TWO ESSAYS ON EMPIRICAL ASSET PRICING

by

Shengzhe Tang

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of the Joseph L. Rotman School of Management
University of Toronto

© Copyright 2018 by Shengzhe Tang
Abstract

TWO ESSAYS ON EMPIRICAL ASSET PRICING

Shengzhe Tang
Doctor of Philosophy
Graduate Department of the Joseph L. Rotman School of Management
University of Toronto
2018

The main theme of my doctoral research is empirical asset pricing. The first chapter of this thesis focuses on the time series predictability of stock returns. I provide improved methods for estimating a predictive regression model system where the future aggregate stock return is regressed on the current value of a single predictor, and the innovations are correlated normal random variables. I propose improved estimators of the predictive slope which practically eliminate finite-sample biases and achieve small mean squared errors. I develop fast computing methods for evaluating the probability distribution of the OLS predictive slope, which enables an exact performance measurement of these estimators. This also facilitates a comparison of several prominent tests of return predictability with respect to their actual test sizes. The usefulness of these econometric methods is illustrated using U.S. equity data.

The second chapter is a joint work with Ruslan Goyenko and Chayawat Ornthanalai. We study the determinants of option illiquidity measured by relative bid-ask spreads of intraday option transactions on S&P 500 firms over an extended period. We find that market makers’ hedging costs significantly impact option illiquidity with the future rebalancing cost dominating the initial delta-hedging cost. Inventory risk and adverse selection also contribute to the cross-sectional variation in option illiquidity, with the latter effect intensifying around information events. We find that option-induced order flows predict their underlying stock returns only when option illiquidity simultaneously increases. This suggests that shocks to option illiquidity help to distinguish abnormal order flows that contain private information from those induced by liquidity trading. We show that a simple stock portfolio strategy contingent on option illiquidity shocks yields a risk-adjusted return of 16.35% per year.
Acknowledgements

I would like to express my sincere thanks to my supervisor Raymond Kan, my dissertation committee members, Chayawat Ornthanalai, and Liyan Yang and my co-author Ruslan Goyenko for their guidance and comments. My special thanks go to Raymond Kan as I am immensely indebted to him for his continuous mentorship and encouragement during my doctoral studies at Rotman School of Management, University of Toronto. My gratitude is also extended to the other Rotman Finance faculty members and Ph.D. colleagues. I gratefully acknowledge the financial support from Rotman School of Management, Society of Actuaries Hickman Scholars Program, Ontario Graduate Scholarship, and Canadian Derivatives Exchange. Finally, I would like to give my profound thanks to my parents for their unwavering support and encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them. Thank you.
Contents

1 Predictive Regressions: Estimation and Inference Methods .............................. 1
  1.1 Introduction ......................................................................................... 1
  1.2 Predictive Regression and Improved Slope Estimators ......................... 5
    1.2.1 Bias of $\hat{\phi}$ and Some Approximations .......................... 6
    1.2.2 MSE of $\hat{\beta}^c$ ................................................................. 12
  1.3 Tests of Return Predictability ............................................................. 14
    1.3.1 Low-bias Standard Error Estimation ......................................... 14
    1.3.2 Stambaugh’s $\hat{\beta}$-test ....................................................... 15
  1.4 The Sizes of Return Predictability Tests .............................................. 18
    1.4.1 Plug-in $\hat{\beta}$-test ................................................................. 18
    1.4.2 A Comparison of Tests of Predictability ................................... 22
  1.5 Empirical Illustration ........................................................................... 26
  1.6 Conclusion ........................................................................................... 30

2 Option Illiquidity: Determinants and Implications for Stock Returns .......... 46
  2.1 Introduction ......................................................................................... 46
  2.2 Empirical Predictions and Related Literature ...................................... 50
    2.2.1 Determinants of Option Illiquidity ......................................... 50
    2.2.2 Option Illiquidity and Informed Trading .................................. 53
  2.3 Data and Variable Constructions ......................................................... 56
    2.3.1 Data and Sample Selection ....................................................... 56
2.3.2 Variable Constructions ........................................... 58
2.4 Determinants of option illiquidity ................................ 61
   2.4.1 Sample Descriptive Statistics ................................. 61
   2.4.2 Hedging Costs, Inventory and Private Information .......... 63
2.5 Private Information .................................................. 65
   2.5.1 The Impact of Information Events ............................. 65
   2.5.2 Option Illiquidity and Stock Returns ......................... 69
   2.5.3 Subperiods .......................................................... 73
2.6 Conclusion ............................................................. 75

Appendices ................................................................. 94

Bibliography .............................................................. 111
List of Tables

1.1 Relative biases and RMSEs of corrected estimators of the AR(1) parameter $\phi$ 32
1.2 Stock return predictive regression results, 1926 – 2016 ................. 34
1.3 Stock return predictive regression results, 1926.01 – 1970.06 ............. 35
1.4 Stock return predictive regression results, 1970.07 – 2016.12 ............. 36

2.1 Summary statistics .......................................................... 77
2.2 Cross correlation matrix of the main variables ............................. 78
2.3 Determinants of ILO ............................................................ 79
2.4 Determinants of change in ILO ............................................. 81
2.5 Option and stock illiquidity around earnings announcement .............. 83
2.6 Portfolio strategies: Order flow and changes in option illiquidity ........ 85
2.7 Portfolio strategies: Order flow and changes in option illiquidity ........ 86
2.8 Portfolio alphas from daily trading strategies: Subperiod analysis ......... 87
2.9 2009–2013 Portfolio strategies: Order flow and changes in ILO ........ 88
C1 Option illiquidity and the magnitude of future stock returns .............. 107
List of Figures

1.1 Bias of $\hat{\phi}$ and its approximations ............................................. 37
1.2 Evaluation of $p$-values ........................................................................... 38
1.3 Actual size of Stambaugh’s nominal 5% left-tail one-sided $\hat{\beta}$-test .... 39
1.4 Actual size of Stambaugh’s nominal 5% right-tail one-sided $\hat{\beta}$-test .... 40
1.5 Actual size of Stambaugh’s nominal 5% two-sided $\hat{\beta}$-test ................. 41
1.6 Cumulative distribution function of $\hat{\beta}$ .................................................. 42
1.7 Actual sizes of nominal 5% left-tail one-sided tests ..................................... 43
1.8 Actual sizes of nominal 5% right-tail one-sided tests .................................... 44
1.9 Actual sizes of nominal 5% two-sided tests ................................................... 45
2.1 Aggregate option illiquidity (ILO) ................................................................. 89
2.2 Option effective spreads v.s. quoted spreads by option type ......................... 90
2.3 Option volume ............................................................................................ 91
2.4 Abnormal volume around earnings announcements ..................................... 92
2.5 Abnormal option and stock illiquidity around earnings announcements ...... 93
List of Appendices

Appendix A ................................................................. 94
Appendix B ................................................................. 100
Appendix C ................................................................. 107
Appendix D ................................................................. 109
Chapter 1

Predictive Regressions: Estimation and Inference Methods

1.1 Introduction

There is a vast literature on whether future aggregate stock returns can be predicted by the current observation of some variables. Running an ordinary least squares (OLS) regression of the stock return on a lagged predictor, more often than not, yields $t$-statistic greater than two and sometimes greater than three. However, researchers find that this constitutes insufficient statistical evidence for establishing return predictability. Indeed, the first-order asymptotic inference theory provides a poor approximation when dealing with finite samples if the predictor is persistent and its innovation is highly correlated with returns (Mankiw and Shapiro, 1986; Stambaugh, 1986).

In this paper, I study the return predictive model system formalized by Stambaugh (1999) where market return is regressed on the lagged value of a predictor, which has a first-order autoregressive dynamic with errors that are contemporaneously correlated with the errors of the return process. I present improved methods for estimating the predictive regression and propose a new testing rule for conducting hypothesis tests of return predictability.

I start with a comprehensive review of the existing predictive slope estimators. Building on the augmented regression method developed by Amihud and Hurvich (2004), I construct new predictive slope estimators that are induced by bias-corrected autocorrelation coefficient estimates. These estimators are practically free of finite-sample biases and have low mean squared errors (MSEs). Measuring the exact performances of these predictive slope estimators requires evaluating the probability distribution and the first
two moments of the ratio of two quadratic forms in normal random variables. I develop improved numerical methods to attain fast computations of the required distribution and density functions, enabling exact evaluations of these performance measures. In contrast with simulation-based performance evaluations in existing studies, the exact performance measures in this paper yield definitive conclusions on the key properties of estimators under various sample sizes and across a wide range of parameter values. I show that when the predictor is not highly autocorrelated, the simple predictive slope estimator induced by the adjusted autocorrelation coefficient developed by Kothari and Shanken (1997) performs the best with a negligible bias and the smallest MSE. With a highly persistent predictor, a newly-constructed slope estimator delivers the best results by capturing the slow-decaying feature of some high-order terms that are missing in existing slope estimators.

Next, I compute the actual rejection rate under the null hypothesis that the predictive slope is zero for several existing tests of return predictability using different assumptions of system parameters and sample sizes. I consider the corrected $t$-statistic based on Amihud and Hurvich’s augmented regression approach of estimating the predictive slope and its corresponding standard error. The analysis also covers a class of significance tests ($\hat{\beta}$-tests) based on the finite-sample distribution of OLS predictive slope, which is first derived by Stambaugh (1999). Extending Stambaugh’s original right-tail one-sided test where $p$-values are evaluated using plug-in OLS parameter estimates, I develop variations of this $\hat{\beta}$-test. In particular, I examine the impact of using various predictive slope estimators to compute the $p$-value on the actual rejection rate under the null and also consider left-tail one-sided and two-sided tests.

This paper introduces a significant methodological improvement on the implementation of the $\hat{\beta}$-test. Stambaugh’s computing method involves solving for eigenvalues of matrix functions which determine the integrand in a numerical integration approach. This method proves to be progressively time-consuming as the sample size grows because the computer runtime for the brute-force eigenvalue-solving is to the cubic order of the sample size. I utilize analytical tractability in an alternative representation of the $p$-value associated with the $\hat{\beta}$-test and achieve a significant improvement in computational efficiency. Equipped with this powerful tool, I conduct a simulation-based comparison of the actual test sizes of the $\hat{\beta}$-tests and corrected $t$-test.

The test size study yields the following findings. First, for each test, the actual rejection rate only depends on the sample size, the autocorrelation of the predictor, and the contemporaneous correlation between the error term of the stock return and that of the predictor. Second, if the predictor is highly persistent and its error term is
highly contemporaneously correlated with that of the stock return, the $\hat{\beta}$-test constructed using the OLS autocorrelation estimate or its sample counterpart delivers the least size distortion in that the nominal significance level is indicative of its actual probability of making a Type I error. Third, under the remaining parameter settings, that is when the predictor is not highly autocorrelated or the correlation between the two error terms is low, Amihud and Hurvich’s corrected $t$-test is a robust choice because its nominal significant level correctly indicates the actual rejection rate under the null. Based on these findings, I propose a new testing rule for making statistical inferences on the predictability of stock returns.

Finally, the proposed estimation and test procedures are illustrated through an empirical study. Using monthly data from 1926 to 2016, I re-examine 18 popular predictors of US stock market returns that have been documented in the literature by applying the proposed statistical inference methods to the data. The list of predictors include various financial ratios, variables of corporate activity, interest rates, and market sentiment. The dependent variable is one-month-ahead total return on the S&P 500 value-weighted index in excess of the 30-day T-bill rate. The empirical findings are as follows. First, the improved estimator results in at least 40% reduction in magnitude from the OLS predictive slope when log dividend-price ratio (DP), log earnings-price ratio (EP), and log book-market ratio (BM) are the single predictor. The remaining predictors see markedly smaller relative changes in the magnitude of their predictive slope estimates as a result of applying the improved estimator. Second, among the candidate predictors, DP, EP, 10-year smoothed EP and BM are the only variables that are sufficiently persistent and exhibit a high correlation between the two error terms in the predictive model. Using $\hat{\beta}$-test, I find that the null hypothesis of no return predictability cannot be rejected when considering these predictors. In fact, there is strong statistical evidence on the return-predicting ability for only one financial ratio: dividend yield (DY) defined as rolling 12-month cumulative dividend paid out on S&P 500 index member stocks divided by the index level at the beginning of the window. I perform robustness checks on the analysis by splitting the sample into two halves. I find that none of the financial ratios can reliably predict next-month stock returns in the second subperiod. Finally, focusing on the second subperiod from July 1970 to December 2016, which may be less prone to data concerns such as structural breaks, I find that only 5 predictors out of the 18 are strong predictors of one-month-ahead stock returns. They are variables capturing stock volatility, cross sectional premium, corporate net issuance, stock short interest and variance risk premium.

This paper makes two main contributions. The first is methodology. The fast com-
puting method developed in this paper facilitates implementation of a class of $\hat{\beta}$-tests, which affords an understanding of the actual sizes (as opposed to the nominal sizes) of various test procedures across a wide range of parameter values. To my knowledge, this is the first paper which utilizes exact $p$-values to study the actual size of a $\hat{\beta}$-test.\footnote{Lewellen (2004) relies on simulation methods to evaluate the $p$-value of Stambaugh’s test.}

The second contribution is to present estimation and inference tools that complement the existing literature on predictive regressions. Assuming that the predictor is stationary, both simulation and analytical studies have shown that the first-order asymptotic theory provides a poor approximations to the actual finite-sample distribution of test statistics when the predictor is persistent and its error term is highly correlated with returns (Nelson and Kim, 1993; Elliot and Stock, 1994; Stambaugh, 1999). Amihud and Hurvich (2004) develop an augmented regression approach to address the finite-sample bias in the OLS predictive slope and the hypothesis testing problem. Lewellen (2004) studies predictors with an autocorrelation coefficient near unity and produces upper bound for the bias in the OLS slope. Subsequent studies examine the empirical evidence for return predictability using predictors that is highly persistent or contains a unit root (Cavanagh, Elliot, and Stock, 1995; Lanne, 2002; Torous, Valkanov, and Yan, 2004; Campbell and Yogo, 2006). Phillips (2015) provides an excellent overview of recent development in the finite sample and asymptotic inferences for predictive regressions. Amihud, Hurvich, and Wang (2009) develop a hypothesis-testing method for multiple-predictor regression in finite samples. They note that with highly persistent predictors, their procedure may lead to over-rejection problems under the null. Assuming a single stationary predictor in the predictive model system, I propose improved predictive slope estimators which outperforms existing ones in finite-sample bias and MSE, and I provide a testing rule for tests of return predictability that are less prone to size distortions.

The rest of the paper is organized as follows. In Section 2, I first review the predictive regression model, and then construct improved predictive slope estimators and provide their exact performance measures. Section 3 starts with a review of the existing tests of return predictability in the literature. I then discuss several extensions of Stambaugh’s $\hat{\beta}$-test and explain how to implement these tests with the powerful computational tool set. In Section 4, the significance tests of predictability are contrasted against each other for their actual rejection probabilities via a Monte Carlo simulation study. Section 5 provides an empirical illustration of the methodological tools developed in this paper using U.S. equity data. I re-examine the empirical evidence for predictability. Section 6 concludes.
1.2 Predictive Regression and Improved Slope Estimators

I consider the following predictive regression model system. Let \( r_t \) be the market excess return, and let \( x_t \) be a candidate predictor. For \( t = 1, \ldots, T \),

\[
r_t = \alpha + \beta x_{t-1} + u_t, \tag{1.1}
\]
\[
x_t = \theta + \phi x_{t-1} + v_t, \tag{1.2}
\]

where the error terms \( (u_t, v_t) \) are i.i.d. bivariate normal with zero mean. Let \( \sigma_u^2, \sigma_v^2, \sigma_{uv}, \rho_{uv} \) respectively denote the variance of \( u_t \), the variance of \( v_t \), the covariance between \( u_t \) and \( v_t \), and their correlation coefficient. I further assume \( |\phi| < 1 \), i.e., \( \{x_t\} \) follows a stationary first-order autocorrelation AR(1) process. In sum, this predictive system features a conditional mean of the next-period market return that is linearly dependent on the current value of a single predictive variable with an AR(1) dynamic. Assuming such a data generating process for the stock return and the candidate predictor has been standard in the literature.\(^2\)

It is well known that if \( \sigma_{uv} \neq 0 \), then the OLS estimator of \( \beta \) has a finite-sample bias and significance tests based on the classical OLS assumptions lead to over-rejection of the null hypothesis of \( \beta = 0 \). Stambaugh (1999) derives the exact finite-sample distribution of the OLS predictive slope estimator, which can be utilized for tests of return predictability, and he also presents a simple plug-in formula for bias correction. More recently, Amihud and Hurvich (2004), henceforth AH, propose a reduced-bias estimation procedure for the predictive regression slope. This procedure is summarized as follows.

First, running OLS regressions on the model equations (1.1) and (1.2) yields OLS estimators \( \hat{\alpha}, \hat{\beta}, \hat{\theta} \) and \( \hat{\phi} \) for \( \alpha, \beta, \theta \) and \( \phi \), respectively. Let \( \hat{u}_t \) and \( \hat{v}_t \) be the fitted OLS residuals. Then \( \hat{\kappa} \equiv \frac{\sum \hat{u}_t \hat{v}_t}{\sum \hat{v}_t^2} \) is a sample-based estimator of \( \kappa \equiv \sigma_{uv}/\sigma_v^2 \). Next, run a return predictive regression that is augmented on the basis of Eq.(1.1) by including a third regressor \( \hat{v}_c \equiv x_t - \hat{\phi}^c x_{t-1} \), where \( \hat{\phi}^c \) is a corrected estimator of \( \phi \). The additional regressor can be interpreted as the fitted residual of the predictor’s AR(1) model using \( \hat{\phi}^c \) as the AR parameter estimate. Let \( \hat{\beta}^c \) be the estimated coefficient of the lagged predictor

\(^2\)This model system was first studied by Stambaugh (1986) and Mankiw and Shapiro (1986), and subsequently by Nelson and Kim (1993), Stambaugh (1999), Lewellen (2004), Amihud and Hurvich (2004).
in the augmented regression. AH show that

\[ \hat{\beta}_c = \hat{\beta} + \hat{\kappa}(\hat{\phi}_c - \hat{\phi}). \]  

(1.3)

Thus \( \hat{\beta}_c \) can be regarded as a corrected estimator of \( \beta \) induced by the correction made to \( \hat{\phi} \). Moreover, AH show that the following equation holds for the bias of \( \hat{\beta}_c \) (Theorem 2 in AH):

\[ E[\hat{\beta}_c] - \beta = \kappa(E[\hat{\phi}_c] - \phi). \]  

(1.4)

This relationship between the two biases implies that if an unbiased estimator of \( \phi \) is found, it is easy to construct an unbiased estimator of \( \beta \). The problem of finding an unbiased estimator of \( \phi \) is yet to be solved. However, if there is a \( \hat{\phi}_c \) with a reduced bias, we can effectively remove the small-sample bias in \( \hat{\beta} \). For this reason, the problem of bias correction for the predictive regression slope is transformed into one for the underlying predictor AR parameter. In the rest of this section, I will take this augmented regression approach and construct estimators of the predictive slope which have desirably a negligible bias and a high level of accuracy.

### 1.2.1 Bias of \( \hat{\phi} \) and Some Approximations

Before searching for bias corrections, a discussion of the bias in \( \hat{\phi} \) is in order. Fix the sample size at \( T \). Under the normality assumption, it is straightforward to show that the bias in \( \hat{\phi} \) is a function of \( \phi \) and \( T \),

\[ h(\phi, T) = E[\hat{\phi}] - \phi = E\left[ \frac{z'MGH^{-1}z}{z'Mz} \right], \]  

(1.5)

where \( G, H \) and \( M \) are \( T \times T \) matrices, respectively defined as

\( G = (\phi^{i-j-1}1_{i<j}) \),

\( H = (\phi^{i-j}/(1-\phi^2)) \),

\( M = I_T - 1_T1_T'/T \), and \( z \sim N(0_T, H) \).\(^3\) Since the bias is the expectation of a ratio of two quadratic forms in multivariate normal random variables, its exact value can be obtained as a single integral.\(^4\) There are fast numerical methods of computing the exact moments of \( \hat{\phi} \), and the cumulative distribution function (cdf)

---

\(^3\)A derivation of this result is provided in Appendix A.

\(^4\)The integration approach starts with Sawa (1972), who provides an integral formula for evaluating the moments of a ratio of quadratic forms in normal variables. Magnus (1986) gives a nice discussion, in particular, the numerical aspect of this method. More recently, Meng (2005) provides an excellent review of this literature, and Bao and Kan (2012) present efficient numerical methods for computing the expectation of the ratio in a general form.
and probability density function (pdf) of \( \hat{\phi} \). The computation of the moments of \( \hat{\phi} \) by integration was first studied by Sawa (1978) and Nankervis and Savin (1988). The efficient procedure in use here for the exact moments is due to Bao and Kan (2012).

Besides computing the exact value of \( h(\phi, T) \), there are a few useful approximations in the statistics literature that have been shown to asymptotically converge to the true value. Under the normality assumption, Marriott and Pope (1954) and Kendall (1954) provide a well-known approximation of \( h(\phi, T) \) to order \( T^{-1} \),

\[
h^{\text{MPK}}(\phi, T) = -\frac{(1 + 3\phi)}{T}. \tag{1.6}
\]

Kiviet and Phillips (2012) show the next higher-order approximation of \( h(\phi) \), to order \( T^{-2} \), is

\[
h^{\text{KP}}(\phi, T) = -\frac{1 + 3\phi}{T} - \frac{\phi(9\phi^2 + 6\phi - 5)}{T^2(1 - \phi^2)}. \tag{1.7}
\]

Nevertheless, as pointed out by Kendall (1954), these expressions have unsatisfactory approximations for \( \phi \) near unity even though further terms to order \( T^{-2} \) or \( T^{-3} \) are included.

To tackle the problem of poor approximating performance when \( \phi \) is close to one, I propose alternative solutions. First, \( h(\phi, T) \) can be approximated by way of a Taylor-series expansion to the second order,

\[
h^{\text{I}}(\phi, T) = \frac{E[U]}{E[V]} - \frac{\text{Cov}[U, V]}{(E[V])^2} + \frac{\text{Var}[V]E[U]}{(E[V])^3}, \tag{1.8}
\]

where \( U = z'\text{MGH}^{-1}z \) and \( V = z'\text{Mz} \). Moreover, an application of the Laplace expansion to order \( T^{-2} \) of the expected ratio of quadratic forms in normal variables, which is suggested by Lieberman (1994), yields the following approximation,

\[
h^{\text{II}}(\phi, T) = h^{\text{I}}(\phi, T) + \frac{\text{E}[(U - E[U])(V - E[V])^2]}{(E[V])^3} - \frac{\text{E}[(V - E[V])^3]E[U] + 3\text{Var}[V]\text{Cov}[U, V]}{(E[V])^4} + \frac{3(\text{Var}[V])^2E[U]}{(E[V])^5}. \tag{1.9}
\]

The required moments of \( U \) and \( V \) and a proof of this expression are presented in Appendix A. Further discussion of these two alternative approximations is in order. Both \( h^{\text{I}} \) and \( h^{\text{II}} \) includes \( \phi^T \) terms. When \( |\phi| \) is small, the exponential effect quickly kicks in. Indeed, it is straightforward to show that \( \lim_{T \to \infty} T(h^{\text{I}} - h^{\text{MPK}}) = 0 \) and \( \lim_{T \to \infty} T^2(h^{\text{II}} - h^{\text{KP}}) = 0 \). However, when \( \phi \) is near unity, it is necessary to explicitly ac-
count for the slow-decaying effect of $\phi^T$, as captured by the two proposed approximations. Two numerical examples are considered below to illustrate this point.

Figure 1.1 visually compares the exact bias in $\hat{\phi}$ and four approximations $h^{MPK}$, $h^{KP}$, $h^I$ and $h^{II}$. Panel A displays the relationship between sample size and the bias in $\hat{\phi}$ and its four approximations for a highly persistent predictor with $\phi = 0.99$, and Panel B plots the relative errors of the approximations. Several observations are made. First, $h^{MPK}$ markedly understates the bias. When the sample size is 60, $h^{MPK} = -0.0662$, about 80% of the exact bias ($h = -0.0822$). Even with a sample size as large as 780, there is still a more than 10% understatement, as shown by the thick solid line in Panel B. Second, $h^{KP}$ severely overstates the bias in small samples, and therefore ignoring $\phi^T$ terms becomes undesirably punitive with highly persistent predictors. Indeed, in a 60-observation sample, $h^{KP}$ is $-0.2011$, more than 2.4 times the exact bias. By contrast, the approximation error of $h^{KP}$ reduces drastically in larger samples, as shown by the dash-dotted line in Panel B. For example, with a sample size greater than 450, the overstatement is less than 10%. Finally, the two approximations proposed here deliver better performances. $h^I$ (dashed-line) significantly outperforms $h^{MPK}$ (thick solid line), as the former accounts for the slow-decaying feature of $\phi^T$ when $\phi$ is near unity. By capturing both higher order terms and this slow-decaying feature, $h^{II}$ (solid line) tracks the exact bias almost perfectly. With a sample size of 60, the relative error is less than 3%. With a sample size greater than 160, the relative error of $h^{II}$ is contained within 0.6% of the exact bias.

When the predictor is moderately autocorrelated, the approximations have more converging performances. Panel C shows that when $\phi$ is 0.7, the four approximations each exhibit a good proximity to the exact bias. In this graph, all five lines virtually overlap under the current scale. Panel D displays the relative error of each approximation as a percentage of the exact bias. With a sample of 60 observations, the difference between any one of the four approximations and the exact bias is within 2% of the exact bias. The relative errors of the approximations quickly diminish to below 0.5% when the sample size is larger than 300, and they rapidly shrinks to zero as sample size grows. Whilst all four approximations have satisfactory accuracy, $h^{II}$ delivers by far the best performance in small samples because it captures all $O(T^{-2})$ terms. $h^{KP}$ also provides a good approximation though it ignores $\phi^T$ terms. Doing so does not appear punitive given $\phi$ is moderately large. I also investigate cases with $\phi < 0.7$. For weakly autocorrelated predictors, all four approximations have negligible tracking errors as the exponential decay is most effective. For brevity these results are not reported in the figure.

In short, the two approximations in existing literature perform poorly with a highly
persistent predictor, as it takes a progressively larger sample for them to achieve satisfactory results. By contrast, both $h^I$ and $h^{II}$ are good approximating functions. The proposed $h^{II}$ performs the best in tracking the exact bias. This is a useful result when the computation time for evaluating the exact bias becomes an issue, which arises when $T$ is large.

**Bias corrections for $\hat{\phi}$**

There are a few existing bias corrections for the OLS estimator of $\phi$ in the econometrics literature. A popular correction, first studied by Sawa (1978), is based on the approximated bias $h^{MPK}$:

$$\hat{\phi}^S = \hat{\phi} - h^{MPK}(\hat{\phi}, T). \tag{1.10}$$

Kothari and Shanken (1997) propose the following corrected estimator of $\phi$ with a bias of order $T^{-2}$, which is further utilized in a bootstrap procedure for the hypothesis tests of the predictive slope $\beta$:

$$\hat{\phi}^{KS} = \hat{\phi} + \frac{1 + 3\hat{\phi}}{T - 3}. \tag{1.11}$$

It is easy to verify that

$$\hat{\phi}^{KS} = \hat{\phi} - h^{MPK}(\hat{\phi}^{KS}, T). \tag{1.12}$$

AH conduct a simulation study on the properties of

$$\hat{\phi}^{AH} = \hat{\phi} - h^{MPK}(\hat{\phi}^S, T). \tag{1.13}$$

These corrections share one common feature. That is, they are each an OLS estimator adjusted by the MPK bias approximation evaluated at some $\phi$ estimate. From the previous analysis on the exact bias in $\hat{\phi}$ and its approximations, we see that the alternative approximations $h^I$ and $h^{II}$ deliver better results than $h^{MPK}$. Motivated by this, we propose three new estimators of $\phi$:

$$\hat{\phi}^I = \hat{\phi} - h^I(\hat{\phi}^{KS}, T), \tag{1.14}$$

$$\hat{\phi}^{II} = \hat{\phi} - h^{II}(\hat{\phi}^{KS}, T), \tag{1.15}$$

$$\hat{\phi}^{III} = \hat{\phi} - h(\hat{\phi}^{KS}, T). \tag{1.16}$$

All six estimators as well as the OLS estimator $\hat{\phi}$ consistently estimate $\phi$. However, their performances in finite samples may differ from one another. Understanding this
difference is important because it provides guidance in empirical applications where only finite samples are concerned.

Towards this goal, I conduct a comprehensive analysis of the exact finite-sample properties for a pool of bias-corrected estimators of $\phi$. For each of the existing and proposed estimators of $\phi$, I first evaluate its bias and MSE utilizing the efficient method developed by Bao and Kan (2012) to compute the moments of a ratio of quadratic forms in normal random variables. Realizations of these estimators of $\phi$ can take values outside the unit circle.\(^5\) Thus I also consider a winsorized version. Specifically, an upper limit of 0.999 and lower limit of $-0.999$ are placed on each corrected estimator.\(^6\)

Next, for the sake of comparison, I present the results on the relative biases and MSEs of the corrected estimators of the AR parameter for different values of $\phi$ and a range of sample sizes. That is, the bias and MSE of each corrected estimator are expressed as a percentage of the bias and MSE of the OLS $\hat{\phi}$, respectively. As shown below, the interpretation of improved performance of the proposed estimators of $\phi$ in relative terms can be readily applied to the induced estimators of $\beta$.

Table 1.1 reports the comparison results. The following discussion begins with the unwinsorized estimators. The first three estimators, namely $\hat{\phi}^S$, $\hat{\phi}^{KS}$ and $\hat{\phi}^{AH}$, are from existing literature, and they each have a bias of order $T^{-2}$ for $|\phi| < 1$. Unlike AH who rely on simulations to decide on the bias-reduction performances of $\hat{\phi}^S$ and $\hat{\phi}^{AH}$, the fast numerical procedure for computing the exact moments of $\hat{\phi}$ enables an accurate and definitive comparison. Table 1.1 indicates that $\hat{\phi}^{KS}$ uniformly outperforms the other two popular existing corrections in reducing the bias and MSE for all $(\phi, T)$-pair examined. Indeed, the incremental improvement of $\hat{\phi}^{KS}$ from $\hat{\phi}^S$ is markedly noticeable in small samples and for large $\phi$’s. Take $T = 60$ and $\phi = 0.99$, for example. When compared with $\hat{\phi}^S$, $\hat{\phi}^{KS}$ has an additional bias reduction of 4–5% of $h(\phi, T)$, the bias of $\hat{\phi}$. By contrast, there is only a small difference in the bias and MSE of $\hat{\phi}^{KS}$ and $\hat{\phi}^{AH}$.

Despite its superior performance among the existing estimators, $\hat{\phi}^{KS}$ still has an unsatisfactory bias-reducing effect in small samples or when $\phi$ is near unity. For instance, in a sample of size $T = 60$, the relative bias of $\hat{\phi}^{KS}$ is 20.6% for $\phi = 0.99$, and it is 13.6% for $\phi = 0.95$. When $\phi = 0.99$, $\hat{\phi}^{KS}$ has a relative bias of 11.8% with the sample size increased to 600.

Now let us examine the performances of the three proposed estimators of $\phi$. $\hat{\phi}^{I}$ and $\hat{\phi}^{II}$

---

\(^{5}\)This can be an unappealing property. For example, when computing the moments and distributional properties of $\hat{\beta}$ using sample data, we require the estimate of $\phi$ between $-1$ and 1.

\(^{6}\)I have also examined the performances of winsorized estimators using the limit pair $(-0.9999, 0.9999)$. The results are insensitive to the change of limits.
are \( \hat{\phi} \) adjusted by approximations to the bias function \( h(\phi, T) \) evaluated at \( \hat{\phi}^{KS} \), while we subtract \( h(\hat{\phi}^{KS}, T) \) from \( \hat{\phi} \) to arrive at \( \hat{\phi}^{III} \). As reported in Table 1.1, there is a steep bias reduction in all three proposed estimators when compared with the three existing ones. Compensated for their good bias reduction performances are the small increases in their MSEs. The contrast between the bias-reducing effect of the new estimators and that of the existing ones is most striking when the predictor is persistent. When \( \phi = 0.99 \), the relative bias of each proposed estimator is less than 6% in a sample of 60 observations. Whilst the magnitude of bias remain small in larger samples, their relative advantages shrink somewhat as the existing estimators catch up. Read vertically, the table shows that with the sample size fixed, for smaller \( \phi \), the new estimators keep its edge by maintaining a small bias, albeit to a lesser degree.

Among the three new estimators, \( \hat{\phi}^{II} \) strikes the most desirable balance between performance and computation complexity. In fact, in almost all \((\phi, T)\) pairs examined, \( \hat{\phi}^{II} \) delivers the best bias-reduction performance with the closest-to-zero biases among all estimators. This desirable property is due to the extra terms of \( \phi^T \) that are kept in the expression of \( h^{II} \). By capturing sufficiently the effect of such terms when \( \phi \) is near unity, \( \hat{\phi}^{II} \) succeeds at reducing bias towards zero. In only a few cases does \( \hat{\phi}^{III} \) outperform with both a smaller bias and a smaller MSE. And in such cases there is only a slight improvement from \( \hat{\phi}^{II} \). Given that the computation burden may be a concern when implementing \( \hat{\phi}^{III} \), it is best to use \( \hat{\phi}^{II} \).

Next, let us consider the winsorized version. Table 1.1 reports the relative performance of the winsorized estimators under column headings R.Bias(w) and R.RMSE(w).\(^7\) A notable effect of winsorization is an increased bias which occurs to all bias-corrected estimators. To understand this effect, we note that all the corrections tend to increase the original OLS \( \hat{\phi} \). Take \( \hat{\phi}^{II} \) for an example. Its sampling distribution is shifted to the right compared to that of \( \hat{\phi} \). This causes \( \text{E}[\hat{\phi}^{II}] \) to be closer to the true \( \phi \). Now, the winsorized \( \hat{\phi}^{II} \) places an upper bound on its realized values, which works against the bias-reducing effect of the raw correction by limiting the contribution of large realizations. Therefore, the mean of the winsorized \( \hat{\phi}^{II} \) is less than that of \( \hat{\phi}^{II} \), and this effect results in a larger bias. Table 1.1 shows that the bias increase of a winsorized estimator relative to its raw version reduces as the sample size grows and as \( \phi \) moves away from unity. In all \((\phi, T)\) pairs, \( \hat{\phi}^{KS} \) continues to outperform \( \hat{\phi}^{S} \) and \( \hat{\phi}^{AH} \) in terms of smaller bias and MSE. Among all six winsorized estimators, \( \hat{\phi}^{II} \) is still the one recommended due to the same reason discussed for the unwinsorized versions.

---

\(^7\)The winsorized version of \( \hat{\phi} \) has a bias and MSE comparable to \( \hat{\phi} \) with negligible improvement, and hence it is omitted for brevity.
In summary, $\hat{\phi}^{\text{KS}}$ delivers the best result among the existing bias-corrected estimators of $\phi$. The proposed new estimators deliver even better bias reduction performances. This observation also holds true for the group of winsorized estimators. Among the proposed estimators, $\hat{\phi}^{\text{II}}$ is the recommended one, for it achieves the best trade-off between performance and computational difficulty.

1.2.2 MSE of $\hat{\beta}^c$

The MSE of an estimator is commonly used as a criterion for judging the accuracy of an estimator. I now analyze the MSE of various bias-corrected estimators of $\beta$. First, a useful result on the MSE of $\hat{\beta}^c$ constructed via Eq.(1.3) is stated in the following proposition. (See Appendix A for a proof.)

