.611, 1.111)</td>
<td>(-0.75, -0.25, 0.75, 1.25)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(5, 5.5, 6.5, 7)</td>
<td>(4, 4.5, 5.5, 6)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(-0.88, -0.38, 0.611, 1.111)</td>
<td>(-0.87, -0.37, 0.625, 1.125)</td>
<td>(-0.83, -0.33, 0.666, 1.166)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(1, 1, 1.5, 2)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(-0.88, -0.38, 0.611, 1.111)</td>
<td>(-0.8, -0.3, 0.7, 1.2)</td>
<td>(-0.8, -0.3, 0.7, 1.2)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(1, 1, 1.5, 2)</td>
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<tr>
<td><strong>Simulated Expert k=2</strong></td>
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<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>(1, 1, 1.5, 2)</td>
<td>(4, 4.5, 5.5, 6)</td>
<td>(7, 7.5, 8.5, 9)</td>
<td>(3, 3.5, 4.5, 5)</td>
<td>(6, 6.5, 7.5, 8)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(-0.8, -0.3, 0.7, 1.2)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(7, 7.5, 8.5, 9)</td>
<td>(5, 5.5, 6.5, 7)</td>
<td>(6, 6.5, 7.5, 8)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(-0.87, -0.37, 0.625, 1.125)</td>
<td>(-0.875, -0.375, 0.625, 1.125)</td>
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<td>(2, 2.5, 3.5, 4)</td>
<td>(1, 1.5, 2.5, 3)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(-0.75, -0.25, 0.75, 1.25)</td>
<td>(-0.83, -0.33, 0.666, 1.166)</td>
<td>(-0.66, -0.16, 0.833, 1.333)</td>
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<td>(1, 1.5, 2.5, 3)</td>
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<tr>
<td>Bldg. height</td>
<td>(-0.85, -0.35, 0.642, 1.142)</td>
<td>(-0.85, -0.35, 0.642, 1.142)</td>
<td>(-0.5, 0, 1, 1.5)</td>
<td>(-0.5, 0, 1, 1.5)</td>
<td>(1, 1, 1.5, 2)</td>
</tr>
<tr>
<td><strong>Simulated Expert k=3</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDVI</td>
<td>(1, 1, 1.5, 2)</td>
<td>(3, 3.5, 4.5, 5)</td>
<td>(1, 1.5, 2.5, 3)</td>
<td>(4, 4.5, 5.5, 6)</td>
<td>(8, 8.5, 9, 9)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(-0.75, -0.25, 0.75, 1.25)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(1, 1.5, 2.5, 3)</td>
<td>(5, 5.5, 6.5, 7)</td>
<td>(7, 7.5, 8.5, 9)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(-0.5, 0, 1, 1.5)</td>
<td>(-0.5, 0, 1, 1.5)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(3, 3.5, 4.5, 5)</td>
<td>(6, 6.5, 7.5, 8)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(-0.8, -0.3, 0.7, 1.2)</td>
<td>(-0.83, -0.33, 0.666, 1.166)</td>
<td>(-0.75, -0.25, 0.75, 1.25)</td>
<td>(1, 1, 1.5, 2)</td>
<td>(6, 6.5, 7.5, 8)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(-0.88, -0.38, 0.611, 1.111)</td>
<td>(-0.87, -0.37, 0.625, 1.125)</td>
<td>(-0.85, -0.35, 0.642, 1.142)</td>
<td>(-0.85, -0.35, 0.642, 1.142)</td>
<td>(1, 1, 1.5, 2)</td>
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</tbody>
</table>
Table C. 3 Simulated matrices converted to Gaussian fuzzy numbers with standard deviation = 1

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<tr>
<th>Simulated Expert k=1</th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
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</thead>
<tbody>
<tr>
<td>NDVI</td>
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<td>(1, 1)</td>
<td>(9, 1)</td>
<td>(9, 1)</td>
<td>(9, 1)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1, 1)</td>
<td>(1, 1)</td>
<td>(4, 1)</td>
<td>(8, 1)</td>
<td>(5, 1)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(1/9, 1)</td>
<td>(1/4, 1)</td>
<td>(1, 1)</td>
<td>(6, 1)</td>
<td>(5, 1)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(1/9, 1)</td>
<td>(1/8, 1)</td>
<td>(1/6, 1)</td>
<td>(1, 1)</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(1/9, 1)</td>
<td>(1/5, 1)</td>
<td>(1/5, 1)</td>
<td>(1, 1)</td>
<td>(1, 1)</td>
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</tbody>
</table>

