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Influence of Particle Size and gradation on the Shear Strength–Dilation Relation of Granular Materials

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Abstract

A close scrutiny of data reported in the literature recommends taking into account the particle-scale characteristics to optimize the precision of well-known empirical Bolton's equations, and imposing particle-sized limits on them. The present paper examines the potential influence of particle size and grading on the shear strength–dilation relation of granular materials from the results of 276 symmetrical direct shear tests. The applicability of physical symmetrical direct shear tests to interpret the plane strain frictional shearing resistance of granular materials was widely discussed using the DEM computer code SiGran. Sixteen different grain-size distribution curves of three different materials were tested at different normal pressures and initial relative densities. It is demonstrated that while the contribution of dilatancy to the shear strength is not influenced by the variation in the coefficient of uniformity C_u in the investigated range, it is significantly decreases with increasing mean particle size D_{50} . The coefficients of Bolton's equations have been, therefore, adjusted to account for D_{50} . A comparison of the predictions by the proposed empirical formulas with ϕ_{ps} and ψ data from literatures shows that accounting for the grain size yields more authentic results.

Key words: Particle size, Shear strength, Dilation, Coefficients, Shape, Flow rule.

Résumé

Un examen minutieux des données rapportées dans la littérature recommande de prendre en compte les caractéristiques de l'échelle des particules pour optimiser la précision des équations empiriques bien connues de Bolton, et leur imposer des limites au niveau de la taille des particules. Le présent article examine l'influence potentielle de la grosseur des particules et de la granulométrie sur la relation force de cisaillement-dilatation des matériaux granulaires à partir des résultats de 276 essais de cisaillement direct symétriques. L'applicabilité des essais physiques de cisaillement direct symétrique pour interpréter la résistance au cisaillement par frottement par déformation plane des matériaux granulaires a été largement discutée en utilisant le code informatique DEM SiGran. Seize courbes de distribution granulométrique différentes de trois matériaux différents ont été testées à différentes contraintes normales et densités relatives initiales. Il est démontré que, bien que la contribution du coefficient de dilatation à la résistance au cisaillement ne soit pas influencée par la variation du coefficient d'uniformité C_u dans la gamme étudiée, elle diminue significativement avec l'augmentation de la taille moyenne des particules D_{50} . Les coefficients des équations de Bolton ont donc été ajustés pour tenir compte du D_{50} . Une comparaison des prédictions par les formules empiriques proposées avec les données ϕ_{ps} et ψ de la littérature montre que la prise en compte de la taille des grains donne des résultats plus authentiques.

Mots clés: Taille des particules, résistance au cisaillement, dilatation, coefficients, forme, loi d'écoulement.

Introduction

Following the pioneer work of Casagrande (1936), a number of theoretical (e.g., Taylor 1948; Skempton and Bishop 1950; Bishop 1954; Newland and Allely 1957; Rowe 1962, 1969; Schofield and Wroth 1968; De Josselin de Jong 1976) and experimental (e.g., Lee and Seed 1967, Roscoe 1970, Bolton 1986, Negussey et al. 1988, Collins et al. 1992) attempts have been made to explain the volumetric deformation of granular assembly upon shearing (i.e. dilatancy). The stress–dilatancy theories based on macroscopic observations can be traced as far back to the early works of Rowe (1962). This relation has been later validated by De Josselin de Jong (1976) with an alternative approach based on the laws of friction. While the importance of confinement, density and stress path on stress–dilatancy relations has been clearly demonstrated in these relations (e.g., Pradhan et al. 1989; Houlsby 1991; Gudehus 1996; Nakai 1997; Wan and Guo 1999; Vaid and Saivathayalan 2000), experimental studies of stress–dilatancy with focus to microstructural issues are scarce (e.g., Oda and Kazama 1998; Wan and Guo 2001a, 2001b; Amirpour Harehdasht et al. 2017a). So that, these equations are not powerful enough to describe aspects of sand behaviour related to its microstructure. In other words, most existing stress–dilatancy theories, except the works of Oda and Kazama (1998) and Wan and Guo (2001a, 2001b), do not address microstructural issues probably due to the paucity of experimental studies in the literature.

