3D Block Modeling of Geomechanical Properties Using Conditional Simulation Method

by

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Abstract

A thorough understanding of in-situ rock mass geomechanical properties is a key factor for a safe and efficient design of a mining project. Geomechanical properties in a mining project are generally quantified using sample data collected during site investigation. In many mining projects, an exhaustive site investigation to characterize the rock mass properties is impossible. Hence, it is crucial to model the spatial variability of rock mass geomechanical properties using the limited data.

This thesis proposes and implements geostatistical simulation approaches for modeling of geomechanical heterogeneity of the rock masses in an iron ore open pit mine. 3D block models for geomechanical properties in different pit areas are generated and verified using statistical methods. The 3D geomechanical models allow the heterogeneity of rock masses to be involved in open pit mine planning and design, as more accurate knowledge of spatial distribution of geomechanical properties promotes safe and economic mine operation.
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Chapter 1
Introduction

1.1 Problem Statement

Like any huge and capital-intensive business, mining companies aim to maximize the long-term profit expected throughout the life-time of their mining operations. To achieve this goal within the current competitive and volatile market in which the commodity prices fluctuate too fast, mining executives would like to optimize the mine-to-mill value chain of the mining operations. Such optimization would be actualized when the expected revenue and throughput of mining operations are maximized while the operational costs are minimized.

The expected revenue and throughput and the operational costs (e.g. drill and blasting, energy, maintenance costs) of mining operations are primarily governed by some exogenous factors such as the (commodity) price and some endogenous factors such as amount of recoverable resource, mining fleet, and so on. Unlike the exogenous factors that are mostly controlled by the market, the endogenous factors are dependent on the managerial and engineering approaches that are practiced in the mining operations. Thus, mining managers and engineers are encouraged to implement optimal mine design and planning methods that would optimize the cash flow of the mining operations. To achieve an optimal mine planning and design, a thorough comprehension of the in-situ rock mass geomechanical quality is crucial because the geomechanical quality of rock masses encounter in a mining project area governs the behavior of the rock when subjected to loads from engineering activities. Such a complete knowledge about the geomechanical behavior of rock mass is primarily acquired through the site characterization process. The site characterization includes evaluation of adequateness and accuracy of collected data, data integration, data interpretation and analysis, and conceptual modeling of the characteristics based on the data analysis and interpretation (Ayalew et al. 2002). A thorough site investigation is not applicable in many mining projects because of different reasons such as the high expenses of a complete investigation, inaccessibility to some parts of rock mass, and lack of trained labors. Therefore, it is highly important to estimate/model the geomechanical quality across the entire in-situ rock mass using the limited data collected during the site investigation program. Such modeling of rock mass geomechanical quality requires that the heterogeneity of geomechanical
properties that characterize the rock mass quality be accurately measured. Understanding the heterogeneity of rock mass geomechanical properties enables the mining managers and engineers to better understand and evaluate the spatial variability of rock mass geomechanical behavior and allows them to identify the high risk zones and the mine sectors that require more exploration and data collection attempts. In open pit mine projects, the models of the geomechanical heterogeneity could improve not only the reliability of pit designs and prediction of the rock mass behaviors during the excavations (i.e. rock mass stability, blast-induced rock mass fragmentation) but also improve the efficiency of comminution process (crushing and grinding behavior of rock). For example, such models can be considered as inputs into the blast design process in order to determine the optimal powder factor of an individual blast required for a target fragmentation (Bye 2011). This would consequently improve the mine-to-mill value chain through enhancing the material loading rates, hauling time and mill throughput.

In open pit mines, the common approach that is practiced for the geomechanical rock mass characterization is to create a deterministic boundary model of the subsurface geomechanical properties. In this approach, the rock masses present in a mine site are divided into a number of geomechanical domains according to the collected data of subsurface condition and engineering judgments (expert knowledge). Each geomechanical domain is assumed to be uniform in which the geomechanical properties are homogeneously distributed. An average value is generally assigned to rock mass properties of each domain and it is assumed that the average value represents for the property across the domain. This common rock mass geomechanical characterization approach is a deterministic method which ignores the local spatial variability of the rock mass properties within a geomechanical domain. The pit designs and mine plans produced using this approach could not be optimal. Assuming homogeneous rock mass properties within a geomechanical domain results in over- or under-estimation of rock mass mechanical properties. Under-estimation of rock mass properties could lead to the under-estimation of safety factors in pit design, entailing shallower pit slope angles and consequently resulting in unnecessary stripping costs and imposing extra time and operational constraints. Over-estimation of rock mass properties can result in pit designs with a higher risk of instability and its related consequences and costs. Both under- and over-estimation of the rock mass geomechanical properties would decrease the expected revenue and throughput and increase the operational costs. Hence, an approach in which the heterogeneity of the rock mass
geomechanical properties are characterized is crucial for the success of open pit mining operations. Such approach should be able to characterize the local variability/heterogeneity of geomechanical properties and consequently provide realistic predictions of the geomechanical properties of rock masses in different areas of the mine.

The present thesis aims to develop geostatistical simulation-based approaches for spatial characterization of the geomechanical properties including rock quality designation (RQD), fracture frequency, joint condition (JCond), uniaxial compressive strength (UCS), and rock mass rating (RMR). The proposed approaches in the thesis are based on the conditional geostatistical simulation technique, sequential Gaussian simulation (SGS). SGS, unlike geostatistical estimation methods, produces equi-probable simulated realizations of rock mass properties in which the smoothing effect observed in estimation techniques’ results is not produced (Deutsch 2002). The rock mass characterization approaches proposed in the thesis are implemented into a case study of an iron ore open pit mine in Quebec, Canada. The results of implementations are 3D block models of above-mentioned geomechanical properties. The 3D block models of each geomechanical property characterizes the heterogeneity or spatial variability of the property in the pit area. The reliability of the developed 3D block models of geomechanical properties are also verified using statistical techniques. Verification results indicate that the proposed approaches are able to characterize the heterogeneity of geomechanical properties where the statistics of the sample data used for modeling are honored.

1.2 Research Objectives and Hypothesis
Any successful earthwork engineering and design project requires a sufficient understanding of the rock mass quality. Therefore, a realistic 3D formulation of the rock mass quality is essential for the safe and effective design and engineering stages of mining projects. Such formulation should capture the heterogeneity of the rock mass properties present in the mine site. Hence, the primary objective of this research is to improve our understanding of heterogeneity of rock mass geomechanical properties. To accomplish this objective, geostatistical simulation-based methodologies are developed and implemented - in a case study of an iron ore open pit mine in Canada - in order to capture and model the spatial distribution of geomechanical properties. In summary, the present research is driven by the following research question: How the spatial variability/heterogeneity of rock mass geomechanical properties can be realistically modeled?
1.3 Research Methodology and Contributions

Briefly, the proposed methodology in this thesis is focused on the 3D block modeling of the geomechanical properties in an open pit mine. For this purpose, geostatistical simulation approach Sequential Gaussian Simulation is applied. The geomechanical drillhole data of an iron ore open pit mine are used as inputs for this analysis. The rock masses encountered in the pit area are firstly divided into six subdomains according their lithology and relative location in the mine formation. Then, two approaches are taken for creating block models of geomechanical properties. In the first approach so called direct approach, block models of RMR are produced using the RMR sample data from the geomechanical borehole dataset. Variogram modeling is firstly performed on the normal scores of RMR sample data within each zone. Then, simulations for each zone is generated using the SGS method. The second approach, indirect approach, utilizes principal component analysis (PCA) to reduce the number of model variables and then generates block models of geomechanical properties, separately for each of the six subdomains. The models within each subdomain are then combined to produce complete block models of geomechanical properties including RMR.

Some important contributions and industrial significance that can be expected from this research project are summarized as follows:

1. A more realistic representation of the rock mass spatial distribution where the heterogeneity of rock mass geomechanical properties are modeled through geostatistical simulation techniques. Such realistic representations can be used as inputs for the optimal pit design which itself can consequently result in:
   a) Optimization of pit slope design in different pit sector;
   b) Maximization of ore recovery (control of ore loss and dilution): the optimal pit slope design can minimize the risk of pit slope failure and consequently minimize its associated costs such as ore dilution and sterilization.
   c) Possible reduction of waste stripping: the waste stripping ratio can be minimized with the optimal pit slope design as the pit design provides the optimum slope angles and mine plan. This can also reduce the environmental impact of the mine operation;
d) Improved safety and liability: the proposed approach allows identification of high-risk zones (zones of poor rock mass quality). This can consequently lead to higher safety and liability of the open pit mine;

2. Identification of zones of strong and weak rock mass quality as well as the areas with inadequate data. This can help mine engineers to:
   a) Minimize drilling and blasting costs: depending on the geomechanical quality (strength, degree of fracturing of rock mass) in each zone, an appropriate drilling and blasting design (e.g. pattern, powder factor) can be established to achieve the target fragmentation and reduced drill and blast costs;
   b) Optimize the sampling campaign: depending on the rock quality in each zone and uncertainty in the spatial modeling, sampling and logging the geomechanical properties can be planned. For example, less sampling will be required for zones that have been estimated with lower uncertainty than the zones of higher estimation uncertainty.

1.4 Organization of Thesis

This thesis consists of five chapters and an appendix. Chapter 1 serves as an introduction to the problem definition, and includes research hypothesis, objectives, methodology and contributions of the research project. In Chapter 2, a literature review on the subsurface rock mass characterization and modeling is provided along with the theoretical background on the geostatistical techniques that are implemented in the research. Chapter 3 elaborates the background information of the case study that is used in this thesis and the exploratory statistical data analysis carried out on the geomechanical drillhole dataset of the case study. This chapter is followed by Chapter 4 where the research methodology is elaborated in details along with the implementation results. Chapter 5 presents a summary of the research methodology and results, and a discussion on the research contributions and limitations, and recommendations for future research. In the section Appendix, the variogram models created in the variography of the regionalized variables of interest in this thesis are presented.
Chapter 2
Modeling Rock Mass Geomechanical Heterogeneity using Geostatistical Methods - A Literature Review

2.1 Introduction

The present chapter focuses on approaches commonly used for characterizing the rock mass’s geomechanical properties and the modeling approaches available for spatial distribution and heterogeneity of the geomechanical properties. The first part of the chapter expresses a background review of rock mass geomechanical characterization/classification methods. Different rock mass geomechanical properties commonly used for geomechanical rock mass classification are elaborated. This is followed by a literature review of approaches used for modeling the heterogeneity of rock mass geomechanical properties with emphasis on geostatistical techniques.

2.2 Rock Mass Characterization/Classification

Rock mass characterization/classification is an integral part of any mining engineering design projects. Rock mass characterization is of special interest in feasibility and preliminary design stages of a mining project where data about the rock mass condition, its stress, and hydrogeological characteristics are limited (Hoek 2004). Rock mass characterization and classification schemes can be used to quantitatively and qualitatively describe a rock mass and estimate the empirical values of engineering properties (e.g. cohesion and friction angle) required for engineering design. Extensive knowledge of rock engineering, understandings of practical limitations and structure of the classification systems are necessary for rock mass characterization/classification (Palmstrom and Broch 2006). Generally, following objectives/outputs are considered for a rock mass characterization/classification program (Bieniawski 1989; Milne et al. 1998; Stille and Palmstrom 2003):

- Recognition of the most important properties and factors influencing on behaviors of rock mass;
- Segmentation of the entire rock mass into regions of homogeneous quality and behavior (geomechanical or geotechnical domains) used for simplifying the engineering design;
- Estimation of rock mass geomechanical properties such as elastic modulus and strength;
• Prediction of rock mass behaviors in different conditions;
• Establishment a basis for understanding each rock mass class’s characteristics;
• Development of quantitative data and guidelines for the detailed engineering design based on each rock mass class;
• Better communication between practitioners involved (e.g. geologist and engineers) through creating a common understanding of the rock behaviour.

In the literature of rock engineering, there are three rock mass classification systems that are broadly used in geomechanical mine design: rock mass rating (RMR) developed by Bieniawski (1976), rock tunneling quality index (Q-system) developed by Barton et al. (1974), and Geological Strength Index (GSI), developed by Hoek et al. (1995). These rock mass classification systems will be discussed in the following sections. As in the current thesis the RMR\textsubscript{76} rock mass classification system (Bieniawski 1976) will be used, this classification scheme is elaborated in great details in the next section. In addition, the other two classification systems are briefly presented along with their evolution, limitations, and applications.

2.2.1 Rock mass rating (RMR) classification system

Rock mass rating or Geomechanics Classification System was initially developed in 1973 by Bieniawski (1973) using 49 case histories. The RMR system was basically developed based on characterizing a number of constitutive geomechanical properties and rating them to classify the rock mass into a number of design classes which represent the overall geomechanical quality of the rock mass. The initial RMR system was devised for the purpose of assessing the stable period for an unsupported span of an underground opening in shale and clay-bearing rocks in presence of water and wetting-drying process (Aksoy 2008; Oliver 1973). Later on, the RMR system was extended and modified several times in 1974, 1975, 1976, 1979, and 1989 to be compatible for further applications in hard rock mining, coal mining, rock slope stability, foundations in rocks, and so on (Bieniawski 1989). Until 1989, RMR was applied and validated in 351 case histories mostly (about 90%) in shallow depths of less than 500 m (Bieniawski 1989). Out of these 351 applications, 62 were in coal mining in the period 1973-1984 and further 78 tunneling and mining projects in the period 1984-1987 (Bieniawski 1989). The RMR database of these 351 case histories indicates that 63%, 20%, and 17% of the rock masses were respectively
sedimentary, igneous, and metamorphic rocks (Suorineni 2014). Table 2-1 compares the different versions of RMR classification system in terms of the parameters importance weight. The modifications made on the RMR system over the time can generally be grouped into four types: number of geomechanical parameters, type/definition of geomechanical parameters, rock mass classes, and ratings.

In the initial release of RMR, the rock mass is classified by weighting of eight geomechanical parameters: intact rock strength, RQD, discontinuity spacing, separation of joints, continuity of joints, groundwater conditions, weathering, and strike and dip orientation for tunnels. In later modifications of RMR (1974, 1975, 1976, and 1989), Bieniawski reduced the number of parameters from eight to six and changed the corresponding importance weightings. In RMR 1975, 1976, and 1989, the six parameters used in RMR are as follows: intact rock strength, RQD, discontinuity spacing, groundwater conditions, joint condition, strike and dip orientation for tunnels.

In RMR 76, rock mass classes are defined in intervals of 20. The point load index is also added to the RMR table as an index for intact rock strength. Besides, roughness is included into the joint condition parameter with increase in its rating compared to 1973 version.

In the latest RMR version developed in 1989, Bieniawski holds the interval of 20 for the rock mass classes. However, a table is added to better define the joint condition using factors such as joint roughness, infilling, and weathering (Milne et al. 1998). Some adjustment factors are also applied for mining induced activities. To quantify joint condition, parameters such as discontinuity persistence, roughness, backfill and degree of deterioration are scored based on the ISRM standards (Aksoy 2008).

In evolution of RMR, changes in importance and weights of parameters are mainly made on joint condition, groundwater conditions, and joint spacing (JS). For example, Bieniawski dropped the rating for joint spacing from 30 in RMR 76 to 20 in RMR 89. Instead, the importance weighting of joint condition and groundwater conditions each was increased five points in RMR 89 compared with RMR 76. The reduction of joint spacing’s rating was due to the fact that RQD and joint spacing were identified highly correlated. Thus, Bieniawski decided not to overly rate the joint spacing parameter in the latest version of RMR.
Table 2-1: Changes in RMR Parameters Ratings in Different Versions (Milne et al. 1998)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Intact Rock strength</td>
<td>10</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>RQD</td>
<td>16</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Discontinuity spacing</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Separation of joints</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Continuity of joints</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Groundwater</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Weathering</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Condition of joints</td>
<td>15</td>
<td>30</td>
<td>25</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Strike and dip orientation</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strike and dip orientation for tunnels</td>
<td>3-15</td>
<td>0-12</td>
<td>0-12</td>
<td>0-12</td>
<td></td>
</tr>
</tbody>
</table>

RMR is a simple and versatile rock mass classification system that has been applied to different mining and civil engineering projects such as coal mining, hard rock mining, and rock slope stability and tunneling (Bieniawski 1988). Unlike its simplicity and versatility, RMR has the following limitations (Bieniawski 1989; Milne et al. 1998):

1. RMR provides relatively conservative results that consequently could entail overdesign of support systems in underground excavations;
2. RMR is roughly insensitive to minor variability in rock mass quality;
3. RMR is dependent on directions that its constitutive geomechanical properties are characterized;
4. RMR is not sensitive enough to each single constitutive parameter since RMR is calculated as summation of ratings assigned to different parameters. For instance, a rock mass can be assigned to a class because of either its low intact rock strength or its low RQD.

In the current thesis, the rock mass classification adopted in the mine site is RMR\textsubscript{76}. Therefore, the focus of this thesis is on RMR\textsubscript{76} for the heterogeneity modeling. To apply the RMR system for rock mass classification, the rock mass should be firstly partitioned into a number of structural zones in a way that certain properties (e.g. lithology, discontinuities spacing, etc.) be almost uniform within each zone. The structural zones are, in most cases, bordered by some geological features such faults, dykes, and shear zones (Bieniawski 1976). Then, values of five constitutive geomechanical parameters RQD, joint spacing, joint condition, groundwater
condition, and intact rock strength are rated for each structural zone. Table 2-2 shows how to rate the constitutive geomechanical parameters, calculate the RMR, and interpret the RMR classes. In Table 2-2, ratings that can be assigned to geomechanical parameters are listed. According to Bieniawski (1976), the ratings should be assigned to each geomechanical parameter, considering the worst conditions rather than average conditions. The intact rock strength is rated from 0 to 15 based on either point load index or the uniaxial compressive strength. The total spectrum of the intact rock strength is divided into seven ranges where a constant rating is assigned to each range. More rates are set to more values of intact rock strength. For example, if UCS=75 MPa (in the range 50-100 MPa), seven is the appropriate rate. RQD in RMR₇₆ is measured through the drillcore logging. RQD rating is within the range of 3 (for the very low RQD values <25%) to 20 (for the very high RQD values 90-100%). For example, for RQD=80% (within the range 75-90%), the appropriate rate is 17. Joint spacing is rated from 5 to 30 where the higher joint spacing values result in higher ratings. Five ranges are defined for joint spacing and the corresponding rating is assigned depending on the range that the value of joint spacing belongs to. For instance, if spacing of joints is within 50-300 mm, 10 is the appropriate rate. It should be noted that ratings for joint spacing is based on the assumption of the rock mass with three joint sets; thus, a conservative rating must be provided for joint spacing of rock masses having less than three joint sets (Bieniawski 1976). Depending on the qualitative interpretation of joint condition (in terms of joint surface, hardness, and shape), five rates can be assigned for this parameter. For example, a rate of 25 is assigned for a discontinuous joint with very rough surfaces, no separation, and hard joint wall rocks. Finally, the groundwater condition is weighted according to the general conditions (in terms of water pressure and content) and inflow rate. An appropriate rate, ranging from 0 to 10, is assigned to the ground water condition based on these general conditions or inflow rate. For example, if a rock mass is completely dry with no inflow rate, the appropriate rate is 10. After assigning the rates for each parameter, they are summated to provide the basic RMR value for each structural zone. The basic RMR value, ranging from 8 to 100, is then adjusted to account for joint orientations (strike and dip) depending on each application type. Table 2-2 section B indicates the adjustment scores for the joint orientations. Since joint orientations adversely function on the stability of excavation, the resulting adjusted RMR is less than or equal to the resulting basic RMR value. Three types of applications tunnels, foundations, and slopes are defined in RMR₇₆. Joint orientations are interpreted qualitatively in
terms of very unfavorable to very favorable orientation. Depending on the engineering application type and the qualitative interpretation of joint orientations, the basic RMR value is penalized by applying the appropriate join orientations rate. Section E of Table 2-2 could be used for the purpose of interpretation of joint orientations for tunneling projects. The resulting adjusted RMR is used then to classify the rock mass within each structural zone. According to section C of Table 2-2, a rock mass is classified as very poor rock (RMR<20), poor rock (RMR: 21-40), fair rock (RMR: 41-60), good rock (RMR: 61-80), and very good rock (RMR: 81-100). For engineering interpretation of rock mass classes, section D is useful. This section suggests values for cohesion and friction angle of a rock mass for each RMR class. Besides, the average stand-up time for tunnels and chambers formed by each rock mass of each RMR class is provided in section D. The constituent parameters of the RMR classification system are discussed in the following sub-sections.

Table 2-2: Ratings for RMRs (Bieniawski 1976)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>RANGES OF VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Strength of intact rock material</td>
<td>Point Load Index</td>
</tr>
<tr>
<td>Uniaxial Compressive Strength</td>
<td>&gt;8 MPa</td>
</tr>
<tr>
<td>RATING</td>
<td>15</td>
</tr>
<tr>
<td>2 Drill core quality - RQD</td>
<td>90-100%</td>
</tr>
<tr>
<td>RATING</td>
<td>20</td>
</tr>
<tr>
<td>3 Spacing of joints</td>
<td>&gt;3m</td>
</tr>
<tr>
<td>RATING</td>
<td>30</td>
</tr>
<tr>
<td>4 Condition of joints</td>
<td>Very rough surfaces</td>
</tr>
<tr>
<td>RATING</td>
<td>25</td>
</tr>
<tr>
<td>5 Ground water</td>
<td>Inflow per 10m tunnel length</td>
</tr>
<tr>
<td>RATING</td>
<td>10</td>
</tr>
</tbody>
</table>

B. RATING ADJUSTMENT FOR JOINT ORIENTATIONS

<table>
<thead>
<tr>
<th>RATING</th>
<th>Very favourable</th>
<th>Favourable</th>
<th>Fair</th>
<th>Unfavourable</th>
<th>Very unfavourable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnels</td>
<td>0</td>
<td>-2</td>
<td>-5</td>
<td>-10</td>
<td>-12</td>
</tr>
<tr>
<td>Foundations</td>
<td>0</td>
<td>-2</td>
<td>-7</td>
<td>-13</td>
<td>-25</td>
</tr>
<tr>
<td>Slopes</td>
<td>0</td>
<td>-5</td>
<td>-25</td>
<td>-50</td>
<td>-60</td>
</tr>
</tbody>
</table>

C. ROCK MASS CLASSES DETERMINED FROM TOTAL RATINGS

<table>
<thead>
<tr>
<th>RATING</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class number</td>
<td>100-81</td>
<td>80-61</td>
<td>60-41</td>
<td>40-21</td>
<td>&lt;20</td>
</tr>
<tr>
<td>Description</td>
<td>Very good rock</td>
<td>Good rock</td>
<td>Fair rock</td>
<td>Poor rock</td>
<td>Very poor rock</td>
</tr>
</tbody>
</table>

D. MEANING OF ROCK MASS CLASSES

<table>
<thead>
<tr>
<th>Class number</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average stand-up time</td>
<td>10 yrs for 5m span</td>
<td>6 mos for 4m span</td>
<td>1 wk for 3m span</td>
<td>5 hrs for 1.5m span</td>
<td>10 min for 0.5m span</td>
</tr>
<tr>
<td>Cohesion of the rock mass</td>
<td>&gt;300 kPa</td>
<td>200-300 kPa</td>
<td>150-200 kPa</td>
<td>100-150 kPa</td>
<td>&lt;100 kPa</td>
</tr>
<tr>
<td>Friction angle of the rock mass</td>
<td>&gt;45°</td>
<td>40-45°</td>
<td>35-40°</td>
<td>30-35°</td>
<td>&lt;30°</td>
</tr>
</tbody>
</table>

E. THE EFFECT OF JOINT ORIENTATIONS IN TUNNELING

<table>
<thead>
<tr>
<th>Strike perpendicular to tunnel axis</th>
<th>Strike parallel to tunnel axis</th>
<th>Dip D-20° irrespective of strike</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive with dip</td>
<td>Dip 45-90°</td>
<td>Dip 20-45°</td>
</tr>
<tr>
<td>Drive against dip</td>
<td>Dip 45-90°</td>
<td>Dip 20-45°</td>
</tr>
</tbody>
</table>

11
2.2.1.1 Intact rock strength

Intact rock strength is one of the parameter for the RMR classification system. Intact rock strength is governed by factors such as rock type, mineral composition, grain or crystal size, alteration, weathering, micro-cracks and stress history. Two major tests for measuring the intact rock strength are UCS and point load index. The measurement captured by either of these two tests can be used in RMR for classification of a rock mass.

UCS test is used to measure the intact rock strength parameter which can be employed for the characterization of intact rock and rock mass classification and consequently estimating the rock mass deformation parameters (ISRM 2007). In this test, a cylindrical specimen of an intact rock is compressively loaded until it breaks. Figure 2-1 shows an exemplary UCS test on a cylindrical intact rock specimen with cross section area $A$, diameter $D$, and height $H$. The value of UCS ($\sigma_c$) is calculated using Equation (2.1) as follows (ISRM 2007):

\[ \sigma_c = \frac{P}{A} = \frac{4P}{\pi D^2} \]  

(2.1)

Figure 2-1: Exemplary UCS test

Where $P$ is the load that causes the failure of the rock specimen. Following considerations must be regarded in the UCS test according to ISRM (2007):

1. **Samples’ specifications:**
   - Samples should be right circular cylinders having the ratio $H/D$ of 2.5-3.0;
• Sides of the cylindrical specimen shall be smooth and free of abrupt irregularities;
• The ends of samples should be flat within 0.02 mm;
• Diameter (D), calculated as the average of diameter in upper-height, mid-height, and lower-height of a specimen, shall be preferably greater than or equal to 51 mm and at least 10 times the largest grain in the intact rock.

2. **Loading:**
   • Uniaxial load should be applied to the specimen continuously at a constant stress rate of 0.5-1.0 MPa/s;
   • Axial load and axial and radial or circumferential strains should be recorded throughout the test.

3. **Failure and replications:**
   • Only test results where failure is through the intact rock rather than defects in the sample should be accepted;
   • At least five replications of each test is required.

4. **Anisotropy:**
   • For anisotropic rocks, UCS tests must be performed several times on cores oriented at different angles to any plane of weakness or foliation;
   • An upper bound and lower bound can be determined for $\sigma_c$.

The UCS results are generally overestimation of the field real strength of the rock since the intact rock samples collected for UCS test are usually not representative and from the better quality parts of rock (Read and Stacey 2009). Resulting values of UCS for an intact rock can be used to qualitatively describe the strength of rock. Table 2-3 shows the classification of a rock based on the value of UCS according to RMR$_{76}$.

<table>
<thead>
<tr>
<th>UCS (MPa)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;200</td>
<td>Very High</td>
</tr>
<tr>
<td>100-200</td>
<td>High</td>
</tr>
<tr>
<td>50-100</td>
<td>Medium</td>
</tr>
<tr>
<td>25-50</td>
<td>Low</td>
</tr>
<tr>
<td>10-25</td>
<td></td>
</tr>
<tr>
<td>3-10</td>
<td>Very Low</td>
</tr>
<tr>
<td>1-3</td>
<td></td>
</tr>
</tbody>
</table>

The Point Load Strength test is an index test that measures the strength ($I_{s(50)}$) of an intact rock (Franklin 1985). Unlike UCS that is only applicable in laboratories to only cylindrical
specimens, the point load strength index test can be performed by a portable equipment in the field as well and on specimens of different shapes such as cores (diametral and axial tests), cut blocks (block test), and irregular lumps (lump test). Figure 2-2 shows a diametral point load test on a core. Figure 2-3 displays the shape requirements for specimens in a diametral point load test. The diametral test is done on core samples of intact rocks with the ratio length to diameter greater than one (L > 0.5 × D). The load should increase continuously and in a constant rate that the specimen breaks within 10-60 seconds. At least 10 tests are required for each sample rock (Franklin 1985). For heterogeneous or anisotropic rocks, more than 10 tests should be run (Franklin 1985). A valid test is such that the failure surface passes through both contact points between the platens and sample.

Figure 2-2: A diametral point load test (Mwanga et al. 2015)

Figure 2-3: Specimen shape requirements for diametral tests (Franklin 1985)
Figure 2-4 displays the shape requirements for specimens in an axial point load test. The axial test is undertaken on core samples of intact rocks. The specimen for this test shall have the ratio length to diameter 0.3 to 1.0 (Franklin 1985). The load should increase continuously and in a constant rate that the specimen breaks within 10-60 seconds. At least 10 tests are required for each sample rock; for heterogeneous or anisotropic rocks, more than 10 tests should be run (Franklin 1985). A valid test is such that the failure surface passes through both contact points between the platens and sample.

