RESOLVING PULSAR EMISSION WITH COSMIC MICROSCOPIES

by

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Radio emission from many pulsars exhibits scintillation - an interference pattern in frequency and time, caused by light travelling through the ionized interstellar medium via multiple paths, separated by tens of astronomical units. If one were able to trace back these paths, one would effectively be able to observe the pulsar from slightly different angles, thus forming a very high angular resolution image of the pulsar. The resolution increases as the scattering region is located closer to the source, similar to a microscope. In this thesis, I aim to measure the emission using two such “microscopes”: the Black Widow pulsar, whose binary companion has an ionized outflow, and the Crab pulsar, which is embedded in a young supernova remnant. I develop a new way to directly measure the impulse response function of a scattering screen, through bright, impulsive pulses. This is used to coherently remove the effects of scintillation in the Black Widow pulsar, revealing intrinsic emission two orders of magnitude shorter than the scattering time. I study the interstellar scattering screen of the Black Widow using Very Long Baseline Interferometry (VLBI), revealing the scintillation to be governed by a single, highly anisotropic screen. The same VLBI data is used to study the radio eclipse of the black widow pulsar, as it is occulted by its companion’s outflow. At 327 MHz, the pulsar is eclipsed in continuum, lending further evidence that the eclipse is caused by absorption, and not through time smearing of pulses. Near the radio eclipse, I discovered highly magnified pulses,
boosted by factors of up to 70 at specific frequencies. The eclipsing material from the companion is acting like a lens, and the strongest events clearly resolve the emission regions, affecting the narrow main pulse and parts of the wider interpulse differently. Finally, I study the scintillation of giant pulses in the Crab pulsar. The dominant scattering screen is within the Crab nebula itself, resulting in spatial resolution of order the size of the light cylinder. The spectra of giant pulses correlate at $\sim 5\%$, implying the giant pulse emission region is resolved, and there is tentative evidence that the scintillation patterns of the main pulse and interpulse are offset in time, which would imply a physical separation of order the light cylinder radius.
Dedicated to my parents
“We may brave human laws, but we cannot resist natural ones.”

- Captain Nemo

“Oh, figures! answered Ned. You can make figures do whatever you want.”

- Ned Land

- Jules Verne, *20,000 Leagues Under The Sea*
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Chapter 1

Introduction

Neutron stars are the compact remnants of massive stars, formed in core-collapse supernovae, first predicted in 1934 (Baade & Zwicky 1934). The first pulsar was discovered by Jocelyn Bell in 1967, as periodic pulses of radio emission (Hewish et al. 1968). As more pulsars were discovered (Pilkington et al. 1968), their origin was soon attributed to highly magnetized, rapidly rotating neutron stars (Pacini 1968).

Pulsars are extremely stable rotators, and as such, the times of arrival of their pulses can be predicted with incredible accuracy. Pulsar timing has enabled many high-precision measurements, notably, tests of general relativity (Hulse & Taylor 1975; Taylor & Weisberg 1982; Kramer et al. 2006), properties of binary orbits, including the first discovered exoplanets (Wolszczan & Frail 1992), and pulsar masses which constrain how matter behaves at the highest densities (Demorest et al. 2010; Antoniadis et al. 2013). A large ongoing international effort is being made to use the most stable pulsars as "timing arrays" (Manchester & IPTA 2013) - using timing residuals to detect gravitational waves on timescales of years.

Pulsars show many propagation effects as they travel through the ionized interstellar medium (ISM). Of particular importance to this thesis is the fact that pulsars “scintillate”, meaning their emission is variable in time and frequency owing to multi-path propagation through the ISM. Pulsars are essentially point sources, and
scintillation is analogous to stars twinkling as they pass through our atmosphere, as the angular size of emission is much smaller than the angular scale of scattering. Very few radio sources are small enough to scintillate, among them being pulsars, fast radio bursts (FRBs) (Masui et al. 2015), and to a lesser degree, active galactic nuclei (Macquart et al. 2013), and gamma-ray burst afterglows (Goodman 1997). While scintillation is often seen as a nuisance which introduces noise in pulsar timing, it can be used as a tool to study the interstellar medium, or to perform high-precision measurements of pulsars themselves.

In this thesis, I will discuss my research aiming to use scintillation as a high-precision angular probe of pulsar emission, and ways to remove the effects of pulsar scattering from radio observations. In the introduction, I will briefly describe background needed to put the further chapters in context, including the basics of pulsar emission, scintillation, and the properties of the sources I will be analysing.

1.1 Pulsar Emission

This section reviews the basics of pulsar emission, while illustrating that it is still largely an open question. For reviews, see chapter 3 in Lorimer & Kramer (2012) and chapter 6 in Condon & Ransom (2016).

As described in this section, most physical properties of pulsars are inferred from the two observables most readily available, their spin period $P$, and their spin-down rate $dP/dt \equiv \dot{P}$. The $P - \dot{P}$ diagram is then a convenient way of visualizing the pulsar population, and their evolutionary tracks (see Figure 1.1, for all pulsars in the Australia Telescope National Facility database).

1.1.1 The Simplest Pulsar Magnetosphere

Pulsars are formed with strong magnetic fields $B \gtrsim 10^{11}\text{G}$, and high spin rates, in the rough range of $P \sim 10 - 100\text{ms}$, as inferred observationally from young pulsars
associated with supernova remnants (Kaspi & Helfand 2002). Their strong B-field combined with rotation leads to an induced electric field, which greatly exceeds the surface gravity, causing charged particles to accelerate off the surface to follow magnetic field lines (Goldreich & Julian 1969). The plasma within the magnetosphere experiences the same $\mathbf{E} \times \mathbf{B}$ force, and can co-rotate with the neutron star. This must break down at the radius for which the rotation velocity is the speed of light, which is defined as the "light-cylinder",

$$R_{\text{LC}} \equiv \frac{c}{\Omega} = \frac{cP}{2\pi}, \quad (1.1)$$

where $\Omega \equiv 2\pi/P$ is the angular rotational frequency. This defines a physical scale of the pulsar magnetosphere, which most emission mechanisms are constrained to (although high energy emission and giant radio pulses may be an exception, which will be touched upon later in this section, and in Chapter 5).

The emission of most pulsars is powered through the loss of angular kinetic energy as they spin-down,

$$\dot{E} = -I\Omega \dot{\Omega} = -4\pi^2 I \dot{P} P^{-3} \approx 4 \times 10^{31} \text{erg s}^{-1} \left( \frac{P}{s} \right)^{-3} \left( \frac{\dot{P}}{10^{-15}} \right), \quad (1.2)$$

where the approximate value uses a canonical moment of inertia of $I = kMR^2 \approx 10^{45} \text{g cm}^2$ obtained for a uniform density sphere with $k = 0.4$, $M = 1.4 M_\odot$, $R = 10 \text{km}$. Assuming magnetic dipole emission, the magnetic field strength on the surface of a pulsar is

$$B_s = \sqrt{\frac{3c^3}{8\pi^2 R^6 \sin^2(\alpha)}} \frac{I}{P^3} \dot{P} \approx 10^{12} \text{G} \left( \frac{\dot{P}}{10^{-15}} \right)^{1/2} \left( \frac{P}{s} \right)^{1/2} \quad (1.3)$$

where $\alpha$ is the angle between the magnetic field and spin axes. It is important to note, however, that perfectly aligned rotators still spin-down, due to sweep-back of the magnetic field lines at the light cylinder boundary (e.g. Spitkovsky 2006). The spin period and its derivative likewise define a characteristic age of the pulsar, assuming
energy lost through dipole radiation,
\[\tau \equiv P/2\dot{P}.\] (1.4)

Pulsars are believed to be born in the upper left portion of this parameter space, with a short spin period, and hence a large energy output (e.g. the Crab pulsar, described in section 1.3.2 and Chapter 5). If they follow a track of constant magnetic field, they quickly (\(\lesssim 1\) Myr) join the main population of \(\sim 1\) s pulsars. The bottom-right portion of the \(P - \dot{P}\) diagram is unpopulated - for a given magnetic field, below a certain rotational period \(P\), the rate of rotation is too low to power the pulsar emission mechanism, forming the so-called "death line" for pulsars (see Ruderman & Sutherland 1975; Bhattacharya & van den Heuvel 1991 for details).

Millisecond pulsars are thought to not form directly as a result of a supernova explosion, but rather are "recycled" pulsars. Accretion from a binary companion transfers angular momentum, which spins up the pulsar to short \(\sim\)ms periods. Millisecond pulsars are usually observed in binaries with degenerate companions (although the existence of isolated pulsars like PSR B1937+21 still poses an evolutionary problem). Additionally, millisecond pulsars are observed to have magnetic fields \(\sim 10^8 - 10^9 G\), 3-4 orders of magnitude lower than the regular pulsar population. Mass transfer from the companion must in some way bury much of the pulsar’s magnetic field. During accretion, radio pulsations are not observable, and these systems are observed as low mass X-Ray binaries.

The ionized accreting material can only enter the pulsar magnetosphere at roughly the Alfven Radius, where the magnetic energy density of the pulsar is equal to the kinetic energy density of the accreting material, assumed to be orbiting at Keplerian velocity (in effect, matter cannot accrete when the corotation velocity is greater than the Keplerian velocity),
\[\frac{B(r)^2}{8\pi} = \frac{1}{2}\rho(r)v^2(r).\] (1.5)
Figure 1.1: $P - \dot{P}$ diagram of pulsars in the Australia Telescope National Facility database. Lines of constant inferred dipole magnetic field strength and characteristic age are overdrawn. The two pulsars studied in detail in this thesis, the Crab pulsar (PSR B0531+21), and the Black Widow pulsar (PSR B1957+20) are labelled.
The density can be expressed in terms of the accretion rate \( \dot{M} = 4\pi r^2 \rho(r) v(r) \). Assuming magnetic dipole emission, and setting the accretion rate to Eddington then sets a lower limit to the rotation period achieved through accretion (see Bhattacharya & van den Heuvel 1991),

\[
P_{\text{eq}} \approx 1.9 \text{ ms} \left( \frac{B}{10^9 \text{ G}} \right)^{6/7}.
\]

This is the so-called "spin-up line", and shows that millisecond periods can only be reached in the presence of a weak magnetic field. The locations of millisecond pulsars in the \( P - \dot{P} \) diagram are broadly consistent with old pulsars which have been spun up through accretion.

### 1.1.2 Pulsar Beam

The main observable of pulsar emission is the pulse profile, the time averaged emission of a pulsar vs. rotational phase. Radio pulse profiles vary wildly between pulsars, and there is no fundamental physical explanation to explain them. There have however, been some empirically driven models which are successful in describing “normal” emission of slow pulsars.

Pulsar radio emission is generally thought to originate in the polar cap, defined as the surface surrounding the magnetic poles containing all of the open field lines. A simple model is to consider the emission to arise from a certain altitude \( r_{\text{em}} \), tangential to the open field lines. This defines a cone of emission with an opening angle governed by the open field lines at that altitude, where lower emission altitudes then correspond to narrower beams. In slow pulsars, it is often observed that the pulse profile narrows towards higher observing frequencies, suggesting that the emission height decreases with observing frequency.

If pulsars all emit from the same emission height \( r_{\text{em}} \), then the emission cones should be wider in short period pulsars, since the size of the magnetosphere is smaller (in detail, this gives a prediction \( \rho \propto \sqrt{P} \), see Lorimer & Kramer 2012 for a derivation).
This relationship observationally holds well for the population of slow pulsars, but breaks down for millisecond pulsars.

Many pulsars show polarization swings across their pulse profile, which can often be explained through the context of the above model, where emission arises in a cone. If emission comes from curvature radiation, it will be highly polarized in the direction of magnetic field lines at the emission point. Then as the cone sweeps along our line of sight, we will see a swing in polarization angle, which depends on how our line of sight crosses through the pulsar beam.

Many pulse profile show multiple components, which cannot be explained with a simple emission cone. This might indicate that the cone is not fully filled in, where only irregular patches within the cone can emit (Lyne & Manchester 1988).

1.1.3 Emission Locations - Magnetospheric Gaps

In the simple picture presented so far, the light cylinder defines a physical boundary for the magnetosphere, and defines which magnetic field lines are closed or open (see Fig. 1.2).

While the exact radio emission mechanism of pulsars is not known, the observed brightness temperatures of pulsars typically require coherent radiation, where bunches of charge emit in phase. Most emission models are believed to occur in “gaps” in the magnetosphere, regions depleted of plasma where a strong $E_\parallel$ can build, accelerating particles to highly relativistic speeds. Curvature radiation from accelerated particles will produce high energy $\gamma$-rays, which can pair produce $e^-$ and $e^+$, producing more $\gamma$-rays which can themselves pair-produce, until the particle energies are below twice the rest mass of the electron. This so-called “pair cascade” can boost the charged particle density by factors of $\sim 10 \sim 10^4$ in these regions, which is necessary for most models of pulsar emission.

The polar cap is the standard region used to explain normal pulsar emission,
described in the last section. Two other such gaps are the outer gap and the slot gap. The outer gap straddles the boundary between open and closed field lines in the outer magnetosphere, near the null line \((\Omega \cdot B = 0)\), which is the boundary between currents of opposite direction. Since this gap is far from the pulsar surface, the relevant magnetic field is the field at the light cylinder,

\[
B_{LC} = B_S \left(\frac{\Omega R}{c}\right)^3 \approx 9.2 \text{ G} \left(\frac{P}{s}\right)^{-5/2} \left(\frac{\dot{P}}{10^{-15}}\right)^{1/2}.
\]

(1.7)

The slot gap (e.g. Harding & Muslimov 2005) is formed below the last open field line, essentially shielded by the pair cascade at the boundary of the polar cap. Both of these models are not thought to be responsible for most regular pulsar emission, but are plausible regions for giant radio pulses, and non-thermal high-energy emission. In particular, giant pulses seem to preferentially occur in systems with a large \(B_{LC}\) (Knight et al. 2006; Bilous et al. 2015), while \(\gamma\)-ray emission appears to be related to the total spin-down energy \(\dot{E}\) (Pétri 2012, see \footnote{https://confluence.slac.stanford.edu/display/GLAMCOG/Public+List+of+LAT-Detected+Gamma-Ray+Pulsars}). It is worth noting that radio emission is energetically insignificant compared to the spin-down luminosity; the majority of energy is emitted in the pulsar wind and high energy photons.

It is entirely plausible that emission can occur beyond the pulsar magnetosphere. Simulations of pulsar magnetospheres (Philippov et al. 2015; Philippov & Spitkovsky 2018) found high energy emission to originate in an equatorial current sheet which extends to a few light cylinder radii.
Figure 1.2: Toy model of magnetosphere, taken from Aliu et al. 2008. The various emission gaps described in this chapter are shown, and the light cylinder radius depicted on this graph is for the Crab pulsar.
1.2 Propagation through the Interstellar Medium

1.2.1 Dispersion through Cold Plasma

The ISM contains cold plasma, which affects the index of refraction of electromagnetic radiation. The index of refraction is

\[ n = \sqrt{1 - \left(\frac{v_p}{v}\right)^2}, \]  

(1.8)

where \( v \) is the observing frequency, \( v_p = \sqrt{\frac{e^2 n_e}{\pi m_e}} \approx 8.5 \sqrt{n_e / \text{cm}^{-3}} \) kHz is the plasma frequency, and \( n_e \) is the electron density (\( e \) and \( m_e \) are the charge and mass of the electron respectively). The signal will not propagate when \( v < v_p \), but this is not typically a concern, as almost all pulsar observations are at above 10 MHz. The refractive index is less than one, meaning the phase velocity \( v_p > c \).

The group velocity \( v_g = nc \) is less than \( c \), leading to a frequency dependent relative group delay, (where we assume \( v_p \ll v \) to Taylor expand the refractive index), of

\[ t = k DM v^{-2}. \]  

(1.9)

Here, \( k \equiv \frac{e^2}{2\pi m_e c} = 4148.808 \text{s pc}^{-1} \text{cm}^3 \text{MHz}^{-2} \) is the dispersion delay constant (Manchester & Taylor 1972), and

\[ DM = \int_0^d n_e dl \]  

(1.10)

is the dispersion measure, the integral of the electron density along the line of sight. Since the group delay is linear in \( n_e \), \( DM \) is simply a measure of the column density between a pulsar and the observer. An example of a dispersed pulse is shown in Figure 1.3.

In the presence of a magnetic field, the index of refraction is different between the left and right-hand circular polarization states. This leads to Faraday rotation, a \( \lambda^2 \) rotation between linear polarization states (however, polarization information is not used in this thesis, refer to Lorimer & Kramer 2012 for more information).
Chapter 1. Introduction

1.2.2 Scintillation basics

Pulsars are observed to scintillate, showing modulation of their intensity in time and frequency (see left side of Figure 1.4). This occurs as the signals take multiple paths through the ISM, acquiring geometric time delays ($\tau$); the electric field we observe on Earth is the summation of these multiple images, which can be in or out of phase with each other.

Considering a single scattered path with some geometric delay $\tau$, light at different frequencies will constructively interfere when the path length difference is a multiple of $\lambda$, creating an interference pattern in frequency with scale $\Delta \nu \sim 1/\tau$. The relative motion of the pulsar, the Earth, and the source of the scattering, cause a Doppler shift $f_D$ between the line of sight and the scattered image. This then leads to an interference pattern in time of $\Delta t \sim 1/f_D$.

The primary observable for scintillation is the dynamic spectrum, which is simply the average pulse spectrum as a function of time, $I(\nu, t)$. While scintillation patterns could be arbitrarily complex, they appear not to be. When taking the power spectrum of $I(\nu, t)$, called the “secondary spectrum”, the power is often found to be quite sparse.
Chapter 1. Introduction

1.2.3 Theory of Secondary spectra

To understand these secondary spectra, I will start by using the "thin-screen approximation", which assumes the scattering happens in a localized region with a small thickness along the line of sight (a schematic is shown in Figure 1.5). We will see after comparing to observed secondary spectra, that this simplifying assumption is a good description of reality. The derivations below follow Walker et al. (2004), and a complimentary derivation is given in Cordes et al. (2006).

Consider a pulsar at a distance $d_{\text{psr}}$ from us, which encounters a scattering screen
Figure 1.5: Diagram of a thin scattering screen being considered in this chapter, with variables labelled.

at a distance $d_{\text{lens}}$. Each point of the screen imparts a phase $\Phi$ to the electric field, and the observed electric field is the sum over the lens:

$$E(r) \propto \int d^2x \exp(i\Phi),$$  \hfill (1.11)

$$\Phi = \phi(x) + \frac{(x - sr)^2}{2R_{Fr}^2},$$  \hfill (1.12)

where $x$ and $r$ are positions in the lens plane and observer plane respectively, $s = 1 - d_{\text{lens}}/d_{\text{psr}}$, $R_{Fr} = \sqrt{\lambda s d_{\text{lens}}}$ is the Fresnel scale. More generally, the observed electric field is the double integral over the source and the lens (Gwinn et al. 1998), but here we treat the pulsar as a point source.