As alternatives to the theoretical stress–dilatancy relations, some researchers defined comprehensive, but simple empirical equations with relatively few material parameters, such as Bolton (1986), Vaid and Sasitharan (1992), and Collins et al. (1992). Bolton (1986) took an experimental approach and proposed the famous empirical flow rule (Eq. (1)) described as operationally indistinguishable from that of Rowe's Equation. The main advantage of this flow rule is that any angle of shearing in excess of the friction angle of loose earth is seen to be due solely to the geometry of the volumetric expansion, which is necessary before shearing can take place.

$$\phi_{ps} - \phi_{cv} = b\psi, \quad b = 0.8 \text{ for plane strain condition.} \quad (1)$$

Bolton (1986) summarized also the previous works performed on strength and dilatancy of sands (e.g., Rowe 1962; Vesic and Clough 1968; Roscoe 1970; Billam 1972) and proposed a relative dilatancy index I_R (Eq. (2)), which is also used to estimate the rate of dilatancy of soil,

thereby the plane strain strength parameters, based on correlation with the relative density D_R , and effective stress, p' (Eq. (3)).

$$I_R = D_R (Q - \ln p') - R = D_R \ln \left(\frac{B \sigma_0}{p'} \right) - 1, \quad R = 1 \quad (2)$$

$$\phi_{ps} - \phi_{cv} = f(D_R, p') = c I_R, \quad c = 3 \text{ for triaxial, and } c = 5 \text{ for plane strain conditions.} \quad (3)$$

where p' is the mean effective stress at failure. The parameter Q relates to the mean effective stress required to suppress dilatancy, and is a function of the average tensile strength of grains, σ_0 (McDowell and Bolton 1998).

Although the applicability of Bolton's relationships in geotechnical practices have been undeniably acknowledged, it is recognized that the degree of their precisions is improvable as they were introduced using extremely limited number of plane strain data. Moreover, it would have been expected that similar to theoretical stress–dilatancy relations, the explicit consideration of the soil grain characteristics (e.g., particle size, particle shape, and particle gradation) strongly affect dilation and strength values (e.g., Oda and Kazama 1998; Wan and Guo 2001a, 2001b; Amirpour Harehdasht et al. 2017a).

A respective analysis of strength data collected from the literature is presented in this paper to discuss the degree of precision of Bolton's equations. It seems that an application of Eqs. (1) – (3) with the constants proposed by Bolton (1986) to sands with different particle-size distribution may under- or over-predict ϕ_{ps} values. The present paper then examines more closely the potential impact of particle size and grading on the existing empirical correlations between the friction angle and the dilation behavior of granular materials in plane strain condition in Eqs. (8) – (9). For this purpose, 276 symmetrical direct shear tests have been carried out on samples made up of basalt beads (rounded particles), and sands consisting of angular particles (Péribonka and Eastmain sands) in the range of 63 to 2000 μm to obtain the values of peak friction ϕ_{ps} and dilation angles ψ over a wide range of normal pressures and initial relative densities. The reliability and applicability of boundary measurements in physical symmetrical direct shear tests used to interpret the plane strain frictional shearing resistance of granular material have been discussed in Amirpour Harehdasht et al. (2017b) and this study. After examining the sensitivity of Eqs. (1) – (3) to the particle characteristics, the coefficients of Bolton's equations have been adjusted to account for particle characteristics, in particular for

D_{50} . A comparison of the predictions by the proposed empirical formulas with ϕ_{ps} -data from the literature are also provided in this paper.