Figure 2-4: Specimen shape requirements for axial tests (Franklin 1985)

Figures 2-5 and 2-6 show the shape requirements for the specimens in the block point load and lump point load tests, respectively. The specimens of size 50 ±35 mm with the shapes demonstrated in Figures 2-5 and 2-6 are appropriate for these tests. The ratio D/W (see Figure 2-5 and Figure 2-6) should be in the range of 0.3-1.0 and preferably close to 1.0.

Figure 2-5: Specimen shape requirements for block test (Franklin 1985)
The point load strength index is calculated using the following equation (Franklin 1985):

\[ I_{s(50)} = F \times I_s \]  \hspace{1cm} (2.2)

Where \( I_s \) and \( F \) represent the uncorrected point load strength index and the size correction factor, respectively. \( I_s \) is calculated as

\[ I_s = \frac{P}{D^2} \]  \hspace{1cm} (2.3)

where \( P \) is the failure load and \( D \) represents the equivalent diameter calculated using Equation (2.4):

\[ D_e = \begin{cases} \frac{D}{4A} & \text{for diametral test} \\ \sqrt{\frac{4A}{\pi}} & \text{for axial, block, lump tests} \end{cases} \]  \hspace{1cm} (2.4)

In Equation (2.4), \( A \) is the minimum cross sectional area of a plane passing through the platen contact points. The size correction factor can be calculated using Equation (2.5) as follows:

\[ F = \left(\frac{D_e}{D_{50}}\right)^{0.45} \]  \hspace{1cm} (2.5)

In assessment of intact rock strength using the point load index tests, followings should be taken into consideration (Read and Stacey 2009):
- Point load index test is not generally applicable for rocks with a UCS value smaller than 25 MPa as the points tend to indent the rock;
- In cases that very scattered results are achieved in the point load index test, more than 100 replications of the test are often necessary to achieve a reliable index.

### 2.2.1.2 Rock quality designation (RQD)

Rock Quality Designation (RQD) is an index of rock quality and core recovery which was initially introduced in 1960s. It is a modified core recovery which shows the percentage of good rock within an interval of a drillhole (Deere and Deere 1988). RQD is measured as the ratio of total length of all pieces of sound fresh, slightly or moderately weathered rock core longer than 100 mm (4 inch) to the total length of core run (Deere and Deere 1988; Milne et al. 1998; Read and Stacey 2009). In core logging, all drill-induced breaks in the core must be ignored (only natural breaks must be counted) (Milne et al. 1998). Figure 2-7 shows a schematic view of the procedure to log the RQD value for a core run.

Figure 2-7: Procedure to measure RQD (Deere 1989)

To log RQD, following factors must be well regarded (Deere and Deere 1988):
Core size:
- The RQD was firstly developed for NX-size cores obtained with double tube core barrels. Two commonly used and mostly recommended core sizes for RQD logging are NX- and NQ-sizes;
- The threshold of 10 cm length is recommended for all core sizes.

Measurement of core lengths:
- The length of core pieces is recommended to be measured along the center-line of cores as depicted in Figure 2-7;
- Drilling breaks should be put together and taken into account as one solid piece;
- Only natural breaks should be counted. If there is any doubt in discerning a drilling break (e.g. in schistose and laminated rocks), the break should be counted as the natural break which consequently results in a conservative RQD;
- RQD must be logged on the site immediately after a core is obtained in order to avoid any artificial fractures and manipulations in cores due to core handling.

Soundness of core pieces:
- According to definition of the RQD, only sound rock pieces of longer 10 cm must be included in calculation of the RQD;
- Based on the degree of weathering of a rock piece, judgment about its soundness could be made;
- It is recommended to count Fresh and Slightly Weathered (Grades I & II) (see the definition in Brown (1978)) rock pieces in RQD calculations. Rock pieces of Moderately Weathered (Grade III) should be counted in the RQD calculation but an asterisk must be used to highlight it.

Length of core run:
- RQD is a function of the core run length. The shorter the core run, the lower the RQD. It is more sensitivity to the core run length;
- Logging RQD is recommended to be performed on 1.5 m core runs

Since its emerge in 1960s, RQD has been extensively applied for preliminary site investigation to identify potential low quality zones (red flag areas) that might be more susceptible and require more site investigation (Deere and Deere 1988). Table 2-4 shows the classification of a rock
mass according to values of RQD. Five classes are defined and a rock mass is ascribed to one of these classes based on the average RQD. The zones with RQD less than 50 (RQD<50) are considered as having poor quality rocks. This could give an indication in early stages of site investigation and design that such poor quality zones require more detailed site investigation.

**Table 2-4: Description of RQD (Bieniawski 1976; Deere 1989)**

<table>
<thead>
<tr>
<th>RQD (%) Range</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;25</td>
<td>Very Poor geomechanical Quality</td>
</tr>
<tr>
<td>25-50</td>
<td>Poor Geomechanical Quality</td>
</tr>
<tr>
<td>50-75</td>
<td>Fair Geomechanical Quality</td>
</tr>
<tr>
<td>75-90</td>
<td>Good Geomechanical Quality</td>
</tr>
<tr>
<td>90-100</td>
<td>Excellent Geomechanical Quality</td>
</tr>
</tbody>
</table>

RQD has been an integral part of major rock mass classification schemes such as RMR. In RMR calculation, RQD rating can be up to 20. Since joint spacing and RQD are highly correlated (Hudson and Harrison 1997; Bieniawski 1979), degree of rock mass jointing can contribute up to 50% of the RMR through combination of RQD and joint spacing ratings. This indicates that RQD is an important constitutive parameter in the RMR classification system.

Unlike its simplicity to log and its wide applications, RQD has the following shortcomings:

- Dependency on measurement directions relative to the joint orientations (Milne et al. 1998; Brown 2003; Hack 2002);
- Insensitivity to changes of joint spacing of greater than 1 m or smaller than 0.1 m (equivalently fracture frequency of greater than 3 m⁻¹) (Milne et al. 1998; Brown 2003; Hack 2002);
- Sensitivity to core handling, drilling operator, and drilling equipment (single, double, triple-tube core barrels can be used) (Hack 2002; Brown 2003);
- Inability to sufficiently characterize rock mass behaviors as it does not take into account other important factors such as joints conditions, joints orientation, etc. (Bieniawski 1976);
- Subjectivity of the measurement, as different core logging operators frequently report different RQD values for a same core run (Read and Stacey 2009);
- Inability to measure the joints trace length (Grenon and Hadjigeorgiou 2003).
2.2.1.3 Joint spacing

Joint spacing, JS, is defined as the mean spacing between discontinuities that are intercepted with a traverse or a drillhole regardless of their orientation. JS can be calculated as the reciprocal of the fracture frequency, FF, which is defined as the number of natural discontinuities (includes all open fractures in a core except those of drilling- or handling-induced fractures) observed in a drillcore per meter (Priest 1993; Hudson and Harrison 1997). In drillcore logging, higher FF indicates a very broken rock mass. Table 2-5 shows the qualitative description of JS according to Bieniawski (1976).

<table>
<thead>
<tr>
<th>Joint Spacing (mm)</th>
<th>Fracture Frequency Range (m⁻¹)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;50</td>
<td>&gt;20</td>
<td>Very Close</td>
</tr>
<tr>
<td>50-300</td>
<td>3.33-20</td>
<td>Close</td>
</tr>
<tr>
<td>300-1000</td>
<td>1.00-3.33</td>
<td>Moderately Close</td>
</tr>
<tr>
<td>1000-3000</td>
<td>0.33-1.00</td>
<td>Wide</td>
</tr>
<tr>
<td>&gt;3000</td>
<td>&lt;0.33</td>
<td>Very Wide</td>
</tr>
</tbody>
</table>

2.2.1.4 Joint condition

Table 2-6 shows the procedure to rate the joint condition parameter, JCond, in RMR_{76}. Joint condition is logged for each individual joint set by quantifying the small-scale irregularities (characterized by roughness) on surfaces of joints, separation of joint walls, infilling nature, and joint wall rock hardness.

<table>
<thead>
<tr>
<th>Joint Condition Description</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very rough surfaces</td>
<td>25</td>
</tr>
<tr>
<td>Not continuous</td>
<td>20</td>
</tr>
<tr>
<td>No separation</td>
<td>12</td>
</tr>
<tr>
<td>Hard joint wall contact</td>
<td>6</td>
</tr>
<tr>
<td>Slickensided surfaces</td>
<td>0</td>
</tr>
<tr>
<td>OR Gouge &lt;5mm thick</td>
<td></td>
</tr>
<tr>
<td>OR Joints open&gt;5 mm</td>
<td></td>
</tr>
<tr>
<td>OR Continuous joints</td>
<td></td>
</tr>
</tbody>
</table>

A joint surface can be characterized in one of the five classes of very rough surface, rough surface, slightly rough surface, smooth surface, and slickensided surface according to irregularities present on the joint surface. Figures 2-8 and 2-9 show examples of the slickensided and rough joint surfaces, respectively. A joint set is classified on the basis of its separation into very tight (<0.1 mm), tight (0.1-1.0 mm), moderately open (1-5 mm), and open (>5 mm). For hardness of the joint wall rock, three categories are defined as: hard rock, medium hard rock, and soft rock.
2.2.1.5 Groundwater condition

Table 2-7 demonstrates the procedure to rate the groundwater condition parameter in RMR$_{76}$. Groundwater condition is rated from zero for the situations with unfavorable water condition to 10 for the case with completely dry rock mass. Two measures can be alternatively used for subjectively understand the rock mass groundwater condition: inflow rate per 10 m tunnel (or excavation) length and the general condition. Using either of these indicators, groundwater condition of a rock mass can be weighted in RMR$_{76}$ calculation (Bieniawski 1976). At one extreme, a completely dry rock mass is corresponding to the negligible inflow per 10 m of the excavation length. At another extreme condition, the general condition of groundwater is interpreted as sever water problem where the inflow rate per 10 m of the excavation length is about more than 125 L/min.

<table>
<thead>
<tr>
<th>Groundwater</th>
<th>Inflow per 10 m tunnel length</th>
<th>None</th>
<th>&lt; 25 L/min</th>
<th>25-125 L/min</th>
<th>&gt;125 L/min</th>
<th>General Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Completely Dry</td>
<td>Moist Only (interstitial water)</td>
<td>Water Under Moderate Pressure</td>
<td>Severe Water Problem</td>
<td></td>
</tr>
<tr>
<td>Rating</td>
<td>10</td>
<td>7</td>
<td>4</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.2.2 Rock tunnelling quality index (Q)

The Rock Tunneling Quality Index or Q was developed in 1974 by Barton et al. (1974). The index was developed after analysis of more than 200 Scandinavian tunnel project records driven in 13 igneous, 24 metamorphic, and nine sedimentary rock types. The purpose of Barton et al.
(1974) was to develop a general rock mass classification scheme for design of rock support requirements in underground tunnels.

The Q-system groups a rock mass into nine classes through characterizing a set of geomechanical properties. For the curse of generality, some parameters used in other classification systems such as the intact rock strength is not explicitly included in calculation of the Q index. The value of Q is calculated by combining the six parameters using the following equation (Barton et al. 1974):

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF}$$  \hspace{1cm} (2.6)

Where:

- **RQD**: total length of all rock pieces longer than 100 mm/total length of core run
- **J_n**: number of joint sets
- **J_r**: joint roughness number
- **J_a**: joint alteration number
- **J_w**: joint water reduction factor
- **SRF**: stress reduction factor

The Q index ranges from 0.001 for exceptionally poor rocks (squeezing-ground) to 1000 for exceptionally good rocks (sound unjointed rock masses). The first quotient $\frac{RQD}{J_n}$ represents a crude measure of block size of the rock mass (structure of the rock mass) with the two extreme values (100/0.5 and 10/20) differing by a factor of 400 (Barton et al. 1974). The second quotient $\frac{J_r}{J_a}$ represents the inter-block shear strength (roughness and frictional characteristics of the joint walls or filling materials). Values of $J_r$ and $J_a$ are respectively corresponding to the roughness and alteration of the weakest joint set or clay-filled discontinuities that are not favorably oriented for stability (Barton et al. 1974). This quotient is rated in favour of rough, unaltered joints in direct contact. The last quotient $\frac{J_w}{SRF}$ is an empirical measure of active stress in which $J_w$ is considered to account for negative effects (e.g. reduction of effective normal stress and softening/ouitwash of clay-filled joints) of water pressure on joint shear strength and consequently the stability of excavation (Barton et al. 1974). According to Barton et al. (1974),
SRF accounts for the loosening loads in clay-bearing rock masses and shear zones, rock stress in competent rocks, and swelling/squeezing loads in plastic incompetent rocks.

Unlike the Barton et al. (1974)’s intention to provide a comprehensive general rock mass classification scheme, Barton et al. (1974) does not suggest use of Q for the special cases such as large unstable wedges (in rock slopes and underground excavations) and presence of major clay-bearing weaknesses and fault zones.

In application of Q system developed by Barton et al. (1974), followings must be taken into account carefully (Palmstrom and Broch 2006; Suorineni 2014):

- Sufficient attention must be paid to all footnotes in the Q tables in order to make the best judgement about the applicability of Q in a project;
- Only the joint set with the least value of \( \frac{J_r}{J_a} \) should be used;
- Q will provide recommendations for only permanent supports;
- Since the database used for development of Q system is primarily for civil tunnels where tunnels are aligned in the best orientation for stability, the joint orientation relative to tunnel axis may not be a significant factor. This is an issue in underground mining where, for example, the drift alignments are not necessarily favorably aligned for stability;
- The support recommendations in the Q system are for the good blasting or excavation practice. Adjustments are necessary for projects with better or poorer drilling/blasting practice;
- The joint set number \( J_n \) cannot clearly define the degree of fracturing in a rock mass as a closely fractured rock mass may have only one joint set while a low fractured rock mass may have many dispersed joint sets;
- One implicit issue in Q is the definition of the block size term \( \frac{RQD}{J_n} \) which requires a rock mass to have at least three continuous joint sets;
- The fraction \( \frac{RQD}{J_n} \) cannot properly represent the block size due to the limitations of RQD and joint set number mentioned in above (e.g. RQD is 1-D, biased regarding the orientation of core log, joint set number does not clearly show the degree of fracturing);
- Q is appropriate for application in the hard jointed rock masses where the mode of failure is structural or brittle.
• The ratio $\frac{I_c}{I_a}$ is useful to characterize the wall contacts and friction as they are critical in the stability of tunnels. One issue of this ratio is that measurement of the wall contact after shearing 0.1 m is difficult in practice;

• Use of Q to evaluate the rock support requirements needs enough care and cautions since the estimations of support by this system is based on average and sometimes inaccurate measurements and interpretations;

• The Q system lacks following parameters: 1) joint orientation that may not be favorable in some excavations 2) joint persistence that highly impacts on the rock mass quality 3) joint aperture that highly effective on the water inflow to the excavation 4) rock strength.

2.2.3 Geological strength index, GSI

In 1995, Hoek et al. (1995) introduced a rock mass classification system, Geological Strength Index or GSI, to handle shortcomings of RMR to reasonably estimate the constant parameters of poorer rocks with RMR less than 25 in using the Hoek-Brown failure criterion and in order to incorporate geological observations into the Hoek-Brown failure criterion to improve the practicality of this criterion (Marinos et al. 2005). GSI is a rock mass classification system that characterizes a rock mass using rock’s geological information and the visual impression of the rock mass’s structure (Marinos et al. 2005). The name ‘geological strength index’ is used to stress the importance of fundamental geological observations about the blockiness of the rock mass and the condition of the joint surfaces to the classification system (Read and Stacey 2009). In GSI, a rock mass is categorized into a class ranging from extremely poor rock mass (GSI≈10) to intact or massive rock (GSI≈100).

GSI has been modified and extended over the time to cover more types of rock masses and geologies. For example, Figure 2-10 shows the GSI chart developed by Marinos and Hoek (2000) in which the laminated/sheared class has been added along with the GSI contours. Since 2000, GSI has also been purposefully adjusted for special cases such as heterogeneous rock masses (Marinos and Hoek 2001).

Recently, Hoek et al. (2013) have quantified the GSI chart despite the original purpose of the GSI development which was to characterize the rock mass only through the direct visual observation of the rock in the field. They justified their quantification approach as GSI has been
being increasingly applied by people with lack of geological understandings (Hoek et al. 2013). Hoek et al. (2013) propose a linear combination of two parameters: joint condition ‘JCond89’ (Bieniawski 1989) and RQD. They used joint condition to account for the joint surface condition. Besides, RQD is considered in the quantification as it could give an interpretation of blockiness.

Some of the recommendations, pros and cons for GSI application presented by Marinos et al. (2005) are as follows:

- GSI is not recommended to be quantified especially in dealing with tectonically disturbed rock masses;
- GSI should not generally be applied to highly anisotropic rock masses in which there is a dominant structural orientation;
- GSI should not be applied to hard rock masses of an excavated face having a few discontinuities spaced as size of the tunnel or slope;
- GSI in the deep hard rock (depth greater than 1000 m) is not recommended since in such conditions the rock mass behave like an intact rock (GSI≈100). The exception is where the tectonic disturbance is important and firmly continues with depth in which GSI could be applied cautiously;
- The discontinuities with filling materials are normally recommended to be categorized into the poor or very poor joint condition in the GSI charts;
- To account for the water conditions of the discontinuities in the GSI, the Fair-Very Poor classes of discontinuities surface conditions in the GSI chart might be needed to shift right;
- A range of values should be considered for the GSI of a rock mass;
2.3 Modelling Spatial Variability and Heterogeneity of Rock Mass Geomechanical Properties

The goal of modeling the spatial variability of geomechanical properties is to identify the heterogeneity of these properties and consequently better characterize the in-situ rock mass conditions for further analysis and designs. Therefore, this section starts with a brief elaboration
of different kinds of rock geomechanical heterogeneity. This will be followed by three approaches that are frequently used for the spatial modeling of the rock properties, specifically the geomechanical properties of interest in this project (RMR and its constituents). These approaches include conventional modeling, random field modeling, and geostatistical modeling.

### 2.3.1 Rock heterogeneity

Rocks and generally geomaterials are well known to be heterogeneous and discontinuous at multiple scales (Khajeh-Mahabadi 2012). Rock heterogeneity can be perceived in different forms *material, structural and geometrical heterogeneity* and at different scales *micro scale, meso* and *macro scale* (Khajeh-Mahabadi 2012; Hamdi 2015). Hamdi (2015) describes the rock heterogeneity in different scales as follow:

- The micro scale heterogeneity (<1 mm, scale of micro-cracks or grain), which primarily influences the mechanical properties of intact rock samples in laboratory scale, comes from the following sources:
  - Material heterogeneity that refers to the presence of various mineral grains each having different properties cemented together with varying contact stiffness;
  - Structural heterogeneity that is due to existence of micro-cracks, pores, and cleavage planes within the intact rock;
  - The geometrical heterogeneity that refers to the heterogeneity in the grain size and shapes.

- The meso scale (few meters, e.g. scale of an underground opening or a bench in an open pit mine) structural heterogeneity of rocks comes primarily from the presence of joints and fractures in the rock matrix. In this scale the material heterogeneity of rocks can be attributed to the heterogeneity of rock types.

- The macro scale (~km, e.g. engineering scale) or regional scale heterogeneity of rocks is usually due to the presence of faults, bedding planes, schistosity, foliation, and large persistent joints. Like meso scale, the material heterogeneity of rocks in this scale can be attributed to the heterogeneity of rock types.

In each scale, the rock behavior is governed primarily by heterogeneities of the corresponding scale (Hamdi 2015). For example, in the laboratory scale the behavior of intact rock is primarily
a function of the micro scale heterogeneity (presence of micro-cracks). In other words, influence of the micro scale rock heterogeneity on its behavior becomes less important in the meso scale.

2.3.2 Conventional modelling of geomechanical heterogeneity

In the conventional method or boundary modeling (Hack et al. 2006), the geomechanical data collected from a mine site are used to deterministically subdivide rock masses encountered in a project area into a number of geomechanical (or geotechnical) domains. In this approach, it is assumed that each domain has a distinct boundary and a specific and recognizable structural fabric and material properties. Read and Stacey (2009) suggest the following features to define each geomechanical domain:

- Mine-scale contacts marking changes in lithology, changes in weathering profiles, and changes in alteration forms;
- Mine-scale faults;
- Mine-scale folded structures and metamorphic structures with particular emphasis on changes in the orientation of the folds and structures;
- Bench and inter-ramp scale faults, folds and metamorphic structures;
- Bench-scale joints, cleavage and micro-structures.

Figure 2-11 illustrates a 2D cross section of a geomechanical model built using the conventional method. This geomechanical model is a superposition of four fundamental models of geology, hydrogeology, structure, and rock materials. Boundaries of geomechanical domains have been defined according to the above-mentioned features. In this figure, the geomechanical domains are marked by numbers. It is generally assumed that the structural, hydrogeological, geomechanical, and rock material properties of one domain are significantly different from other domains.
In the conventional modeling approach, it is assumed that the geomechanical properties are distributed uniformly within each geomechanical domain. The average parameter values of mechanical properties obtained from rock sampling for each domain is considered to be characteristics of rock mass properties in that domain. Thus, the conventional modeling ignores the local heterogeneity of geomechanical properties.

The validity of the assumption that a geomechanical domain is homogeneous depends on the following criteria (Hack et al. 2006):

- The relative variation of the geomechanical property within each geomechanical domain; if variation of geomechanical properties within a geomechanical domain is high relative to the variations in the whole rock mass, the homogeneity of the geomechanical domain may not be valid;
- The type of the engineering project: the more sensitive the project is, the less locally variable the geomechanical properties within a geomechanical domain should be. The sensitivity of the engineering project depends on the type and the regulations. A nuclear power station is an example of a highly sensitive type of the engineering project.
Use of the conventional spatial modeling may have severe negative impacts on the economy of mining and civil engineering projects. At one hand, pit designs based on such models are usually either too conservative or too risky that would endanger the operations’ sustainability. Slope failures due to risky designs (resulting from over-estimation of rock mass properties) or flatter slopes resulting from the conservative designs (due to under-estimation of rock mass properties) would have large negative consequences for the economy of a mining/civil project. At the other hand, applying of the conventional spatial modeling might harm the value chain of mining and civil projects since this approach may incur extra operational costs e.g. drilling and blasting expenses. Ignorance of local variability of geomechanical properties might lead to poor understanding of the rock quality and consequently non-optimal drill and blasting designs. Hence, implementing the conventional heterogeneity modeling in large open pit operations may only be justified in earlier stages of the mine life.

### 2.3.3 Application of random field for modelling geomechanical heterogeneity

In the random field approach, the heterogeneity of in-situ rock materials and/or structures are stochastically simulated using the Monte-Carlo simulation approach. The fundamental assumption is that a rock material or structure property follows a well-known priori probability distribution function. Then, a rock mass is divided into a number of grids and the rock material property is randomly assigned to each grid cell using the Monte-Carlo simulation technique.

The effect of material heterogeneity in soil and rock slope stability has been investigated using the random field approach (Allan et al. 2012; Baker et al. 2011; Griffiths et al. 2009; Griffiths and Fenton 2004). Griffiths and Fenton (2004) used a 2D random field-finite element method to analyze the slope stability. In their heterogeneity modeling, the cohesion parameter is assumed to follow a lognormal distribution and was spatially modeled using the conditional random field approach. The values of cohesion in different grid cells were assigned randomly while an auto-correlation function (based on inter-grid distance) conditioned the cohesion values of every two grid cells. Allan et al. (2012) modeled the heterogeneity of rock mass geomechanical properties in a 2D finite element code (Phase², Rocscience), using a combination of random field and geostatistical estimation method, kriging. In their method, two general scenarios for heterogeneity modeling were considered. The first scenario involved the pure randomly simulation of rock cohesion by assuming that it follows a Gaussian distribution. In the second
scenario, random samples were taken from the same Gaussian distribution and then kriging was employed to characterize the heterogeneity of the cohesion.

Although interesting but the random field method has the following shortcomings:

- It assumes that the probability distribution function of a rock property is known in advance. This assumption requires to collect a sufficiently large data of the geotechnical property which may not be possible in most mine design projects due to high expenses and difficulties in data collection;
- In random field, the structural pattern (anisotropy) of the geomechanical properties is not taken into consideration. Therefore, the random field method ignores the spatial pattern that the geomechanical properties may have. This ignorance could provide simulations that are not realistic.

2.3.4 Applications of geostatistical methods for modelling geomechanical heterogeneity

Geostatistics or theory of regionalized variables, initially developed by Matheron (1963), is a branch of applied statistics that models a natural phenomenon through quantifying the variability of one or more so called regionalized variables (Journel and Huijbregts 1978). Generally, a regionalized variable is a spatial/temporal variable such as ore grade, underground water level, or a geomechanical property that can be used to characterize a natural phenomenon. For example, ore grade as a regionalized variable has been traditionally modeled using the geostatistical applications in order to model the mineralization (Journel and Huijbregts 1978).

The problem of interest in Geostatistics is characterization of the variability of a regionalized variable and consequently estimate the value of the regionalized variable in un-sampled locations using the known values of the sampled locations (Deutsch 2002). The fundamental logic in applications of Geostatistics is the hypothesis that the measured values of geological and geomechanical properties in a project site are correlated to the spatial locations that such measurements are made (Hammah and Curran 2006). Unlike the completely stochastic techniques such as random field simulation that generate only pure random values for the local variation of regionalized variables, geostatistical approaches are able to model both the randomness and spatial structures of a regionalized variable.
There are two types of geostatistical techniques for spatial modeling of regionalized variables: estimation and simulation. The geostatistical estimation methods such as kriging apply a linear weighted estimators to provide the best estimation of an un-sampled location in terms of mean and variance (Chiles and Delfiner 1999). In this linear weighted estimator, weights are assigned to sampled data according to the spatial correlations/similarity, extracted from the semivariograms models' components including: sill, range, and nugget effect. The stronger spatial correlation enables the kriging methods to offer better local estimates. On the other hand, a lower spatial correlation or higher nugget effect will normally lead to the unreliable local estimates. One advantage of the kriging method is that it produces the optimal local estimates (with the least squared errors) and offer the minimized variance for the local estimate (kriging variance). Therefore, depending on the kriging variance, the reliability of the estimate can be assessed. The higher kriging variance indicates the presence of high nugget effect and/or high spatial variability at the scale of distance to the un-sampled location or lack of point data around the un-sampled location (Srivastava 2013). On the other hand, the lower kriging variance is the sign of presence of well correlated sampled data close to the un-sampled location (Deutsch 2002; Srivastava 2013). One major shortcoming of kriging methods is their smoothing effect (Chiles and Delfiner 1999). Estimation techniques such as kriging methods are preferable where the key goal is to have an accurate estimation of a regionalized variable although the produced estimate maps by these methods is smoother than the reality.

The idea of geostatistical simulation is to offer alternate realizations of the random function where the actual values of the regionalized variable are interpreted as one possible realization of the random function (Egaña and Ortiz 2013). Therefore, each realization generated using the geostatistical simulation can be considered as an equi-probable image of the deposit and then can be used for risk assessment and uncertainty quantification (Deutsch 2002). The geostatistical conditional simulation (e.g. sequential Gaussian simulation) provides a more realistic map of the spatial variation of a regionalized variable. The geostatistical conditional simulation approach does not provide a smoother than the reality estimates. The simulation technique is concerned with reproducing the spatial characteristics that relate multiple locations (Egaña and Ortiz 2013). The geostatistical conditional simulation gives a realistic realization of the true in-situ variability in which the sharp changes in the short scales that happen in the reality could be detected and shown (Srivastava 2013). Despite better representing the true spatial variability by geostatistical
conditional simulation method, it is not as accurate as the estimation techniques. Unlike estimation methods, conditional simulation is a preferable method where risk analysis and later decision making and heterogeneity modeling are the primary goals (Goovaerts 1997; Deutsch 2002) Geostatistical conditional simulation provides a set of realizations of the reality with the following characteristics (Dowd 1993; Deutsch 2002):

1. Each realization honors the true sampled data at their locations;
2. Each realization honors the spatial distribution and variability of the sampled data (same semivariograms). In other words, each realization reproduces the same spatial variability as the sample data;
3. Each realization honors the distribution of the sample data in terms of the histogram. In other words, each realization regenerates the same histogram as of the sample data;
4. Each realization honors the co-regionalization in the same way of the sample data. In other words, the relationship between different regionalized variables are honored in each realization.