The integrand $\exp(i\Phi)$ is an oscillatory function, so there will only be large contributions to $E(r)$ where the phase changes slowly. We can then approximate the integral as a sum over stationary phase points where $\nabla \Phi = 0$, where

$$\nabla \phi + \frac{x - sr}{R_{Fr}^2} = 0. \hfill (1.13)$$

Each point will have a contribution $E_i(r) = \sqrt{\mu_i} \exp(i\Phi_i)$, where $\mu_i$ is a magnification which relates to the phase curvature (i.e. $\mu_i$ reflects how large of an area adds coherently on the lens, which is described by the second spatial derivative of $\Phi(x)$).
The dynamic spectrum, the measured intensity of the pulsar as a function of frequency and time, is the square of the electric field,

\[ I(\nu, t) = |E|^2 \approx |\sum_i E_i|^2 = \sum_{i,j} \sqrt{\mu_i \mu_j} \cos \Phi_{ij} \]  

(1.14)

Expanding the phase around a time \( t_0 \) and frequency \( \nu_0 \),

\[ \Phi_{ij} \approx \Phi_{ij}^0 + \frac{\partial \Phi_{ij}}{\partial t} (t - t_0) + \frac{\partial \Phi_{ij}}{\partial \nu} (\nu - \nu_0). \]  

(1.15)

Taking the Fourier transform of \( I(\nu, t) \) gives

\[ \tilde{I}(f_\nu, f_t) \approx \frac{1}{4\pi} \sum_{i,j} \sqrt{\mu_i \mu_j} \left[ e^{i\Phi_{ij}^0} \delta(f_t + f_{t,ij}) \delta(f_\nu + f_{\nu,ij}) + e^{-i\Phi_{ij}^0} \delta(f_t - f_{t,ij}) \delta(f_\nu - f_{\nu,ij}) \right] \]  

(1.16)

The secondary spectrum is the power spectrum of \( I(\nu, t) \), and is the square of the above equation:

\[ A(f_\nu, f_t) = |\tilde{I}(f_\nu, f_t)|^2 \approx \frac{\Delta t \Delta \nu}{(4\pi)^2} \sum_{i,j} \mu_i \mu_j [\delta(f_t + f_{t,ij}) \delta(f_\nu + f_{\nu,ij}) + \delta(f_t - f_{t,ij}) \delta(f_\nu - f_{\nu,ij})] \]  

(1.17)

The secondary spectrum is point symmetric, so we can take only the top plane without losing any information:

\[ A(f_\nu, f_t) \approx \frac{\Delta t \Delta \nu}{(4\pi)^2} \sum_{i,j} \mu_i \mu_j \delta(f_t - f_{t,ij}) \delta(f_\nu - f_{\nu,ij}) \]  

(1.18)

The stationary phase points are then

\[ f_{t,ij} = \frac{\theta \cdot v_{\text{eff}}}{\lambda}, \quad f_{\nu,ij} = \frac{d_{\text{eff}} \theta^2}{2c} - \frac{\phi}{2\pi \nu}. \]  

(1.19)

d_{\text{eff}} and \( v_{\text{eff}} \) are the effective distance and velocity, defined as

\[ d_{\text{eff}} = (1/s - 1) d_{\text{psr}} \]  

(1.20)

\[ v_{\text{eff}} = (1/s - 1) v_{\text{psr}} + v_\odot - v_{\text{lens}}/s. \]  

(1.21)

The second term in \( f_{\nu,ij} \) is the dispersive delay caused by propagating through free electrons in the ISM, and can easily be measured and removed from observations.
(unless $DM$ varies significantly across the lens, as will be seen in Chapter 4). $f_{v,ij} \equiv \tau$ is simply then the geometric delay compared to the line of sight. Likewise, $f_{t,ij} \equiv f_D$ is the fringe rate in time, caused by the motion of the observer through the interference pattern (or, more accurately, caused by the combination of the pulsar, screen, and Earth’s velocity). It can alternatively be thought of as the Doppler shift of the pulsar’s emission between the line of sight and the scattered path. Given $\theta \propto \lambda^2$ (from the refractive index in 1.2.1, $\hat{\alpha} \sim \Delta n \sim \nu^{-2}$), then

$$\tau \propto \lambda^4, \quad f_D \propto \lambda. \quad (1.22)$$

$\tau$ and $f_D$ are then related through their common dependence on $\theta$

$$\tau = \eta f_D^2, \quad \text{with} \quad \eta = d_{\text{eff}}\lambda^2/(2c\nu_{\text{eff}}^2\cos^2(\alpha)) \quad (1.23)$$

The curvature $\eta$ depends on the distance to the screen, the relative velocities of the pulsar, screen and Earth, and the angle between the velocity and the screen. To see a clear parabola in the secondary spectrum is then highly non-trivial: it implies a highly anisotropic scattering screen, with a specific distance, and a constant effective velocity (the last constraint is easiest to explain, as the pulsar velocity often dominates).

**Inverted Arclets**

Equation 1.19 are the stationary phase points between each image and the line of sight, defining the main parabola. More generally, each pair of points interferes (ignoring the dispersive phases):

$$f_{D,ij} = \left(\frac{\theta_i - \theta_j}{\lambda}\right) \cdot v_{\text{eff}}, \quad \tau_{ij} = \frac{d_{\text{eff}}(\theta_i^2 - \theta_j^2)}{2c}. \quad (1.24)$$

The line of sight is a local maximum of $\tau$ between two images. Taking instead images further, or closer to the line of sight will decrease $\tau$, forming a parabola with the same curvature, but inverted.
1.2.4 VLBI

Extending the secondary spectrum, we can think of not just the intensity of a single dish, but the visibility between two telescopes

\[ V(v, t, b) = E_1(v, t) E_2^*(v, t). \] (1.25)

The intensity \( I(v, t) \) can be thought of as the visibility at \( b = 0 \).

This value is called the dynamic cross-spectrum, and contains both an amplitude and an interferometric phase. An analogy to the secondary spectrum can be made, first by Fourier transforming \( V(v, t, b) \)

\[ \tilde{V}(\tau, f_D, b) \approx \frac{1}{4\pi} \sum_{i,j} \exp[i(\Phi_i(-b/2) - \Phi_j(b/2))] \sqrt{\mu_i \mu_j} \delta(\tau - \tau_{ij}) \delta(f_D - f_{D,ij}) \] (1.26)

To find the phase for a particular pixel in \( \tilde{V}(\tau, f_D) \), consider the simplifying case where only one pair of stationary phase points contribute. Then

\[ \Phi_{i,j} = \phi_i - \phi_j + \frac{kd_{\text{lens}}}{2s} \left[ \theta_i^2 - \theta_j^2 + \frac{s}{d_{\text{lens}}} b \cdot (\theta_i + \theta_j) \right] \] (1.27)

The anti-symmetric terms can be removed by computing,

\[ \psi_{i,j}(b) = \Phi_{i,j} + \Phi_{j,i} = \frac{2\pi}{\lambda} b \cdot (\theta_i + \theta_j) \] (1.28)

leaving only the interferometric phase. This is computed in practice by computing

\[ C(f_D, \tau, b) = \tilde{V}(\tau, f_D, b) \tilde{V}(-\tau, -f_D, b), \] (1.29)

which is called the secondary cross-spectrum. The main parabola comes from images interfering with the line of sight, meaning one of \( \theta_i, \theta_j \) is zero, and the remaining angle can be solved for.

Figure 1.6 shows the amplitude and phase of the secondary cross spectrum of PSR B0834+06 from Brisken et al. (2010), as well as the astrometric positions of the points along the main parabola. The resulting image is strikingly linear. This leads
to the immediate question of why scattered images lie in a line at a fixed distance away from us (which is not the topic of my thesis, but will be covered briefly further in the introduction). But it also shows that scattering is understandable and solvable - the complicated scintillation pattern we observe is described by the positions and magnifications of a discrete set of points linearly distributed on the sky. This revelation is very promising for our ability to use interstellar scattering to learn about pulsars.

**Incoherent VLBI**

Spatial information of the scattering screen can be obtained without correlating the electric fields of two stations, but rather just by correlating their dynamic spectra (Galt & Lyne 1972). We will refer to this as the Incoherent Cross-Spectrum, defined as

$$C(f_D, \tau, b) = \tilde{I}_1(\tau, f_D) \tilde{I}_2^*(\tau, f_D).$$

(1.30)

The scintillation pattern has some physical scale at Earth, and will be offset between two stations. For a 1D screen, the phase in the cross-spectrum is simply the baseline projected onto the screen’s axis, divided by the scale of the scintillation pattern at Earth

$$\psi(b) = \frac{b \cos(\alpha_b)}{2\pi \cos(\alpha)} \frac{f_D}{v_{\text{eff}}},$$

(1.31)

where $\alpha_b$ is the angle between $b$ and $\theta$. In this 1D case, the phase is simply linear in $f_D$, independent of $\tau$. With a measurement of $\psi$ at two baselines, one can solve for $\alpha_b$, $v_{\text{eff}} \cos(\alpha)$. The detailed theory of incoherent cross spectra will be detailed in Simard et al. in prep.

**1.2.5 Summary, in terms of observables**

Here, I summarize the observable quantities and what we learn from them. From a single dish, the primary observables are the dynamic spectrum $I(\nu, t) = |E(\nu, t)|^2$ and its power spectrum, the secondary spectrum $A(\tau, f_D) = |\tilde{I}(\tau, f_D)|^2$. The secondary
Figure 1.6: Amplitude (left), and phase (right), of the secondary cross spectrum of PSR B0834+06 at 314.5 MHz from Briskin et al. 2010. The dynamic cross spectrum is measured as $V(v, t, b) = \langle E_1(f,t)E_2^*(v,t) \rangle$ between Arecibo to the Green Bank Telescope, and the secondary cross spectrum is measured as $C(\tau, f_D, b) = \tilde{V}(\tau, f_D, b)\tilde{V}(-\tau, -f_D, b)$. The clear parabolic structure and inverted arclets are seen, and the phase changes smoothly along the parabola, showing that the scattered images are resolved. Bottom: Measured astrometric positions of the points along the main parabola. The scattered images are elongated along a line, while the island of points at $\sim 1$ ms, $-40$ mHz, which lie off the main parabola, are offset.
spectrum gives us a measure of $\tau, f_D$ for each scattered point, and the curvature of the parabola relating them. The four quantities defining the screen are $d_{\text{eff}}, v_{\text{eff}}, \alpha$, and $\theta$, for which we have three measurements.

From a VLBI observation (or through correlating the intensities at two sites), the primary observables are the dynamic cross spectrum $V(v, t) = E_1(v, t)E_2^*(v, t)$ and the secondary cross spectrum $C(\tau, f_D) = \tilde{V}(\tau, f_D, b)\tilde{V}(-\tau, -f_D, b)$. The phase of the parabola of the secondary cross spectrum gives a measure of $\theta$, and we then have sufficient measurements to determine $d_{\text{eff}}, v_{\text{eff}}, \alpha$.

If the distance to the pulsar is known independently, then we have a measure of $d_{\text{lens}}$, and thus know the physical scale of the scattering screen. Similarly, with a pulsar distance and proper motion measurement, we can constrain all terms in $v_{\text{eff}}$.

### 1.2.6 Scintillation as an interferometer

It was quickly realized that scintillation could be used to infer properties of pulsar emission (Lovelace 1970). Scintillation requires pulsars to appear as point sources, i.e. their emission region must be smaller than $\lambda/D$ of the scattering screen. Knowing the physical parameters of the scattering screen then puts constraints on the size of the pulsar emission region, which was immediately used to argue that pulsars must be compact objects (Pilkington et al. 1968; Pacini 1968), and has more recently been used by Gwinn et al. (2012); Johnson et al. (2012) to constrain emission region sizes of the Vela pulsar. Backer (1975) introduced the possibility of using scintillation in a way analogous to an interferometer, not only to constrain emission sizes, but to spatially resolve different pulse components. For instance, if a pulsar has two components, a main pulse and an interpulse, then they could in principle be separated by the scale of the light cylinder, and they could scintillate measurably differently. This is typically a very small effect, measured astrometrically through high S/N, as in Gupta et al. (1999) in PSR B1133+16, Pen et al. (2014) in PSR B0834+06, Smirnova et al. (1996) for a
variety of slow pulsars at low frequency.

Assuming a 1-D screen, then in terms of observables, the resolution at the pulsar along the screen’s axis is

\[
\text{res} = \frac{\lambda}{\theta d_{lens}} (d_{\text{psr}} - d_{\text{lens}}) = \lambda \sqrt{\frac{d_{\text{psr}}}{2cT} \frac{s}{1 - s}}.
\]  

(1.32)

The resolution \( \text{res} \to 0 \) as \( s \to 0 \) (i.e., the screen is at the pulsar), and \( \text{res} \) diverges as \( s \to 1 \) (i.e. the screen is at Earth).

Slightly more physical is to compute the resolution of a fixed deflection angle of the light rays. The relationship between the true angle \( \beta \), the observed position \( \theta \), and the deflection angle \( \hat{\alpha} \) is governed by the lens equation

\[
\theta = \beta + s \hat{\alpha}.
\]  

(1.33)

Taking \( \beta = 0 \) (i.e. the pulsar is the direct line of sight), then simply, \( \theta = s \hat{\alpha} \). The resolution then takes the simple form:

\[
\text{res} = \frac{\lambda}{\hat{\alpha} d_{lens}}. 
\]  

(1.34)

The deflection angle is \( \hat{\alpha} \approx k_{lens} \lambda^2 \), where \( k_{lens} \) encodes the properties of the lens (Simard & Pen 2018), and thus the screen resolution for a fixed deflection angle is

\[
\text{res} = \frac{1}{k_{lens} \lambda d_{lens}}. 
\]  

(1.35)

There are two main takeaways: the resolution of a scattering screen is best when the scattering is closest to the pulsar, and is higher at longer wavelengths (lower observing frequencies).

Using PSR B0834+06 as an example, since it is the one pulsar with a fully solved scattering screen through VLBI. The pulsar and screen distances are \( d_{\text{psr}} \approx 640 \) pc, \( d_{\text{lens}} \approx 420 \) pc respectively. The screen extends to \( \sim 5 \) AU, giving it a resolution at the pulsar of \( \sim 10000 \) km when observed at \( \lambda \approx 90 \) cm, smaller than the pulsar light cylinder of \( R_{LC} \approx 60000 \) km. For comparison, the Crab pulsar is located at
$d_{\text{psr}} \sim 2\,\text{kpc}$, scattered in its nebula, which extends $\sim 1\,\text{pc}$ from the pulsar. Using a scattering time of $\sim 1\,\text{ms}$ at $\lambda \approx 90\,\text{cm}$ gives $\text{res} \approx 200\,\text{km}$, which may be sufficient to resolve pulse components.

This motivates the approach taken in the later chapters of this thesis. I focus on scintillation in pulsars in electron dense environments, where $s \to 0$, to use scintillation to resolve components of the pulse emission. In particular, I study the Crab pulsar, which is embedded within the Crab nebula, and the Black Widow pulsar which is eclipsed by its companion’s ionized outflow.

**Solving orbits through scintillation**

The Doppler frequency of scintillation encodes the proper motion of the pulsar projected onto the axis of the screen. Throughout an orbit, this will manifest in variations in the rate of scintillation. In practice, this is much easier to measure than resolving magnetospheres - pulsar binaries are of order a light second, much larger than typical pulsar magnetospheres. Scintillation can then give an additional orbital constraint, and has been used in practice to solve for the inclination of uncertain systems (Lyne 1984; Ord et al. 2002), although usually with the assumption that scattering is isotropic. Anisotropic scattering was considered by Rickett et al. (2014), who use scintillation to resolve the inclination ambiguity from Shapiro delay in PSR J0737-30, the double pulsar, and measure the orientation of the orbit on the sky. While not the focus of this thesis, pulsar scintillation has high potential as a tool to measure inclinations of systems without measurable post-Newtonian effects, such as black widow or redback systems (e.g. van Kerkwijk et al. 2011).

**1.2.7 Scintillation Mechanism**

Upon the discovery of scintillation in pulsars, it was attributed to diffraction due to turbulence in the interstellar medium (Rickett 1970). Scattering angles of $\theta \sim 10\,\text{mas}$
requires turbulent eddies on physical scales as small as $D \simeq \lambda/\theta \sim 10^7$ m for $\lambda = 1$ m. This is surprisingly small, but some of the observed effects of scintillation, as well as the dependence of scattering time with frequency $\tau \sim \nu^{-4}$, and power extending to high $\tau$ could explained through diffraction caused by electron fluctuations in the ISM described by a Kolmogorov spectrum,

$$P(\kappa) = C_n^2 \kappa^{-11/3}.$$  \hspace{1cm} (1.36)

As a result, this was largely considered the underlying cause of scintillation until the discovery of parabolic arcs in the secondary spectra by Stinebring et al. (2001) (see Gupta 2000 for a review of the evidence for and against Kolmogorov turbulence in the ISM, shortly before the discovery of parabolic arcs). These arcs are difficult to reconcile with the turbulence model, which creates an expectation of isotropic, volume filling scattering.

Refraction was initially dismissed, since it naively requires local electron densities of $n_e \sim 100 \text{ cm}^{-3}$, far greater than the density of free electrons in the ISM $n_e \sim 0.02 \text{ cm}^{-3}$ (from a simple argument of a sphere with electron density $n_e$, $\theta \sim \Delta n$, using $\theta \sim 10 \text{ mas}$, $\lambda = 1$ m as above). Pen & Levin (2014) suggest that sheet-like structures, when seen at grazing incidence, could explain the known properties of scintillation (in particular, they propose waves on current sheets at magnetic field boundaries). Waves on the sheet would increase the bending angle preferentially in one direction, and would lead to multiple scattered images along a line passing through the direct line of sight, analogous to light reflecting along a lake (Fig. 1.7). Simard & Pen (2018) developed this into a predictive theory, which can be tested against observations.

1.2.8 Scintillation / Scattering as source of noise

Interstellar scintillation is a source of noise in pulsar timing. While pulsars are brightest at low frequencies, they become increasingly scatter broadened ($\tau \sim \nu^{-4}$),
Figure 1.7: Stars reflecting off of Lake Traverse, at the Algonquin Radio Observatory (credit Andre Recnik). The light from the star appears elongated along the lake, as small wave crests enhance the bending angle to reflect the light to us. This is analogous to the picture proposed by Pen & Levin 2014, where pulsar emission is scattered by sheet-like structures in the ISM at grazing incidence, enhancing the refraction angle at each crest, resulting in a line of images.
making the profile less sharp, and measured times of arrival of pulses less precise. In addition, variations in $\tau$ create slow, apparent shifts in arrival times. To mitigate this, pulsar timing usually operates in high frequencies, around 1.4 GHz. Even then, the effects of interstellar scattering and scintillation are a source of noise in timing residuals (Stinebring 2013; Cordes 2013).

The curvature of arcs has been observed to be stable over 20 years (Hill et al. 2005), and individual arclets persist for weeks, moving predictably through the secondary spectra along the main parabola due to the pulsar’s proper motion. Using a predictive theory (eg. Simard & Pen 2018), it may be possible to remove scattering.

The effects of pulsar scattering can only be removed, however, if the interstellar impulse response function is known. There are several ways to retrieve the response function of the ISM, through holographic techniques (Walker et al. 2008; Pen et al. 2014), cyclic spectroscopy (Demorest 2011; Walker et al. 2013; Palliyaguru et al. 2015), or directly through intrinsically short giant pulses (Main et al. 2017a). Knowing the response function, it can be coherently removed from observations, resulting in the intrinsic, unscattered pulse profile, and may lead to greatly improved timing residuals.