Reliability of Bolton's Flow Rules to predict ϕ_{ps} Based on Data Reported in the Literature

Before describing the experimental testing program and the results on the potential influence of particle size and grading on the shear strength–dilation relation of granular materials, a brief review is given in this section to discuss the degree of precision of Bolton's equations. In other words, a comparison between the peak friction angle ϕ_{ps} predicted by Bolton's Eqs. (1) – (3) with the ϕ_{ps} data from the literature has been undertaken to investigate if the maximum shear resistance can be precisely expressed without accounting for the particle characteristics. Fig. 1 collects approximately 200 values of ϕ_{ps} measured in different laboratories by different researchers on sands with different D_{50} , and C_u . The data were obtained from plane strain compression tests. The predicted values were calculated having all required values (p' , D_R , ϕ_{cv} , ψ) given in the respective literature. To calculate accurately the I_R value in Eq. (3), the parameter Q was carefully assigned for each material collected from the literature. This parameter takes the value of 10 for quartz and feldspar sands, and reduces for soils comprise of weaker grains with decrease of their average tensile strength σ_0 (Bolton 1986; McDowell and Bolton 1998), as measured by crushing between flat platen (Lee 1992). Bolton (1986) suggested Q value of 8 for limestone, 7 for anthracite, and 5.5 for chalk. It was also confirmed that the average tensile strength of grains is proportional to the yield stress $\bar{\sigma}_0$ as identified from the e -log σ'_v plot, where the rate of particle fracture with increasing stress is maximum (Coop and Lee 1973; Hyodo et al. 2002; McDowell and Bolton 1998; McDowell 2002). So that comparing $\bar{\sigma}_0$ value of materials in Fig. 1 (especially for artificial materials) with those introduced by Bolton (1986) can insure the accuracy of the parameter Q considered.

When Eq. (1) is used to estimate ϕ_{ps} , the scatter of data shown in Fig. 1a from the line described by $\phi_{ps}^{pred} = \phi_{ps}^{meas}$ is quite significant, especially for higher ϕ_{ps} values with an over-estimation reaches 8 degrees. A similar plot (Fig. 1b) is also given for Eq. (3) where the measured ϕ_{ps} versus predicted ϕ_{ps} data shows also a relatively large scatter extending to $\phi_{ps}^{pred} = \phi_{ps}^{meas} + 13^\circ$. In both Figs. 1a and 1b, the majority of the data points are plotted above the bisecting line, showing the over-prediction of ϕ_{ps} values for sands using both equations. In addition, analyzing the data in Fig. 1a shows that ϕ_{ps} values of sands, made up of rounded particles, with small D_{50} ($< 350 \mu m$)

are located at the lower boundary of the group of data points (averaged 1.42 ± 0.78 degrees), while the points lay at the upper boundary (averaged 4.29 ± 2.05 degrees) are attributed to sands with larger D_{50} ($> 700 \mu\text{m}$). Moreover, the dispersed data points in Fig. 1b shows that the over-prediction of ϕ_{ps} value is still large (averaged 4.03 ± 2.95 degrees) when using Eq. (3) instead of Eq. (1). This observation is in agreement with the results reported by Pedley (1990), who observed the over-estimation of plane strain data by Eq. (3), even when applying the suggested minimum cut off stress of $p' = 150 \text{ kN/m}^2$.

The results shown in Figs 1a and 1b demonstrate the relative uncertainty of Eqs. (1) – (3) to precisely predict the soil peak friction angle ϕ_{ps} . However, they do not show a clear tendency concerning the influence of D_{50} and C_u on ϕ_{ps} as the data gathered from the literature are only correspond to a number of ϕ_{ps} data of standard sands and materials having a rather uniform particle size distribution. Therefore, more data of granular materials having different particle size distributions would be beneficial to reflect the potential dependency of ϕ_{ps} values predicted from Eqs. (1) – (3) on particle size distributions and extend these equations based on D_{50} and C_u values. The 276 direct shear tests on 16 different particle-size distributions of different granular materials have been performed on this purpose. Descriptions of the tested materials, testing program, test results and analysis are given next.