2.3.4.1 Theory of regionalized variables

Geostatistics has been developed based on the fundamental practical observation which indicate that in almost all mineral deposits, regionalized variables such as ore grade demonstrate two contradictory characteristics of randomness and structural (Journel and Huijbregts 1978). Figure 2-12 shows the random and structured aspects of a regionalized variable such as a metal grade in an arbitrary direction \( x \). As seen, the value of metal grade fluctuates locally, simultaneously showing a structured increase from the poor zone to the rich zone. At one hand, the values of the metal grade in shorter scales shows a randomness. Therefore, the value of this regionalized variable can be perceived as a locally random variable \( Z(x_i) \). On the other hand, such random variable shows a structured pattern in a regional scale. These two aspects of a regionalized variable such as the metal grade shown in this figure led Matheron and other practitioners of Geostatistics to interpret a regionalized variable as a random function.
Presence of the spatial structure implies that the two random variables corresponding to two locations in the deposit cannot be independent. This means that there is a covariance structure between each pair of variables in two locations. Assuming the regionalized variable in the deposit as a random function, the spatial law of the random function $Z(\mathbf{x})$ ($\mathbf{x} \in \text{Domain}$) for all positive integers $n$ and for all possible combinations of points in the $n$-dimensional space $\mathbb{R}^n$ is defined as follows (Journel and Huijbregts 1978):

$$F_{x_1,x_2,..,x_n}(z_1,z_2,..,z_n) = P(Z(x_1) < z_1, Z(x_2) < z_2, ..., Z(x_n) < z_n)$$

(2.7)

In mining applications, the spatial law of the random function is not required to be known since the first two moments of the spatial law can provide sufficient information about the spatial law and also limited number of sample data does not let to infer the spatial law (Journel and Huijbregts 1978). Therefore, geostatistical tools primarily account on the first two moments of the spatial law along with stationarity assumptions for statistical inference (Journel and Huijbregts 1978).

The first-order moment for a random variable $Z(\mathbf{x})$ at point $\mathbf{x}$ is defined as $E\{Z(\mathbf{x})\} = m(\mathbf{x})$ that indicates the mean of a random variable depends on its location. Three second-order moments in the geostatistics are as follows (Journel and Huijbregts 1978):

1. Variance: The variance of a random variable $Z(\mathbf{x})$ is defined as: $Var\{Z(\mathbf{x})\} = E\{[Z(\mathbf{x}) - m(\mathbf{x})]^2\}$. The variance depends on the location;

2. Covariance: the covariance between two random variables $Z(x_1)$ and $Z(x_2)$ at two different locations of $x_1$ and $x_2$ is defined as $C(x_1,x_2) = E\{[Z(x_1) - m(x_1)][Z(x_2) - m(x_2)]\}$.
The covariance is a measure of spatial correlation/dependence between two random variables at two locations;

3. Variogram: the variogram function is defined as the variance of the increment $[Z(x_1) - Z(x_2)]$: $2\gamma(x_1, x_2) = \text{Var}(Z(x_1) - Z(x_2))$. Variogram is a measure of spatial variability between two locations. $\gamma(x_1, x_2)$ is called semi-variogram.

The first two second-order moments of a regionalized variable depend on the spatial positions. This requires that many realizations for values of $Z(x_1)$ and $Z(x_2)$ be available to be able to infer these moments (Journel and Huijbregts 1978). Since such realizations are never available, further simplifications, stationarity assumptions, are necessary as follows:

1. The strict stationarity (Journel and Huijbregts 1978) indicates that the spatial law of the random function is invariant under the translations. Mathematically this can be expressed as follows: $F_{x_1,x_2,...,x_n}(z_1,z_2,...,z_n) = F_{x_1+\vec{h},x_2+\vec{h},...,x_n+\vec{h}}(z_1,z_2,...,z_n)$ for all translation vectors $\vec{h}$. This assumption, does not provide significant information to be used for the statistical inference about the random function (regionalized variable) as the spatial law of the random function is unknown and the first two moments are enough (Journel and Huijbregts 1978).

2. The second-order stationarity or stationarity of the covariance requires that the random function has a global mean and covariance independent of the location. It is mathematically expressed as follows (Journel and Huijbregts 1978):

- $E[Z(x)] = m, \forall x \in \text{Domain}$. The expectation exists and does not depend on the location $x$;
- $C(x_1, x_2) = E[\{Z(x_1) - Z(x_2)\}] - m^2 = C(\vec{h}), \forall x_1, x_2 \in \text{Domain}$ where $\vec{h} = x_2 - x_1$ is the vector between $x_2$ and $x_1$. The covariance between any two random variables at two locations exists and depends only on the separation vector between the points. This implies that the covariance is independent of the location of $x_2$ and $x_1$.

If the second-order stationarity exists,

$$\text{Var}[Z(x)] = E[\{Z(x) - m(x)\}^2] = C(0) \quad \forall x \in \text{Domain} \quad (2.8)$$
\[
\gamma(\vec{h}) = \frac{1}{2} E \left\{ \left( Z(x + \vec{h}) - Z(x) \right)^2 \right\} = C(0) - C(\vec{h}) \quad \forall x \in \text{Domain} \quad (2.9)
\]

Therefore, stationarity of covariance results in existence of stationarity of variance and variogram (Journel and Huijbregts 1978). In this case, variogram and covariance are two equivalent functions for quantification of the auto-correlation between two variables \(Z(x + \vec{h})\) and \(Z(x)\). Then, the correlogram is defined as

\[
\gamma(\vec{h}) = \frac{c(\vec{h})}{c(0)} = 1 - \frac{\gamma(\vec{h})}{c(0)}.
\]

In case the second stationarity exists, the covariance function \(C(\vec{h})\) cannot take any arbitrary form. The covariance \(C(x_i, x_j), \forall i, j\) must be positive definite as: suppose \(Y = \sum_{i=1}^{n} \alpha_i Z(x_i)\) a linear combination of \(n\) regionalized variables \(Z(x_i), \ i = 1 \ldots n\) where \(\alpha_i\) can take any value. In this case, \(\text{var}\{Y\} = \sum_i \sum_j \alpha_i \cdot \alpha_j \cdot C(x_i - x_j) \geq 0\) (Journel and Huijbregts 1978).

3. The third stationarity assumption, intrinsic stationarity or stationarity of variogram, is an assumption weaker than the second-order stationarity (because it may not necessarily lead to stationarity of covariance). This assumption expresses that the random function has a global mean and location-independent variogram (Journel and Huijbregts 1978). The mathematical formulation of this stationarity is as follows:

- \(E\{Z(x)\} = m, \ \forall x \in \text{Domain}\). The expectation exists and does not depend on the location \(x\).
- \(2\gamma(x_1, x_2) = \text{Var}\{Z(x_1) - Z(x_2)\} = E\{[Z(x_1) - Z(x_2)]^2\} = 2\gamma(h)\) is finite, \(\forall x_1, x_2 \in \text{Domain}\) where \(\vec{h} = x_2 - x_1\) is the vector between \(x_2\) and \(x_1\). The variance of increment exists and does not depend on the location of \(x_1\) and \(x_2\). It depends only on the separation vector between the points.

In case the second stationarity exists, the covariance function \(\gamma(\vec{h})\) cannot take any arbitrary form. The variogram function must be such that \(-\gamma(x_i, x_j), \forall i, j\) be conditionally positive definite as: suppose \(Y = \sum_{i=1}^{n} \alpha_i Z(x_i)\) a linear combination of \(n\) regionalized variables \((x_i), \ i = 1 \ldots n\). In this case, the condition \(\text{var}\{Y\} = C(0) \sum_i \alpha_i \sum_j \alpha_j - \sum_i \sum_j \alpha_i \cdot \alpha_j \cdot \gamma(x_i - x_j) \geq 0\) is only satisfied when \(\sum_i \alpha_i = 0\) which leads to having \(-\sum_i \sum_j \alpha_i \cdot \alpha_j \cdot \gamma(x_i - x_j) \geq 0\) (Journel and Huijbregts 1978).
2.3.4.1.1 Experimental variogram

To quantify the spatial variability (equivalently autocorrelation) of a regionalized variable, it is necessary to calculate variograms (or semi-variograms) in different spatial directions. The natural phenomena generally show anisotropy (direction-dependent continuity) that can be identified by calculation of variograms in different directions in space (Journel and Huijbregts 1978). Since variogram is a population statistics, it should be estimated using the sample data. To estimate the variogram in a direction, experimental variogram is calculated using Equation (2.10):

\[
2\gamma(h) = \frac{1}{N(h)} \sum_{N(h)} [Z(x) - Z(x + h)]^2
\]  

(2.10)

Where \(N(h)\) is the number of data pairs separated with approximate separation lag of \(h\). The experimental variogram at a separation distance \(h\) is the average squared difference of the values approximately separated by \(h\) (Deutsch 2002). For each direction, a number of variogram values are calculated for different lag distances. Since the sampling data may not be perfectly aligned on a direction, tolerances for the direction and separation are usually defined. Figure 2-13 shows how an experimental variogram is calculated in the horizontal plane for the east direction. Tolerances for the azimuth, dip, and the separation distance must be well defined before calculation of the experimental variogram, depending on the data spacing. In addition, bandwidth is generally defined and applied in order to prevent the sampling data of far from the direction (here east direction) to be counted in calculation of the experimental variogram.

![Figure 2-13: Parameters for experimental variogram calculation (Bohling 2005)](image)
2.3.4.1.2 Variogram modelling

To capture the anisotropy, the experimental variograms are calculated and sketched for numerous directions. Figure 2-14 depicts the sill, range, and nugget in a typical variogram so called transition variogram. Three characteristics are identified for a variogram as follows (Deutsch, 2002):

- **Nugget effect** is a measure of short scale variability and can indicate the inherent variability in two adjacent samples. The nugget effect is observed in the variogram graph as a discontinuity at the origin. The sources of nugget effect can be attributed to the sampling errors and microscopic heterogeneity in the scale smaller than the measurement support (e.g. micro scale, mineral scale). The nugget effect is estimated using the downhole variogram;

- **Sill** which is the plateau at which the variogram reaches. Theoretically, the value of sill equals to the variance of data;

- **Range** is defined as the distance at which the variogram reaches the sill. The range of variogram is a measure of autocorrelation of the regionalized variable in a spatial direction. The regionalized variables within the range are considered to be auto-correlated while beyond the range they are generally perceived of no correlation.

The experimental variograms can only give information on the structure of variability in a few directions and distances. In order to apply geostatistical techniques, variograms should be known in all directions and distances (Journel and Huijbregts 1978). Therefore, the variogram must be modeled in a way that the variability structures in all directions are identified. The variogram models are used in the estimation and simulation of a number of realizations for a regionalized variable. The variogram models must be such that the property *conditional positive definiteness* be satisfied (Deutsch 2002). In Geostatistics, a set of variogram models so called licit variogram models, that satisfy this property, are commonly used. Equations (2.11) to (2.14) and Figure 2-15 express a set of commonly used licit variogram models (Deutsch 2002; Journel and Huijbregts 1978; Bohling 2005):
Figure 2-14: Sill, range, and nugget effect in a transition variogram (Bohling, 2005)

Nugget: $\gamma(h) = \begin{cases} 0 & \text{if } h = 0 \\ c & \text{if } h > 0 \end{cases}$ (2.11)

Spherical: $\gamma(h) = \begin{cases} c(1.5\left(\frac{h}{a}\right) - 0.5\left(\frac{h}{a}\right)^3) & \text{if } 0 \leq h \leq a \\ c & \text{if } h > a \end{cases}$ (2.12)

Exponential: $\gamma(h) = c(1 - \exp\left(-\frac{3h}{a}\right))$ (2.13)

Gaussian: $\gamma(h) = c(1 - \exp\left(-\frac{3h^2}{a^2}\right))$ (2.14)

Figure 2-15: Spherical, exponential, and Gaussian variogram models (Bohling 2005)
2.3.4.1.3 Simple kriging

Kriging techniques are collections of linear regression techniques used for estimation of a regionalized variable at an un-sampled location using the information of the variable values at sampled locations. In this section, the mathematical formulation of simple kriging is presented. Simple kriging is the simplest method of the kriging family. All other kriging methods such as ordinary kriging are some variations from simple kriging.

Suppose we want to estimate the value of a regionalized variable at an un-sampled location $z_0$ using the known values of the regionalized variable at $n$ locations $z_1, z_2, \ldots, z_n$, Figure 2-16 (Journel 1989).

![Figure 2-16: Estimating Un-sampled location using values of n sampled locations](image)

Krig proposed following linear estimator for the variable $Z_0$ (Journel 1989):

$$Z_0^* = \lambda_0 + \sum_{\alpha=1}^{n} \lambda_{\alpha} \cdot Z_{\alpha} \quad (2.15)$$

The unbiasedness condition for this estimator requires:

$$E\{Z_0 - Z_0^*\} = E\{Z_0 - \lambda_0 - \sum_{\alpha=1}^{n} \lambda_{\alpha} \cdot Z_{\alpha}\} = E\{Z_0\} - \lambda_0 - \sum_{\alpha=1}^{n} \lambda_{\alpha} \cdot E\{Z_{\alpha}\} = 0 \quad (2.16)$$
\[ E[Z_0] = m_0, \quad \text{and} \quad E[Z_\alpha] = m_\alpha \quad (2.17) \]

Thus,
\[ Z_0^* = m_0 + \sum_{\alpha=1}^{n} \lambda_\alpha \cdot (Z_\alpha - m_\alpha) \quad (2.18) \]

Therefore,
\[ Z_0^* - m_0 = \sum_{\alpha=1}^{n} \lambda_\alpha \cdot (Z_\alpha - m_\alpha) \quad \text{or} \quad (Z_0 - m_0)^* = \sum_{\alpha=0}^{n} a_\alpha \cdot (Z_\alpha - m_\alpha) \quad (2.19) \]

Let's define,
\[ Y = Z_0 - Z_0^* = (Z_0 - m_0) - (Z_0 - m_0)^* = \sum_{\alpha=0}^{n} a_\alpha \cdot (Z_\alpha - m_\alpha) \quad (2.20) \]

Where \( a_0 = 1 \) and \( a_\alpha = -\lambda_\alpha \).

Now we want to minimize \( \text{Var}\{Y\} \) as the estimation error variance.
\[ \text{Var}\{Y\} = \sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} a_\alpha a_\beta C_{\alpha \beta} \quad (2.21) \]

\( C_{\alpha \beta} \) is the covariance between two variables at locations \( \alpha \) and \( \beta \).

Derivating \( \text{Var}\{Y\} = \sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} a_\alpha a_\beta C_{\alpha \beta} \) with regard to each coefficient \( a_\alpha \), we will reach a system of linear equations called the simple kriging system (Equation (2.22)) as follows:
\[ \sum_{\beta=1}^{\beta=0} \lambda_\beta C_{\alpha \beta} = C_{\alpha 0} \forall \alpha = 1 \ldots n, \quad C_{\beta \beta} = C(0) \quad (2.22) \]

Also, the minimized error variance is \( \sigma_{SK}^2 = C_{00} - \sum_{\alpha=1}^{n} \lambda_\alpha C_{\alpha 0} \geq 0 \) which is the estimation error variance for un-sampled location 0.

2.3.4.1.4 Sequential Gaussian simulation, SGS

The most commonly used geostatistical conditional simulation method, SGS, produces multiple equi-probable realizations of a regionalized variable that honor the sample data at their locations, and the variogram/covariance (Deutsch 2002). In this method, the smoothing effect present in the kriging results is removed because of adding some noises to the kriging estimates (through random residual value) and using a random path for simulation (Deutsch 2002). The random path
is used in SGS to prevent the artificial noises/errors that can arise from a preferential predetermined path (Deutsch 2002).

The steps of the SGS algorithm are as follows (Deutsch 2002):

1. Transform the data into the standard normal distribution. This is generally done by mapping the quantiles of experimental cumulative distribution function of the sample data to the cumulative distribution function of a standard Gaussian distribution. Figure 2-17 shows the quantile-to-quantile normal score transformation for sample data of porosity;

2. Apply kriging to find the kriged estimate at location $u$ and its kriging variance $\sigma_k^2$;

3. Draw a random residual $r(u)$ that follows a standard normal distribution with mean zero and variance $\sigma_k^2$;

4. Add the residual to the kriged estimate to have the simulated value for location $u$;

5. Add the simulated value of location $u$ to the pool of data. This would keep a proper covariance structure between the simulated values;

6. Visit all locations in a random path and apply the steps 2 to 5 until all locations are visited and simulated;

7. Back-transform all simulated data values to the original scale.
2.4  3D Block Modelling of Geomechanical Properties Using Geostatistical Methods

A limited number of studies have been done to model the heterogeneity of geomechanical properties particularly using the geostatistical approaches. The early efforts of using geostatistical methods in Geomechanics were done by La Pointe (1980) who applied geostatistics to investigate the spatial distribution of fracture properties to improve the design of rock structures. Since then many researchers have applied the geostatistical interpolation and simulation methods for analysis of spatial distribution of geomechanical properties. These studies can be categorized into two general groups based on the type of geostatistical technique. Majority of the available research studies have applied only geostatistical estimation methods for modeling the geomechanical properties (Vatcher et al. 2016; Abdideh et al. 2014; Ozturk and Simdi 2014; Coli et al. 2012; Marache et al. 2009; Stavropoulou et al. 2007; Oh et al. 2004; Marinoni 2003; Syrjänen and Lovén 2003; Ayalew et al. 2002; Atilla Öztürk and Nasuf 2002;
Other studies have focused on either application of the geostatistical simulation (Egaña and Ortiz 2013; Madani-Efahani and Asghari 2011; Ellefmo and Eidsvik 2009) or applications of both geostatistical estimation and simulation methods (Doostmohammadi et al. 2015; Ferrari et al. 2014; Yu 2010; Marchesi et al. 2009).

Ferrari et al. (2014) used a geostatistical methodology to estimate and simulate the spatial distribution of rock mass rating (RMR) across the 200 km² shallow rock mass of the Italian Central Alps along the San Giacomo Valley, province of Sondrio. They first characterized the geomechanical attributes (RQD, UCS, fracture frequency, and joint condition) using the data of 97 surface mapping. Then, each property was rated to calculate the RMR values of these 97 locations of the Giacomo Valley. Finally, they applied ordinary kriging and SGS to build spatial distribution maps of RMR in the domain of study. Their results show that the RMR map produced using the ordinary kriging method had more continuity and smoother representation while the 100 RMR simulation maps generated using the SGS showed better representations of the local extreme values. Besides, they concluded that the SGS performed better than the ordinary kriging to estimate the RMR values of the newly sampled locations. Despite their interesting geostatistical approach for RMR block modeling, followings can be taken into account as the shortcomings of Ferrari et al. (2014):

- One shortcoming of their method is the lack of geomechanical database. In their approach, only 97 sample data were used for the geostatistical application. The geostatistical techniques require a sufficiently big amount of data in order to do statistical inference. All geostatistical operations such as Variography were implemented using only 97 data;
- In their method, only block models of RMR were produced. They did not generate block models for other geomechanical properties such as RQD and fracture frequency. Having block models for other geomechanical parameters could give more insight about the geomechanical behavior of rock mass.

Egaña and Ortiz (2013) proposed a geostatistical-based methodology to build the block model of the rock mass rating (RMR version 1989). They implemented the sequential Gaussian simulation method using the omni-directional variograms to characterize the spatial variation of
geomechanical properties (rock quality designation RQD, UCS, fracture frequency, joint condition) in a mature underground mine, Chuquicamata in Chile. Then, they combined the resulting realizations of each constituent of RMR using the appropriate ratings to create 3D block models of RMR. They showed the superiority of the proposed method by comparing the results with the deterministic conventional method in the geomechanical design. Two of the shortcomings of the methodology proposed in Egaña and Ortiz (2013) are as follows:

- the anisotropy of the geoemchanical properties have not been taken into consideration because an omni-directional variogram model used for characterization of these attributes’ spatial variability. The regionalized variables such as geomechanical properties (e.g. RQD and RMR) are direction dependent and they behave differently in varying directions. Thus, anisotropy of these variable must be well studied and modeled in the geostatistical applications;
- the validation of the approach taken in Egaña and Ortiz (2013) present poor results in terms of relative error (absolute error divided by mean of variable). For example, relative errors for UCS in their method ranges from 4.4 to 11.5. Besides, absolute errors for UCS block models show a range from 240 MPa to 4400 MPa.

In this thesis, geostatistical simulation-based approaches are presented and implemented for generation of 3D block models of RMR and its constitutive parameters (except groundwater condition) in an open pit mine. The proposed approaches characterize not only the heterogeneity but also anisotropy of these geomechanical parameters.

In chapter 2, the rock mass characterization/classification systems were briefly discussed along with the approaches used for heterogeneity modeling of rock mass geomechanical properties. A literature review of the heterogeneity modeling approaches was also presented with emphasis on the geostatistical techniques. In the next chapter, the iron ore open pit mine in which the proposed geostatistical methodologies are applied will be described. The general background information including the geological and geomechanical attributes and also the results of statistical analysis on the geomechanical data collected in the mine are elaborated in detail.
Chapter 3
Statistical Data Analysis – Mont-Wright Open Pit Mine Complex

This thesis investigated spatial distribution of rock mass geomechanical properties at the Mont Wright open pit mine site. This mine site was selected due to the large amount of geomechanical data collected in the mine since its exploration in 1940s. The current chapter reports on the geomechanical data collected at the Mont-Wright (MW) open pit mine. The mine’s geomechanical dataset was subjected to an exploratory analysis including the cleansing of the dataset collected in the mine and the statistical analysis results were presented and discussed as well.

3.1 Mont-Wright Open Pit Mine Complex

Mont-Wright is a major iron ore deposit that is owned and operated by ArcelorMittal Mines Canada (AMMC) Corporation. Mont-Wright mining complex is located in the Northern Quebec in Canada, 16 km west of the mining town Fermont and approximately 1000 km northeast of Montreal. Mont-Wright can be accessed from the closest city in the south, Baie Comeau, via Highway 389. The mine complex has been in operation since 1975 and totally 1,285 million tonnes of iron ore have been extracted until 2012 (Savard and Jean 2012). The iron ore concentrates are shipped to the AMMC’s pelletizing plant and shipping terminal located at Port Cartier on the north shore of the Gulf of St. Lawrence through a 416 km railway privately owned by AMMC. Mont-Wright’s geographical location is at Latitude of 52° 45’ 20’’ N and Longitude of 67° 17’ 35’’ W. Figure 3-1 demonstrates the Quebec regional location map and the location of Mont-Wright mining complex.

3.1.1 Climate

The climate of Mont-Wright mining complex is sub-Arctic with an average annual temperature of -3.6 ºC (Savard and Jean 2012). The winter is highly freezing with the normal temperature of -40 ºC while the temperature in the summer can be as high as 25 ºC. The average precipitation is 880 mm per year (Savard and Jean 2012).
3.1.2 Pits and baselines

There are several pits and mining faces in the Mont-Wright mining complex. Figure 3-2 shows the location of pits within the mining complex. These pits and mining faces include Hessé, South Hill, Mount Wright, Paul’s Peak, Pits E, B, A, C and C-Prime.

AMMC has partitioned the Mont-Wright formation into three distinct mining areas, Baselines A, B, and C (Savard and Jean 2012), Figure 3-2. Baseline A (BLA) located in the western part of the Mont-Wright and includes four pits as follows:

- Mount-Wright that was depleted in 1989 and has been back-filled with waste;
- South Hill that was depleted in 1999 and is inactive since then;
- North Limb that is presently exhausted and is being filled with the Paul’s Peak wastes;
- Paul’s Peak that is the largest and the only active pit in BLA. Paul’s Peak is planned to be connected with the South Hill in future.

Baseline B (BLB) located in the middle part of the Mont-Wright and comprises pits A, B, C and E. Baseline C (BLC) in the eastern part of the Mont-Wright includes only one pit, C-Prime. Figure 3-2 displays the location of the three baselines in the Mont-Wright in the universal coordinate system (UTM). For each Baseline, a corresponding local coordinate system has been defined and used as well as the UTM. The orientation of these local coordinate systems relative to the UTM is depicted in Figure 3-2. The local coordinates systems for BLA and BLB are respectively rotated horizontally 50° and 5° anti-clockwise relative to the UTM coordinate system. The local coordinate system for BLC is horizontally rotated 20° clockwise relative to the UTM system.
Figure 3-1: Quebec regional location map for MW (Savard and Jean 2012)

Figure 3-2: Pits, baselines and local coordinate systems (Savard and Jean 2012)
3.1.3 Discovery and exploration

Initially, the western region of the Mont-Wright was discovered in 1947 by Dominion Explorers (Savard and Jean 2012). The Mont-Wright deposits were initially claimed by U.S. Steel’s subsidiary Oliver Mining Division (OMD). The preliminary exploration performed by OMD in 1952. In 1950’s, Mont Wright Iron Mines Co. Ltd and United Dominion were also both involved on the eastern part of Mont-Wright performing mapping and small drilling campaign.

The drilling campaign in the Mont-Wright deposits has been primarily conducted using the diamond drilling with NQ cores sizes. The drilling program in MW can historically be separated into the following periods and purposes (Savard and Jean 2012):

- 1953-1966 → Purpose: Exploration
- 1970-1977 → Purpose: Development mainly on BLA
- 1977-1983 → Purpose: Production mainly on BLA
- 1984-1994 → Purpose: Pre-production and production on BLB
- 1994-2010 → Purpose: Production on BLC and Hessé
- 2010-Present → Purpose: Exploration in all sectors

3.1.4 Geology

The Mont-Wright iron ore deposit is a part of the highly folded and metamorphosed southwestern branch of the Labrador Trough where the most important lithology is the specular hematite iron formation containing recrystallized quartz and specular hematite with slight magnetite and iron silicates (Savard and Jean 2012). The complexity of structural geology of the Mont-Wright is primarily due to the presence of severe folding into a series of synclines and anticlines and existence of the secondary folding that have made wide zones of specular hematite, up to 300 m in width (Savard and Jean 2012). This complicated structural folds across east-west and north-south have created, duplicated and repeated iron ore lenses with thickness of more than 100 m in some areas (Savard and Jean 2012).

The iron ore in the Mont-Wright is a coarse specular hematite iron formation with average of 28-32% Fe. The magnetite content is normally less than 5% by weight and the amount of contaminants (e.g. TiO, Al₂O₃, Na₂O, K₂O) in the iron ore is generally low (Jean 1999).
Table 3-1 shows different rock types present and classified in the Mont-Wright deposits. Among these rocks, following three rock types have been identified as the major lithologies primarily forming the Mont-Wright deposits (Jean 1999): Amphibolite, Iron Formation, and Quartzite Mica Schist.

<table>
<thead>
<tr>
<th>Rock Code</th>
<th>Description (Rock Type)</th>
<th>Rock Code</th>
<th>Description (Rock Type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99 (Air)</td>
<td>Air</td>
<td>-99</td>
<td>Unknown</td>
</tr>
<tr>
<td>100 (REM)</td>
<td>Back fill</td>
<td>201</td>
<td>Unknown</td>
</tr>
<tr>
<td>101 (OB)</td>
<td>Over burden</td>
<td>202 (AMPIF)</td>
<td>Amphibolite in iron formation</td>
</tr>
<tr>
<td>102 (AMP)</td>
<td>Amphibolite</td>
<td>203 (WIF)</td>
<td>Uneconomic iron formation in IF</td>
</tr>
<tr>
<td>103 (IF)</td>
<td>Iron formation</td>
<td>204 (QRIF)</td>
<td>Quartzite rock in iron formation</td>
</tr>
<tr>
<td>104 (QR)</td>
<td>Quartzite rock (0 to 40% of mica)</td>
<td>205 (MSIF)</td>
<td>Mica schist in iron formation</td>
</tr>
<tr>
<td>105 (MS)</td>
<td>Mica schist (60 to 100% of mica)</td>
<td>206 (QRMSIF)</td>
<td>Quartz mica schist in iron formation</td>
</tr>
<tr>
<td>106 (QRMS)</td>
<td>Quartz mica schist (40 to 60% of micas)</td>
<td>114 (MAR)</td>
<td>Marble, Dolomite or Calcite</td>
</tr>
<tr>
<td>107 (GN)</td>
<td>Gneiss</td>
<td>208 (PEG)</td>
<td>Pegmatite</td>
</tr>
<tr>
<td>108 (GNF)</td>
<td>Felsic Gneiss</td>
<td>209</td>
<td>Unknown</td>
</tr>
<tr>
<td>109 (GNM)</td>
<td>Mafic Gneiss</td>
<td>302 (AMPIF)</td>
<td>Decontaminated by cut-off on Al&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;3&lt;/sub&gt; et TiO&lt;sub&gt;2&lt;/sub&gt; (Equivalent to 202 by interpretation)</td>
</tr>
<tr>
<td>110 (CNR)</td>
<td>Unrecovered core</td>
<td>303 (WIF)</td>
<td>Cut-off grade of less than 15% of iron for IF (Equivalent to 203 by interpretation)</td>
</tr>
<tr>
<td>113 (CONIF)</td>
<td>Contaminated iron formation (waste)</td>
<td>313 (CONIF)</td>
<td>Decontaminated by cut-off on MnO (Equivalent to 113 by interpretation)</td>
</tr>
</tbody>
</table>

3.2 Paul’s Peak Open Pit Mine

Paul’s Peak is the largest and currently is the only active pit in Baseline A at Mont-Wright. Paul’s Peak has been planned to be expanded into a vast pit where it will be connected to the South Hill. The final Paul’s Peak pit will be corresponding to year 2045 according to the AMMC strategic plans.
Figures 3-3 and 3-4 depict the current Paul’s Peak pit (topography of year 2013) and the final planned Paul’s Peak pit (pit topography in year 2045), respectively. The Paul’s Peak 2013 and final Paul’s Peak pits dimensions are demonstrated in Table 3-2. According to this table, the final planned pit limit for Paul’s Peak will be approximately two times the current Paul’s Peak pit in each of three dimensions. This means that the final pit limit will be roughly eight times bigger than the current pit. Comparing the Paul’s Peak 2013 and Paul’s Peak 2045 reveals that the expansion of the Paul’s Peak will be performed mostly through pushing back the south, north, and west walls. Figure 3-5 depicts a photo of Paul’s Peak 2013 taken on top of the east wall in fall 2013, when the author spent three months at the mine site for data collection.