**Proposition 1.** Let $\omega = \hat{\phi}^c - \hat{\phi}$ be the correction term that only depends on the history of the predictor. Let $\hat{\nu} = [\hat{\nu}_1, \ldots, \hat{\nu}_T]$, and let $\sigma^2_{\varepsilon} = \sigma^2_u - \kappa^2 \sigma^2_v$. Then, the MSE of $\hat{\beta}^c$ has the decomposition:

$$E(\hat{\beta}^c - \beta)^2 = \kappa^2 E(\hat{\phi}^c - \phi)^2 + E[SE_0(\hat{\beta})]^2 + \sigma^2_{\varepsilon} E\left[\frac{\omega^2}{\hat{\nu}'\hat{\nu}}\right],$$

(1.17)

where $SE_0(\hat{\beta})$ denotes the standard error of the estimated coefficient of $x_{t-1}$ in the augmented regression with $\hat{\nu}_t$ as the third regressor.

This MSE breakdown sheds light on the sources of inaccuracy when $\hat{\beta}^c$ is used to estimate $\beta$. Let the three terms on the RHS of Eq.(1.17) be labeled as MSE$_1$, MSE$_2$ and MSE$_3$, respectively from left to right. AH show that the MSE of $\hat{\beta}^c$ is the sum of MSE$_1$ and $E[SE_0(\hat{\beta})]^2$. The latter is the expectation of the squared standard error of the estimated coefficient of $x_{t-1}$ in the augmented regression with $\hat{\nu}_t$ as the third regressor, and therefore it is a correction-dependent term. Proposition 1 goes one step further by decomposing $E[SE_0(\hat{\beta})]^2$ into MSE$_2$, a term that is neutral to correction, and MSE$_3$ a residual term.

All three terms are non-negative. When no correction is made, i.e., $\omega = 0$, the residual term MSE$_3$ disappears, and the MSE of $\hat{\beta}$ is the sum of a scaled MSE of $\hat{\phi}$ and MSE$_2$. With a non-zero correction term, MSE$_3$ is strictly positive. For the six bias-corrected estimators of $\phi$ and their winsorized counterparts, the corresponding correction terms are each of order $T^{-1}$. As $\hat{\nu}'\hat{\nu}$ is asymptotically linear in $T$, it follows that MSE$_4$ is of order $T^{-3}$. Since MSE$_1$ and MSE$_2$ are both $O(T^{-1})$ terms, they are the dominant contributors to the MSE of $\hat{\beta}^c$. 
From the proof of Proposition 1, it is clear that $\text{MSE}[\hat{\beta}^c]$ only depends on $\sigma_u/\sigma_v$, $\phi$, $\rho_{uv}$, and $T$. Note that when normalized by $(\sigma_u/\sigma_v)^2$, each of the three components of this MSE is a function of $\phi$, $\rho_{uv}$ and $T$, exclusively. Without loss of generality, we set $\sigma_u/\sigma_v = 1$ when comparing the contributions of MSE components. It follows that $\text{MSE}_1 = \rho_{uv}^2 \text{MSE}[\hat{\phi}^c]$, and $\text{MSE}_2 = (1 - \rho_{uv}^2)B$, where $B$ is an $O(T^{-1})$ term that only depends on $\phi$ and $T$. The proof of Proposition 1 shows that $B$ is the expectation of the inverse of a positive semi-definite quadratic form in normal random variables. For a given estimator of $\phi$, we can compute and compare the magnitudes of $\text{MSE}_1$ and $\text{MSE}_2$ easily. To put this into perspective, consider the predictive slope estimator constructed based on the winsorized $\hat{\phi}^\Pi$. Suppose that $\phi = 0.99$ and $T = 600$. From Table 1.1, $\text{MSE}[\hat{\phi}^\Pi(w)] = (0.011698 \times 78.02\%)^2 = 8.32 \times 10^{-5}$. Using the fast numerical evaluation for a ratio of quadratic forms, we obtain $B = 1.11 \times 10^{-5}$. Finally, the magnitude of $\rho_{uv}$ determines $\text{MSE}_1$ and $\text{MSE}_2$ as fractions of these two small numbers.

It is also easy to compare the MSE of various predictor slope estimators constructed via the AH augmented regression approach. Due to the correction neutrality of $\text{MSE}_2$, it suffices to rank the MSE of the underlying $\hat{\phi}^c$ for each predictor slope estimator. Table 1.1 shows that all six bias corrections on $\hat{\phi}$ produce smaller MSEs than $\hat{\phi}$. The percentage reduction in MSE of $\hat{\phi}$ is conveniently interpreted as the percentage reduction in $\text{MSE}_1$. For example, when $\phi = 0.99$ in a sample of 60 observations, the reduction induced by using $\hat{\beta}^\KS$ is $50.7\%$ ($= 1 - (70.21/100)^2$). In fact, all other five estimators produce similar percentages of reduction in $\text{MSE}_1$. Table 1.1 also shows that the amount of reduction decreases as the sample size grows and the predictor becomes less persistent. The effect of the sample size is intuitive because $\hat{\phi}$ is a consistent estimator, and the room for accuracy improvement becomes squeezed as $T$ grows. The effect of the persistence of the predictor is also consistent with the fact that the accuracy of $\hat{\phi}$ is an increasing function of $\phi$. For a more weakly autocorrelated predictor, there is less room for accuracy improvement.

The MSE rankings for $\hat{\phi}^c$ are preserved for those of $\hat{\beta}^c$. Among the twelve predictive slope estimators, the ones based on winsorized $\hat{\phi}^c$ provide a more accurate estimation than the unwinsorized ones. Among the winsorized estimators, $\hat{\beta}^\Pi$ has the lowest MSE when $\phi$ is near unity and $T \leq 240$, whereas $\hat{\beta}^\II$ only has a slight underperformance. Otherwise, $\hat{\beta}^\KS$ has the best performance in terms of MSE though $\hat{\beta}^\II$ delivers the smallest bias.

Combining the analysis on bias and that on MSE, we conclude that winsorized $\hat{\beta}^\KS$ and $\hat{\beta}^\II$ are the best estimators with the smallest biases and MSEs. The former has a simple and elegant construction and delivers satisfactory bias and MSE performances across a range of key parameter values. The latter is recommended for near-unity $\phi$ and small sample sizes though its construction is a bit more complex.
1.3 Tests of Return Predictability

This section discusses various tests of return predictability. In the context of the predictive regression model, this amounts to a significance test of the null hypothesis $\beta = 0$. I first generalize the adjusted $t$-statistic developed by AH to accommodate a group of bias-corrected slope estimators. Then I extend Stambaugh’s one-sided test of return predictability to a two-sided test.

1.3.1 Low-bias Standard Error Estimation

In addition to the reduced-bias estimation procedure, AH also construct an adjusted $t$-statistic for performing test of statistical significance on $\beta$. This test statistic requires an appropriate standard error for the predictive slope estimate from the augmented regression. Towards this goal, AH first study the MSE of any $\hat{\beta}^c$ constructed via Eq.(1.3) and demonstrate that the following decomposition holds:

\[
E(\hat{\beta}^c - \beta)^2 = \kappa^2 E(\hat{\phi}^c - \phi)^2 + E[SE_\omega(\hat{\beta}^c)]^2, \tag{1.18}
\]

where $SE_\omega(\hat{\beta}^c)$ is the standard error of the OLS estimator for the coefficient of $x_{t-1}$ in the augmented predictive regression. The subscript in $SE_\omega$ reflects the correction term $\omega = \hat{\phi}^c - \hat{\phi}$ in constructing the bias-corrected estimator of $\phi$.

Motivated by this MSE decomposition, AH propose a low-bias standard error for the special case of $\hat{\beta}^{AH}$. In fact, the AH hypothesis test procedure can be generalized to accommodate any $\hat{\beta}^c$ which has a bias of order $T^{-2}$ or higher and is constructed via Eq.(1.3) with a correction term $\omega$ linear in $\hat{\phi}$. The adjusted standard error of $\hat{\beta}^c$ in this general case is described as follows.

Suppose $\hat{\phi}^c$ is a bias-corrected estimator of $\phi$ constructed by adding to $\hat{\phi}$ a linear correction term $\omega = a_0 + a_1 \hat{\phi}$, where neither $a_0$ or $a_1$ depends on any system parameter. Further, assume $\hat{\phi}^c$ has a bias of order $T^{-2}$. It then follows immediately from Eq.(1.4) that $\hat{\beta}^c$ also has a bias of order $T^{-2}$. Using Eq.(1.18), it is easy to see

\[
Var(\hat{\beta}^c) = \kappa^2 Var(\hat{\phi}^c) + E[SE_\omega(\hat{\beta}^c)]^2 + O(T^{-4})
\]

\[
= (1 + a_1)^2 \kappa^2 Var(\hat{\phi}) + E[SE_\omega(\hat{\beta}^c)]^2 + O(T^{-4}). \tag{1.19}
\]

Next, we observe that $\hat{\kappa}$ is an unbiased estimator of $\kappa$ (Lemma 1 in AH). Based on simulation evidence, AH also propose a heuristic approximation for $Var(\hat{\phi})$ by the squared

---

\(^8\)See Lemma 2 in AH.
SE(\hat{\phi})$, i.e. the standard error of the OLS estimator \( \hat{\phi} \). Hence, by plugging in these statistics, I propose the following approximation for the standard error of the bias-corrected estimator \( \hat{\beta}^c = \hat{\beta} + \hat{\kappa} \omega \):

\[
\hat{SE}(\hat{\beta}^c) = \sqrt{(1 + a_1)^2 \hat{\kappa}^2 \left[ SE(\hat{\phi}) \right]^2 + \left[ SE(\omega^c) \right]^2}.
\] (1.20)

It is trivial to verify that for \( \hat{\beta}^{AH} \) the standard error is evaluated using \( a_1 = 3(T - 1)^{-1} \). Similarly, an approximation for the standard error of \( \hat{\beta}^{KS} \) is produced by setting \( a_1 = 3(T - 3)^{-1} \).

It is worth noting that \( \hat{SE}(\hat{\beta}^c) \) is at least as large as \( SE(\omega(\hat{\beta}^c)) \). The positive difference reflects the additional variability due to the estimation of \( \phi \). If \( \phi \) were known, \( SE(\phi - \hat{\phi}) \) would give the standard error of the corrected estimator \( \hat{\beta} + \hat{\kappa}(\phi - \hat{\phi}) \). In fact, a \( t \)-test based on the AH corrected statistic is appropriate in this known \( \phi \) case. This is due to the following result.\(^9\)

**Proposition 2.** If \( \phi \) is a known parameter, the OLS \( t \)-statistic for the coefficient estimate of \( x_{t-1} \) follows a Student \( t \)-distribution with \( (T - 3) \) degrees of freedom in the augmented predictive regression of \( r_t \) on \( 1, x_{t-1} \) and \( x_t - \phi x_{t-1} \).

In the realistic case where \( \phi \) is known, tests of return predictability against the null \( \beta = 0 \) can be constructed based on the \( t \)-statistic computed as the ratio of a bias-corrected estimator \( \hat{\beta}^c \) to its standard error \( \hat{SE}(\hat{\beta}^c) \). It is of practical importance to evaluate the actual rejection probability of such a test when using the usual cutoff points associated with a standard \( t \)-test. Since by construction the \( t \)-statistic is unitless, the actual rejection probability is a function of \( \phi \), \( \rho_{uy} \) and \( T \).

### 1.3.2 Stambaugh’s \( \hat{\beta} \)-test

Stambaugh (1999) constructs another class of predictability tests based on the probability distribution of the OLS estimator \( \hat{\beta} \). (For ease of exposition, this type of test is labeled as \( \hat{\beta} \)-test henceforth.) The finance and econometrics literature has seen very little use of this test since its first appearance. This is partially due to the numerical difficulty involved in computing the statistic in question. Besides, there is an insufficient understanding of the actual rejection probability and the power of these tests. In what follows, I first review the construction of Stambaugh’s original one-sided \( \hat{\beta} \)-test and consider the extension to a two-sided test. Then I propose a fast numerical method that

---

\(^9\)See Appendix A for a proof.
successfully deals with the computational difficulty mostly encountered in large samples. This method significantly reduces the computer runtime for the simulation-based study of actual rejection probabilities that follows.

The one-sided test constructed by Stambaugh (1999) considers the alternative hypothesis \( \beta > 0 \) against the null \( \beta = 0 \). Intuitively, the critical region (CR) takes the form \( \{ \hat{\beta} > q_R \} \), where \( q_R \) is a real number determined by the probability distribution of \( \hat{\beta} \) under the null and the nominal size of the test. For instance, in the case of size 5% test, \( q_R \) is the \( 1 - 0.05 = 0.95 \) quantile of the \( \hat{\beta} \)'s probability distribution under the null. Hence, understanding the distributional properties of \( \hat{\beta} \) is important. The following result provides some basic properties of the probability distribution of \( \hat{\beta} \).

**Proposition 3.** The probability distribution of \( \hat{\beta} \) only depends on \( \beta, \sigma_u/\sigma_v, \phi, \rho_{uv} \) and \( T \). Let \( F(x; \beta, \sigma_u/\sigma_v, \phi, \rho_{uv}, T) \) be the cdf of \( \hat{\beta} \). Let \( \tilde{\beta} = (\hat{\beta} - \beta)/(\sigma_u/\sigma_v) \). Then,

(i) \( \tilde{F}(x; \phi, \rho_{uv}, T) \equiv F(x; 0, 1, \phi, \rho_{uv}, T) \) is the cdf of \( \tilde{\beta} \).

(ii) \( F(x; \beta, \sigma_u/\sigma_v, \phi, \rho_{uv}, T) = \tilde{F}\left(\frac{x - \beta}{\sigma_u/\sigma_v}; \phi, \rho_{uv}, T\right) \).

(iii) \( \tilde{F}(x; \phi, -\rho_{uv}, T) = 1 - \tilde{F}(x; \phi, \rho_{uv}, T) \).

Appendix A provides a proof of the above results. Proposition 3 shows that \( \beta \) and \( \sigma_u/\sigma_v \) are the location and shape parameters of the distribution of \( \hat{\beta} \), respectively. To put it another way, \( \tilde{\beta} \) is the “standardized” predictor slope estimator. Using Proposition 3, the nominal 5% test’s CR can be re-written in terms of \( \hat{\beta} \)'s associated \( p \)-value: \( \{ F(\hat{\beta}; \beta, \sigma_u/\sigma_v, \phi, \rho_{uv}, T) > 0.95 \} \) or equivalently \( \{ \tilde{F}(\hat{\beta}/(\sigma_u/\sigma_v); \phi, \rho_{uv}, T) > 0.95 \} \).

An importantly pertinent question is whether this test has the intended 5% rejection probability under the null. In addition to \( \beta = 0 \), we need \( \sigma_u/\sigma_v, \phi, \rho_{uv} \) as well as the sample size to evaluate the \( p \)-value associated with each realization of \( \hat{\beta} \). One possibility is to plug the OLS estimates into the \( p \)-value expression, which is the approach taken by Stambaugh (1999). Hence, an implementable CR of the nominal 5% test is

\[
\left\{ \tilde{F}\left(\frac{\hat{\beta}}{\sigma_u/\sigma_v}; \phi, \rho_{uv}, T\right) > 0.95 \right\}.
\]  

As pointed out by Stambaugh (1999), this test can only be carried out if the absolute value of the point estimate of \( \phi \) is less than one. Hence, I winsorize \( \hat{\phi} \) below at \(-0.999\) and above at \(0.999\). It is worth noting that under the null, this CR is parametrically uniquely determined by \( \phi, \rho_{uv} \) and the sample size \( T \). We can see this upon examining of the expression in (1.21). First, it is clear that \( \hat{\phi} \) and \( \hat{\rho}_{uv} \) only depend on \( \phi, \rho_{uv} \) and
Second, Proposition 3 says that \( \hat{\beta} \) only depends on \( \sigma_u/\sigma_v \), \( \phi \), \( \rho_{uv} \) and \( T \). Moreover, since \( \sigma_u/\sigma_v \) is a scaling parameter in both \( \hat{\beta} \) and \( \hat{\sigma}_u/\hat{\sigma}_v \), it follows that the ratio of the two statistics only depend on \( \phi \), \( \rho_{uv} \) and \( T \).

The construction method of Stambaugh’s right-tail one-sided test can be applied to the left tail one-sided and two-sided tests. Analogous to (1.21), the nominal 5% one-sided test of \( \beta < 0 \) against \( \beta = 0 \) has the CR

\[
\left\{ \tilde{F} \left( \frac{\hat{\beta}}{\hat{\sigma}_u/\hat{\sigma}_v}; \hat{\phi}, \hat{\rho}_{uv}, T \right) < 0.05 \right\}.
\]  

(1.22)

The CR of a nominal 5% two-sided test of \( \beta \neq 0 \) versus \( \beta = 0 \) is

\[
\left\{ \tilde{F} \left( \frac{\hat{\beta}}{\hat{\sigma}_u/\hat{\sigma}_v}; \hat{\phi}, \hat{\rho}_{uv}, T \right) < 0.025 \right\} \cup \left\{ \tilde{F} \left( \frac{\hat{\beta}}{\hat{\sigma}_u/\hat{\sigma}_v}; \hat{\phi}, \hat{\rho}_{uv}, T \right) > 0.975 \right\}.
\]  

(1.23)

In addition to Stambuagh’s \( \hat{\beta} \)-test, substituting alternative estimators of \( \sigma_u/\sigma_v \), \( \phi \) and \( \rho_{uv} \) into (1.21)–(1.23) creates other variations. It is then of great interest to understand whether or not using estimates of the parameters rather than their true unknown values in evaluating these CRs results in intended test sizes, and whether bias-corrected estimators of \( \phi \) can correct any potential size distortion.

Before answering these questions, we need to know how to implement the \( \hat{\beta} \)-test, which requires evaluating the cdf of \( \hat{\beta} \). Stambaugh (1999) uses a numerical integration method proposed by Imhof (1961). This method requires solving for eigenvalues of matrix functions in the integrand. Computation becomes an issue for large sample sizes because one needs to deal with high-order matrices inside the integral. This problem poses a serious impediment to any further study of the properties of the test via Monte Carlo simulation, and it perhaps explains why Stambaugh’s \( \hat{\beta} \)-test has seen limited use since its first introduction.

To conquer this computational obstacle, I propose in Appendix B a novel numerical integration approach which derives from the mathematical result by Gil-Pelaez (1951). Although it shares the same theoretical origin with Stambaugh (1999), this method evaluates \( \tilde{F}(x; \phi, \rho_{uv}, T) \) by leveraging the analytical tractability of its integrand, which leads to a significant improvement in computational efficiency.

To illustrate the computational advantage of this novel integration approach, the following example evaluates \( \tilde{F}(0; 0.99, -0.9, T) \) as a function of the sample size \( T \). That is, this numerical example corresponds to the \( p \)-value under the null in the left-tail one-sided test, where the realized \( \hat{\beta} \) is zero with \( \phi = 0.99 \) and \( \rho_{uv} = -0.9 \). Figure 1.2 Panel A
shows that the $p$-value increases from 0.085229 to 0.275587 as the sample size grows from 60 to 1000. This implies that $\hat{\beta}$ becomes more centered around its true value of zero.

When processed by an Intel Core i5-4300U@1.90GHz CPU, the runtime consumed of evaluating each $p$-value under the proposed integration approach is about $1/500$ of a second. Moreover, the CPU runtime does not depend on the sample size $T$. This presents a stark contrast with the existing method which relies on a brute-force spectral decomposition when determining the integrand. Figure 1.2 Panel B plots the CPU runtime of computing the $p$-value against the sample size used. The sample sizes considered range from $T = 60$ to $T = 1000$ in increments of 10, each represented by a dot on the graph. There is a monotonic increasing pattern between runtime and sample size. The runtime increases from 0.0126 second to 3 seconds as the sample size grows from 60 to 1000. In other words, the runtime under the existing method ranges from 6 to more than 1500 times the runtime under the proposed method. Overlaid on the graph is a cubic polynomial curve fitted to the runtime data. This suggests that the number of calculations involved in the brute-force algorithm for eigenvalues is of order $T^3$. Hence, the proposed method has a clear advantage over the existing one.

### 1.4 The Sizes of Return Predictability Tests

This section performs a Monte Carlo simulation study of the actual rejection rate for various return predictability tests under the null. I consider tests of three alternative hypothesis $\beta > 0$, $\beta < 0$ and $\beta \neq 0$. I start with Stambaugh’s $\hat{\beta}$-test and its variations, and examine how the sizes of various $\hat{\beta}$-tests depend on the key system parameters $\phi$ and $\rho_{uv}$, as well as the sample size $T$. Next, I compare the actual sizes of the $t$-tests and $\hat{\beta}$-tests. In what follows, I present analysis for nominal 5% tests only, as this is a commonly used statistical significance level for hypothesis testing. The comparison results for the nominal 1% tests are qualitatively similar, and therefore they are suppressed for brevity.

#### 1.4.1 Plug-in $\hat{\beta}$-test

I first examine the plain vanilla $\hat{\beta}$-tests constructed by plugging in the OLS parameter estimates into the finite-sample distribution of $\hat{\beta}$. Figure 1.3 plots the actual rejection rate under the null hypothesis for the nominal 5% $\hat{\beta}$-test of $\beta = 0$ vs. $\beta < 0$ with the CR in (1.22). Each panel displays the relationship between the actual size of the test and sample size with $\phi$ and $\rho_{uv}$ held fixed. The actual rejection probabilities are evaluated based on 100,000 Monte Carlo simulations.
Three versions of this one-sided $\hat{\beta}$-test are considered. First, the dashed line corresponds to using true $\phi$ and OLS estimates of $\sigma_u/\sigma_v$ and $\rho_{uv}$ in (1.22). Although infeasible, this test helps us understand the impact of using estimated parameters in constructing Stambaugh’s $\hat{\beta}$-test. Across all panels and under different sample sizes, the actual size (dashed line) closely tracks the nominal size 5%, which is obtained exactly if true parameter values are used in (1.22). This indicates that replacing $\rho_{uv}$ and $\sigma_u/\sigma_v$ with OLS estimates while keeping $\phi$ at its true value has little impact on the intended rejection probability. Therefore, the sampling errors of $\hat{\rho}_{uv}$ and $\hat{\sigma}_u/\hat{\sigma}_v$ are small and unimportant in this hypothesis testing context.

Second, I consider CR (1.22) based on $\hat{\phi}$ and true values of $\sigma_u/\sigma_v$ and $\rho_{uv}$, which corresponds to the dotted line in the panels. It is perhaps surprising to observe that $\hat{\phi}$ alone has caused significant size distortions across a large parameter space and under different sample sizes. Indeed, there is sizable over-rejection if $\phi$ is near unity and $\rho_{uv} = 0.9$ as shown in Panel A. The size distortion does not decrease with sample size. In fact, it increases from 5.2% when $T = 60$ to about 8% when $T = 600$ before starting to decrease with sample size. With a less persistent predictor, the hump-shaped pattern is more obvious as shown in Panel C. For example, if $\phi = 0.95$ and $\rho_{uv} = -0.9$, the increasing curve section ends before $T$ reaches 300. When the predictor is $\phi = 0.7$, the actual size is decreasing in sample size as shown in Panel E. However, with sample size as large as 800, there is still a 1% over-rejection. Comparing the three panels (B, D, F) on the right against the ones on the left, we see that with a $\rho_{uv}$ of smaller magnitude there is a reduction in the actual rejection rate under the null hypothesis. This alleviates the test size distortion under most combinations of $\phi$ and sample size.

Third, the solid line corresponds to the feasible $\hat{\beta}$-test which uses the OLS estimates of $\phi$, $\sigma_u/\sigma_v$ and $\rho_{uv}$ to construct CR (1.22). It is clear from the graphs that the solid line and dotted line almost overlap. This once again indicates that the small sampling errors of $\hat{\rho}_{uv}$ and $\hat{\sigma}_u/\hat{\sigma}_v$ do not lead to noticeable size distortions. The size distortion of the feasible $\hat{\beta}$-test is almost entirely due to using $\hat{\phi}$ in place of its true unknown value.

Figures 1.4 and 1.5 study the actual sizes of the right-tail one-sided and two-sided $\hat{\beta}$-tests, respectively. The previous observation on the small sampling errors of $\hat{\rho}_{uv}$ and $\hat{\sigma}_u/\hat{\sigma}_v$ is again applicable to these tests. In addition, there is clear under-rejection in the right-tail tests when $\hat{\phi}$ is used to compute CR (1.21). For the two-sided test, the degree of size distortion lies between the left-tail and right-tail one-sided tests. Size distortion is again most serious when the predictor is persistent as shown by the first four panels in Figures 1.4 and 1.5. Comparing the left panels against the right panels, we see that the impact on the right-tail test of an increase in $\rho_{uv}$ from $-0.9$ to $-0.7$, all else being
equal, is only a small decrease in the actual test size. By contrast, this impact is more noticeable for the two-sided test.

In summary, the size distortions observed of the feasible one-sided and two-sided $\hat{\beta}$-tests are predominantly attributed to the fact that $\phi$ needs to be estimated. By contrast, the OLS estimates of $\rho_{uv}$ and $\sigma_u/\sigma_v$ have small sampling errors and they have minor impact on the actual size of $\hat{\beta}$-test. Thus improving the accuracy of $\hat{\phi}$ used in constructing these tests is a plausible way to remedy the size distortion of the original Stambaugh’s test.

Before going on to analyze more variations of $\hat{\beta}$-test, I examine in the remainder of this subsection Stambaugh’s plug-in $\hat{\beta}$-tests since it is important to understand the sources of their size distortions. The following example provides an illuminating illustration.

Suppose that the true values of the key system parameters are respectively $\phi = 0.99$ and $\rho_{uv} = -0.9$. I conduct the left-tail one-sided test of $\beta = 0$ vs. $\beta < 0$ using a random sample of size 600. To arrive at the actual rejection rate of this test under the null, I need to compute the probability of a random sample that falls in the CR as stated in (1.22) when $\beta = 0$. Although no closed-form expression exists for its evaluation, I can proceed as follows to understand the determinants of this probability. First, it is suitable to replace $\hat{\sigma}_u/\hat{\sigma}_v$ and $\hat{\rho}_{uv}$ in (1.22) with their respective true values owing to their small sampling variations. Operationally, I study the probability of $\{\tilde{F}(\hat{\beta}; \hat{\phi}, -0.9, 600) < 0.05\}$, where $\hat{\beta}$ is the OLS predictive regression slope estimator for a random sample generated from the model system (1.1) and (1.2) with $\beta = 0$, $\phi = 0.99$, $\rho_{uv} = -0.9$, and $T = 600$.

Second, let us examine how the distribution function $\tilde{F}(x; \phi, -0.9, 600)$ varies for different values of $\phi$. Figure 1.6 plots this cdf when $\phi$ is equal to 0.98, 0.99, or 0.995. Panel A shows that $\tilde{F}(x; 0.995, -0.9, 600)$ (solid line) lies below $\tilde{F}(x; 0.99, -0.9, 600)$ (dashed line) in the displayed left tail region. Consequently, the 5% quantile of the former is greater than the 5% quantile of the latter. This implies that for any random sample that falls into the CR (1.22) conditioning on $\hat{\phi} = 0.99$ in the left-tail $\hat{\beta}$-test, the null hypothesis is also rejected by the one-sided test with a CR conditioning on $\hat{\phi} = 0.995$. That is to say, if $A$ denotes the event $\{\tilde{F}(\hat{\beta}; 0.99, -0.9, 600) < 0.05\}$, and $B$ denotes the event $\{\tilde{F}(\hat{\beta}; 0.995, -0.9, 600) < 0.05\}$, then $P(A) < P(B)$. Now, since $P(A) = 0.05$ by construction, it follows that $P(B) > 0.05$. In other words, using a greater-than-0.99 realized $\hat{\phi}$ to construct the 5% nominal CR in (1.22) will result in a greater-than-5% rejection rate under the null. By the same token, since $\tilde{F}(x; 0.98, -0.9, 600)$ (dash-dotted line)

---

10 These parameter values roughly correspond to using dividend price ratio to predict one-month ahead market returns. For example, see Stambaugh (1999) and Amihud and Hurvich (2004).

11 Without loss of generality, I assume $\alpha = \theta = 0$ and $\sigma_u = \sigma_v = 1$. 
line) lies above \( \tilde{F}(x; 0.99, -0.9, 600) \) (dashed line) in the displayed left tail region, it follows that the 5% nominal one-sided test in (1.22) conditioning on a less-than-0.99 \( \hat{\phi} \) will result in a less-than-5% rejection rate under the null.

Third, the actual rejection rate of the test is a weighted average of the conditional rejection rates. As shown by the dotted and solid lines in Figure 1.3 Panel A, there is an upward size distortion when \( \phi = 0.99, \rho_{uv} = -0.9 \) and \( T = 600 \). Thus the upward size distortion induced by greater-than-0.99 realized \( \hat{\phi} \) values outweighs the downward size distortion induced by less-than-0.99 realized \( \hat{\phi} \) values. From the remaining panels of Figure 1.3, it is rather clear that the dominant effect of using \( \hat{\phi} \) that is greater than its true value in left-tail tests is prevalent across different sample sizes and a range of system parameter values.

The same thinking process can be applied to the right-tail one-sided test of \( \beta = 0 \) vs. \( \beta > 0 \). The CR in (1.21) conditioning on a greater-than-0.99 (less-than-0.99) realized \( \hat{\phi} \) once again results in a greater-than-5% (less-than-5%) rejection rate. Indeed, as illustrated in Panel B of Figure 1.6, \( \tilde{F}(x; 0.995, -0.9, 600) \) (solid line) lies above \( \tilde{F}(x; 0.99, -0.9, 600) \) (dashed line) in the displayed right tail region. Consequently, the 95% quantile of the former is less than the 95% quantile of the latter. It follows that any random draw that is rejected conditioning on \( \hat{\phi} = 0.99 \) in the right-tail \( \hat{\beta} \)-test also falls in the CR based on \( \hat{\phi} = 0.995 \). In contrast with the left tail, the direction of the size distortion in the unconditional rejection rate of the right-tail test is the opposite. The downward size distortion induced by less-than-0.99 realized \( \hat{\phi} \) values dwarfs the upward size distortion induced by greater-than-0.99 realized \( \hat{\phi} \) values, which leads to an overall downward size distortion, as evidenced by the dotted and solid lines in Panel A in Figure 1.4.

Last but not least, I consider the actual rejection rate of the two-sided test. It is clear from (1.23) that the actual rejection rate under the null is the sum of the rejection rates of the nominal 2.5% left-tail and right-tail one-sided tests. Although not graphically represented here for the sake of brevity, similar to their nominal 5% counterparts, the former has an upward size distortion while the latter has a downward size distortion. It follows that the nominal 5% two-sided test has a less severe size distortion as the over-rejection in the left-tail test is somewhat offset by the under-rejection in the right-tail test. As a result, the two-sided test size distortion in any of the displayed cases in Figure 1.5 (dotted and solid lines) is visibly smaller than either of the one-sided tests.
1.4.2 A Comparison of Tests of Predictability

In this subsection, I study the actual sizes of various tests of predictability covering two broad classes of tests: \( t \)-tests and \( \hat{\beta} \)-tests. I consider the adjusted \( t \)-statistic resulting from the AH augmented regression procedure in which \( \hat{\phi}^{KS} \) is plugged in as bias-corrected estimator of \( \phi \). This test statistic is referred to as AH-KS \( t \) in the following. In addition, the OLS \( t \)-statistic is also included in the comparison. For \( \hat{\beta} \)-tests, I investigate whether using bias-corrected and more accurate estimators of \( \phi \) to construct feasible \( \hat{\beta} \)-tests can rectify the size distortion observed of the original Stambaugh’s test. The most promising candidates are \( \hat{\phi}^{KS} \) and \( \hat{\phi}^{II} \), which are shown to have small biases and low MSEs in Section 1.2. Similar to \( \hat{\phi} \) in Stambaugh’s test, both estimators are winsorized below at \(-0.999\) and above at \(0.999\) before being plugged into the CR expressions (1.21)–(1.23).

For the sake of brevity, I drop the word winsorized in the following discussion on \( \hat{\beta} \)-tests. In addition to using various estimators of \( \phi \), I use the OLS estimates of \( \sigma_u/\sigma_v \) and \( \rho_{uv} \) in the CR expressions when constructing the feasible \( \hat{\beta} \)-tests. For ease of exposition, the \( \hat{\beta} \)-tests based on \( \hat{\phi} \), \( \hat{\phi}^{KS} \) and \( \hat{\phi}^{II} \) are labeled as \( \hat{\beta}(\hat{\phi}) \), \( \hat{\beta}(\hat{\phi}^{KS}) \), and \( \hat{\beta}(\hat{\phi}^{II}) \), respectively.

Since the properties of these tests only depend on \( \phi \), \( \rho_{uv} \) and sample size \( T \) under the null, to facilitate the comparison, I visually present the actual rejection rates under different combinations of the two system parameters and sample size in Figures 1.7–1.9. Each panel in these figures displays the relationship between the rejection rate under the null and the sample size \( T \) with the pair \((\phi, \rho_{uv})\) held fixed. The discussions will focus on the cases where \( \phi \) takes a value from the set \( \{0.99, 0.95, 0.7\} \) while \( \rho_{uv} \) can be either \(-0.9\) or \(-0.7\). Setting \( \rho_{uv} \) to a negative value is without loss of generality due to the symmetry property of the statistic \( \hat{\beta} \) under the null (Proposition 3). Take \( \rho_{uv} = -0.9 \) for example. It suffices to swap the following discussion on the left-tail test for the one on the right-tail test if we want to produce results for the case of \( \rho_{uv} = 0.9 \), ceteris paribus. It also follows that the result on the two-sided test remains unchanged to this sign change. In what follows, I discuss in turn the left-tail, right-tail and two-sided tests.

1.4.2.1 Left-tail One-sided Tests

First, I examine the rejection rate under the null hypothesis for various nominal 5% tests of \( \beta = 0 \) vs. \( \beta < 0 \). Figure 1.7 Panel A considers the case of a highly persistent predictor and highly correlated error terms in the model system with \( \phi = 0.99 \) and \( \rho_{uv} = -0.9 \). The graph indicates that the test with the least size distortion is the \( \hat{\beta}(\hat{\phi}) \)-test as shown by the thick solid line. The actual test size of \( \hat{\beta}(\hat{\phi}) \) is reasonably close to 5% in a small sample of up to 200 observations. But the size distortion becomes large
in larger samples. By contrast, the other $\hat{\beta}$-tests exhibit even larger size distortions. As shown by the solid line, $\hat{\beta}(\hat{\phi}^{KS})$ has markedly elevated rejection rates while the dashed line indicates that the $\hat{\beta}(\hat{\phi}^{II})$-test has a worsened over-rejection problem. These observations appear at odds with intuition at first. Table 1.1 shows that $\hat{\phi}^{KS}$ and $\hat{\phi}^{II}$ both have a smaller bias and MSE than the raw OLS estimator. When judged by the bias-reduction performance, $\hat{\phi}^{II}$ clearly outperforms $\hat{\phi}^{KS}$ when $\phi = 0.99$ while their MSEs are similarly smaller than that of $\hat{\phi}$. This observation holds regardless of the winsorization treatment. Despite these estimation improvements, neither of the two resulting $\hat{\beta}$-tests rectifies the size distortion of the original Stambaugh’s test. On the contrary, it appears that a more accurate $\phi$ estimator induces a worse size distortion as the dashed line lies above the solid line which in turn lies above the thick solid line.

