STAR CLUSTER FORMATION AND RADIATIVE FEEDBACK

by

Peter H. Jumper

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Department of Astronomy & Astrophysics
University of Toronto

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Abstract

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In this dissertation, I begin by exploring star-cluster forming regions undergoing both mass accretion and stellar feedback. I consider the interplay between accretion, star formation, stellar feedback, turbulent energy-driving, and turbulent decay. The resulting analytical model predicts that the feedback mechanisms are not powerful enough to reverse the flow of accreting matter onto low mass clusters and terminate star cluster formation. Moreover, low-mass clusters will have outflow feedback that significantly drives turbulence. However, as this model makes several assumptions and analytical approximations about the nature of feedback, including radiation feedback in clumpy dust envelopes, I turn my attention to exploring this problem for the remainder of the dissertation. I start from the simpler case of a spherical envelope and address numerical solutions for the radiation forces, relating these to analytical estimates. Radiation reprocessing is important to these models, as infrared radiation escapes from dust more easily than starlight. I conduct these models with the Monte Carlo code Hyperion and the scaling solution code DUSTY. I show that Monte Carlo methods underestimate temperatures and radiation forces when the mean free path of starlight is poorly resolved. I then introduce clumping into these envelopes to study its effects on the captured forces. I find power-law fits to the scaling of three summary parameters (a normalized radial force ($\Phi$), a force-averaged radius, $\langle r \rangle_F$, and a radiative virial term, $\mathcal{R}$) with the intensity of clumping in the envelopes. $\mathcal{R}$ is
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by further exploring the dependence of Monte Carlo solutions with resolution in the
context of AMR density data, varying an imposed level of grid refinement. Coarser
grids suppress channels that helped photon leakage, but also underestimate the
temperature, yielding two opposing effects that may partially cancel against each
other.
“This thesis is dedicated to my parents, Stephen Jumper and Donna Jumper, and to my twin brother, Kevin Jumper, who have always offered me their love, encouragement, and support.”

-Peter Jumper
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Chapter 1

Introduction

Stars are foundational to the fields of astronomy and astrophysics. They were among the first celestial objects known to ancient humanity, and have been studied extensively since those times, be it for the purposes of calendars and celestial navigation as in past times or in various astrophysical applications now. Stars hold a particularly central role in astronomy and astrophysics in that they serve the function of the figurative forges and furnaces of the universe, providing the sites of the nuclear fusion reactions that create heavier elements, and giving off great amounts of energy, exerting powerful forces upon their surrounding environments in the process.

In this dissertation, I explore some of the influences that stars have on their environment. First, I consider the problem of star cluster formation, in which the processes of accretion, turbulence, star formation, and stellar feedback interplay to shape the properties of their cluster. Then, I focus on a particular mechanism of stellar feedback, radiation pressure forces exerted as a result of the starlight, and model these forces with Monte Carlo methods in dusty envelopes, including clumpy environments.
1.1 Star cluster formation

Star formation overwhelmingly occurs within the context of star clusters. While these clusters are themselves embedded within giant molecular clouds (GMCs), studies have suggested that approximately 70% to 90% of the stars within GMCs originate within such cluster environments Lada & Lada (2003). This clustered nature of star formation is quite significant, as the forming stars not only accrete their mass from the accumulated matter gathered into the cluster, thus drawing off matter from a shared reservoir (Zinnecker 1982), but they also in turn inject energy and momentum back into the shared environment through a variety of feedback mechanisms, some of which may also lead to the expulsion of mass from the cluster. Therefore, to more fully understand the problems of star formation, one must develop an understanding of processes occurring both at the cluster scale, which provide the conditions for and facilitate the processes of star formation, and the processes of feedback exerted by the stars within, which may then in turn alter these conditions.

1.1.1 Models of star cluster formation

Several mechanisms have been proposed for explaining the formation and growth of structures embedded within GMCs, including star clusters and the star-forming cores within, and for the accumulation of mass onto these structures. These proposals have included the collision of compressible flows to produce self-gravitating structures (Vazquez-Semadeni et al. 1996), the collapse of virialized, turbulent structures (Myers & Fuller 1992; McLaughlin & Pudritz 1997; McKee & Tan 2003), and Bondi accretion (Naiman et al. 2011; Murray & Chang 2012), the accretion of surrounding GMC material onto a cluster due to relative motion. However, it is another mechanism, the formation of a star cluster by the accretion of material onto a central region from flows along filamentary structures, that will be the focus of Chapter 2.
Several decades of observations have built up our understanding of the filamentary structures that are found embedded within giant molecular clouds. Among the earlier surveys, Schneider & Elmegreen (1979) and Bally et al. (1987) developed catalogs of filaments in regions including the Taurus dark clouds and the Orion molecular clouds, respectively. In more recent years, further observational studies of filamentary structure has suggested that such filamentary regions may be key for the processes of star cluster formation. Filaments have been found to be near-ubiquitous around star clusters, organized such that clusters are found at the intersections of such filaments; this has given rise to the filamentary hub model of star cluster formation (Myers 2009). The abundance of filamentary structures was also supported by the Herschel Gould Belt Survey, which also demonstrated the presence of embedded cores within the filaments (André et al. 2010). (Arzoumanian et al. 2011) further indicated that these filamentary structures tended to have a characteristic width on the order of 0.1 pc. The notion that star cluster formation may be driven by the accretion of materials from filaments into the hub region in which a cluster will form is further supported by studies of such as the $N_2H$ line velocity gradient (Kirk et al. 2013) and chemical measures of cyanopolyynes in the Serpens South cluster Friesen et al. (2013), both of which indicated the acceleration of material along the filaments towards the clusters, driving accretion.

1.1.2 Turbulence in star clusters

Numerous studies have suggested that turbulence is of significant importance to star formation in star clusters. In a seminal paper, Larson (1981) both identified a relationship between the strength of non-thermal, turbulent motions within molecular clouds and the size scale of said regions and that such turbulence could provide support against gravitational collapse in the cloud and substructures within it. Later, Bertoldi & McKee (1992) would characterize the level of turbulent support within a
cluster to its self-gravity with a dimensionless quantity, dubbed the virial parameter, $\alpha = \frac{5\sigma^2 R}{GM}$, where $\sigma$ is the dispersion velocity, $R$ is the radius, $M$ is the mass, and $G$ is the gravitational constant. Further investigations would soon give rise to an emerging new model of turbulence-regulated star formation (Mac Low & Klessen 2004). While the support of turbulence in GMCs and clusters could help prevent the collapse of regions on larger scales, this same turbulence could be invoked to give rise to a lognormal density distribution within these regions, therefore seeding smaller scales regions of overdensity which could self-gravitate strongly enough to collapse despite the turbulent support in the larger region, thus facilitating the creation of star-forming cores (Padoan & Nordlund 2002). Working to account for the star formation rates described by the Kennicutt-Schmidt law (Kennicutt 1998), Krumholz & McKee (2005) developed a analytical model of star formation in regions supported by turbulence, depending on the virial parameter, $\alpha$, and the Mach number, $M$, of the turbulence. Additional studies, including a mix of analytical models and numerical simulations, provide further descriptions of the star formation rate from gravoturbulent fragmentation in the context of turbulence-regulated star formation (Hennebelle & Chabrier 2011; Padoan & Nordlund 2011; Federrath & Klessen 2012).

One of the challenges posed for turbulence-regulated models is accounting for the maintenance of the turbulence necessary to sustain them. The supersonic motions associated with turbulence in star-forming environments decay rapidly, on a timescale on the order of the region’s crossing time or less (Mac Low et al. 1998; Stone et al. 1998). Therefore, in order to maintain the turbulence despite this decay, energy must be injected into the medium to drive continued turbulence. Feedback mechanisms may provide a means to do so, as will be discussed later in this chapter.

As a counterpoint to the model of turbulence-regulated star formation, Clark et al. (2005) proposes that if clusters form in an unbound, short-lived state, lasting on the order of 1-2 cluster crossing times (Elmegreen 2000), such as due to an assembly from
flows, then turbulent support to maintain an equilibrium state would not be necessary. Ochsendorf et al. (2017) also provides a critique of models that utilize a global turbulence, arguing that feedback effects from the stars forming within the clusters must be accounted for to properly capture the star formation rates with GMCs.

1.1.3 Feedback from protostellar outflows

The injection of energy and momentum from stellar feedback processes may play a key role in influencing star formation at both the core and cluster scales. Feedback processes occur by a variety of mechanisms, including but not limited to protostellar outflows and radiation pressure.

Protostellar outflows may play a key role in expelling mass from cores and clusters and driving turbulence. Observational studies have indicated that protostellar outflows are ubiquitous in star forming environments. Parker et al. (1991) found in a sample of low-mass young stellar objects (YSOs) that somewhat less than 70% of the YSOs in the survey showed evidence of having outflows and it was probable that almost all such stars would have such outflows at some point in their lifetimes, a conclusion backed up by subsequent studies. (Bontemps et al. 1996). (Myers et al. 1988) concluded that such outflows had sufficient energy, momentum, duration, frequency, and interactions with surrounding protostellar cores, to serve as a primary mechanism for core disruption. Such outflows may also reduce the protostellar masses and accretion rates attained during the star formation process, as found in numerical simulations (Hansen et al. 2012). Moreover, at larger scales, observations have found that protostellar outflows may sweep up mass from the surrounding cluster environment (Levreault 1984), disrupt cluster-forming regions (Goldsmith et al. 1986), and even break out from the molecular cloud into the intercloud medium (Bally et al. 1994).

Moreover, numerous observational studies and numerical simulations have been conducted to investigate the question of whether protostellar outflows are a viable
means of driving turbulence so as to replenish the turbulence in star-forming environments, offsetting its decay. While an earlier observational study concluded that it was uncertain whether outflows played a major role in driving turbulence (Heyer et al. 1987), and there have been dissenting conclusions asserting that the motions imparted from the outflows damped and failed to drive turbulence (Banerjee et al. 2007), the overall consensus opinion in the literature has tended to be that protostellar outflows do indeed play a key role in driving turbulence. A potential mechanism for this turbulent driving has been proposed by the expansion of cavities inflated by outflows and their subsequent disruption by dynamical instabilities which leads to a randomization in the direction of energy and momentum injection (Quillen et al. 2005; Cunningham et al. 2006b; 2009). Multiple observational studies of outflows have indicated that the energy injection rate from outflows is of a comparable order, such as found in the Serpens (Graves et al. 2010), Serpens South (Nakamura et al. 2011a), and L1688 (Nakamura et al. 2011b). Observations have also indicated that some turbulently-driven regions have sufficiently energetic outflows to unbind and disrupt their clumps (Mottram & Brunt 2012), some lack this amount of energy (Nakamura et al. 2011a), and some presently lack this energy but may attain it with further evolution (Plunkett et al. 2013). These conclusions supporting the importance of outflows to turbulent driving have been supported by a number of numerical simulations (Li & Nakamura 2006; Cunningham et al. 2006a; Nakamura & Li 2007; Carroll et al. 2009).

1.1.4 Radiative feedback in star-forming environments

Stars, like any other object with a temperature above absolute zero, give off radiation in the form of photons. As starlight photons, like all photons, carry energy and momentum, they may have an influence on the surrounding environment when they interact with the material within, including heating the matter and exerting forces upon it. In certain star-forming environments, this may have profound effects.
Radiation feedback from stars is generally most effective in environments with massive stars, as these stars also tend to have higher luminosities, emitting more photons and thus exerting a more powerful influence. This may pose particular challenges to massive star formation by hindering the accretion processes (Wolfire & Cassinelli 1987), although instabilities in the environment may open up optically thin regions through which this radiation may be vented, allowing accretion to continue (Krumholz et al. 2009). Indeed, Krumholz et al. (2009) argue that all main sequence stars with a mass in excess of about $20M_\odot$ will be super-Eddington, with an exerted radiation pressure force in excess of their gravitational force.

Radiation pressure may also be important for rapidly expelling mass from star clusters with high surface densities or total masses (Fall et al. 2010). Although radiation pressure may be unimportant to the dynamics of lower mass clusters, it may also play a prominent role in the dynamics and disruption of massive star clusters (Krumholz & Matzner 2009; Murray 2009; Murray et al. 2010).

1.2 Radiative feedback in dusty envelopes

1.2.1 Overview of radiative reprocessing

A complication that emerges when calculating the strength of the radiative feedback force that a star exerts upon its surrounding environment is that the immediate contribution from the photons emitted directly from the star, sometimes referred to as the direct radiation force, is only a portion of the total resulting radiation force. This is because the photons emitted directly from the star will eventually interact with material within the surrounding environment and in doing so heat that material, leading it to give off increased radiation of its own. This additional radiation excited from the absorption of energy previously radiated is sometimes referred to as reprocessed radiation and is normally emitted as infrared radiation. Thus, the
total radiation force exerted upon a region is a combination of the direct radiation force exerted by the starlight photons and an indirect radiation force exerted by the reprocessed radiation.

The existence of an enhancement of radiation due to the influence of the material in the medium has long been known (O’dell et al. 1967). This is particularly due to the influence of the dust grains present in the media, as they tend to have much higher absorption cross-sections for interaction than does gas. For example, while the Thompson electron-scattering opacity associated with interstellar gas may be on the order of 0.34 cm$^2$/g for solar compositions, dust opacities may be much higher, on the order of $10^4$ cm$^2$/g$_{\text{dust}}$ to $10^2$ cm$^2$/g$_{\text{dust}}$ over the range of 0.1 µm to 5 µm (Draine 2003a); while the gas-to-dust ratio may be on the order of $10^2$, this still allows the dust to be far more effective at interacting with radiation than the gas. This also makes the key point that dusts tend to have a higher opacity to higher-energy, shorter-wavelength light than longer-wavelength infrared light.

Several rough analytical estimates have been developed to approximate the total radiation force exerted, accounting for the influences of radiative reprocessing. In the limiting case where the entire luminosity, $L$, of a star is deposited into the medium, ignoring the effects of reprocessing, then given the speed of light, $c$, a total radiation force of $L/c$ will be exerted. In comparison, for examples of estimates for reprocessing, Krumholz & Matzner (2009) estimates the total reprocessed radiation force in a cluster, $F_{\text{IR}}$, to be on the order of $F_{\text{IR}} \lesssim L/c$, Murray et al. (2010) estimates $F_{\text{IR}} = \tau_{\text{IR}} (L/c)$, where $\tau_{\text{IR}}$ is the optical depth of the region to the (infrared) reprocessed radiation. In Matzner & Jumper (2015), which in an edited form is also presented as Chapter 2 of this dissertation, a single average dust temperature, $T_d$, was used for the estimations of the radiation pressure force.

In addition, there are several numerical methods for calculating the solutions of radiative transfer. Some examples of such methods and techniques include flux-
limited diffusion (FLD) (Levermore & Pomraning 1981), variable Eddington tensor (VET) (Davis et al. 2012), scaling solutions (Ivezic & Elitzur 1997), and Monte Carlo radiative transfer (Whitney 2011).

The consideration of feedback processes in Matzner & Jumper (2015) and the aforementioned rough estimates previously conducted for the force exerted by reprocessed radiation helped provide an initial inspiration to further consider the modeling of radiation pressure forces in dusty environments, as will be discussed in Chapters 3 - 5 of this dissertation.

1.2.2 Monte Carlo radiative transfer

Monte Carlo methods function by representing the outcome of the process as the aggregation of a large number of realizations of outcomes drawn randomly from a distribution. In application to radiative transfer, Monte Carlo methods emit a large number of representative “photon packets”, each representing some portion of the luminosity of a specified source. These photon packets are emitted in random directions, and allowed to propagate for an optical depth randomly drawn from a distribution before interacting with the matter (such as dust) at the location in a scattering event, which changes the direction of the photon packet’s movement, or an absorption event, which removes the photon packet and then causes new photon packets to be emitted from the reprocessing matter at that location based on the temperature or specific energy absorption rate; this dissertation utilizes the latter. Some Monte Carlo methods, including those utilized in this dissertation, also attempted to minimize numerical noise in a weighted Monte Carlo scheme which includes a representation of interactions along the entire path of propagation, utilizing the point that each photon packet actually represents a large ensemble of photons. The Monte Carlo radiative transfer code Hyperion (Robitaille 2011), including modifications detailed in Chapter 3 is utilized for solving the Monte Carlo radiative
transfer problems presented in this dissertation.

1.2.3 Analytical, scaling solution, and Monte Carlo models in spherical envelopes

Chapter 3 presents a version of (Jumper & Matzner 2018a), a paper presently accepted for publication in MNRAS, edited and reformatted for inclusion this dissertation, detailing Monte Carlo radiative transfer models in spherical dust envelopes compared against both scaling solution models and analytical estimates for key parameterizations of the radiation pressure force exerted upon the dust across three regimes in the optical depth: (1.) optically thin to both direct and reprocessed radiation, (2.) optically thick to direct radiation but optically thin to reprocessed radiation, and (3.) optically thick to both direct and reprocessed radiation. Furthermore, the comparisons conducted in this chapter allow an exploration of the resolution-dependent errors in Monte Carlo radiative transfer models. An understanding of such errors may be especially relevant to dynamical models, where they may influence the development of instabilities (Kuiper et al. 2011; Rosen et al. 2016), and where they may also lead to errors in the behaviors of accretion flows (Krumholz 2018).

1.2.4 Clumping in dusty envelopes

Chapter 4 presents an edited and reformatted version of (Jumper & Matzner 2018b) a paper submitted to MNRAS, building upon the work presented in Chapter 3, introduces clumping into dust envelopes, exploring the effects on the total force captured, the characteristic radius of force capturing, and a radiative virial parameter in the resulting envelopes. The chapter provides rules of thumb for characterizing the effects of clumping on these parameters, and finds that the radiative virial parameter is least stochastic in the presence of the inhomogeneities introduced by the clumping.
Chapter 1. Introduction

The key influence of introducing clumping to these envelopes is to open a series of lower-density, optically thin (or thinner) channels between the new areas of enhanced density, facilitating the escape of photons through these channels, a mechanism which may have a major effect on processes of massive star formation (Yorke & Sonnhalter 2002; Krumholz et al. 2005; 2009).

1.2.5 Monte Carlo radiative transfer in AMR grids

Chapter 5 present ongoing work investigating Monte Carlo radiative transfer in the context of adaptive mesh refinement (AMR) models. This chapter provides additional exploration of concepts introduced in Chapters 3 and 4. In the vein of the former, it considers resolution effects on errors in Monte Carlo radiative transfer methods, but does so by adding additional levels of grid refinement around desired regions of interest, rather than across the entire grid. In the vein of the latter, modeling a region with coarser grids suppresses the representation of substructures on scales smaller than cell dimensions. Therefore, as additional refinement levels are added to a region, clumping that was previously hidden on coarser grids becomes revealed on the new, finer grids, influencing the calculated propagation of photons. This chapter makes use of an ORION dataset provided by Anna Rosen in a private communication for a sample AMR density grid and star, which are used as initial condition inputs for the radiative transfer modeling.
Chapter 2

Star Cluster Formation with Stellar Feedback and Large-Scale Inflow

2.1 Preface

This chapter details a paper published in the Astrophysical Journal (ApJ) (Matzner & Jumper 2015)\(^1\). It has been edited from its original form, including to conform to the formatting of this dissertation. Unlike all other papers in this dissertation, for which I was the lead author, I was the second author of this paper. The first author of this paper is Christopher Matzner.

This paper originated as an extension upon an analytical estimation project that I had worked on previously, in which I considered a number of turbulent filaments with some line density (mass per unit length), \(\lambda\), accreting upon a central region that would provide the mass reservoir for a star cluster. This drove star formation to occur at a rate related to the mass of this reservoir and in turn the accretion rate onto the cluster. The internal state of the cluster in this model was determined from the assumptions of approximate virial balance, \(\alpha = \frac{5\sigma(r)^2}{GM(r)} \approx 1\), and another dimensionless parameter,

\(^{1}\)http://iopscience.iop.org/article/10.1088/0004-637X/815/1/68/meta
\( \zeta_c = \frac{GM_{\text{in}}}{\sigma^3} \approx 1 \), relating the rate of mass inflow to the clump velocity dispersion. I approximated the turbulent driving from the accretion rate of the filaments as in §2.5.2 and from protostellar outflows given a characteristic wind momentum per unit stellar mass, \( v_{\text{ch},w} \), based on the star formation from this model, as in §2.6 and §2.6.2. These outflows were also allowed to expel mass from the cluster as in §2.6.3 in the manner described by Matzner & McKee (2000). I also estimated the decay rate of the turbulence on the order of \( \sigma_c^5 / G \) as in §2.5.2.

These estimations provided the core inspirations around which the paper eventually coalesced. During this process, Matzner enhanced many processes with further detail, and added several additional physical processes, including radiative feedback. Matzner also produced all of the figures used in this chapter. Matzner and I both took part in the writing process, with his contributions providing a larger portion. We also both took part in the revision processes.

### 2.2 Chapter abstract

During star cluster formation, ongoing mass accretion is resisted by stellar feedback in the form of protostellar outflows from the low-mass stars and photo-ionization and radiation pressure feedback from the massive stars. We model the evolution of cluster-forming regions during a phase in which both accretion and feedback are present, and use these models to investigate how star cluster formation might terminate. Protostellar outflows are the strongest form of feedback in low-mass regions, but these cannot stop cluster formation if matter continues to flow in. In more massive clusters, radiation pressure and photo-ionization rapidly clear the cluster-forming gas when its column density is too small. We assess the rates of dynamical mass ejection and of evaporation, while accounting for the important effect of dust opacity on photo-ionization. Our models are consistent with the census of protostellar outflows.
in NGC 1333 and Serpens South, and with the dust temperatures observed in regions of massive star formation. Comparing observations of massive cluster-forming regions against our model parameter space, and against our expectations for accretion-driven evolution, we infer that massive-star feedback is a likely cause of gas disruption in regions with velocity dispersions less than a few kilometers per second, but that more massive and more turbulent regions are too strongly bound for stellar feedback to be disruptive.

2.3 Introduction

Star formation is a highly clustered and correlated phenomenon as a consequence of the clumpy nature of massive, turbulent molecular clouds, and because only the densest and most shielded regions within these clouds readily collapse to form stars. For the birth of a star cluster within one of the molecular clumps, there are two major implications: first, that the large-scale flows which assembled the clump can continue to rain down upon it as the star cluster is born; and second, that individual protostars are close enough in space and time to affect one another via jets, winds, radiation, and potentially supernovae. These two effects – input and output, or accretion and feedback – are both capable of driving turbulent motions within the star-forming medium, affecting its dynamics and therefore the rate and nature of star formation, and adding or subtracting mass to the forming cluster. The purpose of this paper is to understand the competing influences of accretion and stellar feedback during a cluster’s formation.

We are motivated by several persistent questions. What is the dominant mode by which matter is assembled: monolithic collapse, colliding flows, or continued accretion? What is the role of inflowing matter in the dynamical evolution of a growing cluster? How important are the effects of stellar feedback – in general, or for
specific observed regions? And, what ultimately ends star cluster formation: a limited supply of bound matter, perhaps, or disruption by stars?

We begin by considering observational results on the structure and dynamics of molecular clouds, which we use to infer how matter accumulates and accretes. The gross properties of a forming cluster are often controlled by accretion, at least until stellar feedback becomes strong. Turning to stellar feedback, we evaluate the effects of protostellar outflows, first with a simple comparison of forces and then by applying analytical results from Matzner & McKee (2000) and Matzner (2007) to an accreting molecular clump. We estimate the direct and indirect forces of radiation pressure and of photo-ionized gas pressure, and comment on the strength of stellar wind pressure, in order to assess the feedback from massive stars. After mapping feedback regimes, we compare against individual low-mass regions NGC 1333 and Serpens South, as well as surveys of massive cluster-forming regions. Finally we draw conclusions about when and whether stellar feedback terminates star cluster formation.

2.4 Mode of mass accumulation

The manner by which matter is gathered and enters the region of cluster formation is an important factor shaping the cluster’s final properties. Since star cluster formation is intermediate in mass and duration between the formation of giant molecular clouds and the formation of individual stars, the scenarios considered on larger and smaller scales have also been adopted for star cluster formation. These include: collapse as a result of colliding flows (Vazquez-Semadeni et al. 1996), Bondi-Hoyle (Naiman et al. 2011) or Bondi (Murray & Chang 2012) accretion, inside-out collapse of a turbulent virialized structure (Myers & Fuller 1992; McLaughlin & Pudritz 1997; McKee & Tan 2003), and the gravitational infall of an initial, possibly filamentary density distribution (e.g., Pon et al. 2011). While there is some overlap, these differ as to whether mass inflow overlaps the period of star formation, as to whether the in-flowing gas is bound
to the cluster-forming region before being incorporated, and also with respect to the
time history and detailed properties (such as density distribution, angular momentum,
and magnetization) of the inflow.

For a couple reasons we believe that observations of the molecular cloud environ-
ments for stellar cluster formation favor the latter options, i.e. inside-out collapse or
infall from an initially filamentary structure, at least in the current Milky Way.

First, molecular clouds are highly filamentary. Filaments are a prominent feature
of observations of molecular clouds (e.g. Schneider & Elmegreen 1979; Bally et al.
1987; Loren 1989; Johnstone & Bally 1999; André et al. 2010; Schneider et al. 2012;
Peretto et al. 2014). Moreover this structure is directly relevant to the formation of
star clusters. As Myers (2009) has stressed, filaments are nearly ubiquitous in the
regions surrounding active star cluster formation; indeed star cluster formation is
often observed at the junctions of molecular filaments (for instance, in the Rosette
molecular cloud; Schneider et al. 2012). In some cases, such as Serpens South, accretion
along filaments has been inferred from molecular line kinematics (e.g., Kirk et al. 2013)
or via chemical signatures (Friesen et al. 2013). Tackenberg et al. (2014), who map
$N_2H^+$ toward 17 Herschel filaments, find several examples of filamentary accretion
flows onto clumps. Filamentary accretion is also a prominent feature in numerical
simulations of massive star formation (e.g., Banerjee et al. 2006), which is analogous
to star cluster formation.

Second, there exists a clear trend for the most massive clumps within molecular
clouds to exhibit the lowest levels of turbulent support against gravity. Bertoldi &
McKee (1992) found that only the most massive regions within several molecular
clouds (which are also the densest and exhibit the highest column density) had
sufficiently small line widths to be considered strongly self-gravitating. Bertoldi &
McKee define the virial parameter $\alpha = 5\sigma(r)^2 r / G M(r)$ to compare turbulence against
gravity. (Here $\sigma(R)$ is the one-dimensional velocity dispersion of a region of radius $r$
and mass $M(r)$). Reviewing a number of more recent works and using more robust dust-derived masses, Kauffmann et al. (2013) verify this trend and show that it extends to remarkably low values of $\alpha$, at least for condensations of quiescent cold gas and dust without prominent signs of star formation. As Kauffmann et al. argue, this implies that the initial conditions for massive star formation are either in a state of imminent collapse or suspended by strong magnetic fields. These conclusions apply equally well to the initial conditions for the creation of a star cluster.

Motivated by these points we shall concentrate on the continuous infall of gravitationally-bound matter, which is organized in a filamentary fashion toward the site of star cluster formation. Several implications are apparent:

– **Infall duration**: The infall of a mass reservoir $M_{\text{in}}(r)$, found within radius $r$ of the cluster formation site at $t = 0$, will last for roughly the initial free-fall time $t_{\text{ff}}(M_{\text{in}}) \approx \left[\frac{\pi^2 r^3}{8GM_{\text{in}}}\right]^{1/2}$ (in the monopole approximation). This duration is intermediate between the inside-out collapse of an initially hydrostatic state, for which infall takes a couple $t_{\text{ff}}(M_{\text{in}})$ because of the forces which balanced gravity in the initial state, and the rapid formation of a cluster by colliding flows of unbound gas.

– **Dynamical age of cluster formation**: Because matter accumulates in a dense central cluster-forming region as matter falls in, the free-fall time on the cluster scale should be significantly shorter than that of the reservoir. Therefore, a proto-cluster’s formation extends for several dynamical times of the its parent clump (e.g., 6.2 clump free-fall times in the fiducial model of § 2.5 [eq. 2.10]), and one should consider the physical state of the cluster-forming region as this happens. In particular, survival for multiple free-fall times strongly favors virial levels of clump turbulence despite sub-virial initial conditions, and this is a key feature of the model we develop below.

– **Specific energy of infall**: Unless the initial conditions are highly magnetized, low values of the initial virial parameter imply that the energy per particle of the infalling matter is close to its initial gravitational potential. If the radius of the cluster-forming
region is small compared to the initial radius, then the inflow speed will be close to the escape velocity.

– Time profile of infall: The filamentary nature of the initial state implies that the mass inflow rate is reasonably constant – neither rapidly increasing nor rapidly decreasing – as the cluster gains most of its mass. We illustrate this below in § 2.4.1 with a simple model that displays a constant rate of accretion. Nevertheless, a decline in the inflow rate or nature of the inflow remains one possible cause for the end of cluster formation.

Given these points, we favor the following model for star cluster formation. The initial conditions correspond to elongated or filamentary molecular concentrations with low virial parameters. While magnetic fields may support these structures locally, magnetic support along their long axes is unlikely (albeit not impossible; Li & Shu 1997). Collapse therefore proceeds in the ‘rapid and violent’, nearly free-fall manner envisioned by Kauffmann et al. (2013). A stellar cluster-forming clump accumulates at the centre of this collapse, undergoes star formation, and evolves under the combined effects of accretion and stellar feedback (Figure 2.1). Because it gains mass over several internal free-fall times, we expect the clump to be virialized, with virial parameter $\alpha_c$ of order unity.

In this scenario, an important factor in a clump’s evolution is its accretion parameter

$$\eta_M = \frac{t \dot{M}_{\text{in}}(t)}{\dot{M}_{\text{in}}(t)},$$

which compares the current rate of accretion to its historical average. This is fixed in the case where the mass and inflow rate are power laws of time, $M_{\text{in}} \propto t^{\eta_M}$ and $\dot{M}_{\text{in}} \propto t^{\eta_M - 1}$. So long as each mass shell enters the cluster in a time $t(M_{\text{in}})$ which

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2 Energy conservation during free-fall collapse implies virial parameters of order unity (Larson 1981; Ballesteros-Paredes 2006; Kauffmann et al. 2013). However during filamentary collapse, infall will be visible primarily as a velocity gradient rather than a line-of-sight velocity dispersion. The virial parameter of the collapsing reservoir will therefore remain less than unity unless $\sigma(r)$ is defined in a way that accounts for velocity gradients.
is proportional to $t_{\text{ff}}(M_{\text{in}})$, any mass distribution with $M_{\text{in}} \propto r^{3-k_\rho}$ collapses with $\eta_M = 6/k_\rho - 2$. This might represent a spherical condensation with density profile $\rho(r) \propto r^{-k_\rho}$ (e.g., McLaughlin & Pudritz 1997; McKee & Tan 2003), but it can also capture the filamentary infall scenario of § 2.4.1. Initial conditions with $k_\rho < 2$ lead to accelerating accretion ($\eta_M > 1$), and those with a marked increase in column density toward the center, i.e. $k_\rho > 1$, have $\eta_M < 4$.

### 2.4.1 Filamentary infall: a simple model

If unstable filamentary structures are indeed the typical initial conditions for star cluster formation, then it is reasonable to introduce a particularly simple model of filamentary infall in which several filamentary structures emanate away from the cluster formation site at $t = 0$. To make the model specific (at some cost in realism), we neglect the motions of gas which created these filaments and arranged them in this way, assuming matter reaches the clump exclusively via filaments, i.e., without any additional accretion of non-filamentary material. If there are filaments stretching away radially in each of $N_{\text{fil}}$ directions, and the average mass per unit length along each is $\lambda_{\text{fil}}$ at $t = 0$, then the reservoir mass is $M_{\text{in}}(r) = N_{\text{fil}}\lambda_{\text{fil}}r$, the monopole free-fall time is $t_{\text{ff}}(r) = \left[\frac{\pi^2}{8N_{\text{fil}}G}\right]^{1/2}r$, and the characteristic infall rate $\dot{M}_{\text{in}} \simeq M_{\text{in}}/t_{\text{ff}}$ is constant ($\eta_M = 1$) with the value

$$\dot{M}_{\text{in}} \simeq \left(\frac{8}{\pi^2}G\lambda_{\text{fil}}^3 N_{\text{fil}}^3\right)^{1/2}. \tag{2.2}$$

The monopole approximation is not appropriate for $N_{\text{fil}} = 2$, which could describe an infinite filament with no acceleration toward $r = 0$. For $N_{\text{fil}} = 1$ the cluster forms at the end of a lone filament, not near the center of mass; see Pon et al. (2011) and Pon et al. (2012) for further discussion. The monopole approximation is reasonably accurate, however, for $N_{\text{fil}} \geq 3$.