Table 3-2: Approximate Dimensions of Current and Final Paul’s Peak

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>West-East Axis (BLA)</td>
<td>1800 m</td>
<td>3700 m</td>
</tr>
<tr>
<td>South-North Axis (BLA)</td>
<td>600 m</td>
<td>1500 m</td>
</tr>
<tr>
<td>Depth</td>
<td>320 m</td>
<td>600 m</td>
</tr>
</tbody>
</table>

Figure 3-6 indicates the relative orientation of coordinate systems UTM and BLA. The horizontal axes of the BLA system are rotated 50º anti-clockwise relative to the UTM system. In both systems, the elevation directions are the same. Table 3-3 gives the information on the approximate boundaries of the ultimate Paul’s Peak pit in year 2045 in the two coordinate systems UTM and BLA.

Table 3-3: Approximate Boundaries of Paul’s Peak in UTM and BLA Coordinate Systems

<table>
<thead>
<tr>
<th>Coordinate</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easting (m)-UTM</td>
<td>611,590</td>
<td>614,825</td>
</tr>
<tr>
<td>Northing (m)-UTM</td>
<td>5,843800</td>
<td>5,847,350</td>
</tr>
<tr>
<td>Easting (m)-BLA</td>
<td>490</td>
<td>4,250</td>
</tr>
<tr>
<td>Northing (m)-BLA</td>
<td>-1,080</td>
<td>580</td>
</tr>
</tbody>
</table>
Figure 3-3: Paul's Peak pit in 2013-view looking east (BLA)

Figure 3-4: Paul's Peak pit in 2045-view looking east (BLA)
Figure 3-5: Paul's Peak 2013-looking west (BLA)

Figure 3-6: The configuration of BLA coordinate system relative to UTM
3.3 Exploratory Data Analysis
This section presents the results of detailed statistical data analysis on the collected geological and geomechanical data in the Paul’s Peak open pit mine. Paul’s Peak has been selected for the case study of this thesis as this pit is the most important pit in the MW which is planned to operate for more than three decades until 2045. All the exploratory data for the Mont-Wright project were provided by the AMMC. In addition, the author spent about three months (in 2013) at the mine site to collect the geomechanical data pertinent for the project. The geological and geomechanical data are primarily in the forms of the borehole datasets, from 2203 drillholes, collected between 1949 and 2013 in MW through different stages of drilling campaigns. The first part of this section is focused on the cleaning the geomechanical borehole dataset of the entire Mont-Wright in order to make a consistent and valid database of the geomechanical properties. In the remaining parts, results of statistical analysis of geomechanical and geological data of the Paul’s Peak pit are presented.

3.3.1 Data cleaning
The geomechanical drillhole dataset carries information of the geomechanical properties and the lithology. The goal of cleaning the geomechanical drillhole dataset was to prepare a reliable dataset having the least ambiguities and inconsistencies. To do so, the geomechanical drillhole dataset of the entire Mont-Wright were cleansed. Four steps were undertaken for cleaning the geomechanical dataset which include:

i. Cut off the extreme long and short core runs from the data set;
ii. Delete the data of lost core runs that have no core recovery values (missing);
iii. Manage the data of core runs with zero core recovery;
iv. Manage the core runs with RQD > Core Recovery.

The Mont-Wright geomechanical data is of the support size of core runs that have different lengths. The first undertaken strategy was to make the sample data to have a consistent support size. This helps that each sample data point almost equally represents a support size. The Mont-Wright geomechanical dataset consists of 141,205 sample data points, each sample point corresponds to a core run. In this dataset, the core runs are within the range 0.1 m to 135.0 m. However, most of the geomechanical core runs have the length of almost 3.0 m. Figure 3-7 shows the histogram of the core runs for the geomechanical dataset of Mont-Wright.
According to Figure 3-7, about 93.48% of the sample data are on the support length within the range of 2.5 m-3.5 m. Besides, 99% of core runs have the length of 1.0 m-5.0 m. The remaining core runs (1% of data), are either very short (<1.0 m) or very long (>5 m). To have support lengths within a limited range, the data of those short and very long core runs were deleted. Thus, data of 1,103 core runs of beyond the range 1.0-5.0 m were discarded. It should be noted that to prevent artificial reduction in the short scale variability (nugget effect), longer than 5.0 m core runs were not broken down into smaller ones. Breaking down the longer core runs into smaller ones would make redundant data and finally reduce the short scale variability (nugget effect) artificially. Figure 3-8 depicts the histogram of core runs (support size) after removing the extremities from the dataset. As can be seen, most of data (94.22%) represent a support size of
3.0 m. After this first step of data cleaning, the remaining dataset includes 139,922 sample data.

The next step in cleansing the dataset was to manage the data of missing cores whose core recovery was recorded as -99. In this regard, data of 585 missing core runs were deleted. These missing core runs were primarily empty of geomechanical properties data except for: a) two core runs with joint condition=0; b) two core runs with UCS=0; and c) 57 core runs with RMR=10.

After this step of data cleansing, the remaining geomechanical data of Mont-Wright contained 139,337 core runs. The next step in the cleaning of the dataset was to manage the data of 1,051 core runs with zero core recovery. To do so, having an understanding of the geomechanical data for these core runs was necessary. Figure 3-9 displays the histogram of the fracture frequency for the 1,051 core runs with zero core recovery. As it can be observed, the fracture frequency of these core runs was either of 0, 10, 16, 20, 25, 35, 50, or 60 m$^{-1}$. The mode of fracture frequency for these core runs is 35 m$^{-1}$. About 89% of these core runs have the fracture frequency of 25 m$^{-1}$ and more. Thus, it can be seen that the zero core recovery for these 1,051 core runs can be attributed to the presence of high fracture frequency. Having such insight, following steps were undertaken for cleansing them:
1. Delete the geomechanical data of two core runs with zero core recovery and zero fracture frequency. Most of geomechanical data of these two core runs, drillholes 2011_109 (interval 197-200 m) and 2013_INC_09 (Interval 26-29 m), are missing;
2. Set 25 m\(^{-1}\) as the fracture frequency of the core runs with zero core recovery and less than 25 m\(^{-1}\) fracture frequency. After modifying the fracture frequency for such core runs, the values of RMR were also adjusted for them. This adjustment was done as RMR is linearly dependent on the fracture frequency and UCS of these zero core recovery drillcores.

![Figure 3-9: Histogram of fracture frequency for core runs of zero core recovery.](image)

The last step of the data cleansing was to manage data of 158 core runs in the geomechanical dataset which were found to have RQD values bigger than core recovery. Based on the definition of RQD and core recovery, the RQD can only be equal or smaller than core recovery. To manage the data of such core runs, following steps were carried out:

1. Geomechanical data of two core runs 67_21 and 67_22 were completely removed as they have mostly been logged with RQD>Core Recovery;
2. The core recovery of core runs belonging to drillholes 92_51 (interval 12-15 m), 2009_13 (interval 88.5-90 m), 2013_DR_P05 (interval 29-32 m), and 90_02 (interval 105-108 m) was set to 100% according to similarity of data with the neighbor core runs in the same drillholes;
3. Data of core runs for the drillhole 83_49 (interval 5.3-8 m) and 92_49 (interval 64-70 m) were completely deleted;
4. For the rest of core runs in which RQD>Core Recovery, the values of RQD and Core Recovery were swapped.

Apart from the main four steps of data cleansing, some minor modifications were also done on the dataset as follows:

- Fracture Frequency values in intervals 3-end of 2009_13 were modified according to the Piteau (2010) report.
- Fracture Frequency value of Interval 26-27.5 m of the drillhole 2009_11 was changed to 20 according to the Piteau (2010) report.

After cleansing the entire Mont-Wright’s geomechanical dataset, 139,234 core runs were remained in the dataset. Only about 1.4% of the original dataset were deleted in the data cleansing. Since the focus of this project was on the Paul’s Peak geomechanical and geological data analysis, the subset of drillhole dataset for the Paul’s Peak pit was selected for further analysis (65,796 out of 139,234 core runs). The Paul’s Peak geomechanical borehole dataset carries the lithological and geomechanical information for only the Paul’s Peak pit.

In Paul’s Peak pit, the geomechanical and lithological data were collected through 871 boreholes since 1949. These drillholes are in different types including regular, piezometer (PZ), drainhole (DR), time-domain reflectometer (TDR), and inclinometer (INC) holes. The regular drillholes have been primarily drilled to measure different geological and geomechanical properties in the pit. The regular drillholes have been initially drilled for the purpose of exploration and orebody definition. Although other types of drillholes (PZ, DR, INC, and TDR) have been made primarily for other purposes, they have been logged for collecting geomechanical data as well. Figure 3-10 shows the spatial locations of the different types of drillholes in the final Paul’s Peak pit. The regular drillholes (colored in red) form the majority of the drillholes in the Paul’s Peak pit with concentration in the middle parts of the pit where the iron ore deposit is placed. Drainholes (in blue) are horizontal holes that have been mainly made perpendicular to the pit walls. Piezometer holes (in green) are also drilled in the pit walls in order to measure and monitor the ground water level. There are 10 inclinometer holes (in black) that have been drilled in the south wall of the
Paul’s Peak pit to monitor the wall displacements. Figure 3-11 depicts the locations of the 871 drillholes’ collar in a plan view (looking down) superimposed on the Paul’s Peak topo map in year 2045. The drillholes have been drilled in an approximately 150 m spacing in the Easting direction (BLA). Besides, the spacing of the drillholes in Northing (BLA) direction is about 50 m. Figure 3-12 displays the drillholes colored by the logged RMR values in the Paul’s Peak 2045 pit in a North-South cross section (Easting 3500±100 in BLA coordinate system). In Figure 3-12, the black strip marks the border of the Paul’s Peak pit in 2045.

Table 3-4 illustrates a basic summary of the Paul’s Peak drillholes information. Table 3-4 indicates that more than 205 km of drillholes have been logged for the geomechanical and geological properties in the Paul’s Peak pit. Besides, 73% of drillholes have been purposefully drilled for the geomechanical and geological logging (regular holes). The drillholes range from 6.6 m to 981.5 m in length. Figure 3-13 shows the histogram of drillholes’ length in the Paul’s Peak pit. This histogram indicates a positive skewed distribution where the majority of drillholes in the Paul’s Peak pit are of the length within the range of 100 m – 250 m.
Figure 3-10: 871 Drillholes in the Paul's Peak pit colored according to their type (looking east in BLA).

Figure 3-11: Location of drillholes' collar superimposed on the Paul's Peak pit 2045 topo map (Looking down)
Figure 3-12: A north-south cross section of the Paul's Peak pit of 2045 with drillholes colored by RMR values (easting 3500±100 in BLA coordinate system)

Table 3-4: Summary Information of Drillholes in the Paul's Peak pit.

<table>
<thead>
<tr>
<th>Number of Drillholes</th>
<th>871</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. Length of DHs (m)</td>
<td>6.6</td>
</tr>
<tr>
<td>Max. Length of DHs (m)</td>
<td>981.5</td>
</tr>
<tr>
<td>Mean of DHs’ Length (m)</td>
<td>235</td>
</tr>
<tr>
<td>Standard Deviation of DHs’ Length (m)</td>
<td>181</td>
</tr>
<tr>
<td>Total DHs’ Length (m)</td>
<td>205,040</td>
</tr>
<tr>
<td>Number of Drillholes per type</td>
<td></td>
</tr>
<tr>
<td>PZ</td>
<td>43</td>
</tr>
<tr>
<td>INC</td>
<td>10</td>
</tr>
<tr>
<td>DR</td>
<td>183</td>
</tr>
<tr>
<td>TDR</td>
<td>0</td>
</tr>
<tr>
<td>Regular</td>
<td>635</td>
</tr>
</tbody>
</table>
3.3.2 Paul’s Peak pit geological data

The borehole dataset from the Paul’s Peak mine include information of the lithology (rock code) per drillhole interval. The objective of collecting and analyzing the geological data for the Paul’s Peak pit was to understand the spatial distribution of rock types in different mine domains.

AMMC used the geological data of Paul’s Peak pit to update the geological block model of the pit in 2013. While the borehole dataset provides the point lithology sample data, the geological block model updated in 2013 gives information of the rock type’s spatial distribution across the Paul’s Peak pit. This geological block model was provided to the author as part of the input data for this research project.

Figure 3-14 shows an isometric view of the Paul’s Peak 2045 pit where the rock types (from the geological block model 2013) are projected on the walls of the pit. This figure indicates that pit walls of the final Paul’s Peak pit are mostly formed by the three major rock types: Amphibolite (code 102), Iron Formation (code 103), and Quartz Mica Schist (code 106). Besides, other rock types such as Gneiss (code 107), Felsic Gneiss (code 108), and Mafic Gneiss (code 109) are the major rock units forming the regions out of the Paul’s Peak pit 2045. Figure 3-15 shows four east-looking cross sections of the geology model in the Paul’s Peak pit. This figure approves that the Paul’s Peak walls are primarily built by the above-mentioned major lithological units. In all of these cross sections, it can be observed that there is a syncline folding with West-East axis (in
BLA) where layers of Iron Formation, Quartz Mica Schist, and Amphibolite rocks are overlaid on each other. Figure 3-16 shows a plan view of the geology distribution on the Paul’s Peak pit 2045. As marked in a dashed line on this figure, there is another folding system with the vertical crest line. Two areas of the Paul’s Peak pit are also depicted in this figure as “Lower Limb” and “Upper Limb”. Presence of these two fold systems in the Paul’s Peak deposit resulted in a complicated geology setting in the deposit. The complexity of geology and existence of folding systems in the Paul’s Peak pit can hint that the geomechanical properties and quality of rock mass would differ across the Paul’s Peak depending on the rock mass position relative to the folds. Figure 3-17 presents the boreholes drilled in the Paul’s Peak area where they are colored according to the lithology logged along the cores. The rock codes can be found in Table 3-1. The colors of the drillholes in Figure 3-17 reveals that the rocks encountered by drillholes are mostly Iron Formation, Amphibolite, Quartz Mica Schist, Mica Schist, and Gneiss. These drillholes are mostly made in order to identify and characterize the ore deposit and consequently mineral resource and reserve modeling. Thus, they have primarily and preferentially been drilled perpendicular to the deposit dip with spacing of roughly 150 m along the Easting (BLA) axis. The orientation of drillholes in the Paul’s Peak deposit has been plotted in an Equatorial (equal angle) stereonet in Figure 3-18. According to this figure, drillholes have been made primarily in dip direction 135° and dip angle 60°-90° in UTM coordinate system.
Figure 3-14: Isometric view-geology mapped on the Paul's Peak 2045 pit walls

Figure 3-15: Cross sections (looking east BLA) of geology model on Paul's Peak 2045 pit.
Figure 3-16: Plan view (looking down) of geology model in Paul’s Peak pit 2045.

Figure 3-17: Drillholes colored based on lithology codes in the Paul’s Peak 2045 pit.
Figure 3-18: Equatorial (equal angle) stereonet plot showing orientation (in UTM) of drillholes in Paul’s Peak

Figure 3-19 displays the total length (in percent) for each rock unit logged in the geomechanical and geological drillhole campaign. According to this figure, the highest amount of sampled rock unit belongs to the Iron formation rock unit (Rock code 103), following by the Amphibolite (rock code 102) and the Quartz Mica Schist (rock code 106). This confirms that these three rock units are the major rocks forming the geology of the pit.
3.3.3 Paul’s Peak pit geomechanical data

The exploratory statistical data analysis was carried out on the cleaned borehole geomechanical data collected from 1949 to 2013 in the area of the Paul’s Peak pit. The geomechanical variables of interest for the statistical analysis include: RQD, fracture frequency, joint condition, UCS, and RMR. All of these parameters have been logged following RMR guidelines. Table 3-5 provides a brief explanation of these geomechanical properties.

<table>
<thead>
<tr>
<th>Geomechanical Property</th>
<th>Range</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQD</td>
<td>0%-100%</td>
<td>Rock Quality Designation</td>
</tr>
<tr>
<td>Fracture Frequency</td>
<td>0-∞</td>
<td>Number of natural joint per meter</td>
</tr>
<tr>
<td>RMR</td>
<td>0-100</td>
<td>Based on Bieniawski (1976)</td>
</tr>
<tr>
<td>UCS</td>
<td>0-450 MPa</td>
<td>Intact rock uniaxial compressive strength</td>
</tr>
<tr>
<td>Joint Condition</td>
<td>0.0-25.0</td>
<td>Condition of joint surface</td>
</tr>
</tbody>
</table>

Exploratory statistical data analysis was performed in order to understand the available geomechanical data, limitations, and inter-correlations between different properties. Table 3-6 shows a summary of simple statistics for the Paul’s Peak borehole geomechanical data. As seen in this table, the number of available samples logged for different geomechanical properties is different. This is due to the fact that some core runs have not been logged for all geomechanical properties. Statistical analysis of the geomechanical borehole data (Table 3-6) indicates that rock masses forming the Paul’s Peak pit are, on average, have a very high intact rock strength with closely spaced jointing. Besides, the quality of rock mass is interpreted as good in terms of average RMR and RQD. Table 3-7 presents the correlation matrix showing the correlation...
coefficient between each pair of geomechanical variables. Each cell of the correlation matrix is colored according to the corresponding correlation coefficient and based on the defined color legend. The correlation matrix indicates that the variable RMR is highly dependent on the three geomechanical properties of RQD, fracture frequency, and joint condition. Besides, it is observed that RQD and fracture frequency are highly but negatively correlated (correlation coefficient = -0.82).

Table 3-6: Basic statistics for geomechanical properties in the Paul's Peak pit.

<table>
<thead>
<tr>
<th>Property</th>
<th>No. of samples</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean/Mode*</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description**</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQD (%)</td>
<td>65796</td>
<td>0</td>
<td>100</td>
<td>80</td>
<td>90</td>
<td>25.0</td>
<td>0.31</td>
<td>Good</td>
</tr>
<tr>
<td>Fracture Frequency (m⁻¹)</td>
<td>65794</td>
<td>0.0</td>
<td>65.0</td>
<td>5.3</td>
<td>4.0</td>
<td>6.7</td>
<td>1.29</td>
<td>Closely Spaced</td>
</tr>
<tr>
<td>UCS (MPa)</td>
<td>65176</td>
<td>5</td>
<td>380</td>
<td>201</td>
<td>193</td>
<td>89.0</td>
<td>0.44</td>
<td>Very High</td>
</tr>
<tr>
<td>Joint Condition</td>
<td>65176</td>
<td>0</td>
<td>25</td>
<td>17/20</td>
<td>20</td>
<td>5.4</td>
<td>0.31</td>
<td>Slightly rough surfaces, separation &lt;1 mm, hard joint wall contact</td>
</tr>
<tr>
<td>RMR</td>
<td>65176</td>
<td>12</td>
<td>100</td>
<td>78</td>
<td>81</td>
<td>16.0</td>
<td>0.21</td>
<td>Good</td>
</tr>
</tbody>
</table>

*Mode is recorded only for the joint condition
**Descriptions are based on Average Values parameters (Defined in RMR 1976)

Table 3-7: Correlation Matrix between Geomechanical Properties in the Paul's Peak pit.

<table>
<thead>
<tr>
<th>Geomechanical Property</th>
<th>RQD (%)</th>
<th>Fracture Frequency (m⁻¹)</th>
<th>UCS (MPa)</th>
<th>Joint Condition</th>
<th>RMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.82</td>
<td>0.43</td>
<td>0.63</td>
<td>0.91</td>
</tr>
<tr>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.82</td>
<td>1.00</td>
<td>-0.33</td>
<td>-0.53</td>
<td>-0.82</td>
</tr>
<tr>
<td>UCS (MPa)</td>
<td>0.43</td>
<td>-0.33</td>
<td>1.00</td>
<td>0.63</td>
<td>0.59</td>
</tr>
<tr>
<td>Joint Condition</td>
<td>0.63</td>
<td>-0.53</td>
<td>0.63</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>RMR</td>
<td>0.91</td>
<td>-0.82</td>
<td>0.59</td>
<td>0.83</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Figures 3-20 to 3-24 present the histograms of geomechanical variables in the Paul’s Peak pit based on the borehole geomechanical dataset. Figure 3-20 shows that the variable RQD is highly and negatively skewed, revealing that most of the logged core runs have good or excellent RQD.
Unlike RQD, fracture frequency is positively skewed where most of the logged rocks are of low fracture frequency. Figure 3-21 indicates that the fracture frequency is mostly within the range 0-8 (m⁻¹) representing moderately to closely spaced jointed rocks. Histogram of UCS, Figure 3-22, shows a bimodal distribution with modes within the range of 120-140 (MPa) and 180-200 (MPa). Histogram of joint condition (Figure 3-23) displays a negatively skewed distribution. Figure 3-24 shows that the distribution of RMR is highly negatively skewed. The negative skewness explicitly means that the quality of logged rocks in the Paul’s Peak pit is mostly of high quality (good or very good rock mass quality in terms of RMR). This observation can also indicate that the RMR values are highly influenced by the values of RQD and fracture frequency.

Figures 3-25 to Figure 3-29 depict a comparison of the geomechanical properties of interest in this project against the major rock units in the Paul’s Peak pit in terms of box plots. The major rock units are Amphibolite (code 102), Iron Formation (code 103), Quartzite rock with 0-40% of mica (code 104), Mica schist with 60-100% of mica (code 105), and Quartz mica schist with 40-60% of micas (code 106). Following observations can be made from these box plots:

- Rock units 103 and 104 have similar median and dispersion for RQD, RMR and Fracture Frequency properties;
- Rock units 103 and 106 have similar median and dispersion for the Joint Condition;
- Among these rock units, Amphibolite shows the least dispersion and range for all the geomechanical properties (except for UCS);
- The rock unit 105 reveals a wide dispersion and range for the fracture frequency, RQD, and RMR while the least dispersion for the UCS;
- Rock units 102 and 104 show that they have mostly the joint condition of 20;
- Iron formation shows the largest dispersion for UCS.
Figure 3-20: Histogram of RQD in the geomechanical borehole data of Paul's Peak

Figure 3-21: Histogram of fracture frequency in the geomechanical borehole data of Paul's Peak

Figure 3-22: Histogram of UCS in the geomechanical borehole data of Paul's Peak

Figure 3-23: Histogram of joint condition in the geomechanical borehole data of Paul's Peak

Figure 3-24: Histogram of RMR in the geomechanical borehole data of Paul's Peak
Table 3-8 summarizes the results of UCS tests previously performed by the AMMC on the representative intact rock samples taken from the mine site. The laboratory tests were done in years 1974, 1983, 1997, and 2010 and the results are presented from the unpublished works (Piteau, 1998, Piteau, 2010). Comparing the borehole UCS datasets and the laboratory UCS
strength tests indicates that the UCS of the borehole dataset is on average larger than the results of the historical UCS tests for all the major rock units. Besides, the UCS values in the borehole geomechanical dataset show more variations than the UCS tests results conducted in the lab. The historical UCS tests (except in 1974) show that the Quartzite Rock with 0-40% Mica (code 104) is the strongest rock unit which this observation is compatible with the geomechanical borehole dataset. In addition, both geomechanical borehole dataset and the UCS test results reveal that the Mica Schist with 60-100% Mica (code 105) has the lowest strength.

Table 3-8: Comparison of UCS values logged in borehole dataset with the historical laboratory UCS test results

<table>
<thead>
<tr>
<th>Rock Type</th>
<th>UCS Laboratory Testing</th>
<th>UCS Dataset from Boreholes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1974</td>
<td>1983</td>
</tr>
<tr>
<td>AMP (102)</td>
<td>137</td>
<td>108</td>
</tr>
<tr>
<td>IF (103)</td>
<td>198</td>
<td>187</td>
</tr>
<tr>
<td>QR (104)</td>
<td>119</td>
<td>237</td>
</tr>
<tr>
<td>MS (105)</td>
<td>61</td>
<td>98</td>
</tr>
<tr>
<td>QRMS (106)</td>
<td>139</td>
<td>161</td>
</tr>
</tbody>
</table>
To explore how RMR values in the geomechanical drillhole dataset have been logged, Figure 3-30 shows the scatter plot between these values and the calculated RMR using the ratings provided in Bieniawski (1976). To calculate RMR values using the RMR76 table, the corresponding ratings for the geomechanical variables were summated assuming the dry ground condition. It should be noted that for rating of the joint spacing in Bieniawski (1976), the fracture frequency which is the inverse of the joint spacing was used. The regression line (in black) and 95% upper and lower limits for regression (in green) also presented in this figure. The correlation coefficient R=0.97 indicates a strong correlation between the RMR calculated using the RMR76 table assuming dry condition (y axis) and the RMR values in the geomechanical borehole dataset (x axis).

In addition, the error (difference between the RMR calculated using the RMR76 table assuming dry condition (y axis) and the RMR values in the geomechanical borehole dataset (x axis) is less than 5 which could be considered negligible.

To check the existence of a linear relationship between the RMR and other geomechanical variables in the geomechanical drillhole dataset, a multi-linear regression analysis was performed. The results of multi-linear regression analysis show the following linear equation:
\[ RMR = 34.22312 + 0.29911 \times RQD - 0.50396 \times Fracture Frequency + 0.01373 \times UCS + 1.15178 \times Joint Condition \quad (3.1) \]

Where \( R^2 = 0.9560 \). Presence of a linear relationship between RMR and other geomechanical variables implies that RMR can be modeled based on a liner relationship with the other variables.

In this chapter, the geological and geotechnical data collected from the Mont-Wright mine site were discussed. The geological and geomechanical data of the iron ore pit were statistically analyzed. In the next chapter, the proposed geostatistical approaches used for heterogeneity modeling of the geomechanical properties including: RMR and its constituent parameters, will be presented along with the implementation results for the Paul’s Peak pit.
Chapter 4
Application of Conditional Simulation Method for Modeling Geomechanical Heterogeneity of Rock Masses in the Paul’s Peak Pit

This chapter expresses the methodologies of applying the conditional simulation approach to the geomechanical borehole dataset in the Paul’s Peak mine in order to model the heterogeneity of geomechanical properties. The SGS method was implemented using the Maptek Vulcan 9.0.0 software (Maptek 2014). The geostatistical simulation approach used in the thesis follows two different routes: direct and indirect approaches. In the direct approach, the values of RMR sample data available in the geomechanical borehole dataset were directly used as inputs to the SGS engine in order to create 3D block models of RMR in the Paul’s Peak pit area. In the indirect approach, each of the RMR geomechanical component parameters such as: RQD, fracture frequency, joint condition, and UCS were separately modeled using SGS and the resulting 3D block models were then combined to build 3D heterogeneity models (block models) of the RMR variable. The indirect approach allowed all the geomechanical properties to be modeled unlike the direct approach where only RMR was modeled. The results of both approaches are presented and validated using appropriate statistics.

4.1 Direct Approach

In the direct approach, five 3D block models of RMR were generated using the SGS method. There is no standard for the number of realizations or block models in the literature. The number of realizations depends on the purpose of study, for example the confidence level accepted by the modeler or decision makers. In this thesis, the purpose of block modeling using SGS is to show that this method might be useful for heterogeneity modeling of geomechanical properties given this fact that geostatistical techniques have been limitedly applied for such applications. Hence, only five block models were created to show that heterogeneity of RMR in the Paul’s Peak pit can be captured using SGS.