Notwithstanding, the analysis in the previous subsection helps to resolve this osten-
sible contradiction. Indeed, both bias-reduction methods correct the bias in the OLS $\hat{\phi}$ essentially by increasing the point estimate through a positive correction term. This positive shift adds to the existing upward size-distorting effect of realized large $\hat{\phi}$ values, which the previous subsection explains using an illustrative example. It then should come as no surprise that using these bias-corrected estimators of $\phi$ to construct $\hat{\beta}$-tests exacerbates the over-rejection problem for the left-tail $\hat{\beta}$ test.

Figure 1.7 further shows that this upward size-distortion problem persists for the $\hat{\beta}$ under a variety of parameter combinations. First, going across the panels, we see that when the magnitude of $\rho_{uv}$ decreases—all else being equal—the actual test sizes of the $\hat{\beta}$-tests are reduced. However, this reduction change appears not very sensitive as $\rho_{uv}$ changes from $-0.9$ to $-0.7$. Second, when the predictor is less autocorrelated, the over-
rejection problem still persists though the difference between the size distortions induced by different estimators of $\phi$ decreases as illustrated in Panels A, C and E. Finally, for theses $\hat{\beta}$-tests, the rankings of their actual test sizes remain unchanged when we vary the values of $\phi$ and $\rho_{uv}$.

Now, let us add the $t$-tests into the comparison. In Panel A with $\phi = 0.99$ and $\rho_{uv} = -0.9$, AH-KS $t$-test (thick dashed line) exhibits a worse over-rejection problem than $\hat{\beta}(\hat{\phi})$. In a sample of 60 observations, the rejection rate is 10% under the null. Contrary to $\hat{\beta}(\hat{\phi})$, the upward size distortion of AH-KS improves as the sample size grows albeit at a slow rate. As the sample size grows to 1000, there is still an elevated rejection rate of 7.5%, which is higher than that of $\hat{\beta}(\hat{\phi})$. On the other hand, the OLS $t$ test exhibits a marked under-rejection problem. The actual rejection rate is less than 2%

\footnote{I have also examined the actual test sizes of these $\hat{\beta}$-tests for a wider range of $\phi$ and $\rho_{uv}$. Their rankings remain unchanged.}
in a sample of up to 1000 observations, which suggests that this left-tail test is excessively conservative.

Comparing the left panels with right panels in Figure 1.7 shows that decreasing the magnitude of $\rho_{uv}$ reduces the size distortion of both $t$-tests. However, the change is small going from $\rho_{uv} = -0.9$ to $\rho = -0.7$. Although not graphically reported, as $\rho_{uv}$ shrinks towards to zero, the two $t$-tests both exhibit a test size close to 5%. This is due to the fact that when $\rho_{uv}$ is zero, the predictive regression equation satisfies the classic OLS regression assumption.

With $\rho_{uv}$ fixed, decreasing $\phi$ alleviates the size distortion of the $t$-tests as we go from top panels to bottom panels in Figure 1.7. When $\phi = 0.95$, AH-KS $t$ starts to exhibit less upward size distortion than $\hat{\beta}(\phi)$ in samples larger than 300. As the predictor becomes less persistent with $\phi = 0.7$, the over-rejection problem of AH-KS $t$ becomes small, and indeed, this test exhibits the least size distortion of all the candidate tests examined.

Consequently, the difference in actual size between the tests is small. The recommendation here is then to use these tests cautiously because all three tests exhibit over-rejection problems. With $\phi = 0.7$, AH corrected $t$-test turns out to have the least size distortion, as the thick solid line is quite close to the 5% horizontal reference line. Therefore, it is recommended in this case.

In sum, no single test consistently exhibits the least size distortion when performing the hypothesis test of $\beta = 0$ vs. $\beta < 0$. When the predictor is persistent and $\rho_{uv}$ is large, none of the tests works satisfactorily in finite samples of up to 1000 observations with a desired rejection rate of 5% under the null. It is the interaction of a large $\phi$ and a large $\rho_{uv}$ magnitude that leads to the poor performances of these tests. When $\phi$ decreases to below 0.7 or $\rho_{uv}$ is close to zero, the AH-KS $t$-test has the least size distortion with only a small risk of over-rejecting which diminishes as the sample size increases.

### 1.4.2.2 Right-tail One-sided Tests

Next, the discussion moves on the right-tail test of $\beta = 0$ vs. $\beta > 0$. This hypothesis testing has been the focus in the existing literature given that the point estimate of the predictive slope often takes a positive value in many real data samples. The analysis presented here provides a better understanding of the actual test sizes of the prominent tests employed in previous studies.

In Figure 1.8, the dash-dotted line representing the OLS $t$-test shows that the standard normal distribution grossly understates the fat right tail of the raw OLS $t$-statistic even in
finite samples when the predictor is persistent and the model error terms are correlated. This confirms the standard knowledge established by Nelson and Kim (1993), Elliot and Stock (1994), and Stambaugh (1999). Due to the interaction of a persistent predictor and highly correlated model error terms, the actual test size of the 5% OLS \( t \)-test is still more than 12% in a sample of 1000 observations.

By contrast, the AH-KS \( t \)-test (thick dashed line) provides the desirable rejection rate close to 5% under the null even when the predictor is persistent and the magnitude of \( \rho_{uv} \) is large. When \( \phi = 0 \), the AH-KS \( t \)-test tends to under-reject. Hence, more credibility can be given to a finding based on the AH-KS \( t \)-test that a predictor is significant at the 5% significance level. Among all the right-tail tests examined, the AH-KS \( t \)-test fares the best delivering the smallest size distortion under various parameter combinations.

Finally, let us take a look at Stambaugh’s \( \hat{\beta} \)-test and its two variations. They all exhibit under-rejection, a situation that appears to improve slowly to sample size increases. Plugging in the two bias-corrected estimators of \( \phi \) does seem to work to some extent by reducing the downward size distortion, which is due to the same mechanism that explains the over-rejection in left-tail \( \hat{\beta} \)-tests. The actual test size based on \( \hat{\phi}^{KS} \) is almost indistinguishable from the one based on \( \hat{\phi}^{H} \). However, there are still serious under-rejection problems suggesting conservatism.\(^\text{13}\)

In short, when performing the right-tail hypothesis testing, AH-KS \( t \)-test exhibits the nearest to intended test size, which is robust under different parameter values. Even with a persistent predictor \( (\phi = 0.99) \) and highly correlated model error terms \( (\rho_{uv} = -0.9) \), the right tail of AH corrected \( t \)-statistic starts to resemble that of the standard normal distribution in a medium-sized sample \( (T > 400) \).

### 1.4.2.3 Two-sided Tests

Last but not least, I consider the two-sided tests, which are illustrated by Figure 1.9. First, the two-sided tests exhibit patterns similar to their left-tail one-sided counterparts. However, the size distortions of these two-sided tests are toned down. This observation can be explained intuitively as follows. Consider \( \hat{\beta} \)-tests with the CR expression in (1.23). It is clear that the CR of the nominal 5% two-sided test is the union of the CRs of the two nominal 2.5% one-sided tests. In an unreported graphical analysis, I find that the nominal 2.5% left-tail \( \hat{\beta} \)-tests have a similar pattern as the \( \hat{\beta} \)-tests in Figure 1.7. On

\(^{13}\)I have also examined the performance of \( \hat{\beta} \)-tests under a wider range of parameter values. Only when \( \rho_{uv} \) is close to zero, the convergence to 5% is reasonably fast in finite samples. However, in these cases, the AH-KS \( t \)-test still exhibits the least size distortion.
the other hand, the nominal 2.5% right-tail \( \hat{\beta} \)-tests remain somewhat flat just like their nominal 5% counterparts in Figure 1.8. It then follows that the sum of two rejection rates under the null – the actual size of the two-sided test – obtained from the nominal 2.5% one-sided tests has a pattern similar to that of the left-tail one-sided test. Moreover, the inability of the bias-corrected estimators of \( \phi \) to rectify the upward size distortion of the two-sided \( \hat{\beta} \)-test results from the larger impact from their respective left-tail rejection rate components. Analogously, the actual size of the \( t \)-tests follows a pattern that lies between the those of the two one-sided tests.

Finally, comparing these tests under different parameter combinations yields the following recommendations. First, when the predictor is highly persistent with \( \phi = 0.99 \), the \( \hat{\beta}(\hat{\phi}) \)-test exhibits the least size distortion in samples larger than 400. Its use should be administered with care as there can still be upward size distortions in samples as large as 1000. Second, as the predictor becomes less persistent, the AH-KS \( t \)-test quickly overtakes \( \hat{\beta} \)-tests as the former has less size distortion though it still produces an over-rejection problem, e.g. when \( \phi = 0.95 \). In such cases, an AH-KS \( t \)-based \( p \)-value may be somewhat understated, and therefore a finding based on the AH-KS \( t \)-test that a predictor is insignificant at the 5% significance level can be considered trustworthy. Third, when the predictor less autocorrelated such as \( \phi = 0.7 \) or \( \rho_{uv} \) is close to zero, the nominal 5% AH-KS \( t \)-test exhibits the intended test size. In such cases, the OLS \( t \)-test works as well.

### 1.5 Empirical Illustration

In this section I use real data to illustrate the estimation and inference methods proposed in the foregoing sections for the common model of predictive regression that is used in various existing studies such as Stambaugh (1999). The dependent variable in the predictive regression is the monthly total return on the S&P 500 value-weighted index in excess of 30-day T-bill rate. I estimate models where stock returns are predicted by popular return predictors in the existing literature which include financial ratios, corporate activity, interest rates and market sentiment. These candidate predictors are examined in their ability to forecast the next-month stock return.

The stock return data start as early as January 1926. The ending month is December 2016, which is dictated by the availability of the predictive variables. Among these, I employ the maintained data set of sixteen predictors of Welch and Goyal (2008) from Amit Goyal’s website.\(^{14}\) Also included are short interest index (SII) studied by Rapach,\(^{14}\)

\(^{14}\)http://www.hec.unil.ch/agoyal/
Ringgenberg, and Zhou (2016) and variance risk premium (VRP) studied by Bollerslev, Tauchen, and Zhou (2008) and Zhou (2009).\textsuperscript{15} The following list provides a brief description for each predictor.\textsuperscript{16}

2. Dividend yield (DY): 12-month moving sum of dividends paid on the S&P 500 index divided by the index level at the start of the 12-month period.
4. 10-year-Earnings-price ratio (E10P): 10-year moving average of real annual earnings on the S&P 500 index divided by the real index level at the end of the 10-year period.
7. Stock Variance (SVAR): Stock variance is computed as sum of squared daily returns on S&P 500 member stocks.
9. Net equity expansion (NTIS): 12-month moving sum of net issues by NYSE-listed stocks divided by the total end-of-year market capitalization of NYSE stocks.
13. Term spread (TMS): LTY minus TBL.
16. Inflation (INFL): Inflation is the Consumer Price Index (All Urban Consumers) from the Bureau of Labor Statistics. Because inflation information is released only in the following month, I substitute the one-month lagged INFL for its contemporaneous value.

\textsuperscript{15}SII is from David Rapach’s website (http://sites.slu.edu/rapachde/home/research). VRP is from Hao Zhou’s website (https://sites.google.com/site/haozhouspersonalhomepage/).
\textsuperscript{16}For details on the original data sources readers are referred to the aforementioned websites.
17. Short interest index (SII): Detrended log aggregate short interest.

For each of the six valuation ratios at the top of this list, I use a logarithmic transformation to reduce its positive skewness.\textsuperscript{17} With most of the predictors on this list, I estimate the model using monthly data series spanning the 91-year period. The exceptions are CSP, SII and VRP. Subject to data availability issues, their respective sample periods are 1937.05–2002.12, 1973.02-2014.12, and 1990.02–2016.12. The estimation and predictability testing results are reported in Table 1.2 and discussed as follows.

First, for each predictor reported in this table, the bias-corrected estimates $\hat{\phi}^{KS}$ and $\hat{\phi}^{II}$ are very close to each other. This is consistent with Table 1.1 which shows that the performances of the two bias-corrected estimators tend to converge as the sample size $T$ grows. The discussion to follow will focus on $\hat{\phi}^{II}$ for brevity. Second, the correction from $\hat{\phi}$ to $\hat{\phi}^{II}$ is small for each predictor, where the change in absolute terms is less than 0.004. Again, this is largely due to the sample size because $4/T$ provides a reasonable upper bound with $T > 1000$. In relative terms, the correction on $\hat{\phi}$ results in a less-than-1% upward adjustment for predictors with a $\hat{\phi}$ close to unity. For weakly autocorrelated predictors, the correction in relative terms is also small. Indeed, the largest percentage adjustment (2.3%) is on LTR.

By contrast, the adjustment from $\hat{\beta}$ to $\hat{\beta}^{II}$ can be large in some cases. With large values of $\hat{\kappa}$, DFY and CSP see the largest corrections from $\hat{\beta}$ to $\hat{\beta}^{II}$ in absolute terms, which are $-0.035$ and $-0.019$, respectively. In relative terms, the largest percentage adjustments are on DE and DP. The bias-corrected estimate of the predictive slope is reduced by more than half of the OLS estimate. This observation has economic relevance. Using the raw OLS estimate is likely to seriously overstate the importance of DE and DP as predictors of stock returns. The bias correction procedures also result in large relative changes in $\hat{\beta}$ for other financial ratios such as DE, BM, EP and E10P. The downward adjustment in the slope estimate ranges from 30% to 50%. By contrast, the adjustment is much smaller in relative terms (less than 10%) for the other predictors.

Next, I consider testing $\beta = 0$ versus $\beta \neq 0$. Using the OLS $t$-test, there are eight predictors that have a statistically significant predictive slope coefficient at the 5% level. Half of these predictors are financial ratios: DP, DY, EP and E10P. Drawing such inferences is based on the assumption that the finite sample bias has become less relevant.

\textsuperscript{17}This is a common practice. See Lewellen (2004) and Amihud and Hurvich (2004).
in a sufficiently large sample where the first-order asymptotics provides a reasonable approxi-

mation. However, as pointed out by the predictability testing rule laid out in the previous section, one needs to consider the persistence of the predictor and the correlation between error terms in the model system when deciding on the appropriate test.

When the predictor is highly persistent (\( \phi > 0.98 \)) and the error terms are highly co-

correlated (\( |\rho_{uv}| > 0.5 \)), it is suitable to perform the \( \hat{\beta} \)-test constructed using the OLS \( \hat{\phi} \). Under other circumstances, the AH-KS \( t \)-test is a robust choice since it is less prone to the over-rejection problem than the OLS \( t \)-test. Only when \( \rho_{uv} \) is close to zero does the OLS \( t \)-test yield correct inferences. In this case, the AH-KS \( t \)-statistic essentially degenerates to the OLS \( t \)-statistic because there will be negligible bias correction due to small \( \hat{\kappa} \), and \( \hat{\text{SE}}(\hat{\beta}) \) is close to \( \text{SE}(\hat{\beta}) \).

Now taking this testing rule to the analysis, we observe that four of the six financial ratios namely, DP, EP, E10P and BM are subject to the \( \hat{\beta} \)-test. For example, DP has a near-unity first-order autocorrelation coefficient with \( \hat{\rho}_{uv} = -0.977 \). In this case, conducting the OLS and AH-KS \( t \)-tests using nominal test sizes may lead to erroneous statistical inferences due to the over-rejection problem, whereas the \( \hat{\beta} \)-test is less prone to this issue. Whilst the OLS \( t \)-statistic implies a statistically significant predictive \( \beta \) for DP with a \( p \)-value of 0.04, the \( \hat{\beta} \)-test produces a \( p \)-value of 0.422 suggesting that this slope coefficient is indistinguishable from zero. The empirical result for the model with E10P as the predictor serves as another example where the over-rejection issue casts doubt on the usual inference based on a \( t \)-test. The two \( t \)-tests both indicate that E10P has a statistically significant ability to forecast stock return one-month ahead with \( p \)-values of 0.004 (OLS) and 0.031 (AH-KS) respectively. By contrast, the \( \hat{\beta} \) test produces an insignificant \( p \)-value of 0.15. The remaining financial ratios and all the other twelve predictors exhibit small \( \hat{\rho}_{uv} \) in magnitude. Thus we employ the AH-KS \( t \)-test. On the whole, Table 1.2 suggests that there is significant statistical evidence on the return-predicting ability of only five predictors: DY, CSP, NTIS, SII and VRP, as highlighted by their \( p \)-values in boldface.

As a robustness check, Tables 1.3 and 1.4 report the estimation and return predictability testing results in the two equal-length subperiods. SII and VRP are excluded in these tables because the first subperiod data are unavailable, and Table 1.2 has already reported their results in the second subperiod. The results on the bias in \( \hat{\beta} \) and its corrected estimates in the two subperiods are qualitatively similar to those in the overall sample period. In relative terms, the largest and economically significant adjustments in \( \hat{\beta} \) are for the financial ratios that are highly persistent and has a small OLS \( \hat{\beta} \) relative to the corresponding \( \hat{\kappa} \). For these financial ratios, the magnitude of the predictive slope
coefficient is markedly smaller after bias correction. Examples are DP, EP, E10P in the two subperiods. Among the other variables, DFY in the first subperiod sees a sizable correction in the predictive slope estimate from $\hat{\beta} = 0.395$ to $\hat{\beta}^{\text{II}} = 0.29$, a 27% reduction. By contrast, the other non-financial-ratio variables see a much attenuated adjustment in the OLS predictive slope estimate.

The return predictability tests in the two subperiods display two different pictures. In both subperiods, DP, EP, E10P and BM have a near-unity first-order autocoeficient estimate $\hat{\phi}$ and a large $\hat{\rho}_{uv}$ in absolute terms, and therefore we should use $\hat{\beta}$ tests. The AH-KS $t$-test is appropriate for the remaining predictors. There are three predictors with a significant predictive slope coefficient at the 5% level in the first subperiod. Among these, DY is the only financial ratio with a $p$-value of 0.013 for the AH-KS test. CSP and NTIS are the other two significant single predictors in this period. In the second subperiod, there is little evidence supporting the predictive ability of any of the financial ratios. Indeed, the $p$-values are markedly larger than 0.3 even with OLS test. Besides the unshown SII and VRP, there are three significant predictors namely SVAR, CSP and DFR in the second subperiod.

In sum, the bias-corrected estimators result in an economically significant reduction in the raw OLS estimate of the predictive slope for financial ratios documented in the finance literature, in the whole sample period and two subperiods. Small relative changes are registered for the other predictors. Based on the predictability testing rule, SVAR, CSP, NTIS, SII, VRP stand out as strong predictors of future stock returns since 1970.

### 1.6 Conclusion

This paper provides improved methods for estimating a predictive regression model system in which the conditional mean of the next-period market return linearly depends on the current value of a single predictive variable and error terms are normal variables. This predictor has a stationary first-order autoregressive dynamic, and its error term is contemporaneously correlated with that of the market return. The assumption of such a data generating process for the stock return and a candidate predictor has been standard in the literature.

Taking the augmented regression approach invented by Amihud and Hurvich (2004), I propose several effectively bias-reducing estimators of the predictive slope which practically eliminate the small-sample bias of the OLS predictive slope and have a high level of accuracy measured by mean squared error. These bias-corrected estimators exhibit
optimal properties under various sample sizes and across a wide range of parameter values covering the empirically relevant case of a highly persistent predictor. Measuring the exact performance of these predictive slope estimators requires evaluating the probability distribution of the ratio of two quadratic forms in multivariate normal random variables. I derive enhanced numerical methods to attain fast computations of performance measures.

I go on to conduct a comprehensive study of the actual sizes of various significance tests in regard to making statistical inferences on the predictive slope using nominal significance levels. I extend the Stambaugh’s one-sided test to a two-sided test, and I also consider an adjusted $t$-statistic based on the AH procedure for a low-bias standard error. The actual rejection rate of a typical test is shown to depend on only two system parameters and the sample size. Drawing evidence from simulations, I propose a testing rule on return predictability depending on how persistent the predictor is and the correlation between contemporaneous return and predictor variables. Finally, I illustrate the use of the proposed estimation tools and testing rule through an empirical study of the return-predicting ability of popular variables documented in the literature.
Table 1.1  Relative biases and RMSEs of corrected estimators of the AR(1) parameter $\phi$

This table reports the relative biases and root mean squared errors (RMSEs) of 6 corrected estimators of the autoregressive parameter, $\phi$, in a stationary AR(1) model with an unknown mean for different values of $\phi$ and sample sizes. Reported for each ($\phi, T$)–pair are the exact bias and RMSE of the least-squares estimator, $\hat{\phi}$, and the bias and RMSE of each corrected estimator relative to (as a percentage of) the bias and RMSE of $\hat{\phi}$, respectively. These percentages are reported under the column headings R.Bias and R.RMSE. The relative biases and RMSEs of the $[-0.999,0.999]$–winsorized versions of the corrected estimators are also reported under the column headings R.Bias(w) and R.RMSE(w), respectively.

<table>
<thead>
<tr>
<th></th>
<th>$T = 60$</th>
<th>$T = 120$</th>
<th>$T = 240$</th>
<th>$T = 600$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w)</td>
<td>(w)</td>
<td>(w)</td>
<td>(w)</td>
<td>(w)</td>
</tr>
<tr>
<td>A. $\phi = 0.99$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\phi}^S$</td>
<td>24.53</td>
<td>70.78</td>
<td>40.18</td>
<td>65.29</td>
</tr>
<tr>
<td>$\hat{\phi}^{KS}$</td>
<td>20.55</td>
<td>70.21</td>
<td>37.94</td>
<td>63.86</td>
</tr>
<tr>
<td>$\hat{\phi}^{AH}$</td>
<td>20.75</td>
<td>70.24</td>
<td>38.05</td>
<td>63.93</td>
</tr>
<tr>
<td>$\hat{\phi}^I$</td>
<td>3.12</td>
<td>72.73</td>
<td>31.31</td>
<td>61.00</td>
</tr>
<tr>
<td>$\hat{\phi}^H$</td>
<td>2.79</td>
<td>74.37</td>
<td>32.54</td>
<td>61.87</td>
</tr>
<tr>
<td>$\hat{\phi}^{HI}$</td>
<td>5.34</td>
<td>72.82</td>
<td>32.47</td>
<td>61.73</td>
</tr>
<tr>
<td>B. $\phi = 0.98$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\phi}^S$</td>
<td>22.49</td>
<td>72.08</td>
<td>35.65</td>
<td>66.45</td>
</tr>
<tr>
<td>$\hat{\phi}^{KS}$</td>
<td>18.42</td>
<td>71.61</td>
<td>33.14</td>
<td>65.09</td>
</tr>
<tr>
<td>$\hat{\phi}^{AH}$</td>
<td>18.62</td>
<td>71.63</td>
<td>33.26</td>
<td>65.16</td>
</tr>
<tr>
<td>$\hat{\phi}^I$</td>
<td>0.93</td>
<td>74.78</td>
<td>25.75</td>
<td>62.70</td>
</tr>
<tr>
<td>$\hat{\phi}^H$</td>
<td>0.89</td>
<td>76.38</td>
<td>27.14</td>
<td>63.53</td>
</tr>
<tr>
<td>$\hat{\phi}^{HI}$</td>
<td>3.24</td>
<td>74.80</td>
<td>27.06</td>
<td>63.38</td>
</tr>
<tr>
<td>C. $\phi = 0.95$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{\phi}^S$</td>
<td>17.89</td>
<td>75.90</td>
<td>24.52</td>
<td>71.80</td>
</tr>
<tr>
<td>$\hat{\phi}^{KS}$</td>
<td>13.57</td>
<td>75.65</td>
<td>21.21</td>
<td>70.82</td>
</tr>
<tr>
<td>$\hat{\phi}^{AH}$</td>
<td>13.78</td>
<td>75.66</td>
<td>21.37</td>
<td>70.87</td>
</tr>
<tr>
<td>$\hat{\phi}^I$</td>
<td>-3.13</td>
<td>80.00</td>
<td>11.73</td>
<td>70.30</td>
</tr>
<tr>
<td>$\hat{\phi}^H$</td>
<td>-2.18</td>
<td>81.24</td>
<td>13.68</td>
<td>70.85</td>
</tr>
<tr>
<td>$\hat{\phi}^{HI}$</td>
<td>-0.59</td>
<td>79.85</td>
<td>13.53</td>
<td>70.71</td>
</tr>
<tr>
<td>D. ( \phi = 0.9 )</td>
<td>( T = 60 )</td>
<td>( T = 120 )</td>
<td>( T = 240 )</td>
<td>( T = 600 )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>(w)</td>
<td>(w)</td>
<td>(w)</td>
<td>(w)</td>
</tr>
<tr>
<td>Bias[( \hat{\phi} )] = -0.067191</td>
<td>Bias[( \hat{\phi} )] = -0.032692</td>
<td>Bias[( \hat{\phi} )] = -0.015958</td>
<td>Bias[( \hat{\phi} )] = -0.006262</td>
<td></td>
</tr>
<tr>
<td>RMSE[( \hat{\phi} )] = 0.105390</td>
<td>RMSE[( \hat{\phi} )] = 0.059509</td>
<td>RMSE[( \hat{\phi} )] = 0.035652</td>
<td>RMSE[( \hat{\phi} )] = 0.019796</td>
<td></td>
</tr>
<tr>
<td>( \hat{\phi}^S )</td>
<td>13.22</td>
<td>81.33</td>
<td>14.52</td>
<td>80.22</td>
</tr>
<tr>
<td>( \hat{\phi}^{KS} )</td>
<td>8.65</td>
<td>81.28</td>
<td>10.25</td>
<td>79.91</td>
</tr>
<tr>
<td>( \hat{\phi}^{AH} )</td>
<td>8.88</td>
<td>81.28</td>
<td>10.46</td>
<td>79.93</td>
</tr>
<tr>
<td>( \hat{\phi}^J )</td>
<td>-4.88</td>
<td>85.58</td>
<td>-0.67</td>
<td>82.01</td>
</tr>
<tr>
<td>( \hat{\phi}^H )</td>
<td>-2.50</td>
<td>85.87</td>
<td>2.03</td>
<td>81.97</td>
</tr>
<tr>
<td>( \hat{\phi}^{HI} )</td>
<td>-2.17</td>
<td>85.23</td>
<td>1.73</td>
<td>81.91</td>
</tr>
<tr>
<td>E. ( \phi = 0.8 )</td>
<td>Bias[( \hat{\phi} )] = -0.058867</td>
<td>Bias[( \hat{\phi} )] = -0.029023</td>
<td>Bias[( \hat{\phi} )] = -0.014359</td>
<td>Bias[( \hat{\phi} )] = -0.005700</td>
</tr>
<tr>
<td></td>
<td>RMSE[( \hat{\phi} )] = 0.109780</td>
<td>RMSE[( \hat{\phi} )] = 0.067237</td>
<td>RMSE[( \hat{\phi} )] = 0.043352</td>
<td>RMSE[( \hat{\phi} )] = 0.025701</td>
</tr>
<tr>
<td>( \hat{\phi}^S )</td>
<td>8.74</td>
<td>88.75</td>
<td>8.75</td>
<td>88.73</td>
</tr>
<tr>
<td>( \hat{\phi}^{KS} )</td>
<td>3.94</td>
<td>88.87</td>
<td>3.95</td>
<td>88.85</td>
</tr>
<tr>
<td>( \hat{\phi}^{AH} )</td>
<td>4.18</td>
<td>88.87</td>
<td>4.19</td>
<td>88.84</td>
</tr>
<tr>
<td>( \hat{\phi}^J )</td>
<td>-3.60</td>
<td>91.16</td>
<td>-3.51</td>
<td>91.06</td>
</tr>
<tr>
<td>( \hat{\phi}^H )</td>
<td>-0.86</td>
<td>90.69</td>
<td>-0.76</td>
<td>90.58</td>
</tr>
<tr>
<td>( \hat{\phi}^{HI} )</td>
<td>-1.21</td>
<td>90.77</td>
<td>-1.13</td>
<td>90.68</td>
</tr>
<tr>
<td>F. ( \phi = 0.7 )</td>
<td>Bias[( \hat{\phi} )] = -0.052538</td>
<td>Bias[( \hat{\phi} )] = -0.026110</td>
<td>Bias[( \hat{\phi} )] = -0.012994</td>
<td>Bias[( \hat{\phi} )] = -0.005180</td>
</tr>
<tr>
<td></td>
<td>RMSE[( \hat{\phi} )] = 0.114750</td>
<td>RMSE[( \hat{\phi} )] = 0.073731</td>
<td>RMSE[( \hat{\phi} )] = 0.049228</td>
<td>RMSE[( \hat{\phi} )] = 0.029965</td>
</tr>
<tr>
<td>( \hat{\phi}^S )</td>
<td>6.66</td>
<td>93.40</td>
<td>6.66</td>
<td>93.40</td>
</tr>
<tr>
<td>( \hat{\phi}^{KS} )</td>
<td>1.75</td>
<td>93.59</td>
<td>1.75</td>
<td>93.59</td>
</tr>
<tr>
<td>( \hat{\phi}^{AH} )</td>
<td>1.99</td>
<td>93.57</td>
<td>1.99</td>
<td>93.57</td>
</tr>
<tr>
<td>( \hat{\phi}^J )</td>
<td>-2.62</td>
<td>94.76</td>
<td>-2.62</td>
<td>94.76</td>
</tr>
<tr>
<td>( \hat{\phi}^H )</td>
<td>-0.44</td>
<td>94.43</td>
<td>-0.44</td>
<td>94.43</td>
</tr>
<tr>
<td>( \hat{\phi}^{HI} )</td>
<td>-0.65</td>
<td>94.50</td>
<td>-0.65</td>
<td>94.50</td>
</tr>
</tbody>
</table>
Table 1.2  Stock return predictive regression results, 1926 – 2016

This table reports the estimation and return predictability test results for the return predictive regression model. The single predictor is assumed to follow an AR(1) process. The dependent variable is the monthly S&P 500 total return in excess of the 1-month T-Bill return from January 1926 to December 2016. The first column lists the popular predictors documented in the finance literature. The first 16 predictors are used by Welch and Goyal (2008). Among these, the first 5 predictors are financial ratios computed for S&P 500 stocks. DP is log dividend-price ratio. DY is log dividend yield. EP is log earnings-price ratio. E10P is log 10-year moving average earnings to price ratio. DE is log dividend-payout ratio. BM is log book-to-market ratio for the Dow Jones Industrial Average. SVAR is the monthly realized variance of S&P 500 stocks. CSP is the cross-sectional beta premium for publicly traded US stocks. NTIS is net equity expansion defined as the ratio of 12-month moving sums of net issues by NYSE-listed stocks to their total market capitalization. TBL is the interest rate on the three-month T-Bill. LTY is the long-term government bond yield. LTR is the return on long-term government bonds. TMS is the long-term government bond yield minus the T-Bill rate. DFY is the difference between Moody’s BAA- and AAA-rated corporate bond yields. DFR is the long-term corporate bond return minus the long-term government bond return. INFL is the one-month lagged log return on the Consumer Price Index for urban consumers. SII is the short interest index for US-listed equities constructed by Rapach et al. (2016). VRP is the variance risk premium constructed by Zhou (2009). The p-values in boldface indicate that the corresponding predictive slope is statistically significant at the 5% level based on the recommended test of $\beta = 0$ vs. $\beta \neq 0$.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>$T$</th>
<th>$\hat{\phi}$</th>
<th>$\hat{\phi}^{KS}$</th>
<th>$\hat{\phi}^H$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}^{KS}$</th>
<th>$\hat{\beta}^H$</th>
<th>$\tilde{\kappa}$</th>
<th>$\tilde{\rho}_{uv}$</th>
<th>OLS $t$-value</th>
<th>AH-KS $t$-value</th>
<th>$\hat{\beta}$-test ($\hat{\phi}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP</td>
<td>1092</td>
<td>0.993</td>
<td>0.997</td>
<td>0.997</td>
<td>0.007</td>
<td>0.004</td>
<td>0.003</td>
<td>-0.963</td>
<td>-0.977</td>
<td>0.040</td>
<td>0.285</td>
<td>0.422</td>
</tr>
<tr>
<td>DY</td>
<td>1092</td>
<td>0.993</td>
<td>0.997</td>
<td>0.997</td>
<td>0.009</td>
<td>0.008</td>
<td>0.008</td>
<td>-0.076</td>
<td>-0.077</td>
<td>0.018</td>
<td>0.022</td>
<td>0.072</td>
</tr>
<tr>
<td>EP</td>
<td>1092</td>
<td>0.987</td>
<td>0.991</td>
<td>0.991</td>
<td>0.009</td>
<td>0.006</td>
<td>0.006</td>
<td>-0.625</td>
<td>-0.766</td>
<td>0.031</td>
<td>0.115</td>
<td>0.213</td>
</tr>
<tr>
<td>E10P</td>
<td>1092</td>
<td>0.994</td>
<td>0.997</td>
<td>0.998</td>
<td>0.012</td>
<td>0.009</td>
<td>0.008</td>
<td>-0.795</td>
<td>-0.658</td>
<td>0.004</td>
<td>0.031</td>
<td>0.150</td>
</tr>
<tr>
<td>DE</td>
<td>1092</td>
<td>0.991</td>
<td>0.995</td>
<td>0.995</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.077</td>
<td>-0.061</td>
<td>0.892</td>
<td>0.936</td>
<td>0.936</td>
</tr>
<tr>
<td>BM</td>
<td>1092</td>
<td>0.993</td>
<td>0.996</td>
<td>0.997</td>
<td>0.006</td>
<td>0.004</td>
<td>0.003</td>
<td>-0.682</td>
<td>-0.785</td>
<td>0.054</td>
<td>0.245</td>
<td>0.368</td>
</tr>
<tr>
<td>SVAR</td>
<td>1092</td>
<td>0.632</td>
<td>0.635</td>
<td>0.635</td>
<td>-0.090</td>
<td>-0.097</td>
<td>-0.097</td>
<td>-2.914</td>
<td>-0.238</td>
<td>0.756</td>
<td>0.735</td>
<td>0.740</td>
</tr>
<tr>
<td>CSP</td>
<td>787</td>
<td>0.979</td>
<td>0.984</td>
<td>0.984</td>
<td>2.078</td>
<td>2.061</td>
<td>2.059</td>
<td>-3.420</td>
<td>-0.037</td>
<td>0.003</td>
<td>0.003</td>
<td>0.010</td>
</tr>
<tr>
<td>NTIS</td>
<td>1080</td>
<td>0.981</td>
<td>0.985</td>
<td>0.985</td>
<td>-0.145</td>
<td>-0.146</td>
<td>-0.146</td>
<td>-0.317</td>
<td>-0.029</td>
<td>0.027</td>
<td>0.025</td>
<td>0.040</td>
</tr>
<tr>
<td>TBL</td>
<td>1092</td>
<td>0.993</td>
<td>0.997</td>
<td>0.998</td>
<td>-0.098</td>
<td>-0.102</td>
<td>-0.102</td>
<td>-1.142</td>
<td>-0.076</td>
<td>0.069</td>
<td>0.058</td>
<td>0.111</td>
</tr>
<tr>
<td>LTY</td>
<td>1092</td>
<td>0.996</td>
<td>0.999</td>
<td>0.999</td>
<td>-0.081</td>
<td>-0.086</td>
<td>-0.086</td>
<td>-2.305</td>
<td>-0.102</td>
<td>0.175</td>
<td>0.146</td>
<td>0.257</td>
</tr>
<tr>
<td>LTR</td>
<td>1091</td>
<td>0.045</td>
<td>0.046</td>
<td>0.046</td>
<td>0.110</td>
<td>0.110</td>
<td>0.110</td>
<td>0.174</td>
<td>0.078</td>
<td>0.107</td>
<td>0.106</td>
<td>0.106</td>
</tr>
<tr>
<td>TMS</td>
<td>1092</td>
<td>0.961</td>
<td>0.965</td>
<td>0.965</td>
<td>0.178</td>
<td>0.178</td>
<td>0.179</td>
<td>0.190</td>
<td>0.013</td>
<td>0.158</td>
<td>0.157</td>
<td>0.175</td>
</tr>
<tr>
<td>DFY</td>
<td>1092</td>
<td>0.975</td>
<td>0.979</td>
<td>0.979</td>
<td>0.399</td>
<td>0.365</td>
<td>0.364</td>
<td>-9.530</td>
<td>-0.269</td>
<td>0.094</td>
<td>0.127</td>
<td>0.162</td>
</tr>
<tr>
<td>DFR</td>
<td>1091</td>
<td>-0.120</td>
<td>-0.119</td>
<td>-0.119</td>
<td>0.135</td>
<td>0.136</td>
<td>0.136</td>
<td>0.629</td>
<td>0.155</td>
<td>0.267</td>
<td>0.266</td>
<td>0.266</td>
</tr>
<tr>
<td>INFL</td>
<td>1091</td>
<td>0.482</td>
<td>0.484</td>
<td>0.484</td>
<td>-0.343</td>
<td>-0.344</td>
<td>-0.344</td>
<td>-0.457</td>
<td>-0.039</td>
<td>0.266</td>
<td>0.264</td>
<td>0.265</td>
</tr>
<tr>
<td>SII</td>
<td>503</td>
<td>0.960</td>
<td>0.967</td>
<td>0.968</td>
<td>0.000</td>
<td>0.005</td>
<td>0.005</td>
<td>0.000</td>
<td>0.030</td>
<td>0.015</td>
<td>0.016</td>
<td>0.030</td>
</tr>
<tr>
<td>VRP</td>
<td>323</td>
<td>0.270</td>
<td>0.275</td>
<td>0.275</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.080</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>
Table 1.3  Stock return predictive regression results, 1926.01 – 1970.06

This table reports the estimation and return predictability test results for the return predictive regression model over the period 1926.01-1970.06. The single predictor is assumed to follow an AR(1) process. The dependent variable is the monthly S&P 500 total return in excess of the 1-month T-Bill return. The first column lists the popular predictors documented in the finance literature. The first 16 predictors are used by Welch and Goyal (2008). Among these, the first 5 predictors are financial ratios computed for S&P 500 stocks. DP is log dividend-price ratio. DY is log dividend yield. EP is log earnings-price ratio. E10P is log 10-year moving average earnings to price ratio. DE is log dividend-payout ratio. BM is log book-to-market ratio for the Dow Jones Industrial Average. SVAR is the monthly realized variance of S&P 500 stocks. CSP is the cross-sectional beta premium for publicly traded US stocks. NTIS is net equity expansion defined as the ratio of 12-month moving sums of net issues by NYSE-listed stocks to their total market capitalization. TBL is the interest rate on the three-month T-Bill. LTY is the long-term government bond yield. LTR is the return on long-term government bonds. TMS is the long-term government bond yield minus the T-Bill rate. DFY is the difference between Moody’s BAA- and AAA-rated corporate bond yields. DFR is the long-term corporate bond return minus the long-term government bond return. INFL is the one-month lagged log return on the Consumer Price Index for urban consumers. The p-values in boldface indicate that the corresponding predictive slope is statistically significant at the 5% level based on the recommended test of $\beta = 0$ vs. $\beta \neq 0$.