It is useful to re-express equation (2.2) in terms of the velocity dispersion within the filament. An infinite, axisymmetric, isothermal filament of sound speed $c_s$, supported
by nothing but gas pressure, has a critical mass per unit length $2c_s^2/G$ (Stodólkiewicz 1963; Ostriker 1964) below which it must be confined by external pressure, and above which homologous collapse ensues until a change in the equation of state causes it to fragment (Inutsuka & Miyama 1992). Normalizing to the critical value for the total velocity dispersion $\sigma_{\text{fil}}$, the mass per length along the filament axis is

$$\lambda_{\text{fil}} = 2\Lambda_{\text{fil}} \frac{\sigma_{\text{fil}}^2}{G}. \tag{2.3}$$

The criticality parameter $\Lambda_{\text{fil}}$ has a maximum of about unity if magnetic forces are negligible. The mass per unit radius is greater than the mass per unit length by a factor $1/\cos(\theta)$ if filaments deviate from the radial direction by an angle $\theta$, and this factor should be absorbed into $\Lambda_{\text{fil}}N_{\text{fil}}$. Combining this definition with equation (2.2),

$$\dot{M}_{\text{in}} \simeq \frac{8}{\pi} \left( \Lambda_{\text{fil}}N_{\text{fil}} \right)^{3/2} \frac{\sigma_{\text{fil}}^3}{G}$$

$$= 1050 \left( \frac{\Lambda_{\text{fil}}N_{\text{fil}}}{4} \right)^{3/2} \left( \frac{\sigma_{\text{fil}}}{0.6 \, \text{km/s}} \right)^3 \dot{M}_\odot \, \text{Myr}^{-1}. \tag{2.4}$$

Compared to the inside-out collapse of an initially static singular isothermal sphere of sound speed $\sigma_{\text{fil}}$ (Shu 1977), the infall rate is higher by the factor $2.6(\Lambda_{\text{fil}}N_{\text{fil}})^{3/2}$. This, along with the fact that filaments display varying degrees of non-thermal support, allows $\dot{M}_{\text{in}}$ to reach the large values required to build a massive star cluster in about a million years, even within this very restricted set of assumptions. Any initial inward motion will increase the accretion rate relative to this estimate.

In extreme cases, the required accretion rate can be very high (e.g. $\sim 1 \, \dot{M}_\odot/\text{yr}$ to build a massive globular cluster), requiring $\sigma_{\text{fil}} \sim 6 \, \text{km s}^{-1}$ in equation (2.4). This can lead to collapse and fragmentation of the filaments before they can accrete (despite magnetic support; see Heitsch & Hartmann 2014).
Table 2.1. Variables and model parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Fiducial value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\text{in}(r)$</td>
<td>...</td>
<td>Initial reservoir mass within $r$</td>
</tr>
<tr>
<td>$\tau_\text{ff}(r)$</td>
<td>...</td>
<td>Free-fall time at $r$, monopole approximation</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>...</td>
<td>Effective density index, $M_\text{in}(r) \propto r^{2-3\kappa}$</td>
</tr>
<tr>
<td>$\eta_{\text{ff}}$</td>
<td>...</td>
<td>Accretion rate parameter: $M_\text{in}(t) \propto \tau_{\text{ff}}^\eta$</td>
</tr>
</tbody>
</table>

Reservoir properties (§ 2.4.4)

| $\lambda_{\text{fil}}$, $N_{\text{fil}}$ | ... | Mass per unit length, number |
| $\Lambda_{\text{fil}}$ | 1 | Criticality parameter, $G\lambda_{\text{fil}}/c_{\text{fil}}^2$ |

Filamentary infall model (§ 2.4.1)

| $\epsilon_{\text{fil}}$, $f_{\text{fil}}$, $\kappa_{\text{fil}}$ | ... | Accretion efficiency, $M_{\text{fil}}/M_{\text{in}}$ |
| $s_{\text{fil}}$ | ... | Gas fraction, $M_{\text{fil}}/M_{\text{c}}$ |
| $a_{\text{fil}}$ | 1.6 | Virial parameter, $5c_{\text{fil}}^2 R_{\text{fil}}/(GM_{\text{fil}})$ |
| $\tau_{\text{fil}}$ | ... | Turbulence-to-accretion parameter, $\tau_{\text{fil}}^\kappa/(GM_{\text{fil}})$ |
| $\eta_{\text{ac}}$ | 0.8 | Efficiency of accretion-driven turbulence |
| $\epsilon_{\text{fil}}$, $s_{\text{fil}}$ | 0.5 | Dimensionless stellar accretion rate, $\epsilon_{\text{fil}} s_{\text{fil}}$ |
| $\text{SFR}_{\text{fil}}$ | 0.03 | Star formation efficiency per free-fall time |
| $v_{\text{fil}}$, $c_{\text{fil}}$, $R_{\text{fil}}$, $M_{\text{fil}}$, $\Sigma_{\text{fil}}$, $\lambda_{\text{fil}}$ | ... | Radius, mass, mean surface density, ejected mass, $M_{\text{in}} - M_{\text{c}}$ |

Clump properties (§ 2.5)

| $r_{\text{th},p}$ | 30 km s$^{-1}$ | Protostellar wind momentum per unit mass |
| $M_{\text{c}}$ | 0.2$M_{\odot}$ | Mean stellar mass |
| $Y$ | ... | 10.1 SFR$_{\text{fil}}/(h_\text{fil}^2 G c_{\text{fil}})$ |
| $G_{\epsilon}$ | 1 | $d \ln M_\epsilon/d \ln M_{\text{fil}}$ |
| $\epsilon_{\text{fil}}$, $f_{\text{fil}}$, $\kappa_{\text{fil}}$ | ... | Mean opacity: flux-averaged; Rosseland, Planck; to dust emission |
| $L_{\text{fil}}$, $S_{\text{fil}}$, $(L_*/M_*)$, $(\mu_\text{fil})$, $\eta_{\text{fil}}$ | ... | Star: luminosity, ionizing output; light-to-mass ratio; $L_*/(4\pi 47 eV)$ |
| $F_{\text{grav}}$, $F_{\text{wind}}$, $F_{\text{rad}}$, $F_{\text{out}}$, $F_{\text{th}}$, $F_{\text{tot}}$ | ... | Forces: gravity, (direct, indirect) radiation, from HII; wind, net outward |
| $F_{\text{grav}}$, $(\Sigma$, $\Sigma_{\text{tot}})$ | ... | Fraction of clump (mass, area) below $\Sigma_{\text{tot}}$, given $\Sigma_{\text{tot}}$ |
| $\Gamma_{\text{grav}}$, $\Gamma_{\text{th}}$, $\Gamma_{\text{out}}$, $\Gamma_{\text{tot}}$ | ... | Eddington parameter of reprocessed starlight; thin limit; thick limit |
| $\Sigma_{\text{crit}}$, $\Sigma_{\text{out}}$, $\Sigma_{\text{in}}$, $\Sigma_{\text{tot}}$, $\Sigma_{\text{grav}}$, $\Sigma_{\text{th}}$, $\Sigma_{\text{out}}$, $\Sigma_{\text{tot}}$ | ... | Critical $\Sigma_{\text{tot}}$ for disruption by direct radiation, for net outward force |
| $\Gamma_{\epsilon}$ | 2 | Direct force enhancement due to innermost re-emission |
| $\psi_{\epsilon}$ | ... | Wind force factor, $F_{\psi_{\epsilon}}/L_*$ |
| $\epsilon_{\text{p,fil}}$, $\epsilon_{\text{p,fil}}$, $T_{\text{fil}}$ | 1.1 | HII: dust opacity $(10^{-21} \text{ cm}^2/\text{H})$ to starlight; temperature $(10^8 \text{ K})$ |
| $\tau_{\text{fil}}$, $\tau_{\text{fil},\text{th}}$, $\tau_{\text{fil,\text{th}}}$ | ... | HII dust optical depth to ion. light; Strömgren value; maximum |
| $c_{\epsilon}$ | 2.0 | Critical $c_{\epsilon}$ for $T_{\text{fil}} = 1$ |
| $\epsilon_{\text{G}}$ | ... | See eqs. (2.33) and (2.34) |

Stellar feedback (§ 2.6.1)

| $M_{\text{out,ind}}, M_{\text{out,evap}}$ | ... | $M_{\text{out}}$ from $F_{\text{rad}}$ due to massive stars, from photo-evaporation |
| $r_{\text{fil}}, F_{\text{grav}}, \text{etc.}$ | ... | Quantities modified by $F_{\text{rad,ind}}$ via $G \rightarrow G' = G/(1 - \Gamma)$ |
2.5 Clump evolution under rapid accretion

Having identified our preferred scenario for the accumulation and continued accretion of matter onto the site of star cluster formation, we now turn to the dynamics of the cluster-formation process itself. We adopt the idealization that there exists a cluster-forming ‘clump’ which serves as a mass reservoir for star formation, and which is dynamically distinct from the matter falling into it. Our distinction between the clump and its accretion flow is corroborated by the recent detection of two power-laws in the column density distributions of molecular clouds (Schneider et al. 2015), in which the lower-column component corresponds to filamentary features, while the high-column component is concentrated in knots at the intersections of filaments.

Despite the filamentary and clumpy nature of accretion, we assume for simplicity that the clump properties can be described with a total (gas and star) mass $M_c$, a radius $R_c$, and one-dimensional velocity dispersion $\sigma_c$ (which can be decomposed into thermal and non-thermal components: $\sigma_c^2 = c_{s,c}^2 + \sigma_{NT,c}^2$). We depict this scenario in Figure 2.1. It is worthwhile to define several additional parameters before discussing the clump’s evolution.

At any time a fraction $f_g$ of the mass is in gas and $1 - f_g$ is in stars, so the gas and star masses are $M_g = f_g M_c$ and $M_* = (1 - f_g) M_c$. These quantities evolve as matter falls into the clump, and collapses into stars or is blown away. If we normalize the rate at which gas is converted into stars to the core’s free fall rate $t_{\text{ff},c}^{-1} = \left[\frac{8GM_c}{\pi^2 R_c^3}\right]^{1/2}$ by the factor $\text{SFR}_{\text{ff}}$, then

$$\dot{M}_* = \text{SFR}_{\text{ff}} \frac{f_g M_c}{t_{\text{ff},c}}. \quad (2.5)$$

Because mass can be lost from the clump as well as added, the clump mass can grow more slowly than mass falls in:

$$\dot{M}_c = \epsilon_{\text{in}} \dot{M}_{\text{in}} \quad (2.6)$$
Figure 2.1: Schematic of the star cluster formation scenario. Inflow, lasting the free-fall time of the filamentary mass reservoir, leads to the accumulation of matter over several free-fall times of a gaseous, star-forming clump, while protostellar outflows and the massive-star feedback (photo-ionization and radiation pressure) tend to eject matter. Both feedback and inflow drive turbulent motions. The clump’s evolution, and that of the star cluster, are determined by a competition between these effects.
where $\epsilon_{\text{in}} \leq 1$. Later on we will calculate $\epsilon_{\text{in}}$ using a model for mass ejection due to stellar feedback.

How might a clump evolve under the influence of accretion and feedback? A clue to the answer lies in the fact that we introduced very few dimensional scales when describing the clump’s growth. If the star formation and stellar feedback do not rapidly change the clump properties, and the infall history is steady ($\eta_{\text{M}}$ does not change rapidly) then we may expect the dimensionless parameters describing clump evolution to be constant, or at least slowly varying. This is reasonable insofar as SFR$_{\text{ff}}$ is reasonably constant (as in the McKee 1989 and Krumholz & McKee 2005 theories, for instance) and when $\epsilon_{\text{in}}$ is not varying rapidly.

Because the clump persists and grows for multiple free-fall times, it must maintain a state close to virial equilibrium, and therefore its virial parameter

$$\alpha_c = \frac{5\sigma_c^2 R_c}{GM_c}$$ (2.7)

should remain reasonably constant. Indeed the molecular clumps which host star cluster formation are observed to contain turbulence comparable to the level required by virial balance. In the Shirley et al. (2003) study of cluster-forming CS clumps traced by water masers, the highest-quality subsample has a median $\alpha_c$ of 1.6. (We adopt this as our fiducial value for the models developed below.) In a theoretical study of accreting molecular clouds, Goldbaum et al. (2011) found that the ram pressure of infall and outflow, both of which have a compressive effect, lead to cloud virial parameters in the range 1 to 3. Compared with the low virial parameters which characterize the initial conditions, this lends credence to the assumption that the phase of active cluster formation (associated with warm dust and turbulence) is distinct from the phase of cold dust accumulation. It also corroborates the notion that the cluster-forming clump is distinct from its stream of accreting matter.

One further dimensionless parameter compares the rate of mass inflow to the
dispersion velocity for a virialized object:

\[ \xi_c = \frac{GM_{\text{in}}}{\sigma_c^3}. \]  

(2.8)

This gives us some measure of the importance of inflow in the dynamical state of the clump. The evolution of \( \xi_c \) must depend on what controls the velocity dispersion \( \sigma_c \).

If _inflow_ drives turbulence, as pictured by Klessen & Hennebelle (2010), one might expect \( \xi_c \) to approach a characteristic value determined by the properties of the inflow – its density, velocity, and magnetic field structure, as well as its rate parameter \( \eta_M \); see § 2.5.2.

Alternatively, if _feedback_ is significant, then \( \sigma_c \) will reflect the driving of turbulence by stars as well as accretion, causing it to take a higher value for a given inflow rate, and causing \( \xi_c \) to take a lower, possibly changing value. (We shall see that protostellar outflows cause only gradual changes in \( \xi_c \).) Mass ejection does not change this conclusion, because we expect it to have very little effect on the infall rate \( \dot{M}_{\text{in}} \). Taking \( \xi_c \) to be constant or slowly varying is therefore reasonable, at least until other feedback effects become important. Without further information, we expect \( \xi_c \) to be of order unity in steady accretion in the absence of stellar feedback, and to be reduced below its ordinary value when feedback stirs turbulence.

If we use the definition of \( \xi_c \) to evaluate the mass inflow rate using the simple filamentary infall model of § 2.4.1, we find that the clump is only moderately more turbulent than the filaments feeding it:

\[ \sigma_c = \frac{1.37(\Lambda_{\text{fil}} N_{\text{fil}})^{1/2}}{\xi_c^{1/3} \sigma_{\text{fil}}}. \]  

(2.9)

The dimensionless ratios defined above are enough to characterize aspects of its evolution, such as the relation between several of its internal time scales:

\[
\begin{pmatrix}
\xi_c \\
\frac{t_{\text{ff,c}}}{t} \\
\frac{M_c}{\dot{M}_*}
\end{pmatrix}
= \begin{pmatrix}
\frac{0.50 \sqrt{\xi_c}}{rac{5\eta_M}{\epsilon_{\text{in}} c_s}} \\
\frac{\frac{5\eta_M}{\epsilon_{\text{in}} c_s}}{f_{\text{SFR}} \xi_c} \\
\frac{0.50 \sqrt{\sigma_c}}{f_{\text{SFR}}}
\end{pmatrix}.
\]  

(2.10)
Here we have introduced $\epsilon_{\text{in}} = M_c / M_{\text{in}}$. There is a reasonably clear separation of time scales: for fiducial values ($\alpha_c = 1.6$, $\xi_c = \eta_M = 1$, $f_g = \epsilon_{\text{in}} = \bar{\epsilon}_{\text{in}} = 0.8$, SFR$_{\text{ff}} = 0.03$) we see that $(t_{\text{ff},c} :: R_c / \sigma_c :: t :: M_c / \dot{M}_*) = (1 :: 1.6 :: 6.2 :: 42)$. For these parameters, the clump is eternally several crossing or free-fall times old.

The comparison of time scales can also be interpreted in terms of the physical extent of the reservoir which feeds the clump. If we are correct that the inflow of matter corresponds to a collapse from the initial state, and takes about one initial free-fall time at each radius, then $t \simeq t_{\text{ff}}(r_0)$ where $r_0$ is the initial radius of this matter at $t = 0$. Given the value of $t / t_{\text{ff},c}$ from equation (2.10), this implies a relation between the clump radius and the initial radius $r_0(t)$ of matter reaching it:

$$r_0(t) / R_c(t) \simeq 4.7 \frac{\eta M^{2/3}}{\alpha_c \bar{\epsilon}_{\text{in}} \xi_c^{2/3}}, \quad (2.11)$$

which is 3.6 for the fiducial values listed above. The clump’s feeding zone is only a few times its own radius at any time.$^3$

Under certain conditions even fewer parameters are required, because the gas mass fraction $f_g$ can be expressed in terms of the others. To see this, first compare the rates of star formation and mass accretion:$^4$

$$\frac{\dot{M}_*}{\dot{M}_c} = \frac{10.1 \text{SFR}_{\text{ff}}}{\alpha_c^{3/2} \bar{\epsilon}_{\text{in}} \xi_c} f_g. \quad (2.12)$$

But if we define $q_* = d \ln M_* / d \ln M_c$, then $\dot{M}_* / \dot{M}_c = q_* M_* / M_c = (1 - f_g) q_*$; therefore

$$f_g = \frac{q_* \alpha_c^{3/2} \bar{\epsilon}_{\text{in}} \xi_c}{10.1 \text{SFR}_{\text{ff}} + q_* \alpha_c^{3/2} \bar{\epsilon}_{\text{in}} \xi_c}. \quad (2.13)$$

If the dimensionless parameters we have listed are truly constant, so that the clump’s growth is self-similar, then $q_* = 1$ because stars form in lock step with the clump. This, along with the other fiducial parameters, implies $f_g = 0.84$, i.e. a stellar mass

---

$^3$ This calculation neglects the difference in time to fall to $R_c$ rather than $r = 0$, and also the possibility that effects which blow out matter also affect the duration of infall.

$^4$ We use decimal coefficients for convenience; here $10.1 = 10^{3/2} / \pi$. 
fraction of 16%. A value of this order is to be expected, given that star formation is assumed to be slow and the clump is permanently only a few free-fall times old.

We pause to review the evolution with mass of clump properties obtained by assuming the dimensionless ratios like $\alpha_c$ and $\xi_c$ are constant, or slowly varying, while $M_{\text{in}} \propto t^{\eta_M} \propto t^6/k_\rho^{-2}$:

$$
\begin{pmatrix}
R_c \\
\sigma_c \\
\Sigma_c
\end{pmatrix}
\propto
\begin{pmatrix}
M_c^{2+\eta_M} \\
M_c^{\eta_M-1} \\
M_c^{\eta_M-3}
\end{pmatrix}
\propto
\begin{pmatrix}
M_c^{\frac{1}{6-k_\rho}} \\
M_c^{\frac{2-k_\rho}{5-k_\rho}} \\
M_c^{\frac{k_\rho-1}{5-k_\rho}}
\end{pmatrix}
$$

(2.14)

where by $\bar{\Sigma}_c$ we mean the clump’s mean column density $M_c/(\pi R_c^2)$. If the initial reservoir has a constant column density ($k_\rho = 1$) then $\bar{\Sigma}_c$ will be constant while $\sigma_c$ increases ($\sigma_c \propto M_c^{1/4}$), whereas if the reservoir is like our filamentary infall model or like a singular isothermal sphere ($k_\rho = 2$), then $\sigma_c$ is constant while the column decreases ($\bar{\Sigma}_c \propto M_c^{-1}$). We expect $1 \leq k_\rho \leq 2$ to bracket the plausible range of values for the main accretion phase, and argued for the upper end of this range in § 2.4. A strong drop in the mass accretion rate would correspond to $\eta_M \ll 1$.

### 2.5.1 Protostellar population

Our parameterization of the growing clump allows us to estimate the population of protostars within it. If we suppose that individual stars acquire their masses at an average rate $\dot{m}_* = \varepsilon_* \xi_* c_{s,c}^3 / G$ (as a result of infall at a rate $\bar{\xi}_* c_{s,c}^3 / G$ of which only $\varepsilon_*$ lands on the star) then the average number of accreting protostars at any time is

$$
\frac{\dot{M}_*}{\dot{m}_*} = \frac{10.1 \text{SFR}_{ff}}{\alpha_c^{3/2}} \frac{f_g}{\varepsilon_* \bar{\xi}_*} \left(\frac{\sigma_c}{c_{s,c}}\right)^3.
$$

(2.15)

Adopting $\varepsilon_* \bar{\xi}_* = 0.5$ and the fiducial values of the other parameters, this becomes $\sigma_c^3/(1.61 c_{s,c})^3$. This result is quite sensitive to the Mach number of clump turbulence, but implies tens of accreting protostars in regions like NGC 1333.
2.5.2 Accretion-driven turbulence: energetics and stability

The expectations laid out above rest on an underlying assumption: that molecular cloud accretion can sustain within it the turbulent, self-gravitating region we call the cluster-forming clump — and further, that the clump’s velocity dispersion $\sigma_c$ is regulated by the dynamics of inflow, around some characteristic value. Is this realistic?

An energetic argument raises doubts. If the accretion flow arrives with close to zero specific energy, as we argued in §2.4, then its inflow velocity will be close to the clump escape velocity $v_{esc,c} = (2GM_c/R_c)^{1/2}$, and the turbulent energy should be created at a rate comparable to the inflow of kinetic energy, $\dot{M}_{\text{in}}v_{esc,c}^2$. If the virial parameter $\alpha_c$ is constant then $\sigma_c \propto v_{esc,c}$ and so the driving rate is of order $\dot{M}_{\text{in}}\sigma_c^2$. However the rate of turbulent dissipation is of order $\sigma_c^5/G$ (assuming the clump is supported by supersonic turbulence). The two rates balance at a specific value for which $\sigma_c^3 \simeq G\dot{M}_{\text{in}}$ (i.e. $\xi_c \simeq 1$). However, the balance is unstable: if the clump is smaller at a given time, then (assuming $\alpha_c$ is the same) its $\sigma_c$ is higher, and dissipation outpaces driving. Conversely, if the clump is expanded, driving outpaces dissipation, and in fact the study of accretion molecular clouds by Goldbaum et al. (2011) shows evidence of unstable behavior (their figure 2). Although the outcome will be complicated by the turbulent nature of the flow and by the renewal of matter during accretion, this energetic instability invites a closer look. Two points suggest that it does not invalidate the notion of a characteristic value of $\xi_c$.

First, we have found that the feeding radius $r_0$ is only a few times larger than the clump radius, even when our parameters take their fiducial values. Accretion shocks and turbulent dissipation involve a loss of energy, which requires an increase in the binding energy, so the clump clearly cannot grow to be comparable in size to its parent region. This places an upper limit on excursions of $R_c$ and $\xi_c$ relative to the state $\xi_c = 1$, and we view this as a consequence of the non-zero energy of the initial state.
Second, a lower limit on $R_c$ and $\xi_c$ comes from non-zero angular momentum in the initial state, which is a feature of any realistic scenario for mass accumulation. If turbulent motions pervade the initial conditions with a virial parameter $\alpha_{\text{res}}$, then a parcel from radius $r_0$ has a characteristic angular momentum $|j| \simeq (\alpha_{\text{res}} GM_c r_0/5)^{1/2}$ and hence, if $j$ is conserved, would orbit at a radius $|j|^2/(GM_c) \sim (\alpha_{\text{res}}/5)r_0$. While some of this angular momentum exists in random motions which can cancel during infall, the cancellation cannot completely erase it. Kratter & Matzner (2006) analyze this cancellation in order to predict disk radii during massive star formation, and find that it reduces $|j|$ on each shell by at most a factor of two (their Appendix A and Table A1), leading to a minimum circularization radius $\sim (\alpha_{\text{res}}/25)r_0$. While there is no evidence of overall rotational support within star clusters, we note that this limiting radius is proportional to $r_0$ and would therefore imply a constant value of $\xi_c$.

These observations suggest that our assumption of a steadily growing clump with a characteristic value of $\xi_c$ is realistic. The process should be examined in greater detail, and for this reason we have begun a set of numerical experiments (Hansen et al. 2015, in prep.). Note, also, that the models of Goldbaum et al. (2011) show no evidence of the energetic instability in clouds with turbulent driving due to star formation. We turn to this topic below.

### 2.6 Feedback from protostellar outflows

Protostellar outflows have long been recognized as a ubiquitous signpost of star formation (Heyer et al. 1987; Parker et al. 1991; Myers et al. 1986; Bontemps et al. 1996). As a source of outward momentum, they can eject matter from the sites of individual star formation (Myers et al. 1988; Nakano et al. 1995; Momose et al. 1996; Velusamy & Langer 1998; Ladd et al. 1998) or the larger clumps in which they are embedded (Levreault 1984; Langer et al. 1986; Goldsmith et al. 1986; Bally et al. 1994). They have also been implicated in the energization of turbulence on clump scales (Quillen et al.
2005; Graves et al. 2010; Covey et al. 2010; Nakamura et al. 2011b,a; Mottram & Brunt 2012; Plunkett et al. 2013) although their influence does not extend to molecular cloud scales (e.g. Arce et al. 2010) as originally proposed (Norman & Silk 1980; McKee 1989). The numerical simulations by several groups (Li & Nakamura 2006; Cunningham et al. 2006a,b; Nakamura & Li 2007; Frank 2007; Nakamura & Li 2008; Cunningham et al. 2009; Carroll et al. 2009; Wang et al. 2010; Cunningham et al. 2011; Hansen et al. 2012; Myers et al. 2013) are broadly consistent with these observational findings, but see Banerjee et al. (2007) for an opposing view and Myers et al. (2014) for important qualifications.

The importance of protostellar outflow feedback in the evolution of a clump is determined largely by the mean protostellar wind momentum per unit mass, $v_{ch,w}$. The net force due to all the winds within a clump is $\dot{M}_* v_{ch,w}$, and, ignoring any dependence of $v_{ch,w}$ on stellar mass and environment, the momentum from a protostar of mass $m_*$ is $m_* v_{ch,w}$. The actual value of $v_{ch,w}$ is quite uncertain; observational estimates are usually below 30 km/s, but Dunham et al. (2013) stress that momentum is often underestimated.

### 2.6.1 Outflow-driven turbulence

A simple comparison of forces shows the potential importance of outflows in stirring turbulence: the characteristic turbulent acceleration for a clump with velocity scale $\sigma_c$ is $\sigma_c^2 / R_c$, corresponding to a characteristic force $M_g \sigma_c^2 / R_c$. Comparing the outflow force to this,

$$\frac{\dot{M}_* v_{ch,w}}{M_g \sigma_c^2 / R_c} = \frac{2.0 \text{SFR}_{\text{ff}} v_{ch,w}}{\alpha_c^{1/2} \sigma_c} = \frac{1.4 \text{ km/s}}{\sigma_c} \left( \frac{1.6}{\alpha_c} \right)^{1/2} \frac{\text{SFR}_{\text{ff}}}{0.03} \frac{v_{ch,w}}{30 \text{ km/s}}. \quad (2.16)$$

This suggests that, for our choice of fiducial parameters, outflows can be a strong influence in clumps with $\sigma_c \lesssim 1.4 \text{ km/s}$; in more turbulent regions they are weak. To
put it another way, so long as the outflow force couples to turbulent motions on the clump scale, we expect outflows to sustain turbulence of order $1.7 \text{ km/s}$ (for fiducial parameters) on their own. Since this is equivalent to the level of accretion-driven turbulence when $\dot{M}_{\text{in}} = 700 \zeta_c M_\odot / \text{Myr}$, both accretion and outflows feedback should be important in supporting regions that gain up to about a thousand solar masses per million years. We estimate that about a quarter of Galactic star formation occurs in such clusters (using the cluster birthrate model of McKee & Williams 1997 and assuming a common formation time of 1 Myr).

Equation (2.16) shows that, in the context of constant accretion, a primary influence of protostellar outflows is to modify the clump velocity dispersion and therefore to reduce the parameter $\zeta_c$ relative to what it would be in the absence of feedback. The virial parameter $\alpha_c$ may also be shifted, although more subtly. This observation corroborates our assertion in § 2.5 that dimensionless parameters like $\alpha_c$ and $\zeta_c$ are likely to be roughly constant during a period of accretion-fed growth.

There are several features of protostellar outflows which should be accounted for when considering their affect on clump turbulence.

First, outflows are discrete events with a characteristic momentum $\bar{m}_* v_{\text{ch},w}$ (where $\bar{m}_*$ is the mean stellar mass) and a wide range of individual intensities. This leads to a characteristic distance $\ell$ on which outflows’ force is applied, and suppresses their effect on clump turbulence (relative to a simple estimate based on force balance, e.g. assuming expression (2.16) to be unity) if $R_c > \ell$. This also introduces $M_c / \bar{m}_*$ as a parameter which can affect the strength of feedback.

Second, outflows are highly collimated: this mitigates the first effect by extending the reach of their momentum injection relative to a model in which they are spherical.

Third, collimation also implies that some of the outflow momentum may be lost from the clump as outflows drive flows that escape the clump entirely.

Fourth, any additional source of turbulence, such as the stirring by an accretion flow, will affect the scale on which outflows deliver their momentum, as this happens
when outflow-driven motions decelerate to the local turbulent speed or wave speed.

A simple model for outflow-driven turbulence that reflects all four effects was presented by Matzner (2007, hereafter M07), and we shall use his equation (24) to estimate outflows’ contribution to $\sigma_c$. We adopt the same stellar initial mass function and wind force structure function as M07 (from Kroupa 2001 and Matzner & McKee 1999a, respectively) when evaluating his function $S(\hat{T})$ in M07’s equation (30), and we also adopt his value of 0.8 for the outflow coupling efficiency (M07’s parameter $\Lambda$). The combined effect of accretion and outflows is depicted in figure 2.2, and in figure 2.3 we estimate the fraction of turbulent energy due to outflows, for accreting clumps with no other form of feedback.

The M07 theory allows us to account for the driving of turbulence by accretion, which influences how outflows interact with the gas. For this we assume that accretion drives turbulence with an acceleration

$$a_{\text{ext}} = \varphi_{\text{acc}} \frac{\dot{M}_\infty v_{\text{esc},c}}{f_g M_c},$$

where $\varphi_{\text{acc}} \leq 1$ describes the efficiency with which accretion drives turbulence. (Our $\varphi_{\text{acc}}$ is similar to the parameter $\varphi$ introduced by Goldbaum et al. 2011.) This definition, along with the definitions of $a_c$ and $\xi_c$, implies

$$\xi_c = \left( \frac{5}{2a_c} \right)^{1/2} \frac{f_g}{\varphi_{\text{acc}}} \frac{a_{\text{ext}} R_c}{\sigma_c^2}, \quad (2.17)$$

and we use this relation to evaluate the relative influence of accretion and protostellar outflows. When feedback is insignificant the M07 theory implies $\sigma_c^2 = c_{s,c}^2 + a_{\text{ext}} R_c$, so that $\xi_c$ takes the unique value $(2.5/a_c)^{1/2} f_g \varphi_{\text{acc}}^{-1}$, assuming $c_{s,c}^2$ can be ignored. Dynamically significant outflows stir higher-velocity turbulence, leading to an appreciable drop in $\xi_c$. Equation (2.17) can be evaluated if the fraction of turbulence supplied by outflows, $(1 - a_{\text{ext}} R_c/\sigma_c^2)$, is known; we plot this quantity in Figure 2.3 using the M07 theory. The result is approximately consistent with our expectation, from equation (2.16), that outflows are strong when $\sigma_c \lesssim 1.4 \text{ km s}^{-1}$, but it is modified somewhat by
Figure 2.2: Combined role of outflows and accretion in the M07 model. A clump of 1000 $M_\odot$ with mean column density 0.2 g cm$^{-2}$ is stirred by weak outflows ($v_{ch,w} = 5$ km/s, thin lines), moderate outflows ($v_{ch,w} = 15$ km/s), or strong outflows ($v_{ch,w} = 40$ km/s, thick lines), as well as no accretion ($\dot{M}_{in} = 0$), moderate accretion ($\dot{M}_{in} = 1000 M_\odot$ Myr$^{-1}$), or strong accretion ($\dot{M}_{in} = 3000 M_\odot$ Myr$^{-1}$). Circles represent the clump radius, where the velocity dispersion $\sigma_c$ and virial parameter $\alpha_c$ are set. Other parameters: SFR$_{ff} = 0.034$; $\varphi_{acc} = 0.75$; $\Lambda = 1$; $c_s = 0.19$ km/s. We adopt the Kroupa IMF and Matzner & McKee (1999a) collimation model, as in M07.
their collimation and escape. For instance, the loss of momentum in escaping outflows is very significant when $M_c \lesssim 100 M_\odot$.