Material and Testing Program

Material

The majority of the present work was performed on basalt microspheres produced by Whitehouse Scientific Ltd, England, in 20 individual narrow distribution grades between 63 and 2000 μm . These gradations were mixed to produce the 16 particle-size distribution curves presented in Fig. 2. The curves are linear in semi-logarithmic scale. The influence of grain size represented by D_{50} on the shearing behavior of the basalt beads was studied on uniform distributions C_1 to C_5 (Fig. 2a) with an identical coefficient of uniformity ($C_u = 1.5$) and D_{50} ranging between 106 to 1000 μm . An investigation into the effects of grading was also performed on the same material using three series of tests (i.e., series 2, 3, and 4 with D_{50} of 212, 600, and 350 μm , respectively) (Fig. 2b). In each series, C_u varies between 1.5 and 5. Values of D_{50} and C_u of the 16 grading curves considered are summarized in Table 1.

In order to investigate whether the trend of the results obtained for the uniform basalt beads could also be observed in other materials with different particle shapes, a number of additional tests was performed on two different sands (Péribonka and Eastmain sands) with different particle angularities and deliberately prepared particle-size distribution curves as shown in Table 1. Péribonka sand is quartz sand with sub-angular to sub-rounded particles, while Eastmain sand is quartz sand made up of angular to sub-angular particles. The Particle shapes of all materials are shown in Fig. 3, where the images for grains smaller than 500 μm were obtained from scanning electron microscope (SEM). The data clearly shows that the particles of Eastmain sand are more angular than those of Péribonka sand, in terms of both particle sphericity and roundness. Sieve analyses on the three tested materials were carried out before and after the tests and no obvious evidence of particle crushing was observed.

Shear Box Testing Device, Specimen Preparation and Testing Procedure

The direct shear apparatus (DSA) is, in particular, one of the oldest and most commonly used laboratory devices, which was constructed by Collin as early as 1846 for slope stability analysis (Skempton 1949). Developing the DSA in its present form dates back to Krey (1926), which later modified by Casagrande (1936) and Terzaghi and Peck (1948) to account for the vertical expansion and contraction measurements. However, the reliability of traditional direct shear interpretation was called into question due to the unknown state of deformation and stress within the shear band and existed ambiguities in interpreting shear strength parameters. So that, different experimental and numerical approaches has been taken through the years to ameliorate the direct shear test results and explore the deformation and strength behavior inside the sample during shearing (e.g., Potts et al. 1987; Jewell 1989; Shibuya et al. 1997; Oda and Kazama 1998; Batiste et al. 2004; Lings and Dietz 2004; Wang et al. 2007; Amirpour Harehdasht et al. 2017b).

Different boundary constraints and developments on the direct shear apparatus have been suggested to obtain peak shear parameters very close to that of an ideal simple shear (Jewell and Wroth 1987; Jewell 1989; Shibuya et al. 1997; Lings and Dietz 2004). In this study, the performance of the symmetrical direct shear apparatus had been optimized by exploring different test configurations (e.g., fix load pad to the upper frame, modified collar attachment, initial gap between the shear-box frames, and the use of thin membrane laid over the interior walls of the frames) to the 55 (mm) $\times$ 55 (mm) direct shear apparatus. These modifications were

suggested to ensure the accuracy of results obtained during shearing. The characteristic most commonly addressed in these studies is the tendency for the load pad and upper frame to rotate during conventional testing. The mechanical details of the shear apparatus developed and examined for this study are described in Amirpour Harehdasht et al. (2017b). During the test, shear load is applied to the lower section of the box in the displacement-controlled fashion at the level of the mid-plane of the sample with the constant velocity of 0.0051 mm/s, while the upper part was restrained against horizontal movement.

