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Experimental Study of Dynamic Bending Failure of Laurentian Granite: Loading Rate and Pre-Load Effects

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ABSTRACT: In deep underground rock engineering projects, rocks are under static pre-load and they may further experience dynamic load due to earthquakes or production blasts. It is thus desirable to consider dynamic failure of rocks subjected to static pre-load. Besides, bending load is commonly encountered near underground openings. Therefore, this study considers the effect of the pre-load on the dynamic bending strength of Laurentian granite (LG). Using a modified split Hopkinson pressure bar system, the semi-circular bend (SCB) method is applied to carry out the bending tests. Five groups of SCB specimens are tested under different pre-loads and loading rates. The results show that under a given pre-load, the flexural tensile strength of LG increases with the loading rate, and decreases with the static pre-load at a given loading rate. The total flexural tensile strength is roughly independent of the pre-load. An empirical equation is used to represent the effects of the loading rate and the pre-load force on the dynamic flexural tensile strength. Furthermore, the flexural tensile strengths measured from SCB tests have higher values than the tensile strengths measured using the Brazilian disc method for the same rock. A non-local failure theory is utilized to quantitatively interpret this discrepancy.

Keywords: Flexural tensile strength, Pre-load, SHPB, SCB, Non-local theory

Introduction

The operation depth in rock engineering has been increasing and thus it is necessary to consider the effect of the static pre-load due to *in situ* stresses on the mechanical properties of underground rocks (Carter 1992; Chen et al. 2016; Wu et al. 2015). Meanwhile, underground rocks may further be exposed to dynamic loads caused by rockburst, blasting and earthquake (Dadfar et al. 2017; Kan et al. 2016; Tang et al. 2010). In this case, the rock dynamic properties superposed on a static pre-load are needed to be addressed (Chen et al. 2016; Li et al. 2008).

There have been many studies on the issue of dynamic strengths and fracture properties for rocks subjected to static pre-loads (Chen et al. 2016, Gong et al. 2010, Li et al. 2008, Zhou et al. 2014). Li et al. (2008) tested the rock specimen subjected to a coupled axial static pre-stress and axial impact load by using a modified split Hopkinson pressure bar (SHPB) system. It showed that the compressive strength of the specimens under coupling loads is higher than their corresponding static or dynamic compressive strength. With a constant axial pre-load stress and increasing impact load, the compressive strength of rock increases initially and then reaches a constant maximum value. Chen et al. (2016) investigated the fracture initiation toughness of rocks subjected to coupled static and dynamic loads by using the notched semi-circular bend (NSCB) method. The results revealed that the dynamic rock fracture toughness decreases with the

static pre-load at a given loading rate. The total fracture toughness, defined as the sum of the dynamic fracture toughness and the initial stress intensity factor calculated from the pre-load, increases with the pre-load at a given loading rate. Zhou et al. (2014) conducted impact tests on rock Brazilian disc (BD) samples under static pre-tensions. They concluded that the dynamic tensile strength of rock-like materials decreases with the increase of the static pre-tension. Wu et al. (2015) adopted the BD method to measure the dynamic tensile strength of rock-like materials under pre-tension. The results showed that the rock dynamic tensile strength decreases with the increase of the pre-tension. It was also observed that under the same pre-tension stress, the dynamic tensile strength increases with the loading rate. However, the total tensile strength of the rock is roughly independent of the pre-tension.

Despite of the existing research efforts on the effect of the pre-load on the dynamic mechanical properties of rocks, the influence of the pre-load on the dynamic flexural tensile strength of rocks remains unknown. However due to *in situ* stresses, bending load is common near underground openings, e.g. flexural tensile stresses are induced at the roof of an underground opening and the failure mode is thus dominated by the flexural tensile failure. It is also worth noting that the flexural tensile strength of rocks is usually higher than the tensile strength obtained from direct tensile tests (Xia et al. 2017; Zhang and Zhao 2013). Hence for the structures subjected to bending load, the safety design based on the tensile strength is too conservative.