1.3 The Pulsars Studied in this Thesis

1.3.1 Black Widow

PSR B1957+20 was the first eclipsing pulsar discovered (Fruchter et al. 1988). It has a very low mass companion, $M_c \sim 0.02M_\odot$, which is heated to $\sim 6000$ K on the side facing the pulsar, and has an ionized outflow which causes the radio eclipse. When first discovered, it was thought that pulsars in such systems could cause their companions to be evaporated completely, thus leaving isolated millisecond pulsars (Fruchter et al. 1988; Phinney et al. 1988, hence the name “Black Widow”, after the spiders who consume their mates). However, for PSR B1957+20, the outflow is no
longer thought to be strong enough (Arzoumanian et al. 1994). The exact nature of the
eclipses is still puzzling. From theoretical arguments, the most plausible explanation
appears to be cyclo-syncrotron absorption (Thompson et al. 1994), yet if that is the
case, it is surprising that at 1.4 GHz the pulsar’s pulsed flux disappears (Fruchter et al.
1990; Ryba & Taylor 1991), while its continuum flux appears not to (Fruchter & Goss

There has been renewed interest in this system, in part because of measurements of
the companion’s radial velocity and lightcurve, combined with pulsar timing, suggests
a high pulsar mass, formally $M_{\text{psr}} = 2.40 \pm 0.12 \, M_\odot$ (van Kerkwijk et al. 2011). Here,
the radial-velocity orbit, and this the mass ratio, is fairly reliable, but the modelling
of the lightcurve, and thus the inclination, may suffer from systematic uncertainties
(ibid.). This led to a VLBI campaign to measure its orbit using scintillation as described
in 1.2.6, as an independent constraint on $i$.

PSR B1957+20 exhibits typical scintillation from the interstellar medium. To use the
scintillation to measure orbital motions requires measuring the screen distance and
orientation through VLBI. In addition, as is the topic in Chapter 4, the companion’s
ionized outflow can act as a lens. Its proximity to the pulsar means that it may be
used to resolve the pulsar’s magnetosphere, and the properties of lensing may give
new constraints on the eclipsing material.

1.3.2 Crab Pulsar

PSR B0531+21 was discovered in 1968 through its emission of single, bright pulses
(Staelin & Reifenstein 1968). It is a young and highly energetic pulsar, being the
remnant of a supernova explosion which was seen in the year 1054, named “the Crab
Pulsar”, as it is the only pulsar in the Crab nebula. The dominant source of radio
emission of the Crab pulsar are its giant pulses, which occur at two rotational phases,
named the main pulse and interpulse. Giant pulses are aligned in phase with X-ray
and $\gamma$-ray emission (see Fig. 1.8), and are typically thought to originate far from the pulsar surface, likely near or beyond the light-cylinder.

The Crab nebula, which is one of the brightest radio sources, dominates the background of most appreciable sized radio telescopes. In addition, the nebula has many filaments caused by Rayleigh-Taylor instability as the pulsar wind pushes into the expanding shell of material from the supernova (Porth et al. 2014). These filaments have electron densities $\sim 1000 \text{ cm}^{-3}$ (Osterbrock 1957), and dominate the temporal scattering and scintillation of the Crab pulsar (Vandenberg et al. 1976; Cordes et al. 2004).

The proximity of the scattering screen to the pulsar, coupled with the enigmatic nature of the Crab’s giant pulses makes it an ideal target to resolve its emission components using scintillation. From physical arguments, we already have an estimate of the location of the scattering, meaning VLBI is not strictly necessary to put a physical scale to the screen’s resolution\textsuperscript{2}.

\textsuperscript{2}Given its proximity to the pulsar, the scattering screen the nebula is not directly resolvable through VLBI. Indeed, VLBI measurements reveal that the angular broadening is dominated by an interstellar scattering screen (Vandenberg et al. 1976), complicating the methods in 1.2.4
Figure 1.8: The pulse profile of the Crab Pulsar from radio to X-rays from Moffett & Hankins 1996. Many emission components appear throughout, but the main pulse and interpulse are prevalent, aligned in phase from low radio frequency (where they are comprised entirely of giant pulses) to X-rays and γ-rays.
1.4 Summary of thesis chapters

My thesis is organized as follows: Chapter 2 and 3 focus on studying interstellar scattering, while Chapters 4 and 5 detail resolving the radio emission of two pulsars with high local electron density. Chapter 2 demonstrates a new method to deconvolve pulses from the impulse response function of the ISM, using bright giant pulses as impulses. Chapter 3 describes work I have done with VLBI visibilities of the Black Widow Pulsar B1957+20, solving the interstellar scattering screen’s orientation and distance, and studying its eclipse in continuum emission. Chapter 4 focuses on the study of extreme plasma lensing events associated with the eclipsing material of the Black Widow pulsar. Chapter 5 focuses on the Crab Pulsar, comparing the scintillation patterns of different pulse components, imparted through scattering in the Crab nebula.
Chapter 2

Descattering of Giant Pulses in PSR B1957+20

Abstract

The interstellar medium scatters radio waves which causes pulsars to scintillate. For intrinsically short bursts of emission, the observed signal should be a direct measurement of the impulse response function. We show that this is indeed the case for giant pulses from PSR B1957+20: from baseband observations at 327 MHz, we demonstrate that the observed voltages of a bright pulse allow one to coherently descatter nearby ones. We find that while the scattering timescale is $12.2 \mu s$, the power in the descattered pulses is concentrated within a span almost two orders of magnitude shorter, of $\lesssim 200 \text{ ns}$. This sets an upper limit to the intrinsic duration of the giant pulses. We verify that the response inferred from the giant pulses is consistent with the scintillation pattern obtained by folding the regular pulsed emission, and

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This Chapter was published in ApJ Letters with bibliographic code 2017ApJ...840L..15M (Main et al. 2017a). It included here verbatim, although some additional results are presented in the appendix, which were not realized at the time of publishing the paper.
that it decorrelates on the same timescale, of 84 s. In principle, with large sets of giant pulses, it should be possible to constrain the structure of the scattering screen much more directly than with other current techniques, such as holography on the dynamic spectrum and cyclic spectroscopy.

2.1 Introduction and Background

When observed at relatively low frequency, pulsars scintillate, showing an interference pattern in frequency and time that arises from multi-path propagation through the interstellar medium. The interference patterns often have clear structure, showing a quadratic dependence of the delay of scattering points as a function of their fringe rate (leading to parabolic arcs in the secondary spectra), which is most readily understood if the scattering is dominated by localized points on a strongly anisotropic screen (Stinebring et al. 2001; Walker et al. 2004; Cordes et al. 2006). This picture was confirmed dramatically by Brisken et al. 2010, who used very long baseline interferometry to show that the scattering screen of PSR B0834+06 appeared on the sky as a collection of points along a single, linear structure.

The above not only provides surprising information about the nature of the interstellar medium, but also offers a remarkable opportunity to study pulsars: given sufficient understanding of the locations of the scattering points, one can use them as an interferometer, which, with baselines of tens of AU, has sub-microarcsecond angular resolution, thus allowing precision astrometry. Indeed, Pen et al. (2014) used the scintillation for PSR B0834+06 to show that the location of the radio emission shifted by a few 10s of km as a function of spin phase.

The scintillation properties of pulsars are typically inferred from their dynamic spectrum, i.e., the intensity of a pulsar’s folded emission as a function of frequency and time. This yields direct information on the amplitudes of the interstellar impulse response function, but to retrieve its phase one has to rely on holographic techniques.
(Walker et al. 2008; Pen et al. 2014). A promising alternative is to use cyclic spectra of pulsars (Demorest 2011), which retain part of the phase information. So far, however, both methods have been shown to work only in specific cases.

In principle, the interstellar response could be measured directly if an object emitted bursts of emission that lasted much shorter than the scattering time. Some pulsars oblige by emitting suitably short “giant pulses.” One of these is the “black widow” pulsar PSR B1957+20 (Knight et al. 2006). In this paper, we show that its giant pulses indeed allow one to measure the interstellar response directly.

### 2.2 Giant Pulses

We recorded 9.5 hr of P-band data of PSR B1957+20 at the Arecibo Observatory, as part of a European VLBI network program (GP 052). The data were taken in four daily 2.4 hr sessions on 2014 June 13–16, recording dual circular polarizations of four contiguous 16 MHz wide bands spanning 311.25 to 375.25 MHz (recorded with the VLBA4 terminal in 2-bit Mark 4 format\(^1\)). We exclude the fourth band from our analysis, as its signal was almost fully filtered out by the receiver, as well as the June 15 data, as these cover the eclipse of the pulsar by the wind of its companion (Fruchter et al. 1988), which hinders our analysis. We are thus left with 7.2 hr of data covering 311.25–359.25 MHz.

No flux calibrators were observed, so we convert to flux based on a nominal system temperature of 120 K and gain of 10 K/Jy for the 327 MHz receiver.\(^2\) With these values, the folded profile yields an average flux of 37 mJy, consistent with the 38 ± 3 mJy found by Fruchter & Goss (1992).

We searched for giant pulses in the de-dispersed time streams of both polarizations, by binning the power in the whole 48 MHz band to 16 \(\mu\)s resolution and flagging

\(^1\)http://www.naic.edu/~astro/aovlbi/
\(^2\)http://www.naic.edu/~astro/RXstatus/327/327greg.shtml
peaks above $12\sigma$ ($\sim 3\,\text{Jy}$). We use custom code to coherently de-disperse in 4s blocks, separately in each 16 MHz sub-band (using a dispersion measure of $29.1162\,\text{pc cm}^{-3}$, tweaked using the folded profile), and properly account for de-dispersion wraparound. We found 247 and 313 pulses in left and right circular polarization, respectively, with 102 of these in common.\(^3\)

In Figure 2.1, we show where the giant pulses arrive relative to the average profile. One sees three clusters, with the first, containing most pulses, coincident with the main pulse, the second coincident with the peak of the interpulse, and the third on the (relatively weak) tail of the interpulse. This distribution is different from what is seen in PSR B1937+21, where the giant pulses are predominantly found at the trailing edges of the pulse components (Cognard et al. 1996; Soglasnov et al. 2004).

The profiles of the giant pulses show a sharp rise and a long tail, suggesting that the pulses are intrinsically short. The tails are well fit by an exponential, with an

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\(^3\)At lower thresholds, many more true giant pulses are present, but separation from the bright tail of the “regular” pulses becomes more difficult, and these fainter pulses are less useful for our purposes here.
Figure 2.2: Left: Profile of the brightest giant pulse in 1 µs bins, summed over all frequencies, with an exponential fit with a timescale of 12.2 µs overlaid on its tail. Right: A segment of the power as a function of frequency for the brightest giant pulse compared with that for the regular emission (determined using 30 s around the pulse, and scaled to match its flux). The bins are 8 kHz wide, the highest resolution possible given the width of the main pulse in the folded profile. The spectra are highly correlated, indicating that the giant pulse and the folded emission have passed through the same interstellar response in this period.

e-folding timescale of 12.2 µs (see Fig. 2.2, lower panel). The pulses are close to 100% polarized, as expected for intrinsically short, single mode emission. They show no preferred polarization direction, and can be strongly linearly or circularly polarized in either direction, in contrast to the folded profile, which is close to unpolarized (Fruchter et al. 1990).

2.3 Scintillation and Scattering

If giant pulses and the regular pulsar emission are both affected in the same way by propagation through the interstellar medium, an immediate expectation is that their frequency power spectra should show similar structure, and that this structure should vary on the same timescale. Comparing the power spectra for the brightest giant pulse with that of the regular emission near it, we indeed find that the spectra
Figure 2.3: Correlations between the voltage streams of a bright pulse pair separated by 1.92 s, for the three 16 MHz bands we observed in (with the two higher frequency bands offset by 1 and 2 units for clarity). This is equivalent to computing the visibility between the two pulses, as if they were a single pulse observed at two telescopes. The power is spread over $\sim 200$ ns, either because of small differences in the interstellar response, or, more likely, because of intrinsic differences in the two pulses. In either case, this spread sets an upper bound on the intrinsic duration of both pulses.

are very similar (see Fig. 2.2). From the de-correlation bandwidth $\Delta \nu = 133$ kHz, we infer a scattering timescale $\frac{1}{2\pi \Delta \nu} \approx 12.0 \mu$s, consistent with our measured timescale from the exponential scattering tail.

A stronger expectation is that the impulse response is the same, i.e., for giant pulses that happen close in time, not just the amplitudes of the impulse response function should match, but also the phases. We first verify this is the case for our closest pair, and then show it is possible to use a giant pulse as a direct measurement of the response function.

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\footnote{The half width at half maximum (HWHM) of the correlation function in frequency at zero time lag of the dynamic spectrum; e.g., Cordes et al. 1990.}
2.3.1 Giant Pulse Intrinsic Width

We directly compare the voltage timestreams of the two closest giant pulses, which are separated by 1.922 s. Both are strongly right-hand polarized, with integrated signal-to-noise of 46 and 18, respectively. We compute visibilities between the two pulses as a function of time lag in each of the three 16 MHz, right-hand circular bands. In Figure 2.3, we show the visibility divided by the geometric mean of the auto-correlations, i.e., a measure of the correlation strength of the voltage streams. We find strong correlation within a narrow envelope, which proves that the response of the interstellar medium is indeed close to identical for these two pulses, and that their intrinsic durations are very short, \( \lesssim 200 \text{ ns} \).

2.3.2 The Interstellar Response Function

The observed data are the convolution of the intrinsic electric field \( E_{\text{int}} \) with the impulse response function \( g \) of the interstellar medium, i.e., \( E_{\text{obs}}(t) = (E_{\text{int}} * g)(t) \).

To the extent that giant pulses approximate impulsive intrinsic emission, it is thus possible to use them to measure the response, with which it should then be possible to undo the effects of scattering and scintillation, at least within the decorrelation time \( t_{\text{corr}} \) on which the response changes.

To verify this, we attempt to measure how well we can “descatter” pairs of giant pulses with each other. We do our analysis in Fourier space, where the pulses can be described as

\[
\tilde{E}_{\text{obs}}(\nu) = \mathcal{F}(E_{\text{obs}}(t)) = \mathcal{F}((E_{\text{int}} * g)(t)) = \tilde{E}_{\text{int}}(\nu) \tilde{g}(\nu) \quad (2.1)
\]

If the intrinsic emission is a delta function at \( t = 0 \), one would have a Fourier spectrum with constant amplitude and zero phase, but because of the interstellar response, the
signal gets mixed between frequency channels, causing amplitudes to change and phases to rotate (but with total power conserved).

In principle, the above suggests we could descatter a giant pulse by dividing by a suitably normalised reference pulse (as long as the two pulses occurred well within the timescale on which the interstellar response \( \tilde{g}(\nu) \) changes, i.e., within \( \ll 84 \text{ s} \); see Sect. 2.3.3). In practice, this is rather noisy, as low-amplitude and thus noisy channels get upweighted and high-amplitude and thus well-measured ones downweighted. An optimal solution to this would involve Wiener deconvolution, in which the different frequency channels are weighted properly using their signal-to-noise ratio. We opted instead, however, to just normalize the reference pulse by its amplitude, i.e., use only the phase information. Beyond simplicity and the knowledge that most of the information is contained in the phases (in terms of power, a fraction \( \sim \pi/4 \); see Sect. 2.3.3), this has the advantage of being complementary to the dynamic spectrum, which considers only the amplitudes. Specifically, dividing a trial pulse by a normalized reference pulse gives,

\[
\frac{\tilde{E}^{\text{trial}}_{\text{obs}}}{|\tilde{E}^{\text{ref}}_{\text{obs}}|} = \left( \frac{\tilde{E}^{\text{trial}}_{\text{int}}}{|\tilde{E}^{\text{ref}}_{\text{int}}|} \right) \left( \frac{\tilde{g}^{\text{trial}}}{|\tilde{g}^{\text{ref}}|} \right),
\]

(2.2)

where we dropped the dependence on \( \nu \) for brevity.

If the pulses have the same response function and the reference pulse is truly impulsive, this reduces to \( \tilde{E}^{\text{trial}}_{\text{int}} |\tilde{g}| \), i.e., one would recover the intrinsic spectrum of the trial pulse multiplied by the amplitudes of the response function. Since the phases are corrected but the amplitudes are not, for an intrinsically short pulse, an inverse Fourier transform should yield a timestream with a pulse which is similarly short but which has reduced amplitude. More generally, since we do not have perfect arrival times, there will be an uncertainty in the time offset (equivalent to a phase gradient in the spectrum), and, since the intrinsic pulses are not true delta functions, the power will only be concentrated within the intrinsic width of the emission.
We apply our method first using our brightest giant pulse as a reference, and its five closest neighbours as trial pulses. Our brightest pulse is strongest in right-circular polarization, with an integrated signal-to-noise of 132. We make Fourier transforms for 32 $\mu$s segments for all pulses, covering the majority of the scattering tail (this corresponds to 1024 real-valued samples and thus 512 channels in each of the three 16 MHz bands). We then descatter and inverse transform as above, and bin and sum the power of the descattered timestreams in 250 ns bins to account for possible intrinsic widths (see Fig. 2.3).

The results are shown in Figure 2.4. For the closest pair, one sees that much of the power of the descattered pulse is contained within a single 250 ns bin, and that the peak intensity of the descattered pulse is more than 10 times stronger than that of the observed, scatter-broadened one. For the next closest pulse, the procedure does not seem to work well, with the descattered pulse having multiple peaks. This is likely intrinsic, since for the further pulses, the descattering does work, though with decreasing efficiency, presumably because the response function becomes increasingly different.

### 2.3.3 Decorrelation Time

The above suggests it should be possible to use the extent to which giant pulses can descatter each other to determine the timescale on which the response function changes. For this purpose, we increase our sample, by selecting as reference pulses all those for which phases can be measured in the Fourier spectrum, i.e., with $\gtrsim 1\sigma$ per voltage sample, corresponding to an integrated signal-to-noise of 40 in the power. This results in 6 and 3 pulses in the right and left circular time streams, respectively (with 1 in common). For each of these, we descatter all pulses within 5 minutes to either side, matching the polarizations. We then sum the power of the descattered timestreams in 1 $\mu$s bins (which should account for any reasonable intrinsic widths),
Figure 2.4: Descattering giant pulses. Top: The profile of the brightest giant pulse in our sample, used to descatter its neighbouring pulses. Lower panels: Nearby giant pulses, ordered by time separation, showing both the profiles of the observed, scatter-broadened pulse (red curve, lowered by 50 Jy), and the descattered pulse (black curve). For all pulses, the signal is binned to 250 ns. One sees that the descattering works less well at larger time differences, for which the interstellar response starts to change significantly. The outlier at $\Delta t = 35.8$ s appears to be formed of a series of bursts, in contrast to the other pulses which appear to be well described by delta functions in this time binning.
and measure the fraction of the total power which is descattered into a single peak to quantify how well the pulses descattered each other.

With our brightest pulse, we find that we can recover up to 70% of the total flux in the descattered peak. For the other, fainter pulses, however, at most half of the power is descattered, as their more limited strength allows only an imperfect measure of the response function. To get a quantitative sense of the imperfections, we test our routine on simulated giant pulses. For the simulations, we use the following two assumptions (which are clearly simplistic but should cover the essence of short intrinsic emission and an exponential scattering tail): that at our time sampling the intrinsic emission of giant pulses can be represented by delta functions, and that the response function $g$ can be described by a normally-distributed random process for which the variance decreases exponentially on the observed 12.2 $\mu$s decay time (normalized to have unity integrated power). Hence, our sets of simulated giant-pulse time streams consists of sets of delta functions of different amplitudes convolved with a given simulated response (of length 32 $\mu$s, i.e., 3072 real-valued samples), with normally distributed measurement noise added to each sample. We run these pulse pairs through our analysis routines to determine the fraction of power that can be de-scattered given two pulses with identical response functions.