### 2.6.2 Outflow driving: energetics and stability

If protostellar outflows are capable of sustaining a virial level of turbulence within a clump, is this balance energetically stable? A simple argument, like the one considered in § 2.5.2, shows that the answer is no. The rate of turbulent energy injection by protostellar outflows is

$$\sim \dot{M}_* v_{\text{ch},\omega} \sigma_c \sim \text{SFR}_\text{ff} \sigma_c^4 v_{\text{ch},\omega} / G,$$

while the rate of turbulence dissipation is

$$\sim \sigma_c^5 / G.$$ Unless there is an effect which causes SFR$_\text{ff}$ to vary rapidly with $\sigma_c$, a balance between these is unstable to variations of $\sigma_c$ away from the equilibrium state. Star formation is observed to be suppressed for low values of the visual extinction (i.e. low $\bar{\Sigma}_c$), but this threshold is well below the columns of interest in star cluster formation.

We infer, therefore, that a clump supported entirely by outflow-driven turbulence is energetically unstable and should oscillate around its equilibrium state, or explode or collapse altogether. The discreteness of individual outflows makes this behavior stochastic, especially in small clumps, as the number of stars formed per free-fall time is

$$\sim 12 \left( M_c / 100 M_\odot \right).$$

(Note that Matzner 1999 argues that this instability leads to over-stable oscillations.)

Similar conclusions hold for feedback effects like H II regions within giant molecular clouds, which also form at a rate proportional to the star formation rate, inject a reasonably constant momentum per unit mass, and involve discrete events. Comparing the results of Krumholz et al. (2006), who do not include accretion, and Goldbaum et al. (2011), who do, we infer that the instability is damped by accretion.

The above argument rests on the negative specific heat of spherical self-gravitating systems: if a loss of energy led $\sigma_c$ to decline, we would have inferred stability rather than instability. Filamentary and planar systems (such as the disks of spiral galaxies)
Figure 2.3: Fraction of clump turbulence driven by outflows rather than accretion, \((1 - a_{\text{ext}} R_c / \sigma_c^2)\), in the M07 model of steady-state driving and decay. This quantity is related to our parameter \(\xi_c\) by equation (2.17). For each combination of \(\sigma_c\) and \(\Sigma_c\), we find the value of accretion driving \((a_{\text{ext}})\) sufficient to maintain turbulence with virial parameter \(a_c = 1.6\) according to M07 equations (20), (21), and (22). The M07 coupling parameter \(\Lambda\) is set to 0.8, and other parameters take their fiducial values. For \(M_c < 100 M_\odot\) fewer than fifteen outflows form per \(t_{\text{ff},c}\) so stochastic effects are strong.
do not have negative specific heats, so stellar feedback leads to stable equilibria in these systems. We have simplified a complex system into just a few degrees of freedom, so we cannot draw firm conclusions on the outcome of the instability without conducting numerical experiments.

2.6.3 Outflow mass ejection

Strong collimation allows outflows to breach the clump and eject matter. As Matzner & McKee (2000, hereafter MM00) explain, an absolute upper limit for the mass ejection rate $\dot{M}_{\text{ej}}$ produced by outflows is obtained imagining that the outflow momentum couples perfectly to motions just fast enough to escape the clump. Since each star emits momentum $m_* v_{ch,w}$, the upper limit of mass ejected by this one star is $M_{\text{ej},*} = m_* v_{ch,w} / v_{esc,c}$. MM00 show that, in the Matzner & McKee (1999a) outflow model, the actual ejected mass is lower by a constant factor, for each outflow strong enough to break free of the clump, but too weak to entirely disrupt it. Taking this factor into account,

$$\frac{M_{\text{ej},*}}{m_*} = \frac{1}{2c_g \ln(2/\theta_0)} \frac{v_{ch,w}}{v_{esc,c}} \lesssim \frac{v_{ch,w}}{12.0 v_{esc,c}}$$

(2.18)

where $\theta_0 \simeq 10^{-2}$ defines a core angle for the outflow, and $c_g \simeq 1.13$ corrects for deceleration as an outflow crosses the clump. The ejected mass per star is inversely proportional to $v_{esc,c}$, implying that the star formation efficiency increases with $\sigma_c$ as seen in Figure 2.4.

Since this ratio is independent of stellar mass, for outflows in this intermediate range, it provides an estimate of the ratio $\dot{M}_*/\dot{M}_{\text{ej}}$ between star formation and mass ejection. An improved estimate must account for other effects, like the confinement of weak outflows within giant clouds, which reduce $\dot{M}_{\text{ej}}$. MM00 evaluate these effects numerically (their figure 2), but we note that they can also be represented using the
functions defined by M07:

\[
\frac{\dot{M}_{\text{ej}}}{\dot{M}_*} = f_g \frac{M_c \dot{\mathcal{I}}_{\text{esc}}}{\dot{M}_*} \left[ 1 - \frac{\dot{\mathcal{S}}(\mathcal{I}_{\text{esc}})}{\dot{\mathcal{S}}_{\text{tot}}} \right].
\] (2.19)

Here \( \mathcal{I}_{\text{esc}} = c_g f_g M_c v_{\text{esc},c} \) is the impulse required to unbind all the gas from the clump (which is also the isotropic-equivalent impulse required to unbind gas in one direction). The M07 function \( \dot{\mathcal{S}}(\hat{I})/\dot{\mathcal{S}}_{\text{tot}} \) is a normalized cumulative distribution of outflow strengths – or more precisely, the strengths of all the individual angular sectors of all the outflows – emitted in the creation of a stellar population (M07 equation 30). In figure 2.4 we compare expression (2.19) to equation (2.18), which provides an approximation for the same quantity.

We now wish to revisit the conditions of an accreting clump for the case in which there is a relation between the rates of mass ejection and star formation, as there is when protostellar outflows are the cause. Using the equality \( \dot{M}_{\text{in}} = \dot{M}_c + \dot{M}_{\text{ej}} \) with equation (2.12), we find

\[
\epsilon_{\text{in}} = 1 - Y \left( \frac{\dot{M}_{\text{ej}}}{\dot{M}_*} \right) f_g
\] (2.20)

where \( Y = 10.1 \frac{\text{SFR}_{\text{ff}}}{(a_c \xi_c)^{3/2}} \). The clump experiences a net loss of mass (\( \epsilon_{\text{in}} < 0 \)) if ejection is stronger than accretion, i.e. if \( \dot{M}_{\text{ej}}/\dot{M}_* > 1/Y \), which is \( 6.7(1.6/a_c)^{3/2} \) for our fiducial parameters.

For the case of a clump undergoing self-similar growth, we can go further by employing equation (2.13), in which we use the definition \( q_* = d \ln M_* / d \ln M_c \simeq 1 \) to relate \( f_g \) and \( \epsilon_{\text{in}} \). This shows that the gas fraction \( f_g \) can be determined as the physically relevant solution of the quadratic equation \( (1 - f_g)q_* [1 - Y(\dot{M}_{\text{ej}}/\dot{M}_*) f_g] = Y f_g \):

\[
f_g = \frac{W - \sqrt{W^2 - 4(\dot{M}_{\text{ej}}/\dot{M}_*)q_*^2 Y}}{2(\dot{M}_{\text{ej}}/\dot{M}_*)q_* Y},
\] (2.21)

where \( W = q_* + Y[1 + (\dot{M}_{\text{ej}}/\dot{M}_*)q_*] \). This result and the corresponding value of \( \epsilon_{\text{in}} \) are plotted in figure 2.5 for the case \( q_* = 1 \).
Figure 2.4: Efficiency of star formation, $\dot{M}_*/(\dot{M}_* + \dot{M}_{ej})$, considering only mass ejection due to outflows. Solid blue lines are an evaluation of equation (2.19), which accounts for the effect of outflow confinement in large clouds. Thin blue lines (with parenthesized values) represent equation (2.18), an approximation from MM00, which does not. Efficiency increases with $\sigma_c$ because each star ejects less mass in a region of higher escape velocity. These curves assume a Kroupa (2001) initial mass function and a Matzner & McKee (1999a) outflow structure with core angle $\theta_0 = 10^{-2}$. 

\begin{align*}
\text{Efficiency per star,} & \quad \frac{\dot{M}_*}{\dot{M}_* + \dot{M}_{ej}} \\
\log_{10} \Sigma_c & \quad (\text{g cm}^{-2}) \\
\log_{10} \sigma_c & \quad (\text{km s}^{-1})
\end{align*}
Figure 2.5: Gas fraction $f_g$ (eq. 2.21, solid lines) and accretion efficiency $\epsilon_{\text{in}}$ (eq. 2.20, dashed lines) as functions of the star formation efficiency and the parameter $Y = 10.1 \frac{\text{SFR}_{\text{eff}}}{(\alpha_c^{3/2} \xi_c^3)}$, for clumps undergoing strictly self-similar growth ($q_\ast = 1$).
Figure 2.6: Gas fraction $f_g$ and accretion time $M_c/\dot{M}_c = t/\eta_M$ for clumps accreting self-similarly under the influence of protostellar outflows (i.e., neglecting massive-star feedback). Outflows affect these curves both by ejecting matter (reducing $\epsilon_{in}$) and by injecting turbulent energy (increasing $\sigma_c$ and reducing $\xi_c$). Other parameters ($v_{ch,vw}, \alpha_c, \phi_{acc}, \text{SFR}_f$) are held fixed at fiducial values, and self-similar growth ($q_* = 1$) is assumed in the calculation of $\epsilon_{in}$ and $f_g$. 

Clump accretion with outflows:

Gas mass fraction $f_g$,

Accretion time

$M_c/\dot{M}_c = t/\eta_M$
In Figure 2.6 we combine these results to make a prediction for the gas fraction $f_g$ and accretion time scale $M_c/\dot{M}_c = t/\eta M$ for clumps undergoing self-similar accretion whilst being afflicted by outflows (and no other feedback). Beginning with the determination of $\dot{M}_{ej}/\dot{M}_*$ (eq. 2.19) and $a_{\text{ext}}R_c/\sigma_c^2$ from the M07 theory, we calculate $\xi_c$ via equation (2.17) and then $\epsilon_{\text{in}}$ and $f_g$ from equations (2.20) and (2.21). The accretion rate is evaluated using $\dot{M}_c = \epsilon_{\text{in}}\dot{M}_{\text{in}} = \epsilon_{\text{in}}\xi_c\sigma_c^{-3}/G$, and used to compute the accretion time scale.

One interesting feature of outflows’ influence that is visible in this figure is that the clump age and accretion time are primarily a function of the clump column $\bar{\Sigma}_c$. For $M_c/\dot{M}_c \simeq 1$ Myr the characteristic column is about $0.3 \text{ g cm}^{-2}$.

The path of an accreting clump through this diagram is determined by the accretion history: for self-similar steady accretion ($k_\rho = 2, \eta_M = 1$) a clump moves downward at nearly constant $\sigma_c$; for accelerating accretion ($k_\rho = 1.5, \eta_M = 2$) the motion is down and to the right, at roughly constant $\sigma_c^2\bar{\Sigma}_c$; and for strongly accelerating accretion ($k_\rho = 1, \eta_M = 4$) it is horizontally to the right, at approximately constant $\bar{\Sigma}_c$.

### 2.7 Massive stellar feedback

Protostellar outflows are relatively unimportant in clusters with velocity dispersions $\sigma_c > 2 \text{ km/s}$. Higher velocity dispersions are the province of massive clusters containing B and O stars, which afflict cluster-forming gas with photon pressure, photo-ionization, stellar winds, and eventual supernovae. The relative importance of these effects on dense cluster-forming gas has been discussed several times (e.g. Krumholz & Matzner 2009, Murray 2009, Murray et al. 2010, Fall et al. 2010, and Dale et al. 2014, among others). Our goal is to review and reconsider these effects in the context of accretion onto the cluster-forming region. In particular, we wish to delineate regions in the parameter space of Figure 2.6 where each effect is most important, and estimate the implications for gas removal.
Our observational points of reference are the cluster-forming regions traced in dust continuum, high-density molecular tracers, and signposts of massive star formation such as water masers, methanol masers, or compact/ultra-compact H II regions (e.g., Plume et al. 1997; Shirley et al. 2003; Walsh et al. 2003; Faúndez et al. 2004; Wu et al. 2005; Dunham et al. 2011; Urquhart et al. 2013). These regions span a wide range of gas mass \( (10^2 - 10^4 M_\odot) \) with column densities \( \sim 0.6 \text{ g cm}^{-2} \pm 0.38 \text{ dex} \) (in the Shirley et al. sample); their infrared emission reflects characteristic dust temperatures \( \sim 32 - 40 \text{ K} \) and luminosity-to-mass ratios \( \sim 70 - 130 \text{ } L_\odot / M_\odot \) (Mueller et al. 2002; Beuther et al. 2002; Faúndez et al. 2004). They are about as likely to show evidence of inflow, in the line profiles of self-absorbed tracers, as are individual low-mass protostars, albeit at much higher flow rates (Wu & Evans 2003; He et al. 2015).

Once sufficiently many stars are born, the complement of massive stars grows, and the strength of their feedback grows as well. Massive-star feedback is initially stochastic, as it depends strongly on the mass of the most massive star in the population, but it becomes more predictable once the cluster samples the initial mass function (IMF) to the highest stellar masses. We illustrate this in figures 2.7 and 2.8, which describe the growth of the initial stellar luminosity-to-mass ratio \( L_*/M_* \) (to its IMF-averaged value \( \sim 10^3 L_\odot / M_\odot \)) and the probability of experiencing a core-collapse supernova (SN), respectively, during the growth of a cluster. Photo-ionization depends on the production rate \( S \) of H-ionizing photons, which grows to \( S \simeq L_* / (47 \text{ eV}) \) for clusters which sample the entire IMF. (For this estimate we use zero-age main sequence luminosities from Tout et al. 1996 and ionization rates from Vacca et al. 1996.) Once its stars have formed, a cluster’s luminosity and ionizing output will be constant for the main-sequence lifetime of its most massive stars, which is most of the time prior to its first supernovae (a few Myr). The detailed properties and evolution of a massive cluster vary somewhat depending on the metallicity, stellar multiplicity (Sana et al. 2012), stellar rotation (Chieffi & Limongi 2013), line blanketing in stellar winds
(Martins et al. 2005), and the form of the stellar initial mass function; however we do not attempt to capture these effects.

We pause to show that, when they exist, massive stars are much brighter than the accretion and contraction luminosity of the low-mass stars. At stellar ages \( t_* \sim \) Myr, low-mass stars are fully convective objects with surface temperatures \( T_{\text{eff}} \approx 4120 \, \hat{m}_{\ast}^{0.13} \) K at stellar mass \( \hat{m}_{\ast} \) \( M_\odot \) (Baraffe et al. 2009). The luminosity of such an object, averaged over its short life, is its binding energy divided by its age, which we compute (e.g., Ushomirsky et al. 1998) to be \( 5.2 \, \hat{m}_{\ast}^{0.51} (\text{Myr}/t_*)^{2/3} \, L_\odot / M_\odot \). Making the simplifying assumption that \( m_\ast \) is chosen from the IMF, we find that the mean luminosity per unit mass of the young stellar objects is

\[
(1.8 \text{ to } 2.8) \left( \frac{\text{Myr}}{t_*} \right)^{2/3} \frac{L_\odot}{M_\odot},
\]

where \( \langle \rangle \) indicates an average over stellar ages, and the range of prefactors reflects the outcome from different IMFs.

The accretion and contraction luminosity of the clump are smaller still, and entirely negligible.

### 2.7.1 Radiation force: direct and indirect

To begin, we compare the direct radiation force against gravity. Because dust illuminated directly by un-extinguished starlight has temperatures \( \gtrsim 150 \) K, and because the clump is optically thick (optical depth \( \sim 20 f_g \Sigma_c / (\text{g cm}^{-2}) \)) to the mid-infrared radiation emitted by these grains, we include this first stage of reprocessing in the ‘direct’ radiation force, boosting it by the factor \( 1 + \phi_{\text{IR}} \approx 2 \). (Further reprocessing is non-local and best incorporated into the ‘indirect’ force discussed below.) For simplicity, assume all the gas is swept into a shell of radius \( R_c \). The weight of the gas

\footnote{We arrive at this estimate by assuming a dust opacity to starlight of \( \sim 10^{-21} \) cm\(^2\) per H atom (Draine 2011), Planck mean opacity from (Semenov et al. 2003), and the starlight flux evaluated at \( r < R_c / 3 \).}
shell is \(f_g(1 - f_g/2)GM_c^2/R_c^2\), and the photon force (including mid-IR reprocessing) is \(L_\ast/c = (1 + \phi_{\text{IR}})(1 - f_g)(L/M)_\ast M_c/c\), so the ratio of the direct radiation force to the weight of gas is
\[
\frac{F_{\text{rad,dir}}}{F_{\text{grav}}} = (1 + \phi_{\text{IR}})\Sigma_{\text{crit}}/\bar{\Sigma}_c,
\]
where
\[
\Sigma_{\text{crit}} = \frac{1/f_g - 1}{1 - f_g/2} \frac{(L/M)_\ast}{\pi G c} = 0.31 \left[\frac{1/f_g - 1}{1 - f_g/2}\right] \frac{(L/M)_\ast}{10^3 L_\odot/M_\odot} \text{g cm}^{-2}. \quad (2.22)
\]
The factor in brackets is unity for \(f_g = 2 - \sqrt{2} = 0.59\), and increases rapidly for smaller gas fractions.

Next we consider the indirect force, which has been approximated in different ways. Krumholz & Matzner (2009) define \(f_{\text{trap,IR}} = F_{\text{rad}}/(L/c)\), and employ a leaky-shell model to estimate \(f_{\text{trap,IR}} \lesssim 1\). Murray et al. (2010) consider instead a closed spherical shell of infrared optical depth \(\tau_{\text{IR}} = \int \bar{\kappa}(r) \rho(r) dr\) (if \(\bar{\kappa}(r)\) is the flux-averaged opacity), and so estimate \(f_{\text{trap,IR}} = \tau_{\text{IR}}\) and \(F_{\text{rad,ind}} = \tau_{\text{IR}} L/c\). Even more sophisticated approaches (e.g., Chakrabarti & McKee 2005) involve radiative transfer solutions in spherical symmetry. We opt for simplicity, on the basis that the local Eddington factor
\[
\Gamma(r) = \frac{\bar{\kappa}(r)L(r)}{4\pi GM(r)c} = \frac{L(r)/M(r)}{10^3 L_\odot/M_\odot} \frac{\bar{\kappa}}{13 \text{cm}^2 \text{g}^{-1}}, \tag{2.23}
\]
which measures the ratio of radiation force to gravity (for flux-averaged opacity \(\bar{\kappa}\)), is modest and bounded. In an optically thick region \(\bar{\kappa}\) is the Rosseland mean; Rosseland mean opacity in the models of Semenov et al. (2003) peaks at a maximum value of \(3 - 10 \text{ cm}^2 \text{ g}^{-1}\), at temperatures \(\sim 100 \text{ K}\) (just cool enough for ices to persist). Considering that \(M(r)\) includes the mass of gas as well as stars, that \((L/M)_\ast\) saturates at \(\sim 10^3 L_\odot/M_\odot\), and that clump dust often temperatures well below 100 K, typical values of \(\Gamma\) are less than unity.
Indeed it suffices, for our purposes, to assign a single average dust temperature $T_d$: the value for which

$$\frac{\sigma r T_d^4 \kappa_{Pl}(T_d)}{1 + 3 \tau_{Pl}(T_d) \tau_R(T_d)/8} = \frac{L}{4M_g} = \frac{1 - f_g}{4f_g} \left( \frac{L}{M} \right)_*, \tag{2.24}$$

where $\sigma_r$ is the Stefan-Boltzmann constant and $\bar{\tau}_{Pl} = \bar{\kappa}_{Pl} \Sigma_c$ and $\bar{\tau}_R = \bar{\kappa}_R \Sigma_c$ are the mean Planck and Rosseland optical depths, respectively. In the optically thin limit, this gives the dust temperature required to radiate the given luminosity per unit mass, $\sigma r T_d^4 = L/(4M_g \bar{\kappa}_{Pl})$, which correctly reproduces 30 K dust for the median source in the Faúndez et al. sample. In the optically thick limit, it approximates the radiation diffusion equation with $\sigma r T_d^4 = (3/8) \tau_R L/(4\pi R_c^2)$; the factor 3/8 derives from a slab model.

To be specific, we adopt the model of composite-aggregate grains with normal silicates from Semenov et al. (2003), for which $\bar{\kappa}_R \simeq 3.0(T_d/100\text{K})^{1.93}$ cm$^2$ g$^{-1}$ and $\bar{\kappa}_{Pl} \simeq 6.1(T/100\text{K})^{1.58}$ cm$^2$ g$^{-1}$ for $30 < T_d < 100$ K; for 100-700 K, $\bar{\kappa}_R$ and $\bar{\kappa}_{Pl}$ are reasonably constant. The temperatures in the optically thin and thick limits are therefore

$$T_{d,\text{thin}} = 47 \left( \frac{L}{M_g} \right)_3^{0.18} \text{K}$$

and

$$T_{d,\text{thick}} = 32 \left( \frac{L}{M_g} \right)_3^{0.48} \Sigma_{g,cgs} \text{K},$$

respectively, so long as $T_d < 100$ K, and the transition from thin to thick occurs for $\Sigma_{g,cgs} = 1.46(L/M_g)_3^{-0.31}$. (Here we employ shorthand: subscript ‘3’ on $L/M$ means units of $10^3L_\odot/M_\odot$, and ‘cgs’ means cgs units, e.g. g cm$^{-2}$.) A good approximation to the solution of equation (2.24) is

$$T_d \simeq (T_{d,\text{thin}}^{3.1} + T_{d,\text{thick}}^{3.1})^{1/3.1}. \tag{2.25}$$

We then estimate $\Gamma$ using an estimate of $\bar{\kappa}$. For the optically thick case $\bar{\kappa} = \bar{\kappa}_R(T_d)$.

For an optically thin clump, the appropriate $\bar{\kappa}$ is the opacity of dust to the glow of
other grains at temperature $T_d$: $\tilde{\kappa} = \tilde{\kappa}_{dd}(T_d)$, where

$$\tilde{\kappa}_{dd} = \frac{\int_0^\infty B_\nu(T_d) \kappa_\nu^2 d\nu}{\int_0^\infty B_\nu(T_d) \kappa_\nu d\nu}. $$

In the adopted dust model, $\tilde{\kappa}_{dd} \simeq 9.0(T_d/100 \, \text{K})^{0.92} \, \text{cm}^2 \, \text{g}^{-1}$ for $30 < T_d < 100 \, \text{K}$.

Combining the thin and thick limits and using equation (2.23), we estimate

$$\Gamma \simeq \Gamma_{\text{thin}} + \Gamma_{\text{thick}}$$

where

$$\Gamma_{\text{thin}} = 0.34 f_\alpha (f_\alpha^{-1} - 1)^{1.16} (L/M)_*^{1.16}$$

and

$$\Gamma_{\text{thick}} = 0.026 f_\alpha (f_\alpha^{-1} - 1)^{1.93} (L/M)_*^{1.93} \Sigma_{c,\text{cgs}}^{1.86}. $$

These expressions are only valid for $T_d < 100 \, \text{K}$. The maximum value of $\Gamma$ (corresponding to $T_d = 100 \, \text{K}$) is $0.23 (L/M_*)^3$, in the optically thick limit, and $0.69 (L/M_*)^3$ in the thin one. Note that $\Gamma$ depends on the stellar IMF, through $(L/M)_*^3$, and on the dust properties. For instance, Semenov et al.’s ‘homogeneous aggregates’ are several times more opaque, and can produce $\Gamma \simeq 1$ for reasonable stellar IMFs.

Because $\Gamma$ is the negative ratio of the indirect radiation force to gravity, for a parcel of dusty gas, we can account for the indirect force by replacing Newton’s constant $G$ with $G' = (1 - \Gamma)G$ in any expression which involves the weight of the gas. For clarity, we add a prime to any quantity (e.g., $\alpha_c'$, $t_{ft'}$) that we modify in this way.

### 2.7.2 Stellar winds

Murray et al. (2010) provide strong arguments that the pressure of shocked stellar winds can never build up enough to overwhelm the direct radiation pressure and photo-ionized gas pressure: leakage of hot gas, or radiative losses enhanced by thermal conduction, sap its strength. Observations of H II regions corroborate this point (Harper-Clark & Murray 2009; Lopez et al. 2011; Pellegrini et al. 2011; Yeh &
Matzner 2012), although these generally address exposed regions on scales larger than our clumps.

We will therefore adopt the conservative estimate that stellar winds transmit no more force to the clump than is imparted to them at the stellar surfaces,

\[ F_w = \phi_w \left( \frac{L}{c} \right) \]  

(2.29)

where \( \phi_w \simeq 0.5 \) (Krumholz & Matzner 2009). By this estimate, wind force is smaller than the force due to mid-infrared reprocessed starlight.

### 2.7.3 Photo-ionization

The pressure of photo-ionized gas (H II) is a potentially important driver of clump motions and of mass loss, which can occur either through dynamical ejection of neutral gas or the evaporation of ionized gas. In the following discussion we adopt an ionized gas temperature \( T_i = 10^4 T_{i,4} \) K, a case-B recombination coefficient \( \alpha_B = 2.63 \times 10^{-13} T_{i,4}^{-0.8} \) cm\(^3\) s\(^{-1}\) (Storey & Hummer 1995), a ratio of stellar luminosity to ionizing photon output \( L_* / S_* = 47 \psi_{47} \) eV, and a dust opacity to starlight of \( 10^{-21} \sigma_{d,i,21} \) cm\(^2\) per H atom (Draine 2011).

With these choices, ionized gas within the clump can exist in a sequence of states corresponding to increasingly intense irradiation. The importance of radiation pressure, and the dust optical depth to starlight \( \tau_{d,i} \) (from the star to the ionization front) both increase along the sequence. We identify three regimes; see Draine (2011) and Figure 1 of Yeh & Matzner (2012). If the intensity of starlight is sufficiently low, one has a classical Strömgren sphere or photo-evaporative flow. In this state most ionizing photons are absorbed by H atoms; radiation pressure is negligible relative to ionized gas pressure; and \( \tau_{d,i} \) < 1. For intermediate intensities, radiation pressure is still small; however \( \tau_{d,i} \simeq 1 \) and dust grains absorb a significant fraction ionizing photons (Petrosian et al. 1972). For high intensities, H II is compressed by radiation
pressure into a thin layer and its dynamical influence is relatively weak; dust consumes up to 70% of the ionizing photons; and $\tau_{d,*i}$ reaches a maximum of $\tau_{d,\text{max}} = 2.0$. (This value depends weakly with the parameters; see Appendix B of Yeh & Matzner 2012 for details on this radiation-confined limit.)

We require an estimate for the additional force $F_i$ due to the photo-ionized gas (for dynamical mass ejection), as well as its volume-averaged density $n_i$ (for the evaporation rate). To calculate these we make reference to an idealized, uniform, dust-free H II region with same radius as the clump, for which the H density is $[3S/(4\pi \kappa_B R_c^3)]^{1/2}$. We find that the optical depth across the reference region is $\tau_{d,\text{St}} = \sigma_c / \sigma_\tau$, where

$$\sigma_\tau = \frac{0.30}{T_i^{0.4}} \left[ \frac{(\alpha_c/1.6) \psi_{47}}{(1 - f_g)(L/M)_\star \sigma_{d,*-21}} \right]^{1/2} \text{km s}^{-1}. \quad (2.30)$$

This indicates that clumps of interest ($\sigma_c > 1 \text{ km/s}$) will have optically thick H II, except when their stars are relatively dim. A simple approximation to the actual dust optical depth of H II regions in the Draine (2011) models, valid in both thick and thin limits, is

$$\tau_{d,*} \approx \tau_{d,\text{max}} \tau_{d,\text{St}} = \frac{\sigma_c}{\sigma_\tau \tau_{d,\text{max}} + \sigma_c}. \quad (2.31)$$

For a filled H II region with radius $R_c$, the volume-averaged ionized gas density $\bar{n}_i$ is related to the optical depth $\tau_{d,*i}$ by $\tau_{d,*i} = \bar{n}_i R_c \sigma_{d,*}$. If the H II is compressed into a thin shell, $\tau_{d,*i} = \bar{n}_i R_c \sigma_{d,*}/3$. An approximation for $\bar{n}_i$, which is valid in both thick and thin limits, is therefore

$$\sigma_{d,*} R_c \bar{n}_i \approx \frac{3}{3} \tau_{d,\text{max}} \sigma_c \frac{2}{3} \tau_{d,\text{max}} \sigma_\tau + \sigma_c. \quad (2.32)$$

We can now calculate the net outward force due to the existence of the H II gas, $F_{\text{II}} = 4\pi R_c^2 \times 2.2 \bar{n}_i k_B T_i$. Computing this, and comparing to the force of starlight,

$$\frac{F_{\text{II}}}{L/c} \approx \frac{\sigma_i^2}{\sigma_c (\sigma_c + 3 \tau_{d,\text{max}} \sigma_\tau)} \quad (2.33)$$
where
\[
\sigma_F = 2.8 \left[ \frac{(a_c/1.6)(\tau_{d,max}/2)T_{i,4}}{(1-f_g)(L/M)_*3\sigma_{d,21}} \right]^{1/2} \text{km s}^{-1}. \tag{2.34}
\]

Equation (2.33) agrees with the approximation employed by Krumholz & Matzner (2009) in the limit $\sigma_c \ll 3\tau_{d,max}\sigma_T$, but is more accurate at higher $\sigma_c$ because it takes dust absorption into account. As a result $F_{II}$ matches $L/c$ for a value of $\sigma_c$ which is lower by about a factor of 2.2 in this theory relative to Krumholz & Matzner’s. But, since radiation pressure dominates in most of the regime where dust absorption is significant, the improvement is important only for a limited range of clump velocity dispersions (about 1 to 4 km s$^{-1}$).

A caveat: in equations (2.30)–(2.34) we have assumed the H II is ionization-bounded with an ionization front radius equal to $R_c$ – the state which divides confinement and blowout. This suffices to estimate the photo-evaporation rate and to identify the criterion for blowout (and perhaps to discriminate compact and extended H II regions), but not for any more detailed predictions about the condition of the H II gas.

One might be puzzled that the effects of H II gas depend directly on the clump’s velocity dispersion $\sigma_c$, so we pause to explain. The first point to note is that the H II dust optical depth, and the ratio of its pressure to the radiation pressure, are both functions of a single parameter (which varies along the sequence of states described above). Second, Krumholz & Matzner (2009) showed that radiation pressure matches H II gas pressure at a characteristic radius proportional to $L^2/S$. For known dimensionless ratios $(L/M)_*$, $(L/S)$, and $f_g$, this corresponds to a radius proportional to $M_c$, and when $a_c$ is known this means a particular value of $\sigma_c$. Therefore, the parameter which controls radiation pressure and dust optical depth in H II regions maps onto $\sigma_c$, when the comparison is made at the clump radius $R_c$.