<table>
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<tr>
<th>Simulated Expert k=2</th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
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</thead>
<tbody>
<tr>
<td>NDVI</td>
<td>(1, 1)</td>
<td>(5, 1)</td>
<td>(8, 1)</td>
<td>(4, 1)</td>
<td>(7, 1)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1/5, 1)</td>
<td>(1, 1)</td>
<td>(8, 1)</td>
<td>(6, 1)</td>
<td>(7, 1)</td>
</tr>
<tr>
<td>Popl. density</td>
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<td>(1/8, 1)</td>
<td>(1, 1)</td>
<td>(3, 1)</td>
<td>(2, 1)</td>
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<td>Tree basal area</td>
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<td>(1/3, 1)</td>
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<td>(2, 1)</td>
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<tr>
<td>Bldg. height</td>
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<td>(1, 1)</td>
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<table>
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<tr>
<th>Simulated Expert k=3</th>
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<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
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</thead>
<tbody>
<tr>
<td>NDVI</td>
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<td>(4, 1)</td>
<td>(2, 1)</td>
<td>(5, 1)</td>
<td>(9, 1)</td>
</tr>
<tr>
<td>Proximity</td>
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<td>(1, 1)</td>
<td>(2, 1)</td>
<td>(6, 1)</td>
<td>(8, 1)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(1/2, 1)</td>
<td>(1/2, 1)</td>
<td>(1, 1)</td>
<td>(4, 1)</td>
<td>(7, 1)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(1/5, 1)</td>
<td>(1/6, 1)</td>
<td>(1/4, 1)</td>
<td>(1, 1)</td>
<td>(7, 1)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(1/9, 1)</td>
<td>(1/8, 1)</td>
<td>(1/7, 1)</td>
<td>(1/7, 1)</td>
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Table C. 4 Aggregated matrix from triangular fuzzy numbers

<table>
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<tr>
<th>Attribute</th>
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<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDVI</td>
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<td>(1/2; 391001/144046; 6)</td>
<td>(1;187133426/35702383;9)</td>
<td>(3; 18130/3211; 9)</td>
<td>(6; 10975/1326; 9)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(1/6; 21661/58797; 2)</td>
<td>(1; 1; 1)</td>
<td>(1; 4; 9)</td>
<td>(5; 130895/19821; 9)</td>
<td>(4; 41536/6349; 9)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(1/9; 8856449/46420925; 1)</td>
<td>(1/9; 1/4; 1)</td>
<td>(1; 1; 1)</td>
<td>(2; 1293367/310893;7)</td>
<td>(1; 228686/55489; 8)</td>
</tr>
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<td>Tree basal area</td>
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<td>(1/9; 19354/127811; 1/5)</td>
<td>(1/7; 286556/1192121; 1/2)</td>
<td>(1; 1; 1)</td>
<td>(1/2; 3219697/1335895;8)</td>
</tr>
<tr>
<td>Bldg. height</td>
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<td>(1/9; 1341/8773; 1/4)</td>
<td>(1/8; 35198/145061; 1)</td>
<td>(1/8; 63122/152133; 2)</td>
<td>(1; 1; 1)</td>
</tr>
</tbody>
</table>

Table C. 5 Aggregated matrix from trapezoidal fuzzy numbers

<table>
<thead>
<tr>
<th>Attribute</th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDVI</td>
<td>(1, 1, 1, 1)</td>
<td>(1, 2.507, 3.336, 6)</td>
<td>(1, 4.573, 5.761, 9)</td>
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<td>(6, 7.773, 8.469, 9)</td>
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<tr>
<td>Proximity</td>
<td>(0, 0.422, 0.923, 2)</td>
<td>(1, 1, 1, 1)</td>
<td>(1, 3.402, 4.573, 9)</td>
<td>(5, 6.099, 7.108, 9)</td>
<td>(4, 6.031, 7.051, 9)</td>
</tr>
<tr>
<td>Popl. density</td>
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<td>(0, 0, 0.777, 1.5)</td>
<td>(1, 1, 1, 1)</td>
<td>(2, 3.637, 4.678, 7)</td>
<td>(1, 3.527, 4.689, 8)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(0, 0, 0.685, 1.25)</td>
<td>(0, 0, 0.652, 1.167)</td>
<td>(0, 0, 0.747, 1.333)</td>
<td>(1, 1, 1, 1)</td>
<td>(1, 2.136, 3.041, 8)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(0, 0, 0.622, 1.143)</td>
<td>(0, 0, 0.655, 1.2)</td>
<td>(0, 0, 0.766, 1.5)</td>
<td>(0, 0, 0.988, 2)</td>
<td>(1, 1, 1, 1)</td>
</tr>
</tbody>
</table>
Table C. 6 Aggregated matrix from Gaussian fuzzy numbers

<table>
<thead>
<tr>
<th>Attribute</th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDVI</td>
<td>(1, 1)</td>
<td>(2.714, 1)</td>
<td>(5.241, 1)</td>
<td>(5.646, 1)</td>
<td>(8.277, 1)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(0.368, 1)</td>
<td>(1, 1)</td>
<td>(4, 1)</td>
<td>(6.604, 1)</td>
<td>(6.542, 1)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(0.191, 1)</td>
<td>(0.250, 1)</td>
<td>(1, 1)</td>
<td>(4.160, 1)</td>
<td>(4.121, 1)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(0.177, 1)</td>
<td>(0.151, 1)</td>
<td>(0.240, 1)</td>
<td>(1, 1)</td>
<td>(2.410, 1)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(0.121, 1)</td>
<td>(0.153, 1)</td>
<td>(0.243, 1)</td>
<td>(0.415, 1)</td>
<td>(1, 1)</td>
</tr>
</tbody>
</table>
Appendix D. Matrix of Gaussian fuzzy numbers after approximation to triangular fuzzy numbers

Weight calculation using fuzzy extent analysis on a Gaussian function require an approximation to a triangular function. Results from approximating Table C.6 to a triangular function are shown in Table D.1

Table D. 1 Matrix of Gaussian fuzzy numbers after approximation to triangular fuzzy numbers