<table>
<thead>
<tr>
<th>Predictor</th>
<th>T</th>
<th>$\hat{\varphi}$</th>
<th>$\hat{\varphi}_{KS}$</th>
<th>$\hat{\varphi}_{II}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}_{KS}$</th>
<th>$\hat{\beta}_{II}$</th>
<th>$\hat{\kappa}$</th>
<th>$\hat{\rho}_{av}$</th>
<th>OLS t p-value</th>
<th>AH-KS t p-value</th>
<th>$\hat{\beta}$-test (\hat{\varphi}) p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DP</td>
<td>546</td>
<td>0.980</td>
<td>0.987</td>
<td>0.988</td>
<td>0.018</td>
<td>0.011</td>
<td>0.010</td>
<td>-0.958</td>
<td>-0.971</td>
<td>0.040</td>
<td>0.212</td>
<td>0.322</td>
</tr>
<tr>
<td>DY</td>
<td>546</td>
<td>0.980</td>
<td>0.987</td>
<td>0.988</td>
<td>0.023</td>
<td>0.022</td>
<td>0.022</td>
<td>-0.091</td>
<td>-0.092</td>
<td>0.010</td>
<td>0.013</td>
<td>0.038</td>
</tr>
<tr>
<td>EP</td>
<td>546</td>
<td>0.978</td>
<td>0.985</td>
<td>0.986</td>
<td>0.023</td>
<td>0.017</td>
<td>0.016</td>
<td>-0.899</td>
<td>-0.936</td>
<td>0.007</td>
<td>0.055</td>
<td>0.167</td>
</tr>
<tr>
<td>E10P</td>
<td>546</td>
<td>0.984</td>
<td>0.991</td>
<td>0.992</td>
<td>0.034</td>
<td>0.028</td>
<td>0.027</td>
<td>-0.792</td>
<td>-0.652</td>
<td>0.000</td>
<td>0.002</td>
<td>0.054</td>
</tr>
<tr>
<td>DE</td>
<td>546</td>
<td>0.995</td>
<td>0.999</td>
<td>0.999</td>
<td>-0.010</td>
<td>-0.010</td>
<td>-0.102</td>
<td>-0.035</td>
<td>0.391</td>
<td>0.371</td>
<td>0.567</td>
<td>0.118</td>
</tr>
<tr>
<td>BM</td>
<td>546</td>
<td>0.977</td>
<td>0.984</td>
<td>0.985</td>
<td>0.027</td>
<td>0.021</td>
<td>0.020</td>
<td>-0.917</td>
<td>-0.914</td>
<td>0.002</td>
<td>0.020</td>
<td>0.118</td>
</tr>
<tr>
<td>SVAR</td>
<td>546</td>
<td>0.719</td>
<td>0.724</td>
<td>0.724</td>
<td>0.386</td>
<td>0.368</td>
<td>0.368</td>
<td>-3.212</td>
<td>-0.230</td>
<td>0.353</td>
<td>0.377</td>
<td>0.378</td>
</tr>
<tr>
<td>CSP</td>
<td>409</td>
<td>0.962</td>
<td>0.972</td>
<td>0.973</td>
<td>2.326</td>
<td>2.294</td>
<td>2.291</td>
<td>-3.335</td>
<td>-0.044</td>
<td>0.024</td>
<td>0.026</td>
<td>0.053</td>
</tr>
<tr>
<td>NTIS</td>
<td>534</td>
<td>0.976</td>
<td>0.984</td>
<td>0.984</td>
<td>-0.265</td>
<td>-0.268</td>
<td>-0.268</td>
<td>-0.366</td>
<td>-0.033</td>
<td>0.009</td>
<td>0.009</td>
<td>0.027</td>
</tr>
<tr>
<td>TBL</td>
<td>546</td>
<td>0.994</td>
<td>0.999</td>
<td>0.999</td>
<td>-0.207</td>
<td>-0.210</td>
<td>-0.210</td>
<td>-0.518</td>
<td>-0.018</td>
<td>0.166</td>
<td>0.161</td>
<td>0.282</td>
</tr>
<tr>
<td>LTY</td>
<td>546</td>
<td>1.002</td>
<td>0.999</td>
<td>0.999</td>
<td>-0.334</td>
<td>-0.322</td>
<td>-0.322</td>
<td>-4.365</td>
<td>-0.076</td>
<td>0.168</td>
<td>0.185</td>
<td>0.245</td>
</tr>
<tr>
<td>LTR</td>
<td>545</td>
<td>-0.038</td>
<td>-0.037</td>
<td>-0.037</td>
<td>0.107</td>
<td>0.108</td>
<td>0.108</td>
<td>0.368</td>
<td>0.086</td>
<td>0.563</td>
<td>0.561</td>
<td>0.560</td>
</tr>
<tr>
<td>TMS</td>
<td>546</td>
<td>0.976</td>
<td>0.984</td>
<td>0.984</td>
<td>0.271</td>
<td>0.267</td>
<td>0.267</td>
<td>-0.616</td>
<td>-0.020</td>
<td>0.324</td>
<td>0.333</td>
<td>0.391</td>
</tr>
<tr>
<td>DFY</td>
<td>546</td>
<td>0.978</td>
<td>0.986</td>
<td>0.986</td>
<td>0.395</td>
<td>0.300</td>
<td>0.290</td>
<td>-13.107</td>
<td>-0.370</td>
<td>0.209</td>
<td>0.341</td>
<td>0.400</td>
</tr>
<tr>
<td>DFR</td>
<td>545</td>
<td>-0.242</td>
<td>-0.242</td>
<td>-0.242</td>
<td>-0.054</td>
<td>-0.054</td>
<td>-0.054</td>
<td>0.434</td>
<td>0.810</td>
<td>0.811</td>
<td>0.810</td>
<td>0.810</td>
</tr>
<tr>
<td>INFL</td>
<td>545</td>
<td>0.417</td>
<td>0.421</td>
<td>0.421</td>
<td>-0.454</td>
<td>-0.455</td>
<td>-0.455</td>
<td>-0.277</td>
<td>-0.026</td>
<td>0.283</td>
<td>0.282</td>
<td>0.282</td>
</tr>
</tbody>
</table>
Table 1.4 Stock return predictive regression results, 1970.07 – 2016.12

This table reports the estimation and return predictability test results for the return predictive regression model over the period 1970.07 – 2016.12. The single predictor is assumed to follow an AR(1) process. The dependent variable is the monthly S&P 500 total return in excess of the 1-month T-Bill return. The first column lists the popular predictors documented in the finance literature. The first 16 predictors are used by Welch and Goyal (2008). Among these, the first 5 predictors are financial ratios computed for S&P 500 stocks. DP is log dividend-price ratio. DY is log dividend yield. EP is log earnings-price ratio. E10P is log 10-year moving average earnings to price ratio. DE is log dividend-payout ratio. BM is log book-to-market ratio for the Dow Jones Industrial Average. SVAR is the monthly realized variance of S&P 500 stocks. CSP is the cross-sectional beta premium for publicly traded US stocks. NTIS is net equity expansion defined as the ratio of 12-month moving sums of net issues by NYSE-listed stocks to their total market capitalization. TBL is the interest rate on the three-month T-Bill. LTY is the long-term government bond yield. LTR is the return on long-term government bonds. TMS is the long-term government bond yield minus the T-Bill rate. DFY is the difference between Moody’s BAA- and AAA-rated corporate bond yields. DFR is the long-term corporate bond return minus the long-term government bond return. INFL is the one-month lagged log return on the Consumer Price Index for urban consumers. The $p$-values in boldface indicate that the corresponding predictive slope is statistically significant at the 5% level based on the recommended test of $\beta = 0$ vs. $\beta \neq 0$.

| Predictor | $T$ | $\hat{\phi}$ | $\hat{\phi}^{KS}$ | $\hat{\phi}^{II}$ | $\hat{\beta}$ | $\hat{\beta}^{KS}$ | $\hat{\beta}^{II}$ | $\hat{\kappa}$ | $\hat{\rho}_{av}$ | OLS $t$-value | AH-KS $t$-value | $\hat{\beta}$-test ($\hat{\phi}$) |
|-----------|-----|-------------|------------------|------------------|---------------|------------------|------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| DP        | 546 | 0.995       | 0.909            | 0.999            | 0.004         | 0.000            | 0.000            | −0.975         | −0.989         | 0.381          | 0.968          | 0.675          |
| DY        | 546 | 0.995       | 0.986            | 0.999            | 0.004         | 0.004            | 0.004            | −0.033         | −0.033         | 0.342          | 0.359          | 0.536          |
| EP        | 546 | 0.998       | 0.998            | 0.999            | 0.002         | −0.001           | −0.001           | −0.366         | −0.566         | 0.648          | 0.826          | 0.922          |
| E10P      | 546 | 0.998       | 0.998            | 0.999            | 0.003         | 0.001            | 0.001            | −0.789         | −0.666         | 0.539          | 0.739          | 0.654          |
| DE        | 546 | 0.985       | 0.992            | 0.993            | 0.003         | 0.002            | 0.002            | −0.074         | −0.096         | 0.631          | 0.701          | 0.733          |
| BM        | 546 | 0.994       | 0.999            | 0.999            | 0.000         | −0.002           | −0.002           | −0.441         | −0.628         | 0.895          | 0.581          | 0.586          |
| SVAR      | 546 | 0.462       | 0.467            | 0.467            | −1.036        | −1.050           | −1.050           | −3.065         | −0.298         | 0.008          | 0.008          | 0.007          |
| CSP       | 378 | 0.954       | 0.964            | 0.965            | 5.243         | 5.225            | 5.224            | −1.717         | −0.013         | 0.015          | 0.016          | 0.029          |
| NTIS      | 546 | 0.982       | 0.989            | 0.990            | −0.044        | −0.046           | −0.046           | −0.309         | −0.028         | 0.640          | 0.623          | 0.650          |
| TBL       | 546 | 0.992       | 0.999            | 0.999            | −0.070        | −0.078           | −0.078           | −1.275         | −0.136         | 0.202          | 0.152          | 0.235          |
| LTY       | 546 | 0.995       | 0.999            | 0.999            | −0.049        | −0.056           | −0.056           | −2.050         | −0.152         | 0.469          | 0.403          | 0.450          |
| LTR       | 546 | 0.053       | 0.055            | 0.055            | 0.117         | 0.118            | 0.118            | 0.139          | 0.098          | 0.053          | 0.053          | 0.052          |
| TMS       | 546 | 0.948       | 0.956            | 0.956            | 0.209         | 0.212            | 0.212            | 0.381          | 0.040          | 0.104          | 0.100          | 0.119          |
| DFY       | 546 | 0.963       | 0.970            | 0.971            | 0.382         | 0.386            | 0.386            | −1.986         | −0.056         | 0.357          | 0.375          | 0.402          |
| DFR       | 546 | −0.037      | −0.035           | −0.035           | 0.262         | 0.263            | 0.263            | 0.738          | 0.249          | 0.039          | 0.038          | 0.038          |
| INFL      | 546 | 0.610       | 0.616            | 0.616            | 0.095         | 0.089            | 0.089            | −1.204         | −0.083         | 0.847          | 0.857          | 0.855          |
Figure 1.1 Bias of $\hat{\phi}$ and its approximations

This figure plots $h(\phi, T)$, the exact bias in $\hat{\phi}$, and its four approximations: $h^{\text{MPK}}$, $h^{\text{KP}}$, $h^I$ and $h^\text{II}$. $h^{\text{MPK}}$ is derived by Marriott and Pope (1954) and Kendall (1954), and $h^{\text{KP}}$ is derived by Kiviet and Phillips (2012). $h^I$ and $h^\text{II}$ are proposed in this study. The two upper panels display the case of a highly persistent predictor with $\phi = 0.99$. The relationship between sample size and the bias and its approximations is presented graphically in Panel A. Panel B plots the relative errors (in percentage) of the four approximations with respect to the exact bias $h(\phi, T)$. Panels C and D display the case of $\phi = 0.7$, in parallel to the first two panels.
Figure 1.2 Evaluation of $p$-values

This figure plots $\hat{F}(0; 0.99, -0.9, T)$ and the computer runtime under the existing brute-force integration algorithm as functions of the sample size $T$. Panel A plots the exact value of $\hat{F}(0; 0.99, -0.9, T)$, i.e. the $p$-value under the null in the left-tail one-sided test, where the realized $\hat{\beta}$ is zero with $\phi = 0.99$ and $\rho_{uv} = -0.9$. Panel B visually reports the computer runtime in seconds (processed by an Intel Core i5-4300U@1.90GHz CPU) of evaluating the $p$-values under the brute-force algorithm. The sample sizes range from $T = 60$ to $T = 1000$ in increments of 10, each represented by a round dot on the graph. The solid line overlaid on the graph is a cubic polynomial curve fitted to the runtime data.
Figure 1.3  Actual size of Stambaugh’s nominal 5% left-tail one-sided $\beta$-test

This figure plots the actual rejection rate under the null hypothesis for the nominal 5% $\hat{\beta}$-test of $\beta = 0$ vs. $\beta < 0$. The rejection probability under the null only depends on $\phi$, $\rho_{uv}$ and sample size $T$. Each panel displays the relationship between actual test size and sample size with $\phi$ and $\rho_{uv}$ held fixed. Three variations of the $\hat{\beta}$-test are considered. The dashed line corresponds to the CR in (1.22) using true $\phi$ and OLS estimates of $\sigma_u/\sigma_v$ and $\rho_{uv}$. The dotted line corresponds to the CR in (1.22) using $\hat{\phi}$ and true values of $\sigma_u/\sigma_v$ and $\rho_{uv}$. The solid line corresponds to the CR in (1.22) using OLS estimates of $\phi$, $\sigma_u/\sigma_v$ and $\rho_{uv}$. The actual rejection probabilities are evaluated based on 100,000 Monte Carlo simulations.
Figure 1.4  Actual size of Stambaugh’s nominal 5% right-tail one-sided $\hat{\beta}$-test

This figure plots the actual rejection rate under the null hypothesis for the nominal 5% $\hat{\beta}$-test of $\beta = 0$ vs. $\beta > 0$. The rejection probability under the null only depends on $\phi$, $\rho_{uv}$ and sample size $T$. Each panel displays the relationship between actual test size and sample size with $\phi$ and $\rho_{uv}$ held fixed. Three variations of the $\hat{\beta}$-test are considered. The dashed line corresponds to the CR in (1.21) using true $\phi$ and OLS estimates of $\sigma_u/\sigma_v$ and $\rho_{uv}$. The dotted line corresponds to the CR in (1.21) using $\hat{\phi}$ and true values of $\sigma_u/\sigma_v$ and $\rho_{uv}$. The solid line corresponds to the CR in (1.21) using OLS estimates of $\phi$, $\sigma_u/\sigma_v$ and $\rho_{uv}$. The actual rejection probabilities are evaluated based on 100,000 Monte Carlo simulations.
Figure 1.5  Actual size of Stambaugh’s nominal 5\% two-sided $\hat{\beta}$-test

This figure plots the actual rejection rate under the null hypothesis for the nominal 5\% $\hat{\beta}$-test of $\beta = 0$ vs. $\beta \neq 0$. The rejection probability under the null only depends on $\phi$, $\rho_{uv}$ and sample size $T$. Each panel displays the relationship between actual test size and sample size with $\phi$ and $\rho_{uv}$ held fixed. Three variations of the $\hat{\beta}$-test are considered. The dashed line corresponds to the CR in (1.23) using true $\phi$ and OLS estimates of $\sigma_u/\sigma_v$ and $\rho_{uv}$. The dotted line corresponds to the CR in (1.23) using $\hat{\phi}$ and true values of $\sigma_u/\sigma_v$ and $\rho_{uv}$. The solid line corresponds to the CR in (1.23) using OLS estimates of $\phi$, $\sigma_u/\sigma_v$ and $\rho_{uv}$. The actual rejection probabilities are evaluated based on 100,000 Monte Carlo simulations.
Figure 1.6  Cumulative distribution function of $\tilde{\beta}$

This figure plots $\tilde{F}(x; \phi, \rho_{uv}, T)$, the cdf of $\tilde{\beta}$, where $\rho_{uv} = -0.9$ and $T = 600$. The cdfs under four values of parameter $\phi$ are considered: 0.995 (solid line), 0.99 (dashed line), and 0.98 (dash-dotted line). Panel A plots the left tails of the probability distributions. Panel B plots the right tails of the probability distributions.
Figure 1.7  Actual sizes of nominal 5% left-tail one-sided tests

This figure displays the actual rejection rate under the null hypothesis for various nominal 5% tests of $\beta = 0$ vs. $\beta < 0$. The tests are three variations of Stambaugh’s $\hat{\beta}$-test, OLS $t$-test and AH-KS corrected $t$-test. Each panel plots the actual test sizes against the sample size with $\phi$ and $\rho_{uv}$ held fixed. The thick solid line corresponds to the $\hat{\beta}$-test with CR (1.22) using OLS estimator $\hat{\phi}$. The solid line corresponds to the $\hat{\beta}$-test with CR (1.22) using bias-corrected estimator $\hat{\phi}^{KS}$ proposed by Kothari and Shanken (1997). The dashed line corresponds to the $\hat{\beta}$-test with CR (1.22) using bias-corrected estimator $\hat{\phi}^{II}$ proposed in this study. All of the three estimators of $\phi$ have been winsorized by the interval $[-0.999, 0.999]$ before being plugged into the CR expressions. In addition, the OLS estimators of $\sigma_u/\sigma_v$ and $\rho_{uv}$ are used in the CR expressions. The dash-dotted line corresponds to the OLS $t$-test. The thick dashed line corresponds to the AH-KS corrected $t$-test. The horizontal dotted line is the 5% reference line. The actual rejection probabilities under the null are evaluated based on 100,000 Monte Carlo simulations.
Figure 1.8  Actual sizes of nominal 5% right-tail one-sided tests

This figure plots the actual rejection rate under the null hypothesis for various nominal 5% tests of $\beta = 0$ vs. $\beta > 0$. The tests are three variations of Stambaugh’s $\hat{\beta}$-test, OLS $t$-test and AH-KS corrected $t$-test. Each panel plots the actual test sizes against the sample size with $\phi$ and $\rho_{\text{uv}}$ held fixed. The thick solid line corresponds to the $\hat{\beta}$-test with CR (1.21) using OLS estimator $\hat{\phi}$. The solid line corresponds to the $\hat{\beta}$-test with CR (1.21) using bias-corrected estimator $\hat{\phi}^{\text{KS}}$ proposed by Kothari and Shanken (1997). The dashed line corresponds to the $\hat{\beta}$-test with CR (1.21) using bias-corrected estimator $\hat{\phi}^{\text{HI}}$ proposed in this study. All of the three estimators of $\phi$ have been winsorized by the interval $[-0.999, 0.999]$ before being plugged into the CR expressions. In addition, the OLS estimators of $\sigma_u/\sigma_v$ and $\rho_{\text{uv}}$ are used in the CR expressions. The dash-dotted line corresponds to the OLS $t$-test. The thick dashed line corresponds to the AH-KS corrected $t$-test. The horizontal dotted line is the 5% reference line. The actual rejection probabilities under the null are evaluated based on 100,000 Monte Carlo simulations.
Figure 1.9  Actual sizes of nominal 5% two-sided tests

This figure plots the actual rejection rate under the null hypothesis for various nominal 5% tests of \( \beta = 0 \) vs. \( \beta \neq 0 \). The tests are three variations of Stambaugh’s \( \hat{\beta} \)-test, OLS \( t \)-test and AH-KS corrected \( t \)-test. Each panel plots the actual test sizes against the sample size with \( \phi \) and \( \rho_{uv} \) held fixed. The thick solid line corresponds to the \( \hat{\beta} \)-test with CR (1.23) using OLS estimator \( \hat{\phi} \). The solid line corresponds to the \( \hat{\beta} \)-test with CR (1.23) using bias-corrected estimator \( \hat{\phi}^{KS} \) proposed by Kothari and Shanken (1997). The dashed line corresponds to the \( \hat{\beta} \)-test with CR (1.23) using bias-corrected estimator \( \hat{\phi}^{III} \) proposed in this study. All of the three estimators of \( \phi \) have been winsorized by the interval \([-0.999, 0.999]\) before being plugged into the CR expressions. In addition, the OLS estimators of \( \sigma_u/\sigma_v \) and \( \rho_{uv} \) are used in the CR expressions. The dash-dotted line corresponds to the OLS \( t \)-test. The thick dashed line corresponds to the AH-KS corrected \( t \)-test. The horizontal dotted line is the 5% reference line. The actual rejection probabilities under the null are evaluated based on 100,000 Monte Carlo simulations.
Chapter 2

Option Illiquidity: Determinants and Implications for Stock Returns

2.1 Introduction

The market microstructure literature has extensively studied the costs of market making in equity markets. It is well established that bid-ask spreads in the stock market increase with dealers’ inventory risk (Amihud and Mendelson (1980); Ho and Stoll (1983); Stoll (1989)) and asymmetric information costs (Copeland and Galai (1983); Glosten and Milgrom (1985); Easley and O’Hara (1987)). Besides understanding the market structure of traded securities, explaining economic sources behind illiquidity, which is conventionally measured by bid-ask spreads, has been the focus of a large body of research. The consensus is that illiquidity affects asset prices.\(^1\)

Recent equity option market literature documents the impact of market frictions and trading costs on equity option prices. Bollen and Whaley (2004) find that option-induced order imbalances exert an impact on option-implied volatilities, suggesting that market makers are not able to absorb all order flows without moving prices. Garleanu, Pedersen and Potesihman (2009) argue that end-users’ net demand pressures significantly affect option

---

prices. Finally, Christoffersen, Goyenko, Jacobs, and Karoui (2018) document the presence of illiquidity premia in individual equity option returns. Therefore, similar to the equity market, transaction costs in the option markets affect asset valuations.

This chain of evidence demands closer examinations of the option market’s illiquidity. While option bid-ask spreads reflect dealers’ total cost of providing liquidity, little is known about the determinants affecting its dynamics. Moreover, because option illiquidity affects option valuations, it should have implications for pricing the underlying assets since the valuations of both securities are related. The implication of option illiquidity for the underlying’s return is particularly important because increasing evidence points to informed trading in the option market.2

Unlike previous studies, we use a comprehensive data set of intraday option trades to empirically validate the economic significance of the determinants that have been theoretically argued to affect option illiquidity. We use the direct measure of option illiquidity, which is the relative (quoted as well as effective) bid-ask spreads obtained from intraday transactions and quotes.

All option exchanges must report their intraday trades for each option series via the Options Price Reporting Authority (OPRA). We obtain intraday option transaction-level data from LiveVol, a commercial data vendor who processes the OPRA data. This includes the national best bid and offer (NBBO) quotes at the time of the trade, the execution price, and the trading volume. The LiveVol data begin in January 2004, and therefore our sample period is from 2004 to 2013. The sample consists of 2,504 trading days. Our empirical analysis focuses on the option contracts written on the firms that make up the S&P 500 index. This sample represents the most liquid and tradable option contracts in the equity option market. Overall, more than 629 million option trades are used in our analyses.

Examining the determinants of option illiquidity, we find that market makers’ cost of establishing initial delta-hedged positions for their contracts significantly widens option bid-ask spreads. Further, as shown by Leland (1985), maintaining the initial-hedged position is risky due to market frictions and because market makers must continually rebalance their positions. We empirically confirm Leland’s (1985) theory that the rebalancing cost significantly widens option bid-ask spreads. In fact, the economic significance of the rebalancing cost dominates the initial delta-hedging cost by about fourfold.

The existing literature has largely ignored or underestimated the effect of rebalancing

2For examples, see Easley, O’Hara and Srinivas (1998), and Pan and Poteshman (2006).
cost on option bid-ask spreads. We believe the difference between our finding and those by previous studies is due to the inclusion of the recent 2009-2013 period, which we are among the first to empirically examine. Importantly, the option market over this sub-period has become more competitive and more liquid as measured by lower bid-ask spreads, lower effective-to-quoted spreads ratios, and higher trading volume.

We also find that option illiquidity significantly increases with the magnitude of option-induced order imbalance, which is used as a proxy for inventory risk. The other important factor affecting option illiquidity is adverse selection cost, which increases with the amount of private-information-driven trading in the underlying security. We measure adverse selection cost faced by option dealers using the probability of informed trading measure, PIN, introduced by Easley, Kiefer, O’Hara, and Paperman (1996).

Our next set of empirical analyses focus on the economic role of option illiquidity in reflecting informed trading. First, we examine how option market makers revise their quotes in response to events that are well-known associates of significant information asymmetry. Motivated by Kim and Verrecchia (1994), we test the prediction that higher option trading volume on earnings announcement dates is accompanied by aggressive widening of option bid-ask spreads because dealers demand compensation for providing liquidity when they run the risk of trading against informed investors. Consistent with their theory, we find that higher abnormal option trading volume around earnings announcements is accompanied by a jump in option bid-ask spreads, often economically large one day before the earnings announcement. This finding holds for both positive and negative earnings surprises. Overall, option illiquidity and trading volume simultaneously spike when information-driven trading in the option market significantly intensifies.

Second, we examine the implication of option illiquidity as a measure of informed trading activity. The market microstructure theories argue that dealers should widen their bid-ask quotes after observing an abnormal order flow, i.e. order imbalance, to compensate for the risk of providing liquidity to informed traders (see e.g., Glosten and Milgrom (1985)). However, as shown by Muravyev (2016), abnormal order flows calculated from option trades do not always signal the presence of informed trading. In particular, he finds that variations in option’ order flows are due to inventory shocks and private information, but importantly, the former effect often dominates. Therefore, we expect option spreads to widen more quickly when abnormal order flows are motivated by informed option trades, and these order flows contain information about their underlying stock returns.

To test the above prediction, we calculate abnormal order flow induced by option trades
using the option-induced order-imbalance (OOI) measure of Hu (2014), which has been shown to positively predict the underlying’s next-day return. We verify the predictive ability of the OOI measure in our sample. Using the regression framework, we find the OOI measure positively and significantly predicts stock returns the next day. However, its predictive ability disappears when we add the cross-interaction term between the OOI measure and change in option illiquidity. In this case, the cross-interacted term becomes the leading variable that predicts the next-day return. Thus, higher OOI, associated with an excess of synthetic long positions on the underlying, positively predicts stock returns when accompanied by increasing option illiquidity. Similarly, lower OOI, associated with an excess of synthetic short positions on the underlying, negatively predicts stock returns when accompanied by increasing option illiquidity. These findings suggest that option order flows predict their underlying stock returns only when the imbalance is driven by informed option trading as signaled by increasing option illiquidity. Therefore, shocks to option illiquidity help to identify option-induced imbalances that are driven by private information from those that are liquidity-demand-driven such as inventory shocks (Muravyev (2016)) and investors’ disagreements (Choy and Wei (2012)).

We find that a daily long-short equity strategy that buys high-OOI stocks and sell low-OOI stocks from 2004–2013 earns a risk-adjusted return of 10% per year. However, the same strategy earns up to 16.35% per year if we focus only on a subset of stocks with aggressively widening option bid-ask spreads. The economic benefit of using option illiquidity to identify abnormal order flow containing private information improves over the recent period when the option market became more competitive, 2009–2013, for which the strategy generates an annualized alpha of 18.7%.³

The profits from the high-minus-low OOI strategy disappear on the second day after portfolio formation if we do not focus on the stocks experiencing significant increases in option illiquidity. This finding suggests that the return-predicting ability of option-induced order flow is transitory. However, we find a significant portfolio alpha of 6.3% per year when trades are executed on the second day after portfolio formation for the strategy that focuses on high-option-illiquidity stocks. Overall, our portfolio trading results show a sizable economic benefit of using changes in option illiquidity to identify option trades that are likely to contain private information.

³An important regulatory change affecting the option market is the Penny Pilot project which was implemented in three phases beginning in 2007. The Penny Pilot project specifies that the quoted tick size of certain option series reduces from five cents to one cent.
The rest of the paper is organized as follows. Section 2.2 reviews literature and outlines our hypotheses. Section 2.3 describes the data and variable constructions. Section 2.4 reports the results on the determinants of option illiquidity. Section 2.5 focuses on private information captured by option illiquidity and its implications for the underlying stock returns. Section 2.6 concludes.

2.2 Empirical Predictions and Related Literature

The empirical literature on option illiquidity is relatively scarce due to the limited availability of comprehensive intraday transaction data necessary for a thorough analysis. This section identifies the main economic determinants affecting option trading costs and develops hypotheses for their empirical tests. Afterwards, we examine implications of option illiquidity for their underlying stock returns.

2.2.1 Determinants of Option Illiquidity

In the Black-Scholes world, market frictions and execution costs associated with option trading are irrelevant since one can perfectly hedge option contracts with shares of the underlying asset. However, in the real world, a perfect hedge is not possible due to model risks (Cetin et al. (2006); Figlewski (1989)), as well as investors’ inability to hedge continuously (Jameson and Wilhelm (1992)). Besides hedging cost, the costs associated with inventory risk Muravyev (2016) and information asymmetry between option dealers and informed investors (Easley et al. (1998)) have been recognized in the literature. As a result, we focus on three fundamental forces as potential determinants of option illiquidity: hedging cost, inventory risk, and private information.

2.2.1.1 Hedging Cost

The option hedging literature is far from conclusive. Existing studies have largely focused on two hedging costs faced by option market makers. The first is the fixed cost of establishing the initial delta-hedged position (Cho and Engle (1999); and Kaul, Nimalendran and Zhang (2004)). The initial delta-hedged position, however, does not immunize market makers against future price changes in the underlying asset. Therefore, in order to keep their positions delta-neutral, market makers must continually rebalance them using
the underlying security. This is referred to as the rebalancing cost (Leland (1985); Engle and Neri (2010)). While the literature generally agrees that the initial delta-hedging cost substantially determines option bid-ask spreads, empirical evidence for the rebalancing cost is less conclusive.

George and Longstaff (1993) find that substantial variations in option bid-ask spreads can be attributed to the premiums for the risk of holding uncovered option positions, suggesting that option dealers cannot hedge completely. Jameson and Wilhelm (1992) show that bid-ask spreads of options increase with their delta and gamma, reflecting the initial hedging cost and the future rebalancing cost, respectively. In a similar vein, Cho and Engle (1999) and Engle and Neri (2010) find that hedging and rebalancing costs are the only important determinants of equity option spreads. On the other hand, De Fontnouvelle et al. (2003), and Kaul et al. (2004) do not find a strong relationship between option effective spreads and their proxies for the rebalancing cost. Chan et al. (2002) also argue that the rebalancing cost should be relatively small.

Contrary to our paper, the aforementioned studies either focus on S&P 100 index option or on a few equity option covering short time spans. Importantly, they do not examine the dynamics of equity option bid-ask spreads after 2002 when option exchanges became integrated. Our paper contributes by providing comprehensive analyses of hedging costs in relation to option illiquidity for the modern-day option market.

Leland (1985) theoretically derives the rebalancing cost for replicating an option contract in the presence of transaction costs and shows that it is proportional to the product of the option vega and the bid-ask spread of the underlying security. Therefore, when quoting an option contract, market makers should account not only for the initial hedging cost, but also for the future rebalancing cost of their initial-hedged position. Thus, our first empirical prediction is as follows.

**Hypothesis 1 (Rebalancing cost).** If option dealers demand compensation for rebalancing their initially hedged position, then the rebalancing cost should positively affect option bid-ask spreads.

The economic significance of rebalancing cost versus initial delta hedging cost is an empirical question that we also examine.

---

4De Fontnouvelle et al. (2003) use two months of data, August and September of 1999 for option on 28 stocks, and Kaul et al. (2004) use one month of data for February 1995 and only CBOE-listed option.

5In 2003, all option exchanges were linked via Linkage, and the National Best Bid and Offer (NBBO) rule was introduced.
2.2.1.2 Inventory

The market microstructure literature suggests that bid-ask spreads of a security should increase with its inventory risk (Amihud and Mendelson (1980); Ho and Stoll (1981)). We empirically test whether this effect holds in the equity option market.