As expected, the strength of the reference pulse determines the average power recovered, while the strength of the (fainter) trial pulse dominates the scatter between different realisations. For our set of reference pulses, we find the simulated recovered fraction ranges from 0.4 to 0.7. From Eq. 2.2, one sees that in the high signal-to-noise limit one should be dominated by the extent to which $\langle |\tilde{g}|^2 \rangle$ is less than one. Since our simulations assume $2\tilde{g}^2$ is distributed as a $\chi^2$ distribution with 2 degrees of freedom, one expects $\langle |\tilde{g}| \rangle^2 = \pi/4$, consistent with our results. For lower signal-to-noise, we find that we can also reproduce our simulated fractions by integrating numerically over probability distributions for both $\tilde{g}$ and the noise (unfortunately, we could not
find a closed-form expression).

In Figure 2.5, we show the fraction of the power that is descattered against time separation for each pulse pair, corrected for the above loss of power (and with error bars reflecting the expected \(1\sigma\) scatter). One sees that at large separation, \(\gtrsim 100\) s, the descattering never recovers much power, while at shorter separation it does, though unequally so. Inspection of the low points around \(\Delta t = 20\) s shows that in those cases the descattering does not lead to a single strong pulse (as for the \(\Delta t = 35.8\) s pulse in Fig. 2.4). This likely reflects intrinsic pulse structure, in either the de-scattered pulses or the reference pulse. Three of the nine reference pulses de-scatter a pulse pair to within a single \(1\mu s\) peak (including our brightest two pulses, shown in Figures 2.3 and 2.4), four of our reference pulses show clear success in de-scattering adjacent pulses but with remaining multi-peaked structure, and the last two reference pulses have no pulse pairs within 84 s separation. We do not have a sufficient number of pulse pairs within the de-correlation timescale to determine which, if any, of our reference pulses are intrinsically wide.

We can compare our decorrelation timescale with that derived using the more traditional way, from the autocorrelation of the dynamic spectrum,

\[
R(\tau) = \frac{\langle (I(t, \nu) - \mu)(I(t + \tau, \nu) - \mu) \rangle}{\sigma^2},
\]

where \(\mu = \langle I(t, \nu) \rangle\), \(\sigma^2 = \langle (I(t, \nu) - \mu)^2 \rangle\), and \(\langle \rangle\) denotes an average over both time and frequency.

We apply this to a dynamic spectrum created for the 9 minutes of data surrounding our brightest giant pulse. We use bins of 4 s, 8 kHz, and 1/32 in phase (where the frequency and phase resolution are set to barely resolve the main pulse; at 8 kHz, we also barely resolve the frequency structure due to scintillation). We define two off gates, with one subtracted from the folded profile to give a pulsed flux, and the other used as an independent measure of the noise in the dynamic spectrum. We
then compute the auto-correlation of the dynamic spectrum, subtracting the auto-
correlation of the noise, and averaging over the central 14 MHz of each band (to avoid
the parts most affected by bandpass variations).

The result is shown in Figure 2.5. One sees that at short times the correlation is
very good (it approaches 0.98 rather than 1, likely because we ignore the frequency
dependence of the noise), and then it decreases smoothly. Taking the decorrelation
time as the lag where the correlation drops by $1/e$, we find $t_{\text{corr}} = 84$ s.

In Section 2.3, we already showed that the spectrum of brightest pulse was similar
to that of the regular emission. With the dynamic spectrum, we can verify this quanti-
tatively, and also check the dependence on lag. For this purpose, we Fourier transform
our brightest giant pulse to the same channelization as the dynamic spectrum (where
8 kHz corresponds to 125 $\mu$s in time, i.e., it covers the full width of the scattering tail;
see Fig. 2.2). We then correlate it against the dynamic spectrum for a range of delays.

At low delay, we find that the giant pulse correlates very strongly with the dynamic
spectrum, at $92 \pm 5\%$ (see Fig. 2.5). This strong correlation independently suggests
a short intrinsic duration of the giant pulse, as intrinsic structure on timescales
comparable to the scattering time will lead to differences in the spectra (as seen for
the scintillation pattern of the Crab’s giant pulses, Cordes et al. 2004). At longer
delays, the correlation drops, and we find $t_{\text{corr}} = 84$ s, the same value obtained from
the auto-correlation of the dynamic spectrum.

Comparing with the points from descattering giant pulses with each other, one
finds the curves derived using the power spectra form a rough upper envelope,
suggesting the decorrelation time is the same. This is not a trivial comparison, since
the auto-correlations take into account the amplitudes of the impulse response function
only, while our descattering only uses the phase information.
Figure 2.5: Degree of correlation of pulsar emission as a function of time, determined using three methods. The first is the standard auto-correlation of the dynamic spectrum (solid black curve; measurement errors smaller than the width of the curve, but larger systematic uncertainties; see Sect. 2.3.3). The second is a cross-correlation of the spectrum of the brightest giant pulse with the dynamic spectrum (dashed red curve; uncertainty estimated from scatter at large $\tau$). The third uses the fraction of the power of a giant pulse that is descattered by using a bright neighbouring giant pulse as a reference measurements of the impulse response (blue points with error bars, with size scaled to reflect the strength of the reference pulse). Here, the power has been corrected for the expected loss given the signal-to-noise ratio of the reference pulse (see Sect. 2.3.3), and the error bars reflect the uncertainty due to noise in the pulse being descattered. Points less than $3\sigma$ above the noise inferred from non-pulse parts of the descattered timestream are shown as upper limits. The square point corresponds to the multi-peaked giant pulse shown in Figure 2.4.
2.4 Ramifications

We have shown that giant pulses in PSR B1957+20 can be used as a direct probe of the impulse response function of the interstellar medium: they allow one to descatter other giant pulses. An immediate result is that we can constrain the typical intrinsic duration of giant pulses of PSR B1957+20 to be very short, \( \lesssim 200 \text{ ns} \).

Having a direct measure of the response also should allow one to verify (and inform) the response inferred from the dynamic spectrum (using holographic techniques; Walker et al. 2008; Brisken et al. 2010) or from cyclic spectra (Demorest 2011; Walker et al. 2013), thus putting those techniques on firmer footing.

One might also hope to use the giant pulses directly to infer how the impulse response changes with time. In general, the de-correlation time is only an average estimate; parts of the impulse response at short delay should change more slowly than parts at large delay. Indeed, longer term, one might hope to use all giant pulses – perhaps aided by the dynamic and cyclic spectra – to measure the evolution of the response as a function of time and frequency. If the scattering towards PSR B1957+20 is dominated by a single, highly anisotropic screen, as was found for PSR B0834+06 (Brisken et al. 2010), then this should allow one to determine amplitudes and phases of individual scattering points directly. Those, in turn, might allow one to resolve the pulsar’s orbit on the sky, as has been done for the pulse emission with spin phase for PSR B0834+06 (Pen et al. 2014).

Unfortunately, only few other pulsars show giant pulses. Among those, we are most excited to apply our technique to the Crab pulsar, since for that source we expect that the main scattering screen, which is in the Crab nebula, will resolve the light cylinder, thus opening up the possibility to determine empirically where the giant pulses originate.
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2.5 Appendix

Here I show some additional results, discovered after this paper was accepted for publication.

Giant Pulse Arrival Times

To compare the arrival times of giant pulses with the averaged pulse profile, we need to take into account the fact that the pulse profile is significantly shifted after being convolved with the impulse response function. Figure 2.1 does not adjust for this fact, leading us to the incorrect conclusion that the giant pulses arrive in the center of the pulse profile. The pulse profile is re-made in Mahajan et al. (2018), adding the scattering time to the phase of the folded profile. The result is plotted in Figure 2.6, showing giant pulse arrivals coincident with the trailing edge of the main pulse.

![Figure 2.6: Pulse profile and giant pulses in B1957+20, with arrival time adjusted for scattering, created by Nikhil Mahajan, and featured in Mahajan et al. 2018](image-url)
Descattering Aligned to the Closest Sample

In the descattering method outlined in the paper, pulses are first channelized to 1 $\mu$s binning (1 MHz channels), and aligned to the maximum of the intensity in this binning. The rise times of the pulses can then be offset on scales less than 1 $\mu$s, which would lead to phase wrapping in frequency when the pulses are descattered. Since pulses within the scintillation timescale correlate coherently (i.e. Figure 2.3), one can avoid this smearing by correlating the electric fields to align them to within a sample. I show an example comparing descattering with the two above methods in Figure 2.7. There is a clear phase gradient in the de-scattered spectra of coarsely aligned pulses, which disappears when the pulses are aligned to the closest sample. The descattered pulse appears only in two time bins when the reference pulse is aligned to the nearest sample, constraining the intrinsic width of both pulses to $\lesssim 21$ ns.

Intrinsic Frequency Structure of Giant Pulses

As seen in Figure 2.5, the spectrum of the brightest giant pulse correlates almost perfectly with the dynamic spectrum surrounding it. This is only true if the intrinsic spectrum of the brightest giant pulse is essentially featureless; intrinsic time structure of giant pulses will give rise to frequency structure of scale $\Delta \nu \sim 1/\Delta t$ (e.g. two subpulses separated by 100 ns would show frequency banding of $\Delta \nu \sim 10$ MHz). The spectra in both polarizations for all of the reference pulses are plotted in Figure 2.8. Many of these pulses show intrinsic, frequency dependent structure, indicative of being comprised of multiple polarized shots. The high correlation found in Fig. 2.5 appears to arise from the brightest pulse being intrinsically very short.
Figure 2.7: Pulse descattering of the brightest pulse with its closest pulse pair, 18 s later. Top: Phase of the descattered pulse in frequency, after applying the descattering method described in the paper. The pulses are originally only coarsely aligned to the closest µs, leading to phase wrapping in frequency, and smearing in time. The size of the points reflects the intensity in each frequency bin of the reference pulse. Middle: Same as top, but with the pulses aligned to the closest sample in the 48 MHz timestream. Bottom: Descattered timestream corresponding to the top two panels. In the case where pulses are aligned perfectly, the pulse is de-scattered to within two samples, or \( \approx 21 \) ns.
Figure 2.8: Coarse spectra of B1957+20's brightest giant pulses, showing intrinsic, frequency dependent structure in some pulses, similar to Bilous et al. 2015.
Chapter 3

Probing the Black Widow pulsar’s scattering screen and eclipse using VLBI

Abstract

The Black Widow pulsar, B1957+20, is a rich system for studying propagation effects, showing interstellar scintillation, and a radio eclipse caused by its companion’s ionized wind. We present the results of a global VLBI campaign on this source at low frequency, using the European VLBI Network, and the Algonquin Radio Observatory. The interference patterns are correlated between the different telescopes, measuring the screen to be highly anisotropic (consistent with 1D), oriented at an angle of $24.1 \pm 1.1^\circ$ on the sky. We infer an effective velocity of $v_{\text{eff}} = 132 \pm 3$ km/s, and effective distance $d_{\text{eff}} \sim 2.2$ kpc. An independent measure of the distance to the

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This Chapter contains work done using visibilities of PSR B1957+20 from a global VLBI campaign. It is not yet published, but the results contained will be used in further analyses.
pulsar, likely through a VLBI parallax measurement, would then fully constrain the scattering lens. The visibilities of Arecibo to Westerbork (the most sensitive baseline in this dataset) are analyzed throughout the pulsed radio eclipse, to see if the pulsar is eclipsed in continuum. No measurable fringe is found throughout the eclipse, showing the eclipse at 327 MHz is likely absorption.

3.1 Scintillation

3.1.1 Dynamic Spectra

As described in Chapter 2, we recorded 9.5 hr of P-band data of PSR B1957+20 for European VLBI network program (GP 052), with telescopes at Arecibo, Westerbork, Jodrell Bank, and the Very Large Baseline Array (VLBA). The data were taken in four daily 2.4 hr sessions on 2014 June 13–16, recording dual circular polarizations of four contiguous 16 MHz wide bands spanning 311.25 to 375.25 MHz (all stations recorded with 2-bits per sample, Arecibo and Jodrell Back are Mark 4, Westerbork is Mark 5b, and the VLBA stations are VDIF). In addition, we observed on June 13, 14, and 16 at the Algonquin Radio Observatory (ARO), with dual linear polarizations, in the 200–400 MHz range. ARO data is stored as 4-bit complex numbers, channelized to 1024 frequency channels using a 4-tap Polyphase Filter Bank (PFB), using a hamming window. To put the data in a format which conforms with the other stations, we invert the PFB using a Wiener deconvolution, cut out all frequencies outside of 311.25 to 375.25 MHz, and store the timestream in four contiguous 16 MHz wide bands, in 2-bit Mark 4 format.

Two separate reductions of the data are used in this chapter - one used to create the visibilities between each pair of sites \( V(v, t) = E_1(v, t)E_2^*(v, t) \), and a separate reduction to make the dynamic spectrum \( I(v, t) \) at each site in higher frequency resolution.
To construct the visibilities, the data were correlated at JIVE, using the SFXC software correlator (all telescopes except for ARO, which is not currently a standard VLBI telescope). The data were binned in 4 s segments, 512 frequency channels per sub-band, and 16 gates across the pulse profile. The data were taken in 9 minute segments, with small breaks for calibrator scans. There were residual phases in these visibilities (likely arising from ionospheric differences at each site), which are modelled and removed using a 2-mode singular value decomposition\(^1\) on each sub-band, in each 9 minute scan.

To be able to measure the fine frequency structure of the scintillation, the baseband data from each site was folded with 2000 channels per IF, and 32 phase bins across the pulse profile. The folded profile at each station is shown in Figure 3.1; the pulsar is detected in each station, although quite poorly in the 25 m VLBA dishes.

\(^1\)A 1-mode SVD will in effect model a linear drift in time and frequency (equivalently, a single fringe peak in \(\tau, f_D\)). A second mode is need to empirically model and remove the effect of drifting fringe rates (see Fig. 3.7)
Figure 3.1: Pulse Profile at all stations, integrated over the first day of observations. While the VLBA dishes all detect the pulsar with poor S/N, they can still be used to construct a meaningful cross-spectrum with Arecibo, since S/N depends on the geometric mean of the two dishes' areas.
Figure 3.2: *Left:* A subset of the dynamic spectrum at Arecibo, for July 13, the first day of observations, in 4 s time bins, and 8 kHz frequency channels. *Right:* The secondary spectrum using the entire Arecibo dynamic spectrum from the same day, with a logarithmic colour scale. A parabolic structure is seen, although inverted arclets are not clearly distinguishable. Significant power extends to the highest delays.
3.1.2 Cross Spectra

Although we in principle have $N \times (N - 1)$ baselines, the only appreciable cross-spectra come from the baselines containing Arecibo, since it dwarfs all other dishes for sensitivity. To increase sensitivity further, rather than correlating the dynamic spectra or each site to Arecibo, we correlate the real part of the visibility of each baseline to Arecibo (e.g. $A(\tau, f_D, b) = Re(\tilde{V}_{wb-ar})\tilde{I}_{ar}$). We refer to this as the visibility cross-spectrum throughout. This is equivalent to correlating Arecibo with a point halfway between Arecibo and a second dish. For VLBA dishes, this is a net positive: the baseline is reduced by a factor of two, but the area of the second effective dish is $\sim 25 \times 300/25^2 = 12$ times larger. The cross-spectrum obtained in this way for Arecibo - Westerbork is shown in Figure 3.3.

The cross-spectra show a linear gradient in $f_D$, independent of $\tau$. This is consistent with a single, linear screen, for which the phase gradient is given by $m = 2\pi b \cos(\alpha_b) / v_{\text{eff}} \cos(\alpha)$, where $\alpha_b$ is the angle between the baseline and the screen (as described in 1.2.4). The slope is measured for each cross-spectrum, averaging over $\tau$ where $|f_D| < 5$ mHz. Combining the slope measurements from each baseline then gives the orientation of the screen of $24.1 \pm 1.1^\circ$ (east of north), and using this orientation, an effective velocity of $v_{\text{eff}} \cos(\alpha) = 132 \pm 3$ km/s, summarized in Figure 3.4.

From these constraints, $d_{\text{eff}}$ can be obtained from the curvature of the secondary spectrum, $a = d_{\text{eff}} \lambda^2 / (2cv_{\text{eff}}^2 \cos^2(\alpha))$. Parabolae in secondary spectra are often well enough defined to be fit by eye (Stinebring et al. 2001; Hill et al. 2003; 2005). A more quantitative approach is to perform a Hodge transform, summing along parabolae of different curvatures (Bhat et al. 2016). However, neither of these approaches work particularly well in our case, since the parabolic structure is not well defined (see Fig. 3.2). The curvature is $\sim 3 - 7 \mu s/mHz^2$ by fitting a parabola by eye, giving a range of $1.3 \text{kpc} \lesssim d_{\text{eff}} \lesssim 3.0 \text{kpc}$. While poorly constrained, this still suggests a screen which is
roughly halfway to the pulsar (i.e. not close to either the pulsar or the Earth).

Consistency check with ARO

Since it was not a standard VLBI station, ARO was not correlated with the other datasets at JIVE. However, it is sensitive enough to measure a cross-spectrum with Arecibo, as are Westerbork and Jodrell Bank. We correlate the dynamic spectra of each site to Arecibo in a 40 minute segment where all four dishes have overlapping data, using the time (phase) offsets between the scintillation patterns to measure the screen angle of $36.5 \pm 6.3^\circ$, $2\sigma$ away from the measurement using the visibility cross-spectra.

3.1.3 Looking Ahead - Measuring the Inclination

The eventual goal of these observations is to measure the inclination, and thus the mass, of the pulsar. The screen constraints need to be used in conjunction with measurements of the scintillation rate across the 9.2 h orbit. This can, in principle, be measured in our dataset, which spans most of the full orbit over four days. However, the effect is expected to be small due to the mass ratio of the pulsar and companion; the pulsar’s orbital motion is $\sim 5$ km/s (van Kerkwijk et al. 2011), compared to the proper motion of $\sim 135$ km/s (Hobbs et al. 2005). At this level, evolution in the secondary spectrum over the four days of measurement may be important to consider, as it could be mistaken as a change in $f_D$ due to orbital motion.

To fully solve the scattering screen, we need a better measurement of the arc curvature, and a measurement of the pulsar distance. The distance can be computed in a standard way, using VLBI parallax. A measurement of the arc curvature would be improved by a secondary spectrum which extends to higher $\tau$. However, we are currently at the limit governed by time-frequency tradeoff; to improve the frequency resolution while still resolving the pulse profile, we would need to construct cyclic spectra (eg. Demorest 2011; Walker et al. 2013).
Figure 3.3: Amplitude (top) and phase (middle) of the visibility cross-spectrum between Arecibo and Westerbork. The phase shows a clear linear gradient as a function of $f_D$, which is the prediction for a single linear screen. Bottom: phase vs. $f_D$ after averaging the complex values of the cross-spectrum along $\tau$. The red lines depict the best fit slope, and the region being fit.
Figure 3.4: UV coverage for the first day of observations, for every baseline connecting Arecibo. The derived screen angle from the visibility cross-spectra, as well as the proper motion vector for 60 s of PSR B1957+20 from Hobbs et al. 2005 are shown. The grey dashed line is perpendicular to the screen, shown for reference.
Chapter 3. VLBI analysis of the Black Widow pulsar

3.2 Scattered Light through Eclipse

3.2.1 The Mystery of the Eclipse

The nature of PSR B1957+20’s radio eclipse is still mysterious, some 30 years after its discovery (Fruchter et al. 1988). The length of the eclipse at 300 MHz is $\sim 10\%$ of the orbital phase, much greater than the extent of the companion, which would eclipse $\sim 2\%$ of the orbit. Cyclotron absorption is proposed as the most plausible eclipse method by Thompson et al. (1994), which requires a 20 G magnetic field at the interface between the companion wind and pulsar wind. This has the added benefit of explaining the length of the eclipse, as the magnetic pressure could balance the pulsar wind. However, while the pulsed emission is seen to be eclipsed from 300 – 1400 MHz (Ryba & Taylor 1991), the continuum emission at 1.4 GHz does not appear to be fully eclipsed, while the continuum emission at 300 MHz is (Fruchter & Goss 1992). This would suggest the disappearance of pulsed flux at 1.4 GHz does not reflect a true eclipse by absorption, but rather smearing by time-delay differences such as scattering or dispersion.