On the other hand, many experimental and numerical studies have been conducted on the direct shear test to investigate the distribution of the stresses and strains within the final failure zone (e.g., Potts et al. 1987; Ni et al. 2000; Masson and Martinez 2001; Cui and O'Sullivan 2006; Wang et al. 2007; Zhang and Thornton 2007). In line with these studies, Amirpour Harehdasht et al. (2017b) also discussed the reliability of boundary measurements in physical symmetrical direct shear tests used in this study using the DEM computer code SiGran (Roubtsova et al. 2011, 2015). Some major conclusion of these studies include: (a) the orientation of the principal stress at the peak state is coaxial with that of the principal strain increment (e.g., Jewell and Wroth 1987; Potts et al. 1987; Masson and Martinez 2001; Wang et al. 2007; Amirpour Harehdasht et al. 2017b); (b) stresses and strains within the final failure zone are fairly uniform to be described by a single state of stress and strain increment, and progressive failure effects are minor (e.g., Jewell and Wroth 1987; Potts et al. 1987; Wang et al. 2007; Amirpour Harehdasht et al. 2017b); (c) the peak shear strength from DSA is very close to that obtained in an ideal simple shear condition, and the deviation of the zero linear extension direction at peak from the horizontal is negligible (e.g., Oda and Konishi 1974; Jewell and Wroth 1987; Potts et al. 1987; Wang et al. 2007; Zhang and Thornton 2007; Amirpour Harehdasht et al. 2017b); (d) below the certain value of the box size to the maximum particle diameter ratio $L/D_{\max}$, the measured friction parameters are reactive to the $L/D_{\max}$ ratio due to the lack of room for the shear zone to develop fully in the box (e.g., Scarpelli and Wood 1982; Palmeira and Milligan 1989; Cerato and Lutenegeger 2006).

While the first three factors, (a) to (c), were extensively studied in Amirpour Harehdasht et al. (2017b), this paper re-examines the specimen scale effect on the macroscopic shearing behavior and the extent of localized shear zone developed inside the adopted 55 (mm) × 55 (mm) symmetrical direct shear apparatus. In addition, the distribution of void ratio inside the sample

and its accurate determination, considering the irregularity of void ratio profile in contacts, are considered in this study while analyzing the shearing behavior of granular materials. These issues are thought to be critical to the main goal of the current research with respect to the determination of dilation-strength behavior of granular material using an appropriate direct shear apparatus.

In this study, all samples of the granular materials were prepared and sheared in dry conditions. For each grading curve, 12 direct shear box tests were performed at four different normal pressures (50, 100, 200 and 400 kPa) and three relative densities ($D_R = 50\%$, 70%, 90%) giving a total of 276 tests. The maximum and minimum void ratios of each distribution were measured in the dry state, according to the method specified by Muszynski (2006). Many conventional (e.g., ASTM 2001a; b; c) and alternative methods (Youd 1973; Oda 1976; Yoshimine et al. 1998) of obtaining limit densities exist, but they generally require a relatively large specimen and were therefore not applicable to the small amount of material available for each distribution (maximum 150 grams). The minimum and maximum void ratios obtained are also summarized in Table 1. Each direct shear specimen, with intended D_R value, was prepared by laying down the dry mixtures carefully in the shear box (to prevent particle segregation) followed by applying the mechanical compaction on the soil deposit. The state of the compactions was monitored by measuring the sample height, until the required density was achieved.

The Effect of Boundaries on Distribution of the Void Ratio inside the Sample

Evaluation of the uniform distribution of void ratio inside the sample and the accurate determination of void ratio that controls the shear in the sample are crucial factors while assessing the strength parameters of a soil deposit during direct shearing. The rigid boundary-particle interface induced due to this method of sample preparation may exhibit local ordering behavior near the walls. This would influence the void ratio of the material in contacts, especially in boundary region above and below the sample. The irregularity in void ratio profiles may extend for a length scale of about several sphere diameters away from the wall (e.g., Roblee et al. 1958; Benenati and Brosilow 1962; Goodling and Khader 1985; Chan and Ng 1986; Hardin 1989; Reyes and Iglesia 1991; Marketos and Bolton 2010; Huang et al. 2014). For example, Benenati and Brosilow (1962) and Goodling and Khader (1985) detected the local oscillations of porosity up to a distance of five sphere diameters from the wall. According to