<table>
<thead>
<tr>
<th>Attribute</th>
<th>NDVI</th>
<th>Proximity</th>
<th>Popl. density</th>
<th>Tree basal area</th>
<th>Bldg. height</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDVI</td>
<td>(-1.146, 1, 3.146)</td>
<td>(0.568, 2.714, 4.86)</td>
<td>(3.096, 5.241, 7.387)</td>
<td>(3.5, 5.646, 7.792)</td>
<td>(6.131, 8.277, 10.423)</td>
</tr>
<tr>
<td>Proximity</td>
<td>(-1.778, 0.368, 2.514)</td>
<td>(-1.146, 1, 3.146)</td>
<td>(1.854, 4, 6.146)</td>
<td>(4.458, 6.604, 8.75)</td>
<td>(4.396, 6.542, 8.688)</td>
</tr>
<tr>
<td>Popl. density</td>
<td>(-1.955, 0.191, 2.337)</td>
<td>(-1.896, 0.25, 2.396)</td>
<td>(-1.146, 1, 3.146)</td>
<td>(2.014, 4.16, 6.306)</td>
<td>(1.975, 4.121, 6.267)</td>
</tr>
<tr>
<td>Tree basal area</td>
<td>(-1.969, 0.177, 2.323)</td>
<td>(-1.995, 0.151, 2.297)</td>
<td>(-1.906, 0.24, 2.386)</td>
<td>(-1.146, 1, 3.146)</td>
<td>(0.264, 2.41, 4.556)</td>
</tr>
<tr>
<td>Bldg. height</td>
<td>(-2.025, 0.121, 2.267)</td>
<td>(-1.993, 0.153, 2.299)</td>
<td>(-1.903, 0.243, 2.389)</td>
<td>(-1.731, 0.415, 2.561)</td>
<td>(-1.146, 1, 3.146)</td>
</tr>
</tbody>
</table>
Appendix E. “Urban Greenness Index” calculated by CMA

Replication of ‘Urban Greenness Index’ by quantifying attributes at the Census Metropolitan Area level. Compared to the 500m x 500m resolution, when using CMAs, more variations are observed between the different fuzzy-AHP methods, especially on the central and north-west areas.

Figure E. 1 Resulting ‘Urban Greenness Index’ calculated by Census Metropolitan Area
Appendix F. Weight variation by fuzzy-AHP method

Weight variation obtained from fuzzy geometric means and fuzzy extent analysis using a triangular function are shown in Figure F.1. To calculate such variations, one of the three pairwise comparison matrices was randomly modified 1000 times, and weights were calculated for the 1000 simulations. For all attributes, fuzzy extent analysis yields higher standard deviations.

Figure F.1 Comparison of 1000 simulated weights and their variation. Weights obtained by fuzzy geometric means and fuzzy extent analysis using a triangular membership function. Diagrams represent mean ± 2SD.
Appendix G. R codes for obtaining attribute data

```r
library(foreign) # to open dbf
library(tableone)

# do not use scientific notation:
options(scipen = 999)


height<-read.dbf("bheight_buffer_500m.dbf", as.is=FALSE)
tree<-read.dbf("tree_buffer_500m.dbf", as.is=FALSE)
popl<-read.dbf("popl_buffer_500m.dbf", as.is=FALSE)
ndvi<-read.dbf("final_ndvi_buffer_500m.dbf", as.is=FALSE)
proximity<-read.dbf("proximity_500m.dbf", as.is=FALSE)

# Grid centroids to identify each buffer
grids<-read.dbf("grid_centroids_500m.dbf", as.is=FALSE)

# Clean datasets to only needed columns
proximity<-proximity[,c(1,4)]
height<-height[,c(11,10,6,7,8)]
popl<-popl[,c(12,3,4,5,7,11,8,9)]
tree<-tree[,c(10,9,6,7)]
ndvi<-ndvi[,c(2,4)]

# Calculating mean and median of height and tree per buffer
height_mean<-aggregate(height[,3:4], by=list(height$Location_n),  FUN=mean, na.rm=TRUE)
height_median<-aggregate(height[,3:4], by=list(height$Location_n),  FUN=median, na.rm=TRUE)

tree_mean<-aggregate(tree[,3], by=list(tree$location_n),  FUN=mean, na.rm=TRUE)
colnames(tree_mean)<-c("Group.1", "tree_mean")
tree_median<-aggregate(tree[,3], by=list(tree$location_n),  FUN=median, na.rm=TRUE)
colnames(tree_median)<-c("Group.1", "tree_median")

# Calculating mean and median of population density per grid
# Population should account for area of each polygon:
popl$area_prop<-popl$Shape_Area/250000 # Proportion of the total grid

# Check which grids are incomplete because are located in border of map, population on them will not be weighted.
complete_grid<-aggregate(popl[,9], by=list(popl$name),  FUN=sum, na.rm=TRUE)
complete_grid$round_area<-round(complete_grid$x, 2) # grids without a 1.00 are in border of map
popl<-merge(popl, complete_grid[,c(1,3)], by.x="name", by.y="Group.1", all=TRUE)
# merge complete grid and popl to indicate which grids are in borders
popl$popl_density<-ifelse(popl$round_area==1.00, popl$Popl_densi*popl$area_prop, popl$Popl_densi)

# Calculating popl. mean and median per grid
popl_sum<-aggregate(popl[,11], by=list(popl$name),  FUN=sum, na.rm=TRUE)
colnames(popl_sum)<-c("Group.1", "popl_sum")

# Calculating popl. mean and median per grid
popl_mean<-aggregate(popl[,11], by=list(popl$name),  FUN=mean, na.rm=TRUE)
colnames(popl_mean)<-c("Group.1", "popl_mean")

# Calculating popl. mean and median per grid
popl_median<-aggregate(popl[,11], by=list(popl$name),  FUN=median, na.rm=TRUE)
colnames(popl_median)<-c("Group.1", "popl_median")
```
# Building a final dataset with each attribute mean and median
height_final <- merge(height_mean, height_median, by="Group.1", suffixes=c(".mean", ".median"))
trees_final <- merge(tree_mean, tree_median, by="Group.1", suffixes=c(".mean", ".median"))
popl_final <- merge(popl_mean, popl_sum, by="Group.1", suffixes=c(".mean", ".sum"))