Each block model is a realization of the RMR spatial distribution showing the heterogeneity of rock mass geomechanical quality. In the direct approach, RMR sample data from the geomechanical borehole dataset of the Paul’s Peak pit were used as inputs to SGS in order to
characterize the rock mass geomechanical heterogeneity. The direct approach includes the following general steps:

- Normalize the RMR sample data;
- Partition the domain of study into subdomains according to lithology and relative location (folding of rock masses);
- Variography analysis for normal scores of RMR for each subdomain;
- Perform SGS on normal scores within each subdomain and then back-transform the results to original space;
- Validate the resulting 3D heterogeneity models (block models) of RMR.

4.1.1 Normal score transformation for RMR

As mentioned in section 2.3.4.5, the SGS technique works with the normal scores. Therefore, it was necessary to transform the RMR sample data into normal scores in order to be compatible with the SGS method. In this step, the RMR sample data were normalized using Gaussian transformation method in Maptek Vulcan 9.0.0 (Maptek 2014). The resulting normal scores of RMR are called N_RMR in the rest of the thesis.

According to Figure 3-24, the original RMR sample data have a skewed distribution. This skewed distribution could indicate that the RMR sample data would show proportional effect or heteroscedasticity which is an intrinsic property of the skewed distributions (Manchuk et al. 2009). Proportional effect is described as the dependency of the variability of a regionalized variable on its magnitude (Oz and Deutsch 2002; Manchuk et al. 2009). The presence of proportional effect implies that a regionalized variable shows more variability in the areas with larger values than lower. This also means that variograms and specially local sill changes across the domain of study (Journel and Huijbregts 1978; Oz and Deutsch 2002; Manchuk et al. 2009). Due to the dependency of variograms to the magnitude of a regionalized variable, inferencing a true variogram model would be difficult in presence of proportional effect (Manchuk et al. 2009). The Variography of the regionalized variables is affected by presence of the proportional effect. Therefore, the proportional effect should be removed from the variables in order to reduce its negative effects on the variogram analysis and consequently the simulation results. It is essential that the proportional effect be removed before applying the geostatistical techniques.
Fortunately, the SGS algorithm is immune to the proportional effect as it applies Gaussian transformation (Oz and Deutsch 2002; Manchuk et al. 2009). To check if the proportional effect exists in the RMR data before and after normal score transformation, the scatter plots of local standard deviation (STD) versus local mean are presented in Figures 4-1 and 4-2. To draw these scatter plots, the entire domain is divided into arbitrary rectangular blocks of 700 × 600 × 300 m. Within each block, standard deviation and mean of the RMR and N_RMR data were calculated. Figure 4-1 indicates that there exists a high correlation between the local STD and local mean of RMR sample data (absolute correlation coefficient = 0.88). This high correlation coefficient would reflect the presence of proportional effect in the RMR sample data. Figure 4-2 displays the scatter plot of local STD vs. mean for the normal scores of RMR (N_RMR). As seen, the absolute correlation coefficient has dropped to 0.3 which means that the proportional effect has been removed after applying the normal score transformation.

4.1.2 Partition the mine domain

In this step, the entire domain of study was partitioned into six subdomains according to the lithology group and relative location within the Paul’s Peak pit. The senior geologist in the
Mont-Wright mine who has been involved in the mine operations for more than 30 years suggests that the mechanical behaviors of rock masses in the Paul’s Peak mine depends on the type of rocks. According to his opinions, there are three groups of rocks in the Paul’s Peak pit with distinguished mechanical behaviors as follows:

- Group I: Iron Formation (rock code: 103);
- Group II: Amphibolite (rock code: 102), Gneiss (rock code: 107), and Felsic Gneiss (rock code: 108);
- Group III: Quartzite rock with 0-40% of mica (rock code: 104), Mica schist (60-100% of mica) (rock code: 105), and Quartz mica schist (40-60% of micas) (rock code: 106).

Based on his recommendation, the whole N_RMR sample data in this study were divided into three subsets according to the above mentioned lithology groups. In addition, the relative location of the geomechanical data in the pit area is an important factor influencing the homogeneity of the sampled data. As Figure 3-16 shows, there is a folding system with vertical axis which divided the entire Paul’s Peak ore deposit into two zones: Upper Limb and Lower Limb. Presence of this folding is expected to effect the geomechanical behavior of the rock masses present in different walls of the Paul’s Peak pit. Thus, each of the three subsets of N_RMR sample data were further partitioned into two subsets according to their locations in either Upper or Lower Limb zone. Consequently, six subdomains were defined and further geostatistical analysis and modeling were done on each of these six subdomains’ sample data. In other words, in the direct approach, N_RMR sample data were clustered into six groups based on lithology group and location. All other geostatistical operations were performed on these six subdomains of N_RMR sample data.

4.1.3 Variography on subdomain’s N_RMR

To capture the spatial variability of RMR normal scores in each subdomain, experimental variograms were calculated. To understand the nugget effect, downhole variograms were first calculated for each subdomain’s N_RMR. Figures A-1 to A-6 in Appendix display the downhole variograms. From these figures, the nugget effect for each subdomain can be estimated as the y-intercept values. Table 4-1 summarizes the amount of sample data and the nugget effect for each subdomain’s N_RMR.
Because of the sedimentary geology of rock mass formation in Paul’s Peak, the principal directions of continuity are believed to be horizontal and vertical. Therefore, experimental variograms were calculated for N_RMR variable for each subdomain. Figures A-7 to A-12 in Appendix show the experimental variograms for N_RMR in each subdomain. In these figures, semi, az, and pl represent the semi-variogram, azimuth, and plunge, respectively. The horizontal variograms have been calculated for directions with azimuth (AZ) intervals of 10º. These figures indicate that the maximum continuity for N_RMR of all lithology groups is about 90º for Lower Limb and 100º-110º for Upper Limb. Table 4-2 summarizes the parameters of variogram models for N_RMR in each subdomain. The variogram models for N_RMR within each subdomain are presented in Figures A-13 to A-18 in Appendix. Generally, two spherical models were used for modeling the variograms of N_RMR for each subdomain.

Table 4-1: Nugget Effect for Each Subdomain’s of N_RMR variable

<table>
<thead>
<tr>
<th>Lithology Group</th>
<th>Lower Limb</th>
<th>Upper Limb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Number of Data</td>
<td>19749</td>
<td>14982</td>
</tr>
<tr>
<td>Nugget Effect</td>
<td>0.20</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 4-2: Parameters of Variogram Models for each Subdomain’s N_RMR

<table>
<thead>
<tr>
<th>Lithology Group</th>
<th>Lower Limb</th>
<th>Upper Limb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>Major Direction of Continuity</td>
<td>Azimuth: 80º Plunge: 0 º</td>
<td>Azimuth: 110º Plunge: 0 º</td>
</tr>
<tr>
<td>Semi-Major Direction of Continuity</td>
<td>Azimuth: 170º Plunge: 0 º</td>
<td>Azimuth: 20º Plunge: 0 º</td>
</tr>
<tr>
<td>Minor Direction of Continuity</td>
<td>Azimuth: 45º Plunge: -90º</td>
<td>Azimuth: 45º Plunge: -90º</td>
</tr>
<tr>
<td>Nugget</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 4-2 (continued):

<table>
<thead>
<tr>
<th>Variogram structure 1</th>
<th>Type</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sill difference</td>
<td>0.55</td>
<td>0.61</td>
<td>0.54</td>
<td>0.15</td>
<td>0.55</td>
<td>0.46</td>
</tr>
<tr>
<td>Range Major (m)</td>
<td>134</td>
<td>320</td>
<td>1202</td>
<td>2350</td>
<td>780</td>
<td>140</td>
</tr>
<tr>
<td>Range Semi-major (m)</td>
<td>134</td>
<td>80</td>
<td>140</td>
<td>2220</td>
<td>646</td>
<td>33</td>
</tr>
<tr>
<td>Range Minor (m)</td>
<td>38</td>
<td>123</td>
<td>21</td>
<td>47</td>
<td>18</td>
<td>143</td>
</tr>
</tbody>
</table>

Table 4-2 (continued):

<table>
<thead>
<tr>
<th>Variogram structure 2</th>
<th>Type</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sill difference</td>
<td>0.25</td>
<td>0.19</td>
<td>0.26</td>
<td>0.65</td>
<td>0.15</td>
<td>0.24</td>
</tr>
<tr>
<td>Range Major (m)</td>
<td>1243</td>
<td>1330</td>
<td>1696</td>
<td>173</td>
<td>183</td>
<td>1001</td>
</tr>
<tr>
<td>Range Semi-major (m)</td>
<td>274</td>
<td>580</td>
<td>650</td>
<td>288</td>
<td>630</td>
<td>911</td>
</tr>
<tr>
<td>Range Minor (m)</td>
<td>85</td>
<td>1566</td>
<td>583</td>
<td>32</td>
<td>679</td>
<td>145</td>
</tr>
</tbody>
</table>
4.1.4 3D block models for RMR

The 3D block models of N_RMR were developed using the sequential Gaussian simulation method. Five realizations/simulations were generated given the knowledge of the variogram models in 4.1.3. Each realization/simulation is a block model. After developing the block models of N_RMR, they were back-transformed to the original space of RMR variable. Each RMR block model has 4,117,203 blocks of 10 m × 10 m × 14 m. This size of blocks for the RMR block modeling was chosen to be the same as the geological block model given by the AMMC. It should be noted that the bench height in the Paul’s Peak pit is 14 m. In the direct approach, the geostatistical simulation was performed given the knowledge of spatial variability of rock type which is presented in the form of geological block model. Figures 4-3 to 4-7 show the cross section views of the five RMR realizations developed using the direct approach. These sections are all made in Easting=3500 m (in BLA coordinate system) closer to the each wall of the ultimate Paul’s Peak pit. In these figures, the black strip marks the border of the final Paul’s Peak pit in year 2045. As it can be seen, the majority of rocks in all realizations are of good and very good quality in terms of RMR\textsubscript{76}. Blue areas indicate the very poor quality rocks which are concentrated in the north part of the current Paul’s Peak pit (pit 2013). This is compatible with the drillhole data seen in Figure 3-12. For example, in all of the cross section views demonstrated in Figures 4-3 to 4-7, the blue area is around the drillhole segments with very poor or poor quality observed in Figure 3-12. Figures 4-8 to 4-12 show isometric views of the developed RMR block models projected on the Paul’s Peak 2045 walls. These figures indicate that the rock quality in the south wall of the Paul’s Peak 2045 pit is mostly very poor to fair quality while in the other areas the quality is better in terms of RMR\textsubscript{76}. It should be noted that there was a big instability in the south wall of the Paul’s Peak pit in 1986. Hence, mining in the south walls of the Paul’s Peak pit should be carried out with more caution due to potential instability.
Figure 4-3: Cross section view (looking east) of RMR (realization 1) developed in direct approach

Figure 4-4: Cross section view (looking east) of RMR (realization 2) developed in direct approach
Figure 4-5: Cross section view (looking east) of RMR (realization 3) developed in direct approach

Figure 4-6: Cross section view (looking east) of RMR (realization 4) developed in direct approach
Figures 4-13 and 4-14 demonstrate probability maps of RMR in a cross section view. In Figure 4-13, the probability of having RMR greater than 40 is mapped in a cross section view while the probability map is presented for RMR greater than 60 in Figure 4-14. According to these figures, in some areas in the north of the current Paul’s Peak pit (pit 2013) the probability of having good quality rocks is less than 0.6 which might indicate that these areas should have a high probability of having poor to fair rocks. In all other areas, rocks show good to very good quality with the high probability indicated in red color. It should be noted that since the produced probability maps are based on only five realizations of RMR, they should be used with caution. The probability maps presented in this thesis are solely used to give an idea about how the uncertainty in geomechanical heterogeneity can be presented and quantified.
Figure 4-8: RMR (realization 1) developed in direct approach mapped on Paul’s Peak 2045

Figure 4-9: RMR (realization 2) developed in direct approach mapped on Paul’s Peak 2045
Figure 4-10: RMR (realization 3) developed in direct approach mapped on Paul’s Peak 2045

Figure 4-11: RMR (realization 4) developed in direct approach mapped on Paul’s Peak 2045
Figure 4-12: RMR (realization 5) developed in direct approach mapped on Paul’s Peak 2045

Figure 4-13: Probability map for generated RMR block models in direct approach, RMR>40

Figure 4-14: Probability map for generated RMR block models in direct approach, RMR>60
4.1.5 Verification of Direct Approach RMR block models

In this section, validity of the created block models of RMR was investigated using statistical techniques. Table 4-3 indicates the summary statistics for the RMR in the developed block models using the direct approach. Table 4-4 shows the summary statistics of the entire RMR sample data (including the area of the pit that has been already mined out), as well as the RMR sample data inside the developed block models. The statistics mentioned for the entire RMR sample data is exactly repeated from Table 3-6. Comparison between the two tables indicates that the created block models of RMR approximately honor the basic statistics of the sample data. For example, the mean of RMR in the created block models is about 81-82 which is close to mean of RMR both from the entire sample data (mean:78) and sample data inside the block model (mean: 80). However, the created block models of RMR in the direct approach show the very good quality on average while the RMR sample data in both entire and in the pit show good quality rock masses.

Figures 4-15 to 4-19 show the histogram of RMR in the five realizations (block models). Comparison between these histograms with the histograms of the entire RMR sample data (Figure 4-20) and histogram of RMR samples intercepted with the block models (Figure 4-21) indicates that the created block models have fairly honored the histograms of sample data in terms of shape and skewness. However, there are some differences between the histograms in terms of bins’ frequencies. For example, the highest frequency in the histograms of the created RMR block models belongs to the bin 90-100 while the sample data of RMR have the highest frequency for RMR within the range of 80-90.

Figure 4-22 and 4-23 show the Q-Q plots between the RMR data in block models and the RMR sample data inside the block model and the entire RMR sample data, respectively. These figures indicate that the created block models slightly overestimate the RMR compared with the RMR sample data. This can be detected as the quantiles of RMR block models’ distributions are above the line of 45°. The degree of overestimation is less for the RMR sample data inside the block model than the entire RMR sample data. This could be expected as the simulated RMR values in the block models would be generally more influenced by the RMR sample data inside the block models than the entire RMR sample data as they are closer to the grid nodes in the SGS.
### Table 4-3: Summary Statistics for the RMR Data in the Five Block Models

<table>
<thead>
<tr>
<th>RMR (Direct Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>3653909</td>
<td>0</td>
<td>100</td>
<td>82</td>
<td>83</td>
<td>14.48</td>
<td>0.18</td>
<td>Very Good</td>
</tr>
<tr>
<td>Realization 2</td>
<td>3653909</td>
<td>0</td>
<td>100</td>
<td>81</td>
<td>82</td>
<td>14.50</td>
<td>0.18</td>
<td>Very Good</td>
</tr>
<tr>
<td>Realization 3</td>
<td>3653909</td>
<td>1</td>
<td>100</td>
<td>82</td>
<td>83</td>
<td>14.50</td>
<td>0.18</td>
<td>Very Good</td>
</tr>
<tr>
<td>Realization 4</td>
<td>3653909</td>
<td>0</td>
<td>100</td>
<td>82</td>
<td>83</td>
<td>14.34</td>
<td>0.17</td>
<td>Very Good</td>
</tr>
<tr>
<td>Realization 5</td>
<td>3653909</td>
<td>0</td>
<td>100</td>
<td>81</td>
<td>83</td>
<td>14.51</td>
<td>0.18</td>
<td>Very Good</td>
</tr>
</tbody>
</table>

*Descriptions are based average value of RMR

### Table 4-4: Summary Statistics for Entire RMR Sample Data and RMR Sample Data Intercepted with Block Models

<table>
<thead>
<tr>
<th>Data</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire RMR Sample Data</td>
<td>65176</td>
<td>12</td>
<td>100</td>
<td>78</td>
<td>81</td>
<td>16.25</td>
<td>0.21</td>
<td>Good</td>
</tr>
<tr>
<td>RMR Sample Data Intercepted</td>
<td>50142</td>
<td>12</td>
<td>100</td>
<td>80</td>
<td>82</td>
<td>15.19</td>
<td>0.19</td>
<td>Good</td>
</tr>
</tbody>
</table>

*Descriptions are based on average value of RMR
Figure 4-15: Histogram of RMR developed in the direct approach (realization 1)

Figure 4-16: Histogram of RMR developed in the direct approach (realization 2)

Figure 4-17: Histogram of RMR developed in the direct approach (realization 3)

Figure 4-18: Histogram of RMR developed in the direct approach (realization 4)
Figure 4-19: Histogram of RMR developed in the direct approach (realization 5)

Figure 4-20: Histogram of entire RMR sample data

Figure 4-21: Histogram of RMR sample data intercepted with block model
Figure 4-22: Q-Q plot between RMR in developed block models using direct approach and RMR sample data intercepted with block models.

Figure 4-23: Q-Q plot between RMR in developed block models using direct approach and entire RMR sample data.
4.2 Indirect Approach

This section provides an explanation of the steps taken in the indirect approach towards developing the 3D block models of RMR. Since the procedure to develop 3D block models of RMR is through developing 3D block models of primary variables forming the RMR including: RQD, fracture frequency, UCS, and joint condition, the approach is called indirect approach. Unlike the direct approach, the block models of RMR were created by combining the block models of RMR constitutive parameters RQD, fracture frequency, UCS, and joint condition while assuming the dry groundwater condition. In the indirect approach, five realizations of each parameter were first developed using the SGS method in Maptek Vulcan 9.0.0 software (Maptek 2014) and the resulting block models were then combined using the ratings mentioned in RMR76 table (Table 2-2). Like the direct approach, each resulting block model has 4, 112, 948 blocks of 10 m × 10 m ×14 m. The sample data of each parameter available in the Paul’s Peak geomechanical borehole dataset were used as inputs to the SGS engine in order to characterize the heterogeneity of each parameter in forms of block models. The general steps taken in the indirect approach are as follows:

- Apply principal component analysis (PCA) and normalize the selected principal components (PCs)
- Partition the domain of study into subdomains according to lithology and relative location;
- Variography of normal scores of the selected principal components for each subdomain;
- Run SGS on normal scores of PCs within each subdomain and then back-transform to original space of parameters to have the block models of geomechanical attributes constituting RMR;
- Rate and combine the created block models of the geomechanical attributes in order to create the block models of RMR;
- Validate the resulting 3D heterogeneity models (block models) of RMR and its constitutive parameters.

The rest of this section provide a detailed elaboration of the steps taken in the indirect approach.

4.2.1 Principal component analysis and normalization

The first step in the indirect approach is to apply the principal component analysis (PCA) on the values of sample data for the constitutive parameters: RQD, UCS, fracture frequency, and joint condition. PCA is a variable reduction method in which the number of variables are reduced into a set of decorrelated principal components (Deutsch 2013; Demsar et al. 2013). PCA has been
applied for the geostatistical applications in previous research such as Cash and Breen (1992), Sánchez-Martos et al. (2001), Satyaji Rao et al. (2010), Liu et al. (2011), de Assis Silva and de Souza Lima (2012), Ou et al. (2012), and Demsar et al. (2013). For the mathematical formulation of PCA, refer to Demsar et al. (2013).

In the indirect approach, PCA is applied in order to have a reduced set of independent variables so called principal components for the curse of faster geostatistical modeling. Since the geomechanical parameters of interest are correlated (refer to the correlation matrix in Table 3-7), they cannot be modeled using the SGS method independently. This inter-correlation requires to apply the methods such as co-simulation that are cumbersome. To avoid the complexity of co-simulation and for faster geostatistical modeling, PCA was applied in the indirect approach. To apply PCA, the values of parameters were initially normalized in order to have a similar scale. To do so, each parameter was transformed into normal scores using Gaussian transformation. The resulting normal scores are symbolized as N_RQD, N_UCS, N_FF, and N_JC for respectively RQD, UCS, fracture frequency, and joint condition. Table 4-5 shows the range of values for the normal scores of parameters. As seen, the resulting normal scores have a similar range about -4.5 to 4.5. To check if the inter-correlation exists for the normal scores, the correlation matrix is presented in Table 4-6. The Table shows that, there exists a high correlation between normal scores of RQD and fracture frequency. The correlation coefficient between other pairs of parameters are also significant that means they need to be decorrelated.

Table 4-7 shows how each PC explains the total variation in the whole data of normal scores. The first principal component (PC1) covers 62% of the total variation of data. This is 22%, 11%, and 5% for the remaining principal components. The first two principal components altogether cover 84% of the whole variation in the normal score data. These two components were selected and used for the rest of geostatistical simulation and modeling. Therefore, the total number of variables for the geostatistical simulation was reduced from four to two. To check the validity of PCA, correlation matrix for the resulting PCs is shown in Table 4-8. According to this table, the resulting PCs are decorrelated. Table 4-9 presents the eigenvectors of PCA and the linear
transformation function which relates each normal score of parameters to the resulting PCs. This table also shows the linear relationships between the PCs and the geomechanical normal scores. This table will be used in the back-transformation from the simulated block models of PCs to the normal scores.

**Table 4-6: Correlation Matrix of Normal Scores of Parameters**

<table>
<thead>
<tr>
<th>Normal Score Variable</th>
<th>Normal Score Variable</th>
<th>Absolute Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N_RQD</td>
<td>N_FF</td>
</tr>
<tr>
<td>N_RQD</td>
<td>1.00</td>
<td>-0.81</td>
</tr>
<tr>
<td>N_FF</td>
<td>-0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>N_JC</td>
<td>0.45</td>
<td>-0.37</td>
</tr>
<tr>
<td>N_UCS</td>
<td>0.41</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

**Table 4-7: Variance Explained By Each PC**

<table>
<thead>
<tr>
<th>Principal Component Number</th>
<th>Variance (%)</th>
<th>Cumulative Variance (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PC1</td>
<td>62</td>
<td>62</td>
</tr>
<tr>
<td>PC2</td>
<td>22</td>
<td>84</td>
</tr>
<tr>
<td>PC3</td>
<td>11</td>
<td>95</td>
</tr>
<tr>
<td>PC4</td>
<td>5</td>
<td>100</td>
</tr>
</tbody>
</table>

**Table 4-8: Correlation Matrix of Principal Components**

<table>
<thead>
<tr>
<th>Correlation Matrix of Principal Components</th>
<th></th>
<th></th>
<th>Absolute Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PC1</td>
<td>PC2</td>
<td>PC3</td>
</tr>
<tr>
<td>PC1</td>
<td>1.00</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>PC2</td>
<td>0.01</td>
<td>1.00</td>
<td>0.01</td>
</tr>
<tr>
<td>PC3</td>
<td>0.02</td>
<td>0.01</td>
<td>1.00</td>
</tr>
<tr>
<td>PC4</td>
<td>-0.02</td>
<td>0.01</td>
<td>-0.00</td>
</tr>
</tbody>
</table>

**Table 4-9: Eigenvectors of PCA and Linear Transformation Equations**

<table>
<thead>
<tr>
<th>Eigenvectors</th>
<th>Linear Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PC1</td>
</tr>
<tr>
<td>N_RQD</td>
<td>0.55</td>
</tr>
<tr>
<td>N_FF</td>
<td>-0.52</td>
</tr>
<tr>
<td>N_JC</td>
<td>0.47</td>
</tr>
<tr>
<td>N_UCS</td>
<td>0.45</td>
</tr>
</tbody>
</table>

N_RQD = 0.55 × PC1 -0.40 × PC2 -0.02 × PC3 + 0.73 × PC4
N_FF = -0.52 × PC1 + 0.52 × PC2 -0.05 × PC3 + 0.68 × PC4
N_JC = 0.47 × PC1 +0.49 × PC2 +0.73 × PC3 - 0.07 × PC4
N_UCS = 0.45 × PC1 +0.58 × PC2 -0.68 × PC3 -0.05 × PC4
After implementing the PCA method, the first two principal components PC1 and PC2 were selected for the further geostatistical modeling. These components were then normalized in order to be compatible with the SGS approach. The Gaussian transformation method was applied to values of PC1 and PC2. The resulting normal scores are N_PC1 and N_PC2. The geostatistical operations such as variography were performed on these normal scores.

4.2.2 Partition the domain for N_PC1 and N_PC2

Like the direct approach, the entire domain of study was partitioned into six subdomains according to the lithology group and relative location within the Paul’s Peak pit. The lithology groups are the same as what was considered in the direct approach as follows:

- Group I: Iron Formation (rock code: 103);
- Group II: Amphibolite (rock code: 102), Gneiss (rock code: 107), and Felsic Gneiss (rock code: 108);
- Group III: Quartzite rock with 0-40% of mica (rock code: 104), Mica schist (60-100% of mica) (rock code: 105), and Quartz mica schist (40-60% of micas) (rock code: 106).

The whole data of N_PC1 and N_PC2 were then subset according to the lithology groups. In addition, each subset of data was further divided according to the relative locations: Upper Limb and Lower Limb. Thus, six subdomains were defined and further geostatistical analysis and modeling were done on each of these six subdomains. In the indirect approach, the normal scores of principal components, N_PC1 and N_PC2 were clustered into six groups as mentioned above and all other geostatistical operations were performed on these six subdomains.

4.2.3 Variography on subdomain’s N_PC1 and N_PC2

Variography was carried out on the data of N_PC1 and N_PC2 for each subdomain. Experimental variograms including the downhole variogram and directional variograms were calculated for each variable N_PC1 and N_PC2 within each subdomain. The nugget effect required for the variogram modeling was captured through the downhole variograms. Figures A-19 to A-24 in Appendix show the downhole variograms calculated for N_PC1 in each subdomain. According to these figures, the nugget effect for N_PC1 in the Lower Limb area is basically less than the Upper Limb area. Figures A-25 to A-30 in Appendix display the downhole variograms for N_PC2 in each subdomain. Basically, the nugget effect in the Lower Limb is less
than Upper Limb for N_PC2. In addition, N_PC1 has the less nugget effect than N_PC2. Tables 4-10 and 4-11 summarize the nugget effect values for N_PC1 and N_PC2, respectively.

| Table 4-10: Nugget Effect for Each Subdomain's N_PC1 |
|----------------------------------|------------------|------------------|--------------------|
| Limb                            | Lower Limb       | Upper Limb       |
| Lithology Group                 | I    | II   | III  | I    | II   | III  |
| Nugget Effect                   | 0.20 | 0.23 | 0.20 | 0.25 | 0.35 | 0.25 |

| Table 4-11: Nugget Effect for Each Subdomain's N_PC2 |
|----------------------------------|------------------|------------------|--------------------|
| Limb                            | Lower Limb       | Upper Limb       |
| Lithology Group                 | I    | II   | III  | I    | II   | III  |
| Nugget Effect                   | 0.35 | 0.40 | 0.30 | 0.35 | 0.50 | 0.40 |

To find the principal directions of continuity for each component normal score within each subdomain, the experimental variograms in different directions in 3D space were calculated. The directional experimental variograms indicate that the direction of maximum continuity for N_PC1 is 80º in Lower Limb and 110º in Upper Limb areas for all lithology groups. Unlike N_PC1, the direction of maximum continuity for N_PC2 ranges from 80º to 100º for both Lower and Upper Limbs. In addition, the direction of minimum continuity is found be vertical for both N_PC1 and N_PC2 in all subdomains. Tables 4-12 and 4-13 present the parameters of variogram structures used for modeling the variogram of N_PC1 and N_PC2 in each subdomain. Two variograms structures, both spherical, were mostly used for the two variables and all subdomains. In some cases, only one variogram model was used for modeling the spatial variability of the variables. For example, variogram of N_PC1 in the subdomain Upper Limb and lithology group I has been modeled only with one variogram structures. As another example, only one variogram structure was used for modeling of the spatial variability of N_PC2 in Upper Limb. The variogram models fit to the principal directions of continuity for each variable (N_PC1, N_PC2) in each subdomain are presented in the appendix (Figures A-31 o A-42).
<table>
<thead>
<tr>
<th>Lithology Group</th>
<th>Major Direction of Continuity</th>
<th>Semi-Major Direction of Continuity</th>
<th>Minor Direction of Continuity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Azimuth: 80º Plunge: 0º</td>
<td>Azimuth: 170º Plunge: 0º</td>
<td>Azimuth: 45º Plunge: -90º</td>
</tr>
<tr>
<td></td>
<td>Azimuth: 80º Plunge: 0º</td>
<td>Azimuth: 170º Plunge: 0º</td>
<td>Azimuth: 45º Plunge: -90º</td>
</tr>
<tr>
<td></td>
<td>Azimuth: 80º Plunge: 0º</td>
<td>Azimuth: 170º Plunge: 0º</td>
<td>Azimuth: 45º Plunge: -90º</td>
</tr>
<tr>
<td></td>
<td>Azimuth: 110º Plunge: 0º</td>
<td>Azimuth: 20º Plunge: 0º</td>
<td>Azimuth: 45º Plunge: -90º</td>
</tr>
<tr>
<td>Nugget</td>
<td>0.20</td>
<td>0.23</td>
<td>0.20</td>
</tr>
<tr>
<td>Range Major (m)</td>
<td>200</td>
<td>250</td>
<td>130</td>
</tr>
<tr>
<td>Range Semi-major (m)</td>
<td>300</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Range Minor (m)</td>
<td>40</td>
<td>83</td>
<td>23</td>
</tr>
</tbody>
</table>

### Table 4-13: Parameters of Variogram Models for Each Subdomain's N_PC2

<table>
<thead>
<tr>
<th>Lithology Group</th>
<th>Major Direction of Continuity</th>
<th>Semi-Major Direction of Continuity</th>
<th>Minor Direction of Continuity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Azimuth: 90º Plunge: 0º</td>
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<tr>
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<td>Azimuth: 0º Plunge: 0º</td>
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<td>0.30</td>
</tr>
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<td>Range Major (m)</td>
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<td>218</td>
</tr>
<tr>
<td>Range Semi-major (m)</td>
<td>163</td>
<td>44</td>
<td>161</td>
</tr>
<tr>
<td>Range Minor (m)</td>
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<td>26</td>
<td>77</td>
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### Variogram structure 1

<table>
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<th>Spherical</th>
<th>Spherical</th>
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<tbody>
<tr>
<td>Sill difference</td>
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<td>0.35</td>
<td>0.70</td>
<td>0.65</td>
<td>0.50</td>
<td>0.60</td>
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<td>Range Major (m)</td>
<td>179</td>
<td>159</td>
<td>218</td>
<td>264</td>
<td>560</td>
<td>244</td>
</tr>
<tr>
<td>Range Semi-major (m)</td>
<td>163</td>
<td>44</td>
<td>161</td>
<td>141</td>
<td>159</td>
<td>133</td>
</tr>
<tr>
<td>Range Minor (m)</td>
<td>26</td>
<td>26</td>
<td>77</td>
<td>105</td>
<td>116</td>
<td>21</td>
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</table>

### Variogram structure 2

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<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
<th>Spherical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sill difference</td>
<td>0.25</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Range Major (m)</td>
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<td>1747</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Range Semi-major (m)</td>
<td>972</td>
<td>418</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Range Minor (m)</td>
<td>260</td>
<td>124</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
4.2.4 Develop 3D block models for RQD, UCS, fracture frequency, and joint condition

The sequential Gaussian simulation technique was applied to create 3D block models of the regionalized variables RQD, fracture frequency, joint condition, and UCS. To do so, five realizations/simulations for N_PC1 and N_PC2, per subdomain, were produced using this technique. The blocks dimension is 14 m (height) by 10 m (width) by 10 m (length).