Note that any evaporative outflow can be affected by gravity if $v_{esc,c}$ becomes large compared to the ionized sound speed (10 km s$^{-1}$). However radiation pressure is strong enough, in this regime, that gravity cannot confine the flow.
Figure 2.7: Effect of finite sampling on the zero-age stellar luminosity-to-mass ratio, which enters the critical column density $\Sigma_{\text{crit}}$ for radiation pressure feedback (eq. 2.22). Here stars are drawn randomly from either a Muench et al. (2002) initial mass function, extended with the Salpeter slope to 120 $M_\odot$, or a Kroupa (2001) IMF; clusters are sorted by mass, and $(L/M)_*$ is recorded using the zero-age main sequence fits of Tout et al. (1996) at solar metallicity.
Figure 2.8: Probability that a supernova has exploded at time $t$, if star formation begins at $t = 0$ and proceeds as $N_* \propto t^{\eta M_*}$, provided stars are drawn randomly from a Kroupa (2001) IMF. An accelerating star formation law (green lines) contains a younger stellar population at fixed $t$, and hence implies later SNe, than a constant rate of star formation (blue lines). The probability of a SN is maximized by assuming that the most massive star forms at $t = 0$ (red dashed lines). Each set of lines has identical contour levels. We use rotating stellar lifetimes from Chieffi & Limongi (2013).
2.7.4 **Energy injection and mass ejection by massive stars**

Combining the direct radiation, stellar winds, and H II gas pressure forces into a net outward force $F_{\text{out}}$, and comparing against the force of gravity (modified by the indirect radiation force), we estimate

$$
\frac{F_{\text{out}}}{F'_{\text{grav}}} \simeq \frac{\Sigma_{\text{crit}}}{(1 - \Gamma) \Sigma_c} \left[ 1 + \phi_{\text{IR}} + \phi_w + \frac{\sigma^2_F}{\sigma_c (\sigma_c + 3 \tau_{d,\text{max}} \sigma_c)} \right]
$$

\[= \frac{\Sigma_{\text{crit,tot}}}{\Sigma_c}. \tag{2.35} \]

We use this combined force to estimate both the creation of turbulent kinetic energy, and the rate of mass ejection, due to massive-star feedback. For the former, we assume that massive-star feedback accelerates feedback at a rate $\varphi_{\text{HM}} F_{\text{out}} / M_g$, where $\varphi_{\text{HM}}$ is a coupling factor. The additional acceleration reduces the external acceleration $a_{\text{ext}}$ required to maintain a given $\sigma_c$, thereby suppressing $\xi_c$ according to equation (2.17).

We expect that $\varphi_{\text{HM}}$ depends on the source of $F_{\text{out}}$: for instance, pho-ionization drives turbulence relatively efficiently by the rocket effect, whereas indirect radiation force is very smooth. In our calculations we employ $\varphi_{\text{HM}} = 0.1$, but we find that the results are not sensitive to this parameter because of the intense mass loss that occurs when $F_{\text{out}}$ becomes strong.

Prior to any supernovae, there are two new types of mass loss due to massive stars: dynamical mass ejection and photo-evaporation. To account for dynamical mass ejection, we posit (e.g., Fall et al. 2010) that matter boils away even when $F_{\text{out}} < F'_{\text{grav}}$, because of inhomogeneities in the distribution of matter within the clump. In particular, we assume that all the clump matter with a radial column less than $\Sigma_{\text{crit,tot}}$ is blown away each free-fall time, i.e.

$$
\frac{\dot{M}_{\text{out,dyn}}}{M_c} = f_g F_M \left( \frac{\Sigma_{\text{crit,tot}}}{\Sigma_c} \right)
$$

where $F_{(A,M)}(x)$ is the (area, mass) fraction of the clump with a column below $x \Sigma_c$. 
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Note that we have assumed that if the forces balance for a gas shell at $R_c$, it will be blown away. The forces of starlight and H II pressure are continuous and scale similarly to the force of gravity, so we do not believe a dynamical correction would be warranted.

In addition to dynamical ejection, photo-ionized H II gas will escape along open lines of sight, which we estimate to cover a fraction $F_A(\Sigma_{\text{crit,tot}}/\bar{\Sigma}_c)$ of the clump’s surface area. Multiplying this surface area times the mean ionized gas density $1.4 m_p n_i$, and assuming an outflow speed equal to the ionized sound speed, we derive the mass ejection rate $\dot{M}_{\text{evap}}$. Comparing this to the clump mass per free-fall time, we find

$$\frac{\dot{M}_{\text{evap,ff,c}}}{M_c} \sim 0.41 \left(\frac{\alpha_c/1.6}{\sigma_{d,\pm 21}}\right)^{1/2} \frac{T_{i,4}^{1/2} 1\text{km/s}}{\Sigma_{c,\text{cgs}} \sigma_c}$$

$$\times F_A \left(\frac{\Sigma_{\text{crit,tot}}}{\bar{\Sigma}_c}\right) \frac{1 + 3 \tau_{d,\text{max}} \sigma_\tau / \sigma_c}{1 + 3 \tau_{d,\text{max}} \sigma_\tau / \sigma_c}.$$ (2.37)

For a clump with decreasing $\bar{\Sigma}_c$, photo-evaporation (eq. 2.37) sets in earlier than dynamical ejection (eq. 2.36), because blowout affects a larger fraction of the area than of the mass (i.e., $F_A(x) > F_M(x)$). However, the characteristic rate of photo-evaporation is several times lower than that of dynamical ejection.

For calculations, we employ a log-normal distribution of column densities, so that

$$F_{(A,M)}(x) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{\ln x \pm \sigma_{\ln x}^2 / 2}{\sqrt{2} \sigma_{\ln x}} \right) \right]$$

(2.38)

where $\sigma_{\ln x}$ is the standard deviation of $\ln x$. For the column distributions of nearby molecular clouds (Lombardi et al. 2008; Froebrich & Rowles 2010) $\sigma_{\ln x}$ is about 0.4, so we adopt this value.

### 2.7.5 Combined models: accretion, outflows, and massive stars

To account for all three effects, we adopt an iterative approach. We start with an Eddington factor $\Gamma = 1$. Beginning with the model for accretion and outflows worked

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6Todd Thompson and Mark Krumholz introduced this mass-loss prescription in the context of mass ejection from starburst galaxies (2014, private communication).
out in §2.6, we estimate the clump luminosity $L$ assuming that $(L/M)_*$ takes its median value for the stellar mass $M_*$ (figure 2.7). We work out $\Sigma_{\text{crit,tot}}/\Sigma_c$ from equation (2.35). We add the mass ejection from massive stars, from equations (2.36) and (2.37), to that from protostellar outflows, and use this in equation (2.21) to determine the self-consistent value of $f_g$. Likewise we add the turbulent kinetic energy generated by massive-star feedback, and use this to re-calculate $\xi_c$. From the values of $f_g$ and $\Sigma_c$ we derive the dust temperature $T_d$ from equation (2.24) and the Eddington factor $\Gamma$ from equation (2.26). To account for indirect force, we replace $G$ with $(1 - \Gamma)G$ in any instance where gravity acts on gas, and start over. Because $\Gamma$ is not large, our solution converges rapidly. Throughout this process we hold $\alpha_c'$ fixed at its fiducial value of 1.6, and adopt the Kroupa (2001) IMF.

The outcome of this exercise is plotted in figure 2.9, in which we outline where in the phase place of growing clusters each effect is dominant, and in figure 2.10, in which we display the gas fraction $f_g$ computed in the manner outlined above. Our models display a sharp transition from gas-rich to gas-poor as massive stellar feedback becomes important, reflecting the fact that gas disruption accelerates as the gas fraction decreases. (Because of this rapid transition, we evaluate $q_*$ in equation 2.21 rather than assuming self-similar growth with $M_* \propto M_c$ and $q_* = 1$; however this modification does not strongly affect the outcome.)

### 2.8 Comparison to observed regions

We compare against two types of regions: individual, relatively small regions NGC 1333 and Serpens South, both of which have been studied in detail, and massive regions in the surveys described by Shirley et al. (2003) and Faúndez et al. (2004).
Figure 2.9: Phase space of feedback for accreting clumps and embedded clusters. Median IMF-sampled values are assumed for the properties of the star cluster. The ‘outflows’ region corresponds to $>50\%$ turbulent energy from outflows; ‘H II’ corresponds to $F_{\text{out}} > F_{\text{grav}}$ and $F_{\text{II}} < F_{\text{out}}/2$; ‘Rad. Pressure’ corresponds to $F_{\text{out}} > F_{\text{grav}}$ and $F_{\text{II}} > F_{\text{out}}/2$; and ‘Supernovae’ corresponds to a greater than $50\%$ chance of a supernova explosion (with $\eta_{M_*} = 2$ and random sampling; see Figure 2.8). Massive star forming regions from Faúndez et al. (2004, points) and Shirley et al. (2003, stars) are plotted for comparison.
Figure 2.10: Gas mass fraction $f_g$ (color and blue contours) in a model for accreting clumps affected by outflows from low-mass stars as well as ionization, stellar winds, and radiation pressure due to massive stars. Note that stellar evolution off the main sequence, important in the lower-right region of this plot, is not taken into account.
2.8.1 NGC 1333

The NGC 1333 star forming region within the Perseus molecular cloud provides a point of comparison for theories of outflow feedback. NGC 1333 hosts 137 known young stellar objects, of which about 39 are of Class 0 or Class I (Gutermuth et al. 2008a). The population of near-infrared excess sources indicates a typical YSO age 1-2 Myr (Lada et al. 1996). Hirota et al. (2008) identify a distance $D = 235 \pm 18$ pc based on maser astrometry, so we adopt this value. We infer from Gutermuth et al. (2008a) that the area of the embedded cluster corresponds to a radius in the vicinity of 0.4 pc, and identify this with the clump radius $R_c$. This is intermediate between the radii of $^{13}$CO and C$^{18}$O(1-0) emission observed by Ridge et al. (2003) (0.46 pc and 0.27 pc, respectively, at $D = 235$ pc) so we adopt an intermediate column density $\Sigma_c = 0.19 \text{ g cm}^{-2}$ and velocity dispersion $\sigma_c = 1.1 \text{ km s}^{-1}$, with uncertainties of about 20%. (The Ridge et al. 2003 analysis corresponds to $\Sigma = (0.2, 0.17) \text{ g cm}^{-2}$ and $\sigma = (1.2, 0.85) \text{ km s}^{-1}$ for the scales probed by the two lines.) For the adopted values, $M_c = 460 M_\odot$ and $\alpha_c = 1.1$. If we assign an average mass of 0.5 $M_\odot$ per YSO (Evans et al. 2009), $f_g \simeq 0.87$. (We note that Arce et al. (2010) identify a larger, more diffuse region with NGC 1333 [$R = 2$ pc, $\sigma = 0.93 \text{ km s}^{-1}$, $\Sigma = 0.02 \text{ g cm}^{-2}$], but since this region has a crossing time of 2 Myr, we associate it instead with an infall or feeding zone.)

Combining FCRAO and CARMA observations of $^{12}$CO and $^{13}$CO(1-0) emission, Plunkett et al. (2013) provide the most sensitive analysis to date of protostellar outflows within NGC 1333, at least within the $(0.48 \text{ pc})^2$ sub-region mapped. Plunkett et al. identify 22 outflow lobes with typical dynamical ages $\sim 5 \times 10^4$ yr and a current, inclination-corrected, total outflow momentum of roughly $35 M_\odot \text{ km s}^{-1}$. Dividing by the dynamical age leads to a net outflow force of roughly $700 M_\odot \text{ km s}^{-1} \text{ Myr}^{-1}$. If this force coupled only to the estimated $\sim 143 M_\odot$ of clump material within region mapped by Plunkett et al., the resulting acceleration of $5 \text{ km s}^{-1} \text{ Myr}^{-1}$ would exceed
the turbulent acceleration $\sigma c^2 / R_c \simeq 3.1 \text{ km s}^{-1} \text{ Myr}^{-1}$. This comparison is not entirely satisfactory, however, because it involves only a portion of the clump.

Arce et al. (2010), who survey the entire Perseus cloud, provide a more complete view. In the NGC 1333 region, Arce et al. (2010) infer at least $74 M_\odot \text{ km s}^{-1}$ in known outflows and new outflow candidates (and reckon that this may underestimate the total momentum by as much as a factor of seven). With a typical dynamical age of $5 \times 10^4 \text{ yr}$, this corresponds to a net outflow force of at least $1480 M_\odot \text{ km s}^{-1} \text{ Myr}^{-1}$ and an outflow-driven clump acceleration scale of at least $3.2 \text{ km s}^{-1} \text{ Myr}^{-1}$. Outflows are clearly relevant and probably sufficient to drive the observed turbulence. Arce et al. and Plunkett et al. come to the same conclusion by comparing outflow, turbulent, and gravitational energies.

The state of NGC 1333 appears to be consistent with the theory of M07 as presented in §2.6. Adjusting our parameters to match the observations ($\alpha_c = 1.1$, SFR$_{ff} = 0.03$, $v_{ch,w} = 21 \text{ km s}^{-1}$), we would predict that 80% of the turbulent energy derives from outflow driving, with accretion providing the remainder. This accretion could be along the dense molecular filament apparent in Sadavoy et al. 2012’s Herschel map of the region, and another, less prominent filamentary structure extending to the east. A rough estimate of the column density of this structure gives about $1.5 \times 10^{22}$ H atoms cm$^{-2}$ across an effective width of 0.2 pc in each filament, for a net mass per unit length $3\lambda_{fil} \simeq 34 M_\odot \text{ pc}^{-1}$. Equation (2.4) then suggests an infall rate of $\sim 77 M_\odot \text{ Myr}^{-1}$.

M07, in contrast, found the current outflows insufficient to drive turbulence in NGC 1333. However M07’s analysis was based on a much lower outflow momentum ($10 M_\odot \text{ km/s}$, from Knee & Sandell 2000 and Quillen et al. 2005). An upward revision of outflow momentum by is perhaps not surprising, considering the corrections for optical depth, finite sensitivity, velocity range, and excitation temperature in observations of protostellar outflows highlighted Dunham et al. (2014), which amount to nearly an order of magnitude typical increase of momentum and force relative to
However, the statistical analyses of the NGC 1333 region by Padoan et al. (2009) and Brunt et al. (2009) give reasons for caution. Brunt et al. use a principal component decomposition of CO maps of the region, finding no hint of local turbulent driving in the $^{13}$CO map and only tentative evidence for turbulent driving on 0.4-0.8pc scales in the C$^{18}$O map (which highlights denser matter on scales of the actual clump). It is unclear whether this result is at odds with our conclusions in § 2.8.1, considering that we invoke a combination of local and large scale (i.e. accretion) driving, and considering that Carroll et al. (2010) identify biases in the principal component method when applied to outflow-driven turbulence.

Padoan et al.’s analysis of the $^{13}$CO map is more starkly at odds with our conclusion of strong outflow feedback, as it employs velocity information (via the velocity coordinate spectrum method of Lazarian & Pogosyan 2006) and shows no indication of any deviation from simulations of turbulence driven isotropically on large scales. This is puzzling, given that the NGC 1333 clump represents a strongly self-gravitating region of the Perseus cloud, which is distinct from the turbulent background in both column density (e.g. Kainulainen et al. 2009) and in the line width-size relation (e.g. Caselli & Myers 1995). Arce et al. (2010) point out that identifying a driving scale is likely to be more difficult by when driving occurs over a range of scales, and indeed this is a critical feature of outflow driving in the M07 theory. Similarly, Carroll et al. (2010) have stressed that driving by collimated outflows leaves a different imprint on turbulent motions than isotropic driving. Clearly the Padoan et al. result merits further investigation.

### 2.8.2 Serpens South

The Serpens South cluster-forming region was discovered by Gutermuth et al. (2008b), who report a total of 91 protostars within the cluster boundary, and a high fraction of
Class I sources indicative of a very young age (0.1-0.3 Myr). The distance is thought to match that of the Serpens main cluster, for which a photometric estimate of $260\pm37$ pc (Straizys et al. 1996) conflicts with a more recent VLBI parallax of $429\pm2$ pc (Dzib et al. 2010). The elongated dust emission implies a gas mass $(420$ to $560)(D/429\text{ pc})^2 M_\odot$ in three concentrations with scale $\sim 0.28\text{ pc}(D/429\text{ pc})$; the velocity dispersion is $0.3$-$0.5$ km s$^{-1}$ from N$_2$H$^+$ (Tanaka et al. 2013) or $1.0$ – $1.3$ km s$^{-1}$ in HCO$^+$ (Nakamura et al. 2011a). With these numbers, $\alpha_c \sim 0.8(429\text{ pc}/D)$.

With its filamentary structure, very young age, and less-than-unity virial parameter (Tanaka et al. 2013), the Serpens South region resembles the initial conditions for cluster formation (§ 2.4) more than our model for a cluster-forming clump. Nevertheless, a burst of outflows accompanies the burst of star formation, so we briefly revisit the question of outflow feedback previously addressed by Nakamura et al. (2011a) and Plunkett et al. (2015). These authors observe different transitions ($^{12}\text{CO}(3-2)$ and $^{12}\text{CO}(1-0)$, respectively) but agree on a total outflow momentum of $(21$ to $25)(D/429\text{ pc})^2 M_\odot \text{ km s}^{-1}$ and outflow dynamical age $\sim 2.4(D/429\text{ pc}) \times 10^4$ yr. This suffices to accelerate the clump matter at a rate $\sim 2$ km s$^{-1}$ Myr$^{-1}$, comparable to the turbulent acceleration $\sim 3$ km s$^{-1}$ Myr$^{-1}$ (both $\propto (D/429\text{ pc})^{-1}$). In addition, there is accretion along and perpendicular to the filament at rates $\sim (50, 210)(D/429\text{ pc}) M_\odot/\text{ yr}$, respectively (Kirk et al. 2013), and this alone suffices to drive $\sigma_c \sim 0.8$ km s$^{-1}$ for $\xi_c \simeq 1$. We therefore concur with previous analyses that the Serpens South proto-cluster is in its initial stages of growth, but is already affected by outflow feedback.

### 2.8.3 Massive star forming regions

Comparing the parameters of the massive cluster forming regions from Faúndez et al. (2004) and Shirley et al. (2003) with the theory outlined here, as in Figure 2.9, we see that most of these regions have parameters for which protostellar outflows are expected to be an important contribution to the turbulent velocity. Very few regions
are found in conditions for which we would expect rapid gas dispersal by H II or by radiation pressure, while some (in the range $1 \text{ km s}^{-1} < \sigma_c < 3 \text{ km s}^{-1}$) are near the boundary of this regime.

However, a striking feature of the selected population is the correlation between $\sigma_c$ and $\bar{\Sigma}_c$, which roughly corresponds to $R_c \propto M_c^{1/4}$ and $\bar{\Sigma}_c \propto \sigma_c^{4/3}$. Regions with high mass and high $\sigma_c$ are found at high column densities, well above the critical column at which radiation pressure becomes important. We interpret this to mean that stellar feedback is not significant in these regions, and that their properties are a consequence of the initial conditions for cluster formation rather than the action of their stars.

### 2.9 Comparison to previous models

Matzner (1999), Matzner & McKee (1999b), Huff & Stahler (2006; 2007), Fall et al. (2010), Zamora-Avilés et al. (2012), and Zamora-Avilés & Vázquez-Semadeni (2014) all present analytical star cluster formation models that overlap ours in some respects, so a comparison is warranted.

Matzner, considering only protostellar outflow feedback, evaluates mass ejection using the MM00 models and accounts for the generation of turbulence by outflow impulses. His models neglected clump accretion and massive-star feedback, but include a treatment of the clump’s energy equation (which we replace with the cruder assumption of a constant virial parameter $a_c$). They display the feedback-driven oscillations discussed in § 2.6.2.

Huff & Stahler solve the clump energy equation within an extended clump, assumed to be instantaneously isothermal but with an evolving effective sound speed. As they neglect feedback and clump accretion, their clumps contract over a couple free-fall times toward an unstable state of high central concentration, while forming stars at an accelerating rate.
Fall et al. consider the same set of feedback phenomena we consider here, but concentrate on the conditions under which a high stellar mass fraction leads to rapid disruption of the remaining gas. Although their analysis is very similar to ours (except for details like dust opacity of H II, for instance), Fall et al. did not consider the evolution of growing star clusters. Fall et al. do not address how a clump might produce massive stars (requiring high column densities) and only later blow away gas (requiring low $\Sigma_c$). We have found that accretion provides a natural explanation, at least for the mass range in which stellar feedback appears to be important.

Zamora-Avilés & Vázquez-Semadeni consider star cluster formation as the end phase of a freely-falling cloud in which star formation proceeds as local regions cross a critical density threshold. Zamora-Avilés & Vázquez-Semadeni account for photo-evaporative mass loss on a star-by-star basis, but neglect the dynamical recoil due to photo-evaporation (Matzner 2002; Krumholz et al. 2006; Goldbaum et al. 2011) as well as the other agents of feedback. Similarly, Zamora-Avilés & Vázquez-Semadeni account for mass gain in the form of accretion from the galactic disk, but neglect the driving of turbulence within the collapsing cloud due to the arrival of fresh material. They find a runaway acceleration of star formation in low-mass clouds ($\lesssim 10^{4.5} M_\odot$), which is prevented by mass loss in more massive clouds.

A distinctive feature of the current work is that we separate the virialized cluster-forming clump from its collapsing reservoir region, and account (in approximate ways) for the driving of turbulence both by accretion and by several forms of stellar feedback. Clumps in our model grow in a roughly self-similar fashion, evolving to larger radii and lower column densities, and becoming increasingly susceptible to massive-star feedback, as they gain mass. Importantly, our model also predicts an accelerating star formation rate so long as the reservoir supplies an accelerating

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[7] We note that Zamora-Avilés & Vázquez-Semadeni (2014) employ the photo-evaporation rate of Franco et al. (1994), which incorrectly associates swept-up mass with ionized mass; see the discussion above Franco et al.’s equation (8) and compare their equation (11) with Matzner (2002)’s equation (19).
mass accretion rate, because the rates are nearly proportional so long as $\epsilon_{\text{in}}$ and SFR$_{\text{ff}}$ change slowly.

### 2.10 Conclusions

In the effort to understand the interaction between gas accretion and stellar feedback, we have developed an approximate model for the growth of a cluster-forming clump by accretion from its environment ($\S$ 2.4 and $\S$ 2.5) and the effects of protostellar outflows ($\S$ 2.6) and massive stars ($\S$ 2.7) on the clump’s evolution. In the presence of ongoing accretion, protostellar outflows are expected to be important for relatively low clump velocity dispersions ($\sigma_c \lesssim 3 \text{ km s}^{-1}$), and remove matter only gradually. In contrast, radiation pressure affects massive regions with low column densities ($\Sigma_c \lesssim 0.3 \text{ g cm}^{-2}$) and causes rapid gas dispersal. Photo-ionization and photo-evaporation enhance the disruptive effects of massive stars in a limited range of velocity dispersions ($1 \text{ km s}^{-1} \lesssim \sigma_c \lesssim 4 \text{ km s}^{-1}$). The well-studied region NGC 1333 appears to be consistent with this analysis, as its complement of protostellar outflows is sufficient to accelerate turbulent motions at nearly the rate required to sustain them (with accretion powering the remainder). Indeed many of the massive regions in the surveys by Faúndez et al. (2004) and Shirley et al. (2003) are expected to be in this state. Many of these regions are close to the condition\(^8\) for disruption by photo-ionization, but few exist where we would expect gas to already have been cleared.

With these results in mind we return to the question of how star cluster formation terminates, because the efficiency and rapidity of this event is crucial to the production of a bound system, and because the birth radius of such a system is crucial to its long-term survival (Parmentier & Kroupa 2011). The combination of accretion and stellar feedback

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\(^8\)We stress that this condition is a stochastic one, because the cluster’s ionization rate depends on its most massive stars, and because the IMF is not completely sampled in regions for which photo-ionization is important (Figure 2.7). Our evaluation in Figures 2.9 and 2.10 used the median $L$ and $S$ at each $M_*$. 

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feedback provides one hypothetical scenario, in which the clump column density \( \Sigma_c \) declines over time while the complement of massive stars grows, until the conditions for disruption by massive-star feedback are met. Accretion is an essential element, because the incorporation of fresh material causes \( \Sigma_c \) to decline (unless the accretion rate accelerates sharply; see § 2.4). However star cluster birth could end otherwise, if the mass reservoir is exhausted before feedback plays any role.

The feedback-termination scenario is likely to be characterized by rapid gas loss. Given that the gas mass fraction remains high prior to gas expulsion (Figure 2.10), rapid gas loss is likely to unbind the stellar population, or at least cause it to expand significantly – as Mathieu (1983) inferred from the radii of embedded and open star clusters. Mass exhaustion, by contrast, would efficiently form bound clusters, unless the initial conditions are very different from our expectation (§ 2.4) of quiescent infall.

Another difference involves the conditions in which most cluster-forming environments are found. If stellar feedback ends star cluster formation, then regions should be found close to the threshold for gas disruption (unless this is precluded by the selection of regions). In the surveys of Faúndez et al. (2004) and Shirley et al. (2003), plotted in Figure 2.9, low-dispersion regions (\( \sigma_c \lesssim 3 \text{ km s}^{-1} \), which represents most of their data) appear to be consistent with feedback termination. However these surveys extend for higher \( \sigma_c \), to column densities well above our estimate of the disruption threshold. Unless this is due to a selection effect, we conclude that reservoir exhaustion, rather than stellar feedback, ends the formation of the most massive clusters in these surveys.

We have simplified our analysis by making several key assumptions, all of which require further scrutiny. We assume that one can divide the cluster-forming ‘clump’ from its accretion flow, and that the clump will evolve through a series of equilibrium states so long as accretion persists. The possibility of energetic instability (§ 2.5.2 and § 2.6.2) gives reasons for caution. Moreover, we have assumed that the evolution
can be approximated with a fixed virial parameter $\alpha_c$, rather than solving the energy equation directly.

Our analysis relies on previous work, especially our own papers on stellar feedback (e.g., MM00, M07, Krumholz & Matzner 2009, Fall et al. 2010, and Goldbaum et al. 2011,) so we end by noting which elements are new. One new element, which is also our motivation, is that we incorporate simultaneous infall and feedback within analytical models for star cluster formation. Another is the introduction of the dimensionless parameter $\xi_c$ (equation 2.8), which helps to make this combination analytically tractable. A third is the observation that the mass ejection by outflows can be expressed analytically (equation 2.19). A fourth is our treatment of radiation pressure feedback, which employs an approximate Eddington factor for the indirect force and provides an estimate of the dust temperature. A fifth is our model for photo-ionization feedback (§ 2.7.3), in which we account for the effects of dust opacity in the ionized gas, and for both photo-evaporation and dynamical disruption (albeit in an approximate way).
Chapter 3

Radiation Forces on Dust Envelopes

3.1 Preface

This chapter details a paper published in Monthly Notices of the Royal Astronomical Society (MNRAS) (Jumper & Matzner 2018a)\(^1\). It has been edited from its original form, including to conform to the formatting of this dissertation. I am the lead and first author of this paper. The second author of this paper is Christopher Matzner.

I conducted all DUSTY (Ivezic et al. 1999) and Hyperion (Robitaille 2011) radiative transfer models detailed in this chapter, including setting up, running, and post-processing the models to produce parameters for my analysis. Matzner utilized the values I calculated to derive the linear fit in the last paragraph of §3.5.1, while I conducted all other analysis of the DUSTY and Hyperion output data. Additionally, I wrote all the modifications of the radiative transfer codes as described.

I created all figures and tables in sections prior to the appendices, while Matzner wrote the appendices and created their sole figure.

I conducted extensive analytical estimates for developing the analytical predictions for the parameters; Matzner also derived his own predictions separately, and after our separate derivations we discussed the predictions at length to reconcile our results and

\(^1\)https://academic.oup.com/mnras/article/480/1/905/5050064
decide which set to adopt. Matzner’s predictions were largely adopted for regions of higher optical depths, with some corrections based on my input. I also proposed the inclusion of the $\Phi_{\text{thin}}$ term, which proved to be necessary for improving the approximation in the intermediate regime.

3.2 Chapter abstract

We address in detail the radiation forces on spherical dust envelopes around luminous stars, and numerical solutions for these forces, as a first step toward more general dust geometries. Two physical quantities, a normalized force and a force-averaged radius, suffice to capture the overall effects of radiation forces; these combine to yield the radiation term in the virial theorem. In addition to the optically thin and thick regimes, the wavelength dependence of dust opacity allows for an intermediate case in which starlight is easily trapped but infrared radiation readily escapes. Scattering adds a non-negligible force in this intermediate regime. We address all three regimes analytically and provide approximate formulae for the force parameters, for arbitrary optical depth and inner dust temperature. Turning to numerical codes, we examine the convergence properties of the Monte Carlo code Hyperion run in Cartesian geometry. We calibrate both Hyperion and our analytical estimate using the DUSTY code, run in spherical geometry. We find that Monte Carlo codes tend to underestimate the radiation force when the mean free path of starlight is not well resolved, as this causes the inner dust temperature, and therefore the inner Rosseland opacity, to be too low. We briefly discuss implications for more complicated radiation transfer problems.

3.3 Introduction

Dusty gas is much more opaque to visible light than ionized gas lacking dust. As a result, radiation pressure forces become dominant in situations where luminous
but sub-Eddington objects, like massive stars and AGN, are surrounded by dusty gas. Examples include individual massive star formation, where radiation forces present an obstacle to stellar accretion (Wolfire & Cassinelli 1987); massive star cluster formation (Fall et al. 2010; Matzner & Jumper 2015), in which matter may be expelled from the cluster-forming zone; the disruption of giant molecular clouds (Krumholz & Matzner 2009; Murray et al. 2010; Hopkins et al. 2012); the initial inflation of giant H II regions (Draine 2011; Lopez et al. 2011; Yeh & Matzner 2012); and the ejection of gas from galaxies (Murray et al. 2011); as well as dust-driven winds and superwinds from AGB stars (Bowen & Willson 1991).

For each of these problems a detailed understanding of radiation forces on dust, and a calibration of numerical methods to estimate these forces, are clearly required. As a step toward a more complete understanding, we focus here on a dramatically simplified case: a spherically symmetric, power-law dust profile surrounding a central light source. We choose this problem several goals in mind. First, we wish to establish a set of measures (output parameters) with which the effects of radiation forces can be summarized. Second, we aim to understand in analytical terms how these measures depend on the innermost dust temperature and optical depth of the envelope (input parameters).

Lastly, we wish to calibrate Monte Carlo (MC) calculations of finite resolution: as we shall show, MC results suffer a type of systematic error when the photon mean free path is not well resolved. Understanding errors in the MC technique will be useful for our future work, in which we use moderate-resolution MC simulations to examine radiation forces in inhomogeneous dust distributions that break spherical symmetry, and then characterize errors in approximate techniques. More generally, resolution-dependent errors are dynamically important within simulations; for instance, in the

\footnote{Dust radiation transfer is more important in luminous H II regions than one might expect, because grain absorption exceeds ionization absorption precisely when radiation forces are strong; see Yeh & Matzner (2012).}
onset of Rayleigh-Taylor instabilities during massive star formation (see Kuiper et al. 2011 and Rosen et al. 2016).

Our spherical problem can be approached by the multi-group radiation transfer code DUSTY (Ivezic et al. 1999), whose adaptive spatial and frequency grids allow it to rapidly converge to a solution that we shall consider to be ground truth. But, DUSTY cannot treat complicated, three dimensional dust distributions. For these we employ the MC code Hyperion (Robitaille 2011), modified to record radiation forces. In our numerical sections we focus on the convergence of radiation force measures, calculated within Hyperion, toward a value determined by DUSTY. This is not meant to be a code comparison (for which it would be more appropriate to run Hyperion in spherical symmetry).

On the analytical front, radiative transfer through spherical dust envelopes, and the corresponding dust temperature profiles, have long been studied in relation to the emergent spectral energy distributions of protostars (Larson 1969; Adams & Shu 1985), star cluster-forming clumps in starburst galaxies (Chakrabarti & McKee 2005), and dusty winds from late-type stars (Ivezic & Elitzur 1995). However, analytical studies (including Krumholz & Matzner 2009, Murray et al. 2010, and our own work: Matzner & Jumper 2015) have so far considered only crude approximations to the radiation force. We will develop more accurate analytical formulae, albeit restricted (for now) to the simple spherical geometry.