While it is difficult to detect if the pulsar signal truly disappears in a single dish ($T_{\text{sys}}$ is enormous compared to the pulsar flux), if the signal is not absorbed throughout eclipse, then it would still be detectable in the visibility between two telescopes. If the light is being scattered, then it’s spatial extent could be measured through its scintillation properties; light scattered at the radius of the companion would likely be resolved by the interstellar screen and would thus not scintillate as much as the unscattered pulsar radiation.

3.2.2 Folded profile leading in and out of eclipse

We begin by studying the single-dish folded profile at Arecibo at ingress and egress, shown in Figure 3.5. As the pulsed intensity fades, the profile does not noticeably
change in width. This makes scattering or dispersion smearing hard to reconcile as explanations for the eclipse. Additionally, $\Delta DM > 0.1$ is needed to smear the pulse by a rotational period, two orders of magnitude higher than the excess $DM$ measured in this region. If the eclipse is not absorption, it would need to be a mechanism which redistributes pulsed power uniformly through pulse profile (analogous to how sunlight is scattered almost uniformly across the sky on a hazy day).

Figure 3.5: B1957+20 pulse profile leading into and out of eclipse. **Bottom:** Folded profile in 1 s bins, 128 phase gates, plotted in square-root scaling to accentuate low brightness features at the eclipse edge. **Top:** Pulsed intensity, taken as the 2nd fundamental of the Fourier transform of the pulse profile. While the pulse is being eclipsed, the intensity lowers without a noticeable increase in scattering time, or smearing through de-dispersion.
3.2.3  Arecibo - Westerbork Visibility into Eclipse

Here, we analyse the VLBI visibility through the eclipse in our data. We restrict ourselves to the Arecibo - Westerbork visibilities, our most sensitive baseline (the same data and reduction described in section 3.1.1).

The phase of the visibility $V(ν, t)$ was measured using the 4 gates encompassing the main pulse and interpulse, and used to calibrate the entire pulse profile. Any residual flux in the off pulse should show up in the real part of the visibility. We fit the pulse profile leading into eclipse as the average pulse profile far from eclipse $(p_c(φ))$, absorbed by a fraction $ε$, with an added constant, i.e.

$$p(φ) = (1 − ε)p_c(φ) + C. \quad (3.1)$$

The expectation for a scattered eclipse is a profile in which the missing pulsed flux is redistributed uniformly across the profile, giving a prediction for the constant of $C = ε⟨p_c(φ)⟩$. The results of our fits are shown in Figure 3.6. The scattered light in the off-pulse regions should have been detected with high significance, yet the background flux is consistent with zero.

In a slightly more direct approach, we sum all 16 gates to form $V(t, ν)$, effectively treating the pulsar as a continuum source, and do a 2D FFT to search for fringes throughout the eclipse. The results of this are shown in Figure 3.7. The fringes disappear in the region that the pulsed eclipse is seen in our data, further confirming a continuum eclipse in our data.

Further work needs to be done to investigate the eclipse mechanism. A natural follow up is to image PSR B1957+20 through eclipse at higher frequencies, to confirm or refute the findings of Fruchter & Goss (1992). As it stands, it would be rather coincidental to have two separate eclipse mechanisms operating over only a factor of 4 in frequency, causing a true eclipse at 300 MHz, and an apparent eclipse at 1.4 GHz through some propagation effect. As it stands, cyclo-synchrotron absorption
remains the best explanation for the eclipse at 300 MHz. The presence of such a large magnetic field should then be easily measurable in how different polarizations propagate through the companion’s outflow (for further discussion, see Chapters 4, 6).

Figure 3.6: Real part of the visibility between Arecibo and Westerbork leading into eclipse. The profile from the three marked regions are summed in time, and fitted with a template profile + a constant, as described in the text. There is no measurable constant flux in the off-pulse region as the pulsar is eclipsed.
Figure 3.7: Top: Phase of Arecibo - Westerbork visibility through eclipse. Bottom: $|\tilde{V}(\tau, f_D)|$, the 2-D Fourier transform of the top plot in 10 minute segments, with logarithmic scaling. This amounts to a $\tau, f_D$ search of the visibility through eclipse.
Chapter 4

Pulsar emission amplified and resolved by plasma lensing in an eclipsing binary

4.1 Main Text

Radio pulsars scintillate because their emission travels through the ionized interstellar medium via multiple paths, which interfere with each other. It has long been realized that the scattering screens responsible for the scintillation could be used as “interstellar lenses” to localize pulsar emission regions (Lovelace 1970; Backer 1975). Most scattering screens, however, only marginally resolve emission components, limiting results to statistical inferences and detections of small positional shifts (Gwinn et al. 2012; Johnson et al. 2012; Pen et al. 2014). Since screens situated close to the source have better resolution, it should be easier to resolve

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emission regions of pulsars located in high density environments such as supernova remnants (Main et al. 2017b) or binaries in which the pulsar’s companion has an ionized outflow. Here, we report events of extreme plasma lensing in the “Black Widow” pulsar, PSR B1957+20, near the phase in its 9.2 hour orbit in which its emission is eclipsed by its companion’s outflow (Fruchter et al. 1988; 1990; Ryba & Taylor 1991). During the lensing events, the flux is enhanced by factors of up to 70–80 at specific frequencies. The strongest events clearly resolve the emission regions: they affect the narrow main pulse and parts of the wider interpulse differently. We show that the events arise naturally from density fluctuations in the outer regions of the outflow, and infer a resolution of our lenses comparable to the pulsar’s radius, about 10 km. Furthermore, the distinct frequency structures imparted by the lensing are reminiscent of what is observed for the repeating fast radio burst FRB 121102, providing observational support for the idea that this source is observed through, and thus at times strongly magnified by, plasma lenses (Cordes et al. 2017).

On 2014 June 13-16, we took 9.5 hours of data of PSR B1957+20 with the 305-m William E. Gordon Telescope at the Arecibo observatory, at observing frequency of 311.25–359.25 MHz (see Methods). These data were previously searched for giant pulses (Main et al. 2017a) – sporadically occurring, extremely bright pulses, which are much shorter than regular pulses ($\lesssim 1 \mu s$ compared to 10s of $\mu s$). While we found many giant pulses at all orbital phases, we noticed that the incidence rate of bright pulses was much higher leading up to and following the radio eclipse.

As can be seen in Figure 4.1, most of the pulses near eclipse do not look like giant pulses but rather like brighter regular pulses – they are bright over a large fraction of the pulse profile, and most tellingly, occur in groups spanning several 1.6 ms pulse rotations, suggesting the underlying events last of order 10 ms. Their properties seem similar to the hitherto mysterious bright pulses associated with the eclipse of PSR
Figure 4.1: **Strong lensing of the pulsar.** 

- **a**, Magnification distributions for the main pulse, in 10s time bins, showing periods of lensing near ingress and egress of the eclipse of the pulsar by its companion’s outflow. The colorbar is logarithmic, to help individual bright pulses stand out. 
- **b**, Pulse profiles as a function of time, averaged over 10 s and in 128 phase bins. Cyan, yellow, and magenta represent contiguous 16 MHz sub-bands, from low to high frequency. Near eclipse, plasma dispersion causes frequency-dependent delays. Gaps in the data correspond to calibrator scans. 
- **c, d**, Enlargements in which each time bin corresponds to an individual, 1.6 ms pulse. One sees extreme, chromatic lensing in which pulses are magnified by an order of magnitude over tens of ms; the brightest event is magnified by a factor of ∼40 across our highest frequency sub-band. In some events, the main pulse and interpulse are clearly affected differently, indicating that different emission regions corresponding to these are resolved by the lensing structures. 
- **e, f**, Enlargements for a quiescent period for comparison.
High-magnification events are often chromatic, as can be seen from the colors in Figure 4.1 (which reflect 16 MHz sub-bands), and is borne out more clearly by the spectra shown in Figure 4.2: some show frequency widths comparable to our 48 MHz band, peaking at low or high frequency, while others show strong frequency evolution, in some cases tracing out a slope in frequency-time space, in others a double-peaked profile.

For many high-magnification events, the magnification is not uniform across the pulse profile. The bottom panel of Figure 4.1 shows this dramatically: at 0.2 s, the main pulse is greatly magnified over 5 pulses, while the entire interpulse is barely affected. In addition, the components of the broad interpulse are often magnified differently from each other, as can be seen most readily in the magnified events in Figure 4.3. Thus, the events resolve the pulsar’s various emission regions.

The most strongly magnified events occur within three specific time spans, each lasting for \( \sim 5 \) minutes, around orbital phase \( \phi = 0.20, \phi = 0.30, \) and \( \phi = 0.32 \) (see Figure 4.1); here, \( \phi = 0.25 \) corresponds to superior conjunction of the pulsar in its 9.2-hour orbit (Fruchter et al. 1988) and the duration of the eclipses at 350 MHz is about 40 to 60 minutes (Fruchter et al. 1988; 1990; Ryba & Taylor 1991), or \( \Delta \phi \simeq 0.1 \) (a sketch of the system geometry is shown in Figure 4.4). In each time span, we observe only small overall frequency-dependent time delays, of order 10 µs, which indicate modest increases in electron column density, with dispersion measures (DM) of \( \sim 10^{-4} \) pc/cm\(^3\) (see Figure 4.1 as well as Figure 4.6). Intriguingly, at times when the delays are longer and the DM thus higher – right before and after eclipse, and in between the two post-eclipse periods of strong lensing, at \( \phi \simeq 0.31 \) – the magnifications are less dramatic, only up to a factor of a few, and correlated over much longer timescales, of order 100 ms. After the main lensing periods, up to \( \phi \simeq 0.36 \), flux variations remain correlated, suggesting even weaker events are still present (examples of lensing in the
Figure 4.2: A range of spectral behavior in lensing events. Power spectra, binned to 3 MHz, of the main pulse in consecutive 1.6 ms rotations, for selected lensing events from eclipse egress. The top panels show the average magnification in each main pulse, and the side panels the magnification spectra of the brightest pulse (i.e., that of pulse number 0).
Figure 4.3: Profiles of lensing events. b, c, d, Frequency-averaged pulse profiles surrounding bright events (using 128 phase bins). One sees that the events resolve the magnetosphere: the main pulse and interpulse are affected differently, as are parts within the interpulse. a, Average pulse profile of a 9 minute quiescent region, scaled up by a factor of 50 for comparison purposes.
aforementioned regions are shown in Figure 4.7).

To determine whether the magnification events could be due to lensing by inhomogeneities in the companion’s outflow, we first measured the excess delay due to dispersion at a time resolution of 2 s, the shortest at which we can measure it reliably (see Methods). We find that during the periods of strong magnification events, the delay fluctuates by $\sim 1 \mu s$ on this timescale. Assuming the relative velocity between the pulsar and the companion’s outflow is roughly the orbital velocity, 360 km/s (van Kerkwijk et al. 2011), the 2 s timescale corresponds to a spatial scale $\Delta x \approx 720$ km. For this scale, the expected geometric delay is $\Delta x^2 / 2ac \approx 0.5 \mu s$ (where $a = 6.4$ lt-sec is the orbital separation and $c$ the speed of light). Since this is comparable to the observed dispersive delays, lensing is expected.

To model the magnified pulses, we treat the signal using a standard wave optics formalism. The electric field received by an observer is the sum of the electric field of the source across the lens plane, with the phase at every point determined by both geometric and dispersive delays. When a large area on the plane has (nearly) stationary phase, the electric field combines coherently leading to a strongly magnified image, with the magnification $\mu$ proportional to the area squared. Since dispersive and geometric delays scale differently with frequency $\nu$, a plasma lens cannot be in focus over all frequency, but will impart a characteristic frequency width. This width will be smaller for larger magnification, with the precise scaling depending on the extent to which the lens is elliptic: $\Delta \nu / \nu \sim 1 / \mu$ for a very elongated, effectively linear lens, and $\Delta \nu / \nu \sim 1 / \sqrt{\mu}$ for a (roughly) circular one. In strong lensing, one generically expects multiple images to contribute, and thus caustics to form, which can lead to a slope in time-frequency space, double-peaked spectra, as well as other interference effects (see Methods). All of these are consistent with the behavior seen in the measured spectra of the magnified pulses in Figure 4.2.

For any lens, different pulse components will be magnified differently if they arise
from regions which have projected separations larger than the lens’s resolution. We can make a quantitative estimate of the lens resolution using the magnifications of the events. Continuing to assume the lens is roughly co-located with the companion, i.e., at the orbital separation \( a \), the resolution of the lens for given magnification would be \( \sim 1.9 R_1 / \mu^{1/2} \) for a linear lens, or \( \sim 1.9 R_1 / \mu^{1/4} \) for a circular one, where \( R_1 \equiv \sqrt{\lambda a / \pi} \), or 23 km at our observing wavelength \( \lambda = c / \nu \approx 90 \text{ cm} \) (see Methods for a derivation). A peak magnification of, e.g., \( \mu = 50 \), then corresponds to a physical resolution of \( \sim 6 \text{ km} \) for a linear lens, or \( \sim 17 \text{ km} \) for a circular one.

The inferred resolution is comparable to the \( \sim 10 \text{ km} \) radius of the neutron star, and substantially smaller than the light-cylinder radius, \( R_{\text{LC}} \equiv c P / 2 \pi = 76 \text{ km} \), where the velocity of co-rotating magnetic fields approaches the speed of light, bounding the magnetosphere in which pulsar emission is thought to originate (Ginzburg & Zhelezniakov 1975). Thus, the lensing offers the opportunity to map the emission geometry.

Qualitatively, since the main pulse and interpulse, as well as parts of the wide interpulse beam, are sometimes magnified very differently, the inferred resolution is a lower limit to both the projected separation between the main pulse and the interpulse, and to the size of the interpulse. Perhaps not unexpectedly, the spatial separations do not seem to map directly to rotational phase: we see similar differences between the main pulse and the interpulse, which are separated by half a rotation, and between parts of the interpulse, which are separated by only \( \sim 0.1 \) rotation. This may indicate that the interpulse consists of multiple components, which are not located close together in space.

Combining the lens resolution with the timescale of the lensing events allows us to constrain the projected relative velocity between the lens and the pulsar. A priori, one might expect the outflow velocity to be slow, in which case the relative velocity would just be the orbital motion of \( \sim 360 \text{ km s}^{-1} \). Given that strong magnification events
typically last about 10 ms, this would then imply a resolution of $\sim 4$ km, not dissimilar from what we find above, thus suggesting that the assumption of a slow outflow velocity is reasonable. By combining different constraints between the duration and frequency widths of lensing events, we can set a quantitative limit to the relative velocity (see Methods for a derivation), of

$$v > 0.5 \frac{R_1}{\Delta t_{\text{HWHM}}} \left( \frac{\Delta \nu}{\nu} \right)^{1/2}_{\text{HWHM}} \gtrsim 360 \text{ km/s},$$

where for the numerical value we use that the tightest constraints come from the shortest, least chromatic events, which have frequency widths $(\Delta \nu / \nu)_{\text{HWHM}} \simeq 0.1$, and durations $\Delta t_{\text{HWHM}} \simeq 10$ ms (measured as Half Width at Half Maximum [HWHM]; see Figure 4.2). This approximate limit equals the relative orbital velocity, which implies that the outflow velocity can be (but does not have to be) small in the rest frame of the companion.

Our results offer many avenues of further research. Ideally, one would use the lensing to map the pulsar magnetosphere. This requires better constraints on the lenses, e.g., from observations over several eclipses at a range of frequencies. Further observations may also shed light on the eclipse mechanism. E.g., if it is cyclo-synchrotron absorption, large magnetic fields, of $\sim 20$ G, are required (Thompson et al. 1994), which would impart measurable polarization dependence of the lensing events. Furthermore, combining density and velocity into a mass-loss rate of the outflow, one can infer the life-time, and thus the final fate, of these systems.

Finally, our observations establish that radio pulses can be strongly amplified by lensing in local ionized material. This adds support for the proposal that fast radio bursts (FRBs) can be lensed by host galaxy plasma, leading to variable magnification, narrow frequency structures, and clustered arrival times of highly amplified (and thus observable) events (Cordes et al. 2017). Evidence for high-density environments includes that FRB 110523 is scattered within its host galaxy (Masui et al. 2015), and that FRB 121102 has been localized to a star-forming region (Tendulkar et al. 2017;
Bassa et al. 2017), in an extreme and dynamic magneto-ionic environment (Michilli et al. 2018). Indeed, the latter’s repeating bursts have spectra (Spitler et al. 2016; Scholz et al. 2016; Law et al. 2017) remarkably similar to those shown in Figure 4.2 (e.g., compare with Figure 2 of ref. Spitler et al. 2016), and, like those bursts, the brightest pulses in PSR B1957+20 are highly clustered in time.

**Author Contributions** R. M. discovered the magnified pulses; wrote the majority of the manuscript; and created the figures. I-S. Y. developed and wrote the sections on wave optics formalism in Methods; M. H. v. K and U-L. P. guided the analysis and interpretation of the results; M. H. v. K. also helped write the manuscript and influenced the content and presentation of the figures; F-X. L., V. C., and K. V. quantified the excess DM associated with the eclipse. N. M. wrote the code used to produce the coherently de-dispersed single pulse profiles used in this analysis, and derived an improved orbital ephemeris relative to which time delays are measured. D. L. worked on criteria to systematically separate giant pulses from pulses magnified by plasma lensing. All authors contributed to the interpretation of results.

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4.2 Methods

Calculating magnifications.

The data were taken in four 2.4 hr sessions on June 13–16, 2014, and were recorded as part of a European VLBI network program (GP 052). The data span 311.25–359.25 MHz, in three contiguous 16 MHz sub-bands, and we read, dedispersed (with a dispersion measure of 29.1162 pc/cm$^3$), and reduced the data as described in Main et al. (2017a). As one extra step, we accounted for the wander in orbital period of PSR B1957+20 by adjusting the time of ascending node in the ephemeris, such that it minimized the scatter of the arrival times of giant pulses associated with the main pulse across all four days of observation.

We define the magnification of a pulse as the ratio of its flux to the mean flux in a quiescent region far from eclipse. Specifically, we construct an intensity profile of each pulse using 128 phase gates, subtract the mean off-pulse flux in each 16 MHz sub-band separately, measure the flux in an 8-gate ($\sim 100 \mu$s) window around the peak location (which we find from folded profiles, averaged in 2 s bins to obtain sufficient signal-to-noise), and divide by the average flux in the same pulse window measured in a 9-minute section far from eclipse.