After generating the realizations of N_PC1 and N_PC2, they were back-transformed into respectively PC1 and PC2. The results would be five realizations of PC1 and PC2. Then, the resulting realizations of PC1 and PC2 were back-transformed into the normal scores space N_RQD, N_UCS, N_JC, and N_FF using the back-transformation linear functions developed in PCA (Table 4-9). Since only the two first major principal components were selected and simulated, the contributions of the third and fourth principal components (PC3 and PC4) were ignored in the equations provided in Table 4-9. Therefore, following equations were applied for the back-transformation purpose:

\[
\begin{align*}
N_{\text{RQD}} &= 0.55 \times \text{PC}_1 - 0.40 \times \text{PC}_2 \\
N_{\text{FF}} &= -0.52 \times \text{PC}_1 + 0.52 \times \text{PC}_2 \\
N_{\text{JC}} &= 0.47 \times \text{PC}_1 + 0.49 \times \text{PC}_2 \\
N_{\text{UCS}} &= 0.45 \times \text{PC}_1 + 0.58 \times \text{PC}_2
\end{align*}
\]  

(4.1)

To achieve the five realizations of the regionalized geomechanical variables, the resulting realizations of N_RQD, N_UCS, N_JC, and N_FF were back-transformed to the original space. Each resulting a 3D block model that shows the heterogeneity of the corresponding geomechanical variables in the domain of study at Paul’s Peak mine.

4.2.4.1 Block models of RQD

Figures 4-24 to 4-28 depict five cross section views of the RQD block models developed in the indirect approach. These figures indicate that the majority of rock mass formation in the Paul’s Peak mine have high rock mass quality in terms of RQD. These high quality rocks are marked in red in the cross section views. Besides, the majority of rocks nearby the south and north walls of Paul’s Peak 2013 walls are of low quality and low RQD that indicates the rock mass in these areas is highly fractured compared with the other areas of the pit. Hence, mining of these areas should be done with caution as there is a chance of instability in these low quality rocks.
Figure 4-24: Cross section view (looking east) of RQD (realization 1) developed in indirect approach

Figure 4-25: Cross section view (looking east) of RQD (realization 2) developed in indirect approach
Figure 4-26: Cross section view (looking east) of RQD (realization 3) developed in indirect approach

Figure 4-27: Cross section view (looking east) of RQD (realization 4) developed in indirect approach

Figure 4-28: Cross section view (looking east) of RQD (realization 5) developed in indirect approach
4.2.4.2 Block models of fracture frequency

Figures 4-29 to 4-33 display five cross section views of the developed block models of fracture frequency in the indirect approach. These figures indicate that the majority of rock mass formation in Paul’s Peak is simulated to be of fracture frequency in the range 1-20 m$^{-1}$ (yellows and greens). In addition, there is a high concentration of high fracture frequency (reds) in the north wall of the current Paul’s Peak pit (pit 2013). This confirms the RQD block models where a low RQD concentration can be seen in the northern areas of the pit. In addition, it is observed that the fracture frequency decreases with the depth (from red to blue blocks).
Figure 4-29: Cross section view (looking east) of fracture frequency (realization 1) developed in indirect approach

Figure 4-30: Cross section view (looking east) of fracture frequency (realization 2) developed in indirect approach
Figure 4-31: Cross section view (looking east) of fracture frequency (realization 3) developed in indirect approach

Figure 4-32: Cross section view (looking east) of fracture frequency (realization 4) developed in indirect approach
4.2.4.3 Block models of joint condition

Figures 4-34 to 4-38 depict five cross section views of the developed block models of joint condition in the indirect approach. These figures indicate that the majority of rock mass formation in Paul’s Peak is simulated to be of having joint condition 20 (slightly rough surfaces, separation <1 mm, and hard joint wall contact). The joint condition in the areas of the pit with low RQD (blue areas in Figures 4-24 to 4-28) and high fracture frequency (red areas in Figures 4-29 to 4-33) has also shown lower joint condition values. This indicates a relatively strong correlation between the RQD and joint condition in the models which follows the relationship observed in the field data (see Table 3-7).
Figure 4-34: Cross section view (looking east) of joint condition (realization 1) developed in indirect approach

Figure 4-35: Cross section view (looking east) of joint condition (realization 2) developed in indirect approach

Figure 4-36: Cross section view (looking east) of joint condition (realization 3) developed in indirect approach
4.2.4.4 Block models of UCS

Figures 4-39 to 4-43 display five cross section views of the developed block models of UCS in the indirect approach. These figures indicate that the majority of rock mass formation in Paul’s Peak pit is simulated to be of high (UCS: 100-200 MPa) and very high (UCS: >200 MPa) intact rock strength. In addition, rocks nearby the current Paul’s Peak south and north walls in the shallower areas are formed by low strength intact rocks.
Figure 4-39: Cross section view (looking east) of UCS (realization 1) developed in indirect approach (the legend values are in MPa)

Figure 4-40: Cross section view (looking east) of UCS (realization 2) developed in indirect approach (the legend values are in MPa)
Figure 4-41: Cross section view (looking east) of UCS (realization 3) developed in indirect approach (the legend values are in MPa)

Figure 4-42: Cross section view (looking east) of UCS (realization 4) developed in indirect approach (the legend values are in MPa)
4.2.5 Development of 3D block models for RMR

After developing 3D block models of the primary geomechanical properties RQD, fracture frequency, UCS, and joint condition, they were combined in order to generate 3D block models for RMR. The ratings in the RMR\textsubscript{76} (Table 2-2) were applied to weight the values of constitutive parameters in their corresponding block models assuming the dry water condition for the pit (a rating of 10 was set for water condition). Figures 4-44 to 4-48 show the cross section views of the resulting five realizations of the RMR block model. As seen, majority of rocks have a very good quality indicated in red. The rocks in the shallower parts near the current Paul’s Peak pit in north wall are also simulated as having lower quality which is compatible with the developed RQD, fracture frequency and UCS block models. In addition, the rock around the south wall of the current Paul’s Peak 2013 is of lower quality than the other areas according to the five cross section views of the RMR block models. This indicates that there is a chance of instability in the south wall as a big failure happened in 1986 in the south wall.
Figure 4-44: Cross section view (looking east) of RMR (realization 1) developed in indirect approach

Figure 4-45: Cross section view (looking east) of RMR (realization 2) developed in indirect approach

Figure 4-46: Cross section view (looking east) of RMR (realization 3) developed in indirect approach
Figures 4-49 to 4-53 demonstrate isometric views of the block models of RMR created in the indirect approach projected on the Paul’s Peak pit 2045 walls. These figures show that the rocks in the south wall of the final Paul’s Peak pit are primarily of lower quality than the other walls. Such observation was seen in the block models of RMR in the direct approach as well. Mining of the rocks in the south wall must be done cautiously due to the lower quality of rocks.

Figures 4-54 and 4-55 show the probability map of having respectively RMR greater than 40 and 60. Based on these figures, majority of rocks in the Paul’s Peak pit have a high probability more than 0.8 to have RMR greater than 40 and 60. However, in the northern part of the current pit in the shallower area this probability decreases which indicates the high probability of lower quality rock mass for those areas. It should be noted that since the produced probability maps are based
on only five realizations of RMR, they should be used with caution. These probability maps are presented in this thesis solely to give an idea about how the uncertainty in geomechanical block modeling can be presented and quantified.

Figure 4-49: RMR (realization 1) developed in indirect approach mapped on Paul’s Peak 2045

Figure 4-50: RMR (realization 2) developed in indirect approach mapped on Paul’s Peak 2045
Figure 4-51: RMR (realization 3) developed in indirect approach mapped on Paul’s Peak 2045

Figure 4-52: RMR (realization 4) developed in indirect approach mapped on Paul’s Peak 2045
Figure 4-53: RMR (realization 5) developed in indirect approach mapped on Paul’s Peak 2045
4.2.6 Verification of Indirect Approach

In this section, the block models for the geomechanical properties developed using the indirect approach were verified using the statistical methods. As mentioned in Chapter 2, SGS produces a number of equi-probable realizations of the reality where the covariance/correlation structure is honored. Table 4-14 shows the correlation matrix for the geomechanical properties in the five realizations developed in the indirect approach. For ease in comparison, each range of absolute correlation was marked with a specific color in these tables. Comparing this correlation matrix with the correlation matrix for the corresponding parameters in the geomechanical borehole dataset (Table 3-7) reveals that the produced block models of geomechanical parameters has kept the correlation structure of the original sample data. This can be insisted through comparing the
correlation matrix of parameters in the block models with the correlation matrix of geomechanical parameters sample data intercepted with the block model (excluding the geomechanical data of the mined out pit from the total geomechanical data collected) (Table 4-15). The comparison indicates that the developed block models of geomechanical properties approximately honor the correlation structure of the sample data. For example, there is a high negative correlation coefficient (~0.90) between simulated RQD and fracture frequency in the developed block models. The correlation coefficient values for such pair of parameters in the entire geomechanical borehole sample dataset and sample data inside the block models are -0.82. Another instance is the correlation coefficient between RMR and UCS in block models developed in the indirect approach is about 0.51 which is close to the corresponding correlation coefficient values in Table 3-7 (0.59) and Table 4-15 (0.52).
### Table 4-14: Correlation Matrix for Geomechanical Parameters Simulated Using Indirect Approach

<table>
<thead>
<tr>
<th>Realization ID</th>
<th>Geotechnical Property</th>
<th>RQD (%)</th>
<th>Fracture Frequency (m⁻¹)</th>
<th>UCS (MPa)</th>
<th>Joint Condition</th>
<th>RMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.91</td>
<td>0.32</td>
<td>0.52</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.91</td>
<td>1.00</td>
<td>-0.22</td>
<td>-0.39</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>0.32</td>
<td>-0.22</td>
<td>1.00</td>
<td>0.66</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Joint Condition</td>
<td>0.52</td>
<td>-0.38</td>
<td>0.64</td>
<td>1.00</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>RMR</td>
<td>0.92</td>
<td>-0.83</td>
<td>0.51</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.91</td>
<td>0.32</td>
<td>0.51</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.91</td>
<td>1.00</td>
<td>-0.21</td>
<td>-0.39</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>0.32</td>
<td>-0.21</td>
<td>1.00</td>
<td>0.66</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Joint Condition</td>
<td>0.51</td>
<td>-0.39</td>
<td>0.65</td>
<td>1.00</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>RMR</td>
<td>0.92</td>
<td>-0.83</td>
<td>0.50</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.90</td>
<td>0.31</td>
<td>0.52</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.90</td>
<td>1.00</td>
<td>-0.23</td>
<td>-0.39</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>0.31</td>
<td>-0.23</td>
<td>1.00</td>
<td>0.65</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Joint Condition</td>
<td>0.52</td>
<td>-0.39</td>
<td>0.65</td>
<td>1.00</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>RMR</td>
<td>0.92</td>
<td>-0.83</td>
<td>0.50</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.90</td>
<td>0.31</td>
<td>0.52</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.90</td>
<td>1.00</td>
<td>-0.22</td>
<td>-0.39</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>0.31</td>
<td>-0.22</td>
<td>1.00</td>
<td>0.67</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Joint Condition</td>
<td>0.52</td>
<td>-0.38</td>
<td>0.66</td>
<td>1.00</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>RMR</td>
<td>0.90</td>
<td>-0.83</td>
<td>0.51</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.91</td>
<td>0.32</td>
<td>0.51</td>
<td>0.91</td>
</tr>
<tr>
<td></td>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.91</td>
<td>1.00</td>
<td>-0.22</td>
<td>-0.35</td>
<td>-0.82</td>
</tr>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>0.32</td>
<td>-0.22</td>
<td>1.00</td>
<td>0.66</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>Joint Condition</td>
<td>0.51</td>
<td>-0.35</td>
<td>0.63</td>
<td>1.00</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>RMR</td>
<td>0.91</td>
<td>-0.82</td>
<td>0.52</td>
<td>0.81</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Absolute Correlation**

- 0.00-0.40
- 0.40-0.60
- 0.60-0.80
- 0.80-0.99
- 1.00
### Table 4-15: Correlation Matrix for Geomechanical Parameters Sample Data intercepted with Block Model

<table>
<thead>
<tr>
<th>Geotechnical Property</th>
<th>RQD (%)</th>
<th>Fracture Frequency (m⁻¹)</th>
<th>UCS (MPa)</th>
<th>Joint Condition</th>
<th>RMR</th>
<th>Absolute Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.82</td>
<td>0.36</td>
<td>0.58</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Fracture Frequency (m⁻¹)</td>
<td>-0.82</td>
<td>1.00</td>
<td>-0.28</td>
<td>-0.50</td>
<td>-0.82</td>
<td></td>
</tr>
<tr>
<td>UCS (MPa)</td>
<td>0.36</td>
<td>-0.28</td>
<td>1.00</td>
<td>0.56</td>
<td>0.51</td>
<td></td>
</tr>
<tr>
<td>Joint Condition</td>
<td>0.58</td>
<td>-0.50</td>
<td>0.56</td>
<td>1.00</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>RMR</td>
<td>0.90</td>
<td>-0.82</td>
<td>0.51</td>
<td>0.79</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

#### 4.2.6.1 Verification of RQD block models

To verify the created block models of RQD, statistical techniques: summary statistics tables, histograms, and Q-Q plots were employed. Table 4-16 presents the simple statistics of RQD in the block models developed in indirect approach. According to this table, values in the RQD block models ranges from the minimum of zero to 100% with the average of 87% and standard deviation about 21%. The quality of rock mass in the block models is on average interpreted as good in terms of RQD (refer to Table 2-4). Comparing these statistics with the simple statistics of the entire RQD sample data and RQD sample data intercepted with the block models (Table 4-17) indicates that indirect approach has on average overestimated RQD values than the sample data as the average of RQD in block models is higher than the sample data. However, the block models of RQD have performed well in prediction of rock quality on average since the rock masses were categorized as good similar to the sample data.

Figures 4-56 to 4-60 show the histogram of RQD for the developed block models in the indirect approach. These histograms indicate that the distribution of RQD in the block models is negatively skewed where about 55% of the RQD data in the block models are greater than 90%. Comparison between these histograms and the histograms of entire RQD sample data from boreholes (Figure 4-61) and RQD sample data inside the block models (excluding the mined out zone) (Figure 4-62) shows that the indirect approach has reproduced histograms in terms of shape and skewness. However, only about 35% and 40% of respectively the entire RQD sample data and RQD sample data inside the block models are more than 90%. This would reflect that
the indirect approach has provided block models of RQD with a tendency to produce bigger values (overestimation). The degree of overestimation can further be investigated using the Q-Q plots. Figure 4-63 shows the Q-Q plot between the RQD distribution in the five block models and the RQD distribution of field sample data intercepted with the block models. As seen, the produced block models overestimate RQD with quantiles bigger than the quantiles of RQD sample data inside the block model. This overestimation is high where the quantiles of RQD sample data are within 20% and 60%. The degree of overestimation is worse when the Q-Q plot is drawn between RQD block models and the entire RQD sample data (Figure 4-64). The statistical methods used for the verification of RQD block models indicate that the indirect approach has not been able to successfully honor the statistics, especially histogram, of RQD sample data.

<table>
<thead>
<tr>
<th>RQD (Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>88</td>
<td>97</td>
<td>20.36</td>
<td>0.23</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>87</td>
<td>97</td>
<td>21.76</td>
<td>0.25</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>87</td>
<td>97</td>
<td>21.46</td>
<td>0.25</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>87</td>
<td>96</td>
<td>21.76</td>
<td>0.25</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>87</td>
<td>97</td>
<td>21.54</td>
<td>0.25</td>
<td>Good</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean

<table>
<thead>
<tr>
<th>Data</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire RQD Sample Data</td>
<td>65796</td>
<td>0</td>
<td>100</td>
<td>80</td>
<td>90</td>
<td>25.06</td>
<td>0.31</td>
<td>Good</td>
</tr>
<tr>
<td>RQD Sample Data Intercepted with block Model</td>
<td>50746</td>
<td>0</td>
<td>100</td>
<td>83</td>
<td>90</td>
<td>24.00</td>
<td>0.29</td>
<td>Good</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-56: Histogram of RQD in block models developed in indirect approach (realization 1)

Figure 4-57: Histogram of RQD in block models developed in indirect approach (realization 2)

Figure 4-58: Histogram of RQD in block models developed in indirect approach (realization 3)

Figure 4-59: Histogram of RQD in block models developed in indirect approach (realization 4)
Figure 4-60: Histogram of RQD in block models developed in indirect approach (realization 5)

Figure 4-61: Histogram of entire RQD sample data

Figure 4-62: Histogram of RQD sample data intercepted with block model
Quantiles of RQD Realizations-Indirect Approach

Quantiles of RQD Sample Data Intercepted with Block Model

Figure 4-63: Q-Q plot between RQD in block models and RQD sample data intercepted with block model

Quantiles of Entire RQD Sample Data

Figure 4-64: Q-Q plot between RQD in block models and entire RQD sample data
4.2.6.2 Verification of fracture frequency block models

The validity of the created block models of fracture frequency was investigated through statistical techniques such as summary statistics tables, histograms, and Q-Q plots. Table 4-18 summarizes the simple statistics of the fracture frequency parameter in the block models developed in the indirect approach. This table shows that the values of fracture frequency in the block models vary from zero to 65 m\(^{-1}\) with the average of 3.60-3.93 m\(^{-1}\) and standard deviation about 6.0 m\(^{-1}\). According to this table, the rock mass forming the Paul’s Peak pit is on average interpreted as rocks with moderately spaced joints (refer to Table 2-5). To verify the simple statistics of the fracture frequency block models, Table 4-19 provides the simple statistics for the entire fracture frequency sample data and fracture frequency sample data inside the block models. Comparing the two tables 4-18 and 4-19 indicates that the indirect approach has produced block models in which the values of the fracture frequency are underestimated. This can be observed as the mean, median, and standard deviation of the fracture frequency data in the block models are lower than the corresponding statistics for the sample data. In addition, the entire sample data of fracture frequency provide an average interpretation of closely spaced jointed rock mass in the Paul’s Peak pit which is different than the block models (moderately spaced). The underestimation of fracture frequency presented in the generated block models would give an overestimation of the overall quality of rock masses.

Figures 4-65 to 4-69 display the histograms of fracture frequency for the produced block models. To check if the indirect approach could reproduce the histogram of fracture frequency in the sample data, Figures 4-70 and 4-71 present respectively the histogram of entire fracture frequency sample data and fracture frequency sample data intercepted with the block models. Histograms of fracture frequency in block models are similar to the sample data histograms in terms of shape and skewness. However, there is a large difference in the frequency of data in the first bin of histograms (0-4 m\(^{-1}\)) between the block models’ histograms and sample data histograms. About 70% of rocks have been populated with the fracture frequency values in the range of 0-4 m\(^{-1}\) while about 50%-55% of the sample data are in this range. This indicates that the fracture frequency has been highly underestimated in the produced block models. For further investigation of how strong was the indirect approach in histogram reproduction, Q-Q plots between the fracture frequency block models and the sample data are presented in Figures 4-72.
The Q-Q plot between the distribution of fracture frequency in the block models and the fracture frequency sample data inside the block models indicates that the distribution of fracture frequency in block models is highly different than such distribution for the sample data inside the block models. Such conclusion can also be seen in Figure 4-73 where the quantiles of fracture frequency in block models are less than corresponding quantiles for the entire fracture frequency sample data. Therefore, it seems that the indirect approach has poorly performed in developing the fracture frequency block models.

Table 4-18: Summary Statistics of Fracture Frequency in the Block Models Developed in Indirect Approach

<table>
<thead>
<tr>
<th>Fracture Frequency (Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>3.93</td>
<td>2.0</td>
<td>5.78</td>
<td>1.47</td>
<td>Moderately Spaced</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>3.89</td>
<td>2.0</td>
<td>5.86</td>
<td>1.50</td>
<td>Moderately Spaced</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>3.63</td>
<td>2.0</td>
<td>6.14</td>
<td>1.69</td>
<td>Moderately Spaced</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>3.72</td>
<td>2.0</td>
<td>6.21</td>
<td>1.67</td>
<td>Moderately Spaced</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>3.60</td>
<td>2.0</td>
<td>6.07</td>
<td>1.69</td>
<td>Moderately Spaced</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean

Table 4-19: Summary Statistics of Fracture Frequency Sample Data

<table>
<thead>
<tr>
<th>Data</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire Fracture Frequency Sample Data</td>
<td>65794</td>
<td>0.0</td>
<td>65.0</td>
<td>5.3</td>
<td>4.0</td>
<td>6.78</td>
<td>1.29</td>
<td>Closely Spaced</td>
</tr>
<tr>
<td>Fracture Frequency Sample Data Intercepted with block Model</td>
<td>50744</td>
<td>0.0</td>
<td>65.0</td>
<td>4.83</td>
<td>3.0</td>
<td>6.74</td>
<td>1.40</td>
<td>Closely Spaced</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-65: Histogram of fracture frequency in block models developed in indirect approach (realization 1)

Figure 4-66: Histogram of fracture frequency in block models developed in indirect approach (realization 2)

Figure 4-67: Histogram of fracture frequency in block models developed in indirect approach (realization 3)

Figure 4-68: Histogram of fracture frequency in block models developed in indirect approach (realization 4)
Figure 4-69: Histogram of fracture frequency in block models developed in indirect approach (realization 5)

Figure 4-70: Histogram of entire fracture frequency sample data

Figure 4-71: Histogram of fracture frequency sample data intercepted with block model
Figure 4-72: Q-Q plot between fracture frequency in block models and fracture frequency sample data intercepted with block models.

Figure 4-73: Q-Q plot between fracture frequency in block models and entire fracture frequency sample data.
4.2.6.3 Verification of UCS block models

Like other parameters, the block models of UCS developed in the indirect approach were investigated using the statistical methods. Table 4-20 presents a summary of basic statistics for UCS in the developed block models. According to this table, UCS data in the block models are in the range 5-380 MPa, with the average about 200 MPa, and standard deviation about 81 MPa. The intact rock strength in the Paul’s Peak pit was also interpreted as very high/high (refer to Table 2-3) according to the mean of UCS values in the block models. Table 4-21 provides the summary statistics of the entire UCS sample data and UCS sample data inside the block models. Comparison between Tables 4-20 and 4-21 shows that the indirect approach has been able to honor the basic statistics of UCS sample data. For example, the average, median, and range of the UCS data in the block models are close to the sample data (both entire data and those inside the block models). In addition, the intact rock has on average been interpreted as very strong in the block models as the sample data.

Figures 4-74 to 4-78 show the histograms for the UCS data in the block models produced using the indirect approach. These histograms are closely similar to the UCS sample data (both entire sample data and sample data inside the block models) shown in Figures 4-79 and 4-80 in terms of shape and frequency of bins. This indicates that the indirect approach has performed well in reproduction of UCS sample data histograms.

Figures 4-81 and 4-82 present the Q-Q plot for the UCS distribution in block models and UCS sample data. As seen, the Q-Q plot is concentrated around the line of 45° which indicates that the produced histograms of UCS models are similar to the histograms of UCS sample data.

<table>
<thead>
<tr>
<th>UCS (Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>200</td>
<td>193</td>
<td>80.48</td>
<td>0.40</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>200</td>
<td>193</td>
<td>81.18</td>
<td>0.41</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>200</td>
<td>193</td>
<td>81.87</td>
<td>0.41</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>197</td>
<td>193</td>
<td>81.15</td>
<td>0.41</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>197</td>
<td>193</td>
<td>81.92</td>
<td>0.42</td>
<td>Very High</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
### Table 4-21: Summary Statistics of UCS Sample Data

<table>
<thead>
<tr>
<th>Data</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire UCS Sample Data</td>
<td>65176</td>
<td>5</td>
<td>380</td>
<td>201</td>
<td>193</td>
<td>89.39</td>
<td>0.44</td>
<td>Very High</td>
</tr>
<tr>
<td>UCS Sample Data Intercepted with block Model</td>
<td>50142</td>
<td>5</td>
<td>380</td>
<td>202</td>
<td>198</td>
<td>88.83</td>
<td>0.44</td>
<td>Very High</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-78: Histogram of UCS in block models developed in indirect approach (realization 5)

Figure 4-79: Histogram of entire UCS sample data

Figure 4-80: Histogram of UCS sample data intercepted with block model
Figure 4-81: Q-Q plot between UCS in block models and UCS sample data intercepted with block models.

Figure 4-82: Q-Q plot between UCS in block models and entire UCS sample data.
4.2.6.4 Verification of joint condition block models

Table 4-22 provides a summary of basic statistics for the joint condition data of the five produced block models in the indirect approach. This table indicates that the majority of blocks are populated by the joint condition value of 20 (slightly rough surfaces, separation <1 mm, hard joint wall contact; refer to Table 2-6). This observation is matched with the sample data statistics as shown in Table 4-23. In both tables, the range, mean, mode, and median values are close to each other. As joint condition is a discrete variable, mode is the better statistic for comparison than the mean. In both tables, the value of mode is 20 which shows that the block models honor the basic statistics of joint condition sample data.

Figures 4-83 to 4-87 show the histogram of joint condition data in the five block models developed using the indirect approach. These histograms are similar to the histograms of sample data, both entire joint condition sample data (Figure 4-88) and joint condition sample data inside the block models (Figure 4-89) in terms of shape and skewness. In the block models, about 80% of blocks are populated with joint condition 20 while such frequency is about 68% and 73% for respectively entire sample data and sample data inside the block models. This gap could imply that histograms of joint conditions have been poorly reproduced in the block modeling using the indirect approach.

Figures 4-90 and 4-91 compare the distribution of joint condition data in block models with the distribution of joint condition sample data in terms of the Q-Q plots. These plots cannot be used for the comparison because of the discrete nature of joint condition variable.