We delineate the problem to be solved below in §3.3.1. In §3.3.2 we introduce two integral quantities of interest, the net radiation force and the force-averaged radius, which can be combined to form a radiation force term in the virial equation. We address the theoretical problem in §3.4 and compare numerical solutions in §3.5.
3.3.1 Physical problem

We consider, for simplicity, a spherically symmetric dust envelope with inner radius $r_{in}$, outer radius $r_{out}$ (set to $4r_{in}$ in our fiducial case, purely for convenience) and radial profile $\rho(r) \propto r^{-k}$ for $r_{in} < r < r_{out}$, for some $k > 1$; in our fiducial case $k = 1.5$. This is representative of the density profile in low and high-mass star formation: van der Tak et al. (2000) finds $k = 1.0$ to $k = 1.5$, Jørgensen et al. (2002) finds $k = 1.3$ to $k = 1.9$, $\pm 0.2$, Shirley et al. (2002) finds $k = 1.8 \pm 0.1$, and Mueller et al. (2002) finds $k = 0.75$ to $k = 2.5$ with a mean value of $\langle k \rangle = 1.8 \pm 0.4$.

We take the spectrum of the central source, $L_{*,\nu}$, to correspond to a blackbody of color temperature $T_* = 5772$ K. We adopt the dust absorption opacity $\kappa_{a,\nu}$ and albedo $a_\nu$ for a dust mixture with $R_V = 5.5$ provided by Draine (2003a;b) from which we compute the total opacity $\kappa_\nu = \kappa_{a,\nu}(1 + a_\nu)$. However, to treat scattered radiation identically in numerical and analytical calculations, we consider only isotropic scattering. This neglects the characteristic mean scattering angle, $\langle \cos(\theta) \rangle$, included in the Draine (2003a;b) dust model, and thus does not consider the effects of realistic phase functions, which may have a preferential direction of scattering. While this is certain to alter our results to a small degree, it also permits a precise comparison between various approaches.

3.3.2 Quantities of Interest

We take the outward luminosity at radius $r$ to be $L(r) = \int_0^\infty L_\nu(r) d\nu$; photon momentum passes $r$ at the rate $L(r)/c$, which is independent of $r$ in static equilibrium (whereas $L_\nu$ can vary with radius). Given an extinction optical depth $\tau_\nu$ (arising from a density and specific opacity, $d\tau_\nu = \rho(r) \kappa_\nu dr$), the radiation force satisfies

$$dF_{rad,\nu} = \frac{L_\nu}{c} d\tau_\nu;$$

(3.1)
Chapter 3. Radiation Forces on Dust Envelopes

the frequency-integrated force within \( r \) is \( F_{\text{rad}}(r) = \int_0^\infty F_{\text{rad,} \nu} d\nu \), and the total outward force is \( F_{\text{rad, tot}} = F_{\text{rad}}(r = \infty) \).

Our first quantity of interest, then, compares the applied radiation force to the photon force:

\[
\Phi \equiv \frac{F_{\text{rad, tot}} L}{c} = \int_0^\infty \frac{L\nu}{L} d\tau_{\nu} d\nu.
\]

This can be less than unity, in the case of an optically thin dust envelope, or much greater than unity if the optical depth is very high. Indeed \( \Phi \) is a luminosity-weighted integral of \( \tau_{\nu} \), as the final expression in equation (3.2) shows.

It is also important to know where the force is applied. For this we introduce a second quantity, the force-averaged radius:

\[
\langle r \rangle_F \equiv \int_0^{F_{\text{rad, tot}}} r dF_{\text{rad}} / F_{\text{rad, tot}}.
\]

We are motivated here by the virial theorem, where the radiation force \( F_{\text{rad}} \) introduces the term \( R = \int r \cdot dF_{\text{rad}} \), equivalent to expression (4.24) of McKee & Zweibel (1992).

For our spherically symmetric problem,

\[
R = \Phi \frac{L}{c} \langle r \rangle_F.
\]

Our model density distributions are described by inner and outer radii \( r_{\text{in}} \) and \( r_{\text{out}} \).

Dimensionless quantities like \( \Phi \) and \( \langle r \rangle_F / r_{\text{in}} \) are functions of the spectral shape of the input radiation (or its color temperature, if it is taken to be a blackbody) and the optical depth \( \tau_{\text{fid}} \) at a chosen reference frequency \( \nu_{\text{fid}} \) (see Rowan-Robinson 1980 and Ivezic & Elitzur 1997).

For brevity we use \( \tau_{\nu}(r) \) to denote the dust extinction optical depth within \( r \), and \( \tau_{\nu} \) to denote the total optical depth through the dust distribution. Therefore \( \tau_{\nu} = \tau_{\nu}(\infty) \).

### 3.4 Analytical predictions

Dusty radiative transfer has two quintessential features: first, the dust opacity \( \kappa_{\nu} \) increases with frequency over the relevant range of \( \nu \); and second, the stellar surface
temperature is much hotter than dust grains can possibly be. Therefore, optical and ultraviolet starlight is always more readily absorbed than the infrared emission from heated grains. As a result, one can distinguish three regimes: I – optically thin to starlight \( \tau_\ast < 1 \) where \( \tau_\ast = \int_0^\infty \tau_\nu(L_\star,\nu/L) \, d\nu \) is the starlight-averaged opacity; II – thick to starlight but thin to dust emission \( \tau_\ast > 1 > \tau_R(T_{d,\text{in}}) \) where \( \tau_R(T_{d,\text{in}}) \) is the Rosseland opacity at the temperature of the innermost dust; and III – optically thick to dust radiation \( \tau_\ast > \tau_R(T_{d,\text{in}}) > 1 \).

I. Optically thin case: Ia. Starlight. Of these, the optically thin case is of course the simplest. The direct starlight luminosity at \( r \) is \( L_{\text{dir}}(r) = e^{\tau_\nu L_\star,\nu} \), and therefore the contribution of direct irradiation to \( \Phi \) is

\[
\Phi_{\text{dir}} = \int_0^\infty (1 - e^{\tau_\nu}) \frac{L_\star,\nu}{L} \, d\nu = 1 - e^{-\tau_\ast} + O(\tau_\ast^2). \tag{3.5}
\]

In the limit \( \tau_\ast \ll 1 \) of very low optical depth, \( e^{-\tau_\nu} \to 1 \) and \( \langle r \rangle_F \) approaches a unique value \( \langle r \rangle_{F,\text{thin}} \); for our truncated power law profile this is

\[
\langle r \rangle_{F,\text{thin}} \equiv \frac{k-1}{2-k} r_{\text{in}}^{2-k} r_{\text{out}}^{k-1} \frac{1-(r_{\text{in}}/r_{\text{out}})^{2-k}}{1-(r_{\text{in}}/r_{\text{out}})^{k-1}}, \tag{3.6}
\]

which is unity for a thin shell \( (r_{\text{out}} = r_{\text{in}}) \) and is intermediate between \( r_{\text{in}} \) and \( r_{\text{out}} \) even for very thick shells, so long as \( 1 < k < 2 \). In our fiducial case \( k = 1.5 \),

\[
\langle r \rangle_{F,\text{thin}} = (r_{\text{in}} r_{\text{out}})^{1/2}.
\]

Ib. Optically thin dust emission. A minor but non-negligible contribution to the total force arises from the interaction of thermal dust radiation with other dust grains. The total infrared luminosity is approximately \( (1 - e^{-\tau_\nu})L \) and the appropriate opacity is \( \kappa_{dd}(T_{d,\text{in}}) \), where

\[
\kappa_{dd}(T) = \frac{\int_0^\infty B_\nu(T) \kappa_\nu \kappa_{a,\nu} \, d\nu}{\int_0^\infty B_\nu(T) \kappa_{a,\nu} \, d\nu} \tag{3.7}
\]

is the opacity of dust grains to the emission of other grains at temperature \( T \). The associated optical depth is \( \tau_{dd,\text{in}} = [\kappa_{dd}(T_{d,\text{in}})/\kappa_{\text{fid}}] \tau_{\text{fid}} \). Note that \( \kappa_{dd}(T) \) is relatively

\footnote{Of course \( \Phi_{\text{dir}} = \tau_\ast + O(\tau_\ast^2) \) would be equally valid, but expression (3.5) is more accurate when \( \kappa_\nu \) varies slowly with \( \nu \).}
large compared to the Rosseland mean, because optically thin radiation is concentrated in frequencies where $\kappa_\nu$ is maximized; otherwise optically thin infrared would be entirely negligible.

The contribution to $\Phi$ due to recaptured infrared emission is therefore approximately

$$\Phi_{\text{IR, thin}} = \frac{\kappa_{a_*}}{\kappa_*} (1 - e^{-\tau_*}) (1 - e^{-\tau_{dd,in}})$$

and we can estimate the total force due to optically thin radiation as

$$\Phi_{\text{thin}} = \Phi_{\text{dir}} + \Phi_{\text{IR, thin}}.$$  

II. Intermediate case: starlight scattering and absorption. As the starlight optical depth increases, $\Phi_{\text{dir}} \to 1$ but scattered starlight adds to the net force and hence a new component $\Phi_{\text{sc}}$ to the force ratio $\Phi$. We address this case in Appendix 3.7 by means of Eddington’s approximation. In the limit $\tau_* \gg 1$ (§ 3.7.1), $\Phi_{\text{dir}} \to 1$ and

$$\Phi_{\text{sc}} \to \left\langle \frac{a_\nu}{1 + \sqrt{3(1 - a_\nu)}} \right\rangle_{L_*} \simeq \frac{a_*}{1 + \sqrt{3(1 - a_*)}}$$

where $\left\langle \cdots \right\rangle_{L_*}$ is a starlight-averaged value, i.e., a frequency average weighted by $L_{*,\nu}$, and $a_* = \langle a_\nu \rangle_{L_*}$.

In the asymptotic case $\tau_R(T_{d,in}) \ll 1 \ll \tau_*$, all the force is applied at the inner boundary and therefore $\langle r \rangle_F / r_{\text{in}} \to 1$. Figure 3.5 demonstrates the dependence of $\Phi_{\text{dir}} + \Phi_{\text{sc}}$ and the force-averaged radius due to (direct + scattered) starlight, as functions of optical depth and albedo.

III. Optically thick case. Here the dust is optically thick to starlight, and also to its own infrared thermal radiation. The direct and scattered radiation and their associated moments (like $\Phi_{\text{dir}}$ and $\Phi_{\text{sc}}$) are as described above, but the self-force due to dust emission becomes appreciable and contributes a new term $\Phi_{\text{IR}}$. In Appendix 3.8 we
compute \( \Phi_{IR} \) using the diffusion approximation, deriving the thick limit

\[
\Phi_{IR,thick} \simeq \frac{4 - \beta}{4(k - 1) + 2\beta} \frac{\kappa_{R, in} r_{in}^\prime}{\kappa_f i d} \tau_{fid} \left[ 1 - \left( \frac{r_{in}}{r_{out}} \right)^{k-1} \right].
\]

The latter expression (in terms of the inner conditions as well as a fiducial opacity and optical depth) is appropriate for comparison to DUSTY simulations. It demonstrates that the normalized radiation force depends on the Rosseland opacity at the inner boundary, and is therefore directly related to the inner dust temperature; this point will be relevant to our analysis of numerical methods.

In the thermal diffusion limit \( \langle r \rangle_F \) takes a limit \( \langle r \rangle_{F,thick} \) given in equation (3.21) of Appendix (3.8).

**Combined formulae.** To combine these asymptotic forms, we propose

\[
\Phi \simeq (1 - e^{-\tau_s}) \left[ 1 + \frac{\kappa_{a,r}^*}{\kappa_s} (1 - e^{-\tau_{dd,in}}) + \Phi_{sc} (1 - e^{-\tau_s}) + \Phi_{IR,thick} \right]
\]

and

\[
\langle r \rangle_F \simeq \left[ r_{in} + \left( \langle r \rangle_{F,thin} - r_{in} \right) e^{-\tau_s} \right] \left( 1 - \frac{\Phi_{IR}'}{\Phi} \right) + \langle r \rangle_{F,thick} \frac{\Phi_{IR}'}{\Phi},
\]

where \( \Phi_{IR}' = (1 - e^{-\tau_s}) \Phi_{IR,thick} \). The radiation contribution \( \mathcal{R} \) in the virial theorem is then given by

\[
\frac{\mathcal{R}}{L/c} \simeq \left[ r_{in} + \left( \langle r \rangle_{F,thin} - r_{in} \right) e^{-\tau_s} \right] \left( \Phi - \Phi_{IR}' \right) + \langle r \rangle_{F,thick} \Phi_{IR}'.
\]

Note that we include a prefactor \( 1 - e^{-\tau_s} \) on \( \Phi_{IR,thick} \) in equation (3.11), since IR luminosity is powered by absorbed starlight. The parameter \( q \) in equation (3.12) and (3.13) does not affect the asymptotes, so it is somewhat arbitrary; we find that \( q \simeq 10 \) optimizes the fit.
3.5 Radiation transfer codes

We now turn to the numerical codes DUSTY and Hyperion. We use fully converged, variable resolution DUSTY simulations to provide precise values of the radiation force measures $\Phi$, $\langle r \rangle_F$, and $R$ across our parameter survey, which we use to check the accuracy of our analytical predictions. Our Hyperion runs serve to explore the convergence properties of moderately resolved, three-dimensional Monte Carlo simulations. This calibration will be useful for numerical hydrodynamics simulations, and also for our own future work on radiation forces in inhomogeneous dust distributions.

3.5.1 DUSTY: adaptive radiation transfer code

Ivezic & Elitzur (1997) found that the radiative transfer equations, if given a specified temperature for the inner edge of the dust envelope and the SED of the luminosity source, could be solved without further reference to dimensional values; everything else could be expressed in terms of dimensionless quantities varying with respect to a dimensionless radial profile. Using these methods, DUSTY (Ivezic et al. 1999) provides solutions to the radiative transfer problem parameterized by the inner dust temperature $T_{d,\text{in}}$, the ratio $n$ relating the outer and inner radii such that $r_{\text{out}} = nr_{\text{in}}$, the density power law index $k$, the optical depth $\tau_{\text{fid}}$ at a fiducial frequency $\nu_{\text{fid}}$, and the spectral distributions of the central source ($L_{*}\nu$) and of the dust properties ($\kappa_{\nu}$, $a_{\nu}$).

We inspect these solutions to construct $\Phi$ and $\langle r \rangle_F$.

To calculate these solutions, DUSTY creates a self-refined grid along the radial profile through an iterative process. Starting at the inner cavity wall, $r/r_{\text{in}} = 1$, DUSTY creates an initial set of radial grid points such that for each pair of consecutive points, each of three regulating quantities changes by less than some specified increment or ratio. These quantities are the increment of the optical depth to the fiducial frequency as a fraction of the total depth $\tau_{\text{fid}}$ (default: 0.3), the ratio of their radii (default: 2.0), and the ratio of their densities (default: 4.0). We preserve the latter two of these
defaults, while adopting a different value for the maximum fractional increment of the optical depth, 0.025, as a starting value which we will soon vary. Once the initial grid has been generated, DUSTY begins the self-refinement by calculating the bolometric flux through the grid, checking whether the flux is conserved and converged within some specified accuracy parameter (default: 0.05). If it is not, or if too large a fraction of the allowed accumulated error is accrued between a single pair of points, DUSTY adds additional grid points in the vicinity of these problematic points to further refine the problem, iterating this procedure until an accepted grid is produced. DUSTY calculates the radiative transfer solutions by determining the spectral energy density with an integral equation (Ivezic & Elitzur 1997), utilizing a temperature profile derived from the assumption of radiative equilibrium.

Adopting a starting flux accuracy parameter of 0.01 for a quick parameter survey of DUSTY’s convergence, we vary both this value and the maximum fractional increment of the optical depth to explore this space. For these, we hold $T_{d,in}$ fixed at 1500 K (roughly the dust sublimation temperature) and set $\tau_{fid} = 10$, which for $c/v_{fid} = 1.95 \mu m$ corresponds to $\tau_* = 54.7$. The results are shown in Tables 3.1 and 3.2. DUSTY’s solutions appear to converge as the requested tolerances are made more strict and more grid points are required to determine the solution. Fitting the results with linear functions of the tolerances, we infer $\Phi \rightarrow 5.183$ and $\langle r \rangle \rightarrow 1.596 r_{in}$ as these tend to zero.

### 3.5.2 Hyperion: Monte Carlo radiation transfer code

Hyperion (Robitaille 2011) represents the alternative Monte Carlo technique. Monte Carlo codes like Hyperion model radiation transfer by propagating a large number of photon packets throughout the medium, allowing them to scatter or be absorbed and re-emitted in a number of interactions.
Table 3.1: Convergence of DUSTY radiative transfer for the problem with \( T_{d,\text{in}} = 1500 \, \text{K} \) and \( \tau_{\text{fid}} = 10 \) at \( c/\nu_{\text{fid}} = 1.95 \, \mu\text{m} \). Here we vary the requested flux conservation accuracy, holding all other control parameters and physical parameters of the problem constant. For a requested accuracy of 0.01 or worse, other control parameters provide more stringent refinement criteria and the solution is unchanged. At higher requested accuracy, more grid points are finally required, and \( \Phi \) varies slightly (a 1% change as the requested accuracy changes over an order of magnitude). Meanwhile, \( \langle r \rangle_F/r_{\text{in}} \) remains insensitive to the required flux accuracy.

<table>
<thead>
<tr>
<th>Req. Accuracy</th>
<th>Grid Points</th>
<th>( \Phi )</th>
<th>( \langle r \rangle_F/r_{\text{in}} )</th>
<th>Flux Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>73</td>
<td>5.20</td>
<td>1.59</td>
<td>( 3.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>0.005</td>
<td>60</td>
<td>5.22</td>
<td>1.59</td>
<td>( 1.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.01</td>
<td>57</td>
<td>5.25</td>
<td>1.59</td>
<td>( 3.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.05</td>
<td>57</td>
<td>5.25</td>
<td>1.59</td>
<td>( 3.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.10</td>
<td>57</td>
<td>5.25</td>
<td>1.59</td>
<td>( 3.6 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

Table 3.2: DUSTY convergence for the same problem as Table 3.1, here varying the maximum step in optical depth between consecutive grid points relative to the total optical depth, \( \max \Delta \tau/\tau \), while holding all other control variables at default values. Allowing 8 times larger steps, the number of steps decreases by a factor of 4.5; \( \Phi \) changes by about 1.3% from its starting value, while \( \langle r \rangle_F/r_{\text{in}} \) chances by about 0.63% from its starting value.

<table>
<thead>
<tr>
<th>Max ( \Delta \tau/\tau )</th>
<th>Grid Points</th>
<th>( \Phi )</th>
<th>( \langle r \rangle_F/r_{\text{in}} )</th>
<th>Flux Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0125</td>
<td>108</td>
<td>5.23</td>
<td>1.59</td>
<td>( 3.7 \times 10^{-3} )</td>
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<tr>
<td>0.025</td>
<td>57</td>
<td>5.25</td>
<td>1.59</td>
<td>( 3.6 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.05</td>
<td>33</td>
<td>5.29</td>
<td>1.59</td>
<td>( 3.3 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.1</td>
<td>24</td>
<td>5.32</td>
<td>1.58</td>
<td>( 4.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>0.2</td>
<td>23</td>
<td>5.32</td>
<td>1.58</td>
<td>( 6.6 \times 10^{-3} )</td>
</tr>
</tbody>
</table>
Hyperion has the advantage of being able to handle arbitrary density distributions with a choice of several coordinate geometries. Thus, it can solve more types of problems than can DUSTY, which is limited to spherically symmetric or slab geometries. We adopt a Cartesian coordinate geometry here to maintain consistency with a subsequent paper, in which we will generate inhomogeneities as clumps in the dust envelopes; this process is simplified in such a geometry. However, Hyperion is far slower than DUSTY for the spherically symmetric problem, which they can both solve; and while it can accept hierarchically refined density distributions, it cannot adaptively re-grid.

In its base form, Hyperion calculates the specific energy absorption rate in the medium due to the radiative transfer. This allows one to derive properties such as temperatures, the spectral energy distribution, and resulting images, but disregards and discards vector information. Therefore, to calculate the radiation forces exerted by the photons, we modified Hyperion to record and track the specific force (an acceleration, the force per unit mass) exerted upon the material in each cell as a vector with Cartesian components. The specific force in each cell is built up as a summation of the contributions by two mechanisms from each photon packet: their interaction events with the dust, and their propagation through the dust. The former includes all scattering and absorption events, which are treated as momentum transfer events based on the change in the packet’s energy and velocity vector. The latter reflects the weighted Monte Carlo technique of Lucy (1999), which deposits force along the entire path of propagation. Our approach is similar to, but independent of, the work by Harries (2015).

Whereas DUSTY’s simulations are parameterized by $T_{d,in}$ and $\tau_{fid}$, Hyperion’s are parameterized by dimensional quantities like distance, luminosity, and density (as defined on the computational grid). The dust emissivity and mean opacities are tabulated as a function of the specific energy absorption rates based on the absorption
Table 3.3: Variation of Hyperion’s predictions for Φ as the numbers of photon packets per cell are varied, in the same problem as before. Cases labeled “Low”, “Medium”, and “High” correspond to averages of 10, $10^6/32^3 = 30.5$, and 100 photon packets per cell, respectively.

<table>
<thead>
<tr>
<th># Cells</th>
<th>Low: Φ</th>
<th>Medium: Φ</th>
<th>High: Φ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32^3$</td>
<td>4.735</td>
<td>4.734</td>
<td>4.734</td>
</tr>
<tr>
<td>$64^3$</td>
<td>4.932</td>
<td>4.933</td>
<td>4.932</td>
</tr>
<tr>
<td>$128^3$</td>
<td>5.056</td>
<td>5.058</td>
<td>—</td>
</tr>
<tr>
<td>$256^3$</td>
<td>5.125</td>
<td>5.124</td>
<td>—</td>
</tr>
</tbody>
</table>

and emission of energy, and the temperatures may be calculated in turn as a function of these rates, converging toward the ideal solution just as DUSTY does. We choose $L_*=L_\odot$ (an arbitrary choice that can be adjusted without loss of generality), adopt the same stellar spectrum and dust properties, and then set the dust density coefficient and $r_{\text{in}}$ (here $2.15 \times 10^{12}$ cm in the case of $\tau_{\text{fid}} = 10$) so that the converged values of $T_{d,\text{in}}$ and $\tau_{\text{fid}}$ match the DUSTY calculation.

Hyperion’s results must converge in two ways. First, there must be sufficiently many photon packets propagated through the grid for the dust temperature distribution to settle toward equilibrium. Although small-number statistics certainly add variance to the dust temperatures, we found that the recommended 30 packets per grid cell was easily sufficient (Tables 3.3 and 3.4). Indeed, integral quantities like Φ and $\langle r \rangle_F$ appear to be remarkably insensitive to discrete sampling of the photon field.

Second, the dust density distribution must be sufficiently resolve the physical problem. Here, convergence is not as rapid. Varying the grid resolution (Table 3.5) we observe significant variation in the integral quantities, with only a slow convergence (error $\propto (\text{resolution})^{-0.73}$) that is, reassuringly, toward the DUSTY result. As the number of photon packets must increase if the average number of packets per cell is to be maintained as a constant, highly accurate MC force evaluations can be quite
Table 3.4: Like Table 3.3, but for $\langle r \rangle_F / r_{in}$.

<table>
<thead>
<tr>
<th># Cells</th>
<th>Low: $\langle r \rangle_F / r_{in}$</th>
<th>Medium: $\langle r \rangle_F / r_{in}$</th>
<th>High: $\langle r \rangle_F / r_{in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32$^3$</td>
<td>1.67</td>
<td>1.67</td>
<td>1.67</td>
</tr>
<tr>
<td>64$^3$</td>
<td>1.63</td>
<td>1.63</td>
<td>1.63</td>
</tr>
<tr>
<td>128$^3$</td>
<td>1.61</td>
<td>1.61</td>
<td>—</td>
</tr>
<tr>
<td>256$^3$</td>
<td>1.60</td>
<td>1.60</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 3.5: A convergence study of Hyperion with grid resolution, and a comparison with DUSTY (for a flux conservation accuracy parameter of 0.001 and a maximum $\Delta \tau / \tau_{max}$ of 0.025), for a physical problem created to match that in Tables 3.1 and 3.2. Note that $T_{d,in}$ converges toward the desired value (1500 K) as $\Phi$ and $\langle r \rangle_F$ converge.

<table>
<thead>
<tr>
<th>Resolution</th>
<th>$\Phi$</th>
<th>$\langle r \rangle_F$</th>
<th>$T_{d,in}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>32$^3$</td>
<td>4.73</td>
<td>1.67</td>
<td>1209</td>
</tr>
<tr>
<td>64$^3$</td>
<td>4.93</td>
<td>1.63</td>
<td>1296</td>
</tr>
<tr>
<td>128$^3$</td>
<td>5.06</td>
<td>1.61</td>
<td>1372</td>
</tr>
<tr>
<td>256$^3$</td>
<td>5.12</td>
<td>1.60</td>
<td>1429</td>
</tr>
<tr>
<td>DUSTY</td>
<td>5.20</td>
<td>1.59</td>
<td>1500</td>
</tr>
</tbody>
</table>

costly. However, as we discuss below, we consider Hyperion’s under-estimation of the radiation force to be a predictable systematic effect.

Criteria for Monte Carlo convergence

Spatial resolution affects Monte Carlo radiation transfer codes like Hyperion in two distinct ways. One is familiar to all numerical simulations: a physical problem is defined by only a few numbers per grid cell, so the problem itself (e.g., the spherical power-law dust density profile) converges to its ideal form only in the limit of infinite
resolution. Hyperion, for instance, treats the dust as uniform within each cell rather than enforcing our model $\rho \propto r^{-k}$.

A second source of error arises from lack of resolution of the photon mean free path. This can sometimes be handled in the diffusion approximation, but our problem is defined by a zone of starlight absorption in which dust reaches the maximum temperature $T_{d,\text{in}}$, and the peak wavelength of emission from this layer sets the Rosseland optical depth of the envelope. This is most relevant in the optically thick regime, which is also where starlight is absorbed very close to $r_{\text{in}}$. If the deposition of starlight energy is diluted by a lack of resolution, $T_{d,\text{in}}$ will be systematically underestimated. The consequence is an artificial lowering of the Rosseland opacity $\kappa_R(T_{d,\text{in}})$, as observed in Table 3.5. The radiation force $\Phi$ should be underestimated by the same factor, as in equation (3.10), even if the rest of the radiation transfer problem is handled perfectly.

We illustrate the resulting systematic underestimation of Monte Carlo forces and $T_{d,\text{in}}$ in Figure 3.1 for the case of $\tau_{\text{fid}} = 10$ and $T_{d,\text{in}} = 1500$ K, comparing against our “ground truth” solution provided by DUSTY. Such an underestimation may be of particular relevance to radiative hydrodynamical simulations, where the characteristic scales of the hydrodynamical processes and the mean free path for the radiation often differ significantly; in such cases, additional resolution is required to properly capture the radiative transfer forces.

The figure also compares each resolution with well resolved DUSTY runs in which $T_{d,\text{in}}$ is adjusted to agree with each Hyperion run (as opposed to the 1500 K target value). We see that the tendency for Hyperion to underestimate $T_{d,\text{in}}$ at coarse linear resolution explains much of the discrepancy in $\Phi$.

This suggests that the effect of discretization should be much less noticeable at lower optical depths for which the starlight mean free path is not so short. We tested this out with Hyperion runs of varying resolution and with $\tau_{\text{fid}}$ as low as
Figure 3.1: An investigation of the force convergence in Hyperion Monte Carlo simulations of varying resolution (black circles). In MC runs, $\Phi$ approaches an asymptote (top-right black star), identified using fully-resolved DUSTY simulations with the same dust column and inner temperature (1500 K). Lower resolution MC runs underestimate $\Phi$. This is partly because the inner temperature is underestimated when the mean free path of starlight is not resolved, as we demonstrate by connecting each run with a DUSTY simulation (black stars) of the same $T_{d,\text{in}}$ (dashed lines) and total dust column. Yellow, orange, red, and dark red lines show the locus of DUSTY solutions for 1200, 1300, 1400, and 1500 K, respectively.
10^{-3}. As expected, the discrepancy in each parameter depends strongly on optical depth. For \( \Phi \), we find that for \( \log_{10} \tau_{\text{fid}} = (-3, -2, -1, 0, 1) \) that the 32\(^3\) results underestimate DUSTY’s \( \Phi \) by (0.57%, 1.6%, 0.7%, 2.8%, 8.7%) whereas the 256\(^3\) results are (0.11%, 0.12%, 0.23%, 0.87%, 1.2%) low, assuming a required flux accuracy of 0.001.

Resolution affects Hyperion’s determination of \( \langle r \rangle_F \) as well, but not as strongly: \( \langle r \rangle_F \) decreases by 4.2% going from 32\(^3\) to 256\(^3\) in the model with \( \tau_{\text{fid}} = 10 \). This undoubtedly reflects the fact that \( \langle r \rangle_F \) approaches a unique asymptotic value in each of the three regimes discussed in §3.4, so it should be relatively insensitive to errors.

Finally, as noted in Equation 3.4, for our problem, \( \mathcal{R} = \Phi T c \langle r \rangle_F \). Therefore, it is unsurprising that in the optically thick regime, the errors \( \mathcal{R} / (r_{\text{in}} L / c) \) track in a similar manner to those of \( \Phi \).

The variation of these errors with optical depth is visualized in Figure 3.2. The radiative fluxes at the in radii \( r_{\text{in}} \) for these models are listed in Table 3.6.

### 3.5.3 Parameter space survey

With the scaling behaviors examined, we now examine the overall behaviour of the radiation transfer solutions. Figures 3.3 and 3.4 present the variation of \( \Phi \) and \( \langle r \rangle_F \), respectively, across the range \( 10^{-3} < \tau_{\text{fid}} < 10^2 \) and 100 K < \( T_{d,\text{in}} < 1500 \) K. We plot
Figure 3.2: The magnitude of fractional difference in the values of $\Phi$, $\langle r \rangle_{F}/r_{in}$, and $R_{c}/(r_{in}L/c)$ as found with the Monte Carlo code Hyperion at varying resolutions, versus DUSTY models with a required flux accuracy parameter of 0.001.
DUSTY results (with the adopted parameters) against $\tau_*$ in panel (a) of Figure 3.3, and against $\tau_R(T_{d, in}) = [\kappa_R(T_{d, in})/\kappa_*] \tau_*$ in panel (b); the two plots are meant to illustrate that $\Phi$ depends on $\tau_* = 5.47 \tau_{\text{fid}}$ in the optically thin regime and on $\tau_R(T_{d, in})$ in the thick regime. Figure 3.4 makes a similar point. These dependences are hard-wired into the analytical predictions, equations (3.11) and (3.12), which we overplot in each figure. (To avoid clutter we plot these predictions only for $T_{d, in} = 100$ K and 1500 K.)

These figures make very clear that the three radiation transfer regimes discussed in §3.4 apply to the real problem as well as the idealized one. As for our analytical predictions: while for low temperatures the error in $\Phi$ is large, with a peak of $\approx 50\%$ for 200 K at $\tau_* = 274$ over the 100 K and 200 K range, there is also a peak error of $\approx 30\%$ at $\tau_* = 547$ at a temperature of 500 K on the range of 300 K to 800 K, and a peak error of 8.5% also at $\tau_* = 547$ at a temperature of 900 K over the range of 900 K -1500 K.