# Making the final attribute dataset with all mean and medians
attribute_data <- merge(height_final, trees_final, by="Group.1", all=TRUE)
attribute_data <- merge(attribute_data, ndvi, by.x="Group.1", by.y="Location_n", all=TRUE)
attribute_data <- merge(attribute_data, proximity, by.x="Group.1", by.y="OBJECTID", all=TRUE)
attribute_data <- merge(attribute_data, popl_final, by="Group.1")
colnames(attribute_data) <- c("Buffer", "mx_hgt_mean", "avg_hgt_mean", "mx_ht_med","avg_ht_med", "tree_mean", "tree_median", "ndvi_mean", "proximity", "popl_mean", "popl_sum")
attribute_data[is.na(attribute_data)] <- 0

# Merge with grids centroid to have coordinates and perform clustering analysis
attribute_data <- merge(attribute_data, grids[,c(3:5)], by.x="Buffer", by.y="Name")
# Save file to then perform clustering
# write.csv(attribute_data, file="Attribute raw values per 500m grid.csv", row.names=FALSE)

# Percentiles ----
# Open file just created
attribute_data <- read.csv("Attribute raw values per 500m grid.csv")
# Normalizing values
attributes.zscores <- data.frame(attribute_data$Buffer,lapply(attribute_data[,2:11], FUN=scale), attribute_data$longi, attribute_data$lati)
summary(attributes.zscores[,2:11])
attributes.percentiles <- data.frame(attributes.zscores$attribute_data.Buffer, lapply(attributes.zscores[,2:11], FUN=pnorm), attributes.zscores$attribute_data.longi, attributes.zscores$attribute_data.longi)
summary(attributes.percentiles[,2:11])
colnames(attributes.percentiles)[c(1,12,13)] <- c("Buffer", "long", "lat")

# Merging raw and percentile attribute datasets
final.attributes <- merge(attribute_data, attributes.percentiles[,1:11], by="Buffer", suffixes=c("_raw", ",_percentile"))
# write.csv(final.attributes, file="raw_perc_attributes_grid500m.csv", row.names=FALSE)

# Clustering ----
setwd("C:/Users/User/Documents/thesis/2018/final grid files/Clustering by grids")
popl_lowcl <- read.dbf("low_30p_population.gis.dbf", as.is=TRUE)
attributes <- read.csv("raw_perc_attributes_grid500m.csv")

# Delete not significant clusters
popl_lowcl$CLUSTER <- as.factor(popl_lowcl$CLUSTER)
popl_lowcl <- popl_lowcl|(pobl_lowcl$CLUSTER==9 | pobl_lowcl$CLUSTER==10 | pobl_lowcl$CLUSTER==11 )

# Remove repeated grids
pobl_lowcl <- unique(pobl_lowcl[,c(1,7)])
# Merge cluster number with attribute file
attributes <- merge(attributes, pobl_lowcl, by.x="Buffer", by.y="LOC_ID", all.x=TRUE)
summary(attributes$LOC_OBS)
write.csv(attributes, file="final_attribute_data_Aug16.csv", row.names=FALSE)

#Table one
#Open final all attributes dataset just created above
attributes<-read.csv("raw_perc_attributes_grid500m.csv")

#attributes$CLUSTER<-as.factor(attributes$CLUSTER)

names(attributes)

#Raw values
cont.raw<-c("avg_hgt_mean_raw","avg_ht_med_raw","tree_mean_raw","tree_median_raw",
           "ndvi_mean_raw","proximity_raw","popl_sum_raw")
raw.table <- CreateTableOne(vars = cont.raw, data = attributes)

#Percentile values
cont.percentile<-c("avg_hgt_mean_percentile","avg_ht_med_percentile","tree_mean_percentile","tree_median_percentile",
                   "ndvi_mean_percentile","proximity_percentile","popl_sum_percentile")
cat.percentile<-"CLUSTER"
percentile.table <- CreateTableOne(vars = cont.percentile, data = attributes)

#Results
raw.table
percentile.table

#Correlations after multiplying by weights
library(corrplot)
library(RColorBrewer)

colnames(attributes)[c(17,19,20,21,23)]<-c("Building Height","Tree basal area","NDVI level","Proximity green spaces","Population density")

#Raw values
m1<-cor(attributes[,c(9,11,13,14,15)])