Roblee et al. (1958) and Reyes and Iglesia (1991), for a randomly packed sample with regular shaped particles, the porosity decreases regularly from one at the wall to the average porosity at about two particles diameters. Hardin (1989) suggested that for container sizes normally used in practice, boundary effect are negligible for silt and clay soils and for cohesionless soils containing more than 10% finer than 0.074 mm. Marketos and Bolton (2010) observed a porosity variation of 2-3% relative to the mean in the regions extend as far as 2 and 6 diameters from bottom and top rigid boundaries, respectively. The influence of the wall becomes less important as particles irregularity increases (e.g., Roblee et al. 1958), the grain size distribution broadens (e.g., Goodling and Khader 1985; Hardin 1989; Reyes and Iglesia 1991), or the density of the samples decreases (e.g., Thadani and Peebles 1966).

In this study, an investigation was conducted based on the DEM simulations of C_5 polydisperse sample at a constant relative density D_R of approximately 90%. According to the literature reviewed above, the C_5 polydisperse dense sample would be subjected to the most local void ratio irregularity at the boundaries among other distributions in this study. These simulations provided data on the distribution of void ratio inside the sample in comparison with the void ratio calculated at the boundaries. Three-dimensional (3D) dense sample of 102248 basalt spheres was subjected to different normal pressures (50, 200, and 400 kPa) and sheared in virtual symmetrical DSA using the computer code SiGran (Roubtsova et al. 2011, 2015, Amirpour Harehdasht et al. 2017b). To achieve confident conclusions about the micromechanics response of samples in direct shear tests, quantitative validations of the DEM models were attempted by relating the physical tests to the corresponding DEM simulations. Details can be found in Amirpour Harehdasht et al. (2017b).

To determine the void ratio distribution inside each sample, its volume was divided into slices which extended throughout the whole sample in three directions (Fig. 4). In our simulations, each slice had a thickness of 1000 μm , equal to the D_{50} of C_5 distribution. The void ratio within a given slice was accurately determined in SiGran by introducing the solid volume as the summation of the volume of fully-enclosed particles and that of intersected boundary particles whose fractions outside the zone were subtracted. The distribution of void ratio of each sample before and during shearing are shown in Fig. 5.

Two different mean void ratio values are presented in Fig. 5, using void ratio results of all slices. The overall average void ratio was calculated considering the voids created adjacent to the rigid

walls while the inner average void ratio was calculated excluding the voids ratio of the close to the walls. It is clear from Fig. 5 that the void ratios in the boundary regions are higher than the overall and the inner average values. Fig. 5 shows also a heavily damped decaying behavior extending from the walls to a length of about 2 to 3 D_{50} of the soil sample. However for the three values of normal stresses considered, the overall average values are only around 1% higher than the inner average values before shearing, increase to less than 2% during the peak and ultimate states. In addition for each simulation, the void ratio calculated from the boundary measurements, using the weight and total volume of the shear box sample, is located between two average values and deviated less than 0.5%, 1%, and 1.5% from inner average value before shearing, during the peak, and ultimate states, respectively.

Macro-Scale Experimental and Micro-Scale DEM Simulations of Direct Shear Specimen Scale Effects

Since Parsons (1936), specimen scale effect in the direct shear apparatus on the shear strength parameters of granular soils has been examined through experimental and numerical studies (e.g., Scarpelli and Wood 1982; Mühlhaus and Vardoulakis 1987; Palmeira and Milligan 1989; Oda et al. 2004; Cerato and Lutenegeger 2006; Jacobson et al. 2007; Zhang and Thornton 2007; Gutierrez and Wang 2010). The scale effect problem arises when the artificial restraints, introduced by the shear box rigid boundaries, effect the deformation and strength behavior of soils during shearing. According to these studies, two specific issues should be carefully evaluated, namely the effect of box size on the macroscopic shear stress ratio and the extent of localized shear zone developed inside the box. The first issue determines and recommends an appropriate DSA size that could yield the true shear strength parameters of a soil (e.g., Palmeira and Milligan 1989; ASTM 2001d; Cerato and Lutenegeger 2006; Zhang and Thornton 2007), whereas the second issue essentially concerns the mechanism responsible for the observed scale effects (e.g., Roscoe 1970; Scarpelli and Wood 1982; Mühlhaus and Vardoulakis 1987; Oda et al. 2004; Alshibli and Hasan 2008; Gutierrez and Wang 2010). Studies in the past three decades showed that the measured friction angle generally decreases or remains constant with increasing box size above a certain ratio of the box size to particle diameter, depending on the type of sand and the relative density (e.g., Jewell and Wroth 1987; Cerato and Lutenegeger 2006; Gutierrez and Wang 2010). While, micromechanics-based analysis indicated that the local and global aspects of fabric change and failure are the major mechanisms responsible for the