3.6 Conclusions

We start with our key findings. First, for a spherical dust envelope the overall importance of radiation force is captured by a couple integral quantities: a normalized radial force $\Phi$ (formally equivalent to the net flux-averaged optical depth) and a force-averaged radius $\langle r \rangle_F$. These combine to give the radiation term in the virial theorem, $R \equiv \int r \cdot dF = \Phi \langle r \rangle_F L/c$, and so they directly affect the dynamical evolution when radiation forces matter at all. We stress this point because radiation forces are frequently described only in terms of a net force. Assuming the force is applied on the largest scales gives an overestimate for $R$, because $\langle r \rangle_F$ tends to be of order the innermost radius, or (in the optically thin case) intermediate between inner and outer radii.
Figure 3.3: (a): Plot of the $\Phi_{\text{rad}}$ parameter for radiation pressure force against the optical depth to starlight, $\tau_\star$, for 765 DUSTY models (with 15 different inner dust temperatures each at 51 different maximum optical depths), compared against analytical approximations for $\Phi_{\text{rad}}$ utilizing the direct, scattering, and diffusion regimes. The corresponding values of $\tau_{\text{fid}}$ range from 0.001 to 100. The inner dust temperatures range from 100 K to 1500 K, at an interval of 100 K. All models assume a geometry with $r_{\text{out}} = 4r_{\text{in}}$ and $k = 1.5$. The 100 K (lower) and 1500 K (upper) contours are highlighted in red, with the intermediate contours shown in gray. Also shown in blue is an analytical approximation for $\Phi_{\text{rad}}$ for 100 K and 1500 K models. We see the convergence of the analytical approximation and the DUSTY solution in the direct radiation and the diffusion limits. In the intermediate scattering regime, the analytical approximation underestimates DUSTY. (b): The same $\Phi_{\text{rad}}$ contours, but now plotted against the axis of $\tau_{R,\text{in}}$, the Rosseland mean optical depth of the dust to radiation reprocessed at the inner temperature. Once again, the 100 K (leftmost) and the 1500 K (rightmost) contours of the models are highlighted in red.
Figure 3.4: Plot of the $\langle r \rangle_F$ parameter for radiation pressure force against the optical depth to starlight, $\tau_*$, for 765 DUSTY models (15 different inner dust temperatures each at 51 different maximum optical depths), compared against analytical approximations across the direct, scattering, and diffusion regimes. The corresponding values of $\tau_{\text{fid}}$ range from 0.001 to 100. The inner dust temperatures ranges over an interval from 100 K to 1500 K with 100 K increments. All models assume a geometry of $n = 4$ and $k = 1.5$. The 100 K (lower) and 1500 K (upper) contours are highlighted in red, with the intermediate contours shown in gray. Also shown in blue is an analytical approximation for $\langle r \rangle_F / r_{\text{in}}$ for 100 K and 1500 K models.
Second, the difference in opacity between starlight and thermal infrared radiation opens an intermediate regime in which starlight is easily scattered and absorbed, but thermal emission escapes. For this reason, the radiation transfer problem breaks into thin, intermediate, and thick regimes. Each of these can be understood analytically in its asymptotic form, and we provide approximate formulae for $\Phi$, $\langle r \rangle_F$, and $R$ that combine these forms into a single expression valid to a few tens of percent (and within 10% at higher temperatures).

Third, we calibrate the accuracy with which force parameters are determined in three-dimensional Monte Carlo simulations of moderate spatial resolution, by comparing results from the Hyperion MC code (run in Cartesian geometry) against highly resolved calculations by DUSTY (run in spherical symmetry). Although MC forces converge toward the physical solution, this comparison reveals that Monte Carlo underestimates radial forces when the starlight mean free path is not well resolved. Part of this error comes from the fact that stellar luminosity is then deposited in too thick a layer, leading to an underestimate of the maximum dust temperature. Because of the wavelength dependence of dust opacity, the net result is an underestimate of the net radiation force.

This may generate a tension in the design of Monte Carlo simulations, between the desire to resolve the starlight mean free path and the need to allocate numerical resources. This is particularly relevant to hydrodynamic models for which resolving the mean free path of starlight would be prohibitive. One solution would be to locally refine the grid using the starlight mean free path as a refinement criterion. Another would be to implement a sub-grid model for the dust temperature profile. A third would be to apply a correction factor to remove the systematic effect of poor resolution on the radiation forces.

Although we have only considered spherically symmetric dust profiles, we can comment on non-spherical effects. Clearly, segregating dust into clumps and opening
paths of lower optical depth will reduce the trapping of radiation, lowering the net radiation force and $\Phi$, especially in the optically thick regime. On the other hand, the same effects tend to increase the radial scale $\langle r \rangle_F$ on which radiation forces are applied, potentially by a large factor. The net effect on $R$ is not immediately obvious. We intend to return to these questions in a future publication on non-spherical and inhomogeneous density distributions.

3.7 Chapter appendix A: scattered light

Here we consider the scattered starlight. Because in our numerical investigations we implement isotropic scattering, the scattered light is close to isotropic at all radii. It can therefore be treated with Eddington’s approximation, in which the specific intensity in direction $\hat{n}$ is a linear function of $\mu = \hat{n} \cdot \hat{r}$ at every radius. The equations are exactly the same as those presented by Rybicki & Lightman, with the exception that the direct illumination by starlight adds a new source of scattered radiation. In this section, for additional clarity, we rename the total optical depth of the dust envelope $\tau_{\text{max}}$ and the local optical depth from the centre as $\tau_\nu$.

Defining $\tau_\nu = (3\epsilon_\nu)^{1/2} \tau_\nu$, where $(1 - \epsilon_\nu) = a_\nu$ is the albedo, the mean scattered intensity $J_\nu$ satisfies

$$J_\nu'' - J_\nu + (S_{\text{dir}, \nu} + B_\nu) = 0$$

(3.14)

where prime denotes $d/d\tau_\nu$, $B_\nu$ is the thermal radiation at the local dust temperature, and

$$S_{\text{dir}, \nu} = \frac{1 - \epsilon_\nu}{\epsilon_\nu} \frac{L_{\nu, s}}{(4\pi)^2 r^2} e^{-\tau_\nu}$$

(3.15)

is the additional source term. With the inner boundary condition $J'(0) = 0$ corresponding to no net scattered flux at the origin (we take $\tau_\nu$ increasing outward), equation (3.14) has the explicit solution

$$J_\nu = C_\nu \cosh(\tau_\nu) + \int_0^{\tau_\nu} [S_{\text{dir}, \nu}(\tau') + B_\nu(\tau')] \sinh(\tau_\nu - \tau') d\tau'.$$

(3.16)
The integration constant $C_ν$ is determined by the condition of zero incoming flux at the outer boundary. In the two-stream approximation as described by Rybicki & Lightman this condition is $3^{1/2}J_ν + dJ_ν/dτ_ν = 0$, which corresponds to $J_ν + ε_ν^{1/2}J'_ν = 0$, at the maximum effective optical depth $τ_{ν,max}$. This implies

$$C_ν = \int_0^{τ_{ν,max}} \left[ \sinh(τ_{ν,max} - τ') + ε_ν^{1/2} \cosh(τ_{ν,max} - τ') \right] \left[ S_{dir,ν}(τ') + B_ν(τ') \right] dτ'.$$

(3.17)

Because the star is much hotter than the dust, and we are concerned here with the peak frequencies for scattered starlight, we neglect $B_ν$ in practice. This has the benefit that equations (3.16) and (3.17) can be evaluated directly, without solving self-consistently for the dust temperature distribution.

Our goals involve the radiation force due to scattered starlight. For this we require the scattered flux, which in Eddington’s approximation is given by $F_{sc,ν} = -(4π/3)dJ_ν/dτ_ν = -4π(ε_ν/3)^{1/2}J'_ν$, where

$$J'_ν(τ_ν) = C_ν \sinh(τ_ν) - \int_0^{τ_ν} \left[ S_{dir,ν}(τ') + B_ν(τ') \right] \cosh(τ_ν - τ') dτ'.$$

The luminosity of this radiation is $L_{sc,ν} = 4πr^2F_{sc,ν}$, and its differential force is $dF_{sc,ν} = (L_{sc,ν}/c)dτ_ν$. Its normalized total force at frequency $ν$ is then $Φ_{sc,ν} = F_{sc,ν}/(L_*/c)$, and its force-averaged radius is $⟨r⟩_{F_{sc,ν}} = (∫ r dF_{sc,ν})/F_{sc,ν}$. Another ingredient is the relation between $r$ and $τ$, for which the simple cloud model $ρ = ρ_{in}(r_{in}/r)^k$ implies $r = r_{in} (1 - τ/τ_{∞,ν})^{-1/(k-1)}$ where $τ_{∞,ν} = ρ_{in}r_{in}κ_ν/(k - 1)$. Finally, we have two equivalent expressions for the maximum optical depth:

$$τ_{ν,max} \simeq \left[ 1 - \left( \frac{r_{in}}{r_{out}} \right)^{k-1} \right] τ_{∞,ν} = \frac{κ_ν}{κ_{fid}} τ_{fid}. $$

The outcome of this analysis is plotted in Figure 3.5 for our fiducial case ($k = 1.5$, $r_{out} = 4r_{in}$). We see that the force is increased by about a factor of two over the direct force of starlight for reasonably high albedos, and that the force is applied at a location that is at most a couple times the inner boundary, decreasing toward $r_{in}$.
Figure 3.5: Normalized force (top) and force-averaged radius (bottom) for direct and scattered starlight, using Eddington’s approximation for the scattering, for the case $k = 1.5, r_{\text{out}} = 4r_{\text{in}}$, as the optical depth increases. Both of these trends are markedly different from the diffusive force due to dust emission.

3.7.1 A1: Limit of high optical depths

The case of a very optically thick cloud (to starlight) is of particular interest, as scattering is most important in this regime. So long as the cloud is also effectively optically thick, so that $\hat{\tau}_{\nu,\text{max}} \gg 1$, the force due to scattered light becomes analytical, because all the light is absorbed near the inner boundary and one can take $r = r_{\text{in}}$, independent of $\tau_{\nu}$. Then $S_{\text{dir},\nu} = S_0 e^{-\tau_{\nu}}$ where $S_0 = (\epsilon_{\nu}^{-1} - 1) L_{\nu,\nu} / (4\pi r_{\text{in}})^2$. The requirement $J' \to 0$ as $\hat{\tau}_{\nu} \to \infty$ implies $C_\nu = \int_0^\infty S(\hat{\tau}_{\nu}) e^{-\hat{\tau}_{\nu}} d\hat{\tau}_{\nu} = S_0 / [1 + (3\epsilon_{\nu})^{-1/2}]$, so $J' = -S_0(3\epsilon_{\nu})^{1/2}(e^{-\tau_{\nu}} - e^{-\hat{\tau}_{\nu}}) / (1 - 3\epsilon_{\nu})$. Computing the force and comparing to the photon momentum, the ratio is

$$\Phi_{\text{sc},\nu} = \frac{1 - \epsilon_{\nu}}{1 + (3\epsilon_{\nu})^{1/2}}$$

and in this limit, all of the starlight momentum is transferred to the cloud ($\Phi_{\text{dir},\nu} = 1$). Equation (3.18) shows that when scattered light is absorbed near the inner boundary, its force is comparable to that of the direct radiation. (For the net force of scattered
radiation to become significantly larger, the scattered photons must penetrate beyond
the inner radius.)

We note that, at the inner boundary, the mean intensity of scattered radiation is
\((1 - \epsilon_v) / (\epsilon_v + \sqrt{\epsilon_v/3})\) times that of the direct starlight in this limit. Backscatter should
therefore push the sublimation radius outward by the factor \(\left[ (1 + \sqrt{3/\epsilon_\star}) / (1 + 1/\sqrt{3\epsilon_\star}) \right]^{1/2}\).

### 3.8 Chapter appendix B: diffusion of thermal infrared light

At the risk of rehashing familiar material (e.g., Krumholz & Matzner 2009 eq. 33), we
consider \(\tau_{R}(T_{d, in}) \gg 1\) and work in the diffusion approximation, \(dP_{\text{rad}} = -L d\tau_{R} / (4\pi r^{2} c)\)
where \(d\tau_{R} = \kappa_{R} \rho \, dr\) for Rosseland opacity \(\kappa_{R}(T)\). Integration yields the profile of
temperature and radiation pressure \(P_{\text{rad}} = aT^{4}/3\):

\[
\int_{P_{\text{rad}}(r_{\text{ph}})}^{P_{\text{rad}}(r)} \frac{dP_{\text{rad}}}{\kappa_{R}} = \frac{L}{4\pi c} \int_{1/r_{\text{ph}}}^{1/r} \rho \, d(r^{-1}).
\]  
(3.19)

The effective photosphere \(r_{\text{ph}}\) is the radius from which the Rosseland optical depth to
infinity is roughly unity and the temperature is set by the Stefan-Boltzmann relation
\(L \simeq 4\pi r_{\text{ph}}^{2}\sigma_{\text{SB}} T(r_{\text{ph}})^{4}\) so that \(P_{\text{rad}}(r_{\text{ph}}) \simeq L / (3\pi r_{\text{ph}}^{2} c)\). Note that, for high enough
optical depth, \(r_{\text{ph}}\) approaches the outer boundary \(r_{\text{out}}\) if one exists. If we adopt an
opacity power law \(\kappa_{R}(T) \propto T^{\beta}\) (in addition to the density power law \(\rho(r) \propto r^{-k}\)) then,
integrating equation (3.19),

\[
P_{\text{rad}} / \kappa_{R} = \frac{1 - \beta/4}{1 + k} \frac{L\rho}{4\pi cr} \left[ 1 - \left( \frac{r}{r_{\text{ph}}} \right)^{k+1} \right] + \frac{L}{3\pi r_{\text{ph}}^{2}\kappa_{R}(r_{\text{ph}})c}.
\]  
(3.20)

The last term is of order \((r_{\text{ph}}/r)^{-2}(\kappa_{R}r)^{-1}\) relative to everything else: it can safely
be ignored in regions of high optical depth. Likewise the second term in brackets is
negligible for \((r/r_{\text{ph}})^{k+1} \ll 1\), so it can often neglected near the inner boundary. One
is then left with the inner power law profile
\[
P_{\text{rad}} \kappa_R \sim \frac{(1 - \beta/4)L\rho}{4\pi(1 + k)cr}
\]
in which \( T \propto r^{-(k+1)/(4-\beta)} \).

The force distribution is particularly simple as \( dF_{\text{IR}} = (L/c)d\tau_R \) in the diffusion approximation, so \( \Phi_{\text{IR}} = \tau_R \) modulo a small offset arising from the photosphere and optically thin region. The inner power law solution suffices to estimate \( \Phi_{\text{IR}} \), because the force is concentrated in the densest, hottest regions near the inner boundary; this gives equation (3.10).

The force-averaged radius \( \langle r \rangle_F \) is also determined by central conditions, although not to the same degree as \( \Phi_{\text{IR}} \): \( \langle r \rangle_F \) takes the optically thick limit
\[
\langle r \rangle_{\text{F, thick}} = \frac{\int_{r_{\text{in}}}^{r_{\text{ph}}} [1 - (r/r_{\text{ph}})^{k+1}] \frac{\beta}{4-\beta} r^{1-\frac{\beta (k+1)}{4-\beta}+k} dr}{\int_{r_{\text{in}}}^{r_{\text{ph}}} [1 - (r/r_{\text{ph}})^{k+1}] \frac{\beta}{4-\beta} r^{1-\frac{\beta (k+1)}{4-\beta}+k} dr} \to 2\frac{(k - 1) + \beta}{4 - \beta} r_{\text{in}}.
\] (3.21)

The second expression is valid only insofar as the inner power law solution holds to radii well beyond \( \langle r \rangle_F \), and so should not be used in our fiducial problem.
Chapter 4

The Dependence of Radiation Forces on Clumping in Dusty Envelopes

4.1 Preface

This chapter details a paper published in Monthly Notices of the Royal Astronomical Society (MNRAS) (Jumper & Matzner 2018b)\(^1\). It has been edited from its original form, including to conform to the formatting of this dissertation. I am the lead and first author on the paper. The second author is Christopher Matzner. I conducted, post-processed, and analyzed all simulations conducted for this paper and created all the figures and tables within. Matzner wrote Appendix B of the paper and assisted in the revision processes by streamlining and/or rewording some sections.

4.2 Chapter abstract

Dust barriers effectively capture the photon momentum of a central light source, but low-density channels, along with re-emission at longer wavelengths, enhance its

\(^1\)https://academic.oup.com/mnras/article/480/4/4424/5069409
Chapter 4. The Dependence of Radiation Forces on Clumping in Dusty Envelopes

We use Monte Carlo simulations to study the effects of inhomogeneity on radiation forces imparted to a dust envelope around a central star. We survey the strength and scale of an inhomogeneous perturbation field, as well as the optical depth of its spherical reference state. We run at moderate numerical resolution, relying on our previous resolution study for calibration of the associated error. We find that inhomogeneities matter most when their scale exceeds the characteristic mean free path. As expected, they tend to reduce the net radiation force and extend its range; however, there is significant variance among realizations. Within our models, force integrals correlate with the emergent spectral energy distribution, given a specified set of dust properties. A critical issue is the choice of integral measures of the radiation force: for strong deviations from spherical symmetry the relevant measures assess radial forces relative to the cloud centre of mass. Of these, we find the virial term due to radiation to be the least stochastic of several integral measures in the presence of inhomogeneities.

4.3 Introduction

Radiation pressure forces are suggested to play a key role in several contexts where massive stars interact with interstellar matter. The capture of photon momentum by dust grains is responsible for superwind mass loss from asymptotic giant branch stars (Bowen & Willson 1991), poses a formidable barrier to massive star formation (Wolfire & Cassinelli 1987), and may be influential in the removal of matter during massive star cluster formation and the disruption of molecular clouds (Fall et al. 2010; Murray et al. 2010). The effect of radiation forces on H II regions (Krumholz & Matzner 2009; Draine 2011; Yeh & Matzner 2012) has been cited as a crucial precondition for the successful deposition of supernova energy on galactic scales (Hopkins et al. 2014), and radiation has been directly implicated in disk support (Thompson et al. 2005; Thompson 2009).
Chapter 4. The Dependence of Radiation Forces on Clumping in Dusty Envelopes and galactic winds (Zhang & Thompson 2012; Faucher-Giguère & Quataert 2012) driven by starbursts or AGN. At the same time, there are counterexamples in which radiation forces are found to be relatively weak, especially in star cluster formation and molecular cloud disruption (Dale et al. 2005; Pellegrini et al. 2011; Matzner & Jumper 2015).

We wish to delineate the influence of radiation pressure forces, but we must be mindful of several complications. First, the opacity of dust grains is strongly wavelength dependent, allowing luminosity to escape a region more easily once it is absorbed and re-radiated by grains. Second, conclusions can depend on the choice of numerical method, as studies of super-Eddington galaxy disks (Krumholz & Thompson 2013; Davis et al. 2014) and radiation-dominated accretion disks (Hirose et al. 2009; Jiang et al. 2013) demonstrate. Lastly, most dusty environments are anisotropic or inhomogeneous, easing the escape of luminosity through channels of low optical depth; this is a critical feature of massive star formation models (Yorke & Sonnhalter 2002; Krumholz et al. 2005; 2009).

We addressed the first two complications in a previous paper (Jumper & Matzner 2018b, hereafter JM18A), in which we used Hyperion Monte Carlo simulations (Robitaille 2011) of dust transfer through spherically symmetrical envelopes and surveyed physical and numerical parameters while comparing to a separate method (DUSTY: Ivezic et al. 1999). Monte Carlo radiative transfer solutions are formally exact in the limit of very high resolution and numbers of photon ‘packets’. We found that poor resolution of the mean free path introduces a predictable error, but that the radiation forces in various optical depth regimes are understandable in terms of direct, scattered, and re-emitted radiation.

In this work, we extend these models with the introduction of inhomogeneities that break the spherical symmetry of the problem, and explore their influence on the radiation force and its distribution. These considerations may be particularly
relevant in application to analytical approximations of systems undergoing radiative feedback, where the effect of reprocessed radiation may be described in a “trapping factor” (Chakrabarti & McKee 2005; Krumholz & Matzner 2009; Murray et al. 2010; Matzner & Jumper 2015). The models conducted within this paper illustrate how such trapping factors may vary with the properties of the clumping in the surrounding region, which may help inform choices in models. Furthermore, these models may also provide insight into the variation with clumping properties for both where within the region the trapped force is deposited and a summary of how the internal structure of the region is evolving at a given instant as a consequence of these forces. These properties will be related to the parameters used in this paper as introduced in §4.4.4.

4.4 Physical problem and implementation

4.4.1 Parent dust envelopes

Our analysis involves modifying a centrally-illuminated, spherically symmetric dust dust envelope, which we call the “parent envelope”, with a clumpy “contrast field”. For the parent envelopes we adopt the truncated power-law dust profiles we analyzed previously (JM18A). These consist of a dust density satisfying $\rho \propto r^{-1.5}$ from an inner radius $r_{\text{in}}$ to an outer radius of $4r_{\text{in}}$. Dust opacity follows the Draine (2003a) model of carbonaceous silicates, although we simplify the scattering processes to be isotropic. The governing parameters for radiation transfer through the parent envelope are:

- $T_*$, the colour temperature of the central light source, set to 5772 K;
- The density power law and ratio of outer to inner radii, set to $-1.5$ and 4, respectively;
- the innermost dust temperature $T_{\text{in}}$, set to 1500 K in this study; and
Table 4.1: Parent Envelopes with $T_{\text{in}} = 1500$ K

<table>
<thead>
<tr>
<th>$\log_{10} \tau_*$</th>
<th>Flux at $r_{\text{in}}$ ($10^6$ erg cm$^{-2}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1.0$</td>
<td>240</td>
</tr>
<tr>
<td>$-0.5$</td>
<td>216</td>
</tr>
<tr>
<td>$0.0$</td>
<td>171</td>
</tr>
<tr>
<td>$0.5$</td>
<td>125</td>
</tr>
<tr>
<td>$1.0$</td>
<td>96.8</td>
</tr>
<tr>
<td>$1.5$</td>
<td>76.6</td>
</tr>
<tr>
<td>$2.0$</td>
<td>55.6</td>
</tr>
</tbody>
</table>

- the optical depth to starlight, $\tau_*$, for which we consider values of $10^{-1}$ to $10^{1.5}$ in half-decade increments.

These parameters suffice to determine the flux at $r_{\text{in}}$, as listed in in Table 4.1. The inner radius $r_{\text{in}}$ itself depends on the central luminosity $L_*$, which we arbitrarily set to $L_\odot$. The inner dust density is determined by $r_{\text{in}}$ and $\tau_*$.  

4.4.2 Contrast fields

We create inhomogeneous dust envelopes by multiplying the parent envelope by a log-Gaussian contrast field with a definite power law spectral index. Our choice of a log-Gaussian (lognormal) field is motivated from the general observation that star-forming environments tend to be turbulent and that the results of numerous numerical studies have shown that such turbulent environments tend to give rise to a log-Gaussian probability distribution function in density (Padoan & Nordlund 2002 and references therein). While we do not develop inhomogeneities self-consistently from the dynamics of turbulence, our approach maintains a level of realism while
allowing us to study the influence of a small set of parameters describing the dust density.

The procedure for generating the contrast fields is detailed in Appendix 4.6. It starts with one of six definite random seeds to specify realizations named “Case A” through “Case F”; uses this to generate amplitudes and phases for a Gaussian random field; specifies mode amplitudes according to a power law spectrum ($\propto k^\beta$ for $-2 \leq \beta \leq -1$ in increments of 0.25). The field is then multiplied by an amplitude factor ($0 \leq q \leq 1$ in steps of 0.2). This yields a trial contrast field which is multiplied by the parent envelope density. Finally, $\rho(r)$ and its contrast field $P$ are obtained by renormalizing the result to preserve the mass and mean optical depth of the parent envelope.

Models are parameterized by $\tau_*$, the random seed, the spectral slope $\beta$, and the strength of the contrast field. Rather than use our input parameter $q$, we prefer the ‘clumping factor’ $\langle P^2 \rangle / \langle P \rangle^2$ as a measure of its strength.

A feature of our approach is that the random seed sets the initial phases of the modes that enter $P$, while the spectral slope $\beta$ determines their relative amplitudes. Therefore, each case (A through F) generates smooth distributions of any force parameter as $q$ or $\beta$ is changed continuously.

**4.4.3 Numerical implementation**

We initialize the parent envelopes in the Monte Carlo radiative transfer code Hyperion (Robitaille 2011) at fixed $128^3$ resolution, using a total of $6.4 \times 10^7$ photon packets, for an average of $\approx 30.5$ photon packets per cell. As described in JM18A, we modified Hyperion to directly calculate the specific radiation force (radiation pressure force per unit mass). Following from the method of Lucy (1999), each given photon packet is assigned a target optical depth to propagate to before an interaction may occur to alter its path, either by scattering or by absorption and re-emission. During this process,
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Two forms of energy deposition occur: continuous absorption during the propagation along the entire path to the point of interaction, and a localized transfer of momentum at this point based on the difference in energy and velocity (including direction) between the incident photon and the scattered or re-emitted photon that emerges. The resulting transfers were calculated in terms of the radiation’s “specific force”, or the force exerted per unit mass (an acceleration). Integrating the specific force with the properties of the envelope summarizes the force-capturing in the envelope in terms of a group of parameters, as discussed in §4.4.4.

Our analysis from JM18A allows us to calibrate the degree to which results quoted here will be affected by the finite resolution of our runs. Comparing Hyperion runs of varying Cartesian resolution against converged spherical solutions from DUSTY (Ivezic et al. 1999), we found there that the force will be underestimated at 128$^3$ resolution, but by less than 1% for $\tau_* < 5.5$, increasing to a few percent for $\tau_* \sim 10^2$. These errors are partly due to an under-representation of the inner dust temperature in cases where the starlight mean free path is unresolved. We do not know how the error depends on inhomogeneity of the dust distribution, but we have no reason to think it should be sensitive. The force error is quite small compared to the change in force due to clumping, as we shall see in §4.5.2.

### 4.4.4 Force parameters

In JM18A we introduced three integral quantities to measures forces: $\Phi$, the ratio of the total applied outward radiation force $F_{\text{rad}}$ to the photon force of the stellar luminosity; $\langle r \rangle_F$, the force-averaged radius; and $R$, the radiation term in the virial theorem. These remain useful for inhomogeneous distributions of dust and luminosity, but we must be careful to adopt the correct origin for the radial vector $r$ that appears in their definitions. In Appendix 4.7 we demonstrate that the centre of mass (CM) is
the correct origin for the definition of $\mathcal{R}$, and therefore we define

$$\Phi = \frac{\int \hat{r} \cdot d\mathbf{F}_{\text{rad}}}{L/c},$$

(4.1)

$$\langle r \rangle_F = \frac{\int r \cdot d\mathbf{F}_{\text{rad}}}{\int \hat{r} \cdot d\mathbf{F}_{\text{rad}}},$$

(4.2)

and

$$\mathcal{R} = \int r \cdot d\mathbf{F}_{\text{rad}} = \Phi \langle r \rangle_F \frac{L}{c},$$

(4.3)

where $\hat{r} = r/|r|$ is the radial unit vector, and $r$ is measured relative to the CM, i.e., $r_{\text{cm}} = 0$. We identify the CM by assuming a constant dust-to-gas ratio. We also define dimensionless versions of the latter parameters: $\langle \tilde{r} \rangle_F = \langle r \rangle_F / r_{\text{in}}$ and $\tilde{\mathcal{R}} = \mathcal{R} / (r_{\text{in}} L/c)$.

Note that $\Phi$ and $\mathcal{R}$ will be positive when the radiation force points away from the CM, i.e. when the cloud is illuminated from within, and will tend to be negative when it is illuminated from without. Note also that only radial forces enter these integrals. Tangential forces can affect the motion of the CM (see Appendix 4.7), and can contribute indirectly to the virial theorem for internal cloud motions by enhancing its internal kinetic energy.

These quantities characterize the properties of the radiation force. The “trapping factor” used in previous works is, for an optically-thick envelope, $\Phi = 1$. The force-averaged radius $\langle r \rangle_F$ describes where in the envelope the photon force is deposited. These combine to give $\mathcal{R}$, the net dynamical influence of radiation as measured by the virial theorem; see Appendix 4.7.

### 4.5 Results and discussion

In our parameter survey, we vary the properties of the dust envelopes for six random seeds, seven optical depths ($\tau_* = 10^{-1}$ to $10^{2.0}$), five spectral indices $\beta$, and six amplitude parameters $q$. This amounts to 1260 combinations, although as all $q = 0$
combinations reduce to the parent envelope, there are only 1057 unique physical problems.

In §4.5.1 we provide examples of envelopes within our parameter space as well as the force, specific force, and temperature within each model. In §4.5.2 we characterize the scaling of the force-capturing parameters with envelope optical depths and anisotropies. We connect the force parameters to the emergent spectral energy distribution in §4.5.3.

4.5.1 Envelope visualization: anisotropy variation and radiative capture

We illustrate the patterns generated by Cases A–F in Figure 4.1, showing each case for constant values of $\tau_\ast = 10$, $\beta = -2.0$, and $q = 1.0$. We then take one of these cases, Case B, and demonstrate how the contrast factor changes as the other parameters are varied: $\beta$ and $q$ in Figures 4.2 and 4.3 respectively. The former illustrates the variation of the spatial extent of the clump structure with $\beta$, with the clump sizes growing as $\beta$ becomes more negative. The latter varies the amplitude parameter $q$, making the clumps and pores more (larger $q$) or less (smaller $q$) intense while preserving their locations. In the limiting case of $q = 0$, $\mathcal{P} = 1$ at all points and the parent envelope is obtained.

We conducted a Monte Carlo model, using Hyperion, to solve the radiative transfer for each dust envelope in our survey, obtaining the specific forces and temperatures in each cell. These forces were then used to calculate the force-capturing parameters ($\Phi$, $\langle r \rangle_F$, and $\mathcal{R}$) and their dimensionless forms. We also illustrate the force, specific force, and temperatures, shown in Figures 4.4, 4.5, 4.6, calculated for slices of constant $\tau_\ast = 10$, $\beta = -2.0$, and $q = 1.0$, varied over the different seeds, to provide an example of their realizations. Relating the forces seen in these figures to the clumping $\mathcal{P}$ shown in Figure 4.1, we observe that the total force captured per cell strongly
Figure 4.1: Slices of the $\log_{10}$ of the contrast field $P$ through the center of the dust envelope in the $yz$-plane for the realizations of the six random seeds, labeled Cases A-F, for constant values of $\tau_*=10.0$, $\beta=-2.00$, and $q=1.0$. 
Figure 4.2: Slices of the log_{10} of the contrast field $\mathcal{P}$ through the center of the dust envelope in the $yz$-plane for the random seed used in Case B of Figure 4.1, holding $\tau_\star = 10.0$ and $q = 1.0$ constant and varying $\beta$. Additionally, the unperturbed parent envelope (with $\mathcal{P} = 1$ everywhere) is included in the upper-left panel of the figure for reference. As $\beta$ grows more negative, the spatial extent of the clumps and pores grows as well.
Figure 4.3: Slices of the $\log_{10}$ of the contrast field $P$ through the center of the dust envelope in the yz-plane for the random seed used in Case B of Figure 4.1, holding $\tau_* = 10.0$ and $\beta = -2.00$ constant and varying $q$. The unperturbed parent envelope (with $P = 1$ everywhere) coincides with the $q = 0$ case. As $q$ grows, the envelopes’ clumps become denser and pores become rarer, both while preserving their positions.
correlates with highly overdense or ‘clumped’ (large $P$) regions (Figure 4.4), while the specific force (acceleration) in the cells tracks with strongly underdense or ‘porous’ ($P \ll 1$) regions instead (Figure 4.5). Although our models are static, this suggests that in a dynamical model the radiation forces would act to intensify the existing density contrasts, as the material in lower-density porous regions will accelerated more strongly and preferentially driven away. In addition, we find that the temperature, calculated as a function of the dust’s specific energy absorption rate, also is higher in the porous region and lower in both highly clumpy regions and the shadowed regions exterior to them (Figure 4.6).

We note that Figures 4.4 and 4.5 indicate that some regions, particularly where there is heavy shadowing from prominent clumps to the interior, received no photon packets (and thus no forces). While running the model with a larger number of photon packets may have allowed packets to reach these regions, the fact that they received none of the $6.4 \times 10^7$ photon packets used in the model (representing an average of about 30.5 packets/cell for the $128^3$ box) suggests that the contributions of these regions to the overall solution is negligible. This absence is not seen in Figure 4.6, where the Hyperion temperature function has a floor of 0.1 K.

### 4.5.2 Influence of inhomogeneities on the radiation force

We now present our results on the influence of inhomogeneities on the capture of radiative forces, as summarized in the parameters’ dimensionless forms: $\Phi$, $\langle \tilde{r} \rangle_F$, and $\tilde{R}$.