Muravyev (2016) advocates a significant inventory risk in the option market as measured by dealers’ order imbalances and documents their effects on option returns. Bollen and Whaley (2004) find that order imbalances have a significant impact on option-implied volatilities because of liquidity providers’ inability to costlessly absorb larger positions. Garleanu et al. (2009) applied the inventory risk models (e.g., Ho and Stoll (1983), and Grossman and Miller (1988)) to the option market and show that end-user demand pressures contemporaneously affect option prices. Their main empirical prediction is that a net demand shock in option contracts increases option prices by an amount proportional to the variance of an unhedged part of the option. In other words, a net demand shock should have an additive effect on bid-ask spreads after accounting for the hedging costs. We test this empirical prediction next.

Hypothesis 2 (Inventory). Option bid-ask spreads should increase with the magnitude of option-induced order imbalances (positive or negative).

We calculate option-induced order imbalance (OOI) measure following Hu (2014), and use it as the proxy for an inventory shock.

2.2.1.3 Private Information

The information asymmetry literature shows that security dealers widen bid-ask spreads to compensate for the risk of providing liquidity to investors with private information, i.e., adverse selection cost (Glosten and Milgrom (1985)). We test this empirical prediction in the option market.

Hypothesis 3 (Private information). Option bid-ask spreads should increase with the measure of adverse selection.

We estimate the degree of private information in the underlying stock with the probability of informed trading measure, PIN, introduced by Easley, Kiefer, O’Hara, and Paperman
A higher level of PIN in the underlying stock should reflect a higher adverse selection cost for option dealers, and thus a wider bid-ask spread.\(^6\)

### 2.2.2 Option Illiquidity and Informed Trading

As liquidity providers, option market makers revise their quotes in response to trades initiated by informed and uninformed traders. However, when the market is dominated by informed traders, adverse selection arises as the primary concern of option dealers thereby pressuring them to widen their quotes more aggressively (Glosten and Milgrom 1985). As a result, large positive changes to option bid-ask spreads may signal the arrival of informed trading activities. We develop two hypotheses examining to what extent option illiquidity reflects the level of informed trading and contribute to the debate on the presence, as well as the impact, of informed trading in the option market.

Black (1975) argues that informed investors are attracted to the option market because they can gain higher leverage. However, empirical evidence of informed trading in the option market is quite mixed. Vijh (1990) finds that trading in options is largely driven by differences of opinion rather than private information. Cho and Engle (1999) fail to find any link between option bid-ask spreads and trading volume, therefore, concluding in favor of the differences of opinion hypothesis. More recently, Muravyev, Pearson and Broussard (2013) find no economically significant price discovery in the option market.

By contrast, using a different methodology, Chakravarty, Gulen and Mayhew (2004) find that the option market significantly contributes to price discovery. Easley, O’Hara and Srinivas (1998) develop an asymmetric information model under which informed traders choose to trade in both the option and stock markets. The authors test their empirical prediction and show that signed option volume can predict stock returns.

The existing studies also differ in how to empirically identify trades that originate from informed option traders. Several studies advocate the use of metrics derived from directional option trading volume, e.g, Bollen and Whaley (2004), and Pan and Poteshman (2006). However, Chan, Chung and Fong (2002) argue that information in the option market is reflected mostly via quote revisions rather than changes in volume.

---

Option-induced order imbalance, arguably, proxies for inventory risk, as well as adverse selection concern to option dealers. Muravyev (2016) argues that in the daily data, option-induced order imbalances mainly capture inventory risks with economically insignificant adverse selection components. On the other hand, Hu (2014) advocates private information as the driver of option-induced order imbalances. We examine when it is likely for changes in option-induced order imbalances to reflect private information in Hypothesis 5.
2.2.2.1 Option Illiquidity Around Information Events

The existing literature argues that the benefits of trading in option are highest around corporate events when the value of private information is the largest.\footnote{See Cao, Chen, and Griffin (2005), Augustin, Brenner and Subrahmanyam (2014) for evidence of informed option trading ahead of mergers and acquisitions. Similarly, evidence of informed option trading in the IPO aftermarket is documented in Chemmanur, Ornthanalai, and Kadiyala (2015).} Therefore, we apply the event-study methodology to examine how option bid-ask spreads change during the period when we are most likely to observe informed option trades. Our proxy for the information event is the date of the earnings announcement. Kim and Verrecchia (1994) argue that financial accounting disclosures such as earnings announcements induce higher information asymmetry because they facilitate informed judgments. Their model predicts that on earnings release dates, asset illiquidity should increase, i.e., wider bid-ask spreads, while trading volume rises sharply as well. This prediction starkly differs from inventory-risk models (see e.g., Amihud and Mendelson (1980), and Ho and Stoll (1983)), which argue that option trading volume and option spreads are negatively correlated.

Our event-study approach is similar to Amin and Lee (1997), who empirically examined option trading around earnings announcements. They find higher volume around earnings releases, but they also find economically insignificant increases in option bid-ask spreads which are inconsistent with the informed trading hypothesis. Choy and Wei (2012) argue against informed trading around earnings announcements and advocate that speculative trading and differences of opinion drive an increase in trading volume. The authors, however, do not analyze option bid-ask spreads. Motivated by Kim and Verrecchia (1994), our next empirical prediction is:

**Hypothesis 4 (Information events).** Around earnings announcements, higher trading volume is associated with a higher participation rate by informed traders and thus should be met by higher option bid-ask spreads.

2.2.2.2 Implication for Stock Returns

If informed investors trade in the option market, their trades would reveal the direction of information and subsequent return pattern of the underlying stock. For example, buying a call or selling a put indicates a synthetic long position in the underlying and tends to convey positive information about future stock prices. Alternatively, selling a call or buying a put may signal negative information. Motivated by this intuition, there exists a growing
literature that uses directional, i.e., “signed”, option trades to infer future information about the underlying security.\textsuperscript{8}

Using signed option trades, Bollen and Whaley (2004) show that the net option-buying pressure, defined as the difference between buyer-initiated and seller-initiated option trades, has a contemporaneous price impact on the shape of the option-implied volatility curve. Hu (2014) derives a slight variation of Bollen and Whaley’s (2004) measure referred to as the option-induced order imbalance (OOI) and shows that it positively predicts stock returns the next day.\textsuperscript{9} Hu (2014) argues that positive (negative) OOI reflects synthetic net buying (selling) pressure and thus positive (negative) private information, which explain the predictive ability of OOI for stock returns.

However, as argued by Chan, Chung and Fon (2002) and Muravyev (2016), the net demand pressure calculated using signed option trades (e.g., OOI) is an imperfect measure of informed-trading activity in the option market. Chan, Chung and Fong (2002) find support for the return predictability using option bid-ask quote revisions, but not with signed option volume. They argue that information in the option market is reflected mostly via quote revisions rather than changes in volume because informed investors often trade with limit orders due to high option trading costs. As a result, option bid-ask quotes rather than option trades reflect the extent of private information. Relatedly, Muravyev (2016) shows that option-induced demand pressure reflects the aggregate net demand of both informed and liquidity-driven trades, but importantly, the liquidity-driven component dominates with the information-driven component being economically insignificant.

This debate raises an important question as to why prior studies find that option-induced demand pressure (e.g., OOI) positively predicts the underlying stock returns. We argue that the return predictability due to option-induced demand pressures manifests when trading in the option market is predominantly motivated by private information, which can be identified using shocks to option bid-ask spreads (Chan, Chung and Fong 2002). This occurs because as market makers suspect informed trading, they aggressively widen their securities’ bid-ask spreads. Therefore, we expect changes in option illiquidity to proxy for the extent of private information embedded in current option trades, regardless of whether the signal is positive

\textsuperscript{8}Such studies include Easley, O’Hara and Srinivas (1998), Pan and Poteshman (2006), and Ge, Lin and Pearson (2015).

\textsuperscript{9}Bollen and Whaley (2004) scale their daily option-induced demand pressure by the option trading volume, while Hu (2014) scales it by the number of shares outstanding of the underlying stock. In both studies, each option trade is weighted by the absolute value of its option’s delta to express demand in stock equivalent units.
or negative.

Following the argument above, we hypothesize that the positive relationship between option-induced demand pressure, OOI, and future stock returns emerges when option bid-ask spreads simultaneously and significantly widen, indicating that option trades during this period are dominated by private information. On the other hand, when changes to option bid-ask spreads are relatively small, the OOI measure should have little to no predictive power for the underlying stock returns because option trades during this period are mostly motivated by either disagreement or liquidity-driven demand. We summarize the empirical predictions of the last hypothesis below:

**Hypothesis 5** (Impact on stock returns). *Higher (lower) option-induced order imbalances accompanied by increasing option illiquidity identify informed trading on positive (negative) information and predict positive (negative) stock returns. Alternatively, higher (lower) option-induced order imbalances accompanied by relatively lower option illiquidity identify disagreement or liquidity-demand-driven trades and have no information about future stock returns.*

### 2.3 Data and Variable Constructions

#### 2.3.1 Data and Sample Selection

The data used in this study are drawn from several sources. We obtain intraday option transaction data from LiveVol, intraday stock transactions from the NYSE’s TAQ database, and daily stock return and volume data from CRSP.

Our sample covers exchange-listed option contracts written on firms that are included in the S&P 500 index from January 2004 to December 2013. There are a total of 2,504 trading days. The time period corresponds to the coverage of the available LiveVol data. The monthly history of the S&P 500 index constituents is drawn from COMPUSTAT. In any given month, we consider all firms that constitute the S&P 500 index, and keep all firm-day observations that also appear in the CRSP daily stock file.

10 Roll, Schwartz and Subrahmanyam (2010) study the price impact of information content in option volume. Here, they use absolute stock return as the dependent variable because they do not have directional information on the option trading volume. Similar to them, we cannot “sign” option bid-ask spreads. We verify the absolute-return predictability using changes in option illiquidity in Appendix B Table C1.
Our main data source is the intraday option transaction data obtained from LiveVol. Similar to the NYSE TAQ database, the LiveVol data contain trades and quotes on each option series, which is uniquely identified by its underlying stock, option type (call or put), expiration date, and strike price. LiveVol provides national best bid and offer (NBBO) quotes associated with each transaction. Other intraday transaction-level information includes trade price and trade size (number of contracts) for each option series. We apply a list of filters to this data set. First, we focus on option series, of which the daily closing mid-quote is at least 10 cents, and the quoted spread does not exceed 50% of the transacted price. Second, we remove trades that are canceled or recorded outside the regular trading hours (9:30 a.m. – 4:00 p.m. EST). Next, we retain option trades that meet all the following conditions: (1) trade price $\geq$ intrinsic value; (2) trade size $> 0$; (3) prevailing best quotes satisfy $0 < \text{bid} < \text{ask} < 5 \times \text{bid}$; (4) trade price $\leq 2 \times \text{mid-quote}$. After these filters, 626,348,046 out of the 678,159,426 raw trade transactions, or 92.4% of the raw data, remain.

In January 2007, major option exchanges such as CBOE and ISE initiated the Penny Pilot project. The exchange rules stipulate that for a participating underlying stock, any associated option series should be quoted in increments of 1 cent if below $3.00 and in increments of 5 cents otherwise.\(^{11}\) Whereas options written on a non-participating underlying stock should be quoted in increments of 5 cents if below $3.00 and in increments of 10 cents otherwise. Therefore, we place an additional filter by checking whether option bid and ask quotes conform to this rule. After this additional screening, 626,298,280 trades remain.

We obtain intraday trades and quotes on the underlying stocks from the NYSE TAQ database. In order to obtain bid and ask quotes associated with each stock trade, we merge the consolidated trade files with the NBBO data. We require that each trade recorded during the regular trading hours (9:30 a.m. – 4:00 p.m. EST) is matched with the prevailing NBBO quotes at least one second before the trade’s timestamp. Next, we purge the merged intraday stock data by deleting records that are reported out-of-sequence or with special settlement conditions (condition code= Z, O, L, G, W). Finally, we place the following standard filters on the trade records: (1) $0.01 \leq \text{bid} < \text{ask}$; (2) $\text{ask} – \text{bid} \leq 3.00$; and (3) $0.01 \leq \text{trade price} < 1.5 \times \text{mid-quote}$.

Using intraday LiveVol and TAQ data, we compute option-implied volatilities and option sensitivities, i.e., Greeks, which we use to measure the hedging costs faced by option market makers. First, we match each time-stamped option trade with its underlying stock’s prevailing NBBO quotes recorded in the TAQ database at least one second before. The option

\(^{11}\)See CBOE Rule 6.42.
price and the underlying stock price used in the calculation are based on the option trade recorded in LiveVol and its underlying stock’s prevailing mid quote from TAQ, respectively.

Second, we employ a binomial-tree option-pricing model and solve for the implied volatility of each option trade. To account for the effect of stock dividends, we use the dividend schedule of each underlying firm obtained from OptionMetrics.\textsuperscript{12} Next, for each option transaction, we calculate option delta, gamma and vega based on the implied volatility obtained previously. Finally, for each option series, we aggregate the transaction-level implied volatility, delta, gamma, and vega at the daily level by calculating volume-weighted (number of contracts) averages of their respective intraday values.

On a given day, the number of option series on an underlying stock can be quite large. Therefore, we group options written on the same underlying stock into subsets indexed by option type, i.e., call vs. put, with our main tests focused on options with short-to-medium maturities defined as those with 30-182 calendar days to expiration. Options with extreme moneyness are deleted. We define moneyness as in Bollen and Whaley (2004), using options’ average delta on day $t$ and retaining calls with $1/8 < \delta \leq 7/8$, and puts with $-7/8 < \delta \leq -1/8$. The final sample size is 260,583,909 trades. For future reference, we define a unique firm-option-type combination as an \textit{option class}.

\subsection*{2.3.2 Variable Constructions}

\textbf{Option and Stock Illiquidity}

We measure illiquidity in the option market using relative bid-ask spreads at the time of option trades. For each option transaction, we calculate two measures of relative bid-ask spreads: \textit{relative effective spreads} and \textit{relative quoted spreads}. The relative effective spread is defined as twice the absolute difference between the trade price and the mid-point of the prevailing NBBO quotes, divided by this mid-quote. The \textit{relative quoted spread} is the quoted bid-ask spread divided by the mid-point of the prevailing NBBO quotes.

Because our main empirical analyses rely on cross-sectional regressions at the daily level, we construct option illiquidity measures (ILO) on a daily basis. For a given option class, we compute daily ILO as the dollar-volume-weighted average of the relative option effective (or quoted) spreads. Thus, the contribution to ILO from each option trade in the option

\textsuperscript{12}For details on the numerical procedure for computing the implied volatility of an American option, see Hull (2011) Ch.20.
class is proportional to its dollar trade amount. We use two measures of option illiquidity, ILO, throughout this paper: one calculated using effective spreads, and the other calculated using quoted spreads. In addition to the option illiquidity measure, we compute $OptVolume$ as the total number of contracts traded in the option class during the day.

We measure stock illiquidity (ILS) using stock relative effective bid–ask spreads. The daily relative effective spread is calculated as the dollar-volume-weighted average of relative effective spreads of the underlying stock intraday trades.

**Option Trade Imbalance**

For each recorded option transaction, we classify its trade direction following the Lee and Ready (1991) algorithm. Trade records are first subject to the ‘quote test’ – assigned as buyer-initiated if it is executed above the mid-quote, and as seller-initiated if it is below the mid-quote. For trades where the ‘quote test’ is not applicable or inconclusive due to missing quotes or the trade price being equal to the mid-quote, we conduct the ‘tick test’ by looking back at the previously recorded trades.

In order to account for the size in each trade direction, we multiply each buyer-initiated (or seller-initiated) trade amount by its option delta to express it in the underlying stock equivalent units (see also Bollen and Whaley (2004)). We then aggregate buyer-initiated trades and seller-initiated trades at the daily level.

For each day, we calculate net option-induced demand pressure for each underlying stock as the difference between daily aggregate buyer-initiated and aggregate seller-initiated trades. We follow Hu (2014) and normalize daily option-induced demand pressure of each underlying stock by the number of shares outstanding and refer to it as the option-induced order imbalance, OOI. The OOI measure is calculated daily for each stock in our sample.

**Option Hedging Costs**

For a given option class on day $t$, we calculate two hedging cost variables. The first is the cost associated with initial hedging, i.e., a delta-hedged position. An option delta is the first partial derivative of option price with respect to the price of its underlying. It indicates the amount of shares in the underlying stock that the option writer must buy or sell in order to immunize the position against a small change in the price of the underlying. Similar to
Cho and Engle (1999), among others, we measure delta-hedging cost using percentage delta, \(\%\text{DELTA}\), which is defined as

\[
\%\text{DELTA} \equiv \left| \frac{\partial C}{\partial S} \right| \frac{S}{C} = |\Delta| \frac{S}{C},
\]

(2.1)

where \(\Delta\) is the daily volume-weighted average option delta, \(S\) is the closing price of the underlying, and \(C\) is the daily volume-weighted average option price. We use the percentage delta, rather than the raw delta, to have its magnitude economically comparable across different option price levels. A call option delta is always positive while a put option delta is always negative. We therefore apply the absolute sign in Eq.(2.1) in order to capture the magnitude. We can interpret \(\%\text{DELTA}\) as the absolute option price elasticity with respect to the underlying stock price.

A delta-hedged option position provides immunity against changes in its value temporarily. Immunization against further changes in value requires continuous rebalancing of the hedged portfolio. Leland (1985) shows that the cost associated with continuous rebalancing can be very high, and it depends on the option contract’s sensitivity to changes in volatility, as well as the liquidity of the underlying. Specifically, Leland (1985) shows that the cost of future rebalancing of the initially hedged option position in dollar terms is proportional to \(v \cdot \text{ILS}\), where \(v \equiv \partial C / \partial \sigma\) is the option vega, and ILS is the underlying stock illiquidity.\(^{13}\)

In order to apply the rebalancing cost to cross-sectional tests, we scale the dollar rebalancing cost by the volume-weighted average option price. This procedure allows us to measure rebalancing costs in percentage units (%RBC) of the traded dollar amount, and makes it comparable to the percentage delta variable described previously. For our empirical analyses, we calculate the rebalancing cost for each option class each day, which we define as

\[
\%\text{RBC} = \frac{v}{C} \text{ILS},
\]

(2.2)

where \(v\) is the daily volume-weighted average option vega.

\(^{13}\)Note that the rebalancing cost defined using option vega, \(v\), is related to the cost of gamma hedging an option position. This is because option gamma, \(\Gamma\), and vega, \(v\), are related by the relationship \(v = \Gamma S^2 \sigma^2 T\), where \(S\) is the underlying price, \(\sigma\) is the implied volatility, and \(T\) is the time-to-maturity of the option contract.
PIN

We use the intraday stock data to calculate the probability of the information-driven trading (PIN) measure developed by Easley, Kiefer, O’Hara, and Paperman (1996). We first aggregate the number of market buy and sell orders on each day by using the Lee and Ready (1991) algorithm to determine the trade direction for each stock trade. The resulting daily buy and sell order counts are then used to estimate the probability of information-based trading (PIN). We explain the details of the PIN model and its estimation procedure in D.

2.4 Determinants of option illiquidity

2.4.1 Sample Descriptive Statistics

Table 2.1 presents summary statistics of option illiquidity and trading activity variables. Panel A reports the time-series averages of the cross-sectional distribution of relative effective and quoted spreads and other key variables for calls and puts. Effective spreads (ILOE), on average, are wider for a representative call (6.5%) compared to a representative put (5.8%). Consistent with trades taking place inside the quotes, quoted spreads (ILOQ) are higher (8% and 7.2% for call and put options, respectively). On average, ILOE and ILOQ are slightly higher for calls compared to puts. The average trading volume (number of option contracts) and number of trades per day for calls significantly exceed those for puts. For example, the average daily call volume is 2,208 contracts, and the put volume is 1,546 contracts, a difference of approximately 30%. Overall, call options are more actively traded.

Figure 2.1 plots daily cross-sectional averages of option illiquidity calculated using effective spreads (ILOE) and quoted spreads (ILOQ). For comparison, we also plot daily cross-sectional averages of the illiquidity measure of the underlying stocks in the bottom panel. Overall, we observe a gradual improvement in options liquidity, especially for effective spreads, from the start to the end of our sample with a common spike during 2008–2009 crisis. The illiquidity of the underlying stocks, on the other hand, remains fairly stable through our sample period outside the 2008–2009 crisis.

In order to compare the time-series dynamics of option effective versus quoted spreads, we plot the daily aggregate effective-to-quoted spread ratio in the top panels of Figure 2.2. We define effective-to-quoted ratio as the ratio of option effective spreads to option quoted spreads. The ratio below one would suggest that option trades, on average, are executed
within the quoted bid-ask spreads. The top-left and top-right panels of Figure 2.2 represent the daily volume-weighted averages across firms for calls and puts, respectively. In the beginning of our sample, this ratio is approximately one for both calls and puts, suggesting that almost all transactions occur at the quoted spreads. However, by the end of our sample this ratio decreases to below 0.8. This finding suggests that increasing competition among market makers forces them to trade at more competitive prices.

Alternatively, the quality of trades’ execution can be observed by looking at the fraction of trades executed within the quoted spreads. Any trades executed outside the quoted bid-ask spreads can be considered non-competitive, perhaps due to the inability of liquidity providers to absorb excess demands. The bottom panels of Figure 2.2 plot daily fractions of option trades that are executed within the quoted spreads for puts and calls. The results show that the fraction of trades that occur within quoted spreads gradually increases towards the end of the sample, yet remains below 80%. Overall, we find that the quality of trading in the option market gradually improves through our sample.

We compute daily cross-sectional average option volume, measured as the number of contracts for each option type on each day. Figure 2.3 plots its time-series dynamic. As we move towards the end of the sample, trading volume increases significantly. One of the highest spikes in volume is observed in the second half of 2008 after the Lehman Brothers’ collapse. Overall, we observe higher trading volume for both calls and puts in the second half of the sample, 2009-2013. For the remaining analyses, we use the natural log transformation of $\text{OptVolume}$.

Panel B of Table 2.1 reports summary statistics for stock illiquidity averaged across firms in the S&P 500 index. The average effective spread is 8 basis points. The maximum of 1.36% is reached during the 2008 financial crisis (see the bottom panel of Figure 2.1).

Table 2.2 reports time-series averages of the cross-sectional pairwise correlations for the main variables. ILOE and ILOQ are highly positively though not perfectly correlated with the magnitude of 0.88 for calls and 0.85 for puts. Although ILOQ is indicative of the bid and ask prices at which dealers are willing to trade, it is not always binding. In fact, certain trades do occur outside the bid-ask quotes. We therefore include both ILOE and ILOQ in our analyses throughout the paper for completeness.

Focusing on option illiquidity calculated using effective spreads, ILOE, we find that it is positively and significantly correlated with the two hedging cost variables %DHC and %RBC. For call (put) options, their correlations reach 0.30 (0.29) and 0.47 (0.41), respectively.
Option-induced order imbalance, OOI, has very little to no correlation with ILOE. The stock illiquidity measure, ILS, significantly correlates with ILOE, although the magnitude is small, ranging between 0.13 for puts and 0.16 for calls. The low correlation between ILOE and ILS suggests that the option illiquidity is not largely driven by the underlying stock illiquidity. Table 2.2 also shows that OptVolume is negatively and significantly correlated with ILOE. This is expected since higher trading volume leads to lower order processing costs for dealers, and thus lower bid-ask spreads. We find that PIN has a positive and significant correlation with ILOE of 0.13 for both calls and puts, suggestive of informed trading being an important concern facing option dealers.

As expected, we find the correlations between ILOQ and various variables in Table 2.2 carry the same sign and are comparable to those calculated for ILOE. This finding suggests that factors influencing quoted and effective bid-ask spreads in the option market are fairly similar.

### 2.4.2 Hedging Costs, Inventory and Private Information

We run the Fama-Macbeth regressions for 1,134,312 firm-day observations for each of the two option types: calls and puts. Table 2.3 reports regression results. The dependent variable in Panel A is option illiquidity calculated using quoted spreads (ILOQ), while for Panel B, the dependent variable is option illiquidity calculated using effective spreads (ILOE). Our main variables of interest are %DHC, which captures the initial hedging cost; %RBC, which is the measure of future rebalancing cost (Leland (1985)); |OOI|, which is the absolute value of option-induced order imbalance and captures inventory shocks, either positive or negative, to option dealers (Chordia, Roll and Subrahmanyam (2002)); and PIN, which measures the level of private information on the underlying stock.

Among other control variables, we include lagged option spreads, ILO(t-1), to account for persistence in option illiquidity, and OptVolume to control for cross-sectional differences in option trading activity. We also control for trading activity in the underlying market by including stock return, lagged stock return, and 5-day moving average absolute stock return, MA5|RET|. The latter controls for the volatility of the underlying stock.

First, consider call options and ILOQ, Panel A. Here, %DHC, %RBC, |OOI| and PIN have positive and significant impact on quoted spreads. Combining the coefficients in Panel A with the standard deviations of these variables reported in Table 2.1, suggests that one standard deviation shock to %DHC results in 0.9% increase in ILOQ, and a similar shock to
%RBC leads to a 3.7% increase in ILOQ, i.e., more than four times the magnitude of initial delta hedge impact. Similar magnitudes are observed for puts, and for ILOE (see Panel B). Different from De Fontnouvelle et al. (2003) and Kaul et al. (2004), we find that rebalancing cost is not only statistically significant but its impact substantially exceeds that of delta-hedging. This new evidence highlights the role of rebalancing cost as one of the leading concerns for market makers in the modern U.S. equity option market. It also provides overwhelming support to our Hypothesis 1.

Further, after controlling for hedging costs, we observe a significant impact of both option-induced order imbalance and private information on options spreads. Here, a one-standard-deviation shock to $|\text{OOI}|$ and PIN leads to 0.57% and 0.32% increase in ILOQ, respectively. Although smaller but comparable to the initial delta-hedge in economic magnitudes, these variables add to bid-ask spreads variability after accounting for all hedging costs. Therefore, the hedging cost theories of Cho and Engle (1999), and Engle and Neri (2010) do not fully explain variations in option illiquidity as shown by the data. Further, our findings confirm that inventory shocks (Muravyev (2016)) as well as private information (Easley et al. (1998)) significantly contribute to the costs of market making. These results are consistent with Garleanu et al. (2009) net demand pressures effect on options prices, and also support our Hypotheses 2 and 3.

Among the control variables in Table 2.3, we find that returns of the underlying stocks affect put and call options differently. Call option bid-ask spreads decrease when their underlying stock prices increase, while put bid-ask spreads decrease when their underlying stock prices decrease. Interestingly, we find that option illiquidity is negatively related to its stock illiquidity. The coefficients on ILS are negative and significant across puts and calls. This finding suggests that investors use the option market as an alternative trading venue for the underlying stocks experiencing high illiquidity.

Table 2.4 presents predictive regression results for changes in option illiquidity.\textsuperscript{14} Comparing the results against the estimates in Table 2.3, we find the coefficients on hedging variables, and order imbalance flip signs from positive to negative. This finding suggests the impact of hedging costs and inventory risk on option illiquidity is transitory, and the negative coefficients that we observe are due to the mean-reverting characteristics of their variables. However, we find that coefficients on PIN remain positive and significant. In other words, asymmetric information appears to be a long-lasting concern for option dealers, forcing them to continue widening their bid-ask spreads.

\textsuperscript{14}Similar to Chordia, Roll and Subrahmanyam (2002), in predictive regressions we use changes in illiquidity.
Among other variables, we find that option volume is negative and significant. This too supports the information theory of Easley et al. (1996) that higher trading volume decreases information asymmetry. It is also consistent with order-processing costs hypothesis which predicts that transaction costs are lower when trading volume increases.

While the volatility of the underlying stock, MA5|RET|, has a positive and significant impact on option spreads in Table 2.3, this effect is reversed in Table 2.4. This finding is consistent with the mean-reverting nature of volatility; a high volatility period is followed by a decrease in volatility.

Table 2.4 shows that past stock returns predict option illiquidity differently for puts and calls. The coefficients on RET(t-1) and RET(t-2) are negative for call options, suggesting that call option liquidity improves when the underlying stock is performing well. On the other hand, for put options, the coefficients on RET(t-1) and RET(t-2) are positive suggesting that put option liquidity improves when the underlying stock price is falling. We conjecture that an increasing share price generates interest in the synthetic long position, i.e., buying calls and selling puts, leading to higher end-user demand for call options, but lower end-user demand for put options. As a result, order-processing cost decreases for call options resulting in narrower bid-ask spreads, while for puts, we observe the opposite effect.

Notice that adjusted $R^2$ values in our Table 2.3 range from 45% to 55%, suggesting that a significant variation of option bid-ask spreads are explained by hedging, inventory, private information, and other controls we use. Overall, the rebalancing cost dominates all other variables in terms of economic magnitude on a day-to-day basis. This result suggests that a substantial portion of option illiquidity reflects the premium for the risk that option market makers must continually hedge their positions after they have initiated option contracts. We also conclude from Tables 2.3 and 2.4 that the effects of hedging costs and inventory risk on option illiquidity are short-lived, while for private information, it has a lasting impact.

In the next section we first directly test the private information hypothesis and then its application for stock returns.

### 2.5 Private Information

#### 2.5.1 The Impact of Information Events

Kim and Verrecchia (1994) argue that market participants use their private informed judgements to process earnings announcements. This stimulates their willingness to engage
in trading activity and exacerbates the information asymmetry between informed traders and market makers. As a result, during earnings news releases, the market becomes less liquid, i.e., wider bid-ask spreads, even though we observe significant increases in trading volumes.

Empirical evidence on the behavior of option bid-ask spreads around earnings announcements is rather limited. Amin and Lee (1997) examine option volume and bid-ask spread behavior for 1988–1989 sample for 141 firms and find significant abnormal volume preceding earnings announcements and on the announcement day, but no economically meaningful changes in option bid-ask spreads. Therefore, while their evidence on trading volume supports informed trading, their finding on bid-ask spread changes is not consistent with the Kim and Verrecchia’s (1994) theory.

We use S&P 500 firms’ earnings announcements with available options traded from January 2004 to December 2013. We obtain earnings announcement information from I/B/E/S unadjusted files. We define an event window as [–10,10], with day 0 as the event day. We identify the pre-event window [–42,–21] relative to the announcement date. All variables of interest are reported in abnormal values computed as the estimate on day \( t \) less its corresponding average value computed over the pre-event window.

We classify earnings announcements as either a negative or a positive surprise based on cumulative abnormal returns (CAR) to the earnings announcement. We calculate CAR over the three-day window [–1,1] as:

\[
\text{CAR}_j = \sum_{t=-1}^{+1} (R_{jt} - R_{mt}),
\]

where \( R_{jt} \) is the raw return of stock \( j \) on day \( t \), and \( R_{mt} \) is CRSP NYSE/AMEX/NASDAQ value-weighted index return on day \( t \). We then use standardized cumulative abnormal return (SCAR), computed as \( \text{SCAR}_j = \text{CAR}_j / \sqrt{3}\sigma_j \), where the standard deviation \( \sigma_j \) of abnormal returns in the denominator is computed over the [–42,–10] pre-event window. We quintile-sort stock price reactions to earnings announcements based on SCAR, with the fifth quintile identified as a positive earnings surprise and the first quintile identified as a negative earnings surprise.\(^{16}\)

\(^{15}\)If an earnings announcement takes place after trading hours, then the event date is the next trading day.
\(^{16}\)As a robustness check, we verify our results are qualitatively similar when classifying earnings surprise using SUE (standardized unexpected earnings). SUE is defined as the difference between actual earnings per share (EPS) and consensus EPS forecast (median), normalized by the standard deviation of analyst forecasts.
Figure 2.4 plots event-study results for abnormal option trading volume (left panels) and abnormal stock trading volume (right panels) around earnings announcements. We plot results separately for call options (solid line) and put options (dotted line). The top panel reports results averaged across all earnings announcements. The middle (bottom) panel reports results for positive (negative) earnings surprises, respectively.

Confirming the results reported in previous studies, we find that trading volume spikes several days before the earnings announcement date for options as well as for their underlying stocks. The highest abnormal trading volume is observed on the day of earnings releases. In terms of magnitude, we find that changes in abnormal trading volume is much larger for stocks than for put and call options.

Figure 2.5 presents event-study results of abnormal option bid-ask spreads (both ILOQ and ILOE), as well as stock bid-ask spreads (ILS) around earnings announcements. For both call and put options, we find that on the event day 0, option spreads increase substantially. We also observe a large increase in spreads on the pre-event day −1, suggesting informed trading one day before an announcement. This pattern holds for both positive and negative earnings surprises, i.e., option bid-ask spreads widen regardless of whether the signal is positive or negative. Looking at abnormal stock illiquidity, we find an economically small increase in stock bid-ask spreads on the days before and on earnings announcements. Therefore, despite observing a tremendous change in abnormal stock trading volume around earnings announcements (see Figure 2.4), stock bid-asks spreads do not widen substantially.

Table 2.5 tabulates the economic and statistical significance of the results reported in Figure 2.1. Consider Panel A, which reports the results for all earnings announcements. Both quoted ILOQ and effective spreads $ILOE$ significantly increase on the day of earnings announcement. In terms of economic magnitude, the increase in abnormal quoted bid-ask spreads ranges between 74 bps for call options to 81 bps for put options. These values indicate relative increases of approximately 11% and 14% for ILOQ and ILOE, respectively, on the event day. Importantly, Table 2.5 shows that option bid-ask spreads significantly increase one day preceding the announcements. On the event day −1, the relative option quoted spreads increase by 22 bps for calls, and by 31 bps for puts, which translate to 3.4% and 5.3% incremental increases above the mean. This finding evidently supports the presence of informed trading in the option market in anticipation of announcement news. Looking at abnormal stock bid-ask spreads (ILS), Panel A of Table 2.5 shows that changes in stock bid-ask spreads are economically trivial relative to changes in option bid-ask spreads, confirming the results illustrated in Figure 2.5.
Panels B and C of Table 2.5 present results for positive and negative earnings surprises, respectively. On average, we find that relative option bid-ask spreads started increasing on the day before earnings announcement, with the highest level reached on the announcement day, except for calls. For positive earnings surprises (see Panel B), relative quoted spreads for calls are highest on the day before the event, i.e., event day \(-1\), with the magnitude of 25 bps. The second highest value is realized on the announcement day, i.e., event day \(0\), with the magnitude of 17 bps. For put options, we find that the highest increase in ILOQ and ILOE is observed on the event day, with magnitudes of 101 bps and 90 bps, respectively. The substantially larger increase in put option spreads than call option spreads in Panel B is consistent with our finding in Table 2.3 that put bid-ask spreads are positively related to their underlying returns. On positive earnings announcement days, stock prices respond positively and strongly to the news. Given the evidence that, on average, investors are net-sellers of equity options (see e.g., Garleanu et al. (2009), and Christoffersen et al. (2018)), the higher abnormal put spreads in response to positive stock returns are consistent with increased selling activity of put options by investors to create synthetic-long positions on the underlying stocks.

Conversely, for negative earnings surprises (see Panel C), the highest increase in option illiquidity is observed for calls. Here, abnormal increases in relative quoted and effective spreads are 120 bps and 107 bps, respectively. This finding can be attributed to investors taking a synthetic short position in the underlying by increasingly selling calls, resulting in higher call bid-ask spreads. Interestingly, we observe abnormally high call spreads persisting up to 5 days after the announcement.

Table 2.5 also reports results for abnormal effective spreads of underlying stocks, ILS, around earnings announcements. Similar to the option market, ILS increases on the announcement day, but the economic magnitude is trivial relative to changes in options bid-ask spreads.