#percentiles
m2<-cor(attributes[,c(17,19,20,21,23)])

corrplot(m1, method="color", type="upper", col=brewer.pal(n=8, name="RdBu"),tl.col="black", tl.srt=45)
corrplot(m1, method="number", type="upper", col=brewer.pal(n=8, name="RdYlBu"),tl.col="black", tl.srt=45)

#This one!
corrplot(m2, method="color", type="upper", col=brewer.pal(n=8, name="RdBu"),tl.col="black", tl.srt=45)
cor.test(attributes$`Building Height`, attributes$`Population density`)
cor.test(attributes$ndvi_level.median.raw, attributes$basal_area.median.raw)
cor.test(attributes$`Building Height`, attributes$`NDVI level`)
cor.test(attributes$`Population density`, attributes$`NDVI level`)
cor.test(attributes$`NDVI level`, attributes$`Tree basal area`)
cor.test(attributes$`NDVI level`, attributes$`Proximity green spaces`)

Appendix H. R codes for AHP and fuzzy-AHP

#FAHP Process shown on Gaussian fuzzy numbers
library(FuzzyAHP)
library(rgdal)
library(psych)

#Creating fuzzy matrices
#Function to create random fuzzy matrices
fuzzy_randommatrix<-function(rows, cols) {
  matrixname<-round(matrix(runif(25,1,9),rows,cols))
  matrixname[lower.tri(matrixname)]<-NA
  matrixname<-pairwiseComparisonMatrix(matrixname)
}

#Run function
set.seed(300)
m1.f<-fuzzy_randommatrix(5,5)
set.seed(302);m2.f<-fuzzy_randommatrix(5,5)
set.seed(2000)
m3.f<-fuzzy_randommatrix(5,5)
print(m1.f); print(m2.f); print(m3.f)

#Aggregating Gaussian fuzzy matrices using Geometric mean.
m1<-m1.f@values
m2<-m2.f@values
m3<-m3.f@values

#Dataframe to store values
all.values<-data.frame(matrix(NA, nrow = 25, ncol = 3)); colnames(all.values)<-c("m1","m2","m3")

#Extracting values from each matrix #Its moving per columns
i = 1
for (j in 1:5){
  for (k in 1:5){
    all.values$m1[i]<-m1[j,k]
    all.values$m2[i]<-m2[j,k]
    all.values$m3[i]<-m3[j,k]
    i <- i + 1
  }
}

#Calculating geometric means
all.values$row.gmean<-(all.values$m1*all.values$m2*all.values$m3)^(1/3)

##Building matrix with modal values
overall.gaussian<-matrix(all.values$row.gmean,5,5, byrow = TRUE)
overall.gaussian1<-overall.gaussian
overall.gaussian1<-round(overall.gaussian1,3)
#Convert to fuzzy gaussian with sd=1
for (j in 1:5){
  for (k in 1:5){
    overall.gaussian1[j,k]<-paste("(",overall.gaussian1[j,k],", 1)")
  }
}
#Calculating $l$ (left bound)

```r
# l = mu - sd(sqrt(-ln(alpha)))
alpha <- 0.01
sd <- 1
all.values$left <- all.values$row.gmean - (sd * sqrt(-log(alpha)))
all.values$right <- all.values$row.gmean + (sd * sqrt(-log(alpha)))
```

#Storing the left values as min values, gmean as modal, and right values as max values (cause are treated as triangular fuzzy numbers now)

```r
goverall.min <- matrix(all.values$left, 5, 5, byrow = TRUE); goverall.min <- round(goverall.min, 3)
goverall.modal <- matrix(all.values$row.gmean, 5, 5, byrow = TRUE); goverall.modal <- round(goverall.modal, 3)
goverall.max <- matrix(all.values$right, 5, 5, byrow = TRUE); goverall.max <- round(goverall.max, 3)
```

#Store in list

```r
gfuzzy.mtx <- list(goverall.min, goverall.modal, goverall.max)
```

#Store as fuzzy matrix overwriting in a fuzzy class object.

```r
final_gaussian <- fuzzyPairwiseComparisonMatrix(m1.f)
final_gaussian@fnMin <- goverall.min;
final_gaussian@fnModal <- goverall.modal;
final_gaussian@fnMax <- goverall.max
```

#Reorganize matrix

```r
pretty.gaussian <- matrix(NA, 5, 5)
for (j in 1:5){
  for (k in 1:5){
    pretty.gaussian[j,k] <- paste("\(,final_gaussian@fnMin[j,k],\),",final_gaussian@fnModal[j,k],\"",final_gaussian@fnMax[j,k],\"\)
  }
}
```

#matrix saved in: C:\Users\User\Documents\thesis\2018\results_for_index\FAHP Matrices\FAHP matrices.xlsx

#In order to calculate weights using fuzzyAHP package function, need to set the diagonal of all min, modal and max to 1:

```r
v1diagonal = c(1,1,1)
for (i in 1:5){
  for (j in 1:5){
    if(i==j){
      final_gaussian@fnMin[i,j] = v1diagonal[1]
      final_gaussian@fnModal[i,j] = v1diagonal[2]
      final_gaussian@fnMax[i,j] = v1diagonal[3]
    }
  }
}
```

#Removing values smaller than 0

```r
for (i in 1:5){
  for (j in 1:5){
    intensity = final_gaussian@fnMin[i, j]
    if(intensity<0){
      final_gaussian@fnMin[i,j] = 0
    }
  }
}
```