Moreover, because the core-based semi-circular bend (SCB) specimen is easier to make and the SCB flexure tests are easily conducted and relatively inexpensive, a large database of rock flexural tensile strength measured from the SCB method for both quasi-static (Coviello et al. 2005) and dynamic loads (Dai et al. 2008; Dai et al. 2010; Dai et al. 2013; Dai and Xia 2010) is widely used in rock engineering and mining industry. Therefore, it is desirable to investigate the dynamic flexural tensile failure of rock-like materials under static pre-load by using the SCB method.

In this study, a modified SHPB system was utilized to perform dynamic flexural tensile strength tests on the rock specimens under five different pre-load conditions. The SCB specimens were employed to investigate the dynamic flexural tensile strength. In addition, the flexural tensile strength measured from SCB-SHPB test with pre-load is compared with the tensile strength measured from the BD method with the pre-load under similar loading rates. For all different pre-load conditions, the discrepancy between the dynamic flexural tensile strength and tensile strength is quantitatively interpreted by using a non-local failure theory.

This paper is organized as follows. After the introduction, Section 2 introduces the experimental methodology and the results along with discussions are presented in Section 3. Section 4 summarizes the entire paper.

Methodology

Pre-load SHPB System and Specimen Preparation

A modified SHPB system is used to perform the dynamic SCB tests with a static pre-load. As shown in Fig. 1a, this modified SHPB system consists of a traditional SHPB system and the pre-load system. The traditional SHPB system includes a striker bar, an incident bar and a transmitted bar (Zhou et al. 2012). The specimen is sandwiched between the incident bar and the transmitted bar, and the striker bar is launched by a gas gun. The pre-load system developed by Wu et al. (2015) was utilized in this study. The pre-load system is mainly composed of a pressure chamber and a rigid mass at the incident bar end. Two tie-rods are used to connect the chamber and the rigid mass. The pressure chamber is connected to the hydraulic control system through a high-pressure pipeline. High hydraulic pressure derived from oil in the chamber provides an axial pre-load to the piston and the axial pre-load is then transmitted into the bar and specimen. The pre-load is controlled by the hydraulic pressure. Compared with the pre-load systems innovated by other researchers (Chen et al. 2016; Frew et al. 2010; Zhou et al. 2014), the main advantage of the pre-load system in this study is that the total length of the compressed bars is much shorter and thus less prone to buckling (Wu et al. 2015).

The recording system includes strain gauges, a dynamic strain meter and an oscilloscope. Two sets of strain gauges are mounted on the incident bar and transmitted

bar, respectively. The history of strain gauge signals during the dynamic SCB test is recorded by a digital oscilloscope from the Wheatstone bridge circuits after amplification.

Insert Fig. 1 here.

The SCB specimen was used in the dynamic flexural strength tests with a static pre-load. The configuration of the SCB specimen is shown in Fig. 1b, where R is the radius of the half disk and B is the thickness of the rock disc. The span between two supporting pins is $S = 27.5$ mm. Failure is initiated at the failure spot O on the specimen due to dynamic bending load.

In order to prepare the SCB specimen, rock cores with a nominal diameter of 50 mm ($2R$) are first directly drilled from the same rock block and then sliced into disk-shaped specimens with an average thickness of $B = 25$ mm. All discs are polished to a length variation better than 0.5% of their thickness (Zhou et al. 2012). Thereafter the discs are cut into semicircular specimens for dynamic SCB tests.

The SCB specimens are made of Laurentian granite (LG), which is taken from the Laurentian region of Quebec City, Canada. The dominant minerals in LG are feldspar (60%), quartz (33%) and biotite (3-5%). The mineral grain size of LG varies from 0.2 to 2 mm. The average grain size of feldspar, quartz and biotite and is 0.4 mm, 0.5 mm and 0.3 mm, respectively. The physical and mechanical properties of LG are summarized in Table 1 (Dai et al. 2010; Nasser and Mohanty, 2008; Xia et al. 2017).