Overall, we find an empirical support for Kim and Verrecchia’s (1994) theory in the option market, which supports the empirical prediction of our Hypothesis 4. By looking at changes in option illiquidity, we find evidence of informed trading in the option market before and on earnings announcement dates.
2.5.2 Option Illiquidity and Stock Returns

This section examines the implications of option illiquidity for their underlying returns, which addresses Hypothesis 5 of the paper.

Existing studies find that option-induced demand pressure, e.g., Hu’s (2014) OOI, by capturing private information positively predicts the underlying stock returns. However, Muravyev (2016) finds that variations in option net-buying pressure, on average, are due to liquidity-driven trades and not due to private information, which raises the question of the source of its ability to predict future returns of the underlying. We address this question by showing that the documented predictive power of option-induced demand pressure comes mostly from circumstances where order flow is influenced by trading activities of informed investors. In other words, while fluctuations in option net-buying pressure, on average, are due to liquidity-driven trades or investors’ disagreements, for certain periods, trading by informed investors intensifies, leading the net-buying demand pressure to contain information about the underlying.

Specifically, we use changes in option illiquidity to identify the underlying securities for which the adverse selection issue arises as the leading concern for option market makers. This approach is motivated by the market microstructure theories of information asymmetry (see e.g., Glosten and Milgrom (1985), and Copeland and Galai (1983)). In these models, the securities’ dealers infer the risk of trading against informed traders by observing traders’ quotes and in response, widen their bid-ask spreads to compensate for potential losses on informed trades.

2.5.2.1 Portfolio Sorting Results

We measure option-induced demand pressure using Hu’s (2014) order imbalance measure, OOI.\textsuperscript{17} The summary statistics in Table 2.1 shows that OOI is, on average, negative but close to zero. The magnitude of OOI measure is almost identical across calls and puts. Similar to Hu (2014) and Bollen and Whaley (2004), we aggregate the OOI measure across calls and puts in order to capture the aggregate net-buying demand on the underlying stock.

A higher level of OOI measure indicates an excess demand for synthetic long positions relative to short positions on the underlying stock. Alternatively, a lower OOI level suggests

\textsuperscript{17}We verify that our conclusions are qualitatively similar when we use other variations of net-buying pressure calculated from signed option trades. For instances, the options’ order imbalance measure, OIB, of Bollen and Whaley (2004), and the raw option net-buying pressure (i.e., order flow).
that the selling pressure dominates, which points towards an excess demand for synthetic short positions.

Hu (2014) finds that the OOI measure positively predicts return of the underlying on the next day. We confirm this finding using a portfolio-sorting strategy. Panel A of Table 2.6 reports the results. On each day $t$, we tercile-sort S&P 500 stocks in the sample based on their OOI level, from low (portfolio 1) to high (portfolio 3). Then, on the next day $t+1$, we calculate the value-weighted average returns for these three portfolios. We repeat this analysis for the full sample period from 2004 to 2013. Table 2.6 reports portfolio alphas calculated using the Fama-French-Carhart four-factor model.

As expected, we find that high OOI portfolio has a positive and significant next-day risk-adjusted alpha of 2.38 bps ($t = 4.85$), while low OOI portfolio has a negative and significant alpha of $-1.65$ bps ($t = 3.25$). A self-financing high-minus-low strategy based on sorted OOI portfolios yields an alpha of 4.02 bps ($t = 6.72$) per day, or equivalent of about 10% annually.

We next consider a double-sorting portfolio strategy based on OOI and changes in option illiquidity. Panels B and C of Table 2.6 report the results for option illiquidity measure calculated using quoted spreads (ILOQ), and effective spreads (ILOE), respectively. We use changes in option illiquidity to alleviate its persistence and to account for the fact that information in the option market is also revealed via quote revisions (Chan et al. (2002)). According to Hypothesis 5, we expect that directional change in OOI measure to contain information about the underlying when accompanied by increasing option illiquidity. Alternatively, when there is little change in option illiquidity, the directional change in OOI measure is likely due to liquidity-driven trades and contains little information about the future underlying stock price.

Focusing on the results with ILOQ in Panel B, we find the highest portfolio alpha of 3.37 bps ($t = 2.12$) for the high-OOI portfolio with the largest increase in ILOQ. The alpha of the High-OOI portfolio with the smallest change in ILOQ is significant but very small economically, i.e., 1.5 bps ($t = 2.04$). The difference in portfolio alphas of the largest and the smallest $\Delta$ILOQ portfolios for the high-OOI category is statistically significant at the 5% level. This finding suggests the positively large return following a high OOI day is observed mostly among stocks with higher probability of informed trading as measured by changes in their option illiquidity. We find a consistent set of results when looking at the low OOI portfolios in Panel B; these are stock portfolios with excess demand for synthetic
short positions. The alpha for the double-sorting portfolio with the largest \( \Delta ILOQ \) is \(-3.13\) \((t = 4.08)\), while for the lowest \( \Delta ILOQ \), the alpha is near zero and statistically insignificant.

Overall in Panel B, we find the OOI measure has the best predictive ability for the next-day return when we focus on portfolios with the largest increase in option illiquidity, i.e., among stocks with the highest likelihood of informed trading. The alphas of the portfolio in the lowest \( \Delta ILOQ \) group (Column A) do not monotonically increase with the OOI measure. However, we find that portfolio alphas of the largest \( \Delta ILOQ \) group (Column C) monotonically increase with the OOI measure. A long-short strategy based on OOI measure that uses only stocks with the largest \( \Delta ILOQ \) earns a daily alpha (see \( \text{Alpha}_{\text{OOI}\times\text{ILO}} \)) of 6.5 bps \((t = 6.55)\), or equivalently 16.5% per year. This portfolio alpha is larger than the 4.02 bps (see \( \text{Alpha}_{\text{OOI}} \) in Panel A) earned from the long-short strategy based on single-sorting the OOI measure that uses all underlying stocks; the p-value for the difference is 0.007. Collectively, the findings in Table 2.6 confirm the prediction of Hypothesis 5 that the return predictability due to option-induced demand pressure is confined to stocks experiencing an increased of informed option trading.

Panel C reports alphas from the double-sorting portfolio strategy based on OOI and \( \Delta ILOE \). We find that the results in Panels B and C are qualitatively similar.

2.5.2.2 Option Illiquidity and Stock Returns: Regression Analysis

Portfolio sorting results reported above provide univariate estimates of the economic magnitude of informed trading captured by changes in option illiquidity. However, they do not allow us to control for other variables such as underlying stock illiquidity, firm size, volatility, and other variables which have been shown to explain daily stock returns. In this section, we apply the regression analysis to further verify the prediction of Hypothesis 5.

Table 2.7 reports the Fama-MacBeth regression results examining the predictability of stock returns using the ILO measure and the change in option illiquidity. The dependent variable here, \( \text{Ret}_{i,t+1} \), is the one-day ahead return on stock \( i \). Our independent variables of interest include the current level of option-induced order imbalance, \( \text{OOI}_{i,t} \); the tercile-ranked change in option illiquidity, \( \Delta ILO\text{\_ranked}_{i,t} \). We use the daily cross-sectionally ranked change in ILO instead of the raw daily change in ILO in order to mitigate the effect of outliers found in the ILO variables as shown in Table 2.1. Importantly, Figure 2.1 shows the liquidity in the option market is improved throughout our sample period. Therefore, raw changes in the ILO measure are not meaningfully comparable between the beginning and end of our
sample. The other independent variable of interest that we focus on is the cross-interaction term \( OOI_{i,t} \times \Delta ILO_{ranked,i,t} \). This interacted variable captures the predictive ability of the OOI measure at different levels of \( \Delta ILO_{ranked,i,t} \) when changes in the underlying stock’s option illiquidity are at lowest, average, and largest levels.

We include a host of control variables in the regressions. We control for changes in the underlying stock illiquidity using \( \Delta ILS_{ranked,i,t} \), which is calculated by ranking cross-sectional changes in ILS daily into three increasing groups. We use ranked change in ILS in order to stay consistent with our change in the option illiquidity variable, \( \Delta ILO_{ranked,i,t} \). We control for lagged stock returns using \( Ret_{i,t} \), and stock return momentum using \( Ret_{i,[-5, -1]} \). In addition, we control for uncertainties in the stock market and in the option market using realized volatility, \( RRV_{i,t} \), and option-implied volatility, \( IV_{i,t} \), respectively. \( RRV_{i,t} \) is a daily range-based proxy for the realized volatility of the underlying stock, defined as the difference between the underlying stock’s intraday high and low prices divided by the closing stock price. \( IV_{i,t} \) is the implied volatility for the underlying stock, calculated as the average implied volatilities of the call-put pair with 30 calendar days to maturity reported in the standardized option file from OptionMetrics. The remaining control variables include firm size and option trading volume.

The first regression specification in Table 2.7, Column (I), examines the predictive ability of the raw OOI measure for the underlying returns. This finding confirms the results in Hu (2014) that option-induced demand pressure positively predicts one-day ahead return of the underlying stock. In Columns (II) and (III), we examine the predictive ability of the change in option illiquidity calculated using quoted spreads (ILOQ), while in Columns (IV) and (V) we examine the predictive ability of the change in option illiquidity calculated using effective spreads (ILOE).

The regression specifications in Columns (II) and (IV) examine the predictive ability of the \( \Delta ILO_{ranked,i,t} \) variable. As expected, we do not find that \( \Delta ILO_{ranked,i,t} \) predicts the underlying return. This finding is consistent with the market microstructure theory of information asymmetry (see e.g., Glosten and Milgrom (1985)), and our results in Table 5 which show that option bid-ask spreads widen with the arrival of positive and negative news about the underlying firm. As a result, the positive and negative information effects cancel each other out resulting in an insignificant predictive coefficient for \( \Delta ILO_{ranked,i,t} \).

Columns (III) and (V) of Table 2.7 present results for the regression specification with the cross-interacted variable, \( OOI_{i,t} \times \Delta ILO_{ranked,i,t} \). In these two specifications, we find
that $OOI_{i,t} \times \Delta ILO_{ranked,i,t}$ is positive and highly significant, while $\Delta ILO_{ranked,i,t}$ is negative and highly significant. These findings are consistent with the double-sorting portfolio strategy results reported in Table 2.6. First, by looking at the positive coefficient on $OOI_{i,t} \times \Delta ILO_{ranked,i,t}$, we observe that $OOI_{i,t}$ positively predicts the next-day return of the underlying stock that has a large increase in option illiquidity, $\Delta ILO_{ranked,i,t}$. This confirms the prediction of our Hypothesis 5.

Second, the coefficient on $\Delta ILO_{ranked,i,t}$ can be interpreted as the predictive ability of changes in option illiquidity for the baseline case when $OOI_{i,t}$ level is very low, i.e., there is an excess demand for synthetic short positions on the underlying. The negative coefficient on $\Delta ILO_{ranked,i,t}$ suggests that when there is an aggregate synthetic net-selling pressure in the underlying stock, increasing options bid-ask spreads would indicate that the selling pressure is driven by informed trades which convey negative information about the underlying stock. This finding is consistent with the double-sorted portfolio results in Panels B and C of Table 2.6. Looking specifically at the rows labeled (1) where we observe “low OOI” levels, we find that portfolio alphas decrease with increasing $\Delta ILO_{ranked}$.

In Table 2.7, we find that the coefficient on $OOI_{i,t}$ in Columns (III) and (V) is not significant when the cross-interacted variable $OOI_{i,t} \times \Delta ILO_{ranked,i,t}$ is included in the regressions. For these two specifications, the coefficient on $OOI_{i,t}$ can be interpreted as the predictive ability of the OOI measure when option bid-ask spreads decrease, i.e., lowest $\Delta ILO_{ranked,i,t}$ tercile. This finding is consistent with Hypothesis 5, which predicts that variations in the OOI measure contains information about the underlying stock only when option illiquidity increases, i.e., when option dealers aggressively widen their spreads to compensate for the risk of trading against informed investors.

### 2.5.3 Subperiods

Table 2.8 summarizes the economic profits from trading on the simple high-minus-low OOI-sorted portfolios (Strategy I) versus the high-minus-low OOI-sorted portfolios that utilize only stocks in the highest $\Delta ILOQ$ tercile (Strategy II). Column (I) reports portfolio alphas of the Strategy I, $\text{Alpha}_{OOI}$, while Column (II) reports portfolio alphas of the Strategy II for various subperiods and specifications.

The first row of Table 2.8 reports the high-minus-low alphas from the two strategies for the full 2004–2013 sample period, which are replicated from Table 2.6. The difference between the two strategies is 2.48 bps per day (or 6.25% per year) and significant at the one
percent level. We next divide our sample into two subperiods with equal length: 2004–2008, and 2009–2013. The second row reports results for the 2004–2008 subperiod, which overlaps with the sample of Hu (2014). Here, the gain of using Strategy II vs. Strategy I is only marginal, 5.67 bps ($t = 3.85$) vs. 4.40 bps ($t = 4.92$). However, for the later subperiod, i.e., 2009–2013, the difference is striking. Strategy I results in daily alpha of 3.47 bps ($t = 4.35$), while Strategy II yields daily alpha of 7.42 bps ($t = 5.62$) or 18.7% per year. This strong performance of Strategy II that we observe after 2008 is likely due to the fact that the option market became significantly more liquid, due to increasing competition among option market makers, as well as regulatory effects of the Penny Pilot project.\footnote{The Penny Pilot project begins in three phases starting in 2007. It’s intention is to decrease the tick size of option quotes trade below $3$ from five-cent increment to one-cent increment} The reduction in options trading cost likely push informed traders to increasingly use the option market as trading venue for their private information instead of the equity market. As a result, changes in option bid-ask spreads increasingly signal trades that contain private information. This could explain why the double-sorting strategy that uses change in ILO performs significantly well in the latter half of the sample relative to the single-sorted strategy that relies on OOI alone.

To further separate the transitory effect from the private information effect, the last row, row (4), reports results for high-minus-low alphas assuming that investors can only enter to buy or sell the underlying stocks on day $t + 1$, i.e., on the second day after portfolio formation. The results for Strategy I are no longer significant. Consistent with private information story, Strategy II still provides significant spreads of 2.5 bps ($t = 2.55$), or 6.3% per year. The results are qualitatively similar for ILOE, but not reported here for brevity.

Muravyev (2016) shows that order flows are persistent in the option market. That is, if we observe an excess demand for a synthetic long position today, we will likely observe it again tomorrow. As a result, the persistence of options’ order flow mechanically generates return predictability on the underlying because it is well known that contemporaneously, order flows and stock returns are positively correlated. In other words, the predictive ability of the OOI measure on stock returns is merely transitory and should not persist beyond the first day. Our findings that the OOI measure does not predict the underlying return beyond the next day is consistent with the results of Muravyev (2016). However, the fourth row in Table 2.8 shows that if we form high-minus-low OOI-sorted portfolios that utilize stocks with increasing option illiquidity, the return predictability persists beyond the first day. This finding suggests that changes in option illiquidity help to identify a subset of
underlying securities where informed option trading is most likely to take place.

Table 2.9 further explores the 2009–2013 sub-sample results for double-sorting strategy reported in Table 2.6. As we already mentioned, this second half of our sample experiences significantly improvements in option liquidity. The improvement in options liquidity can be partly attributed to the Penny pilot program which was implemented in three phases beginning in 2007 and ending in 2008, representing about 50% of industry trading volume. The Penny Pilot program significantly reduced options trading cost by specifying quoting increments of one cent for options trading at less than $3.00 and increments of five cents for options trading at $3.00 or more. In 2009, the SEC expanded the pilot project to additional 500 securities. We identify that 40% of the stocks in our sample are a part of Penny Pilot program. Thus from 2009 and onwards, almost half of our sample experiences significant improvement in liquidity. Further, Amihud, Mendelson and Lauterbach (1997) documented that there are positive externalities that spillover from the pilot stocks (i.e, those subject to trading increment reduction) to non-pilot stocks. This positive liquidity spillover effects likely explain the overall option liquidity improvement across all S&P500 stocks in the second half of our sample (2009–2013). This sub-sample, both in terms of cross-sectional selection and time span, differs from Hu (2014) sample and can further explain the differences in the results we find.

Panel A of Table 9 reports double-sorting results using ∆ILOQ, and Panel B uses ∆ILOE. Consider first Panel A. Here, the highest OOI portfolio and the highest ∆ILOQ portfolio has risk adjusted alpha of 4 bps ($t = 4.05$) per day, and the lowest OOI and the highest ∆ILOQ portfolio has the risk-adjusted alpha of $-3.4$ bps ($t = -3.27$) per day, Panel A. Note that for the low and medium ∆ILOQ portfolios, we do not largely observe any significant results. The high-minus-low OOI difference for the medium ∆ILOQ portfolio, albeit statistically significant, it is economically very small, approximately 2 bps ($t = 2.09$). The results are qualitatively similar in Panel B. Thus, the ability of OOI to predict stock returns, as stated in Hypothesis 5, is driven by stocks with significantly increasing option illiquidity.

2.6 Conclusion

Using intraday transaction data of individual equity option written on the S&P 500 index constituents from 2004 to 2013, we empirically examine two research questions. First, we analyze the determinants of option dealers’ market making costs, measured by relative effective and quoted spreads. Unlike previous literature, we use the largest cross-section and the
longest time-series of option intraday transaction data to conduct our analyses and highlight the economic and statistical significance of variables affecting option illiquidity. Second, we study and explain the implications of option illiquidity for the return predictability of the underlying stock.

We find that hedging costs are the largest component of option bid-ask spreads in terms of economic magnitudes with future rebalancing cost dominating the initial delta hedging cost. This finding contributes to a deeper understanding of the market microstructure of the US option market. Besides hedging costs, we find that option-induced demand pressure and private information are other main sources of risks for option market makers, which they normally absorb by quoting wider bid-ask spreads.

Finally, we study the implications of private information captured by option illiquidity for stock returns. Consistent with market microstructure theories of asymmetric information, we find that higher option-induced order imbalance (excess demand for synthetic long positions) and higher option illiquidity positively predict next day stock returns. On the other hand, lower option-induced order imbalance (excess demand for synthetic short positions) and higher option illiquidity negatively predict stock returns.

We do not find that option-induced order imbalance contains information about the underlying stock when option illiquidity decreases. This finding suggests that option-induced net buying pressure reflects trades of informed investors only when option dealers aggressively widen their spreads in response to the increased likelihood that they are trading against informed investors.
Table 2.1 Summary statistics

We report time-series averages of cross-sectional statistics of the main variables used in our study. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database. Panel A reports descriptive statistics for option-related variables separately for call and put options. ILOQ is the daily option illiquidity measure defined as the dollar-volume-weighted average relative quoted spreads of intraday option trades. ILOE is the daily option illiquidity measure defined as the dollar-volume-weighted average relative effective spreads of intraday option trades. %DHC is the daily percentage initial hedging cost associated with delta-hedging an option contract. %RBC is the daily percentage rebalancing cost of for maintaining the initially hedged option position (Leland(1985)). OOI is the daily option-induced order imbalance measure calculated following Hu (2014). OptVolume is the daily number of option contracts traded (in thousands). Num. of trades is the daily number of option transactions executed in each option category. Panel B reports descriptive statistics of stock-related variables used in our study. ILS is the daily underlying stock illiquidity measure defined as the average relative effective spreads of intraday stock trades. RET is the return on the underlying stock. MA5|RET| is the average absolute stock return over the past 5 trading days. PIN is the probability of information–based trading in the underlying stock computed on a quarterly basis for each firm (Easley et al (1996)).

### Panel A: Option-related variables for call and put options

<table>
<thead>
<tr>
<th>Variable</th>
<th>Call options (472 Firms)</th>
<th>Put options (443 Firms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Stdev</td>
</tr>
<tr>
<td>ILOE</td>
<td>6.5%</td>
<td>6.8%</td>
</tr>
<tr>
<td>ILOQ</td>
<td>8.0%</td>
<td>7.7%</td>
</tr>
<tr>
<td>%DHC</td>
<td>11.04</td>
<td>4.76</td>
</tr>
<tr>
<td>%RBC</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td>OOI</td>
<td>0.0000</td>
<td>0.0002</td>
</tr>
<tr>
<td>OptVolume</td>
<td>2.208</td>
<td>7.367</td>
</tr>
<tr>
<td>Num. of trades</td>
<td>120</td>
<td>376</td>
</tr>
</tbody>
</table>

### Panel B: Underlying stock variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Stdev</th>
<th>Min</th>
<th>Max</th>
<th># firm count</th>
</tr>
</thead>
<tbody>
<tr>
<td>ILS</td>
<td>0.08%</td>
<td>0.09%</td>
<td>0.02%</td>
<td>1.36%</td>
<td>500</td>
</tr>
<tr>
<td>RET</td>
<td>0.05%</td>
<td>1.74%</td>
<td>-10.12%</td>
<td>11.10%</td>
<td></td>
</tr>
<tr>
<td>MA5</td>
<td>RET</td>
<td></td>
<td>1.49%</td>
<td>0.85%</td>
<td>0.15%</td>
</tr>
<tr>
<td>PIN</td>
<td>9.1%</td>
<td>3.4%</td>
<td>0.6%</td>
<td>42.4%</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.2 Cross correlation matrix of the main variables

We report time-series averages of daily cross-sectional pairwise correlation coefficients of the main variables used in this paper. Panels A and B report results for variables calculated using call options and put options, respectively. ILOQ is the daily option illiquidity measure defined as the dollar-volume-weighted average relative quoted spreads of intraday option trades. ILOE is the daily option illiquidity measure defined as the dollar-volume-weighted average relative effective spreads of intraday option trades. %DHC is the daily percentage initial hedging cost associated with delta-hedging an option contract. %RBC is the daily percentage rebalancing cost of maintaining the initially hedged option position (Leland(1985)). OOI is the daily option-induced order imbalance measure calculated following Hu (2014). OptVolume is the natural logarithm of the daily number of option contracts traded. ILS is the daily underlying stock illiquidity measure defined as the average relative effective spreads of intraday stock trades. RET is the return on the underlying stock. MA5|RET| is the average absolute stock return over the past 5 trading days. PIN is the probability of information-based trading in the underlying stock computed on a quarterly basis for each firm (Easley et al (1996)). ***, **, and * indicate significance at the 1, 5, and 10 percent levels, respectively, based on Newey-West adjustment for autocorrelation up to 45 lags.

**Panel A: Call Options**

<table>
<thead>
<tr>
<th></th>
<th>ILOQ</th>
<th>ILOE</th>
<th>%DHC</th>
<th>%RBC</th>
<th>OOI</th>
<th>OptVolume</th>
<th>ILS</th>
<th>RET</th>
<th>MA5</th>
<th>RET</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ILOE</td>
<td>0.88***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%DHC</td>
<td>0.32***</td>
<td>0.30***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%RBC</td>
<td>0.48***</td>
<td>0.47***</td>
<td>0.29***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OOI</td>
<td>0.01***</td>
<td>0.01***</td>
<td>-0.00***</td>
<td>0.01***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OptVolume</td>
<td>-0.43***</td>
<td>-0.38***</td>
<td>-0.20***</td>
<td>-0.16***</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ILS</td>
<td>0.16***</td>
<td>0.16***</td>
<td>-0.32***</td>
<td>0.60***</td>
<td>0.01***</td>
<td>0.03***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RET</td>
<td>-0.01***</td>
<td>-0.01***</td>
<td>0.02***</td>
<td>-0.01***</td>
<td>0.08***</td>
<td>0.02***</td>
<td>-0.01***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA5</td>
<td>RET</td>
<td></td>
<td>-0.07***</td>
<td>-0.06***</td>
<td>-0.45***</td>
<td>-0.06***</td>
<td>0</td>
<td>0.20***</td>
<td>0.29***</td>
<td>-0.01**</td>
<td></td>
</tr>
<tr>
<td>PIN</td>
<td>0.13***</td>
<td>0.12***</td>
<td>-0.05***</td>
<td>0.09***</td>
<td>-0.00***</td>
<td>-0.16***</td>
<td>0.17***</td>
<td>0</td>
<td>0</td>
<td>0.07***</td>
<td></td>
</tr>
</tbody>
</table>

**Panel B: Put Options**

<table>
<thead>
<tr>
<th></th>
<th>ILOQ</th>
<th>ILOE</th>
<th>%DHC</th>
<th>%RBC</th>
<th>OOI</th>
<th>OptVolume</th>
<th>ILS</th>
<th>RET</th>
<th>MA5</th>
<th>RET</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ILOE</td>
<td>0.85***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%DHC</td>
<td>0.31***</td>
<td>0.29***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%RBC</td>
<td>0.43***</td>
<td>0.41***</td>
<td>0.24***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OOI</td>
<td>0</td>
<td>-0.00***</td>
<td>0.00*</td>
<td>0.01***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OptVolume</td>
<td>-0.38***</td>
<td>-0.33***</td>
<td>-0.19***</td>
<td>-0.13***</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ILS</td>
<td>0.13***</td>
<td>0.13***</td>
<td>-0.32***</td>
<td>0.59***</td>
<td>0.01***</td>
<td>0.03***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RET</td>
<td>0.00***</td>
<td>0.00***</td>
<td>0.01***</td>
<td>0.02***</td>
<td>0.08***</td>
<td>0.02***</td>
<td>-0.01***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MA5</td>
<td>RET</td>
<td></td>
<td>-0.08***</td>
<td>-0.07***</td>
<td>-0.44***</td>
<td>-0.06***</td>
<td>0</td>
<td>0.20***</td>
<td>0.29***</td>
<td>-0.01**</td>
<td></td>
</tr>
<tr>
<td>PIN</td>
<td>0.13***</td>
<td>0.11***</td>
<td>-0.06***</td>
<td>0.08***</td>
<td>-0.00***</td>
<td>-0.16***</td>
<td>0.16***</td>
<td>0</td>
<td>0</td>
<td>0.07***</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.3  Determinants of ILO

This table reports coefficient estimates from daily Fama-MacBeth cross-sectional regressions. The dependent variables are the daily option illiquidity measures ILOQ and ILOE. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30—182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database. Panels A and B report results for ILOQ and ILOE regressions, respectively. The independent variables include proxy for the probability of informed trading, market-making costs, and option-induced demand pressure faced by option dealers, as well as control variables. We consider two hedging cost variables affecting option market makers. %DHC is the daily percentage initial hedging cost associated with delta-hedging an option contract. %RBC is the daily percentage rebalancing cost of for maintaining the initially hedged option position (Leland(1985)). |OOI| is the daily absolute value of option-induced order imbalance calculated following Hu (2014). PIN is the probability of information-based trading in the underlying stock (Easley et al (1996)) calculated at the quarterly frequency. OptVolume is the natural logarithm of the number of option contracts traded. ILO(t-1) is the one-day lagged measure of option illiquidity (ILOQ or ILOE). RET is the return of the underlying stock. RET(t-1) is the return of the underlying stock on the previous trading day. MA5[RET] is the average absolute stock return over the past 5 trading days; it proxies for the stock volatility level. Day Count reports the number of daily cross-section regressions used for calculating the Fama-MacBeth coefficient estimates. Avg. cross section is the average sample size of daily cross-sectional regressions. Newey-West t-statistics adjusted for autocorrelation up to 45 lags are reported in square brackets below each estimate. ***, **, and * indicate significance at the 1, 5, and 10 percent confidence levels.
Table 2.3 (continued)

<table>
<thead>
<tr>
<th>Hedging costs</th>
<th>Panel A. Relative quoted spreads:</th>
<th>Panel B. Relative effective spreads:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call</td>
<td>Put</td>
</tr>
<tr>
<td>%DHC</td>
<td>0.0019***</td>
<td>0.0023***</td>
</tr>
<tr>
<td></td>
<td>[17.64]</td>
<td>[21.27]</td>
</tr>
<tr>
<td>%RBC</td>
<td>16.1051***</td>
<td>14.4994***</td>
</tr>
<tr>
<td></td>
<td>[30.29]</td>
<td>[33.59]</td>
</tr>
<tr>
<td>Demand pressure</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>O</td>
<td>O</td>
</tr>
<tr>
<td></td>
<td>[31.52]</td>
<td>[36.25]</td>
</tr>
<tr>
<td>Private information</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PIN</td>
<td>0.0945***</td>
<td>0.0934***</td>
</tr>
<tr>
<td></td>
<td>[9.55]</td>
<td>[10.06]</td>
</tr>
<tr>
<td>Other controls</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OptVolume</td>
<td>-0.0037***</td>
<td>-0.0030***</td>
</tr>
<tr>
<td>ILS</td>
<td>-20.2360***</td>
<td>-16.3433***</td>
</tr>
<tr>
<td>ILO(t-1)</td>
<td>0.3639***</td>
<td>0.3581***</td>
</tr>
<tr>
<td></td>
<td>[46.63]</td>
<td>[40.50]</td>
</tr>
<tr>
<td>RET</td>
<td>-0.0323***</td>
<td>0.0139***</td>
</tr>
<tr>
<td></td>
<td>[-6.75]</td>
<td>[2.85]</td>
</tr>
<tr>
<td>RET(t-1)</td>
<td>-0.0223***</td>
<td>-0.0174***</td>
</tr>
<tr>
<td>MA5</td>
<td>RET</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[23.55]</td>
<td>[23.10]</td>
</tr>
<tr>
<td>Day Count</td>
<td>2504</td>
<td>2504</td>
</tr>
<tr>
<td>Avg. cross section</td>
<td>453</td>
<td>413</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.531</td>
<td>0.471</td>
</tr>
</tbody>
</table>
Table 2.4  Determinants of change in ILO

This table reports coefficient estimates from daily Fama-MacBeth cross-sectional regressions. The dependent variables are changes in the equity option illiquidity measures: $\Delta ILOQ$ and $\Delta ILOE$ on day $t$. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE's TAQ database. Panels A and B report results for $\Delta ILOQ$ and $\Delta ILOE$ regressions, respectively. The independent variables include proxy for the probability of informed trading, market-making costs and demand pressures faced by option dealers, as well as control variables. %DHC(t-1) is the one-day lagged percentage initial hedging cost associated with delta-hedging an option contract. %RBC(t-1) is the one-day lagged percentage rebalancing cost of for maintaining the initially hedged option position (Leland(1985)). $|OOI(t-1)|$ is the one-day lagged absolute value of option-induced order imbalance calculated following Hu (2014). PIN is the probability of information-based trading in the underlying stock (Easley et al (1996)). OptVolume(t-1) is the natural logarithm of the number of option contracts traded on day $t-1$. $\Delta ILS(t-1)$ is the change in the underlying stock illiquidity measure on day $t-1$. $\Delta ILO(t-1)$ is the change in the equity option illiquidity measure on day $t-1$. RET(t-1) and RET(t-2) are daily returns of the underlying stock on day $t-1$ and $t-2$, respectively. MA5|RET|(t-1) is the lagged average absolute stock return over the past 5 trading days. Day Count reports the number of daily cross-section regressions used for reporting the Fama-MacBeth regression estimates. Avg. cross section is the average sample size of daily cross-sectional regressions. Newey-West $t$-statistics adjusted for autocorrelation up to 45 lags are reported in square brackets below each estimate. ***, **, and * indicate significance at the 1, 5, and 10 percent confidence levels.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Panel</th>
<th>Coefficient</th>
<th>SE</th>
<th>T-Statistic</th>
<th>p-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>%DHC(t-1)</td>
<td>A</td>
<td>-0.03</td>
<td>0.02</td>
<td>-1.50</td>
<td>0.130</td>
</tr>
<tr>
<td>%RBC(t-1)</td>
<td>A</td>
<td>0.04</td>
<td>0.02</td>
<td>2.10</td>
<td>0.034</td>
</tr>
<tr>
<td>$</td>
<td>OOI(t-1)</td>
<td>$</td>
<td>A</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>PIN</td>
<td>A</td>
<td>0.01</td>
<td>0.01</td>
<td>0.70</td>
<td>0.482</td>
</tr>
<tr>
<td>OptVolume(t-1)</td>
<td>A</td>
<td>0.02</td>
<td>0.01</td>
<td>2.10</td>
<td>0.034</td>
</tr>
<tr>
<td>$\Delta ILS(t-1)$</td>
<td>A</td>
<td>-0.03</td>
<td>0.02</td>
<td>-1.50</td>
<td>0.130</td>
</tr>
<tr>
<td>$\Delta ILO(t-1)$</td>
<td>A</td>
<td>0.04</td>
<td>0.02</td>
<td>2.10</td>
<td>0.034</td>
</tr>
<tr>
<td>RET(t-1)</td>
<td>A</td>
<td>-0.02</td>
<td>0.01</td>
<td>-2.00</td>
<td>0.042</td>
</tr>
<tr>
<td>RET(t-2)</td>
<td>A</td>
<td>-0.01</td>
<td>0.01</td>
<td>-1.00</td>
<td>0.319</td>
</tr>
<tr>
<td>MA5</td>
<td>RET</td>
<td>(t-1)</td>
<td>A</td>
<td>-0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Day Count</td>
<td>A</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. cross section</td>
<td>A</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%DHC(t-1)</td>
<td>B</td>
<td>-0.03</td>
<td>0.02</td>
<td>-1.50</td>
<td>0.130</td>
</tr>
<tr>
<td>%RBC(t-1)</td>
<td>B</td>
<td>0.03</td>
<td>0.02</td>
<td>1.50</td>
<td>0.130</td>
</tr>
<tr>
<td>$</td>
<td>OOI(t-1)</td>
<td>$</td>
<td>B</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>PIN</td>
<td>B</td>
<td>0.01</td>
<td>0.01</td>
<td>0.70</td>
<td>0.482</td>
</tr>
<tr>
<td>OptVolume(t-1)</td>
<td>B</td>
<td>0.02</td>
<td>0.01</td>
<td>2.10</td>
<td>0.034</td>
</tr>
<tr>
<td>$\Delta ILS(t-1)$</td>
<td>B</td>
<td>-0.03</td>
<td>0.02</td>
<td>-1.50</td>
<td>0.130</td>
</tr>
<tr>
<td>$\Delta ILO(t-1)$</td>
<td>B</td>
<td>0.04</td>
<td>0.02</td>
<td>2.10</td>
<td>0.034</td>
</tr>
<tr>
<td>RET(t-1)</td>
<td>B</td>
<td>-0.02</td>
<td>0.01</td>
<td>-2.00</td>
<td>0.042</td>
</tr>
<tr>
<td>RET(t-2)</td>
<td>B</td>
<td>-0.01</td>
<td>0.01</td>
<td>-1.00</td>
<td>0.319</td>
</tr>
<tr>
<td>MA5</td>
<td>RET</td>
<td>(t-1)</td>
<td>B</td>
<td>-0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>Day Count</td>
<td>B</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg. cross section</td>
<td>B</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.4 (continued)

<table>
<thead>
<tr>
<th>Hedging costs</th>
<th>Panel A: Change in relative quoted spreads: $\Delta ILOQ(t)$</th>
<th>Panel B: Change in relative effective spreads: $\Delta ILOE(t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>%DHC(t-1)</td>
<td>-0.0014*** [-20.23] -0.0014*** [-20.04]</td>
<td>-0.0010*** [-14.65] -0.0008*** [-12.58]</td>
</tr>
</tbody>
</table>
| Demand pressure | \midrule
| Private information | \midrule
| PIN           | 0.0060* [1.89] 0.0087** [2.15]                            | 0.0037** [1.99] 0.0059** [2.33]                            |
| Other controls | \midrule
| OptVolume (t-1) | -0.0003*** [-3.05] -0.0001 [-1.56]                     | -0.0003*** [-3.83] -0.0001 [-2.08]                     |
| $\Delta ILO$ (t-1) | -0.3343*** [-60.77] -0.3228*** [-49.50]            | -0.4557*** [-148.6] -0.4529*** [-147.6]            |
| RET (t-1)     | -0.0339*** [-6.72] 0.0511*** [8.86]                    | -0.0433*** [-10.27] 0.0432*** [10.09]                  |
| RET (t-2)     | -0.0001 [-0.03] 0.0315*** [7.95]                      | -0.0109*** [-2.86] 0.0241*** [7.84]                     |
| MA5 $|RET| (t-1) | -0.3895*** [-18.53] -0.3490*** [-18.08]                | -0.2627*** [-15.19] -0.2188*** [-14.06]                |
| Day Count     | 2497 2497                                                | 2497 2497                                                |
| Avg. cross section | 441 395                              | 441 395                              |
| Adj. R$^2$    | 0.1859 0.174                                           | 0.2707 0.2679                                           |
Table 2.5 Option and stock illiquidity around earnings announcement

We report event-study results for abnormal ILOQ, ILOE, and ILS over [−5,+5] trading-day windows around earnings announcement dates. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). Abnormal option illiquidity is calculated by subtracting daily ILOQ (ILOE) from its time-series average calculated over the [−42,−21] days relative to the event date, i.e. pre-event window. We consider option series with maturity between 30–182 calendar days. ILOQ is the daily option illiquidity measure defined as the dollar-volume-weighted average relative quoted spreads of intraday option trades. ILOE is the daily option illiquidity measure defined as the dollar-volume-weighted average relative effective spreads of intraday option trades. Abnormal stock illiquidity is calculated similarly by subtracting daily ILS from its time-series average calculated over the [−42,−21] days relative to the event date. ILS is the daily average relative effective spreads of intraday stock trades. Panel A reports event-study results for all earnings announcements. Panels B and C report event-study results for positive earnings surprises and negative earnings surprises, respectively. In each panel, we report results for abnormal ILOQ, ILOE, and ILS in basis points (bps). Results for options are reported separately for call and put options. We measure the magnitude of earnings surprises using the standardized cumulative abnormal returns of the underlying stock over the window [−1,1] relative to the event. The t-statistics, reported in square brackets, are based on the Hall (1992) skewness-corrected transformed normal test.