To construct the spectra of lensing events, we start by binning power spectra in 3 MHz bins. We correct these approximately for the bandpass and the effects of interstellar scintillation (which still has a small amount of power on 3 MHz scales) by dividing them by the average spectra in the 15 seconds before and after (excluding the lensing event itself). This time span is chosen to be safely less than the timescale of $\sim 84$ s on which the interstellar scintillation pattern varies (Main et al. 2017a), ensuring the dynamic spectrum is stable on these scales. With 30 s of data, it is also very well measured: each 3 MHz channel has $S/N \simeq 150$. 
Evidence that strong plasma lensing must occur.

To obtain the properties of the lensed images, and in particular to determine whether we are in the “strong” or “weak” lensing regime – i.e., whether or not multiple images are formed – we use the basic principles of wave optics, considering path integrals of the electric field over a thin lens (Born & Wolf 1999). In our case, the phase of an electromagnetic wave going through different paths has contributions from both geometric and dispersive time delays,

\[ \phi(x, y) = \phi_{GM}(x, y) + \phi_{DM}(x, y), \] (4.2)

where \( x \) and \( y \) are in the lens plane. For a geometrically thin lens, i.e., with thickness along the line of sight much smaller than the separation \( a \), the geometric contribution to the phase can be written as,

\[ \phi_{GM} = \frac{1}{2} \left( \frac{x^2 + y^2}{a^2} \right) \left( \frac{a}{\lambda} \right) = \frac{x^2 + y^2}{2R_{Fr}^2}, \] (4.3)

where \( R_{Fr} \) is the Fresnel scale,

\[ R_{Fr} \equiv \sqrt{\lambda a} \approx 40 \text{ km}, \] (4.4)

with \( \lambda \) the wavelength of the radiation. For the numerical value, we used \( \lambda = c/\nu = 90 \text{ cm} \) (where \( \nu \) is the observing frequency) and assumed that the lensing material was associated with the companion and thus at the orbital separation of \( a \approx 6.4 \text{ lt-sec} \). (Note that by using wavelengths and frequencies of full cycles, our phases are in cycles too, not radians.)

The dispersive contribution arises from the signal propagating through the lens’s extra dispersion measure (electron column density) \( \Delta DM = \int n_e dz \) (with \( n_e \) the electron number density),

\[ \phi_{DM}(x, y) = -\frac{k_{DM}}{\nu} \Delta DM(x, y), \] (4.5)
where \( k_{\text{DM}} = e^2 / 2\pi m_e c = 4148.808 \text{s pc}^{-1} \text{cm}^3 \text{MHz}^2 \) (Manchester & Taylor 1972). The minus sign arises because in a plasma the phase velocity is greater than the speed of light.

Integrating over different paths, one effectively selects regions where the electric fields are coherent; these lead to the final images. For instance, if the extra electrons are distributed uniformly (in the \( x-y \) plane after the \( z \) integral), then \( \phi_{\text{DM}} \) is approximately constant and the total phase has a stationary point around \( x = y = 0 \), i.e., around the line of sight. Furthermore, all paths which are less than the Fresnel scale away from this central path have similar phase, and thus one recovers the general result that the area of the lens that contributes scales with \( R_{\text{Fr}}^2 \).

For the lensing to be strong, a minimum requirement is that changes in the geometric and dispersive phase are of similar magnitude. Since dispersion also leads to overall time delays, differences in pulse arrival time give a handle on inhomogeneities in the lensing material. We measure pulse arrival times using the usual procedure of fitting pulse profiles to a high signal-to-noise template (from a quiescent period). We fit profiles in the three bands separately, and convert the weighted average to \( \Delta \text{DM} \) using standard equations. We find that the inferred \( \Delta \text{DM} \) shows both the expected large-scale variations (Fruchter et al. 1988) and significant variability down to the shortest time scales, of 2s, at which we can measure it reliably (see Fig. 4.6). During the periods in which the strongly magnified events occur, we find intrinsic variability corresponding to variations in delay of \( \Delta t_{\text{DM}} \sim 1 \mu \text{s} \), which suggests the inhomogeneities correspond to differences in the dispersion phase of \( \Delta \phi_{\text{DM}} = \nu \Delta t_{\text{DM}} \sim 300 \text{cycles at } 2 \text{s timescales} \).

To compare this to the geometric phase, we need to translate the timescale of 2s to a length scale. Assuming the relative motion between the pulsar and the lensing material is dominated by the orbital motion, of 360 km/s, the spatial scale is 720 km. Using Eq. (4.3), this corresponds to \( \Delta \phi_{\text{GM}}(720 \text{ km}) \sim 200 \text{cycles} \).
The fact that $\Delta \phi_{\text{DM}}$ is comparable and slightly larger than $\Delta \phi_{\text{GM}}$ at the 720 km distance scale guarantees that somewhere around this scale, slopes in $\phi_{\text{DM}}$ and $\phi_{\text{GM}}$ will sometimes cancel. It also implies that multiple stationary points with $\nabla \phi = 0$ are likely. Since the phases result from a continuous function of $x$, $y$ and $\nu$, the stationary points must emerge or annihilate in pairs, leading to so-called caustics, where one has not just $\nabla \phi = 0$ but also $\det(\partial_i \partial_j \phi) = 0$ (Nye 1999).

Given the above, strong lensing is a natural cause of the strongly magnified events we observe.

**A perfect lens model.**

To calculate properties of the lensed images, we would need to perform the path integral. This is not possible since it requires the value of $\phi_{\text{DM}}$ of the full two-dimensional lens plane at a resolution better than the Fresnel scale, while our time-delay measurements are limited to the one-dimensional trajectory of the pulsar, and to an order of magnitude larger scales.

Hence, for our analysis we use a simplified model, amenable to wave-optics analysis, in which each strong lensing event is associated with an elliptical perfect lens, i.e., there is a single focal point to which all paths within the lens contribute perfectly coherently, and all paths outside the lens do not contribute at all. As shown in Figure 4.5, our model is parametrized by the location of the focal point relative to the centre of the lens, $(X, Y)$, the semi-major and semi-minor axes of the lens, $(\Delta x_{\text{lens}}, \Delta y_{\text{lens}})$, and the orientation relative to the projected pulsar trajectory.

In the special case of a centred circular lens with radius $R$, i.e., $\Delta x_{\text{lens}} = \Delta y_{\text{lens}} = R$ and $X = Y = 0$, the setup would produce an Airy disk. By comparing this to the actual path-integral of an unlensed image (i.e., with $\phi_{\text{DM}} = 0$), we can identify the unlensed case with a circular lens with radius $R_1 \equiv R_{\text{Fr}}/\sqrt{\pi}$.

Integrating the electric field over an elliptical lens, the magnification (defined as
the increase in intensity $\mu = I / \langle I \rangle$, where $I = E^2$, will be proportional to the area squared,

$$\mu = \left( \frac{\Delta x_{\text{lens}}}{R_1} \right)^2 \left( \frac{\Delta y_{\text{lens}}}{R_1} \right)^2.$$  \hspace{1cm} (4.6)

Before continuing, we note that at first glance it might appear puzzling for the magnification to be proportional to image area squared, since that seems to violate energy (flux) conservation. However, the image is also more highly beamed; what we calculated is the magnification in the center of the beam. Viewed from an angle, a linear phase term is induced across the originally coherent region. Thus, when the region is larger, it is easier to become incoherent. The solid angle scales as $\Delta \Omega \propto \Delta x_{\text{lens}}^{-1} \Delta y_{\text{lens}}^{-1}$, and thus total energy (flux) is indeed proportional to the area of the lens.

**Differential magnification and chromaticity.**

We now proceed to derive the physical resolution of the lens. The above magnification is reached only when the source is exactly at the focal point. When the emission region is some distance away from the focal point, paths from it to different parts of elliptical region will have extra phase differences. For instance, when a source is separated from the focal point in the $x$ direction by $x_{\text{source}} \equiv (X - x_s)$ (where $x_s$ is measured relative to the centre of the lens), there is a phase difference across the lens of

$$\Delta \phi = \frac{\Delta x_{\text{lens}} x_{\text{source}}}{R_1^2}.$$ \hspace{1cm} (4.7)

When this phase difference reaches order one, the image from the new source location will no longer be magnified. Defining the resolution as the offset for which total
cancellation happens, we find from an explicit integral over the elliptical lens,\(^1\)

\[
x_{\text{res}} \simeq \frac{1.9R_1^2}{\Delta x_{\text{lens}}},
\]

\[
y_{\text{res}} \simeq \frac{1.9R_1^2}{\Delta y_{\text{lens}}},
\]

Hence, total cancellation occurs on an elliptical beam with semi-minor and semi-major axes \(x_{\text{res}}\) and \(y_{\text{res}}\) that scale inversely to the semi-major and semi-minor axes of the lens.

We can write the above in terms of the magnification for a specific lens model. We will consider two extremes, the first a very anisotropic lens, effectively one-dimensional, for which \(\Delta y_{\text{lens}} = R_1\). For this case,

\[
x_{\text{res}} \simeq \frac{1.9R_1}{\mu^{1/2}}.
\]

In the opposite direction, we consider a circular lens, with \(\Delta x_{\text{lens}} = \Delta y_{\text{lens}}\). For this case,

\[
x_{\text{res}} = y_{\text{res}} \simeq \frac{1.9R_1}{\mu^{1/4}}.
\]

Here, dividing by the distance \(a\) and writing in terms of diameter \(D = 2\Delta x_{\text{lens}}\), one recovers the usual relation for the angular resolution of a circular lens, \(1.22\lambda/D\).

A similar result can be derived in frequency space. Given the different frequency dependencies of the geometric and dispersive phases, a lens cannot be fully in focus across all frequencies, but will have some characteristic frequency width. To show this, we first assume that the focal point is co-located with the centre of the elliptical lens, i.e., \((X, Y) = (0, 0)\), and that perfect coherence occurs at some frequency \(\nu_c\), i.e., that at \(\nu_c\) one has,

\[
\phi_{\text{GM}}(x, y, \nu_c) = -\phi_{\text{DM}}(x, y, \nu_c) = \frac{\nu_c(x^2 + y^2)}{2ac}.
\]

\(^1\)With a simple rescaling, the elliptical integral can be written as the circular one leading to an Airy disk.
The total phase at a nearby frequency is then given by

$$\phi(x, v_c + \Delta v) = \left( \frac{v_c + \Delta v}{v_c} - \frac{v_c}{v_c + \Delta v} \right) \frac{v_c(x^2 + y^2)}{2ac},$$

(4.13)

the dominant term of which is linear in $\Delta v$. If this term is of order one within the elliptical region, the image will no longer be magnified. Perfect cancellation does not happen in this case, but one can define characteristic width. Using half-width at half-maximum, one finds,

$$\left( \frac{\Delta v}{v_c} \right)_{\text{HWHM}} = \frac{2.9 R_1^2}{\sqrt{3\Delta x_{\text{lens}}^4 - 2\Delta x_{\text{lens}}^2 \Delta y_{\text{lens}}^2 + 3\Delta y_{\text{lens}}^4}}. \quad (4.14)$$

In terms of the magnification, we find for the special case of an effectively one-dimensional lens,

$$\left( \frac{\Delta v}{v_c} \right)_{\text{HWHM}} \approx \frac{1.7}{\mu}, \quad (4.15)$$

while for a circular lens,

$$\left( \frac{\Delta v}{v_c} \right)_{\text{HWHM}} \approx \frac{1.5}{\mu^{1/2}}. \quad (4.16)$$

When the focal point is not in the center of the elliptical region, there will be linear terms proportional to $X \Delta v$ and $Y \Delta v$ that also contribute to the phase. They make the dependence on $\Delta v$ even steeper, which means that Eq. 4.14 is an upper bound on the frequency width of magnified images.

It is possible for the frequency and position shifts to cancel each other, so that after moving the source by some distance, it is still strongly magnified at a nearby frequency. That leads to a slope of strongest magnification in time-frequency space. Following a derivation similar to that outlined above for the behavior with frequency and position separately, we find that the slope is related only to the offset of the source from the focal point. For instance, for the case that the source is traveling along the semi-major axis of the lens, i.e., along a trajectory $(x_s, y_s) = (x_s(t), 0)$, we find

$$\frac{dv}{dx_s} = \frac{v_c}{2x_s}. \quad (4.17)$$
Finally, it is possible that the pulsar is close to the focus of more than one lens at the same time. As the pulsar moves, these multiple focal points lead to multiple images which change intensity and interfere with each other. This can lead to a wide variety of spectral behavior, and might well be responsible for the multi-peaked structures seen in some panels of Figure 4.2.

A lower bound on the relative velocity.

Within the perfect lens approximation, one can derive a lower bound on the relative, transverse velocity between the lens and the pulsar using the durations and frequency widths of the lensing events.

The duration of strongly magnified events is predicted to be,

\[ \Delta t_{\text{HWHM}} \simeq \frac{0.7 R_1^2}{\sqrt{\Delta x_{\text{lens}}^2 v_x^2 + \Delta y_{\text{lens}}^2 v_y^2}} > \frac{0.7 R_1^2}{v \Delta x_{\text{lens}}}, \]  

(4.18)

where the inequality corresponds to making the most conservative estimate, that the lens is effectively one-dimensional, extended in the \( x \) direction, and that the full velocity \( v \) is directed along it.

In the frequency space, the same event will have its width bounded from above by Eq. (4.14),

\[ \left( \frac{\Delta \nu}{\nu} \right)_{\text{HWHM}} < \frac{1.8 R_1^2}{\Delta x_{\text{lens}}^2}. \]  

(4.19)

Combining the two limits to eliminate \( \Delta x_{\text{lens}} / R_1 \), we find,

\[ v > 0.5 \frac{R_1}{\Delta t_{\text{HWHM}}} \left( \frac{\Delta \nu}{\nu} \right)_{\text{HWHM}}^{1/2}. \]  

(4.20)

Lensing consistency checks.

Our analysis in terms of lensing was motivated by the fact that the bright pulses before and after eclipse do not look like giant pulses, that they occur at specific orbital phases where DM fluctuations are large, and that, in the majority of events, the entire
profile increases in flux (although not always by the same factor across the profile),
with the enhancements lasting for several pulses.

Consistent with the expectations of plasma lensing, in which lenses focus light
but arrive with associated “shadows” where light is scattered away from our direct
line of sight, we find that in regions with many strongly lensed pulses, the average
flux received is unchanged (see Figure 4.7). Also consistent with plasma lensing is
that the strongly magnified pulses are highly chromatic, often peaking at high or low
frequencies, and sometimes showing slopes in frequency time space or interference
patterns characteristic of caustics.

Finally, as a concrete example, the peak magnifications of $\mu \sim 70$ are comparable
to the magnifications expected for plasma lensing. The measured dispersive time
delay changes provide direct evidence that the geometric and dispersive phases $\phi_{GM}$
and $\phi_{DM}$ can cancel over distances of $\sim 720$ km. The cancellation will likely happen
mostly in one spatial direction; setting $\Delta x_{\text{lens}} = 720$ km and $\Delta y_{\text{lens}} = R_1$, our perfect
lens model predicts a magnification of order a hundred, comparable to the observed
value.

**Data Availability.** The data underlying the figures is available in text files at
https://github.com/ramain/B1957LensingData

**Code Availability.** The raw data were read using the baseband package: https://
//github.com/mhv/baseband
### 4.3 Extended Data

![Shock geometry diagram](image)

Figure 4.4: **Shock geometry.** The brown dwarf companion is irradiated by the pulsar wind, causing it to be hotter on the side facing the pulsar (van Paradijs et al. 1988), and inflated, nearly filling its Roche lobe (Reynolds et al. 2007). Outflowing material is shocked by the pulsar wind, leaving a cometary-like tail of material. This tail is asymmetric because of the companion’s orbital motion, which leads to eclipse egress lasting substantially longer than ingress. The companion and separations are drawn roughly to scale, while the pulsar is not – the light cylinder radius of 76 km would be indistinguishable on this figure. The inclination of the system is conservatively constrained to $50 < i < 85$ deg (van Kerkwijk et al. 2011).
Figure 4.5: **Geometry of a lensing region.** An almost edge-on (left) and a face-on (right) view of the lensing geometry. We assume that the source is at a separation equal to the semi-major axis $a$ of the binary system and moves on a trajectory parallel to the lens plane. A source at the focal point $(X, Y)$ will illuminate the entire elliptical lens coherently, leading to strong magnification. In general, the focal point may not be at the center of the ellipse (although it is likely within it), and the source trajectory $[x_s(t), y_s(t)]$ may not intersect the focal point or the elliptical region.
Figure 4.6: Dispersion measure near the radio eclipse. Shown is the excess dispersion measure relative to the interstellar dispersion, with insets focusing on regions of strong lensing. The excess dispersion is estimated from delays in pulse arrival times (see Methods); at our observing frequency of 330 MHz, $\Delta DM = 0.001 \text{ pc cm}^{-3}$ corresponds to a delay of $38 \mu s$. The scatter around the curves is intrinsic. During periods of strong lensing, it is at a level of $2.6 \times 10^{-5} \text{ pc cm}^{-3}$, corresponding to variations in delay time of $1 \mu s$. 
Figure 4.7: **Pulse profiles and magnification distributions.** Each row shows pulse profiles for 10 s segments (left, in 128 phase bins), and the corresponding magnification distributions. The segments are taken from: a, a quiescent region, showing a log-normal magnification distribution (repeated using a dotted line in b–e for comparison); b, eclipse ingress, showing some lensing events, reflected in the tail to high magnification; although hard to see, the average flux is reduced to $\sim 70\%$ by absorption in eclipsing material); c, the first post-eclipse lensing period, showing extreme magnifications but little change in average flux; d, in between the two post-eclipse strong lensing periods, showing only weak lensing on relatively long, $\sim 100$ ms timescales; e, the second post-eclipse period of strong lensing.
Chapter 5

Resolving the emission location of the Crab pulsar’s giant pulses

Abstract

The Crab pulsar has striking radio emission properties, with the two dominant pulse components – the main pulse and the interpulse – consisting entirely of giant pulses. The emission is scattered in both the Crab nebula and the interstellar medium, causing multi-path propagation and thus scintillation. We study the scintillation of the Crab’s giant pulses using phased Westerbork data at 1668 MHz. We find that giant pulse spectra correlate at only $\sim 5\%$, much lower than the $1/3$ correlation expected from a randomized signal imparted with the same impulse response function. In addition, we find that the main pulse and the interpulse scintillate significantly differently, and appear to be offset in time and frequency. These lines of evidence would suggest

An earlier version of this work was submitted to ApJ Letters, and is available on the arxiv pre-prints at arXiv:1709.09179 (Main et al. 2017b). We have expanded the paper based upon the substantive comments raised by the reviewer, and plan to submit a new revised manuscript to ApJ, rather than ApJ Letters. This chapter is the revised manuscript, in a version close to re-submission.
both that the giant pulse emission regions are extended, and that the main pulse and interpulse arise in physically distinct regions, which are, assuming the scattering takes place in the nebular filaments, of order a light cylinder radius (as projected on the sky). With further VLBI and multi-frequency data, it should be possible to measure the distance to the scattering screens, the size of giant pulse emission regions, and the physical separation between the pulse components.