Measurement Principles

The rigid mass and the pressure chamber are first connected through two tie-rods, and the hydraulic chamber is then pressurized to the desired value. The pressure is transmitted to the bar system and the static pre-load is applied to the SCB specimen because the rigid mass constrains the leftward axial motion of the bars through a flange on the incident bar (Fig. 1a). When the desired pre-load is achieved in the SCB specimen, the impact of striker bar on the free end of the incident bar generates the dynamic loading in the incident bar. The incident stress wave ε_i propagates along the incident bar before it reaches the flange. The stress wave propagation is not affected by the flange since the impact motion of the bars is rightward (Wu et al. 2015). When the incident stress wave arrives at the specimen, a reflected stress wave ε_r and a transmitted stress wave ε_t are produced. These three waves are captured by the strain gauges on the incident and transmitted bar surfaces (Fig. 2). Since the oscilloscope channels for the strain gauges are set to AC coupling in our tests, only the strain change could be measured, i.e., the strain caused by the static pre-load in the transmitted bar is not recorded in the signal during the dynamic tests (Chen et al. 2016). The solid line in Fig. 2 represents the signal of the incident wave and reflected wave and the dash line delineates the signal of the transmitted wave. The voltage of the signal on the transmitted bar starts from zero, which indicates that the strain caused by the pre-load is not included in the signal.

Insert Fig. 2 here.

Determination of the dynamic flexural tensile strength requires the calculation of the normal forces on two ends of the specimen. Those forces in the incident end (P_1) and the transmitted end (P_2) can be determined (Zhou et al. 2012):

$$P_1(t) = P_{d1}(t) + P_0 = AE[\varepsilon_i(t) + \varepsilon_r(t)] + A\sigma \quad (1)$$

$$P_2(t) = P_{d2}(t) + P_0 = AE\varepsilon_t(t) + A\sigma \quad (2)$$

where P_0 is the static force on the bars due to the pre-load; P_{d1} and P_{d2} are the dynamic force history on the bars; A and E are the cross-sectional area and Young's modulus of the bars, respectively; σ is the static pressure on the bars caused by the pre-load. Since only the stress change was measured, the static force ($P_0 = A\sigma$) on the bars was not recorded in the signal and the forces on two ends of specimen recorded by the strain gauges are P_{d1} and P_{d2} .

According to the suggested method by the International Society for Rock Mechanics (ISRM), the pulse shaping technique placed at the free end of the incident bar (Fig. 1a) was applied to achieve the dynamic force balance (Dai et al. 2008; Zhou et al. 2012). Based on Eqs. (1) and (2) and the AC coupling used in the signal record, the force balance condition ($P_1(t) \approx P_2(t)$) for dynamic SCB tests should be written as: $AE[\varepsilon_i(t) + \varepsilon_r(t)] \approx AE\varepsilon_t(t)$. Figure 3a shows the forces on both ends of the SCB specimen in a typical pre-load dynamic SCB-SHPB test. The forces (P_1 and P_2) on both

ends of the specimen are approximately equal during the whole dynamic loading period (Fig. 3a), which is the same as the traditional SCB-SHPB test (Chen et al. 2016). Consequently, the dynamic force balance on both loading ends of specimen is achieved, i.e. $P_1 \approx P_2$. The dynamic force balance has been critically assessed for all pre-load dynamic SCB-SHPB tests in this study.

Insert Fig. 3 here.

Determination of Dynamic Flexural Tensile Strength

With the dynamic force balance in the SCB-SHPB tests, the flexural tensile stress history at the failure spot O in the SCB specimen can be determined as (Dai et al. 2008; Dai et al. 2013; Dai and Xia 2010):

$$\sigma_f(t) = \frac{P(t)}{\pi BR} \cdot Y\left(\frac{S}{2R}\right) \quad (3)$$

where $P(t)$ is the time varying loading force in the specimen, which equals to the transmitted force $P_2(t)$ ($\approx P_1(t)$) in dynamic SCB-SHPB tests. Y is a function of the dimensionless geometry parameter $S/2R$, and is calibrated with static finite element analysis according to the method proposed by Dai et al. (2008). A commercial finite element analysis software ANSYS was used to determine $Y(S/2R)$, which can be written as a polynomial fitting formula:

$$Y = 2.348 + 2.517\left(\frac{S}{2R}\right) + 4.795\left(\frac{S}{2R}\right)^2 \quad (4)$$

Hence, for our configuration, $S/2R = 0.55$ and $Y = 5.16$. The dynamic flexural tensile strength σ_f in our SCB-SHPB tests can be calculated using the maximum value of the loading force ($P_{\max}$).