### Panel A: All earnings announcements

<table>
<thead>
<tr>
<th>Event day</th>
<th>Abnormal Option quoted spread (bps)</th>
<th>Abnormal Option effective spread (bps)</th>
<th>Abnormal Stock effective spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call options</td>
<td>Put options</td>
<td>Call options</td>
</tr>
<tr>
<td>-5</td>
<td>0.38 [0.14]</td>
<td>1.70 [0.64]</td>
<td>-0.60 [-0.22]</td>
</tr>
<tr>
<td>-4</td>
<td>0.50 [0.17]</td>
<td>0.08 [0.03]</td>
<td>3.52 [1.25]</td>
</tr>
<tr>
<td>-3</td>
<td>-2.32 [-0.82]</td>
<td>-1.07 [-0.38]</td>
<td>-0.05 [-0.01]</td>
</tr>
<tr>
<td>-2</td>
<td>2.06 [0.74]</td>
<td>1.97 [0.74]</td>
<td>2.98 [1.16]</td>
</tr>
<tr>
<td>0</td>
<td>74.43 [28.83]</td>
<td>81.43 [36.04]</td>
<td>67.83 [29.18]</td>
</tr>
<tr>
<td>2</td>
<td>-4.56 [-1.50]</td>
<td>-5.30 [-1.92]</td>
<td>-5.31 [-1.83]</td>
</tr>
<tr>
<td>4</td>
<td>-7.26 [-2.39]</td>
<td>-14.00 [-4.88]</td>
<td>-6.91 [-2.53]</td>
</tr>
</tbody>
</table>
Table 2.5 (continued)

Panel B: Positive earnings announcements surprises

<table>
<thead>
<tr>
<th>Event day</th>
<th>Abnormal Option quoted spread (bps)</th>
<th>Abnormal Option effective spread (bps)</th>
<th>Abnormal Stock effective spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call options</td>
<td>Put options</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>-6.1 [-0.97]</td>
<td>-11.06 [-1.88]</td>
<td>-5.12 [-0.88]</td>
</tr>
<tr>
<td>-4</td>
<td>-5.93 [-0.91]</td>
<td>-13.54 [-2.30]</td>
<td>-0.36 [-0.04]</td>
</tr>
<tr>
<td>-3</td>
<td>2.78 [0.45]</td>
<td>-2.02 [-0.31]</td>
<td>10.48 [1.77]</td>
</tr>
<tr>
<td>-2</td>
<td>3.64 [0.60]</td>
<td>-6.15 [-1.07]</td>
<td>4.62 [0.85]</td>
</tr>
<tr>
<td>1</td>
<td>-52.03 [-7.29]</td>
<td>15.67 [2.70]</td>
<td>-39.92 [-5.96]</td>
</tr>
<tr>
<td>3</td>
<td>-59.82 [-9.85]</td>
<td>-20.82 [-3.51]</td>
<td>-50.38 [-8.89]</td>
</tr>
<tr>
<td>4</td>
<td>-60.62 [-9.24]</td>
<td>-17.21 [-2.59]</td>
<td>-46.32 [-6.52]</td>
</tr>
</tbody>
</table>

Panel C: Negative earnings announcements surprises

<table>
<thead>
<tr>
<th>Event day</th>
<th>Abnormal Option quoted spread (bps)</th>
<th>Abnormal Option effective spread (bps)</th>
<th>Abnormal Stock effective spread (bps)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Call options</td>
<td>Put options</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>-0.36 [-0.04]</td>
<td>10.99 [1.90]</td>
<td>2.18 [0.37]</td>
</tr>
<tr>
<td>-4</td>
<td>1 [0.16]</td>
<td>7.66 [1.27]</td>
<td>8.75 [1.49]</td>
</tr>
<tr>
<td>-3</td>
<td>-0.88 [-0.13]</td>
<td>2.23 [0.39]</td>
<td>5.75 [0.96]</td>
</tr>
<tr>
<td>-2</td>
<td>-0.27 [-0.04]</td>
<td>4.03 [0.70]</td>
<td>9.09 [1.62]</td>
</tr>
<tr>
<td>-1</td>
<td>17.83 [3.02]</td>
<td>34.72 [6.29]</td>
<td>22.6 [4.25]</td>
</tr>
<tr>
<td>0</td>
<td>120.18 [22.55]</td>
<td>58.24 [12.19]</td>
<td>107.46 [22.70]</td>
</tr>
<tr>
<td>1</td>
<td>54.31 [8.90]</td>
<td>-12.26 [-2.31]</td>
<td>47.66 [8.61]</td>
</tr>
</tbody>
</table>
Table 2.6  Portfolio strategies: Order flow and changes in option illiquidity

The sample consists of S&P 500 index firms that have options traded on their underlying from January 2004 to December 2013 (2,504 trading days). This table reports the risk-adjusted returns or alphas, in basis points per day, on the equally weighted portfolios of stocks ranked by their respective daily option-induced order imbalances, OOI, (Hu (2014)), and daily change in option illiquidity ΔILOQ (or ΔILOE). At the market close of each day, stocks are sorted into portfolios by their OOI value and held for the next trading day. Panel A reports results from a single-sorting portfolio strategy based on OOI. Panel B reports results from a double-sorting portfolio strategy: First, stocks are sorted into three groups by OOI, and then are independently sorted into three groups by option illiquidity measure (ΔILOQ). All resulting nine (3 × 3) portfolios are to be held for the next trading day. Panel C repeats the strategy in Panel B using ΔILOE. The risk-adjusted returns are reported in the form of Fama-French-Carhart four-factor adjusted returns. The square brackets contain the t-statistics. ***, **, and * indicate significance at the 1, 5, and 10 percent levels, respectively.

Panel A: Portfolio alpha (t+1) sorted by OOI

<table>
<thead>
<tr>
<th>OOI rank</th>
<th>α_{t+1}</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) High OOI</td>
<td>2.38 ***</td>
<td>[4.85]</td>
</tr>
<tr>
<td>(2)</td>
<td>1.06 ***</td>
<td>[2.68]</td>
</tr>
<tr>
<td>(1) Low OOI</td>
<td>-1.65 ***</td>
<td>[-3.25]</td>
</tr>
<tr>
<td>(3)−(1) Alpha_{OOI}</td>
<td>4.02 ***</td>
<td>[6.72]</td>
</tr>
</tbody>
</table>

Panel B: Portfolio alpha (t+1) double-sorted by OOI and ΔILOQ

<table>
<thead>
<tr>
<th>OOI rank</th>
<th>ΔILOQ rank</th>
<th>α_{t+1}</th>
<th>t-stat</th>
<th>α_{t+1}</th>
<th>t-stat</th>
<th>α_{t+1}</th>
<th>t-stat</th>
<th>(A)−(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) High OOI</td>
<td>(A) Low</td>
<td>1.48 **</td>
<td>[2.04]</td>
<td>2.30 ***</td>
<td>[3.15]</td>
<td>3.37 ***</td>
<td>[2.12]</td>
<td>[2.02]</td>
</tr>
<tr>
<td>(2)</td>
<td>(B) Mid</td>
<td>1.54 ***</td>
<td>[2.88]</td>
<td>0.79</td>
<td>[1.48]</td>
<td>0.74</td>
<td>[1.32]</td>
<td>[-1.33]</td>
</tr>
<tr>
<td>(1) Low OOI</td>
<td>(C) High</td>
<td>-0.42</td>
<td>[-0.57]</td>
<td>-1.38 *</td>
<td>[-1.94]</td>
<td>-3.13 ***</td>
<td>[-4.08]</td>
<td>[-2.98]</td>
</tr>
</tbody>
</table>

Panel C: Portfolio alpha (t+1) double-sorted by OOI and ΔILOE

<table>
<thead>
<tr>
<th>OOI rank</th>
<th>ΔILOE rank</th>
<th>α_{t+1}</th>
<th>t-stat</th>
<th>α_{t+1}</th>
<th>t-stat</th>
<th>α_{t+1}</th>
<th>t-stat</th>
<th>(A)−(C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) High OOI</td>
<td>(A) Low</td>
<td>1.59 **</td>
<td>[2.16]</td>
<td>1.94 ***</td>
<td>[2.67]</td>
<td>3.60 ***</td>
<td>[4.79]</td>
<td>[2.10]</td>
</tr>
<tr>
<td>(2)</td>
<td>(B) Mid</td>
<td>1.59 ***</td>
<td>[2.96]</td>
<td>0.53</td>
<td>[1.05]</td>
<td>0.98 *</td>
<td>[1.73]</td>
<td>[-0.98]</td>
</tr>
<tr>
<td>(1) Low OOI</td>
<td>(C) High</td>
<td>-0.75</td>
<td>[-1.00]</td>
<td>-1.79 **</td>
<td>[-2.46]</td>
<td>-2.31 ***</td>
<td>[-3.14]</td>
<td>[-1.70]</td>
</tr>
<tr>
<td>(3)−(1) Alpha_{OOIΔILO}</td>
<td>2.34 **</td>
<td>[2.35]</td>
<td>3.73 ***</td>
<td>[4.13]</td>
<td>5.91 ***</td>
<td>[6.08]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2.7 Portfolio strategies: Order flow and changes in option illiquidity

This table reports results for the stock return predictability using daily Fama–MacBeth cross-sectional regressions. The dependent variable is the 1-day ahead return on the underlying stock $i$, $\text{RET}_{i,t+1}$. The predictors include: Option-induced order imbalance on day $t$ ($\text{OOI}_{i,t}$), cross-sectional rank of daily change in ILO ($\Delta \text{ILO}_{\text{ranked},i,t}$), and an interaction term between OOI and $\Delta \text{ILO}_{\text{ranked}}$ ($\text{OOI}_{i,t} \times \Delta \text{ILO}_{\text{ranked},i,t}$). Control variables are also included. For stock $i$ on day $t$, $\Delta \text{ILS}_{\text{ranked},i,t}$ is the cross-sectional rank of daily change in ILS. $\text{RET}$ is the daily return on the underlying stock. $\text{RET}_{[-5,-1]}$ is the cumulative return on the underlying stock from day $t-5$ to day $t-1$. $\text{RRV}$ is a daily range-based proxy for the realized volatility of the underlying stock, defined as the difference of the underlying stock’s intraday high and low price divided by the closing stock price. $\text{IV}$ is the implied volatility for the underlying stock, calculated as the average implied volatilities of the call-put pair with 30 calendar days to maturity reported in the standardized option file from OptionMetrics. $\ln(\text{OptVolume})$ is the natural logarithm of the number of option contracts traded. $\ln(\text{size})$ is the natural logarithm of the market value of the underlying stock. Day count reports the number of daily cross-section regressions used for reporting the Fama-MacBeth regression estimates. Avg. cross section is the average sample size of daily cross-sectional regressions. Newey-West $t$-statistics adjusted for autocorrelation up to 45 lags are reported in square brackets below each estimate. ***, **, and * indicate significance at the 1, 5, and 10 percent levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>(I)</th>
<th>(II)</th>
<th>(III)</th>
<th>(IV)</th>
<th>(V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Dependent variable: $\text{RET}_{i,t+1}$ (%)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{OOI}_{i,t}$</td>
<td>0.0211***</td>
<td>0.0017</td>
<td>0.0072</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[7.70]</td>
<td>[0.30]</td>
<td>[1.28]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{ILO}_{\text{ranked},i,t}$</td>
<td>-0.0006</td>
<td>-0.0077***</td>
<td>0.0001</td>
<td>-0.0048**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-1.03]</td>
<td>[-3.49]</td>
<td>[0.21]</td>
<td>[-2.50]</td>
<td></td>
</tr>
<tr>
<td>$\text{OOI}<em>{i,t} \times \Delta \text{ILO}</em>{\text{ranked},i,t}$</td>
<td>0.0035**</td>
<td>0.0024**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[3.39]</td>
<td>[2.54]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Control variables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta \text{ILS}_{\text{ranked},i,t}$</td>
<td>-0.0012*</td>
<td>-0.0012*</td>
<td>-0.0012*</td>
<td>-0.0012*</td>
<td>-0.0013*</td>
</tr>
<tr>
<td></td>
<td>[-1.72]</td>
<td>[-1.74]</td>
<td>[-1.80]</td>
<td>[-1.81]</td>
<td>[-1.86]</td>
</tr>
<tr>
<td>$\text{RET}_{i,t}$</td>
<td>-0.0055*</td>
<td>-0.0041</td>
<td>-0.0055*</td>
<td>-0.0041</td>
<td>-0.0054</td>
</tr>
<tr>
<td>$\text{RET}_{i[-5,-1]}$</td>
<td>-0.0038***</td>
<td>-0.0038***</td>
<td>-0.0038***</td>
<td>-0.0038***</td>
<td>-0.0038***</td>
</tr>
<tr>
<td></td>
<td>[-2.82]</td>
<td>[-2.86]</td>
<td>[-2.78]</td>
<td>[-2.85]</td>
<td>[-2.78]</td>
</tr>
<tr>
<td>$\text{RRV}_{i,t}$</td>
<td>-0.0002***</td>
<td>-0.0002***</td>
<td>-0.0002***</td>
<td>-0.0002***</td>
<td>-0.0002***</td>
</tr>
<tr>
<td></td>
<td>[-5.80]</td>
<td>[-5.58]</td>
<td>[-5.60]</td>
<td>[-5.65]</td>
<td>[-5.59]</td>
</tr>
<tr>
<td>$\text{IV}_{i,t}$</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>[1.24]</td>
<td>[1.17]</td>
<td>[1.19]</td>
<td>[1.20]</td>
<td>[1.22]</td>
</tr>
<tr>
<td>$\ln(\text{OptVolume})_{i,t}$</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0000</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>[0.26]</td>
<td>[0.03]</td>
<td>[0.10]</td>
<td>[0.02]</td>
<td>[0.08]</td>
</tr>
<tr>
<td>$\ln(\text{size})$</td>
<td>-0.0058</td>
<td>-0.0055</td>
<td>-0.0055</td>
<td>-0.0053</td>
<td>-0.0052</td>
</tr>
<tr>
<td></td>
<td>[-1.14]</td>
<td>[-1.05]</td>
<td>[-1.04]</td>
<td>[-1.02]</td>
<td>[-0.98]</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.0656</td>
<td>0.1070*</td>
<td>0.1040*</td>
<td>0.1010*</td>
<td>0.0852</td>
</tr>
<tr>
<td></td>
<td>[1.08]</td>
<td>[1.74]</td>
<td>[1.69]</td>
<td>[1.67]</td>
<td>[1.35]</td>
</tr>
<tr>
<td>Day Count</td>
<td>2,491</td>
<td>2,469</td>
<td>2,469</td>
<td>2,469</td>
<td>2,469</td>
</tr>
<tr>
<td>Avg. cross section</td>
<td>483</td>
<td>470</td>
<td>470</td>
<td>471</td>
<td>471</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>10.30%</td>
<td>10.27%</td>
<td>10.35%</td>
<td>10.26%</td>
<td>10.35%</td>
</tr>
</tbody>
</table>
Table 2.8  Portfolio alphas from daily trading strategies: Subperiod analysis

This table reports the risk-adjusted returns or alphas, in basis points per day, on the equally weighted portfolios of stocks ranked by their respective daily option order imbalance OOI (Hu (2014)), and/or daily change in option illiquidity measures $\Delta$ILOQ for different subperiods as well as the results for the skip-one-day trading strategy for the entire time sample. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). At the market close of each day, stocks are sorted into portfolios by their OOI value and held for the next trading day. Column (I) reports results from a single-sorting portfolio strategy based on OOI. Column (II) reports results from a double-sorting portfolio strategy based on OOI and $\Delta$ILOQ. The risk-adjusted returns are calculated using the Fama-French-Carhart four-factor adjusted returns. The square bracket underneath each estimate reports the $t$-statistics. ***, **, and * indicate significance at the 1, 5, and 10 percent levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Alpha_{OOI} Value</td>
<td>Alpha_{OOI/ILOQ} Value</td>
<td>Alpha_{OOI/ILOQ} Value</td>
<td>Alpha_{OOI/ILOQ} Value</td>
</tr>
<tr>
<td>t-stat</td>
<td>t-stat</td>
<td>t-stat</td>
<td>t-stat</td>
</tr>
<tr>
<td>0.59 [1.03]</td>
<td>2.49 ** [2.55]</td>
<td>1.90** [2.42]</td>
<td></td>
</tr>
</tbody>
</table>
Table 2.9 2009–2013 Portfolio strategies: Order flow and changes in ILO

The sample consists of S&P 500 index firms that have options traded on their underlying from January 2009 to December 2013 (1,252 trading days). This table reports the risk-adjusted returns or alphas, in basis points per day, on the equally weighted portfolios of stocks ranked by their respective daily option-induced order imbalances OOI (Hu (2014)), and daily change in option illiquidity ∆ILOQ (or ∆ILOE). At the market close of each day, stocks are sorted into portfolios by their OOI value and held for the next trading day. Panel A reports results from a single-sorting portfolio strategy based on OOI. Panel B reports results from a double-sorting portfolio strategy: First, stocks are sorted into three groups by OOI, and then are independently sorted into three groups by option illiquidity measure (∆ILOQ). All resulting nine (3 × 3) portfolios are to be held for the next trading day. Panel C repeats the strategy in Panel B using ∆ILOE. The risk-adjusted returns are reported in the form of Fama-French-Carhart four-factor adjusted returns. The square brackets contain the \( t \)-statistics. ***, **, and * indicate significance at the 1, 5, and 10 percent levels, respectively.

**Panel A: Portfolio alpha (t+1) sorted by OOI**

<table>
<thead>
<tr>
<th>OOI rank</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) High OOI</td>
<td>2.00 ***</td>
<td>[3.07]</td>
</tr>
<tr>
<td>(2)</td>
<td>1.25 **</td>
<td>[2.49]</td>
</tr>
<tr>
<td>(1) Low OOI</td>
<td>-1.47 **</td>
<td>[-2.13]</td>
</tr>
<tr>
<td>(3)–(1) AlphaOOI</td>
<td>3.47 ***</td>
<td>[4.35]</td>
</tr>
</tbody>
</table>

**Panel B: Portfolio alpha (t+1) double-sorted by OOI and ∆ILOQ**

<table>
<thead>
<tr>
<th>OOI rank</th>
<th>( \Delta ILOQ ) rank</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) High OOI</td>
<td>(A) Low</td>
<td>1.08</td>
<td>[1.31]</td>
<td>1.19</td>
<td>[1.29]</td>
<td>3.99 ***</td>
<td>[4.05]</td>
</tr>
<tr>
<td></td>
<td>(B) Mid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) High</td>
<td>3.35 ***</td>
<td>[3.49]</td>
<td>3.35 ***</td>
<td>[3.49]</td>
<td>2.37</td>
<td>[2.02]</td>
</tr>
<tr>
<td></td>
<td>(C) – (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Low OOI</td>
<td>(A) Low</td>
<td>1.43 *</td>
<td>[1.96]</td>
<td>0.69</td>
<td>[0.93]</td>
<td>1.47 **</td>
<td>[2.07]</td>
</tr>
<tr>
<td></td>
<td>(B) Mid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) High</td>
<td>0.45</td>
<td>[0.47]</td>
<td>-1.21</td>
<td>[-1.30]</td>
<td>-3.43 ***</td>
<td>[-3.27]</td>
</tr>
<tr>
<td></td>
<td>(C) – (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Low OOI</td>
<td>(A) Low</td>
<td>0.83</td>
<td>[0.67]</td>
<td>2.40 **</td>
<td>[2.09]</td>
<td>7.42 ***</td>
<td>[5.62]</td>
</tr>
<tr>
<td></td>
<td>(B) Mid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) High</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Panel C: Portfolio alpha (t+1) double-sorted by OOI and ∆ILOE**

<table>
<thead>
<tr>
<th>OOI rank</th>
<th>( \Delta ILOE ) rank</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
<th>( \alpha_{t+1} )</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3) High OOI</td>
<td>(A) Low</td>
<td>0.84</td>
<td>[0.83]</td>
<td>1.94 **</td>
<td>[2.05]</td>
<td>3.35 ***</td>
<td>[3.49]</td>
</tr>
<tr>
<td></td>
<td>(B) Mid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) High</td>
<td>2.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) – (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Low OOI</td>
<td>(A) Low</td>
<td>1.37 *</td>
<td>[1.92]</td>
<td>0.95</td>
<td>[1.43]</td>
<td>1.36 *</td>
<td>[1.80]</td>
</tr>
<tr>
<td></td>
<td>(B) Mid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) High</td>
<td>-0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) – (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1) Low OOI</td>
<td>(A) Low</td>
<td>0.45</td>
<td>[0.46]</td>
<td>-2.04 **</td>
<td>[-2.22]</td>
<td>-2.36 **</td>
<td>[-2.30]</td>
</tr>
<tr>
<td></td>
<td>(B) Mid</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) High</td>
<td>-2.37</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(C) – (A)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3)–(1) AlphaOOILOE</td>
<td>0.39</td>
<td>[0.30]</td>
<td>3.98 ***</td>
<td>[3.34]</td>
<td>5.70 ***</td>
<td>[4.33]</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.1 Aggregate option illiquidity (ILO)

We plot daily cross-sectional averages of the option illiquidity measures ILOQ and ILOE and the daily cross-sectional averages of underlying stock illiquidity measure ILS (bottom panel). For an individual firm, ILS is defined as the daily average relative effective spreads of intraday stock trades. The sample consists of S&P 500 index firms that have options traded on their underlying from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database.
Figure 2.2 Option effective spreads v.s. quoted spreads by option type

The top row shows daily cross-sectional averages of the ratio of effective–to–quoted option spreads separately for calls and puts. A ratio below one indicates that trades are executed well within the quotes. The bottom row shows daily fraction of option trades that are executed within the quoted bid–ask spreads separately for calls and puts. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database.
Figure 2.3 Option volume

We plot daily average option volume separately for calls and puts. Daily average volume is calculated by summing transaction volumes in each option category. The sample consists of S&P 500 index firms with exchange-listed options from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database.
Figure 2.4 Abnormal volume around earnings announcements

This figure plots event-study results of option and stock abnormal trading volumes over the \([-10,+10]\)-day window around earnings announcements. The left-hand and right-hand panels plot the results for option and stock trading volumes, respectively. Option trading volume is measured as the number of contracts traded (in thousandth units). Stock trading volume is measured as the level of shares turnover (trading volume over the number of shares outstanding). Abnormal option (or stock) volume is calculated as the daily average volume less its time-series mean over the pre-event window \([-42,-21]\). For option trading volume, we report results separately for call (solid line) and put (dotted line) options. The top panel plots event-study results for all earnings announcements news. The middle- and bottom-panels plot results for earnings announcements that are classified as positive surprises, and negative surprises, respectively. We measure the magnitude of earnings surprise using the standardized cumulative abnormal returns of the underlying stock over the three-day window \([-1,1]\) relative to the event date. The sample consists of S&P 500 index firms that have options traded on their underlying from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database.
Figure 2.5 Abnormal option and stock illiquidity around earnings announcements

This figure plots event–study results for option illiquidity (ILO) and stock illiquidity (ILS) over the $[-10,+10]$ days window relative to earnings announcement dates. ILOQ is the daily option illiquidity measure defined as the dollar-volume-weighted average relative quoted spreads of intraday option trades. ILOE is the daily option illiquidity measure defined as the dollar-volume-weighted average relative effective spreads of intraday option trades. Abnormal option (or stock) illiquidity measure is calculated as its the daily level less its time-series mean over the pre-event window $[-42,-21]$. Results are reported separately for call options (solid line), put options (dotted line), and underlying stocks (dashed-dotted line). The top panel plots event-study results for all earnings announcements news. The middle- and bottom-panels plot results for earnings announcements that are classified as positive surprises, and negative surprises, respectively. We measure the magnitude of earnings surprise using the standardized cumulative abnormal returns of the underlying stock over the three-day window $[-1,1]$ relative to the event date. The sample consists of S&P 500 index firms that have options traded on their underlying from January 2004 to December 2013 (2,504 trading days). Intraday option trades and quotes are obtained from LiveVol. We focus on transactions on option contracts with maturity of 30–182 calendar days. Intraday stock data are obtained from NYSE’s TAQ database.
Appendix A

This appendix presents the proofs for the propositions in Chapter 1.

Proof of Eq. (1.5). Let \( x_\cdot = [x_0, \ldots, x_{T-1}]' \). Let \( v_0 = 1 - \phi^2 \right)^{\frac{1}{2}} x_0 \), and let \( v_\cdot = [v_0, \ldots, v_{T-1}]' \) and \( v = [v_1, \ldots, v_T]' \). Define

\[
J_T = \begin{bmatrix} 0_{T-1} & I_{T-1} \\ 0_{T-1} & 0'_{T-1} \end{bmatrix}, \quad K = \begin{bmatrix} 1 - \phi^2 & 0 & \cdots & 0 \\ -\phi & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\phi & 1 \end{bmatrix}.
\]

(A1)

It is straightforward to arrive at the following derivation:

\[
\hat{\phi} - \phi = \frac{x_\cdot'Mv}{x_\cdot'Mx_\cdot} = \frac{v_\cdot'K^{-1}Mv}{v_\cdot'K^{-1}MK^{-1}v_\cdot}.
\]

(A2)

where the second equality follows from \( x_\cdot = K^{-1}v_\cdot + \mu x 1_T \). Next, take expectation on both sides of (A2) conditioning on \( v_\cdot \), and use the identities \( E[v|v_\cdot] = J_T v_\cdot \) and \( GK' = J_T \). It follows that

\[
E[\hat{\phi} - \phi|v_\cdot] = \frac{v_\cdot'K^{-1}ME[v|v_\cdot]}{v_\cdot'K^{-1}MK^{-1}v_\cdot} = \frac{v_\cdot'K^{-1}MJ_Tv_\cdot}{v_\cdot'K^{-1}MK^{-1}v_\cdot} = \frac{v_\cdot'K^{-1}MGK'v_\cdot}{v_\cdot'K^{-1}MK^{-1}v_\cdot}.
\]

(A3)

Finally, let \( z = K^{-1}v_\cdot / \sigma_v \) and note that \( H = (K'K)^{-1} \). The result then follows.

The Expression in Eq. (1.9). The required moments are given by

\[
E[U] = -f_1, \quad (A4)
\]

\[
E[V] = T(1 - \phi^2)^{-1} - f_4, \quad (A5)
\]
\[
\text{Cov}[U, V] = 2f_2 - 4f_3 + 2f_1f_4, \quad (A6)
\]
\[
\text{Var}[V] = 2T(1 - \phi^2)^{-2} + 4(1 - \phi^2)^{-1}(\phi f_2 - f_4 - 2\phi f_3) + 2f_4^2, \quad (A7)
\]
\[
\text{E}[(V - \text{E}[V])^3] = 8(f_6 - 3f_7 + 3f_4f_5 - f_4^3), \quad (A8)
\]
\[
\text{E}[(U - \text{E}[U])(V - \text{E}[V])^2] = 2\phi^{-1} \{(1 - \phi^2)\text{E}[(V - \text{E}[V])^3] - 4\text{Var}[V]\} \quad (A9)
\]
where
\[
f_1 = T^{-1}1_T G 1_T = \frac{1}{1 - \phi} + \frac{\phi^T - 1}{T(1 - \phi)^2}, \quad (A10)
\]
\[
f_2 = \text{tr}(GH) = \frac{\phi[T(1 - \phi^2) + \phi^2T - 1]}{(1 - \phi^2)^3}, \quad (A11)
\]
\[
f_3 = T^{-1}1_T GH1_T = \frac{T(1 - \phi^2)(1 + \phi + \phi^T) - (1 - \phi^T)(1 + 3\phi + 3\phi^2 - \phi^{T+1})}{T(1 - \phi)^2(1 - \phi^2)^2}, \quad (A12)
\]
\[
f_4 = T^{-1}1_T H1_T = \frac{1 + 2\phi f_1}{1 - \phi^2}, \quad (A13)
\]
\[
f_5 = T^{-1}1_T H^21_T = \frac{f_4 + 2\phi f_3}{1 - \phi^2}, \quad (A14)
\]
\[
f_6 = \text{tr}(H^3) = \frac{T[1 + \phi^2(4 + \phi^2 + 6\phi^2T)]}{(1 - \phi^2)^5} - \frac{6\phi^2(1 + \phi^2)(1 - \phi^2T)}{(1 - \phi^2)^6}, \quad (A15)
\]
\[
f_7 = T^{-1}1_T H^31_T = \frac{T\phi^{T+1}}{(1 - \phi^2)^2(1 - \phi^2)^3} + \frac{(1 + \phi)^4 + \phi^{T+1}(5 + 8\phi + 5\phi^2 - 4\phi^{T+1})}{(1 - \phi^2)^4(1 - \phi^2)^2}
\]
\[
- \frac{2\phi(1 - \phi^T)\{3 + \phi[9 + \phi^{2T+1} - 3\phi^T(1 + \phi + \phi^2) + \phi(14 + 9\phi + 3\phi^2)]\}}{T(1 - \phi^2)^5(1 - \phi)^2}. \quad (A16)
\]

**Derivations of (A4)–(A9).** Equations (A4)–(A9) are derived using the results on the moments of products of quadratic forms in normal variables.\(^1\) It is straightforward to show that
\[
\text{E}[U] = \text{tr}(MG) = -\frac{1_T^T G 1_T}{T}, \quad (A17)
\]
\[
\text{E}[V] = \text{tr}(MH) = \frac{T}{1 - \phi^2} - \frac{1_T^T H 1_T}{T}, \quad (A18)
\]
\[
\text{Cov}[U, V] = 2\text{tr}(MGHM) = 2\text{tr}(GH) - \frac{41_T^T G 1_T}{T} + \frac{2(1_T^T G 1_T^T)(1_T^T H 1_T)}{T^2}, \quad (A19)
\]
\[
\text{Var}[V] = 2\text{tr}((MH)^2) = 2\text{tr}(H^2) - \frac{41_T^T H^2 1_T}{T} + \frac{2(1_T^T H 1_T)^2}{T^2}, \quad (A20)
\]

\(^1\)See Magnus (1978).
E[(V - E[V])³] = 8 \text{tr}((MH)³), \quad \text{(A21)}
E[(U - E[U])(V - E[V])²] = 8 \text{tr}(MG(MH)²). \quad \text{(A22)}

In addition, using the fact that
\[
H = \frac{1}{1 - \phi^2} [I_T + \phi(G + G')],
\]
we have
\[
\frac{1_T' H 1_T}{T} = \frac{1}{1 - \phi^2} + \frac{2\phi 1_T' G 1_T}{T(1 - \phi^2)},
\]
\[
\text{tr}(H^2) = \frac{T}{(1 - \phi^2)^2} + \frac{2\phi \text{tr}(GH)}{1 - \phi^2},
\]
\[
\frac{1_T' H^2 1_T}{T} = \frac{1_T' H 1_T}{T(1 - \phi^2)} + \frac{2\phi 1_T' G H 1_T}{T(1 - \phi^2)}.
\]

It can be readily shown that
\[
1_T' G 1_T = \frac{T(1 - \phi) + \phi^T - 1}{(1 - \phi)^2},
\]
\[
\text{tr}(GH) = \frac{\phi[T(1 - \phi^2) + \phi^2 T - 1]}{(1 - \phi^2)^2},
\]
\[
1_T' G H 1_T = \frac{T(1 - \phi^2)(1 + \phi + \phi^T) - (1 - \phi^T)(1 + 3\phi + 3\phi^2 - \phi^T + 1)}{(1 - \phi)^2(1 - \phi^2)^2}.
\]

**Proof of Proposition 1.** Let $\text{SE}_\omega(\hat{\beta}^c)$ be the standard error of the estimated coefficient of $x_{t-1}$ in the augmented regression with $\hat{v}_t^c$ as the third regressor, and therefore it is a correction-dependent term. Lemma 2 in Amihud and Hurvich (2004) says that
\[
\text{E}(\hat{\beta}^c - \beta)^2 = \kappa^2 \text{E}(\hat{\beta}^c - \phi)^2 + \text{E}[\text{SE}_\omega(\hat{\beta}^c)]^2.
\]
Hence, it suffices to show that the second term can be further decomposed.

Define $\mathcal{F}_x$ as the information set generated by $x_0, \ldots, x_T$. Let $e_t = u_t - \kappa v_t$. It follows that $e_t$'s are independent of $\mathcal{F}_x$, and they are i.i.d. normal with zero mean. Recall $x_t = \theta + \phi x_{t-1} = \hat{\phi}^c x_{t-1} + \hat{v}_t^c$. It follows that
\[
y_t = \alpha + \beta x_{t-1} + \kappa v_t + e_t
\]
\[ = \alpha - \kappa \theta + [\beta + \kappa (\hat{\phi}^c - \phi)]x_{t-1} + \kappa \hat{v}_t^c + e_t. \quad \text{(A31)} \]

Due to the normality and strict exogeneity of the error term, the standard OLS regression result implies that \( E[\hat{\sigma}_e^2|\mathcal{F}_x] = \sigma_e^2 \), where \( \hat{\sigma}_e^2 \) is the SSR of this augmented regression divided by \( (T - 3) \).