5.1 The Unusual Properties of the Crab Pulsar

The Crab pulsar is one of the most unusual radio pulsars, and has been the subject of much observational and theoretical research (for reviews, see Hankins & Eilek 2007; Eilek & Hankins 2016). The two dominant components to its radio pulse profile, the main pulse and the low-frequency interpulse (simply referred to as the interpulse for the remainder of this paper), appear to be comprised entirely of randomly occurring giant pulses – extremely short and bright pulses of radio emission showing structure down to ns timescales and reaching intensities over a MJy (Hankins & Eilek 2007). Only the fainter components of the pulse profile – such as the precursor (to the main pulse) – are similar to what is seen for regular radio pulsars.

The main pulse and interpulse are aligned within 2 ms with X-ray and \( \gamma \)-ray components (Moffett & Hankins 1996; Abdo et al. 2010). Since pair production strongly absorbs \( \gamma \)-ray photons inside the magnetosphere, this suggests both components arise far from the neutron-star surface, with possible emission regions being the various magnetospheric “gaps” (Romani & Yadigaroglu 1995; Muslimov & Harding 2004; Qiao et al. 2004; Istomin 2004) or regions outside the light cylinder (Philippov & Spitkovsky 2018). While similar in their overall properties, the main pulse and interpulse have differences in detail. In particular, the interpulse has a large scatter in its dispersion measure compared to the main pulse, possibly suggesting that it is observed through a larger fraction of the magnetosphere (Eilek & Hankins 2016). In addition, it appears
shifted in phase and shows “banding” in its power spectra above 4 GHz, with the spacing proportional to frequency (Hankins & Eilek 2007).

The Crab pulsar, like many pulsars, exhibits scintillation from multi-path propagation of its radio emission. The scattering appears to include both a relatively steady component, arising in the interstellar medium, and a highly variable one, originating in the the Crab nebula itself, with the former responsible for the angular and the latter for (most of) the temporal broadening (Rankin & Counselman 1973; Vandenberg 1976; Popov et al. 2017; Rudnitskii et al. 2017). The proximity of the nebular scattering screen to the pulsar implies that, as seen from the pulsar, the screen extends a much larger angle than would be the case if it were far away (for a given scattering time). Therefore, the scintillation pattern is sensitive to small spatial scales, of order $\sim 2000 \text{ km}$ at our observing frequency (see Sect. 5.4.1), comparable to the light-cylinder radius $r_{\text{LC}} \equiv cP/2\pi \sim 1600 \text{ km}$.

The high spatial resolving power also implies that, for a given relative velocity between the pulsar and the screen, the scintillation timescale is short. Indeed, from the scintillation properties of giant pulses, Cordes et al. (2004) infer a de-correlation time of $\sim 25 \text{ s}$ at 1.4 GHz. Karuppusamy et al. (2010) compare the scintillation of pulses within a single pulse rotation, finding that main pulses weakly correlate with interpulses. In this paper, we compare the scintillation structure of the main pulse and the interpulse in more detail. We find that the scintillation pattern is only weakly correlated between pulses, suggesting an extended emission region. The main pulse and interpulse scintillate significantly differently, and appear to be offset in time, which would indicate that, as projected on the sky, the locations at which their emission originates differ on the scale of the light cylinder.
Chapter 5. Resolving the Crab pulsar’s giant pulses

5.2 Observations and Data Reduction

We analyse 6 hours of phased Westerbork data, and 2.5 hours of simultaneous Arecibo data, that were taken as part of a RadioAstron observing run on 2015, January 10–11 (Popov et al. 2017). The data cover the frequency range of 1652–1684 MHz, and consist of both circular polarizations in two contiguous 16 MHz channels, recorded using standard 2-bit Mark 5B format (Westerbork), and VDIF (Arecibo). The use of a telescope with high spatial resolution is particularly beneficial in studies of the Crab pulsar, as it helps to resolve out the Crab nebula, effectively reducing the system temperature from 830 Jy (for the integrated flux at 1.7 GHz) to 165 Jy and 275 Jy, for Westerbork and Arecibo, respectively (Popov et al. 2017).

To search for giant pulses, we coherently dedispersed\(^1\) the data from the two channels to a common reference frequency, and summed the power from both channels and both polarizations in 8 \(\mu\)s bins. We flagged peaks above 6\(\sigma\) in the Westerbork data, corresponding to \(\sim 45\) Jy, as giant pulses, finding 29332 events, i.e., a rate of \(\sim 1.3\) s\(^{-1}\). We show some sample giant pulses in Fig. 5.1, along with the folded profile.

5.3 Scintillation Properties

With the phased Westerbork array, our pulse detection rate is sufficiently high that it becomes possible to compute a traditional dynamic spectrum by summing intensities as a function of time. We do this first below, as it gives an immediate qualitative view of the scintillation. A more natural choice for pulses which occur randomly in time, however, is to parametrize variations as a function of \(\Delta t\), the time separation between pulses (Cordes et al. 2004; Popov et al. 2017). Hence, we continue by constructing correlation functions of the spectra, as functions of both time and frequency offset.

\(^1\)Using a dispersion measure of 56.7716 pc cm\(^{-3}\) appropriate for our date (taken from http://www.jb.man.ac.uk/~pulsar/crab.html; Lyne et al. 1993). We took care to read sufficient extra data to avoid dededispersion wrap-around.
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Figure 5.1: Top: Average pulse profile at Westerbork across our 6 hour observation, derived by folding the dedispersed data in 128 phase bins. The slight dips around the main and interpulse in the folded profile are an artefact from the 2-bit digitization (it effectively clips the signal when a bright giant pulse arrives). Bottom Panels: Pulse profiles and spectra of the two brightest main pulses (black) and interpulses (red) in 250 ns bins; the spectra contain 8 $\mu$s centered on the peak, in 125 kHz channels. Note the large intrinsic differences between pulse profiles - shot noise in the pulse profiles manifests as frequency structure, meaning the frequency structure seen is a combination of the scintillation pattern and intrinsic structure.
Figure 5.2: Part of the dynamic spectrum inferred from the main pulse by summing individual giant pulse spectra at 250 kHz resolution in 4 s bins. The total flux in each time bin was normalized to remove the effects of variable pulse brightness. The random occurrence of giant pulses and their variable flux means that the noise properties of the time bins are heterogeneous, and that some bins have no flux.

5.3.1 The Dynamic Spectrum of the Main Pulse

During our observation, the scattering time was relatively small - smaller than the intrinsic duration of the pulses, such that we can resolve scintillation patterns in frequency. We can thus construct the dynamic spectrum $I(t, \nu)$ by simply summing giant pulse spectra. Since we wish to observe the scintillation pattern rather than the vast intrinsic intensity variations between giant pulses, we normalize each time bin by the total flux within that bin. While there will still be structure in the dynamic spectrum owing to the intrinsic time structure of the giant pulses (Cordes et al. 2004), any features in frequency which correlate in time should only be associated with scintillation. We show a 20 minute segment of the dynamic spectrum in Figure 5.2. While noisy, the dynamic spectrum shows scintillation features. They are resolved by our time and frequency bin sizes of 4 s and 250 kHz, respectively, but only by a few bins, suggesting that the scintillation timescale and bandwidth are larger than our bin sizes by a factor of a few (consistent with $\nu_{\text{decorr}} = 1.35 \pm 0.02$ MHz, $t_{\text{scint}} = 10.1 \pm 0.2$ s measured below).
5.3.2 Correlation Functions

The correlation function between two spectral intensity streams \( I_1(t, \nu) \) and \( I_2(t, \nu) \) can be written as,

\[
R(\Delta t, \Delta \nu) = \frac{\langle (I_1(t, \nu) - \mu_1)(I_2(t + \Delta t, \nu + \Delta \nu) - \mu_2) \rangle}{\sigma_1 \sigma_2},
\]

where \( \Delta t \) and \( \Delta \nu \) are offsets in time and frequency, \( \mu_1 \) and \( \mu_2 \) are averages of \( I_1 \) and \( I_2 \) over time and frequency, and \( \sigma_1 \) and \( \sigma_2 \) estimates of the standard deviation.

To infer the scintillation bandwidth and timescale, one usually uses the auto-correlation of the dynamic spectrum, but for pulses randomly spaced in time, it is easier to calculate covariances for pulse pairs and then bin by time separation \( \Delta t \) (Cordes et al. 2004). We then correlate each pulse pair (averaging eight 1\( \mu \)s spectra around each peak, taking care to account for the contributions of noise to variance in the spectra, see Appendix 5.5), giving an estimate of \( R(\Delta t, \Delta \nu) \) for a single value of \( \Delta t \), the time separation of the pulses. We then sum these correlated spectra in equally spaced bins of \( \Delta t \) to construct our average correlation function. In Fig. 5.3, we show the result, both for correlations between main pulse pairs and for correlations between main pulse and interpulse pairs (there are insufficient giant pulses associated with the interpulse to calculate a meaningful correlation function from those alone).

The main pulse spectra decorrelate on a scale of \( \nu_{\text{decorr}} = 1.35 \pm 0.02 \) MHz in frequency, and on a scale of \( t_{\text{scint}} = 10.1 \pm 0.2 \) s in time.\(^2\) The timescale is somewhat shorter than the value of 25 \( \pm \) 5 s found at 1.475 MHz by Cordes et al. (2004), and the difference in frequency does not account for the difference (for \( t_{\text{scint}} \propto \nu \), our measurement corresponds to 9.07 \( \pm \) 0.16 s at 1.475 GHz). Differences are expected for observations at different epochs, however, as the scattering in the nebula is highly variable (Rankin & Counselman 1973; Lyne & Thorne 1975; Isaacman & Rankin 1977; Isaacman & Rankin 1977; Isaacman & Rankin 1977).

\(^2\)We adopt the usual convention, defining \( \nu_{\text{decorr}} \) and \( t_{\text{scint}} \) as the values where the correlation function drops to 1/2 and 1/\( e \) respectively.

The amplitude of the correlation is surprisingly low, which is investigated further in section 5.3.4. We construct the correlation from main pulses in Arecibo, shown on the right hand side of Fig. 5.3. The correlation is seen, with similar scintillation timescale bandwidth and amplitude, lending confidence that the correlation seen in Westerbork is not a systematic product of the telescope.

The decorrelation bandwidth obtained in this paper differs significantly from what was calculated in Popov et al. (2017) for the same dataset. Our methods differ, however - they auto-correlate giant pulse spectra between their left and right circular polarizations, while we correlate pulse pairs. If we adopt the cutoff of SN > 22 as in Popov et al. (2017), correlate left and right circular polarizations and fit a single exponential, then we measure $\nu_{\text{decorr}} = 0.42 \, \text{MHz}$, much closer to their value of $\nu_{\text{decorr}} = 0.32 \, \text{MHz}$. However, a two-exponential fit is a much better fit to the data, giving two distinct scales of $\nu_{\text{decorr,1}} = 0.98 \, \text{MHz}$, $\nu_{\text{decorr,2}} = 0.16 \, \text{MHz}$; the above explanation is consistent with our results if the small bandwidth $\nu_{\text{decorr}}$ is caused by intrinsic pulse structure (correlating only within a single pulse’s spectrum), and the wide bandwidth $\nu_{\text{decorr}}$ is the scintillation bandwidth (correlating between pulse pairs within $t_{\text{scint}}$).

In Fig. 5.3, one sees that the main pulse to interpulse correlation function is different from the main-pulse autocorrelation, being offset in time and frequency by about $-1.3 \, \text{s}$ and $-0.6 \, \text{MHz}$, respectively, and having a lower maximum correlation. To quantify the significance of these differences, we use simulated cross-correlations. For these, since we have many more giant pulses during the main pulse than the interpulse, we simply take 528 random main pulses (the number of interpulses above $16\sigma$) and correlate these with the other 6401 main pulses. We repeat this 10000 times, and fit each subset with a 2D Gaussian, allowing for offsets in time and frequency.
Figure 5.3: **Left:** Images: Cross-correlations $R(\Delta t, \Delta \nu)$ of pulse dynamic spectra, between giant pulses in the main pulse with themselves (top) and with giant pulses in the interpulse (bottom). The correlation between main-pulse giant pulses is point-symmetric by construction (i.e., $R(\Delta t, \Delta \nu) = R(-\Delta t, -\Delta \nu)$), but this is not the case for the correlation between interpulse and main pulse. **Side panels:** a 3-bin and 5-bin wide average of main correlations through the best fit $\Delta t, \Delta \nu$, respectively. Blue dotted lines are the same cuts through the 1D Gaussian fits.

**Right:** Same as left side, but instead using pulses at Arecibo. The S/N is drastically lower in this example, owing both to the higher $T_{\text{sys}}$ value at Arecibo (leading to fewer detected pulses, with lower S/N), and the shorter observation time. The red dotted line is the overlay of the MP-MP correlation at Westerbork. There is a slight discrepancy on the amplitude, which is likely due to uncorrected instrumental effects in the Arecibo data: after dividing by background to remove the frequency response of giant pulses, wide bandpass features at Arecibo still show up in the cross-correlations.
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\(\Delta t_0\) and \(\Delta \nu_0\). Comparing these with the value fit to the interpulse to main-pulse correlations (see Fig. 5.4), the differences are significant: none of the simulated data sets have larger \(\Delta \nu_0\), larger widths in frequency and time, and none have as small an amplitude. The measured time offset \(\Delta t_0\) is only marginally significant.

The reduced amplitude makes it somewhat difficult to estimate uncertainties on time and frequency offsets. We estimate them by scaling the standard deviations from the simulations by the ratio of the main simulated amplitude to the observed one, yielding \(\Delta t_0 = -1.3 \pm 0.6\) s and \(\Delta \nu_0 = -0.62 \pm 0.08\) MHz.

5.3.3 Secondary Spectra

Pulsar scintillation is often best studied in terms of its conjugate variables \(\tau\) and \(f_D\), through their secondary spectrum \(A(\tau, f_D) = \tilde{I}_1(\tau, f_D) \tilde{I}_2^*(\tau, f_D)\) (e.g. Stinebring et al. 2001; Brisken et al. 2010). The secondary spectrum is simply the Fourier transform of the correlation function \(R(\Delta \nu, \Delta t) = I_1(\nu, t) \ast I_2(\nu, t)\). For the MP-MP correlation, \(A(\tau, f_D)\) is purely real, but in the MP-IP correlation, any time or frequency offsets in the correlation function will manifest as a phase gradient in \(f_D, \tau\). We form secondary spectra for both correlations, after padding by 60 zero bins in time, shown in Fig. 5.5.

The MP-IP secondary spectrum is dominated by a phase gradient in \(\tau\), arising from the frequency offset in the correlation function. Removing a linear phase gradient in \(\tau\) shows a marginally significant phase gradient in \(f_D\).

5.3.4 The Surprisingly low Correlation Coefficient

Giant pulses are on average a few \(\mu s\) in duration, comprised of many smaller, unresolved "nanoshots" (e.g. Hankins & Eilek 2007). However, if all nanoshots originate from the same projected physical location, they should all be imparted with the same impulse response function. To test this, we begin by selecting only pulses above 16\(\sigma\), and channelize them to time and frequency binnings of 1\(\mu s\) and 500 kHz, respectively.
Figure 5.4: Bottom-left: Corner plot of the best-fit parameters of the simulated interpulse to main-pulse correlation function, obtained by fitting a two-dimensional Gaussian. Simulated correlation functions are constructed from randomly drawn sets of giant pulses from the main pulse (with the same sample size as that available for the interpulse), correlated with the full main pulse sample. The dotted lines show the best fit to the actual MP-IP correlation. None of the simulated time and frequency widths are as large as the MP-IP values, and none of the simulated amplitudes are as low. Top-right: The MP-IP correlation, and 3 example simulated MP-IP plots for comparison.
Figure 5.5: Top-left: Secondary spectrum of the main pulse. Top and bottom-right: Amplitude and phase of the main pulse - interpulse cross-spectrum. Note the amplitude and phase are point symmetric, and point anti-symmetric, by definition. Bottom-left: Averaged phase of the top half of the MP-IP cross-spectrum, after rotating out the best-fit linear phase gradient in $\tau$. Note that the error bars are correlated, due to zero-padding $R(\Delta v, \Delta t)$ before Fourier transforming.
This implies that they have an integrated signal-to-noise ratio of $\gtrsim 2$ in each 500 kHz channel, and that the scintillation bandwidth is just resolved.

We then center all pulses, and correlate spectra from the first and second 4 $\mu$s half of each giant pulse (i.e. summing four 1 $\mu$s spectra in each half), shown in Fig. 5.6. At $\Delta \nu = 0$, the spectra correlate at $\sim 6\%$. The intrinsic shot noise structure of giant pulses can cause additional frequency structure, lowering the correlation, but this cannot reduce a correlation to below 1/3 (Cordes et al. 2004, Appendix 5.5).

The $\sim 6\%$ correlation between the two halves of each giant pulse is well below 1/3, yet is still larger then the $\sim 4\%$ correlation for MP-MP. This could be explained if individual pulses come from a smaller part of the full extended emission region. The low correlations would be consistent if the emission region is larger than the resolution of the scattering screen. This would also help explain the lower amplitude and increased time and frequency widths of the MP-IP correlation compared to the two halves of each giant pulse: if both the MP and IP have similar emission sizes, then their correlation would be extended by a factor of $\sqrt{2}$.

\section*{5.4 Ramifications}

\subsection*{5.4.1 Spatial Resolution of Scattering Screen}

The size and location of the scattering screen is not precisely known, but a model in which the temporal scattering occurs in the Crab nebula is favoured by VLBI measurements showing the visibility amplitude is constant through the scattering tail (Vandenberg et al. 1976) as well as the short scintillation timescale (Cordes et al. 2004). The geometric time delay is

$$\tau = \frac{\theta^2 d_{\text{eff}}}{2c}, \quad \text{with} \quad d_{\text{eff}} = \frac{d_{\text{psr}} d_{\text{lens}}}{d_{\text{psr}} - d_{\text{lens}}},$$ (5.2)

where $\theta$ is the angle the screen extends to as seen from Earth, and $d_{\text{psr}}$ an $d_{\text{lens}}$ are the distances to the pulsar and the screen, respectively. The scattering screen can be seen
Figure 5.6: Spectral correlations between two 4 \( \mu s \) halves of the giant pulses. Top: Correlation \( R(\Delta \nu) \) averaged over all giant pulses. Bottom: Histogram of all values of \( R(\Delta \nu = 0) \).

as a lens, with physical size \( D = \theta d_{lens} \) and corresponding angular resolution \( \lambda / D \), giving a physical resolution at the pulsar of \( \Delta x = (d_{psr} - d_{lens}) \lambda / \theta d_{lens} \), or, in terms of the scattering time \( \tau \),

\[
\Delta x = \lambda \left( \frac{d_{psr} - d_{lens}}{2c\tau} \frac{d_{psr}}{d_{lens}} \right)^{1/2}.
\] (5.3)

Assuming the scattering is dominated by the nebula, we have \( d_{lens} \approx d_{psr} \) and hence for the known scattering time \( \tau \approx 160 \) ns (from \( \tau = 1/2\pi \Delta \nu, \Delta \nu \approx 1 \) MHz), the dominant unknown is the distance between the pulsar and the screen.

Since scattering requires relatively large differences in electron density, it cannot happen inside the pulsar-wind filled interior of the Crab nebula, which must have very low density. For a reasonable bulk magnetic field of \( 10^{-4} \) G, the emitting electrons are very relativistic, with \( \gamma \approx 10^{6} \). The radio emitting electrons have a density of \( n_e \approx 10^{-5} \) cm\(^{-3} \) (Shklovsky 1957), implying that the refractive index deviates from
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unity by a tiny amount,
\[ \Delta n \approx \left( \frac{v_p}{\nu} \right)^2 \sim 10^{-32}, \]
where \( v_p = \left( 4\pi e^2 n_e / \gamma m_e \right)^{1/2} \) is the plasma frequency, and \( \nu \) is the observed radio frequency.