According to the dynamic flexural tensile stress history calculated using Eq. (3), the loading rate is determined by the time evolution of the flexural tensile stress in the SCB specimen. A typical dynamic flexural tensile stress-time curve in the SCB specimen is shown in Fig. 3b, in which there is an almost linear regime in the curve. Based on the suggested method for determining the dynamic properties of rocks by ISRM, the slope of this region (the dash line in Fig. 3b) is defined as the loading rate (Zhou et al. 2012).

Results and Discussions

Effect of Loading Rate and Pre-load

In this study, five groups of SCB specimens are tested under different pre-load conditions and the corresponding pre-loads in SCB specimens are 0, 20%, 40%, 60% and 80% of the static critical load, which is a critical external load on the SCB specimen when the failure occurs under a static load. Therefore, the static flexural tensile stress σ_{pre} at the failure point O in the SCB specimen due to the pre-load on the specimen can be calculated using Eq. (3), and the pre-load on the SCB specimen is defined and specified

via the σ_{pre} value. The σ_{pre} values of SCB specimens in this study are 0 MPa, 5.24 MPa, 10.48 MPa, 15.72 MPa and 20.96 MPa, respectively.

There are two types of flexural tensile strength considered in this study: the dynamic flexural tensile strength $\sigma_{f_Dynamic}$ of rocks (which is directly derived from dynamic loading force in SCB-SHPB tests), and the total flexural tensile strength σ_{f_Total} (which is the summation of the dynamic flexural tensile strength and the flexural tensile stress σ_{pre} due to the pre-load stress). The loading force used in Eq. (3) for calculating $\sigma_{f_Dynamic}$ is the dynamic force $P_{Dynamic}$, and $\sigma_{f_Total} = \sigma_{f_Dynamic} + \sigma_{pre}$. Therefore $\sigma_{f_Dynamic}$ represents the dynamic loading capacity of a rock specimen under a certain static pre-load while σ_{f_Total} indicates the total loading capacity

Figure 4a and b illustrate $\sigma_{f_Dynamic}$ and σ_{f_Total} versus loading rate under different pre-loads, respectively. It is obvious that both $\sigma_{f_Dynamic}$ and σ_{f_Total} increase with the loading rate, revealing the phenomenon of rate dependency that is common for rock-like materials, including rocks (Chen et al. 2016; Wu et al. 2015; Xia and Yao 2015; Zhang and Zhao 2013), concretes (Cusatis 2011, Fujikake et al. 2006; Yao et al. 2017) and ceramics (Brar et al. 1988; Field et al. 2004).

Insert Fig. 4 here.

Apart from the rate dependency discussed above, Figure 4a also shows that the $\sigma_{f_Dynamic}$ of LG decreases with the increase of the pre-load at the same loading rate. For

example, when the loading rate is approximately 1400 GPa/s, the $\sigma_{f_Dynamic}$ of LG is 64.37 MPa if the SCB specimen does not have a pre-load. However, the $\sigma_{f_Dynamic}$ of LG at the similar loading rate are 58.9 MPa with 5.24 MPa pre-load, 57.86 MPa with 10.48 MPa pre-load, 52.23 MPa with 15.72 MPa pre-load, 46.85 MPa with 20.96 MPa pre-load. This phenomenon is consistent with the results reported by Wu et al. (2015) for dynamic tensile strength of the same rock subjected to pre-load. The decrease of dynamic flexural tensile strength can be attributed to the opening of microcracks when the specimen bears the pre-load stress.

Figure 4b shows that the σ_{f_Total} of LG are almost the same under the same loading rate regardless of the magnitude of the pre-load applied to the SCB specimen. The similar phenomenon was reported for dynamic tensile strength of LG under the static pre-tension using BD specimens (Wu et al. 2015). This may be caused by more microcracks generated and activated under higher pre-loads. The increase of activated microcracks leads to a more viscous material that is more sensitive to the loading rate. The pre-load weakening effect is offset by this increased rate sensitivity and thus the total flexural tensile strength is roughly independent of the pre-load (Wu et al. 2015).