We begin with Figures 4.7, 4.8, 4.9, which plot the variation of our parameters against a "clumping factor", $\langle P^2 \rangle / \langle P \rangle^2$ (right panels), and relate these to the variation with $\tau_*$ in the parent envelopes (left panels). The curves on the left panels are computed with another code, DUSTY, as described in JM18A. Our clumping factor is analogous to the definition by Owocki & Sundqvist (2018).
Figure 4.4: Slices of the log\(_{10}\) of the force exerted on the dust in units of g cm s\(^{-2}\), \(F\), through the center of the dust envelope in the yz-plane for the realizations of the six random seeds, Cases A-F, as introduced in Figure 4.1 holding \(\tau_\ast = 10.0\), \(\beta = -2.00\), and \(q = 1.0\) constant. Some regions were heavily shielded by clumps and received no photons.
Figure 4.5: Slices of the log_{10} of the specific force (acceleration) exerted on the dust in units of cm s^{−2}, \( \mathcal{F} \), through the center of the dust envelope in the yz-plane for the realizations of the six random seeds, Cases A-F, as introduced in Figure 4.1 holding \( \tau_s = 10.0 \), \( \beta = -2.00 \), and \( q = 1.0 \) constant. Some regions were heavily shielded by clumps and received no photons.
Figure 4.6: Slices of the dust temperature (K) through the center of the dust envelope in the yz-plane for the realizations of the six random seeds, Cases A-F, as introduced in Figure 4.1 holding $\tau_\ast = 10.0$, $\beta = -2.00$, and $q = 1.0$ constant. The physical dimensions of the dust envelope were set to establish an inner wall temperature of 1500 K for the parent envelopes, as calculated by DUSTY, and the temperature determination routine has a floor temperature of 0.1 K.
Insofar as converged DUSTY results align with the high-resolution limit of Hyperion runs (a result from JM18A), the effect of finite resolution can be seen in these figures as a slight vertical offset between the dashed lines on the left panel and the dots on the right panel. This offset is barely visible, and clearly quite small.

In these figures, all curves sharing the same seed are plotted with the same colour, and all curves plotted display six values at a given optical depth for the parameter in question, holding $\beta$ and seed values constant and allowing the amplitude parameter $q$ to vary, which in turn also varies $\langle P^2 \rangle / \langle P \rangle^2$. We see that in many cases the scaling can be approximated as a power law in $\langle P^2 \rangle / \langle P \rangle^2$. We discuss such power-law fits later in the paper.

We observe that in the majority of cases, $\Phi$ and $\tilde{R}$ diminish as a function of the contrast clumping factor, while $\langle \tilde{r} \rangle_F$ increases. The results for $\Phi$ and $\langle \tilde{r} \rangle_F$ are as expected, as the concentration of matter into clumps leaves underdense porous regions in the spaces between them, which allow photons to easily leak through to escape, thus bypassing the locally optically thicker regions established by the clumps. Thus, less force tends to be captured overall, and the capturing which does occur tends towards further out radii. Regarding $R$, in the limiting case of a spherically symmetric envelope, $R = \Phi \langle r \rangle_F L / c$, so whether $R$ increases or decreases with the introduction of anisotropies depends on whether $\Phi$ or $\langle \tilde{r} \rangle_F$ scales more strongly and dominates the calculations. A question posed in JM18A is now answered: we see in Figure 4.9 that $\tilde{R}$ diminishes with the introduction of clumping.

However, we also observe that in the cases of the realizations from certain seeds, the scaling behaviors behave differently from the manner described above. This is seen particularly prominently in Figures 4.7 and 4.8 ($\Phi$ and $\langle \tilde{r} \rangle_F$), especially in Case A (red curves) to a lesser extent in Case C (gold curves). Figure 4.1 helps us understand why this anomalous behavior occurred. We see that in both cases the clumps are positioned such that much of the inner cavity wall, where the photons first arrive at the dust
envelope, abuts these clumps, although much more prominently in Case A. The local increase in optical depth leads to a net increase in force. Although in both cases there are lower-density regions on other sides of the cavity wall, it appears in both cases the particular clump geometry and position has produced a net enhancement in the force capturing, in contrast to the reduction found in the other cases. Returning to Figure 4.7, we also note that this effect is most prominent when the parent envelope is optically thin and diminishes as the parent envelope becomes optically thick. This is most likely attributable to high contrast clumps creating locally optically thick regions in the anisotropic envelopes that readily capture photons that would have easily escaped from the thin parent envelopes.

Observing these figures, we also find that $R$ is less sensitive to dependence on the particular realization of the generating seed that the other parameters, as seen in the much narrower dispersion of its curves. This outcome makes sense, as $R = \Phi \langle r \rangle_F L/c$. As has been established, $\Phi$ tends to decrease as anisotropies are introduced, while $\langle r \rangle_F$ behaves in the opposite manner and tends to increase. Any variations in a particular realization of the effects these anisotropies have on the radiative capturing from the typical result for a given set of properties will then tend to affect these parameters in the opposite directions, so when we consider that they may be considered as factors of $R$, there is a greater tendency towards these variations largely canceling. Thus the reduced dispersion in solutions for $R$ is reasonable.

We now consider power-law fits of the form $\Phi = \Phi_0 (\tau_s) (\langle P^2 \rangle / \langle P \rangle^2)^{b_\Phi}$, and likewise for $\langle r \rangle_F$ and $R$. We average these power-law indices ($\beta_\Phi$, $\beta_r$, and $\beta_R$, respectively) over random seeds to explore trends with $\beta$ and $\tau_s$. We demonstrate the resulting fitting in Figure 4.10 for the $\tau_s = 10.0$ and $\beta = -2.00$ for each force-capturing parameter.

We conducted this procedure for all 35 $\tau_s$-\$\beta$ combinations, each of which carried the information for 36 parameter models (varying over 6 seeds and 6 amplitude factors
Figure 4.7: **Left:** DUSTY models for $\Phi$ for a collection of envelope optical depths $\tau_*$ from JM18A (solid line) and for this parameter survey (dots). **Right:** The variation of $\Phi$ with the clumping factor, $\langle P^2 \rangle / \langle P \rangle^2$, for all models in the parameter space. All models of the same random seed case are plotted with the same colour. In this and the two following figures, any error due to finite resolution is visible as a vertical offset between dots in the left and right panels.
Figure 4.8: Left: DUSTY models for $\langle \tilde{r} \rangle_F / \tau_*$ for a collection of envelope optical depths $\tau_*$ from JM18A (solid line) and for this parameter survey (dots). Right: The variation of $\langle \tilde{r} \rangle_F / \tau_*$ with the clumping factor, $(\langle P^2 \rangle / \langle P \rangle^2)$, for all models in the parameter space. All models of the same random seed case are plotted with the same colour.
Figure 4.9: Left: DUSTY models for $\tilde{R}$ for a collection of envelope optical depths $\tau_*$ from JM18A (solid line) and for this parameter survey (dots). Right: The variation of $\tilde{R}$ with the clumping factor, $\langle P^2 \rangle / \langle P \rangle^2$, for all models in the parameter space. All models of the same random seed case are plotted with the same colour.
Figure 4.10: For $\tau_s = 10.0$ and $\beta = -2.00$, an examination of the dependence on the three force-capturing parameters on the clumping factor. The black line represents an overall power-law fit to the data, derived from the average index of the power-law fits for seed Cases A-F.
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We present these results in the form of contour plots for the power-law fit indexes, shown in Figures 4.11, 4.12, and 4.13.

We observe that the power-law fit to the scaling of $\Phi$ with clumping becomes more negative with increasingly negative $\beta$ (and thus a larger characteristic size scale of the clumps). This result is reasonable; the larger clumps come alongside larger channels between the clumps, which facilitates the easier escape of photons between these channels. The opposite is true for small scale pores with less negative $\beta$ values.

In the lower-left corner of the figure, where $\beta$ and $\tau^*$ have their smallest magnitudes, we also see that there is a regime where the power-law index of fit is a very weakly positive value, rather than a negative value. As discussed previously, Case A and to a lesser extent Case C exhibited a similar behavior in the low $\tau^*$ regime, enhancing the force capturing rather than diminishing it. However, the remaining seed realizations, Cases B, D, E, and F, behaved in the opposite manner, reducing the force capture as the clumping increased. These opposing sets of values nearly cancel in the averaging, so there is little net effect of clumping in this regime.

We also observe that the index of fit for $\Phi$ also becomes more negative (the force diminishes more strongly with increasing clumping) as we move towards larger optical depths in the regime from optically thin envelopes to envelopes of several optical depths. This also makes intuitive sense, as clumping should have no effect in an optically thin envelope, while channeling is important in an optically thick one.

However, we observe a deviation from this trend, with a local minimum in the strength of the scaling ($b_\phi$ becomes less negative) around $\tau^* = 10.0$. This is probably an effect of the optical depth, as the infrared opacity, being an order of magnitude lower than the optical opacity, is about unity in this regime. However we have not ruled out the possibility that resolution effects also play a role; these depend on the resolution of the mean free path, as we discussed in JM18A. The anomalous behavior is observed in all three of our force parameters (Figures 4.11-4.13).
Figure 4.11: Contour plot of the average power-law exponent, $b_{\Phi}$, for the scaling of $\Phi$ with the clumping factor, $\langle P^2 \rangle / \langle P \rangle^2$, varying with both $\log_{10} \tau_*$ and $\beta$. We indicate the parameter values of our data points with black dots and interpolate the contour lines from these data points.

Returning to the remaining figures, Figure 4.12 shows that the scaling of the dimensionless force-averaged radius, $\langle \tilde{r} \rangle_F$ also intensifies with increasing envelope optical depth over the same regimes as found for $\Phi$; as before, the presence of channels to bypass otherwise thick regimes means that their effect has increased over thin areas. However, in contrast to our above example, we find that the scaling depends much more weakly on the size of the clumps as characterized by $\beta$. In Figure 4.13, we see that $\mathcal{R}$ behaves in a manner very similar to that already described for $\Phi$. 
Figure 4.12: Contour plot of the average power-law exponent, $b_r$, for the scaling of $\langle \tilde{r}\rangle_F$ with the clumping factor, $\langle P^2 \rangle / \langle P \rangle^2$, varying with both $\log_{10} \tau_*$ and $\beta$. We indicate the parameter values of our data points with black dots and interpolate the contour lines from these data points.
Figure 4.13: Contour plot of the average power-law exponent, $\beta_R$, for the scaling of $\bar{R}$ with the clumping factor, $\langle P^2 \rangle / \langle P \rangle^2$, varying with both $\log_{10}\tau_*$ and $\beta$. We indicate the parameter values of our data points with black dots and interpolate the contour lines from these data points.
4.5.3 SEDs from the model envelopes

We now briefly consider the spectral energy distributions (SEDs) that would be observed from our model dust envelopes. We are motivated by the observation that, when inhomogeneities are strong enough to affect the force integrals, they also tend to change the SED by allowing relatively short-wavelength radiation to escape; however, our results cannot be considered diagnostics because we only consider one dust model, and we neglect the possibility of protostellar outflow cavities (Whitney & Hartmann 1993). All SEDs presented in this subsection may be scaled as a function of the input stellar luminosity and the distance of the observer from the envelopes; we adopt fiducial values of \( L = L_\odot \) and \( d = 10 \text{pc} \). For each model in our parameter space, the SED is calculated for 108 wavelengths and six orthogonal viewing angles.

In Figure 4.14, we examine how the SED varies with the clumping factor \( \langle P^2 \rangle / \langle P \rangle^2 \) for three optical depths (\( \log_{10} \tau_* = -0.5, 0.5, 1.5 \)) and for \( \beta = -1, -2 \). The depths presented here are chosen to represent the three key regimes of the radiative transfer problem: the thin regime, the intermediate regime (thick to direct starlight radiation but thin to reprocessed radiation), and the thick regime. As expected, the peak of the SED is shifted to longer wavelengths from reprocessing as we progress from the thin to thick regimes, and the reprocessing feature around 10 \( \mu \text{m} \) becomes less distinct in this thick regime, where the initially reprocessed radiation itself undergoes further reprocessing. However, as seen in the right side of the figure, in envelopes with large clumping factors, less reprocessing occurs, as some photons are able to escape through the pores. In turn, more spectral energy is found at these cases closer to the original peak wavelengths, an outcome which intensifies as the clumping factor (and thus also the prominence of the pores between the clumps) increases.

Additionally, we consider in Figure 4.15 how \( \tilde{R} \) varies with the SED slope, \( \log(\lambda F_\lambda[10.2 \mu\text{m}]/\lambda F_\lambda[1.02 \mu\text{m}]) \), to gain insight regarding what an observer may be able to tell about \( \tilde{R} \) from observing the SED. The particular wavelengths chosen...
for this slope in our figure coincide with the closest points on the wavelength grid realized by our Hyperion models to 10 μm and 1 μm. We observe that for optically thin regions, $\tilde{R}$ varies largely independently of the SED slope; while increasing the clumping factor will tend to decrease $\tilde{R}$, the SED slope will not vary significantly. This outcome could be anticipated, as the change in the SED slope between points is driven by reprocessing, which is not a dominant process in optically thin regions. However, in contrast, as we progress to the optically thick regions, we find a much greater variation of the SED slope with the clumping factor; this accords with our expectations, as well as from our consideration of the SEDs presented in Figure 4.14.

Thus, for optically thick regions, examining the SED slope for the region may offer insight into the intensity of the clumping and the radiative virial parameter within that region if the optical depth of the region and the dust properties are known. However, as a key caveat, since this analysis has been conducted for a particular test model geometry and anisotropic variations away from it, the precise outcomes may vary in more complex environments.

### 4.5.4 Relation to prior works

We now take a brief digression to relate our work in this paper to that of prior studies in the field of radiative transfer in clumpy environments. In this paper, we randomly generate a density contrast field spanning over several orders of magnitude and apply this field to an initially power-law density distribution. This contrasts with the popular “two-phase” approximation used in papers such as Wolf et al. (1998), Városi & Dwek (1999), and Scicluna & Siebenmorgen (2015), which in its archetypal form assigns a distinct, single density to the clump regions and another, lower density to the porous regions surrounding them, greatly simplifying the mathematical descriptions of these regions. In addition to the two-phase approximation, some papers further characterize the behavior of radiative transfer in these environments with the mega-grain model,
Figure 4.14: For three selected parent envelope optical depths, $\log_{10} \tau_\ast = \{-0.5, 0.5, 1.5\}$, and two values $\beta = \{-1.0, 2.0\}$, $\lambda F_\lambda$ for model SEDS across a range of clumping factors in the fiducial case of a stellar luminosity $L = L_\odot$ and a distance from the observer of 10 pc. We organize the data into six bands in the clumping factors, each of equal logarithmic intervals (with a multiplicative factor of $\approx 1.804$ across the band). We then darkly shade the region between the 25th and 75th percentiles within each band, and lightly shade and hatch the regions outside this interval. The listed band boundaries in the legend have been rounded to one decimal place.
Figure 4.15: Plot of $\hat{R}$ vs. the SED slope from 1-10 $\mu$m, calculated from the average of the ratio over six viewing angles, across the parameter space. We label the regimes for each parent envelope optical depth within our space, and assign each its own data marker. The colour of the data point shown indicates the logarithm of the their clumping factor ($\langle P^2 \rangle / \langle P \rangle^2$). Points with the same $\tau_c$, $\beta$, and random seed are connected with a thin line.
which treats dense clumps like very large dust particles with an associated cross-section for interaction with radiation (Városi & Dwek 1999; Varosi & Dwek 1999). Other models utilize continuous stochastic (Hegmann & Kegel 2003), fractal (Varosi & Dwek 1999; Watson et al. 2009), or simulation-generated (Owocki & Sundqvist 2018) distributions that provide more varied density fields. Our randomly generated density contrast fields are more closely related to this latter set of models.

A key difference our work and that presented in these previous papers is the quantities that are focused upon. Generally speaking, the emphases of the above cited papers has been the determination of an effective optical depth for the modeled regions and/or characterizing the transmission of fluxes or spectral energy distributions through these regions. In contrast, this paper specifically focuses on the effects of the clumping on the radiation pressure forces captured within the volume of the dust envelope. We find that clumping is partly, but not completely, equivalent to a reduction in the overall optical depth.

### 4.5.5 Conclusions

We begin with our key findings. First, the capture of the radiation force in dust envelopes may be summarized with three integral quantities: a normalized force, $\Phi$, a force-averaged radius, $\langle r \rangle_F$, and a radiative virial term, $\mathcal{R}$. Each of these makes reference to the centre of mass, so radiative forces alone do not suffice to determine them; one must also know the mass distribution. We adopt a constant gas-to-dust ratio when specifying the CM, but other choices are possible.

Second, as we introduce inhomogeneities into the dust envelope, the parameters $\Phi$, $\langle r \rangle_F$, and $\mathcal{R}$ will vary as a function of both the strength and scale of the perturbations, as well as the base optical depths of the originating symmetric envelopes. We find that these variations can be approximated as power-law behaviors scaling with the degree of introduced clumping, characterized as $\langle P^2 \rangle / \langle P \rangle^2$, where $P$ is the contrast factor.
between the inhomogeneous and base states in each cell. Generally, the normalized force $\Phi$ diminishes with increasing clumping, as the porous regions formed between the clumps facilitates the leaking of radiation from the envelope. These porous regions also lead to an increase in the force-averaged radius, as the photons that still are captured tend to leak further before this occurs. The radiative virial term, $\mathcal{R}$, is proportional to $\Phi\langle r \rangle_F$ and tends to vary like $\Phi$ because the variation of $\langle r \rangle_F$ is generally weaker. However, $\mathcal{R}$ is significantly less sensitive to variations in the clumping caused by realizations of our random contrast field.

Third, the sensitivity of our force measures to clumping depends on the physical scale of the clumps as well as the overall optical depth. These scalings are demonstrated in contour plots in Figures 4.11, 4.12, and 4.13. The scaling tends to be at weakest for all parameters in optically thin envelopes, where most photons would escape regardless of whether or not the clumping was present. In contrast, the scaling tends to increase as we increase the optical depths of the envelopes, as the porous regions introduced alongside the clumping provide bypasses through which the photons may leak.

However, for all three force-capturing parameters studied, a temporary reversal is found in this trend in the region of $\tau_* \sim 10$ before returning to its previous behavior at even larger optical depths. This is probably due to the fact infrared dust radiation is marginally optically thick in this regime, although finite-resolution effects could also be contributing.

The power-law scaling of the force parameters also weakens for small-scale inhomogeneities and strengthens for large-scale inhomogeneities, for $\Phi$ and $\mathcal{R}$, while in contrast $\langle r \rangle_F$ is fairly insensitive to the scale of the clumping for regimes below a few optical depths. Nonetheless, as the scaling remains relatively weak throughout the entire parameter space, strong clumping is necessary to have a major impact on the capture and escape of radiation in the envelope.
We briefly consider the variation of $\mathcal{R}$ and the clumping factor with the mid-to-near-infrared SED slope. There is little relation between these in the optically thin regime. In the thick regime the SED slope may offer insight into the degree of clumping and the virial term $\mathcal{R}$ in the envelope, although outflow cavities and uncertainties in the dust model would also affect the emergent SED. Insofar as our model represents real clumps, infrared radiation provides some information on the force parameters. However we have not considered effects, like outflow channels, that may dominate the emergent SED while also affecting the radiative forces.

In this study, we conducted our radiative transfer for static realizations, and have not considered the dynamical evolution of envelopes. However, we have observed, as demonstrated in Figures 4.4 and 4.5, that while the force captured per cell is higher in clumpy regions, the specific force (acceleration) exerted upon each cell is larger in lower-density, porous regions. This observation may become significant in dynamical modeling, where its sets the expectation that the material remaining in the porous space between the clumps will also be the material most strongly accelerated by the radiation forces. In turn, this should contribute to the further evacuation of material from the porous inter-clump regions, either expelling the material or driving it into new clumps. Thus, we may anticipate that the overall degree of clumping in the region should intensify over time, and with it the impact of this on the further capture and escape of photons.

4.6 Chapter appendix A: contrast fields

In this subsection, we will outline our procedure for generating our contrast fields, labeled as $P(r)$, which we apply multiplicatively to the parent envelopes to introduce inhomogeneities, yielding a new density field, $\rho_P(r) = \rho(r)P(r)$. In all cases, we impose the condition that for a parent envelope of mass $M_{\text{initial}}$, the new asymmetrical
envelope produced from the application of the contrast field conserves the original mass, such that \( \int \rho P(r) dV = M_{\text{initial}} \).

We generate \( P(r) \) as realizations from a set of random seeds, \( S \), in a procedure taking direction from Lewis & Austin (2002) and an iterative log-normal random density field generator \(^2\). In this paper, we adopt \( S = \{0, 10, 20, 30, 40, 50\} \) for our seeds, which we shall herein refer to Cases A-F respectively, to initialize the Python random number generator. Once so initialized, we generate an \( N^3 \) cell cube of Gaussian random noise with a mean value of \( \mu = 0.0 \) and \( \sigma = 0.01 \); in this paper, \( N = 128 \). This random noise provides the basis for the underlying patterns realized in the contrast field \( P(r) \).

We then take a numerical fast Fourier transform of this Gaussian noise, converting it to a wavenumber space. We then also generate a spectrum \( E \) for the wavenumber space which we will multiply against this noise. The purpose of this spectrum is to impose a characteristic scale to the physical extent of the clumps produced by the noise; by varying the index given to this spectrum, we can produce smaller or larger clump structures. To this end, we choose a corner of the wavenumber space cube to serve as an origin, then for that corner’s octant measure the distance the distance in cell lengths of each cell from that corner as \( k \). We then assign \( E = k^\beta \) for all cells in that octant; for this paper, we consider the cases of \( \beta = \{-1, -1.25, -1.5, -1.75, -2.0\} \). Next, we reflect these values across axes of symmetry to all other octants of the cube. Afterwards, we set \( E = 1 \) at the origin.

After multiplying the noise and the spectrum together, we return the result to real space by performing a numerical inverse Fourier transform upon the cube in wavenumber space. This produces an underlying contrast field, \( P_0 \), which we then operate upon. Particularly, we wish to explore the effects on the radiation transfer when the intensity of the clumping produced by the contrast field is varied, so we

multiply $\mathcal{P}_0$ by a set of possible amplitude factors, $q = \{0.0, 0.2, 0.4, 0.6, 0.8, 1.0\}$, yielding new fields $q\mathcal{P}_0$. Then, to roughly approximate a lognormal distribution for the contrasts, we exponentiate the result, transforming the fields to $\exp(q\mathcal{P}_0)$. Finally, we apply a normalization factor to create a final contrast field $\mathcal{P}$ where the condition $\int \rho \mathcal{P}(r) dV = M_{\text{initial}}$ is enforced.

At this point, the completed contrast field $\mathcal{P}$ may be applied to parent envelopes to introduce inhomogeneities in accordance to our spectrum parameter $\beta$ and the amplitude factor $q$. Note that in the case of $q = 0$, $\exp(q\mathcal{P}_0) = 1$ for all points in the envelope, in which case applying $\mathcal{P}$ will simply return the input parent envelope.

### 4.7 Chapter appendix B: virial analysis for internal motions

We wish to supply a brief justification for our definition of the radiation virial term $\mathcal{R}$ and its relation to a cloud’s centre of mass (CM), so we review here a result from elementary mechanics. Let $\mathbf{r}(dm)$ be the position vector of a mass element $dm$ in an inertial reference frame, so that $d\mathbf{F} = \mathbf{\ddot{r}} dm$ is the differential of force. Let $\mathbf{r}_{\text{cm}}$ be the centre of mass, defined by $M\mathbf{r}_{\text{cm}} = \int \mathbf{r} dm$ (where $M = \int dm$ is the cloud mass and all integrals extend over the cloud volume and surface), and let $\mathbf{\hat{r}} = \mathbf{r} - \mathbf{r}_{\text{cm}}$ be the position relative to the cloud CM, so that $\int \mathbf{\hat{r}} dm = 0$.

The trace of the cloud moment-of-inertia tensor is $I = \int \mathbf{r}^2 dm$. The virial theorem (in Lagrangian form: McKee & Zweibel 1992) follows from writing $\dddot{I}/2 = \int \mathbf{r}^2 dm + \int \mathbf{r} \cdot \mathbf{\ddot{r}} dm$, then recognizing the former term as twice the kinetic energy, and the latter, $\int \mathbf{r} \cdot d\mathbf{F}$, as a series of energies and surface terms. Given our definitions,

$$I = M\mathbf{r}_{\text{cm}}^2 + \int \mathbf{\hat{r}}^2 dm \equiv I_{\text{cm}} + \hat{I}$$

(4.4)

and therefore $I = I_{\text{cm}} + \hat{I}$. The first of these terms can be written $I_{\text{cm}} = M\mathbf{r}_{\text{cm}}^2 + \mathbf{r}_{\text{cm}} \cdot \mathbf{F}$, which is the virial theorem for the motion of the CM. Defining the internal kinetic
energy $\hat{T} = \frac{1}{2} \int \dot{\hat{r}}^2 \, dM$, the second term is
\[
\begin{align*}
\hat{I} &= \int \dot{\hat{r}}^2 \, dM + \int \dot{\hat{r}} \cdot (\hat{r} - \hat{r}_{cm}) \, dm \\
&= 2\hat{T} + \int \dot{\hat{r}} \cdot dF + \hat{r}_{cm} \cdot \int \dot{\hat{r}} \, dm
\end{align*}
\tag{4.5}
\]
in which the final term is zero by virtue of the definition of $\hat{r}$.

Equations 4.4 and 4.5 express the fact that the internal motions of the cloud, which represent structural changes, separate cleanly from the CM motion and obey their own virial theorem in which position relative to the CM is the relevant radial vector.

Like all the other virial terms, the net radiation virial term $\mathcal{R} = \int \mathbf{r} \cdot d\mathbf{F}_{\text{rad}}$ decomposes into two terms: one that appears in the CM virial theorem for $\ddot{I}_{cm}$, which is $\mathcal{R}_{cm} = \mathbf{r}_{cm} \cdot F_{\text{rad}}$; and another that appears in the internal virial theorem for $\ddot{\hat{I}}$, which is
\[
\hat{\mathcal{R}} = \int \dot{\hat{r}} \cdot d\mathbf{F}_{\text{rad}}.
\tag{4.6}
\]
As we are primarily concerned with the effect of radiation forces on the internal cloud structure, rather than the magnitude of a radiation-driven rocket effect, we omit the hat and refer to $\hat{\mathcal{R}}$ as $\mathcal{R}$ in the main text.

Finally, we note that forces between cloud elements are governed by Newton’s third law, so that their contribution to $\mathbf{F}$ is zero and so is their contribution to $\ddot{I}_{cm}$; for such forces $\ddot{I}$ does not depend on the centre-of-mass location. Radiation forces are not in this category, because of the reaction force that can accumulate if radiation is incident from one direction, or preferentially escapes in another. So, for radiation, the internal virial theorem depends on a cloud’s CM and thus on its mass distribution. A study like ours must therefore specify the distribution of mass as well as that of opacity, in order to unambiguously define $\hat{\mathcal{R}}$. In our calculation of $\mathbf{r}_{cm}$ we make the simplifying assumption that dust density traces mass density.
Chapter 5

Radiation Forces in AMR Grids

5.1 Preface

This chapter details research in progress, currently unpublished, that I have conducted regarding radiative transfer modeling in more complex environments represented in adaptive mesh refinement (AMR) grids. This chapter makes use of data from an ORION dataset for massive star formation provided by Anna Rosen in a private communication to provide initial conditions for models within, particularly density structures. Additionally, Rosen provided a zero-age main sequence properties calculator for estimating certain properties used to provide the initial conditions of the luminosity source.

5.2 Chapter abstract

Adaptive mesh refinement (AMR) methods are commonly utilized in hydrodynamical simulations to resolve processes over a wide range of scales. These methods identify regions requiring increased resolution and introduce more highly-refined grids in these areas, while allowing regions that do not require such resolution to remain more
coarsely resolved. Utilizing density information from an AMR gridded dataset, I conduct Monte Carlo simulations to study the resolution effects of AMR data on radiative transfer solutions. Failing to sufficiently resolve regions may lead to the suppression of inhomogeneities in the density structure, particularly lower-density channels which would facilitate the escape of photons, thus contributing to an overestimation of the photon momentum captured. However, other resolution-based effects may compete with this influence. As the resolution becomes coarser, the maximum temperature of the central region around the star diminishes, which may cause reprocessed photons to be emitted at longer wavelengths. This will lead them to experience lower opacities to their wavelengths, allowing them to escape more easily and thus contributing to an underestimation of the captured photon momentum. Therefore, these two effects with coarsening resolution, the suppression of channels and the underestimation of temperatures, behave in opposition to each other.

5.3 Introduction

Many astrophysical processes can stretch across a wide range of physical scales. For example, in the problem of star cluster formation, one may consider properties and processes both on scales much larger and much smaller than that of the cluster itself. For instance, in the former case, there is the context of the surrounding giant molecular cloud (GMC) in which the cluster is embedded, while in the latter case, the formation of and feedback from the individual constituent stars may also be relevant.

The wide range of scales possible in such problems may pose a great challenge to conducting numerical simulations of these problems, thanks to the existence of a number of stability criteria for running hydrodynamical and self-gravitating hydrodynamical problems with common numerical schemes. For instance, Courant-Friedrichs-Lewy condition (Courant et al. 1928) places limitations on how much a flow may be allowed to advect within a given timestep, often requiring smaller timesteps
as the speed of the flow increases. Likewise, the Jeans criterion of Truelove et al. (1997) places a requirement for the number of cells that must be used for resolving the Jeans length (Jeans 1902) of a region, so as a region becomes denser and the Jeans length contracts, a more finely-resolved grid becomes necessary. Such criteria place additional demands upon computational resources, but often only a specific region within the model requires the maximum level of resolution specified by these conditions. Therefore, it is desirable to implement a scheme that saves computational effort by only assigning additional resolution to those areas which need it, rather than making a blanket application of the maximum required resolution across the entire computational grid. To meet this desire, the process of adaptive mesh refinement (AMR) was developed. AMR methods evaluate the state of the simulation throughout its evolution, automatically adding or removing grids with additional refinement to regions as necessary (Berger & Colella 1989), and have been applied to a variety of astrophysical problems, such as the collapse and fragmentation of molecular clouds (Klein 1999).

In this chapter, I consider a dataset provided in a private communication by Anna Rosen, describing a density distribution defined in terms of an AMR grid. The dataset depicts a snapshot from a scenario for massive star formation, involving the collapse of a turbulent, unmagnetized core, with radiation feedback handled with radiation modules from ORION, the code which generated the data. ORION is a modular AMR code including hydrodynamics, gravity, and radiation; refer to Krumholz et al. (2007) for a list of numerous methods papers contributing to each of ORION’s individual modules, and to Rosen et al. (2017) for information on a new radiation solver, HARM², recently added to it. ORION is a proprietary code, and as such is not available for use in the study described in this chapter. However, the dataset is still of use for providing a source of initial conditions to Hyperion (Robitaille 2011), a Monte Carlo radiative transfer code, which I have modified as described in Chapter 3.
In this chapter, I will utilize the AMR density data provided in Rosen’s ORION dataset to investigate the resolution effects of AMR data grids [herein used for the rest of the chapter to refer to both data grids produced by an AMR code and to static data grids structured in the same manner as an AMR grid, with multiple levels of refinement] on radiative transfer solutions determined from Monte Carlo methods. However, I will first repeat a calculation of a spherical dust envelope conducted in Jumper & Matzner (2018a) (Chapter 3) to demonstrate that Hyperion’s AMR calculations will provide a result consistent with those found previously in a fixed-resolution Cartesian geometry, to serve as a test-problem for validation before proceeding.

5.4 Physical problems

Throughout the problems described in this chapter, the radiative transfer parameters previously established in Chapters 3 and 4 are utilized, specifically adopting the latter’s center-of-mass centered implementation of them.