Let \( x_\cdot = [x_0, \ldots, x_{T-1}]' \), and let \( \hat{v}^c = [\hat{v}_0^c, \ldots, \hat{v}_T^c]' \). Let \( \hat{M}^c \) be the projection matrix w.r.t. \( \mathcal{R}[1_T, \hat{v}^c] \). It follows that

\[
E[SE_\omega(\hat{\beta}^c)]^2 = E \left[ \frac{\hat{\sigma}_e^2}{x_\cdot' \hat{M}^c x_\cdot} \right] = \sigma_e^2 E \left[ \frac{1}{x_\cdot' \hat{M}^c x_\cdot} \right]. \quad \text{(A32)}
\]

Let \( M \) be the projection matrix w.r.t. \( \mathcal{R}[1_T] \). Also note that the following identity holds for any real number \( \omega \).

\[
\frac{1}{x_\cdot' \hat{M}^c x_\cdot} = \frac{1}{x_\cdot' M x_\cdot} + \frac{\omega^2}{\hat{v}' \hat{v}}. \quad \text{(A33)}
\]

To see this, let \( \bar{x}_\cdot = M x_\cdot \), and let \( \hat{v} = [\hat{v}_1, \ldots, \hat{v}_T]' \). Define \( P_{[1_T, \hat{v}^c]} \) as the projection matrix w.r.t. \( \mathcal{R}[1_T, \hat{v}^c] \), and let any other projection matrix be denoted similarly with the subscript giving the basis of the vector subspace onto which the projection is made. Then,

\[
P_{[1_T, \hat{v}^c]} = P_{[1_T, \hat{v} - \omega \hat{x}_\cdot]} = P_{1_T} + P_{M(\hat{v} - \omega \hat{x}_\cdot)} = P_{1_T} + P_{\bar{v} - \omega \bar{x}_\cdot}. \quad \text{(A34)}
\]

The last equality holds because \( \hat{M} \hat{v} = \hat{v} \). It follows that

\[
\hat{M}^c = M - P_{\bar{v} - \omega \bar{x}_\cdot} = M - \frac{(\hat{v} - \omega \bar{x}_\cdot)(\hat{v} - \omega \bar{x}_\cdot)'}{(\hat{v} - \omega \bar{x}_\cdot)'(\hat{v} - \omega \bar{x}_\cdot)}. \quad \text{(A35)}
\]

It then suffices to compute \( 1/(x_\cdot' \hat{M}^c x_\cdot) \) using (A35). Straightforward algebra yields (A33). Taking expectation on both sides of (A33) and using (A32) yield

\[
E[SE_\omega(\hat{\beta}^c)]^2 = \sigma_e^2 E \left[ \frac{1}{x_\cdot' M x_\cdot} \right] + \sigma_e^2 E \left[ \frac{\omega^2}{\hat{v}' \hat{v}} \right]. \quad \text{(A36)}
\]

Finally, the desired MSE breakdown in (1.17) holds because the first term on the RHS of (A36) equals \( E[SE_0(\hat{\beta})]^2 \). This follows from the same argument that leads to (A32). Here, we only need to consider the augmented regression using the uncorrected OLS residual as the third regressor. The argument is omitted for brevity. This completes the proof.
Proof of Proposition 2. Let us define \( e_t = u_t - \kappa v_t \). Then \( e_t \sim \mathcal{N}(0, \sigma_e^2) \), with \( \sigma_e^2 = \sigma_u^2 - \kappa^2 \sigma_v^2 \), and \( e_t \) is independent of \( v_t \). Moreover, since \((u_t, v_t)\) are i.i.d. bivariate normal across \( t \), it follows that \( e_t \) is independent of \( x_s, s = 0, 1, \ldots, T \). Next, we note \( x_t - \phi x_{t-1} = \theta + v_t \), which follows from AR(1) equation (1.2). Therefore, the DGP of \( r_t \) can be written as

\[
\begin{align*}
    r_t &= \alpha + \beta x_{t-1} + \kappa v_t + e_t \\
    &= \alpha - \kappa \theta + \beta x_{t-1} + \kappa(x_t - \phi x_{t-1}) + e_t.
\end{align*}
\]

(A37)

Since \( e_t \) is strictly exogenous, the assumptions of the classic regression model hold. The result then follows from the normality of the error term.

Proof of Proposition 3. Let \( x_- = [x_0, \ldots, x_{T-1}]' \), \( r = [r_1, \ldots, r_T]' \), and \( u = [u_1, \ldots, u_T]' \). Let \( 1_T \) be a \( T \times 1 \) vector of ones. Putting (1.1) into vector form \( r = \alpha 1_T + \beta x_- + u \) and substituting this into the expression of \( \hat{\beta} \) yield

\[
\hat{\beta} = x_- ' M r / x_- ' M x_- = x_- ' M(\alpha 1_T + \beta x_- + u) / x_- ' M x_- = \beta + x_- ' M u / x_- ' M x_- .
\]

(A38)

Let \( \mu_x = \theta / (1 - \phi) \). Let \( I_T \) be the identity matrix of order \( T \), and let \( M = I_T - 1_T 1_T' / T \). Now, define

\[
\tilde{\beta} = \tilde{z}' A \tilde{z} / (\tilde{z}' B \tilde{z}),
\]

where

\[
\tilde{z} = \begin{bmatrix} u / \sigma_u \\ (x_- - \mu_x 1_T) / \sigma_v \end{bmatrix}, \quad A = \begin{bmatrix} 0_{T \times T} & M / 2 \\ M / 2 & 0_{T \times T} \end{bmatrix}, \quad B = \begin{bmatrix} 0_{T \times T} & 0_{T \times T} \\ 0_{T \times T} & M \end{bmatrix}.
\]

(A40)

It follows that

\[
\hat{\beta} = \beta + (\sigma_u / \sigma_v) \tilde{\beta}.
\]

(A41)

Next, we derive the distribution of \( \tilde{\beta} \). Define \( v_0 = (1 - \phi^2)^{\frac{1}{2}}(x_0 - \mu_x) \). Let \( v_- = [v_0, \ldots, v_{T-1}]' \). It follows that

\[
\begin{bmatrix} u \\ v_- \end{bmatrix} \sim \mathcal{N} \left( 0_{2T}, \begin{bmatrix} \sigma_u^2 1_T & \sigma_{uv} 1_T \\ \sigma_{uv} 1_T' & \sigma_v^2 I_T \end{bmatrix} \right),
\]

(A42)
where
\[ J_T = \begin{bmatrix} 0_{T-1} & I_{T-1} \\ 0 & 0'_{T-1} \end{bmatrix}. \quad (A43) \]

Define two constant square matrices of order \( T \) as follows. Let \( G = (\phi^{j-i-1} \mathbb{1}_{i<j}) \), \( H = (\phi^{j-i}/(1 - \phi^2)) \). We then have \( H = (K'K)^{-1} \) and \( G = J_TK'^{-1} \), where
\[
K = \begin{bmatrix} \sqrt{1 - \phi^2} & 0 & \cdots & 0 \\ -\phi & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\phi & 1 \end{bmatrix}. \quad (A44)
\]

Note that \( K^{-1}v = x - \mu_x 1_T \). This, combined with the definition of \( \tilde{z} \) and the joint normal distribution in (A42), implies that \( \tilde{z} \sim N(0_{2T}, V) \), where
\[
V = \begin{bmatrix} I_T & \rho_{uv}G' \\ \rho_{uv}G & H \end{bmatrix}. \quad (A45)
\]

It then follows from Eq.(A39) that \( \tilde{\beta} \) is the ratio of two quadratic forms in joint normal random variables. Further, noting Eq.(A41), we conclude that \( \tilde{\beta} \) only depends on \( \beta, \sigma_u/\sigma_v, \phi, \rho_{uv} \) and \( T \). Moreover, the results in (i) and (ii) follow immediately.

Finally, for result (iii), it suffices to show that \( \tilde{\beta}_1 = -\tilde{\beta} \), where \( \tilde{\beta}_1 \equiv \tilde{w}'A\tilde{w} / (\tilde{w}'B\tilde{w}) \) and
\[
\tilde{w} \sim N \left( 0_{2T}, \begin{bmatrix} I_T & -\rho_{uv}G' \\ -\rho_{uv}G & H \end{bmatrix} \right). \quad (A46)
\]

Let \( \tilde{z} = [\tilde{z}_1', \tilde{z}_2'] \), where the two subvectors are of length \( T \). Define \( \tilde{w}_1 = -\tilde{z}_1 \) and \( \tilde{w}_2 = \tilde{z}_2 \). Then \( \tilde{w} \equiv [\tilde{w}_1', \tilde{w}_2'] \) follows the multivariate normal distribution in Eq.(A46). It is then straightforward to verify that
\[
\tilde{\beta}_1 = \frac{\tilde{w}_1'M\tilde{w}_2}{\tilde{w}_2'M\tilde{w}_2} = -\frac{\tilde{z}_1'M\tilde{z}_2}{\tilde{z}_2'M\tilde{z}_2} = -\tilde{\beta}. \quad (A47)
\]

This completes the proof.
Appendix B

This appendix discusses the computation of the cdf and pdf of $\hat{\beta}$. In view of Proposition 3, it suffices to derive the distribution and density functions of $\hat{\beta}$. Fix $\phi, \rho_{uv}$ and $T$. The result obtained by Gil-Pelaez (1951) is the main vehicle for computing $\tilde{F}(x)$, the cdf of $\hat{\beta}$. Let $\zeta(s,t)$ be the characteristic function of $(\tilde{z}'A\tilde{z}, \tilde{z}'B\tilde{z})$. Define $\xi \equiv \tilde{z}'(A - xB)\tilde{z}$, and let $\psi(s;x)$ be the characteristic function of $\xi$. Since $\tilde{z}'B\tilde{z} > 0$ a.s., we have

$$\tilde{F}(x) = P[\xi < 0] = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im} \psi(s;x)}{s} \, ds = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im} \zeta(s, -xs)}{s} \, ds, \quad (B1)$$

where the second equality is due to Gil-Pelaez (1951), and the third equality follows from $\psi(s;x) = \zeta(s, -xs)$.

The integrals can be evaluated numerically in many scientific computing softwares. Imhof (1961) proposes a generic method for computing the first integral in Eq. (B1) which involves solving for eigenvalues inside the integrand. This poses a runtime challenge for matrices of higher order if a brute-force spectral decomposition is solved for inside the integral. For the problem at hand, as shown below, an analytically tractable expression can be obtained of the integrand if we consider the second integral in Eq. (B1). This substantially reduces runtime. In what follows, we discuss the computation of the second integral.

First, we show that the limit exists at the left end of the integral. Applying L'Hôpital’s rule yields

$$\lim_{s \to 0} \frac{\text{Im} \zeta(s, -xs)}{s} = \text{Im} \psi'(0) = E[\xi] = \text{tr}((A - xB)V), \quad (B2)$$

It is straightforward to verify that

$$\text{tr}(AV) = -\frac{\rho_{uv}}{1 - \phi} + \frac{\rho_{uv}(1 - \phi^T)}{T(1 - \phi)^2}, \quad (B3)$$
\[ \text{tr}(BV) = \frac{T-1}{1-\phi^2} - \frac{2\phi}{(1-\phi)(1-\phi^2)} \left[ 1 - \frac{1-\phi^T}{T(1-\phi)} \right]. \quad (B4) \]

Next, we algebraically derive \( \zeta(s, t) \). Recall \( \tilde{z} \sim (0_{2T}, V) \), and \( A \) and \( B \) are constant symmetric matrices. Then,

\[ \zeta(s, t) = |V|^{-\frac{1}{2}}|V^{-1} - 2isA - 2itB|^{-\frac{1}{2}}. \quad (B5) \]

Applying matrix algebra, we obtain the inverse of \( V \) as

\[ V^{-1} \equiv \begin{bmatrix} V^{11} & V^{12} \\ V^{21} & V^{22} \end{bmatrix}, \quad (B6) \]

where

\[ V^{11} = \begin{bmatrix} \frac{1}{1-\rho_{uv}}I_{T-1} & 0_{T-1} \\ 0'_{T-1} & 1 \end{bmatrix}, \quad (B7) \]

\[ V^{12} = -\rho_{uv}V^{11}GH^{-1} \]

\[ = \frac{\rho_{uv}}{1-\rho_{uv}^2} \begin{bmatrix} \phi & -1 & 0 & \cdots & 0 \\ 0 & \phi & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \phi & -1 \\ 0 & \cdots & 0 & 0 & 0 \end{bmatrix}, \quad (B8) \]

\[ V^{22} = H^{-1} + \rho_{uv}^2H^{-1}G'V^{11}GH^{-1} \]

\[ = \frac{1}{1-\rho_{uv}^2} \begin{bmatrix} 1-\rho_{uv}^2 + \rho_{uv}^2\phi^2 & -\phi & 0 & \cdots & 0 \\ -\phi & 1+\phi^2 & -\phi & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & -\phi & 1+\phi^2 & -\phi \\ 0 & \cdots & 0 & -\phi & 1 \end{bmatrix}. \quad (B9) \]

Except for a few elements, \( V^{-1} \) is almost like a Toeplitz matrix. With this result, it is straightforward to show that

\[ |V| = \frac{|H|}{|V^{11}|} = \frac{(1-\rho_{uv}^2)^{T-1}}{1-\phi^2}. \quad (B10) \]
Using (B6) and the definition of $A$ and $B$, we have
\[ V^{-1} - 2isA - 2itB = \begin{bmatrix} V^{11} & V^{12} - isM \\ V^{21} - isM & V^{22} - 2itM \end{bmatrix}. \] (B11)

Plugging (B10) and (B11) into (B5), after some simplification, yields the following:
\[ \zeta(s, t) = (1 - \phi^2)^\frac{1}{2}|C(s, t)|^{-\frac{1}{2}}, \] (B12)

where
\[ C(s, t) = V^{22} - 2itM - (V^{21} - isM)(V^{11})^{-1}(V^{12} - isM) \]
\[ = H^{-1} + s^2M(V^{11})^{-1}M - 2itM - is\rho_{uv}(MGH^{-1} + H^{-1}G'M). \] (B13)

Using (B12) in the last expression of (B1), we obtain
\[ \tilde{F}(x) = \frac{1}{2} - \frac{(1 - \phi^2)^{\frac{1}{2}}}{\pi} \int_{0}^{\infty} \frac{\text{Im}|C(s, -xs)|^{-\frac{1}{2}}}{s} \, ds. \] (B14)

Now, the derivation continues towards getting a closed form expression of the integrand in the above line. Partition $C(s, -xs)$ as
\[ C(s, -xs) = \begin{bmatrix} c_{1,1} & a' & c_{1,T} \\ a & \Delta & b \\ c_{T,1} & b' & c_{T,T} \end{bmatrix}, \] (B15)
with the following submatrices
\[ c_{1,1} = 1 + \frac{[T(T - 1) - 1](1 - \rho_{uv}^2)}{T^2} s^2 + 2i(\rho_{uv}\phi s + xs) \left(1 - \frac{1}{T}\right), \] (B16)
\[ c_{1,T} = c_{T,1} \]
\[ = -\frac{T - \rho_{uv}^2}{T^2} s^2 + \frac{-2xs + (1 - \phi)\rho_{uv}s}{T}, \] (B17)
\[ c_{T,T} = 1 + \frac{(T - 1)(T - \rho_{uv}^2)}{T^2} s^2 + 2i \left[ \frac{\rho_{uv}s}{T} + \left(1 - \frac{1}{T}\right)xs \right], \] (B18)
\[ a = -\phi e_1 + \frac{\rho_{uv}^2 - T(1 - \rho_{uv}^2)}{T^2} s^2 - i \left[ \rho_{uv}s e_1 - \frac{(1 - 2\phi)\rho_{uv}s - 2xs}{T} \right], \] (B19)
\[ \mathbf{b} = -\phi e_{T-2} - \frac{T - \rho_{uv}^2 s^2}{T^2} - i \left( \rho_{uv}s e_{T-2} - \frac{(2 - \phi)\rho_{uv}s - 2xs}{T} \right), \] (B20)

\[ \Delta = \mathbf{D} - g1_{T-2}1_{T-2}', \] (B21)

where \( e_1 = [1, \ 0_{T-3}]', \ e_{T-2} = [0_{T-3}, \ 1]', \ g = \left[ \frac{T(1-\rho_{uv}^2)-\rho_{uv}^2 s^2 - 2i((1 - \phi)\rho_{uv}s - xs)}{T} \right], \) and \( \mathbf{D} \) is a \((T-2) \times (T-2)\) symmetric tri-diagonal Toeplitz matrix with elements

\[ D_{i,i} = 1 + \phi^2 + (1 - \rho_{uv}^2)s^2 + 2is(\phi\rho_{uv} + x), \] (B22)

\[ D_{i,i+1} = D_{i,i-1} = -\phi - i\rho_{uv}s. \] (B23)

Using the structure of \( \mathbf{C}(s, -xs) \), we obtain

\[ |\mathbf{C}(s, -xs)| = |\Delta|[(c_{1,1} - a'\Delta^{-1}a)(c_{T,T} - b'\Delta^{-1}b) - (c_{1,T} - a'\Delta^{-1}b)^2]. \] (B24)

The terms in (B24) can be evaluated algebraically by way of the following results.

\[ |\Delta| = |\mathbf{D}|(1 - g1_{T-2}'\mathbf{D}^{-1}1_{T-2}) \] (B25)

\[ \Delta^{-1} = D^{-1} + g(1 - g1_{T-2}'\mathbf{D}^{-1}1_{T-2})^{-1}\mathbf{D}^{-1}1_{T-2}1_{T-2}'\mathbf{D}^{-1} \] (B26)

Substituting the above into (B24), after simplification, we observe that the following terms need to be evaluated in order to compute \( |\mathbf{C}(s, -xs)| \): \( |\Delta|, \ 1_{T-2}'\mathbf{D}^{-1}1_{T-2}, \ e_1'\mathbf{D}^{-1}1_{T-2}, \ e_1'\mathbf{D}^{-1}e_1, \ e_{T-2}'\mathbf{D}^{-1}1_{T-2}, \ e_{T-2}'\mathbf{D}^{-1}e_{T-2}, \ e_1'\mathbf{D}^{-1}e_{T-2}. \)

To proceed, we utilize the following standard results on a symmetric tri-diagonal Toeplitz matrix. Let \( \mathbf{D} = \mathbf{U}\Lambda\mathbf{U}' \), where \( \Lambda = \text{Diag}(\lambda_1, \ldots, \lambda_{T-2}) \) and \( \mathbf{U} = (u_{k,j}) \), we have for \( k = 1, \ldots, T-2, \ j = 1, \ldots, T-2, \)

\[ \lambda_j = 1 + \phi^2 + (1 - \rho_{uv}^2)s^2 + 2is(\phi\rho_{uv} + x) - 2(\phi + i\rho_{uv}s) \cos \left( \frac{j\pi}{T-1} \right), \] (B27)

\[ u_{k,j} = \left( \frac{2}{T-1} \right)^{\frac{1}{2}} \sin \left( \frac{j\pi k}{T-1} \right), \] (B28)

It follows that

\[ |\mathbf{D}| = \prod_{j=1}^{T-2} \lambda_j, \] (B29)

\[ \mathbf{D}^{-1} = \mathbf{U}\Lambda^{-1}\mathbf{U}'. \] (B30)
In addition, we have
\[ T^{-2} \sum_{j=1}^{T-2} u_{k,j} = \begin{cases} \left( \frac{2}{T-1} \right)^{\frac{1}{2}} \cot \left( \frac{k\pi}{2T-2} \right) & \text{for odd } k, \\ 0 & \text{for even } k. \end{cases} \] (B31)

Therefore, we have
\[ d_1 \equiv 1_T' D^{-1} 1_{T-2} = \frac{2}{T-1} \sum_{k=1}^{\lfloor (T-1)/2 \rfloor} \frac{\cos^2 \left( \frac{(2k-1)\pi}{2T-2} \right)}{\lambda_{2k-1}}, \] (B32)
\[ d_2 \equiv e_1' D^{-1} 1_{T-2} = \frac{2}{T-1} \sum_{k=1}^{\lfloor (T-1)/2 \rfloor} \frac{\cot \left( \frac{(2k-1)\pi}{2T-2} \right) \sin \left( \frac{(2k-1)\pi}{T-1} \right)}{\lambda_{2k-1}}, \] (B33)
\[ d_3 \equiv e_1' D^{-1} e_1 = \frac{2}{T-1} \sum_{j=1}^{T-2} \frac{\sin^2 \left( \frac{j\pi}{T-1} \right)}{\lambda_j}, \] (B34)
\[ d_4 \equiv e_1' D^{-1} e_{T-2} = \frac{2}{T-1} \sum_{j=1}^{T-2} (-1)^{j-1} \frac{\sin \left( \frac{j\pi}{T-1} \right)}{\lambda_j}. \] (B35)

By symmetry, we have \( e_1' D^{-1} 1_{T-2} = d_2 \) and \( e_{T-2}' D^{-1} e_{T-2} = d_3 \). With these results on \( D \), it is straightforward to derive analytic expressions of the terms in (B24). The final expression of (B24) is omitted for brevity.

It is useful to write (B24) using real arithmetic. Write the polar representation of \( \lambda_j \):
\[ \lambda_j = \tau_j (\cos \theta_j + i \sin \theta_j). \]
\[ \tau_j = \left( a_j^2 + b_j^2 \right)^{\frac{1}{2}}, \] (B36)
\[ \theta_j = \arctan \left( -\frac{b_j}{a_j} \right). \] (B37)

It follows that
\[ |D|^{-\frac{1}{2}} = \prod_{j=1}^{T-2} \lambda_j^{-\frac{1}{2}} = h_1 (\cos \eta_1 + i \sin \eta_1), \] (B38)
where \( h_1 = \prod_{j=1}^{T-2} \tau_j^{-\frac{1}{2}} \) and \( \eta_1 = -\frac{1}{2} \sum_{j=1}^{T-2} \theta_j \). In order to avoid underflow problem, we should compute \( h_1 \) using \( \exp \left( -\frac{1}{2} \sum_{j=1}^{T-2} \tau_j \right) \). In addition, write the polar representation for
the remaining factors in $|\mathbf{C}(s, -xs)|$:

$$1 - g_1 T^{-1} \mathbf{1}_{T-2} = h_2 (\cos \eta_2 + i \sin \eta_2), \quad (B39)$$

$$\left( c_{1,1} - a' \Delta^{-1} a \right) (c_{T,T} - b' \Delta^{-1} b) - (c_{1,T} - a' \Delta^{-1} b)^2 = h_3 (\cos \eta_3 + i \sin \eta_3). \quad (B40)$$

We can then write $\text{Im} |\mathbf{C}(s, -xs)|^{-\frac{1}{2}} = h_1 h_2^{-\frac{1}{2}} h_3^{-\frac{1}{2}} \sin[\eta_1 - (\eta_2 + \eta_3)/2]$. This concludes the discussion of the cdf.

Only a bit more work is needed for the pdf, which is obtained by differentiating the cdf of $\hat{\beta}$:

$$\tilde{f}(x) = \frac{1}{\pi} \int_0^\infty \text{Im} \frac{\partial \mathbf{C}}{\partial t}(s, -xs) \, ds$$

$$= \frac{1}{\pi} \left(1 - \phi^2 \right)^{\frac{1}{2}} \int_0^\infty \text{Re} \left( |\mathbf{C}(s, -xs)|^{-\frac{1}{2}} \text{tr}(\mathbf{C}(s, -xs)^{-1} \mathbf{M}) \right) \, ds. \quad (B41)$$

Let

$$\mathbf{C}(s, -xs)^{-1} \equiv \begin{bmatrix} c_{11} & C_{12} & c_{13} \\ C_{21} & C_{22} & C_{23} \\ c_{31} & C_{32} & c_{33} \end{bmatrix}, \quad (B42)$$

we have

$$\text{tr}(\mathbf{C}(s, -xs)^{-1} \mathbf{M})$$

$$= \text{tr}(\mathbf{C}(s, -xs)^{-1}) - \frac{\mathbf{1}_{T}' \mathbf{C}(s, -xs)^{-1} \mathbf{1}_T}{T}$$

$$= c_{11} + c_{33} + \text{tr}(\mathbf{C}_{22}) - \frac{c_{11} + 2c_{31} + c_{33} + 21_{T-2}' \mathbf{C}_{21} + 21_{T-2}' \mathbf{C}_{23} + 1_{T-2}' \mathbf{C}_{22} \mathbf{1}_{T-2}}{T}. \quad (B43)$$

The terms in (B43) are computed as follows. First, let us define

$$\tilde{d}_1 = c_{1,1} - a' \Delta^{-1} a, \quad (B44)$$

$$\tilde{d}_2 = c_{1,T} - a' \Delta^{-1} b, \quad (B45)$$

$$\tilde{d}_3 = c_{T,T} - b' \Delta^{-1} b, \quad (B46)$$

$$\tilde{d}_4 = \tilde{d}_1 \tilde{d}_3 - \tilde{d}_2^2. \quad (B47)$$
With these quantities, it is straightforward to verify, after some algebraic simplification, that

\[
C_{11} = \frac{\tilde{d}_3}{d_4}, \quad (B48)
\]

\[
C_{21} = \frac{1}{d_4} \Delta^{-1} (\tilde{d}_2 b - \tilde{d}_3 a), \quad (B49)
\]

\[
C_{22} = \Delta^{-1} + \frac{1}{d_1} \Delta^{-1} a a' \Delta^{-1} + \frac{\tilde{d}_1}{d_4} \Delta^{-1} \left( b - \frac{\tilde{d}_2}{d_1} a \right) \left( b - \frac{\tilde{d}_2}{d_1} a \right)', \quad (B50)
\]

\[
C_{23} = \frac{1}{d_4} \Delta^{-1} (\tilde{d}_2 a - \tilde{d}_1 b), \quad (B51)
\]

\[
c_{31} = -\frac{\tilde{d}_2}{d_4}, \quad (B52)
\]

\[
c_{33} = \frac{\tilde{d}_1}{d_4}. \quad (B53)
\]

It follows from (B50) that

\[
\text{tr}(C_{22}) = \text{tr}(\Delta^{-1}) + \frac{a' \Delta^{-2} a}{d_1} + \frac{\tilde{d}_1}{d_4} \left( b - \frac{\tilde{d}_2}{d_1} a \right)' \Delta^{-2} \left( b - \frac{\tilde{d}_2}{d_1} a \right), \quad (B54)
\]

\[
1_{T^{-2}}' C_{22} 1_{T^{-2}} = 1_{T^{-2}}' \Delta^{-1} 1_{T^{-2}} + \frac{(1_{T^{-2}}' \Delta^{-1} a)^2}{d_1} + \frac{\tilde{d}_1}{d_4} \left[ 1_{T^{-2}}' \Delta^{-1} \left( b - \frac{\tilde{d}_2}{d_1} a \right) \right]^2. \quad (B55)
\]

As with the cdf, the integrand in (B41) of the pdf can be expressed in real mathematics. Write the polar representation of (B43) as tr\((C^{-1} M) = h_4 (\cos \eta_4 + i \sin \eta_4)\). Then,

\[
\text{Re} \left( |C(s, -xs)|^{-\frac{1}{2}} \text{tr}(C(s, -xs)^{-1} M) \right) = h_1 h_2^{-\frac{1}{2}} h_3^{-\frac{1}{2}} h_4 \cos \left( \eta_1 - \frac{1}{2} \eta_2 - \frac{1}{2} \eta_3 + \eta_4 \right). \quad (B56)
\]

This concludes the discussion of the pdf.
Appendix C

This appendix reports additional results that are not part of the main text of Chapter 2.

Table C1 Option illiquidity and the magnitude of future stock returns

This table reports results for the absolute stock return predictability using daily Fama-MacBeth cross-sectional regressions. The dependent variable is the 1-day ahead absolute stock return on the underlying stock $i$, $|\text{Ret}_{i,t+1}|$. The main variable of interest is $\Delta\text{ILO}_{\text{ranked},i,t}$, which proxies for the change in option illiquidity (ILO) from time $t-1$ to time $t$. On each day, we cross-sectionally rank changes in ILO into 10 deciles from the lowest change, i.e., $\Delta\text{ILO}_{\text{ranked},i,t} = 1$, to the largest change, $\Delta\text{ILO}_{\text{ranked},i,t} = 10$. We include host of control variables in the regression specification. For stock $i$ on day $t$, $\Delta\text{ILS}_{\text{ranked}}$ is the cross-sectional rank of daily change in stock illiquidity, ILS; we use decile sort for consistent with $\Delta\text{ILO}_{\text{ranked},i,t}$. RET is the return on the underlying stock. RRV is a daily range-based proxy for the realized volatility of the underlying stock, defined as the difference of the underlying stock’s intraday high and low price divided by the closing stock price. IV is the implied volatility for the underlying stock, calculated as the average implied volatilities of the call-put pair with 30 calendar days to maturity reported in the standardized option file from OptionMetrics. $\ln(\text{OptVolume})$ is the natural logarithm of the number of option contracts traded. $\ln(\text{size})$ is the natural logarithm of the market value of the underlying stock. The row labeled Day count reports the number of daily cross-section regressions used for reporting the Fama-MacBeth regression estimates. The row labeled Avg. cross section reports the average sample size of daily cross-sectional regressions. Newey-West $t$-statistics adjusted for autocorrelation up to 45 lags are reported in square brackets below each estimate. ***, **, and * indicate significance at the 1, 5, and 10 percent levels, respectively.
<table>
<thead>
<tr>
<th></th>
<th>Quoted bid-ask spreads</th>
<th>Effective bid-ask spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(I)</td>
<td>(II)</td>
</tr>
<tr>
<td>( \Delta ) ILO ranked(_{i,t} )</td>
<td>0.0012**</td>
<td>0.0010**</td>
</tr>
<tr>
<td></td>
<td>[2.42]</td>
<td>[2.40]</td>
</tr>
<tr>
<td>( \Delta ) ILS(_{ranked,i,t} )</td>
<td>-0.0003</td>
<td>-0.0003</td>
</tr>
<tr>
<td></td>
<td>[-0.40]</td>
<td>[-0.42]</td>
</tr>
<tr>
<td>RET(_{i,t} )</td>
<td>0.3860*</td>
<td>0.3890**</td>
</tr>
<tr>
<td></td>
<td>[1.95]</td>
<td>[1.97]</td>
</tr>
<tr>
<td>RRV(_{i,t} )</td>
<td>0.0009***</td>
<td>0.0009***</td>
</tr>
<tr>
<td></td>
<td>[23.34]</td>
<td>[23.36]</td>
</tr>
<tr>
<td>IV(_{i,t} )</td>
<td>0.0058***</td>
<td>0.08846***</td>
</tr>
<tr>
<td></td>
<td>[23.36]</td>
<td>[23.47]</td>
</tr>
<tr>
<td>ln(OptVolume)(_{i,t} )</td>
<td>0.0122***</td>
<td>0.0124***</td>
</tr>
<tr>
<td></td>
<td>[39.93]</td>
<td>[39.70]</td>
</tr>
<tr>
<td>ln(size)</td>
<td>-0.0059</td>
<td>-0.0060</td>
</tr>
<tr>
<td></td>
<td>[3.83]</td>
<td>[3.73]</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.0585</td>
<td>0.0623</td>
</tr>
<tr>
<td></td>
<td>[-1.06]</td>
<td>[-1.04]</td>
</tr>
<tr>
<td>Day Count</td>
<td>2,469</td>
<td>2,469</td>
</tr>
<tr>
<td>Avg. cross section</td>
<td>470</td>
<td>471</td>
</tr>
<tr>
<td>Adj. R(^2)</td>
<td>16.79%</td>
<td>16.78%</td>
</tr>
</tbody>
</table>
Appendix D

This appendix explains how PIN is estimated with intraday stock data in Chapter 2. Details about the underlying market microstructure model can be found in Easley, Kiefer, O’Hara and Paperman (1996) and Easley, Hvidkjaer, and O’Hara (2002). Assuming that the market buy and sell orders arrive according to a Poisson process, the model implies that on any given day $t$, the likelihood of observing the number of buy trades $B_t$ and the number of sell trades $S_t$ is given by

$$L(\theta|B_t, S_t) = \alpha (1 - \delta) e^{-\left(\mu + \epsilon_b\right)B_t} \frac{\left(\mu + \epsilon_b\right)^B_t}{B_t!} e^{-\epsilon_s S_t} \frac{\epsilon_s^S_t}{S_t!}$$

$$+ \alpha \delta e^{-\epsilon_b B_t} \frac{\epsilon_b^{B_t}}{B_t!} e^{-\left(\mu + \epsilon_s\right)S_t} \frac{\left(\mu + \epsilon_s\right)^S_t}{S_t!} + (1 - \alpha) e^{-\epsilon_b B_t} \frac{\epsilon_b^{B_t}}{B_t!} e^{-\epsilon_s S_t} \frac{\epsilon_s^S_t}{S_t!},$$

where the parameter vector $\theta = (\alpha, \delta, \mu, \epsilon_b, \epsilon_s)$ represents the structural parameters of the model, and $\alpha$ denotes the probability of a private information event on the day. Given an information event day, the probability of a bad news is $\delta$, and the probability of good news is $1 - \delta$. Informed traders submit orders only on information event days with the arrival rate $\mu$ and act to their informational advantage. Uninformed investors trade every day and submit buy orders with the arrival rate $\epsilon_b$ and sell orders with the arrival rate $\epsilon_s$.

Under the assumption of independence between days, the joint likelihood of observing a daily time series of buy and sell order counts $M = \{(B_t, S_t), t = 1, ..., T\}$ is then the product of the individual likelihoods:

$$L(\theta|M) = \prod_{t=1}^{T} L(\theta|B_t, S_t).$$

The PIN measure is then defined by
\[ PIN = \frac{\alpha \mu}{\alpha \mu + \epsilon_b + \epsilon_s}, \]  

which equals the probability that the opening trade comes from an informed trader.

To estimate the structural parameters, we maximize the joint likelihood in D2 over the parameter space numerically. The daily time series of buy and sell order counts are gathered and PIN estimates are produced for the subsample of firms which are listed on NYSE and AMEX because the specialist market structure is close to the PIN model feature. (see Easley, Kiefer, O’Hara and Paperman 1996). Vega (2006), Lin and Ke (2011) and Yan and Zhang (2012) express concerns about estimating the parameters by using optimization software packages. The optimization solution is sensitive to the initial values fed into the search algorithm and may yield boundary solutions frequently. Depending on the magnitude of the buy and sell counts, the likelihood function D2 may not be computed because of numerical overflow. Generally speaking, the larger the number of trades, the more difficult it is to obtain MLE estimates for PIN.

We employ the estimation method developed by Yan and Zhang (2012) who run the optimization procedure using a grid search algorithm.\(^1\) These authors show that their method increases the probability of delivering valid PIN estimates, and generally makes the estimates more reliable. We run the estimation procedure for an underlying stock every calendar quarter, requiring that there are at least 50 days of trading on that stock during the quarter. Hence, for each firm, PIN is updated every quarter.

\(^1\)We greatly acknowledge the help from Yuxing Yan and Shaojun Zhang who provided us with their computer code for estimating PIN.
Bibliography


Chemmanur, T., C. Ornthanalai, and P. Kadiyala, 2015, Options on initial public offerings, Working Paper, University of Toronto.


Meng, Xiao-Li, 2005, From unit root to Stein’s estimator to Fisher’s $k$ statistics: If you have a moment, I can tell you more, *Statistical Science* 20, 141–162.


Pearson, N., A. Poteshman, and J. White, 2006, Does option trading have a pervasive impact on underlying stock prices?, Working Paper, University of Illinois at Urbana–Champaign.


Stambaugh, Robert F., 1986, Bias in regressions with lagged stochastic regressors, Unpublished Manuscript, University of Chicago, Chicago, IL.


Zhou, Hao, 2009, Variance risk premia, asset predictability puzzles, and macroeconomic uncertainty.