Failure Pattern of the SCB Specimens

Figure 5 compares five typical recovered SCB specimens from dynamic SCB-SHPB tests. The specimen in Fig. 5a was damaged at a pre-load of 0 MPa and 880 GPa/s. The

specimens in Fig. 5b and c were impacted with loading rates of 1856 GPa/s and 2711 GPa/s respectively without a pre-load. These three specimens were completely split into two roughly identical halves by a damage zone enclosing the main crack. It can be seen that the width of damage zone increases with the increase of the loading rate. This phenomenon may be explained as follows. The higher the loading rate is, the less time the microcracks have for propagation and coalescence until failure. As a result, more and more microcracks are involved in the dynamic fragmentation process with the increase of loading rate, resulting in the increase of the damage zone. This phenomenon is consistent with the results of other rock-like materials reported in the literature (Dai et al. 2008; Dai et al. 2013).

In addition, the specimen in Fig. 5d was loaded by a pre-load of 10.48 MPa before the impact with a loading rate of 975 GPa/s and the specimen in Fig. 5e was impacted with loading rate of 911 GPa/s under a pre-load of 20.96 MPa. These two specimens were also completely split into two roughly identical halves by a damage zone enclosing the main crack. Furthermore, the specimens in Fig. 5a, d and e are at the almost same loading rate. It indicates that at the almost same loading rate, the damage zone of the SCB specimen increases with the pre-load. More microcracks activated by the increased pre-load are available to be involved in the fragmentation process, leading to the increase of the damage zone.

Insert Fig. 5 here.

Empirical Model for Flexural Strength Based on Loading Rate and Pre-Load Effects

In order to completely describe the complicated dependence of the dynamic flexural tensile strength $\sigma_{f_Dynamic}$ on the pre-load and loading rate, the following empirical formula is proposed:

$$\frac{\sigma_{f_Dynamic}}{\sigma_{f0}} = 1 - \left(\frac{\sigma_{pre}}{\sigma_{f0}} \right) + \alpha \left(\frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^\beta \quad (5)$$

where σ_{f0} is the quasi-static flexural tensile strength (here $\sigma_{f0} = 26.2$ MPa (Dai et al. 2010)), $\dot{\sigma}$ is the loading rate, $\dot{\sigma}_0$ is the reference loading rate and equals 0.001 GPa/s, α and β are data fitting constants. The term of $(\sigma_{pre}/\sigma_{f0})$ represents the pre-load effect and the term of $\alpha(\dot{\sigma}/\dot{\sigma}_0)^\beta$ indicates the effect of loading rate. When there is no pre-load ($\sigma_{pre} = 0$), Eq. (5) will degenerate to $\sigma_{f_Dynamic}/\sigma_{f0} = 1 + \alpha(\dot{\sigma}/\dot{\sigma}_0)^\beta$, which only reflects the effect of loading rate. When the specimen is loaded by the quasi-static condition ($\dot{\sigma} \approx 0$), Eq. (5) will be written as $\sigma_{f_Dynamic}/\sigma_{f0} = 1 - (\sigma_{pre}/\sigma_{f0})$, which shows the relationship between the pre-load and the flexural tensile strength under the quasi-static condition. Particularly, the flexural tensile strength is the static flexural tensile strength when $\dot{\sigma} \approx 0$ and $\sigma_{pre} = 0$.

Furthermore, because $\sigma_{f_Total} = \sigma_{f_Dynamic} + \sigma_{pre}$, the total flexural tensile strength σ_{f_Total} can be deduced from Eq. (5) as:

$$\frac{\sigma_{f_Total}}{\sigma_{f0}} = \frac{\sigma_{f_Dynamic} + \sigma_{pre}}{\sigma_{f0}} = 1 - \left(\frac{\sigma_{pre}}{\sigma_{f0}} \right) + \left(\frac{\sigma_{pre}}{\sigma_{f0}} \right) + \alpha \left(\frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^\beta = 1 + \alpha \left(\frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^\beta \quad (6)$$

It is seen that the second term expresses the rate dependence effect of the total flexural tensile strength.