Instead, the only plausible location for the temporal scattering is in the optically emitting filaments in the Crab Nebula. These filaments develop due to the Raleigh Taylor instability: as the pulsar wind pushes on the shell material, the contact discontinuity accelerates (Chevalier 1977) leading to the RT instability and formation of filaments (Porth et al. 2014).

With 3-dimensional models fit to spectroscopic optical data of the Crab Nebula, Lawrence et al. (1995) find the filaments reside in the range 0.3–0.75 pc when using a nominal pulsar distance of 2 kpc (given the full range of distances, \( 1.4 \lesssim d_{\text{psr}} \lesssim 2.7 \text{kpc} \) from Trimble 1973, this implies filaments in the range 0.2–1 pc). Assuming \( d_{\text{psr}} - d_{\text{lens}} \approx 0.5 \text{pc} \), then \( \Delta x \approx 2300 \text{ km} \) (for the full range of possibilities \( 1400 \lesssim \Delta x \lesssim 3200 \text{ km} \)). Thus, the resolution of the scattering screen is comparable to the light-cylinder radius of the Crab pulsar, \( R_{\text{LC}} \equiv cP / 2\pi = 1600 \text{ km} \).

From Fig. 5.3, the low correlation values point to emission regions which are at least of order the resolution element of the screen, or, equivalently, of order the light cylinder radius. Additionally, the nominal time offset between the main pulse and interpulse of \( \sim 1 - 2 \text{ s} \) is \( \sim 10 - 20\% \) of the scintillation timescale, which would suggest that the emission locations are separated by hundreds of km. We could turn the measured time offset into a physical separation given a relative velocity between the pulsar and the screen. Unfortunately, this is not known, though we can set limits from the proper motion. The proper motion of the Crab pulsar relative to its local standard of rest is measured to be \( 12.5 \pm 2.0 \text{ mas/yr} \) in direction \( 290 \pm 9 \text{ deg} \) (east of north, Kaplan et al. 2008), where the uncertainties attempt to account for the uncertainty in the velocity of its progenitor, and, therewith, of the nebular material. At
an assumed distance of 2 kpc, the implied relative velocity of the pulsar is $\sim 120 \text{ km/s}$, and non-radial motions in the filaments can be up to $\sim 70 \text{ km/s}$ (Backer et al. 2000). The time delay between pulse components would then suggest a projected separation between the interpulse and main pulse emission regions of $\sim 60 - 250 \text{ km}$.

### 5.4.2 Fully Measuring the Separation

A major uncertainty in the estimate of the spatial separation between the main pulse and the interpulse is the geometry of the lens. From studies of the scintillation in other pulsars, the scattering screens in the interstellar medium are known to be highly anisotropic, as demonstrated most dramatically by the VLBI observations of Brisken et al. (2010). If the same holds for the nebular scattering screens, this implies that our resolution elements are similarly anisotropic. Since the orientation relative to the proper motion is unknown, the physical distance between the main and interpulse regions could be either smaller or larger than our estimate above. Since the scattering varies with time, it may be possible to average out these effects.

Furthermore, all values relating to the scattering screen include the uncertain distance to the Crab pulsar, suggesting that a parallax distance would improve our constraints. In addition, the rough location of the scattering in the filaments is a physical argument, and would be greatly improved through a direct measurement.

The distance to the screen(s) can be constrained through VLBI and through scintillation measurements across frequency. VLBI at space-ground baselines (Rudnitskii et al. 2016) or at low frequencies (Kirsten et al., in prep.) can help constrain the angular size of the scattering in the interstellar medium. This in turn can constrain the size of the nebular screen; the visibility amplitudes will only decrease below 1 when the scattered image of the pulsar is not point-like to the interstellar screen. In addition, two scattering screens will impart two distinct scattering times only when they do not resolve each other (Masui et al. 2015). The transition frequency for the two
scintillation timescales to become apparent in the spectra will give a size measurement of the nebular screen.

Applying this same analysis across different frequencies, or in times of different scattering in the nebula will also help to quantify both the separation of the main pulse and interpulse, and the size of the emitting regions of both components. The correlation function of spectra is a crude measurement - it is fourth order in the electric field, and the scintillation pattern is contaminated with intrinsic pulse substructure. A much cleaner measurement can hopefully be made in the regime where the duration of giant pulses is much less than the scattering time, akin to the coherent method of de-scattering pulses used in Main et al. (2017a).

We associate the scattering screen with the Rayleigh-Taylor filaments in the pulsar wind nebula (Porth et al. 2014). These filaments appear when the pulsar wind pushes and accelerates freely expanding envelope. This stage terminates after few thousand years when the reverse shock from the interaction between the supernova remnant at the ISM reaches the pulsar wind nebula (Gelfand et al. 2009, see review by Slane 2017). Thus we expect such scattering events to be specific for pulsar wind nebulae during a fairly short period - sufficiently young for the reverse shock not to reach the pulsar wind nebula, but sufficiently advanced to have RT-induced filaments.

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5.5 Appendix: Correcting Noise Biases in the Correlation Coefficient

The correlation coefficient between two pulse intensity spectra \( I_{1,2}(v) \) can be generally defined as,

\[
R(I_1(v), I_2(v)) = \frac{\langle (I_1(v) - \mu_1)(I_2(v) - \mu_2) \rangle}{\sigma_1 \sigma_2},
\]

(5.5)

where \( \langle \ldots \rangle \) indicates an average over frequency, and \( \mu \) and \( \sigma \) are measures of the average and variations around it, respectively. Typically, one chooses \( \mu_P = \langle I_P(v) \rangle \) and \( \sigma_P = s_P \equiv \langle (I_P - \mu_P)^2 \rangle^{1/2} \) so that, in the absence of noise, \( R = 1 \) for two pulses with intrinsically identical frequency structure. In the presence of some measurement noise \( \sigma_n \), one could approximate \( \sigma_P^2 = s_P^2 - \sigma_n^2 \), but this holds only for normally-distributed noise, not for our case of intensity spectra.

Here, we derive an expression valid for our case, where we wish to ensure that \( R = 1 \) for two pulses that are sufficiently short that we can approximate them as delta functions, and that are affected by the interstellar medium the same way, i.e., have the same impulse response function \( g(t) \). In that case, the measured electric field of a giant pulse is,

\[
E_P(v) = A_P g(v) + n(v),
\]

(5.6)

where \( A_P \) is the amplitude of the pulse’s delta function in the Fourier domain, and \( g(v) \) and \( n(v) \) are the Fourier transforms of the impulse response function and the measurement noise, respectively. The measured intensity is then

\[
I_P(v) = E_P^2(v) = A_P^2 g^2(v) + n^2(v) + 2A_P |g(v)||n(v)| \cos(\Delta \phi(v)),
\]

(5.7)

where \( \Delta \phi(v) \) is the phase difference between \( n(v) \) and \( g(v) \).

The expectation value for the average is,

\[
\mu_P = \langle I_P \rangle = A_P^2 \langle g^2 \rangle + \langle n^2 \rangle,
\]

(5.8)
where we have dropped the dependencies on frequency for brevity, and used that the cross term averages to zero since $\langle \cos(\Delta \phi) \rangle = 0$. Hence, the expectation value for the standard deviation is,

$$s_p^2 = A_p^4 \left[ \langle g^4 \rangle - \langle g^2 \rangle^2 \right] + \langle n^4 \rangle - \langle n^2 \rangle^2 + 4A_p^2 \langle g^2 n^2 \cos^2(\Delta \phi) \rangle,$$

where we have again omitted terms that average to zero. The last term does not average to zero because of the squaring: it reduces to $2A_p^2 \langle g^2 \rangle \langle n^2 \rangle$, since $g$ and $n$ are independent and $\langle \cos^2(\Delta \phi) \rangle = 1/2$.

For two pulses differing only by noise, the numerator of $R$ is

$$\langle (I_1(v) - \mu_1)(I_2(v) - \mu_2) \rangle = A_p^2 A_2^2 \left[ \langle g^4 \rangle - \langle g^2 \rangle^2 \right].$$

(5.10)

Thus, for an unbiased estimate of $R$, we need to estimate $\sigma_p = A_p^2 \left[ \langle g^4 \rangle - \langle g^2 \rangle^2 \right]^{1/2}$. We can do this by also measuring the properties of the background, which, if it is dominated by measurement noise with the same properties as the pulse, has $\mu_B = \langle n^2 \rangle$ and $s_B^2 = \langle n^4 \rangle + \langle n \rangle^2$ (this will underestimate the noise if the pulse is strong enough to raise the system temperature; for that case, however, the noise is small anyway). With this, it follows that to make estimates of $R$ free of noise bias, we should use,

$$\sigma_p^2 = s_p^2 - s_B^2 - 2(\mu_B - \mu_P)\mu_B.$$

(5.11)

All of the above is true for correlating the intensity of one polarization. Using the total intensity $I = I_L + I_R = I_X + I_Y$, under the assumption that the noise is not correlated between the polarizations, the expectation value for the standard deviation is

$$s_p^2 = A_p^4 \left[ \langle g^4 \rangle - \langle g^2 \rangle^2 \right] + \langle n^4 \rangle - \langle n^2 \rangle^2 + 2A_p^2 \langle g^2 n^2 \cos^2(\Delta \phi) \rangle,$$

(5.12)

and the noise free estimate is

$$\sigma_p^2 = s_p^2 - s_B^2 - (\mu_P - \mu_B)\mu_B.$$

(5.13)
with the cross-term differing by a factor of 2. Adding more samples with independent noise, the cross-term further diminishes, and can be treated as Gaussian in the limit of large N.

To test the above, we simulated identical delta function giant pulses with different noise in the manner described in Main et al. (2017a). We find that using the above estimates, the correlation coefficients between these pulses indeed average to unity when the impulse response functions are the same. Trying a slightly more realistic simulation, forming giant pulses with $N$ fully polarized shots, with random amplitudes (drawn from a normal distribution) and random phases, the correlation decreases, saturating at $R = 1/3$ for large ($\gtrsim 10$) $N$, in line what is expected from the derivation in Cordes et al. (2004).
Chapter 6

Conclusions & Future Work

In this thesis, I resolve the emission regions of two pulsars in electron dense environments. I place quantitative constraints on the size and separation of different pulse components in these systems, a step towards the ultimate goal of creating an map of pulsar emission. I also show that the impulse response function of the ISM can be directly measured through bright, short pulses, which has potential uses to remove the effects of scattering, or as a way to probe pulsar emission in the scatter-broadened regime.

The results presented in this thesis lead to many logical avenues to follow. To conclude, I describe some of the related topics that I believe should be pursued.

6.1 Scattering in Supernova Remnants / Pulsar Wind Nebulae

Much of this thesis was based on the idea that scattering at close proximity to pulsars has improved resolution compared to typical interstellar scattering. One could continue by targeting other such pulsars in supernova remnants (Lorimer et al. 1998), or pulsars such as PSR B0355+54, which may be scattered in a bow shock between the pulsar wind and the ISM (Xu et al. 2018).
There is much follow up work to be done on the Crab as well. In Chapter 5, the giant pulse emission region is resolved, and the separation between the main pulse and interpulse is tentatively measured. More observations at a range of resolutions (i.e. different observing frequencies, or scattering timescale) could be used to measure the size of the emission region, by seeing the inferred resolution at which the correlation reaches 1/3.

While it is easiest to resort to a physical argument to place the scattering within the Crab nebula’s filaments, there is still the prospect of solving the screen using VLBI. The reason we cannot do that currently is that $\tau$ is measured for the nebula, while $\theta$ is measured for scattering in the ISM. However, the timescale of the ISM should show up easily in the profile of scattered giant pulses, where the convolution of both screens leads to a rise time. This would fully constrain the ISM’s scattering screen, which itself may be able to resolve the nebular scattering screen, giving a measure of its distance from the pulsar (although, depending on how isotropic the nebular screen it, its orientation is another complicating factor).

The radio profile of the Crab pulsar becomes particularly interesting above 2 GHz, where the high-frequency interpulse appears, showing regularly spaced banding in frequency, possibly owing to propagation effects through the magnetosphere (Hankins & Eilek 2007). Constraints on its location and size from scintillation would be of great interest.

### 6.2 Eclipsing Systems

The results in Chapter 4 all came from studying a single eclipse of one pulsar, in one frequency range. A logical follow-up is to observe more eclipses of PSR B1957+20, in different frequency bands. There are many other eclipsing systems, one of which is already known to have highly magnified pulses associated with its eclipse (Bilous et al. 2011). The effects of lensing can be searched for in other systems to test how
ubiquitous it is, and to understand the physical conditions that lead to strong lensing events. While individual pulses may not be detectable in all systems, the effects of lensing can still be searched for through flux which correlates in time.

Modelling the lensing events can give constraints on the outflow velocity, and can lead to measured mass loss rates, giving insights into the final fate of these systems. In addition, studying the effects of polarization can probe the magnetic field of the lensing material, which could be easily measurable if the eclipse is caused by cyclotron absorption (and will be more easily measured in highly polarized systems, unlike PSR B1957+20).

In Chapter 4, we used a simple model for the plasma lenses applied in a simple way, using characteristic magnification, frequency and time widths of a few bright lensing events to constrain emission locations and the outflow velocity of the wind. We still need to develop lensing theory, and apply it in a more quantitative way. Even starting with the perfect lens model in Chapter 4, we can parametrize the dynamic spectra of lensing events in terms of the size and orientation of the lens, the trajectory of the pulsar, and fit the model to data to constrain velocity. Expanding the model, wave optics will need to be considered, and compared to data; with sufficiently high-sensitivity observations, the DM structure can be measured to the Fresnel scale, which can be used as an input to lensing models.

### 6.3 Holography

It would be greatly useful to have a general method to retrieve the impulse response function of scintillation. Currently, there are methods which can work given certain optimal conditions. Giant pulses work, but most sources simply do not emit them. Cyclic spectroscopy has been shown to accurately retrieve the impulse response function in PSR B1937+21 (Demorest 2011; Walker et al. 2013) and in simulated data (Palliyyaguru et al. 2015), and while it is in principle a general method, it has in practice
not been successful in extracting \( g(t) \). Phase-retrieval from the secondary spectrum has also been shown to work for PSR B0834+06 (Walker et al. 2008; Pen et al. 2014), but the iterative methods currently employed generally have difficulty converging.

PSR B1937+21 seems an ideal system to target in detail to compare the different phase-retrieval methods, to hopefully understand their similarities and modes of failure. In particular, it is the source in which cyclic spectroscopy is known to accurately extract the impulse response function, it shows clear parabolic arcs in the secondary spectrum (Walker et al. 2013), and has bright, intrinsically short giant pulses (Cognard et al. 1996; Soglasnov et al. 2004).

### 6.4 Testing Models of Scintillation

While the mechanism behind scintillation is still unknown, we have now at least one model which makes testable predictions for the evolution of secondary spectra in time and frequency (Simard & Pen 2018). There are pulsars with isolated “echoes”, single dominant images which trail the pulse profile (e.g. Crab pulsar, Backer et al. 2000; Lyne et al. 2001, PSR B2217+47, Michilli et al. 2018). Tracking these echoes seems like a clean test for theories, as they are essentially single points in \( \tau, f_D \), and should move in a predictable way; for sheets, the echoes must pass through the origin of the secondary spectrum, while this is not a requirement for scattering from isolated points, which can pass through \( f_D = 0 \) at finite \( \tau \). The same is true for the evolution of arclets in secondary spectra, and the sheet model makes additional predictions on the angular position of scattering points as a function of frequency. An ideal experiment would be a multi-epoch, VLBI monitoring of such pulsars.
6.5 Binary orbits

Scintillation remains a promising way to study pulsar orbits. As described in section 1.2.6, it may be used to measure the inclination and masses in physically interesting systems, such as the black widow pulsar.

However, in systems with a full binary solution, it may also be possible to use the additional constraint from scintillation to measure a precise distance to the pulsar. This would be of great use for relativistic binaries, which are used to test General Relativity (two ideal candidates are PSR J0737-30 Kramer et al. 2006; Rickett et al. 2014, and PSR J1141-6545, Ord et al. 2002; Venkatraman Krishnan et al. 2017). In particular, the GR test based on the orbital decay $\dot{P}$ is limited by uncertainty in pulsar distance (eg. Stairs et al. 2002).

Precise distances to pulsars used in timing arrays have the potential to greatly improve sensitivity to detecting gravitational waves; distance measurements to within a gravitational wavelength ($\sim 1 - 10$ lyr) would allow the use of the term corresponding to gravitational waves incident on the pulsar, rather than just the term for waves incident on Earth. A logical starting point / proof of concept for this would be PSR J0437-4715, which has a measured VLBI parallax distance of $156.3 \pm 1.3$ pc (Deller et al. 2008), is in a 5.75 day orbit with a white dwarf companion, and has one of the most precise timing solutions of any pulsar.

6.6 Fast Radio Bursts

Pulsars are not the only scintillation sources known, as Fast Radio Bursts (FRBs) have been observed both to be scatter broadened, and to scintillate (Masui et al. 2015).

An exciting use of FRBs is to map the electron distribution of the universe (McQuinn 2014). A measurement of an FRB’s location and redshift, combined with its DM gives the total electron column to a specific spot in the universe. Many such mea-
surements allows one to create a 3D distribution of electrons in the nearby universe. However, this is uncertain to the DM contribution within the FRB’s host, which can be a significant fraction of the observed DM (Tendulkar et al. 2017), or from a special dense environment (Connor et al. 2016). It would be of interest to know to what we can learn about environments surrounding FRBs through scintillation / scattering, and if the local contribution of DM can be determined.

It has been proposed that FRBs can be lensed by plasma in their host galaxies (Cordes et al. 2017). It would be interesting to see if we can successfully apply the same lensing theory we developed for the Black Widow pulsar on FRB 121102 (the repeating FRB), given measurements of its variable DM and RM (especially given the similarities between the spectra of bursts from FRB 121102 and the magnified events in PSR B1957+20). With CHIME FRB coming online, there will be hundreds to thousands of FRBs discovered in the coming years, and there are likely to be more repeating FRBs found, and FRBs localized, for which to study.
Bibliography


Antoniadis, J., et al. 2013, Science, 340, 448


Baade, W., & Zwicky, F. 1934, Physical Review, 46, 76


Cordes, J. M. 2013, Classical and Quantum Gravity, 30, 224002


Goodman, J. 1997, New Astronomy, 2, 449


Lorimer, D. R., & Kramer, M. 2012, Handbook of Pulsar Astronomy


Lyne, A. G. 1984, Nature, 310, 300
Manchester, R. N., & IPTA. 2013, Classical and Quantum Gravity, 30, 224010
Masui, K., et al. 2015, Nature, 528, 523
Nye, J. F. 1999, Natural focusing and fine structure of light: caustics and wave dislocations


Popov, M. V., Rudnitskii, A. G., & Soglasnov, V. A. 2017, Astronomy Reports, 61, 178


Rudnitskii, A. G., Karuppusamy, R., Popov, M. V., & Soglasnov, V. A. 2016, Astronomy Reports, 60, 211
Rudnitskii, A. G., Popov, M. V., & Soglasnov, V. A. 2017, Astronomy Reports, 61, 393
Slane, P. 2017, ArXiv e-prints
Stinebring, D. 2013, Classical and Quantum Gravity, 30, 224006