##Calculate weights by arithmetic means using Chang’s method----

```r
fhap.gau.weights <- calculateWeights(final_gaussian)
```

fhap.gau.weights
gaus_w1 <- c(fahp.gau.weights@fnMin[1], fahp.gau.weights@fnModal[1], fahp.gau.weights@fnMax[1])

# Modal value dominancy method to defuzzify


# PLOTTING g.mean w.

for (i in 1:5) {
  delta1[i] <- (fahp.gau.weights@fnModal[i] - fahp.gau.weights@fnMin[i])/10
  delta2[i] <- (fahp.gau.weights@fnMax[i] - fahp.gau.weights@fnModal[i])/10

  geom.w.gau[,i] <- c(seq(fahp.gau.weights@fnMin[i], fahp.gau.weights@fnModal[i], delta1[i]),
                     seq((fahp.gau.weights@fnModal[i]+delta2[i]),fahp.gau.weights@fnMax[i], delta2[i]))
}

geom.w.gau.plot <- gather(geom.w.gau, key=Attribute, value=`Attribute Weight`, 1:5, factor_key=TRUE)

# Weights by Extent analysis ----

# Removing negative numbers and making main diagonal 1 in all min, modal and max matrices:

g.fuzzy.mtx[[1]] <- final_gaussian@fnMin; g.fuzzy.mtx[[2]] <- final_gaussian@fnModal; g.fuzzy.mtx[[3]] <- final_gaussian@fnMax

# 1. Sum across the rows of g.fuzzy.mtx

rowSums(g.fuzzy.mtx[[1]]), rowSums(g.fuzzy.mtx[[2]]), rowSums(g.fuzzy.mtx[[3]]))

# 2. Sum all l's, all m's and all u's

g.sum.sums.a <- c(colSums(gsum.a))

# 3. rearrange as u, m and l instead of l,m,u

gsum.sum.a <- c(gsum.sum.a[3], gsum.sum.a[2], gsum.sum.a[1])

# 4. Multiply values as in formula 7 (l*1/u, etc...)

# to get S,S2,S3,S4, and S5:

g.s <- data.frame(matrix(NA, nrow = 5, ncol = 3)); colnames(g.s) <- c("1","m","u")

j=1
for (i in 1:nrow(g.s)) {
    g.s$l[j] <- gsum.a[i,1]/gsumsum.a[1]
    g.s$m[j] <- gsum.a[i,2]/gsumsum.a[2]
    g.s$u[j] <- gsum.a[i,3]/gsumsum.a[3]
    j = j + 1
}

# 5 Return all S back to gaussian by formula 38 on Hefyn H et al. or equations 47 and 48 from my thesis (they are the same)
g.s$sd.l <- (g.s$m - g.s$l) / sqrt(-log(alpha))
g.s$sd.u <- (g.s$u - g.s$m) / sqrt(-log(alpha))

# rearrange g.s:
g.s <- g.s[c(4,2,5)]

# 6. Calculate degrees of possibility of S1 > S2, etc...
S1S2 <- ifelse(g.s$m[1] >= g.s$m[2], 1, exp(-((g.s$m[1] - g.s$m[2])^2) / (g.s$sd.u[1]^2 + g.s$sd.l[2]^2)))
S1S3 <- ifelse(g.s$m[1] >= g.s$m[3], 1, exp(-((g.s$m[1] - g.s$m[3])^2) / (g.s$sd.u[1]^2 + g.s$sd.l[3]^2)))
S1S4 <- ifelse(g.s$m[1] >= g.s$m[4], 1, exp(-((g.s$m[1] - g.s$m[4])^2) / (g.s$sd.u[1]^2 + g.s$sd.l[4]^2)))
S1S5 <- ifelse(g.s$m[1] >= g.s$m[5], 1, exp(-((g.s$m[1] - g.s$m[5])^2) / (g.s$sd.u[1]^2 + g.s$sd.l[5]^2)))

# V(S2 > S1)
S2S1 <- ifelse(g.s$m[2] >= g.s$m[1], 1, exp(-((g.s$m[2] - g.s$m[1])^2) / (g.s$sd.u[2]^2 + g.s$sd.l[1]^2)))
S2S3 <- ifelse(g.s$m[2] >= g.s$m[3], 1, exp(-((g.s$m[2] - g.s$m[3])^2) / (g.s$sd.u[2]^2 + g.s$sd.l[3]^2)))
S2S4 <- ifelse(g.s$m[2] >= g.s$m[4], 1, exp(-((g.s$m[2] - g.s$m[4])^2) / (g.s$sd.u[2]^2 + g.s$sd.l[4]^2)))
S2S5 <- ifelse(g.s$m[2] >= g.s$m[5], 1, exp(-((g.s$m[2] - g.s$m[5])^2) / (g.s$sd.u[2]^2 + g.s$sd.l[5]^2)))