According to the experimental data of σ_{f_Total} shown in Fig. 4b, Genetic algorithm, an effective method to find the best solution based on natural selection, is used to determine the parameters α and β . The fitting curve is shown in Fig. 4b and it is seen that the function fits well to the data points of σ_{f_Total} . The values of the fitting parameter are listed in Table 2. Furthermore, we substituted these two fitting values into Eq. (5) and the fitting curves for $\sigma_{f_Dynamic}$ under different pre-loads and loading rates are derived from Eq. (5) and shown in Fig. 4a. Expectedly, these fitting curves in Fig. 4a provide good predictions of the data points of $\sigma_{f_Dynamic}$. Thus, the dynamic and total flexural tensile strengths of rocks with pre-loads are predicted by Eqs. (5) and (6), respectively.

Empirical Model for Tensile Strength Based on Loading Rate and Pre-Load Effects

The dynamic tensile strengths of LG under different pre-tensions and loading rates are reported by Wu et al. (2015) using the BD method and the SHPB system. However, an empirical formula was not developed to quantitatively describe the influence of the pre-load and loading rate on the tensile strength. Hence in this study the dynamic tensile strengths of LG in the literature (Wu et al. 2015) are quantitatively analyzed via the

similar empirical formula and the complicated dependence of the dynamic tensile strength on the pre-load and loading rate is also completely described by the following equation:

$$\frac{\sigma_{t_Dynamic}}{\sigma_{t0}} = 1 - \left(\frac{\sigma_{t_pre}}{\sigma_{t0}} \right) + \alpha \left(\frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^\beta \quad (7)$$

where $\sigma_{t_Dynamic}$ is the dynamic tensile strength, σ_{t0} is the quasi-static tensile strength (here $\sigma_{t0} = 12.8$ MPa (Wu et al. 2015)), σ_{t_pre} is the pre-tension at the center of the BD specimen, $\dot{\sigma}$ is the loading rate, $\dot{\sigma}_0$ is the reference loading rate and equals 0.001 GPa/s, α and β are data fitting constants. Similarly, the pre-load effect is described by the term of $(\sigma_{t_pre}/\sigma_{t0})$ and the loading rate dependence is represented through the term of $\alpha(\dot{\sigma}/\dot{\sigma}_0)^\beta$. When $\sigma_{t_pre} = 0$, the loading rate dependence is written as $\sigma_{t_Dynamic}/\sigma_{t0} = 1 + \alpha(\dot{\sigma}/\dot{\sigma}_0)^\beta$. The relationship between the pre-load and the static tensile strength is expressed as $\sigma_{t_Dynamic}/\sigma_{t0} = 1 - (\sigma_{t_pre}/\sigma_{t0})$ when $\dot{\sigma} \approx 0$.

Moreover, similar to the dynamic SCB tests, the total tensile strength σ_{t_Total} of LG from the BD method is the sum of $\sigma_{t_Dynamic}$ and σ_{t_pre} . The total tensile strength σ_{t_Total} of LG in function of the pre-load and loading rate can be deduced from Eq. (7):

$$\frac{\sigma_{t_Total}}{\sigma_{t0}} = \frac{\sigma_{t_Dynamic} + \sigma_{t_pre}}{\sigma_{t0}} = 1 - \left(\frac{\sigma_{t_pre}}{\sigma_{t0}} \right) + \left(\frac{\sigma_{t_pre}}{\sigma_{t0}} \right) + \alpha \left(\frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^\beta = 1 + \alpha \left(\frac{\dot{\sigma}}{\dot{\sigma}_0} \right)^\beta \quad (8)$$

In the above equation, the second term expresses the loading rate dependence of the total tensile strength. Similarly, when there is no dynamic loading, the total tensile strength of rocks is the quasi-static tensile strength. Based on the experimental results of

total tensile strength in Fig. 6b, the values of α and β for BD tests are obtained from the data fitting and tabulated in Table 2. Figure 6b illustrates the fitting curve and it is obvious that the function fits well to the data points of σ_{t_Total} . In addition, we substituted these fitting values of α and β into Eq. (7) and the fitting curves for $\sigma_{t_Dynamic}$ under different pre-loads and loading rates are derived from Eq. (7) and shown in Fig. 6a. Expectedly, these fitting curves in Fig. 6a provide good predictions of the data points of $\sigma_{t_Dynamic}$. Thus, both the dynamic and total tensile strengths of rocks under different pre-loads and loading rates are predicted by Eqs. (7) and (8), respectively.