specimen scale effect (e.g., Scarpelli and Wood 1982; Potts et al. 1987; Oda and Kazama 1998; Oda et al. 2004; Wang et al. 2007; Gutierrez and Wang 2010).

To evaluate the potential scale effects on strength parameters obtained from the $55 \times 55 \times 30$ (mm^3) shear box, the macroscopic results of C_5 samples of basalt beads and pérignon sand, with the largest particle diameters among other distributions, were compared with those of the $88.5 \times 88.5 \times 40$ (mm^3) shear box. While the length, the height, and the aspect ratio ($L/D_{\max}$) of the small box were approved as the suitable dimensions to perform direct shear test (e.g., Parsons 1936; Palmeira and Milligan 1989; Oda and Kazama 1998; ASTM 2001d), some studies (e.g., Cerato and Lutenegger 2006; Zhang and Thornton 2007; Gutierrez and Wang 2010) recommended using the bigger box to avoid scale effect during shearing. For both groups of materials, the dense samples ($D_R = 90\%$) were sheared at the same rate, under normal stresses of 50, 100 and 200 kPa. The effects of box size on the shear stress ratio τ/σ_n , and the vertical displacement against the shear displacement of basalt beads and pérignon sand are shown in Fig. 6. It can be seen for a given material and applied normal stress that there is a slight difference of 3% maximum between peak stress ratios obtained from both boxes, and the shear displacement corresponding to the peak stress ratios are close to each other. This means that similar magnitudes of boundary displacement are required to fully mobilize material strength. On the other hand, at the peak state, the dilation rate induced by the local fabric change is nearly the same for both cases at a given normal stress, as suggested by similar values of vertical displacements of the top wall incurred at each peak state. In other words, the extent of the granular portion which is influenced by the boundary movement to establish a shear band and sustain the maximum stress until the peak stress state was slightly changed by the shear box size.

This study also employed the micromechanics-based analysis to examine either the mechanism underlying the formation of the fabric inside the shear zone is uniform or the peak state ensues from the progressive failure inside the $55 \times 55 \times 30$ (mm^3) samples. For this purpose, the formation mechanism of the shear band inside the samples with particle size distribution C_5 was visualized through the spatial distribution of particle rotations (e.g., Dyer and Milligan 1984; Jewell and Wroth 1987; Potts et al. 1987; Bardet 1994; Oda and Kazama 1998; Zhang and Thornton 2007; Amirpour Harehdasht et al. 2017b) together with the void ratio along their lengths and heights (e.g., Oda and Kazama 1998; Batiste et al. 2004; Oda et al. 2004), which

were determined using the DEM computer code SiGran method previously developed by (Roubtsova et al. 2011, 2015). The former was extensively explained in Amirpour Harehdasht et al. (2017b), where the rotational distribution energy of the three simulations under normal stresses of 50, 200 and 400 kPa at their peak and ultimate stress states were clearly represent a pronounced and concentrated shear band in the mid-plane of the box.