# V(S3 > S1)
S3S1 <- ifelse(g.s$m[3] >= g.s$m[1], 1, exp(-((g.s$m[3] - g.s$m[1])^2) / (g.s$sd.u[3]^2 + g.s$sd.l[1]^2)))
S3S2 <- ifelse(g.s$m[3] >= g.s$m[2], 1, exp(-((g.s$m[3] - g.s$m[2])^2) / (g.s$sd.u[3]^2 + g.s$sd.l[2]^2)))
S3S4 <- ifelse(g.s$m[3] >= g.s$m[4], 1, exp(-((g.s$m[3] - g.s$m[4])^2) / (g.s$sd.u[3]^2 + g.s$sd.l[4]^2)))
S3S5 <- ifelse(g.s$m[3] >= g.s$m[5], 1, exp(-((g.s$m[3] - g.s$m[5])^2) / (g.s$sd.u[3]^2 + g.s$sd.l[5]^2)))

# V(S4 > S1)
S4S1 <- ifelse(g.s$m[4] >= g.s$m[1], 1, exp(-((g.s$m[4] - g.s$m[1])^2) / (g.s$sd.u[4]^2 + g.s$sd.l[1]^2)))
S4S2 <- ifelse(g.s$m[4] >= g.s$m[2], 1, exp(-((g.s$m[4] - g.s$m[2])^2) / (g.s$sd.u[4]^2 + g.s$sd.l[2]^2)))
S4S3 <- ifelse(g.s$m[4] >= g.s$m[3], 1, exp(-((g.s$m[4] - g.s$m[3])^2) / (g.s$sd.u[4]^2 + g.s$sd.l[3]^2)))
S4S5 <- ifelse(g.s$m[4] >= g.s$m[5], 1, exp(-((g.s$m[4] - g.s$m[5])^2) / (g.s$sd.u[4]^2 + g.s$sd.l[5]^2)))

# V(S5 > S1)
S5S1 <- ifelse(g.s$m[5] >= g.s$m[1], 1, exp(-((g.s$m[5] - g.s$m[1])^2) / (g.s$sd.u[5]^2 + g.s$sd.l[1]^2)))
S5S2 <- ifelse(g.s$m[5] >= g.s$m[2], 1, exp(-((g.s$m[5] - g.s$m[2])^2) / (g.s$sd.u[5]^2 + g.s$sd.l[2]^2)))
S5S3 <- ifelse(g.s$m[5] >= g.s$m[3], 1, exp(-((g.s$m[5] - g.s$m[3])^2) / (g.s$sd.u[5]^2 + g.s$sd.l[3]^2)))
S5S4 <- ifelse(g.s$m[5] >= g.s$m[4], 1, exp(-((g.s$m[5] - g.s$m[4])^2) / (g.s$sd.u[5]^2 + g.s$sd.l[4]^2)))

# 7. Choose the minimum for each comparison
w1 <- min(S1S2, S1S3, S1S4, S1S5)
w2 <- min(S2S1, S2S3, S2S4, S2S5)
w3 <- min(S3S1, S3S2, S3S4, S3S5)
w4 <- min(S4S1, S4S2, S4S3, S4S5)
w5 <- min(S5S1, S5S2, S5S3, S5S4)

# 8. Normalize weights by geometric means
weights <- c(g.w1, g.w2, g.w3, g.w4, g.w5)
total <- g.w1 + g.w2 + g.w3 + g.w4 + g.w5
chang.weights <- c(g.w1/g.total, g.w2/g.total, g.w3/g.total, g.w4/g.total, g.w5/g.total)
chang.weights
# Also normalize by setting different alpha cut values using fuzzy weights in s

gau.alpha.w <- g.s

gau.alpha.w$sdl.alpha <- gau.alpha.w$m - gau.alpha.w$sd.l
gau.alpha.w$sdu.alpha <- abs(gau.alpha.w$sd.u - gau.alpha.w$m)

# Weights also defuzzified by beta distribution on discussion

tri.w.beta <- list(
    (tri_w$tri_w1[1] + tri_w$tri_w1[2] + tri_w$tri_w1[3]) / 3,
    (tri_w$tri_w2[1] + tri_w$tri_w2[2] + tri_w$tri_w2[3]) / 3,
    (tri_w$tri_w3[1] + tri_w$tri_w3[2] + tri_w$tri_w3[3]) / 3,
    (tri_w$tri_w4[1] + tri_w$tri_w4[2] + tri_w$tri_w4[3]) / 3,
    (tri_w$tri_w5[1] + tri_w$tri_w5[2] + tri_w$tri_w5[3]) / 3)

# names(def.tri.w) <- c("def.tri_w1","def.tri_w2","def.tri_w3","def.tri_w4","def.tri_w5")
Appendix I. R codes to generate “Urban Greenness Indices”

library(rgdal)

# Open grids as shapefile to include here all the summarized info at the end
setwd("C:/Users/User/Documents/thesis/2018")
grids <- readOGR(layer="500mgrid_projected", dsn="final grid files")

#cmas <- readOGR(layer="TO CMA polygons", dsn="Only Toronto CMA map")
#cmas@data <- cmas@data[c(1,2,10,11)]

# Open final all attributes dataset just created above
attributes <- read.csv("final_attribute_data_Aug16.csv")

# Leaving only columns that will be used:
# Adding values for low population vs high population cluster (Low popl = 1 to give higher weight)
attributes$clusterforindex <- ifelse(is.na(attributes$LOC_OBS), 0.5, 1)