Insert Fig. 6 here.

Reconciliation of Flexural Tensile Strength with Tensile Strength

The total flexural tensile strengths and the total tensile strengths of LG under different loading rates and pre-loads are compared in Fig. 7. It is obvious that under all pre-loads, the total flexural tensile strength measured through the SCB method is higher than the total tensile strength measured through the BD method for a given loading rate. This phenomenon has also been observed in the static tensile measurements with a stress gradient around the potential failure spot (Coviello et al. 2005; Hudson et al. 1972; Lajtai 1972; Mellor and Hawkes 1971). Moreover, Zhao and Li (2000) reported the higher value of dynamic strength from their three-point flexural test as compared to that obtained by the BD test. Xia et al. (2017) and Dai et al. (2013) found the dynamic flexural tensile

strength of LG is higher than dynamic tensile strength from the BD test, and proposed a theoretical model to reconcile this difference.

Insert Fig. 7 here.

In this study, due to the achievement of dynamic equilibrium in the SCB specimen, a non-local failure model can be applied to reconcile the discrepancy between the measured total flexural tensile strengths from SCB tests and the measured total tensile strengths from BD tests (Carter 1992; Lajtai 1972; Van de Steen and Vervoort 2001). The details of the non-local failure model can be found in references (Carter 1992; Lajtai 1972; Van de Steen and Vervoort 2001). In this theory, the failure of materials occurs when the average local stress over a characteristic material length δ along the prospective fracture path equals to the tensile strength of materials. The relationship between the flexural tensile strength σ_f (stress at the failure spot when the specimen fails) and the resulting tensile strength σ'_t is determined as $\sigma_f/\sigma'_t = 1.6345$ by using a characteristic material length δ (as shown in Fig. 8). The characteristic material length is related to the characteristic length of material microscopic structures and cannot be directly measured (Carter 1992; Dai et al. 2010; Lajtai 1972; Van de Steen and Vervoort 2001; Xia et al. 2017), thus we assume $\delta = 6$ mm in this study as suggested for the same rock in the earlier work (Dai et al. 2010; Van de Steen and Vervoort 2001).

Insert Fig. 8 here.

Similar to early work, the strength is corrected by using the non-local theory, while it is unnecessary to convert the loading rate because the failure in the SCB specimen is at the center of the original disc (i.e., point O in Fig. 1b) and the loading rate derived from the stress history at this point is valid (Dai et al. 2010; Xia et al. 2017). Hence, the corrected total tensile strength σ'_{t_Total} for all pre-loads can be deduced from the measured total flexural tensile strength σ_{f_Total} . The results are also shown in Fig. 7. It can be seen that the corrected total tensile strengths from the total flexural tensile strengths are consistent with those measured from BD tests.

Conclusions

In this work, we utilized a pre-load SHPB system with a SCB specimen to measure the dynamic flexural tensile strength of rocks subjected to different pre-loads. Five groups of SCB specimens are tested under a pre-load of 0, 20%, 40%, 60% and 80% of the maximum static load and with the loading rate ranging from 0 to 2750 GPa/s.

Pulse shaping technique was used in all dynamic SCB-SHPB tests to facilitate dynamic force equilibrium. Finite element method was employed to obtain equations calculating flexural tensile strength. The results indicate that, under a given pre-load, there is a significant enhancement in dynamic flexural tensile strength with the increase of the loading rate. The dynamic flexural tensile strength decreases with the increase of the pre-load stress at a given loading rate due to the opening of microcracks by the

pre-load. It is worth noting that the total flexural tensile strength of the rock specimen, defined as the sum of the dynamic flexural tensile strength and initial flexural tensile stress calculated from the pre-load, is roughly independent of the pre-load.