<table>
<thead>
<tr>
<th>Joint Condition (Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean/Mode</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>18/20</td>
<td>20</td>
<td>4.66</td>
<td>0.26</td>
<td>Slightly rough surfaces, separation &lt;1 mm, hard joint wall contact (based on mode value)</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>18.0/20</td>
<td>20</td>
<td>4.78</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>18/20</td>
<td>20</td>
<td>4.86</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>18/20</td>
<td>20</td>
<td>4.94</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>18/20</td>
<td>20</td>
<td>4.99</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>No. of Data</td>
<td>Min.</td>
<td>Max.</td>
<td>Mean/Mode</td>
<td>Median</td>
<td>STD</td>
<td>CV</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>-------------</td>
<td>------</td>
<td>------</td>
<td>-----------</td>
<td>--------</td>
<td>------</td>
<td>------</td>
<td>-------------------------------------------------</td>
</tr>
<tr>
<td>Entire Joint Condition Sample Data</td>
<td>65176</td>
<td>0</td>
<td>25</td>
<td>17/20</td>
<td>20</td>
<td>5.41</td>
<td>0.31</td>
<td>Slightly rough surfaces, separation &lt;1 mm, hard joint wall contact (based on mode value)</td>
</tr>
<tr>
<td>Joint Condition Sample Data Intercepted with block Model</td>
<td>50142</td>
<td>0</td>
<td>25</td>
<td>18/20</td>
<td>20</td>
<td>4.85</td>
<td>0.27</td>
<td>-</td>
</tr>
</tbody>
</table>
Figure 4-83: Histogram of joint condition in block models developed in indirect approach (realization 1)

Figure 4-84: Histogram of joint condition in block models developed in indirect approach (realization 2)

Figure 4-85: Histogram of joint condition in block models in indirect approach (realization 3)

Figure 4-86: Histogram of joint condition in block models developed in indirect approach (realization 4)
Figure 4-87: Histogram of joint condition in block models developed in indirect approach (realization 5)

Figure 4-88: Histogram of entire joint condition sample data

Figure 4-89: Histogram of joint condition sample data intercepted with block model
Quantiles of Joint Condition Realizations - Indirect Approach

Figure 4-90: Q-Q plot between joint condition in block models and joint condition sample data intercepted with block models

Quantiles of Entire Joint Condition Sample Data

Figure 4-91: Q-Q plot between joint condition in block models and entire joint condition sample data
4.2.6.5 Verification of RMR block models

Table 4-24 gives a summary statistics for the RMR data of the five simulated RMR block models developed in indirect approach. According to this table, the simulated RMR block models have data within the range 2 to 98, mean 82, median 87, and standard deviations around 13. The rock mass in the Paul’s Peak pit was also simulated to have very good quality (refer to Table 2-2 section C) in terms of RMR in the five block models. Table 4-25 presents the summary statistics for the whole RMR sample data and also those samples inside the block models. Comparison between these tables indicates that the simulated RMR data has a wider range, bigger mean, larger median, and smaller standard deviations than the sampled data. The difference between these statistics of block models and sample data can be considered negligible regarding the scale of RMR. For example, only a difference 82-78=4 exists between mean of RMR in block models and mean of entire RMR sample data. In addition, the simulated block models of RMR could provide a well prediction of average rock quality as RMR sample data interpreted with block models is on average in a very good quality class. It should be noted that although the rock mass quality is interpreted on average as good according to sample RMR data inside the block models, 80 is the border of two classes good and very good quality (Table 2-2 section C). Therefore, the simulated RMR block model could honor the simple statistics of the sample data. The block models of RMR developed in the indirect approach show also similar statistics to the RMR data in the block models developed in the direct approach (Table 4-4).

Figures 4-92 to 4-96 show the histograms of RMR data in the block models developed in the indirect approach. These histograms are similar to histograms of the entire RMR sample data (Figure 4-97) and RMR sample data inside the block models (Figure 4-98) in terms of shape and skewness. Histograms of RMR in block models are especially similar to corresponding histogram for the RMR sample data inside the block models as the order of frequency of bins are hold. For example, bins 80-90, 90-100, and 70-80 are of the highest frequency of data in both RMR block models and RMR sample data inside the block models. This order is different for the entire RMR sample data 80-90, 70-80, and 90-100. Despite the similarity in the histograms, there is a high concentration of RMR within 80-90 for the block models (about 60%) which is highly different than the sample data histograms which is less than 35%. This might indicate that the indirect approach could not reproduce the histogram of RMR for especially higher values of
RMR. To further investigate the RMR distributions, Q-Q plots between RMR data of block models and sample data are presented in Figures 4-99 and 4-100. These Q-Q plots show an overestimation in the simulation and a discrepancy between the produced RMR histograms and sample data histograms.

The results of verification for the indirect approach reveals the fact that this indirect approach could not provide block models of geomechanical properties in which the statistics of the sample data are completely honored. Therefore, it is required to investigate the reasons that have caused such discrepancy in the simulation using the indirect approach. In the next section, the indirect approach is adjusted in order to provide better and more realistic block models of geomechanical properties.

<table>
<thead>
<tr>
<th>Table 4-24: Summary Statistics of RMR in the Block Models Developed in Indirect Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RMR (Indirect Approach)</strong></td>
</tr>
<tr>
<td>Realization 1</td>
</tr>
<tr>
<td>Realization 2</td>
</tr>
<tr>
<td>Realization 3</td>
</tr>
<tr>
<td>Realization 4</td>
</tr>
<tr>
<td>Realization 5</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean

<table>
<thead>
<tr>
<th>Table 4-25: Summary Statistics of RMR Sample Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
</tr>
<tr>
<td>Entire RMR Sample Data</td>
</tr>
<tr>
<td>RMR Sample Data Intercepted with block Model</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-92: Histogram of RMR in block models developed in indirect approach (realization 1)

Figure 4-93: Histogram of RMR in block models developed in indirect approach (realization 2)

Figure 4-94: Histogram of RMR in block models developed in indirect approach (realization 3)

Figure 4-95: Histogram of RMR in block models developed in indirect approach (realization 4)
Figure 4-96: Histogram of RMR in block models developed in indirect approach (realization 5)

Figure 4-97: Histogram of entire RMR sample data

Figure 4-98: Histogram of RMR sample data intercepted with block model
Quantiles of RMR Realizations-Indirect Approach

Quantiles of RMR Sample Data Intercepted with Block Model

Figure 4-99: Q-Q plot between RMR in block models and RMR sample data intercepted with block models

Realization 1
Realization 2
Realization 3
Realization 4
Realization 5
Line of 45 Degrees

Quantiles of Entire RMR Sample Data

Figure 4-100: Q-Q plot between RMR in block models and entire RMR sample data

Realization 1
Realization 2
Realization 3
Realization 4
Realization 5
Line of 45 Degrees
4.2.7 Adjusted Indirect Approach

Verification of the indirect approach showed that this approach could not perform well in repeating the histogram of some geomechanical variables such as fracture frequency, RQD, and RMR. Therefore, the author conducted a thorough investigation on why the indirect approach failed in reproduction of histogram and how to improve the results of simulation. The diagnostic investigation demonstrated that the average values of simulated N_PC1 and N_PC2 for some subdomains were drastically different than their sampled data (normal scores of PC1 and PC2 in the sample data), despite the fact that their histograms were highly similar in terms of shape.

One way to adjust the indirect method to reproduce the histograms is to shift the histogram of the resulting simulated variables (Deutsch 2013). Therefore, the histograms of resulting simulated N_PC1 and N_PC2 for each subdomain was shifted to be closer to the corresponding histograms for the sample data of N_PC1 and N_PC2. Table 4-26 summarizes the average values of N_PC1 and N_PC2 per subdomain for the sample data and the five realizations. The reds values highlight what histograms must be translated towards the sample data. For example, the average of N_PC1 in the sample data for the Lower Limb and lithology group I is -0.08. The average value of N_PC1 in the first realization for the same subdomain is 0.51. There is 0.59 difference between the two average values. Thus, all simulated values of N_PC1 in this subdomain were subtracted a constant value (here 0.49) to have a histogram very close to the corresponding sample data. To keep the variation in the resulting simulation data, values of all realization for a subdomain were subtracted by a constant values. For example, all realizations of N_PC1 in the subdomain (Lower Limb and lithology group I) were deducted by 0.49. Table 4-27 shows the average values of adjusted histograms for N_PC1 and N_PC2 per subdomain. As seen, the average values for the simulated normal scores are now closer to the sample data.
Table 4-26: Average Values of $N_{PC1}$ and $N_{PC2}$ in Sample Data and Realizations in Indirect Approach Per Subdomain

<table>
<thead>
<tr>
<th>Principal Component</th>
<th>Limb</th>
<th>Lithology Group</th>
<th>Sample Data</th>
<th>Realization ID</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$N_{PC1}$</td>
<td></td>
<td></td>
<td>I</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>Lower Limb</td>
<td></td>
<td>I</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II</td>
<td>0.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>Upper Limb</td>
<td></td>
<td>I</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II</td>
<td>-0.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III</td>
<td>0.26</td>
</tr>
<tr>
<td>$N_{PC2}$</td>
<td></td>
<td></td>
<td>I</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>II</td>
<td>-0.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>III</td>
<td>0.15</td>
</tr>
</tbody>
</table>

After applying this adjustment, all other steps such as back-transformation were done exactly as in the indirect approach. In the remaining parts of this section, the results of adjusted indirect approach block modeling are presented.

4.2.7.1 Block models of RQD in Adjusted Indirect Approach

Figures 4-101 to 4-105 display five cross section views of the block models of RQD developed based on the adjusted indirect approach. These figures indicate that the majority of blocks are simulated with the high RQD greater than 90% (reds in the figures). Besides, rocks nearby the current Paul’s Peak south wall and also in the northern part of the pit are mostly simulated as poorer quality marked with blues and yellows.
Figure 4-101: Cross section view (looking east) of RQD (realization 1) developed in adjusted indirect approach

Figure 4-102: Cross section view (looking east) of RQD (realization 2) developed in adjusted indirect approach
Figure 4-103: Cross section view (looking east) of RQD (realization 3) developed in adjusted indirect approach

Figure 4-104: Cross section view (looking east) of RQD (realization 4) developed in adjusted indirect approach
4.2.7.2 Block models of fracture frequency in Adjusted Indirect Approach

Figures 4-106 to 4-110 present five cross section views of the fracture frequency block models in the adjusted indirect approach. In all of these cross sections, the rocks close to the south wall and also northern part inside the current Paul’s Peak have been more or less simulated as very large fracture frequency (reds). Comparison between the cross section views of fracture frequency block models developed in the adjusted indirect approach with the corresponding block model developed using the indirect approach (Figures 4-29 to 4-33) reveals that the rocks located in lower east of the sections have been simulated more fractured in the adjusted indirect approach than the indirect approach. This can imply that the adjusted indirect approach has produced block models in which rocks are simulated on average more fractured.
Figure 4-106: Cross section view (looking east) of fracture frequency (realization 1) developed in adjusted indirect approach

Figure 4-107: Cross section view (looking east) of fracture frequency (realization 2) developed in adjusted indirect approach
Figure 4-108: Cross section view (looking east) of fracture frequency (realization 3) developed in adjusted indirect approach

Figure 4-109: Cross section view (looking east) of fracture frequency (realization 4) developed in adjusted indirect approach
4.2.7.3 Block models of joint condition in Adjusted Indirect Approach

Figures 4-111 to 4-115 display five cross section views of the developed block models of joint condition in the adjusted indirect approach. These figures indicate that the majority of rock mass formation in the Paul’s Peak pit was simulated to be of having joint condition 20 (Slightly rough surfaces, separation <1 mm, and hard joint wall contact). In addition, rocks nearby the current Paul’s Peak south wall and in the northern part part of the pit are having poorer joint condition where the RQD is low and fracture frequency is high.
Figure 4-111: Cross section view (looking east) of joint condition (realization 1) developed in adjusted indirect approach

Figure 4-112: Cross section view (looking east) of joint condition (realization 2) developed in adjusted indirect approach
Figure 4-113: Cross section view (looking east) of joint condition (realization 3) developed in adjusted indirect approach

Figure 4-114: Cross section view (looking east) of joint condition (realization 4) developed in adjusted indirect approach
4.2.7.4 Block models of UCS in Adjusted Indirect Approach

Figures 4-116 to 4-120 display five cross section views of the developed block models of UCS in the adjusted indirect approach. These figures indicate that the majority of rock mass formation in Paul’s Peak was simulated to be of high (UCS: 100-200 MPa) and very high (UCS: >200 MPa) intact rock strength. In addition, rocks nearby the current Paul’s Peak south wall in the shallow areas are formed by low strength intact rocks. Therefore, the areas in northern part of Peal’s Peak is simulated to have lower intact rock strength and high fractured rock masses. This would imply that those areas should have poor quality rock mass in terms of RMR.
Figure 4-116: Cross section view (looking east) of UCS (realization 1) developed in adjusted indirect approach (the legend values are in MPa)

Figure 4-117: Cross section view (looking east) of UCS (realization 2) developed in adjusted indirect approach (the legend values are in MPa)
Figure 4-118: Cross section view (looking east) of UCS (realization 3) developed in adjusted indirect approach (the legend values are in MPa)

Figure 4-119: Cross section view (looking east) of UCS (realization 4) developed in adjusted indirect approach (the legend values are in MPa)
4.2.7.5 Block models of RMR in Adjusted Indirect Approach

Figures 4-121 to 4-125 show five cross section views of the resulting five realizations of the RMR block model developed in the adjusted indirect approach. As it can be seen, majority of rocks have a good to very good rock mass quality. The rock masses in the shallower parts near the current Paul’s Peak north wall are also simulated as having lower quality which is compatible with the developed RQD, fracture frequency and UCS block models. In addition, low RMR in the area of south wall indicates that this wall is susceptible to instability. As mentioned before, a big failure happened in 1986 in the south wall of the Paul’s Peak pit. Hence, low RMR in south wall of Paul’s Peak and historical instability could confirm the high chance of instability.

Comparing with the corresponding cross section views of the RMR block models developed in the direct approach (Figures 4-3 to 4-7) and indirect approach (Figures 4-44 to 4-48), the RMR block models in the adjusted indirect approach have more rocks simulated with good quality (greens). This indicate that the adjusted indirect approach has some blocks with good quality while they were simulated as very good quality in the direct and indirect approaches. Comparing the cross section views of RMR block models with the cross section view of drillholes (Figure 4-126) indicates that the built RMR block models have followed the RMR sample data.
The isometric views of the RMR block models developed in the adjusted indirect approach are demonstrated in Figures 4-127 to 4-131. In these figures, the Paul’s Peak pit walls are colored according to their RMR values. These figures show that the rocks in the south wall of the final Paul’s Peak pit is primarily of lower quality which conform to the majority of historical failures happened in the south wall of the pit in the past.

Figures 4-132 and 4-133 show the probability maps of having respectively RMR greater than 40 and 60. Based on these figures, majority of rocks in the Paul’s Peak pit have a high probability, more than 0.8 to have RMR greater than 40 and 60. However, in the north part of the current pit 2013, in the shallower area, this probability decreases which indicates the high probability of lower quality in terms of RMR for those areas. It should be noted that since the produced probability maps are based on only five realizations of RMR, they should be used with caution.
Figure 4-121: Cross section view (looking east) of RMR (realization 1) developed in adjusted indirect approach

Figure 4-122: Cross section view (looking east) of RMR (realization 2) developed in adjusted indirect approach
Figure 4-123: Cross section view (looking east) of RMR (realization 3) developed in adjusted indirect approach

Figure 4-124: Cross section view (looking east) of RMR (realization 4) developed in adjusted indirect approach
Figure 4-125: Cross section view (looking east) of RMR (realization 5) developed in adjusted indirect approach

Figure 4-126: A north-south cross section of the Paul’s Peak pit 2045 with drillholes colored by RMR values (section easting 3500±100 in BLA coordinate system)
Figure 4-127: RMR (realization 1) developed in adjusted indirect approach mapped on Paul’s Peak 2045

Figure 4-128: RMR (realization 2) developed in adjusted indirect approach mapped on Paul’s Peak 2045
Figure 4-129: RMR (realization 3) developed in adjusted indirect approach mapped on Paul’s Peak 2045

Figure 4-130: RMR (realization 4) developed in adjusted indirect approach mapped on Paul’s Peak 2045
Figure 4-131: RMR (realization 5) developed in adjusted indirect approach mapped on Paul’s Peak 2045
4.2.7.6 Verification of Adjusted Indirect Approach

In this section, the block models for the geomechanical properties developed using the adjusted indirect approach were investigated using the statistical methods. The first verification check was to investigate if the produced block models of the geomechanical parameters could honor the correlation structure between the parameters. Table 4-28 shows the correlation matrix for the geomechanical properties in the five realizations developed in the adjusted indirect approach. For
ease in comparison, each range of absolute correlation was marked with a specific color in these tables. Comparing this correlation matrix with the correlation matrix of the corresponding parameters in the geomechanical borehole dataset (Table 3-7) reveals that the produced block models of geomechanical parameters have honored the correlation structure of the original sample data. This can be emphasized through comparing the correlation matrix of parameters in the block models with the correlation matrix of geomechanical parameters sample data intercepted with the block model (Table 4-15). For example, there is a high negative correlation coefficient (~-0.85) between simulated RQD and fracture frequency in the block models developed using the adjusted indirect approach. The correlation coefficient values for such pair of parameters in the entire geomechanical borehole sample dataset and sample data inside the block models are -0.82. In another instance, the correlation coefficient between RMR and UCS in block models developed in the adjusted indirect approach is about 0.51 which is close to the corresponding correlation coefficient values in Table 3-7 (0.59) and Table 4-15 (0.52).

In the adjusted indirect approach, the correlation coefficients for each pair of parameters have also been approached to the sample data correlation matrix relative to the indirect approach. For example, the correlation coefficient between RQD and RMR is 0.90 which is same as the corresponding correlation coefficient in the sample data inside the block models than the indirect approach (0.92). Another example is the correlation coefficient between UCS and joint condition (about 0.55) in the adjusted indirect approach which is much closer to the correlation coefficient in the sample data inside the block models (0.56) than the indirect approach (0.67). In addition, the correlation matrix of the adjusted indirect approach is similar to the sample data correlation matrix in terms color legend which confirms that the correlation structure is honored. Therefore, the adjusted indirect approach has shown improvement relative to the indirect approach in keeping the correlation structure between parameters.
Table 4-28: Correlation Matrix for Geomechanical Parameters Simulated Using Adjusted Indirect Approach

<table>
<thead>
<tr>
<th>Realization ID</th>
<th>Geomechanical Property</th>
<th>RQD (%)</th>
<th>Fracture Frequency (m(^{-1}))</th>
<th>UCS (MPa)</th>
<th>Joint Condition</th>
<th>RMR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RQD (%)</td>
<td>1.00</td>
<td>-0.85</td>
<td>0.33</td>
<td>0.56</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>Fracture Frequency (m(^{-1}))</td>
<td>-0.85</td>
<td>1.00</td>
<td>-0.30</td>
<td>-0.51</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>UCS (MPa)</td>
<td>0.33</td>
<td>-0.30</td>
<td>1.00</td>
<td>0.54</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>Joint Condition</td>
<td>0.56</td>
<td>-0.51</td>
<td>0.54</td>
<td>1.00</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>RMR</td>
<td>0.90</td>
<td>-0.83</td>
<td>0.51</td>
<td>0.78</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Absolute Correlation

- 0.00-0.40
- 0.40-0.60
- 0.60-0.80
- 0.80-0.99
- 1.0
4.2.7.6.1 Verification of RQD block models

To verify the created block models of RQD, statistical techniques: summary statistics tables, histograms, and Q-Q plots were employed. Table 4-29 summarizes the simple statistics of the RQD parameter in the block models developed in the adjusted indirect approach. According to this table, values of RQD in the block models ranges from the minimum of zero to 100% with the average of 83-84% and standard deviation about 24%. The quality of rock mass in the block models is on average interpreted as good in terms of RQD (refer to Table 2-4). Comparing these statistics with the simple statistics of the entire RQD sample data and RQD sample data intercepted with the block models (Table 4-17) indicates that the adjusted indirect approach has produced marginally overestimated block models of RQD than the sample data as the average of RQD in block models is slightly higher than the sample data. However, the block models of RQD have performed well in prediction of rock quality on average since the rocks are categorized as good similar to the sample data. Comparing the simple statistics of RQD block models in the adjusted indirect approach with the indirect approach shows that the adjusted indirect approach performs better. The average and standard deviation of RQD in the block models developed in the adjusted indirect approach are closer to the corresponding statistics of sample data (both entire and those inside the block models) than the produced block models in the indirect approach. Therefore, adjustment of the simulated N_PC1 and N_PC2 histograms in the adjusted indirect approach has improved the quality (in terms of simple statistics) of sequential Gaussian simulation for generation of RQD block models.

Figures 4-134 to 4-138 show the histograms of RQD in the RQD block models developed in the adjusted indirect approach. These histograms indicate that the distribution of RQD in the block models is negatively skewed where about 42% of the RQD data in the block models are greater than 90%. Comparison between these histograms and the histograms of entire RQD sample data (Figure 4-139) and RQD sample data inside the block models (Figure 4-140) shows that the adjusted indirect approach has been successfully able to reproduce histograms in terms of shape, skewness, and bins’ frequency. As can be seen in these figures, the produced RQD block models’ histograms are closely parallel to the histograms of sample data, especially the sample data inside the block models. For example, about 15%, 5%, 18%, and 43% of blocks in the adjusted indirect approach are having RQD values within the respective ranges of 80%-85%,
85%-90%, 90%-95%, and 95%-100%. These frequencies are about 17%, 3%, 16%, and 34% for entire RQD sample data and 13%, 4%, 18%, and 41% for RQD sample data inside the block models. The degree of similarity between the histograms of block models developed in the adjusted indirect approach and sample data has been improved. In the indirect approach about 55% of blocks were simulated to have RQD of 90%-100% (about 15-20% difference from the corresponding bin’s frequency in the histograms of sample data). However, such difference is about 1-5% in the adjusted indirect approach.

Figure 4-141 shows the Q-Q plot between the distribution of RQD data in the five block models using adjusted indirect model and the RQD sample data inside the block models. The produced block models slightly underestimate the quantile less than 60% and slightly overestimate the quantiles greater than 60%. This is unlike the indirect approach in which the quantiles are all highly overestimated. Comparing with indirect approach, the Q-Q plot of Figure 4-141 is a significant improvement. This improvement can be identified as Q-Q plot of Figure 4-141 changes tightly around the line of 45°. The Q-Q plot of indirect approach (Figure 4-63) shows substantially higher variations around the line of 45°. Such improvement is also observed from comparison of Q-Q plot of Figure 4-142 and Q-Q plot of Figure 4-64. Therefore, verification of RQD block models developed in adjusted indirect approach reveals drastic improvement over the indirect approach.

<table>
<thead>
<tr>
<th>RQD (Adjusted Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>84</td>
<td>93</td>
<td>24.48</td>
<td>0.29</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>84</td>
<td>93</td>
<td>24.87</td>
<td>0.30</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>83</td>
<td>93</td>
<td>25.51</td>
<td>0.31</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>83</td>
<td>92</td>
<td>25.81</td>
<td>0.31</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>0</td>
<td>100</td>
<td>83</td>
<td>93</td>
<td>25.60</td>
<td>0.31</td>
<td>Good</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean

Table 4-29: Summary Statistics of RQD in the Block Models Developed in Adjusted Indirect Approach
Figure 4-134: Histogram of RQD in block models developed in adjusted indirect approach (realization 1)

Figure 4-135: Histogram of RQD in block models developed in adjusted indirect approach (realization 2)

Figure 4-136: Histogram of RQD in block models developed in adjusted indirect approach (realization 3)

Figure 4-137: Histogram of RQD in block models developed in adjusted indirect approach (realization 4)
Figure 4-138: Histogram of RQD in block models developed in adjusted indirect approach (realization 5)

Figure 4-139: Histogram of entire RQD sample data

Figure 4-140: Histogram of RQD sample data intercepted with block model
Figure 4-141: Q-Q plot between RQD in block models and RQD sample data intercepted with block models (after adjustment)

Figure 4-142: Q-Q plot between RQD in block models and entire RQD sample data
4.2.7.6.2 Verification of fracture frequency block models

To verify the created block models of fracture frequency, statistical techniques including summary statistics tables, histograms, and Q-Q plots were employed. Table 4-30 summarizes the simple statistics of the fracture frequency parameter in the block models developed in the adjusted indirect approach. According to this table, values of fracture frequency in the block models ranges from the minimum zero to 65 m\(^{-1}\) with the average about 5 m\(^{-1}\) and standard deviation about 7.8 m\(^{-1}\). The rock mass in the block models is on average interpreted as closely spaced jointed in terms of joint spacing (refer to Table 2-5). Comparing these statistics with the simple statistics of the entire fracture frequency sample data and fracture frequency sample data inside the block models (Table 4-19) indicates that the adjusted indirect approach has on average produced block models with similar statistics to statistics of the sample data. In addition, the block models of fracture frequency have on average performed well in prediction of rock quality since the rocks are categorized into close spaced jointed rock mass in block models and the sample data. Comparing the simple statistics of fracture frequency block models in the adjusted indirect approach with the indirect approach shows that the adjusted indirect approach performs better. The average of fracture frequency in the block models developed in the adjusted indirect approach are closer to the sample data mean (both entire and those inside the block models) than the produced block models in the indirect approach.

Figures 4-143 to 4-147 show the histogram of fracture frequency in the block models developed in the adjusted indirect approach. These histograms indicate that the distribution of fracture frequency in the block models is positively skewed where about 51% of the fracture frequency data in the block models are less than 4 m\(^{-1}\). Comparison between these histograms and the histograms of entire fracture frequency sample data (Figure 4-148) and fracture frequency sample data inside the block models (Figure 4-149) shows that the adjusted indirect approach has been successfully able to reproduce histograms in terms of shape, skewness, and bins’ frequency. The produced fracture frequency block models’ histograms are closely parallel to the histograms of sample entire, especially the sample data inside the block models. For example, about 50%, 28%, and 7%, of blocks in the adjusted indirect approach are having fracture frequency values within the respectively ranges of 0-4 m\(^{-1}\), 4-8 m\(^{-1}\), and 8-12 m\(^{-1}\). These frequencies are about 48%, 33%, and 8% for entire fracture frequency sample data and 55%, 33%, and 5% for fracture
frequency sample data inside the block models. The degree of similarity between the histograms of block models developed in the adjusted indirect approach and sample data has been improved. In the indirect approach about 70% of blocks are simulated to have fracture frequency of 0-4 m$^{-1}$ (about 15-20% difference from the corresponding bin’s frequency in the histograms of sample data). However, such difference is about 2-5% in the adjusted indirect approach.

Figure 4-150 shows the Q-Q plot between the fracture frequency data in the five block models using adjusted indirect method and the fracture frequency sample data inside the block models. The produced block models slightly underestimate the quantile less than 20 m$^{-1}$ and slightly overestimate the quantiles thereafter. This is unlike the indirect approach in which the quantiles are all highly underestimated. Comparing with indirect approach, the Q-Q plot of Figure 4-150 is an improvement. This improvement can be identified as Q-Q plot of Figure 4-150 changes around the line of 45°. The Q-Q plot of indirect approach in Figure 4-72 shows substantially higher variations around the line of 45°. Such improvement is also observed from comparison of Q-Q plot of Figure 4-151 and Q-Q plot of Figure 4-75. Therefore, verification of fracture frequency block models developed in the adjusted indirect approach reveals intensive improvement over the indirect approach.