5.4.1 Test problem: spherical dust envelope

For the test problem, as in Jumper & Matzner (2018a) and Chapter 3, a spherically-symmetric dust envelope surrounding a central star, with a blackbody color temperature of $T_\ast = 5772$ K, a density distribution of $\rho(r) \propto r^{-k}$ for $r_{in} < r < r_{out}$ for $k = 1.5$ and $r_{out} = 4r_{in}$, and the same dust properties as provided by Draine (2003a;b) as utilized in those chapters, provides the initial conditions for the model. In contrast to the models in Jumper & Matzner (2018a), which utilized a fixed-resolution grid, the grid used in this model utilizes two grid levels. The coarser level, referred to as the base level, is a $32^3$ cell cube covering the same spatial extent and possessing the same resolution as the $32^3$ model from Jumper & Matzner (2018a). The finer level,
referred to here as the refined level, covers a cube sharing a common center with the base level (the star) and with spatial extents of \( \frac{1}{2} \) the length in each direction; thus, it occupies a volume \( \frac{1}{8} \) of the total. For this model, I adopt a factor of eight (8) change in resolution between the refinement levels, such that a \( 128^3 \) cell cube resolves this region, with four times the number of cells per side resolving dimensions with half the total length. A total of \( 10^6 \) photon packets are utilized in this model, representing an average of about 30.5 photon packets per base level cell in the model.

### 5.4.2 Radiation forces in an ORION dataset

Rosen’s provided ORION dataset depicts a cube with sides of 0.4 pc and a mass of about 117.1 \( M_\odot \) contained within its grid cells. I approximate the mass therein as entirely a gas mass, and so must convert the associated densities from a gas density to a dust density to determine the optical depths posed by the dust in the radiative transfer problem. I adopt a dust model described in Draine (2003a;b), as previously adopted in Chapters 3 and 4, making the same approximations as previously discussed. Then, I assume \( \frac{M_{\text{gas}}}{M_{\text{dust}}} = 105.1 \) as in this dust model to make the conversion from gas densities to dust densities. The resulting density structure is represented by an AMR grid with six levels. The coarsest level, herein referred to as the base level or the 0th refinement level, has a grid resolution of \( 128^3 \). The data represented by grids on each subsequent, more refined level of resolution (referred to as refinement levels) have dimensions of half the length of the prior level.

Additionally, as a simplification and to facilitate my intent to explore resolution effects in Monte Carlo modeling, I make use of a covering grid algorithm provided by the yt package (Turk et al. 2011) to extract density data from specified regions within the dataset and to impose fixed resolutions (a particular refinement level) over the resulting grids. Data from cells in grids on the same refinement level as that requested are preserved unchanged by this process; the cells of grids more coarsely refined than
requested are split into smaller cells, while the opposite (merger) is done where the grid resolution is finer than requested. Hyperion allows for multiple levels of AMR grids to be defined that overlap over the same location, but calculates the radiative transfer solution on the most finely-resolved grid available at a particular region. Since all the grids input in this model are treated with covering grids utilizing the same underlying data, only at different degrees of resolution, this allows me to input several layers of grids to represent a hierarchy of resolutions of this data. Furthermore, by omitting the covering grids for higher refinement levels, I can explore resolution effects by forcing the calculations to utilize a representation of the density data on a coarser level.

In the models conducted in this chapter, I utilize a maximum of six covering grids, layered upon each other in a hierarchy as described above, to provide the densities based on the Rosen dataset to the Hyperion radiative transfer calculation. The base level covering grid encompasses the entirety of the dataset, and shares its resolution, $128^3$ cells, with the grid dimensions of the coarsest AMR grid of the Rosen dataset. The covering grids for the next five refinement levels (1st - 5th) are initialized with sides of half the length of the previous level, while retaining $128^3$ cell resolution within these smaller regions, thus doubling the spatial resolution per unit length on each level. The center of the first refinement level is placed slightly offset from the center of the overall grid, in consideration of the sink particle star’s own offset from the center, such that it may be located near the center of this refined level. All deeper refinement levels extend over a central cubical region embedded within the previous refinement level.

In addition to the mass contained within the dataset’s grid cells, the Rosen dataset also includes a sink particle, representing a zero-age main sequence star mass of $26.3M_\odot$, which will provide the luminosity source in this problem. The star is located near but not exactly at the center of the cube, with offsets of $-1.19 \times 10^{16}$
cm, $-2.18 \times 10^{16}$ cm, and $-0.70 \times 10^{16}$ cm from the center in the three coordinate directions. Utilizing a zero-age main sequence properties calculator provided by Rosen along with the dataset, I calculate that this source has a luminosity of $L = 3.33 \times 10^{38}$ erg/s, or $8.70 \times 10^4 L_\odot$, a radius of $4.96 \times 10^{11}$ cm, or $7.12 R_\odot$, and an effective surface temperature of $3.71 \times 10^4$ K. These values are used as inputs to define the luminosity in the Hyperion models of Monte Carlo radiative transfer conducted in this chapter.

In this chapter, I conduct models using covering grids extending down to a maximum of 0, 2, 3, 4, and 5 refinement levels. For the majority of these models, I utilize $6.4 \times 10^6$ photon packets, corresponding to an average of about 3.05 photon packets per cell in the base level covering grid, a lesser amount than used in the previous chapters. This choice was made to reduce the computational time necessary to complete these models. To evaluate the impacts of this choice, I also ran a model with a maximum of 5 refinement levels and $64 \times 10^6$ photon packets, corresponding to an average of about 30.5 photon packets per cell in the base level.

5.5 Results and discussion

5.5.1 Test problem

Figure 5.1 illustrates the AMR grid structure initialized for the spherical dusty envelope described in §5.4.1. As previously noted, the refined grid in this test problem covers a cube with sides of half the length as that of the base level. The configuration of $32^3$ cells in the base level and $128^3$ cells in the refined level provides the inner region with comparable resolution to that found in the $256^3$ model of Jumper & Matzner (2018a) and Chapter 3 for the given optical depth ($\tau_{\text{fid}} = 10.0$ per the conventions of that chapter, or an optical depth to starlight of $\tau_\star = 54.7$). In contrast to the $256^3$ model in that prior chapter, which utilized $5.12 \times 10^8$ photon packets to maintain a standard value of $10^6$ packets per $32^3$ cells (about 30.5 packets/cell), this test problem only
Table 5.1: Results of the test problem for a spherically symmetric dusty envelope comparing a fixed-resolution grid model, using $5.12 \times 10^8$ photon packets, against an AMR grid model representing the same density distribution, using $10^6$ photon packets. The listed values for the parameters $\Phi$, $\langle \hat{r} \rangle_F$, and $\hat{R}$ are rounded to two decimal places. These values are found to be identical within the rounding for each parameter.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Phi$</th>
<th>$\langle \hat{r} \rangle_F$</th>
<th>$\hat{R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed-Resolution</td>
<td>5.12</td>
<td>1.60</td>
<td>8.19</td>
</tr>
<tr>
<td>AMR</td>
<td>5.12</td>
<td>1.60</td>
<td>8.19</td>
</tr>
</tbody>
</table>

utilizes $10^6$ packets, applying that standard value against its base level rather than its refined level. A slice of the resulting specific force in the envelopes is illustrated as an example in Figure 5.2. I find that both models find identical results to two decimal places in the parameters $\Phi$, $\langle \hat{r} \rangle_F$, and $\hat{R}$, as shown in Table 5.1; thus, the test problem is successful.

### 5.5.2 Resolution study

First, to give visualization examples of the AMR grid used in this study and the specific forces captured by the radiative transfer in it, I provide visualization slices of these quantities, utilizing a lognormal scale. Each of the illustrations presented here represents the AMR grid model including covering grids for the base level plus all five further refinement levels. In Figure 5.3, I display the density, while in Figures 5.4 and 5.5 I show the specific forces for the $6.4 \times 10^6$ and $6.4 \times 10^7$ photon packet models respectively.

Next, I consider the behavior of dimensionless parameters characterizing the force capturing in the environment. As in Jumper & Matzner (2018a) and Jumper & Matzner (2018b) (refer to Chapters 3 and 4), I make use of the dimensionless parameter $\Phi$ to characterize the total outward force exerted relative to the dust’s center of mass. I also
Figure 5.1: Slice of the density, through the center, in units of g cm$^3$, for an AMR grid representation of a model previously considered with a fixed-resolution grid in Jumper & Matzner (2018a) and Chapter 3, a spherically symmetric dust envelope with an optical depth of $\tau_{\text{fid}} = 10.0$. This corresponds to an optical depth to starlight of $\tau_* = 54.7$. The coarse grid, labeled as “Base Level” is resolved with $32^3$ cells. The fine grid, labeled as “Refined Level” is resolved with $128^3$ cells for dimensions of half the length on each side; it has eight times the resolution of the base level grid.
Figure 5.2: Slice of the magnitude of the specific force in units of cm s$^{-1}$, through the center, for the model shown in Figure 5.1. The specific force is calculated at the highest provided resolution within a given region. Thus, the central, square-shaped void seen in the slice in the left panel corresponds to the region where the specific force is instead calculated at the refined level, which is displayed in the right panel. In contrast, the circular, central void seen in the right panel corresponds to the location of the empty, central cavity embedded within the envelope.
Figure 5.3: Slice of log$_{10}$ of density in g cm$^{-3}$ for the Rosen dataset provided in private communication, across six levels (the base level and five refined levels) of an AMR grid model. The base level slice is taken through the region containing the sink particle (slightly off-center), while the remaining slices are through the center of their respective grids. For the region included within each level, the data has been smoothed with a covering grid algorithm from yt. The base level grid has dimensions of 0.4 pc on a side, and each level of refinement beyond this halves the lengths of its dimensions. As each refinement level resolves its region to $128^3$ cells, the grid resolution thus doubles with each level.
Figure 5.4: Slice of the $\log_{10}$ of the magnitude of the specific force, in units of cm s$^{-1}$, for the model shown in Figure 5.3, for a Hyperion model using $6.4 \times 10^6$ photon packets, corresponding to a total of approximately 3.05 photon packets per cell at the base level resolution of $128^3$. For a given region, the calculation of the specific force is made at the most highly refined grid included for that region; here, this is imposed by the positions of the embedded yt covering grids. Thus, regions for which a more highly-refined grid is available appear in these slices as white, squared-shaped regions; the specific force within is portrayed at the more highly-refined levels. The locations of the slices follow the same convention described in Figure 5.3.
Figure 5.5: Slice of the log\(_{10}\) of the magnitude of the specific force, in units of cm s\(^{-1}\), for the same model shown in Figure 5.3 and 5.4, except using \(6.4 \times 10^7\) photon packets, corresponding to a total of approximately 30.5 photon packets per cell at the base level resolution of 128\(^3\), tenfold the number used in Figure 5.4. The locations of the slices follow the same convention described in Figure 5.3.
Table 5.2: Variation of $\Phi$ and $R / (L_c)$ across the listed models. The models utilize the same underlying density data, but vary how many levels of refinement the most resolved covering grids in the model are allowed to make, thus altering the spatial resolution with which the input density structures are represented. There is also a model covering down to the fifth refinement level for two different numbers of photon packets.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Phi$ [dimensionless]</th>
<th>$R / (L_c)$ [in cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Refinements, $6.4 \times 10^7$ Photon Packets</td>
<td>13.74</td>
<td>$3.43 \times 10^{18}$</td>
</tr>
<tr>
<td>5 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>13.74</td>
<td>$3.43 \times 10^{18}$</td>
</tr>
<tr>
<td>4 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>13.49</td>
<td>$2.38 \times 10^{18}$</td>
</tr>
<tr>
<td>3 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>14.54</td>
<td>$1.73 \times 10^{18}$</td>
</tr>
<tr>
<td>2 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>20.20</td>
<td>$1.52 \times 10^{18}$</td>
</tr>
<tr>
<td>0 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>13.13</td>
<td>$7.58 \times 10^{17}$</td>
</tr>
</tbody>
</table>

consider an expression of the radiative virial term, $R$; dividing out the photon force of the luminosity, $L_c$, I obtain $R / (L_c)$, which has units of cm.

Table 5.2 demonstrates the variation of these parameters across the models conducted in this study. First, comparing the two models including five refinement levels (in addition to the base level), which vary only in the number of photon packets used in the modeling, one finds that both $\Phi$ and $R / (L_c)$ are identical within the two decimal places listed in the table. Thus, varying the number of photon packets by a factor of ten did not have a significant effect on the outcome of the model.

Next, considering the variation of these parameters with the maximum number of utilized refinement levels, one finds that $R / (L_c)$ diminishes as the maximum refinement level also decreases. At first, this seems to run contrary to the expectations set by Jumper & Matzner (2018b) and Chapter 4, in which I showed that $\hat{R}$ tended to decrease (along with $\Phi$) as the degree of clumping in the environment increases, as the channels opened up between these clumps facilitated the escape of photon momentum.
Coarser grids, as found at lower levels of refinement, will tend to suppress clumping with the region contained by making smaller-scale variations which would have contributed to it in a more resolved grid.

Indeed, such a suppression of clumping is seen in the grids used in this chapter, as shown in Tables 5.3 and 5.4. In both tables, I consider a clumping factor, $\langle \rho^2 \rangle / \langle \rho \rangle^2$, to characterize the degree of clumping within a region. As this factor increases, mass will become increasingly concentrated in dense clumps, but will also allow the opening of channels between these clumps. In the former table, I consider the difference between the clumping factor found in a particular refinement level, and the clumping factor in the region of the previous refinement level in which the current refinement level is embedded. In each case, the coarser grid from the level above the specified one represents less clumping than is found at the deeper, specified level, as would be expected. This effect also becomes more prominent for each successive pair of consecutive coarser levels. In the latter table, I consider a similar problem, but instead compare the clumping in the portion of the covering grid which coincides with the region containing the 5th refinement level, and compare the clumping within this region at a coarser resolution to the actual value found in the 5th refinement level’s covering grid ($\langle \rho^2 \rangle / \langle \rho \rangle^2 = 45.25$). This latter comparison is relevant in that it concerns itself with the region in the vicinity of the star, where any early leaking of photons may help some of them escape to further out, cooler regions, where they may be absorbed and reprocessed at lower temperatures and wavelengths, thus facilitating their further escape from the region; suppressing this effect should tend to lead to an overestimation of the captured force.

In addition, $\Phi$ also does not appear to behave in the manner that one would expect simply from the suppression of clumping and the associated escape of photons through channels, which would also tend to lead to an overestimation of $\Phi$ with decreasing resolution. However, here, the actually observed behavior of $\Phi$ is more
Table 5.3: Comparison of the clumping factor $\langle \rho^2 \rangle / \langle \rho \rangle^2$ in the covering grid used in the AMR grid model at the specified level against the clumping factor for only those cells whose location coincides with the embedded position of the covering grid at the next highest level of refinement. Compare values in the left column to the values in the next row in the right column (visually, diagonally to the right of their position) to find the two representations of the clumping in that space: a finer grid (left column) and a coarser grid (right column, next row). One sees that the coarser grids suppress the representation of the clumping (as measured by $\langle \rho^2 \rangle / \langle \rho \rangle^2$).

<table>
<thead>
<tr>
<th>Grid Level</th>
<th>Grid $\langle \rho^2 \rangle / \langle \rho \rangle^2$</th>
<th>Grid’s $\langle \rho^2 \rangle / \langle \rho \rangle^2$ for Region of Next Refinement</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th Refinement</td>
<td>45.25</td>
<td>–</td>
</tr>
<tr>
<td>4th Refinement</td>
<td>51.88</td>
<td>40.40</td>
</tr>
<tr>
<td>3rd Refinement</td>
<td>53.97</td>
<td>39.86</td>
</tr>
<tr>
<td>2nd Refinement</td>
<td>68.19</td>
<td>30.93</td>
</tr>
<tr>
<td>1st Refinement</td>
<td>181.29</td>
<td>36.82</td>
</tr>
<tr>
<td>Base Level</td>
<td>631.68</td>
<td>94.73</td>
</tr>
</tbody>
</table>
Table 5.4: Variation of the clumping factor $\langle \rho^2 \rangle / \langle \rho \rangle^2$ for a covering grid at a specified refinement level, only including cells which have the location of the deepest (5th refinement) level’s covering grid embedded within. (Each refinement level has a grid with sides half as long as in the previous level). This explores how well-resolved clumping located in the regions close to the star’s position (in the 5th refinement level) is at coarser levels. As channels between clumps may allow photon momentum to leak more easily away from the star, the suppression of such clumping may also suppress this effect, driving the model towards an overestimation of photon forces. The value of $\langle \rho^2 \rangle / \langle \rho \rangle^2$ for the fifth level covering grid is 45.25.

<table>
<thead>
<tr>
<th>Grid Level</th>
<th>Grid’s $\langle \rho^2 \rangle / \langle \rho \rangle^2$ for Region of 5th Level Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th Refinement</td>
<td>45.25</td>
</tr>
<tr>
<td>4th Refinement</td>
<td>40.40</td>
</tr>
<tr>
<td>3rd Refinement</td>
<td>30.79</td>
</tr>
<tr>
<td>2nd Refinement</td>
<td>16.83</td>
</tr>
<tr>
<td>1st Refinement</td>
<td>8.30</td>
</tr>
<tr>
<td>Base Level</td>
<td>6.64</td>
</tr>
</tbody>
</table>
complicated than for $\mathcal{R}/\left(\frac{L}{c}\right)$, for which one observes a decrease in the parameter value at in each case. Instead, one finds that: (1.) the $\Phi$ value of this model slightly decreases between the 5 refinements and the 4 refinements models, from 13.74 to 13.49, (2.) then increases as one moves to the 3 refinements and 2 refinements models (14.54 and 20.20), and (3.) decreases once again as it moves to the 0 refinements, obtaining a value of 13.13.

In this case, what may account for the behaviors exhibited by $\mathcal{R}/\left(\frac{L}{c}\right)$ and $\Phi$? Recalling Jumper & Matzner (2018a) (see Chapter 3), a potential explanation is that the reduced resolution of the coarser grids may lead to a systemic underestimation of the temperatures in the central regions nearby the star. In turn, any photons absorbed and reprocessed at those lower temperatures would have a lower wavelength and energy than they would have had if reprocessed at higher temperatures. In turn, the lower wavelengths would tend to help the photon escape from the environment more readily [due to experience reduced optical depths to its wavelength], while also causing it to deposit less energy in the event that it interacted in the environment. A similar effect was previously discussed in the context of channels in the environment allowing photons to potentially escape to regions with a lower temperature, but a key difference between these scenarios is that the channeling effect is a physical outcome suppressed by coarsely-resolved grids, while the temperature underestimation effect discussed in this paragraph is a numerical error brought on by insufficient resolution.

To explore the potential of this effect, I examine the maximum temperatures found in the regions around the star (considering the highest level covering grid in each model). As shown in Table 5.5, the maximum temperature found in any of the models is 3289 K, occurring for the models including the maximum used number of refinement levels (5); this is as expected. Then, as one moves to increasingly coarse grids, the maximum temperature falls to 2438 K with 4 levels, 1523 K and 1544 K with 3 and 2 levels, and 870 K with 0 levels of refinement (the base level).
Considering these temperatures in combination with the suppression of clumping as discussed earlier provides possible insights into the seemingly unusual behavior of the $\Phi$ parameter in these models. The difference between the clumping factor for the near-star region of 4th refinement level and the clumping factor found for the 5th refinement level is relatively small (40.40 vs. 45.25). Therefore, the change in the channel-dominated effects of photon escape due to the suppression of clumping in this case may be comparatively small in this case. In the same transition between models, the maximum temperature attained drops by 851 K, or about 25.9%. One expects the decrease in the achieved temperature to contribute to reducing the calculated value of $\Phi$, while the suppression of clumping should contribute to increasing it; it appears that the temperature-dominated effects may dominate in this case.

As one proceeds to the next model, with a maximum at the 3rd refinement level, the suppression of clumping increases significantly, while the temperature drop from the previous model is of a comparable amount (915 K between the 4th and 3rd refinement models, vs. 851 K between the 5th and 4th refinement models). The growing prominence of the clumping suppression may account for the increase in $\Phi$ in this interval.

For the next pair of models, moving from the 3rd refinement model to the 2nd refinement model, the maximum temperature achieved changes relatively little, so little change is anticipated from the temperature-dominated errors (unlike in prior cases), while the clumping and channel suppression continues unabated with the coarser grids. Thus, it appears likely that the large increase in $\Phi$ in this step is a channel-dominated effect, which would indeed be expected to increase in response.

Finally, for the base level model with no refinement levels, the captured $\Phi$ value decreases again, to the smallest value found in this examination, at 13.13 This is probably again a temperature-dominated effect, and the maximum temperature has indeed fallen to 870 K by this point.
If this understanding of the competing nature of temperature-dominated and channel-dominated effects is correct, then it also appears that temperature-driven effects may possibly also account for the unexpected behavior of $\mathcal{R} / (\frac{L}{c})$, in which it decreased with decreasing clumping rather than increasing. Bringing temperature effects into consideration, the reduced temperature in coarser grids may reduce the radiative force $F_{\text{rad}}$ exerted throughout the grid, the driving $\mathcal{R}$ and $\mathcal{R} / (\frac{L}{c})$ lower in term. In this case, it appears that $\mathcal{R}$ may be more strongly influenced by temperature effects than clumping effects when varying the resolution.

A key caveat to the preceding arguments is that although an examination of the values discussed within may hint at a driving information, more information and analysis from a larger number of models, is needed to further explore and test this notion, as well as to quantifying the scaling of such behaviors.

In consideration of such effects, what steps could be taken to combat such problems in a similar AMR grid problem? As discussed in Jumper & Matzner (2018a), the temperature underestimation effects found in the Monte Carlo method manifest in optically thick regions where the mean free path of the starlight is not adequately resolved; these effects weaken as the resolution around the star is improved. Thus, adding additional grids of higher refinement level around the star could help capture the stellar temperature more accurately, and thus minimize the force capturing error due to this component. However, for extremely optically thick regions, this may require a large number of additional refinement levels; while this may be feasible for a radiative transfer snapshot as in these models, it may prove to be prohibitive in dynamical models. Such an increase in local refinement would also help resolve the channel-dominated error, but only in cases where additional clumpy structure was being hidden by the use of a coarser level that available â€’ such as induced in from the smoothing of the density structures into coarser grids through use of the covering grid in this chapter.
Table 5.5: For the same models presented in Table 5.2, the variation in the dimensionless force-capturing parameter $\Phi$ and the maximum temperature achieved in the region surrounding the star.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\Phi$ [dimensionless]</th>
<th>$T_{\text{max}}$ [in K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 Refinements, $6.4 \times 10^7$ Photon Packets</td>
<td>13.74</td>
<td>3289</td>
</tr>
<tr>
<td>5 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>13.74</td>
<td>3289</td>
</tr>
<tr>
<td>4 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>13.49</td>
<td>2438</td>
</tr>
<tr>
<td>3 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>14.54</td>
<td>1523</td>
</tr>
<tr>
<td>2 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>20.20</td>
<td>1544</td>
</tr>
<tr>
<td>0 Refinements, $6.4 \times 10^6$ Photon Packets</td>
<td>13.13</td>
<td>870</td>
</tr>
</tbody>
</table>

**5.6 Conclusions**

In this chapter, I have explored the variations in calculated radiation pressure forces, as summarized in a pair of parameters, $\Phi$ and $R$ (expressed in the form of $R/(L/c)$) as grid-resolutions are varied in an AMR grid model. As the resolution of the region decreases, a systemic underestimation of maximum temperatures in the region ensues, reinforcing conclusions from previous works. This effect, which can contribute to a reduction in radiation pressure forces, may have a key driving effect on the behavior of $R$, which is shown to decrease as the density structures represented on the grid are represented on increasingly coarser grids. However, these temperature-dominated effects do not appear sufficient to account for the behavior of $\Phi$ in response to these changing resolutions. Instead, it appears that a second mechanism, dominated by the suppression of channels at lower resolutions, may be contributing to these outcomes, providing an opposing effect.

The competing nature of the influences from temperature-dominated and channel-dominated effects may provide challenges in the understanding errors from radiative
transfer techniques in AMR grids, as this competition may lead to partial cancellations of opposing error contributions for the captured forces. Additional modelling over a wider variety of simulations and initial conditions may help provide further insight on this problem. Imposing additional refinement levels around the star to help resolve the mean free path of the starlight may help address the temperature error found in Monte Carlo methods, but may be prohibitive in very optically thick models, especially for dynamically models.
Chapter 6

Conclusions

6.1 Conclusions

I developed an approximate model for the growth of a star cluster-forming clump by accretion from its environment, incorporating the interactions between gas accretion, feedback, and turbulent driving and decay within the cluster. In addition to the virial parameter, $\alpha$, I adopt the dimensionless parameter $\xi_c$ (equation 2.8) for a virialized clump to help complete these relations in an analytically feasible manner. For a constant virial parameter, the turbulence in the clump scales proportionately with the clump’s escape speed, $\sigma_c \propto v_{esc,c}$, resulting in turbulent driving on the order of $\dot{M}_{\text{in}} \sigma_c^2$. I also find that turbulent decay scales with $\sigma_c^5/G$ and achieves balance with turbulent driving for $\xi_c \approx 1$. Assuming a characteristic wind momentum per unit mass, $v_{ch,w}$, this model estimates the force of protostellar winds due to outflows exerted on the clump as $\dot{M}_* v_{ch,w}$, driving turbulence to $\sim \dot{M}_* v_{ch,w} \sigma_c \sim \text{SFR}_f \sigma_c^4 v_{ch,w}/G$. Furthermore, as in Matzner & McKee (2000), I consider an upper limit to the mass expulsion per star as a consequence of the outflow momentum. A key prediction resulting from this analytical model is that the feedback from clusters will not be strong enough to expel mass at a sufficient rate to overcome and reverse the accretion flow onto low-mass clusters. Therefore, stellar feedback will not shut down low-mass
cluster formation through this mechanism. Nonetheless, low-mass clusters will have outflow feedback that will drive significant turbulence within them.

I conducted Monte Carlo radiative transfer models in Hyperion, along with scaling solution models in DUSTY, to characterize the capturing of photon momentum in dusty envelopes and the radiation forces that resulted. I utilized a number of integral quantities for characterizing the importance of these forces and their effects on the surrounding environment. These included a normalized radial force, $\Phi$, a force-averaged radius, $\langle r \rangle_F$, and a radiative virial term, $\mathcal{R} \equiv \int \mathbf{r} \cdot d\mathbf{F}$, which relates to the dynamical effect of the radiation force.

Using comparisons of the highly-resolved DUSTY results for such parameters (in spherical geometry) against varying resolutions of Hyperion Monte Carlo models, I developed calibrations for the accuracy with which these force parameters were determined by Monte Carlo methods. Moreover, I showed that under-resolution of the spatial grids for Monte Carlo methods relative to the starlight mean free path resulted in a systemic underestimation of the resulting temperatures, manifesting as the starlight photons were deposited over too thick of a layer. In turn, this temperature underestimation was associated with a concurrent underestimation of the radiation forces captured by this envelope. This error may operate through two complementary mechanisms. First, the lower temperatures achieved by the dust will lead to the emission of photons with lower energies, so they will deposit less energy during their interactions of the dust. Second, the photons emitted from these cells will tend to escape from the region more easily, as their reduced wavelengths will tend to ease their escape from the region. To address this error, it may be desirable to locally-refine the grid in the vicinity of the luminosity source to resolve the starlight mean free path.

I also investigated analytical models for the radiation force in a spherical dusty envelope, and compared the results against numerical radiative transfer simulations. As radiation reprocessing in response to the absorption of starlight photons by dust
leads to the emission of infrared radiation, and since this infrared radiation in turn interacts with the dust far less readily than did the starlight, this leads to the emergence of three key regimes. The first is the thin regime, where the envelope is optically thin to both starlight and infrared radiation. The second is an intermediate regime, where the envelope is optically thick to starlight but remains optically thin to the reprocessed infrared radiation. The third is the thick regime, where the envelope is optically thick to both starlight and infrared radiation.

Next, I showed how the force parameters would vary in response to the introduction of inhomogenities, including clumping and channels, to previously spherically-symmetric dusty envelopes, holding the masses of the envelopes constant under this transformation. I characterized the introduced inhomogenities as a function of their strength, summarized by a clumping factor $\langle P^2 \rangle / \langle P \rangle^2$, where $P$ is the contrast factor between the inhomogeneous and base states in each cell, and the scale of the perturbations, in terms of the index of a generating spectrum. Although there was variation between individual realizations of the applied inhomogenities, generally the normalized radial force $\Phi$ and the radiative virial term $R$ would diminish with an increasing clumping factor, while the force-averaged radius would increase. Of these, the radiative virial term was significantly less sensitive to variations in the realizations of the clumping, and thus may be the more useful summary parameter in such environments.

Fitting the scaling of these force measures to power laws, I further characterize the dependence of these measures on the strength and scale of the clumping in their environment by interpolating the data with a contour plot, providing rule-of-thumb estimations. The power-law indices of the scaling for the various measures are at their greatest magnitudes in optically-thick regimes, which conforms to expectations, as the channels opened in such regions between the clumps may allow the photons to bypass very optically thick regions. In contrast, the power-law scaling was weakest
in optically thin regimes. Nonetheless, the scaling with clumping was relatively weak in all cases. Moreover, the power-law scaling of $\Phi$ and $R$, tends to weaken further in the case of small-scale (in size) inhomogenities, while it intensifies with large-scale inhomogenities. In contrast, the force-averaged radius, $\langle r \rangle_F$, remains relatively insensitive to the size scale up to regions of a few optical depths.

Additionally, I briefly consider the influence of clumping in such envelopes on the resulting SEDs that would be observed, characterized by the SED slope. In optically thick regions, the SED slope tends to vary with the clumping factor, while there is minimal relation between these values at low optical depths.

Finally, I bring together both resolution and clumping effects for Monte Carlo models in the context of adaptive mesh refinement (AMR) grid data and in varying the imposed levels of refinement across the region. As before, I find that under-resolving the spatial grid leads to an underestimation of the dust temperatures calculated by the model, which may tend to reduce the forces exerted on the environment. However, as the same imposition of under-resolution also suppresses clumping on scales smaller than the new, now coarser grid cells, this same variation also hinders the ability of photons to escape through channels of relatively lower optical depth; this mechanism promotes a tendency to overestimate forces. Thus, these temperature-dominated and channel-dominated errors tend to promote conflicting affects. More work is necessary to determine the conditions under which one of these effects will tend to overpower the other; apparent instances of such an overpowering have been noted for both mechanisms.

### 6.2 Future work

Several simplifications were adopted as assumptions in this work; further work on the subject may further refine the understanding of the problems discussed within by relaxing these assumptions, or by adding additional physics neglected in this
dissertation. Here, I consider some of the further investigations that could be pursued to address these.

The analytical model of star cluster formation presented in this dissertation assumes a virialized state of equilibrium, and uses this to estimate properties of the clump, including particularly for setting the radius of the clump (and in turn, properties derived using the clump radius), rather than solving using an energy equation. I neglect radial density distributions within the clump, as well as any role the driven-turbulence may play in seeding high-density regions to promote the formation of star-forming cores (Padoan & Nordlund 2002), which may promote an enhancement of the star formation rate.

In my models of radiation transfer in spherical envelopes, I use a consistent dust model throughout, which future work may consider varying, and as a simplifying assumption I ignore the influence of gas entirely. While dust absorbs radiation far more readily than gas, this nonetheless ignores any contributions that the gas may make, and also neglects the influence of any potential gas-dust coupling effects. Magnetic fields are likewise neglected.

Furthermore, I assume a constant underlying geometry for the spherical envelopes, with a ratio of $r_{\text{out}}$ to $r_{\text{in}}$ adopted for convenience rather than a particular physical model of such envelopes. Future work may wish to adopt more realistic proportions for such envelopes as well as the possibility of varying these proportions to isolate any potential geometrical effects. A similar variation may be of interest for the density power-law. Additionally, as a simplification, I have treated all scattering as isotropic in these models; future work may desire to reintroduce anisotropies to the scattering process.

All radiative transfer models conducted in this dissertation provide snapshots of the radiative transfer solution for a particular set of initial conditions, and do not follow the subsequent dynamical evolution the regions that the radiation feedback forces
exert. While Monte Carlo methods have traditionally been fairly computationally expensive, acceleration schemes have been in development (Harries 2015) which may help significantly speed such models, offering to make dynamical models with Monte Carlo radiative transfer become increasingly tenable.
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