An empirical equation is proposed to represent the effect of loading rates and pre-loads, and the results show that the equation can depict the trend of the experimental data from both SCB and BD tests. Furthermore, the total flexural tensile strength measured from SCB tests has a higher value than the total tensile strength measured using the BD method under similar loading rates. A non-local failure theory is used to quantitatively rationalize this discrepancy. The corrected total tensile strengths from the total flexural strengths are consistent with those measured from BD tests.

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Figure Captions:

Fig. 1. (a) Pre-load SHPB system and (b) SCB specimen (Not to scale).

Fig. 2. Signal captured from the strain gauges on the SHPB bars.

Fig. 3. (a) Dynamic force balance and (b) loading rate determination in a typical pre-load dynamic SCB-SHPB test.

Fig. 4. The (a) dynamic and (b) total flexural tensile strength versus loading rate for different pre-loads.

Fig. 5. Typical recovered SCB specimens (the scale in the picture is 10 mm). (a) Pre-load: 0 MPa and loading rate: 880 GPa/s; (b) Pre-load: 0 MPa and loading rate: 1856 GPa/s; (c) Pre-load: 0 MPa and loading rate: 2711 GPa/s; (d) Pre-load: 10.48 MPa and loading rate: 975 GPa/s; (e) Pre-load: 20.96 MPa and loading rate: 911 GPa/s.

Fig. 6. The (a) dynamic and (b) total tensile strength with BD specimens versus loading rate for different pre-loads (after Wu et al. (2015)).

Fig. 7. Comparison of strengths from dynamic SCB test and BD test with pre-load (Wu et al. (2015)) as well as the reconciliation by non-local failure model.

Fig. 8. Tensile stress along the prospective fracture path (x) in SCB sample (x is the distance of a point along the prospective fracture path to the failure spot of the SCB sample).

Table 1. Summary of physical and mechanical properties of Laurentian Granite.

Density	Porosity	Young's modulus	Poisson's ratio	UCS	Brazilian tensile strength	Flexural tensile strength	K_{IC}
g/cm^3	%	GPa		MPa	MPa	MPa	$\text{MPa}\cdot\text{m}^{1/2}$
2.63	0.64	92	0.21	259	12.8	26.2	1.5

Table 2. Summary of the values of data fitting parameters.

	α	β
SCB	1.6421×10^{-4}	0.6467
BD	1.2548×10^{-4}	0.6979

Fig. 1. (a) Pre-load SHPB system and (b) SCB specimen (Not to scale).

61x19mm (300 x 300 DPI)

Fig. 2. Signal captured from the strain gauges on the SHPB bars.

140x106mm (300 x 300 DPI)

Fig. 3. (a) Dynamic force balance and (b) loading rate determination in a typical pre-load dynamic SCB-SHPB test.

69x24mm (300 x 300 DPI)

Fig. 4. The (a) dynamic and (b) total flexural tensile strength versus loading rate for different pre-loads.

68x24mm (300 x 300 DPI)

Fig. 5. Typical recovered SCB specimens (the scale in the picture is 10 mm). (a) Pre-load: 0 MPa and loading rate: 880 GPa/s; (b) Pre-load: 0 MPa and loading rate: 1856 GPa/s; (c) Pre-load: 0 MPa and loading rate: 2711 GPa/s; (d) Pre-load: 10.48 MPa and loading rate: 975 GPa/s; (e) Pre-load: 20.96 MPa and loading rate: 911 GPa/s.

120x78mm (300 x 300 DPI)

Fig. 6. The (a) dynamic and (b) total tensile strength with BD specimens versus loading rate for different pre-loads (after Wu et al. (2015)).

76x28mm (300 x 300 DPI)

Fig. 7. Comparison of strengths from dynamic SCB test and BD test with pre-load (Wu et al. (2015)) as well as the reconciliation by non-local failure model.

99x48mm (300 x 300 DPI)

Fig. 8. Tensile stress along the prospective fracture path (x) in SCB sample (x is the distance of a point along the prospective fracture path to the failure spot of the SCB sample).

161x175mm (300 x 300 DPI)