# Calculated weights
classic <- list(0.48, 0.3, 0.125, 0.156, 0.037); names(classic) <- c("ndvi", "prox", "popl", "tree", "height")
gmean.tri <- list(0.44, 0.34, 0.17, 0.07, 0.05); names(gmean.tri) <- c("ndvi", "prox", "popl", "tree", "height")
gmean.trap <- list(0.38, 0.28, 0.16, 0.10, 0.08); names(gmean.trap) <- c("ndvi", "prox", "popl", "tree", "height")
gmean.gaus <- list(0.46, 0.31, 0.15, 0.09, 0.07); names(gmean.gaus) <- c("ndvi", "prox", "popl", "tree", "height")
extent.gm.tri <- list(0.34, 0.29, 0.21, 0.12, 0.02); names(ent.gm.tri) <- c("ndvi", "prox", "popl", "tree", "height")
extent.gm.gaus <- list(0.35, 0.33, 0.19, 0.07, 0.04); names(ent.gm.gaus) <- c("ndvi", "prox", "popl", "tree", "height")
extent.a.tri <- list(0.4, 0.32, 0.17, 0.06, 0.03); names(ent.a.tri) <- c("ndvi", "prox", "popl", "tree", "height")
extent.a.gaus <- list(0.4, 0.32, 0.17, 0.07, 0.03); names(ent.a.gaus) <- c("ndvi", "prox", "popl", "tree", "height")

# Create dataframe with all the indices
urban.index <- attributes[,c(1,17,19,20,21,25)]

# Classic AHP
urban.index$classic.AHP <- ((attributes$ndvi_mean_percentile * classic[[1]]) +
                             (attributes$proximity_percentile * classic[[2]]) +
                             (attributes$clusterforindex * classic[[3]]) +
                             (attributes$tree_median_percentile * classic[[4]]) +
                             (attributes$avg_ht_med_percentile * classic[[5]]))

# Geometric means
urban.index$gmean.tri <- ((attributes$ndvi_mean_percentile * gmean.tri[[1]]) +
                           (attributes$proximity_percentile * gmean.tri[[2]]) +
                           (attributes$clusterforindex * gmean.tri[[3]]) +
                           (attributes$tree_median_percentile * gmean.tri[[4]]) +
                           (attributes$avg_ht_med_percentile * gmean.tri[[5]]))

urban.index$gmean.trap <- ((attributes$ndvi_mean_percentile * gmean.trap[[1]]) +
                         (attributes$proximity_percentile * gmean.trap[[2]]) +
                         (attributes$clusterforindex * gmean.trap[[3]]) +
                         (attributes$tree_median_percentile * gmean.trap[[4]]) +
                         (attributes$avg_ht_med_percentile * gmean.trap[[5]]))

urban.index$gmean.gaus <- ((attributes$ndvi_mean_percentile * gmean.gaus[[1]]) +
                        (attributes$proximity_percentile * gmean.gaus[[2]]) +
                        (attributes$clusterforindex * gmean.gaus[[3]]) +
                        (attributes$tree_median_percentile * gmean.gaus[[4]]) +
                        (attributes$avg_ht_med_percentile * gmean.gaus[[5]]))
(attributes$tree_median_percentile * gmean.gaus[[4]]) +
(attributes$avg_ht_med_percentile * gmean.gaus[[5]])

#Extent analysis + geometric means
urban.index$extentgm.tri <- ((attributes$ndvi_mean_percentile * extentgm.tri[[1]]) +
(attributes$proximity_percentile * extentgm.tri[[2]]) + (attributes$clusterforindex * extentgm.tri[[3]]) +
(attributes$tree_median_percentile * extentgm.tri[[4]])
+ (attributes$avg_ht_med_percentile * extentgm.tri[[5]])

urban.index$extentgm.gaus <- ((attributes$ndvi_mean_percentile * extentgm.gaus[[1]]) +
(attributes$proximity_percentile * extentgm.gaus[[2]]) + (attributes$clusterforindex * extentgm.gaus[[3]]) +
(attributes$tree_median_percentile * extentgm.gaus[[4]])
+ (attributes$avg_ht_med_percentile * extentgm.gaus[[5]])

#Extent analysis + alpha cuts
urban.index$extenta.tri <- ((attributes$ndvi_mean_percentile * extenta.tri[[1]]) +
(attributes$proximity_percentile * extenta.tri[[2]]) + (attributes$clusterforindex * extenta.tri[[3]]) +
(attributes$tree_median_percentile * extenta.tri[[4]])
+ (attributes$avg_ht_med_percentile * extenta.tri[[5]])

urban.index$extenta.gaus <- ((attributes$ndvi_mean_percentile * extenta.gaus[[1]]) +
(attributes$proximity_percentile * extenta.gaus[[2]]) + (attributes$clusterforindex * extenta.gaus[[3]]) +
(attributes$tree_median_percentile * extenta.gaus[[4]])
+ (attributes$avg_ht_med_percentile * extenta.gaus[[5]])

summary(urban.index[,8:14])

#Save index dataset
getwd()
write.csv(urban.index, file="urban_index_by_grid_final.csv", row.names=FALSE)

#Merge index value to each grid
urban_greenness_grid_shapefile <- merge(grids, urban.index, by.x="Id", by.y="Buffer")

#Save as shapefile to open in arcmap
setwd("C:/Users/User/Documents/thesis/2018")
writeOGR(urban_greenness_grid_shapefile, dsn="final grid files", layer="Urban Greenness Shapefile by grid Aug 16", driver="ESRI Shapefile")