### Table 4-30: Summary Statistics of Fracture Frequency in the Block Models Developed in Adjusted Indirect Approach

<table>
<thead>
<tr>
<th>Fracture Frequency (Adjusted Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>4.9</td>
<td>3.0</td>
<td>7.45</td>
<td>1.52</td>
<td>Closely Spaced</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>4.96</td>
<td>3.0</td>
<td>7.55</td>
<td>1.52</td>
<td>Closely Spaced</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>5.03</td>
<td>3.0</td>
<td>7.83</td>
<td>1.55</td>
<td>Closely Spaced</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>5.12</td>
<td>0.3</td>
<td>7.92</td>
<td>1.55</td>
<td>Closely Spaced</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>0.0</td>
<td>65.0</td>
<td>4.72</td>
<td>3.0</td>
<td>7.77</td>
<td>1.65</td>
<td>Closely Spaced</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-143: Histogram of fracture frequency in block models developed in adjusted indirect approach (realization 1)

Figure 4-144: Histogram of fracture frequency in block models developed in adjusted indirect approach (realization 2)

Figure 4-145: Histogram of fracture frequency in block models developed in adjusted indirect approach (realization 3)

Figure 4-146: Histogram of fracture frequency in block models developed in adjusted indirect approach (realization 4)
Figure 4-147: Histogram of fracture frequency in block models developed in adjusted indirect approach (realization 5)

Figure 4-148: Histogram of entire fracture frequency sample data

Figure 4-149: Histogram of fracture frequency sample data intercepted with block model
Quantiles of Fracture Frequency Realizations—Adjusted Indirect Approach

Figure 4-150: Q-Q plot between fracture frequency in block models and fracture frequency sample data intercepted with block models (after adjustment)

Quantiles of Entire Fracture Frequency Sample Data

Figure 4-151: Q-Q plot between fracture frequency in block models and entire fracture frequency sample data
4.2.7.6.3 Verification of UCS block models

Table 4-31 presents a summary of basic statistics for UCS in the developed block models using adjusted indirect approach. According to this table, UCS data in the block models are in the range 5-380 MPa, with the average of 200-203 MPa, and standard deviation about 80 MPa. The intact rock strength in the Paul’s Peak pit are also interpreted as very high (refer to Table 2-3) according to the mean of UCS values in the block models. Comparison between the simple statistics of UCS block models and the simple statistics of UCS sample data (Table 4-21) shows that the adjusted indirect approach has been able to honor the basic statistics of UCS sample data. For example, the average, median, and range of the UCS data in the block models are close to the sample data (both entire data and those inside the block models). In addition, the intact rock strength has on average been interpreted as very high in the block models as the sample data. The adjusted indirect approach has performed similar to the indirect approach for the UCS block models in terms of simple statistics. The similarity can be identified as they have provided UCS block models with similar basic statistics. Therefore, the adjusted indirect approach has not noticeably improved the quality of UCS block modeling.

Figures 4-152 to 4-156 show the histograms for the UCS data in the block models produced using the adjusted indirect approach. These histograms are closely similar to the UCS sample data (both entire sample data and sample data inside the block models) shown in Figures 4-157 and 4-158 in terms of shape and frequency of bins. This indicates that the indirect approach has performed well in reproduction of UCS histograms.

Figures 4-159 and 4-160 present the Q-Q plot for the UCS distribution in block models and UCS sample data. As seen, the Q-Q plot is concentrated around the line of 45° which indicates that the produced histograms of UCS are similar to the histograms of UCS sample data. The Q-Q plots in Figures 4-159 and 4-160 are similar to the Q-Q plots of indirect approach (Figures 4-81 and 4-82). Thus, the adjusted indirect approach has produced UCS block models that the histograms and distributions are similar to the indirect approach.
Table 4-31: Summary Statistics of UCS in the Block Models Developed in Adjusted Indirect Approach

<table>
<thead>
<tr>
<th>UCS (Adjusted Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>202</td>
<td>193</td>
<td>79.46</td>
<td>0.39</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>201</td>
<td>193</td>
<td>80.46</td>
<td>0.40</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>203</td>
<td>193</td>
<td>81.11</td>
<td>0.40</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>200</td>
<td>193</td>
<td>80.13</td>
<td>0.40</td>
<td>Very High</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>5</td>
<td>380</td>
<td>201</td>
<td>193</td>
<td>80.84</td>
<td>0.40</td>
<td>Very High</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-152: Histogram of UCS block models developed in adjusted indirect approach (realization 1)

Figure 4-153: Histogram of UCS in block models developed in adjusted indirect approach (realization 2)

Figure 4-154: Histogram of UCS in block models developed in adjusted indirect approach (realization 3)

Figure 4-155: Histogram of UCS in block models developed in adjusted indirect approach (realization 4)
Figure 4-156: Histogram of UCS in block models developed in adjusted indirect approach (realization 5)

Figure 4-157: Histogram of entire UCS sample data

Figure 4-158: Histogram of UCS sample data intercepted with block model
Figure 4-159: Q-Q plot between UCS in block models and UCS sample data intercepted with block models (after adjustment)

Figure 4-160: Q-Q plot between UCS in block models and entire UCS sample data
4.2.7.6.4 Verification of joint condition block models

Table 4-32 provides a summary of basic statistics for the joint condition data of the five produced block models in the adjusted indirect approach. This table indicates that the majority of blocks are populated by the joint condition value of 20 (slightly rough surfaces, separation <1 mm, hard joint wall contact; refer to Table 2-6). This observation is matched with the sample data statistics as shown in Table 4-23. The statistics range, mean, mode, and median values are close to corresponding statistics of the sample data. As joint condition is a discrete variable, mode is the better statistics for comparison than the mean. The block models of joint condition developed using the adjusted indirect approach, all have the mode of 20 which is same as the sample data’s mode for the joint condition. Thus, the join condition block models have honored the basic statistics of the sample data used for the simulation. In other words, the adjusted indirect approach has successfully produced block models of joint condition that repeat the sample data statistics. Comparison of the basic statistics for the joint condition block models in the adjusted indirect approach with such models developed in the indirect approach (Table 4-22) indicates that the adjusted indirect approach has worked similar to the indirect approach in terms of basic statistics. Thus, a significant improvement in the statistics is not observed in the adjusted indirect approach than the indirect approach.

Figures 4-161 to 4-165 show the histogram of joint condition data in the five block models developed using the adjusted indirect approach. These histograms are similar to the histograms of sample data, both entire joint condition sample data (Figure 4-166) and joint condition sample data inside the block models (Figure 4-167) in terms of shape, skewness, and bins’ frequency. Figures 4-161 to 4-165 show that about 2%, 1%, 15%, and 78% of blocks are populated with joint condition respectively 0, 6, 12, and 20. The corresponding frequencies are 4%, 3%, 21%, and 68% for entire joint condition sample data and 2%, 2%, 17% and 74% for the sample data inside the block models. As seen, the difference between the bins’ frequencies for the block models and sample data are not significant. This negligible difference indicates that the adjusted indirect approach has been able to successfully produce the join condition block models which honor the sample data histograms. Comparison of histograms for the joint condition block models in the adjusted indirect approach with ones in the indirect approach reveals a negligible
difference in terms of bins’ frequencies (3%, 2%, 15%, and 80% of blocks in the indirect approach are respectively 0, 6, 12, and 20).

Figures 4-168 and 4-169 present the Q-Q plots for the joint condition block models of adjusted indirect approach and respectively entire joint condition sample data and joint condition sample data inside the block models. As seen, the Q-Q plots are around the line of 45º which indicates the suitability of adjusted indirect approach in joint condition block modeling. Figures 4-168 and 4-169 look similar to such Q-Q plots of indirect approach. Hence, a significant difference between the adjusted indirect approach and indirect approach is not identifiable for the joint condition block modeling.

Table 4-32: Summary Statistics of Joint Condition in the Block Models Developed in Adjusted Indirect Approach

<table>
<thead>
<tr>
<th>Joint Condition (Adjusted Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean/Mode</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>18/20</td>
<td>20</td>
<td>5.10</td>
<td>0.29</td>
<td>Slightly rough surfaces, separation &lt;1 mm, hard joint wall contact (based on mode value)</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>17/20</td>
<td>20</td>
<td>5.22</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>17/20</td>
<td>20</td>
<td>5.28</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>17/20</td>
<td>20</td>
<td>5.36</td>
<td>0.31</td>
<td></td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>0</td>
<td>25</td>
<td>17/20</td>
<td>20</td>
<td>5.40</td>
<td>0.31</td>
<td></td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-165: Histogram of joint condition in block models developed in adjusted indirect approach (realization 5)

Figure 4-166: Histogram of entire joint condition sample data

Figure 4-167: Histogram of joint condition sample data intercepted with block model
Quantiles of Joint Condition Realizations - Adjusted Indirect Approach

Figure 4-168: Q-Q plot between joint condition in block models and joint condition sample data intercepted with block models

Quantiles of Joint Condition Sample Data Intercepted with Block Model

- Realization 1
- Realization 2
- Realization 3
- Realization 4
- Realization 5
- Line of 45 Degrees

Figure 4-169: Q-Q plot between joint condition in block models and entire joint condition sample data

Quantiles of Entire Joint Condition Sample Data

- Realization 1
- Realization 2
- Realization 3
- Realization 4
- Realization 5
- Line of 45 Degrees
4.2.7.6.5 Verification of RMR block models

Table 4-33 gives a summary statistics for the RMR data of the five simulated RMR block models developed in adjusted indirect approach. According to this table, the simulated RMR block models have data within the range 18 to 100, mean 79-80, median 87, and standard deviations around 16. The rock mass in Paul’s Peak is also simulated to have good quality (refer to Table 2-2 section C) in terms of average RMR in the five block models. The statistics of RMR block models developed in the adjusted indirect approach are highly similar to the statistics of the entire RMR sample data and sample data inside the block models (Table 4-25). Therefore, it can be concluded that the produced RMR block models using the adjusted indirect approach have honored the statistics of sample data. Comparison between the basic statistics of the RMR block models developed in the adjusted indirect approach with the statistics in the indirect approach shows that the adjusted indirect approach has outperformed since the average and standard deviation in the adjusted indirect approach are closer to the samples data’ average and standard deviation than the indirect approach.

Figures 4-170 to 4-174 show the histograms of RMR data in the block models developed in the adjusted indirect approach. These histograms are strongly similar to histograms of the entire RMR sample data (Figure 4-175) and RMR sample data inside the block models (Figure 4-176) in terms of shape, skewness, and bins’ frequency. For example, about 22%, 33%, 27% of the blocks in the RMR block models developed in the adjusted indirect approach are within the RMR ranges of respectively 70-80, 80-90, and 90-100. These frequencies are similar to corresponding frequencies in the entire RMR sample data histogram (respectively 25%, 32%, and 23%) and RMR sample data inside the block models (respectively 22%, 33%, and 28%). Therefore, differences in the bins’ frequencies between histograms of RMR block models in the adjusted indirect approach and histograms of sample data are negligible. This in fact indicates that the adjusted indirect approach has been able to successfully repeat the histograms of sample data used in the Sequential Gaussian simulation. Comparing the histograms of RMR block models developed in the adjusted indirect approach with ones in the indirect approach shows superiority of the adjusted indirect approach to the indirect approach in histogram reproduction. To prove this superiority, it is enough to mention that there is a significant difference between bins’ frequencies of the histogram of RMR block models produced in the indirect approach and
bins’ frequencies of sample data histograms. For example, there are about 12%, 29%, and 8% difference between frequencies of RMR in the indirect approach block models and the entire sample data for the respectively bins of 70-80, 80-90, and 90-100 RMR. Such difference are about 6%, 28%, and 13% for frequencies of RMR in the indirect approach block models and the RMR sample data inside the block models for corresponding bins.

Figures 4-177 and 4-178 present the Q-Q plots used for comparing the distributions of RMR in the block models developed in the adjusted indirect approach with the distributions for the sample data. As seen, the Q-Q plots are around the line of 45º especially for the RMR sample data inside the block models. This fact shows that the distribution of RMR in the block models are highly similar to the sample data distributions. Thus, the adjusted indirect approach has succeeded in honoring the RMR sample data histograms. In comparison, the adjusted indirect approach has provided better Q-Q plots for RMR than the indirect approach since its Q-Q plots are closer to the line of 45º. In this sense, the adjusted indirect approach is an improvement of the indirect approach.

<table>
<thead>
<tr>
<th>RMR (Adjusted Indirect Approach)</th>
<th>No. of Data</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>Median</th>
<th>STD</th>
<th>CV</th>
<th>Description*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Realization 1</td>
<td>4267468</td>
<td>18</td>
<td>100</td>
<td>80</td>
<td>87</td>
<td>15.78</td>
<td>0.20</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 2</td>
<td>4267468</td>
<td>18</td>
<td>100</td>
<td>80</td>
<td>87</td>
<td>16.08</td>
<td>0.20</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 3</td>
<td>4267468</td>
<td>18</td>
<td>100</td>
<td>79</td>
<td>87</td>
<td>16.46</td>
<td>0.21</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 4</td>
<td>4267468</td>
<td>18</td>
<td>100</td>
<td>79</td>
<td>87</td>
<td>16.51</td>
<td>0.21</td>
<td>Good</td>
</tr>
<tr>
<td>Realization 5</td>
<td>4267468</td>
<td>18</td>
<td>100</td>
<td>79</td>
<td>87</td>
<td>16.64</td>
<td>0.21</td>
<td>Good</td>
</tr>
</tbody>
</table>

*Descriptions are based on mean
Figure 4-170: Histogram of RMR in block models developed in adjusted indirect approach (realization 1)

Figure 4-171: Histogram of RMR in block models developed in adjusted indirect approach (realization 2)

Figure 4-172: Histogram of RMR in block models developed in adjusted indirect approach (realization 3)

Figure 4-173: Histogram of RMR in block models developed in adjusted indirect approach (realization 4)
Figure 4-177: Q-Q plot between RMR in block models and RMR sample data intercepted with block models.

Figure 4-178: Q-Q plot between RMR in block models and entire RMR sample data.
4.3 Selection of Block Modeling Approach

In this chapter, two SGS-based approaches were proposed and implemented for generation of heterogeneity models (block models) of geomechanical properties. In the first approach called direct approach, RMR sample data were used as inputs to SGS in order to create five realizations of RMR heterogeneity. In the second approach, indirect approach, block models of four geomechanical parameters: RQD, fracture frequency, UCS, and joint condition were separately developed and then the resulting block models were combined to provide block models of RMR. Due to the relatively poor performance of the indirect approach for reproductions of statistics of sample data, the indirect approach was then adjusted in order to present more realistic and valid block models of geomechanical properties (adjusted indirect approach). Interestingly, the weakness of rock mass in the ultimate Paul’s Peak south wall was identified in all the approaches. Figure 4-179 shows the south wall of Paul’s Peak in 2013. In this figure, the area surrounded by the red dashed line indicate the failure of rock mass historically happened. This failure and weakness of the south wall rock mass simulated in all the realizations produced in either of the three approaches, direct, indirect, and adjusted indirect approaches, confirm that the mining of the Paul’s Peak south wall must be carried out cautiously to prevent possible instabilities.

One question left is to select the best block modeling approach. In this thesis, the best block modeling approach was selected according to its performance to provide realistic simulations of the real heterogeneity of geomechanical properties. Such realistic approach would offer the block models of geomechanical parameters which honor the statistics of the sample data best. In this sense, the adjusted indirect approach is selected as the best block modeling approach for the heterogeneity modeling of geomechanical properties at Paul’s Peak Mine. Comparisons made in verification of block models of adjusted indirect approach showed the superiority of this approach over the indirect approach in terms of statistics. Apparently, the RQD, fracture frequency, and RMR block models developed in the adjusted indirect approach outperformed the ones generated in the indirect approach in terms of statistics. In comparison with the direct approach, the adjusted indirect approach shows the following advantages:

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• The adjusted indirect approach provides not only models of heterogeneity for RMR but also for its constitutive parameters (except water condition). Therefore, a better and more complete picture of the rock mass quality is obtained in the adjusted indirect approach while in the direct approach, only block models of RMR are developed.

• The adjusted indirect approach would provide better realizations of RMR heterogeneity in terms of statistics. The simple statistics of RMR block models developed in the adjusted indirect approach are closer to the sample data statistics than the direct approach. Specifically, mean and standard deviation of RMR in block models of the adjusted indirect approach are closer to the corresponding statistics of the sample data than the direct approach. In addition, the histograms for RMR block models developed in the adjusted indirect approach are much closer to the sample data histograms. There is a significant improvement in the Q-Q plots for RMR in the adjusted indirect approach over the direct approach. Therefore, the adjusted indirect approach would totally outperform the direct approach.

Figure 4-179: Paul’s Peak south wall failure
Chapter 5
Summary, Conclusions, and Future Research

In this thesis, 3D block models showing the heterogeneity of geomechanical parameters were built using the Sequential Gaussian simulation technique. This chapter provides the reader with a summary of the research, limitations, contributions, and recommendations for future research.

5.1 Summary of Research

In the present thesis, geostatistical conditional simulation technique SGS were applied to quantify the heterogeneity of geomechanical properties including RQD, fracture frequency, UCS, joint condition, and RMR for an open pit iron ore mine. Geomechanical borehole data from 1949 to 2013 were collected and used for creating block models of the geomechanical properties. The geomechanical borehole dataset was cleaned in order to be ready for the geostatistical operations. Two basic approaches were proposed for block modeling of the geomechanical data, direct and indirect approaches. In the direct approach, the RMR sample data available in the geomechanical borehole dataset were utilized in order to create 3D block models of RMR. The 3D RMR block models were then compared to the sampled data using statistical methods. In the indirect approach, 3D block models of each constitutive geomechanical parameter of RMR except groundwater condition were separately modeled using the SGS method. To do so, PCA was used to produce smaller set of uncorrelated variables for the curse of faster simulation. All geostatistical operations were made on the first two principal components. Then, block models of each geomechanical parameter were built using the created block models of principal components. Finally, resulting block models of constitutive parameters were rated according to the RMR76 table in order to provide 3D block models of RMR. Since the generated block models of geomechanical properties developed in the indirect approach did not successfully honor the sample data statistics, this approach was then adjusted. In the adjusted indirect approach, histograms of simulated data were shifted towards the histogram of the sample data so that the resulting heterogeneity models of geomechanical properties reproduce the statistics of the sample data. The verification step carried out on the adjusted indirect approach showed the superiority of this adjusted indirect approach over the indirect and direct approaches. Therefore, the results of adjusted indirect approach were recommended for assessment of the spatial variation of geomechanical properties.
5.2 Limitations of Research

Geostatistical techniques and most specifically simulation methods have been limitedly applied for modeling the heterogeneity of rock mass geomechanical properties in mines. There are some challenges and obstacles that have made the slow adoption of geostatistical techniques in the geomechanical and geotechnical projects. These challenges include lack of geomechanical data in many mining projects, subjectivity in data collection, homogeneity of data, non-additivity of geomechanical variables, direction dependency of geomechanical variables, and upscaling.

Lack of geomechanical data: Geostatistical approaches require sufficient amount of data for the statistical inference. Unlike ore grade data, which is generally closely and systematically spaced across the ore body, geomechanical information is often scattered or widely dispersed in the area of study, which can make it difficult to apply geostatistical approximation techniques properly. The amount of geomechanical data is usually short because:

- Limited budget allocated to geomechanical borehole drilling;
- Most core runs from containing ore are typically split for assay as soon as possible after drilling. This makes geomechanical logging difficult and destroys opportunities for collecting valuable geomechanical data.
- Conventional geomechanical core logging process is a very time intensive activity.

In the Paul’s Peak mine, majority of the exploration drillholes have been logged for geomechanical purposes prior to assay analysis. Therefore, adequate amount of input sample data were available for the present study for application of geostatistical techniques in spatial modeling of the geomechanical attributes.

Subjectivity: there is a significant degree of subjectivity in geotechnical data collection during the site investigation. The subjectivity of geomechanical data collection may come from two sources: subjective in definitions of the geomechanical properties and estimation of a geomechanical variable using other measured ones. Subjectivity in definitions of a geomechanical property would be identified where two loggers report two different values for a geomechanical variable of a same core run. For instance, two different core loggers could interpret the definition of the sound fresh, slightly or moderately weathered piece of rock differently in logging RQD. Use of other variables measured data in logging one geomechanical variable could make biases in the data and
consequently biased models of heterogeneity. For example, estimating values of UCS for a rock using an experimental regression between the point load strength index and the UCS is one example of such logging bias. In the Paul’s Peak data, the geomechanical database used for the geostatistical modeling have been logged by several loggers between 1949 and 2013. It is recognized that all those loggers did not have the same level of experience and knowledge. However, it was practically impossible to remove the uncertainties associated to data subjectivity from the database.

**Homogeneity:** one of the major challenges in applications of geostatistical techniques is inhomogeneity of the data that are used for modeling. Homogeneity of the data is generally required for the statistical inference. Journel and Huijbregts (1978) suggest that homogeneity or representativeness of data used for the geostatistical modeling must be constant through the time and space. In mature mining projects such as the Paul’s Peak mine, homogeneity of the geomechanical data over time and space are difficult to guarantee. The inhomogeneity of geomechanical data in time and space can be attributed to varying technologies and equipment used in different periods of time. Newer core drilling technologies and the use of triple tube core barrel would lead to better core recovery and generally better logged values for RQD and fracture frequency than the old technologies. Therefore, temporal/spatial homogeneity of data might be acquired only if constant methodologies are followed through the time and space for the drilling, sampling, and logging of geomechanical variables. In the present thesis, the entire domain of study was divided into six subdomains according to lithology groups and relative locations in order to reduce the inhomogeneity and have a more homogeneous data.

**Additivity:** geostatistical estimation methods such as ordinary kriging estimates the values of a regionalized variable such as ore grade using some sort of arithmetic averaging of the values in the sampled locations. These methods primarily assume that the regionalized variable being estimated is additive or linear. A regionalized variable is additive where all linear combinations of its values have the same meaning as it (Journel and Huijbregts 1978). The additivity assumption is valid for regionalized variables such as the ore grade measured in percent or gr/ton where the support is constant (Journel and Huijbregts 1978). Unlike the ore grade, the geomechanical variables such as RQD and RMR are not additive due to the fact that their linear average would not necessarily represent the true value of them. For example, a weighted linear
combination of RMR values logged in different sectors of a rock mass would not represent the true RMR of the entire rock mass. The non-linearity of RMR arises as well from this fact that RMR is calculated through a non-linear combination of its constituent variables (Egaña and Ortiz 2013). In calculation of RMR, each geomechanical variable (RQD, UCS, joint spacing, joint condition, and water condition) is rated according to the class that its value belongs to. Another factor that makes a regionalized variable non-additive is the support change effect. The support change effects on the additivity of the RQD, fracture frequency, and consequently RMR. The support in the geotechnical logging is usually considered as the core run length. The average of two RQD values logged for two different core runs lengths cannot represent the true RQD of these core runs. Therefore, it is essential that the geomechanical variables be logged and reported in consistent and constant core runs. In the present thesis, the geomechanical data were logged in varying support lengths. To avoid the non-additivity due to the support length changes, only those sample data with the support length 1-5 m were used for geostatistical modeling.

**Direction Dependency:** geomechanical variables such as joint spacing (or fracture frequency), and RQD, that are measured along a drillhole or a scanline survey are direction dependent. Depending on the structural heterogeneity of rock mass, the values of RQD and fracture frequency logged in the process of core logging can significantly vary when the orientation of the drillhole or the scanline traverse changes. Since RQD and joint spacing are the two components in the calculation of RMR, this variable can also show directional behavior. To address the direction dependency in spatial modeling of geomechanical variables, it would be ideal if these geomechanical variables be logged using the drillholes / scanline traverses in various orientations. This allows better detection of rock mass geomechanical anisotropy. In practice, this may not be always possible as generally the drillholes are oriented with respect to the orientation of ore body. For the Paul’s Peak mine, majority of the geomechanical sample data were logged in a preferred direction (orientation) that did not let the author to reduce the effects of direction dependency of the geomechanical variables.

**Upscaling:** upscaling from the point estimates to the 3D block models is a vital step in the modeling of the regionalized variables. The upscaling of the estimated values of ore grade into 3D blocks is a routine in the geostatistical modeling. This is due to the fact that ore grade is an additive regionalized variable and the arithmetic average of the estimated values within a block
can truly represent the upscaled value of the 3D block. However, upscaling of the geomechanical variables is a challenging and complex task (Dunham and Vann 2007) due to the non-linearity or non-additivity nature of these variables.

5.3 Recommendation for Future Research

In the present thesis, geostatistical conditional simulation technique was applied for the heterogeneity modeling of geomechanical properties. In this regard, 3D block models of geomechanical properties RQD, fracture frequency, UCS, joint condition, and RMR were developed and verified. For further investigate the applicability and strength of the proposed geostatistical methodologies in the present thesis, followings are recommended for future research:

i. Implement the geostatistical estimation approaches (e.g. ordinary kriging) for modeling the geomechanical properties. It would be of interest if the results of block models for geomechanical properties developed using geostatistical estimation techniques are compared with the models developed using the geostatistical simulation approaches. This comparison, could identify how the geostatistical simulation approaches employed in the present thesis work compared with the geostatistical estimation approaches;

ii. Develop discrete fracture network (DFN) models that characterize the structural heterogeneity of rock masses in the domain of study. DFN modeling is a stochastic approach for explicitly simulation of the 3D joint network. The results of the DFN modeling will be a number of equi-probable realizations of the joint network. The DFN methodology seeks to describe the rock mass fracture system in statistical ways through constructing a series of discrete fracture objects based upon field observations of geometrical fracture characteristics such as: size, orientation and intensity. The results of DFN modeling are 3D models showing the heterogeneity of joints in terms of the fracture frequency, volumetric fracture intensity, and so on. In the present thesis, 3D block models of fracture frequency were developed using the geostatistical simulaion approaches. It would be of interest to compare how the proposed approaches in the thesis work against the 3D DFN models;

iii. Linking the developed block models of geomechanical properties to the numerical models (i.e. finite difference, discrete element models) allows a stochastic slope stability analysis of different pit sectors. The results of the block models in the present thesis can be used as input
data for numerical modeling of pit slope stability and design. Implementation of numerical slope stability analysis using the developed heterogeneity models of geomechanical properties would give the opportunity to the design engineers to investigate the influence of spatial variability and anisotropy of rock masses on the mechanics of rock slope failure.


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Appendix
Figure A-1: Downhole variogram for N_RMR in Lower Limb and for lithology group I

Figure A-2: Downhole variogram for N_RMR in Upper Limb and for lithology group I

Figure A-3: Downhole variogram for N_RMR in Lower Limb and for lithology group II

Figure A-4: Downhole variogram for N_RMR in Upper Limb and for lithology group II

Figure A-5: Downhole variogram for N_RMR in Lower Limb and for lithology group III

Figure A-6: Downhole variogram for N_RMR in Upper Limb and for lithology group III
Figure A-7: Horizontal variograms for N_RMR of Lower Limb and lithology group I
Figure A-8: Horizontal variograms for N_RMR of Upper Limb and lithology group I
Figure A-9: Horizontal variograms for N_RMR of Lower Limb and lithology group II
Figure A-10: Horizontal variograms for N_RMR of Upper Limb and lithology group II
Figure A-11: Horizontal variograms for N_RMR of Lower Limb and lithology group III
Figure A-12: Horizontal variograms for $N_{RMR}$ of Upper Limb and lithology group III
Figure A-13: Variogram Modeling for N_RMR of Lower Limb and lithology group I

Figure A-14: Variogram modeling for N_RMR of Upper Limb and lithology group I

Figure A-15: Variogram modeling for N_RMR of Lower Limb and lithology group II

Figure A-16: Variogram modeling for N_RMR of Upper Limb and lithology group II

Figure A-17: Variogram modeling for N_RMR of Lower Limb and lithology group III

Figure A-18: Variogram modeling for N_RMR of Upper Limb and lithology group III
Figure A-19: Downhole variogram for N_PC1 in Lower Limb and for lithology group I

Figure A-20: Downhole variogram for N_PC1 in Upper Limb and for lithology group I

Figure A-21: Downhole variogram for N_PC1 in Lower Limb and for lithology group II

Figure A-22: Downhole variogram for N_PC1 in Upper Limb and for lithology group II

Figure A-23: Downhole variogram for N_PC1 in Lower Limb and for lithology group III

Figure A-24: Downhole variogram for N_PC1 in Upper Limb and for lithology group III
Figure A-25: Downhole variogram for N_PC2 in Lower Limb and for lithology group I

Figure A-26: Downhole variogram for N_PC2 in Upper Limb and for lithology group I

Figure A-27: Downhole variogram for N_PC2 in Lower Limb and for lithology group II

Figure A-28: Downhole variogram for N_PC2 in Upper Limb and for lithology group II

Figure A-29: Downhole variogram for N_PC2 in Lower Limb and for lithology group III

Figure A-30: Downhole variogram for N_PC2 in Upper Limb and for lithology group III
Figure A-31: Variogram modeling for N_PC1 of Lower Limb and Lithology Group I

Figure A-32: Variogram modeling for N_PC1 of Upper Limb and lithology group I

Figure A-33: Variogram modeling for N_PC1 of Lower Limb and lithology group II

Figure A-34: Variogram modeling for N_PC1 of Upper Limb and lithology group II

Figure A-35: Variogram modeling for N_PC1 of Lower Limb and lithology group III

Figure A-36: Variogram modeling for N_PC1 of Upper Limb and lithology group III
Figure A-37: Variogram modeling for N_PC2 of Lower Limb and lithology group I

Figure A-38: Variogram modeling for N_PC2 of Upper Limb and lithology group I

Figure A-39: Variogram modeling for N_PC2 of Lower Limb and lithology group II

Figure A-40: Variogram modeling for N_PC2 of Upper Limb and lithology group II

Figure A-41: Variogram modeling for N_PC2 of Lower Limb and lithology group III

Figure A-42: Variogram modeling for N_PC2 of Upper Limb and lithology group III