In the literature, significant research efforts have focused on tracking the shear band formation using theoretical derivations (e.g., Mühlhaus and Vardoulakis 1987; Vardoulakis and Aifantis 1991) and experimental observations (e.g., Roscoe 1970; Oda and Kazama 1998; Batiste et al. 2004; Oda et al. 2004; Alshibli and Hasan 2008). According to Oda and Kazama (1998), Oda et al. (2004) and Alshibli and Hasan (2008), the localization of the shear band is mainly characterized by the formation of extremely large voids which caused by the large dilatancy taking place inside the shear zone. In Fig. 5, the formation of the shear band is manifested in the increasing of the void ratio on three directions, during peak and ultimate states. Comparing the void ratio variation along the height of the samples at peak and initial states reveals that the increase of the void ratio is confined to a zone of $10-12 D_{50}$, with the peak at the mid-plane of the box, while remains unchanged close to the top and bottom boundaries during the shearing. The thickness of the shear zone is in agreement with literature, suggesting the thickness of $10 D_{50}$ (e.g., Roscoe 1970; Bridgwater 1980; Scarpelli and Wood 1982; DeJaeger 1991; Cerato and Lutenege 2006; Gutierrez and Wang 2010). The increase of the void ratio in y direction is also accompanied by the steady increase of void ratio of slices along the length and width of the samples, confirming the formation of uniform shear zone perpendicular to these directions. The uniform distribution of particle rotational energy in Amirpour Harehdasht et al. (2017b) and void ratio inside the virtual samples in current study, support Jewell and Wroth (1987) and Potts et al. (1987) statements that within the shear zone the deformation is fairly uniform and progressive failure effects are minor.

Test Results

Investigating the Effect of Grading Characteristics on the Direct Shear Friction Angle and the Dilation Angle

To efficiently evaluate and improve Eqs. (1) – (3) to reflect the particle characteristics (e.g. particle size, particle shape, and particle gradation), the peak direct shear friction angle ϕ_{ds} and

the maximum dilation angle ψ have to be deduced directly from the direct shear tests obtained from boundary measurements through Eqs. (4) – (5). ϕ_{ds} is obtained from boundary measurements of the average horizontal shear stress τ_{xy} and the average vertical normal stress σ'_{yy} , using:

$$\tan \phi'_{ds} = \frac{\tau_{yx}}{\sigma'_{yy}} \quad (4)$$

Assuming that all dilations occur within a uniform shear zone of thickness t (e.g., Oda and Konishi 1974; Oda and Kazama 1998; Batiste et al. 2004; Lings and Dietz 2004; Zhang and Thornton 2007), ψ can be estimated from the rate of change of vertical displacement, v_y , with respect to horizontal displacement, v_x through:

$$\tan \psi = \frac{-d\varepsilon_{yy}}{d\gamma_{yx}} = \frac{dv_y/t}{dv_x/t} = \frac{dv_y}{dv_x} \quad (5)$$

The variations of ϕ_{ds} as a function of D_{50} for all the distributions of basalt beads are presented in Fig. 7a at different combination of normal pressures and initial relative densities. It is clear from the results presented in this figure that the material peak strength decreased with the increase of D_{50} at the dense state ($D_R = 90\%$) and the lowest normal pressure of 50 kPa. With the increase in the applied normal stresses and the initial void ratios of the tested samples, the peak strengths converged towards an approximate constant value irrespective of the D_{50} of interest. At the same D_{50} , Fig. 7a shows also that the increase in C_u yields a minor decrease in ϕ_{ds} ; namely within 2 degrees, in agreement with earlier results reported by Kirkpatrick (1965) and Zelasko et al. (1975) who observed the minor effect of C_u on the shear strength of granular materials. The ψ values for all distributions at two different combinations of normal pressures and initial relative densities are presented in Fig. 7b. As the majority of samples exhibited contractive behavior at the highest normal stress (400 kPa) and the lowest relative density ($D_R = 50\%$), the ψ results of samples at this condition is not presented in this figure. Similar to the results presented in Fig. 7a, it is obvious from Fig. 7b that the ψ values are also influenced by particle size, as well as the material initial density and stress level. The experimental data shows also that for a given D_{50} , ψ decreased steadily as a function of the normal pressure with the medium-dense samples had a much lower ψ than the dense samples.

At large horizontal displacement when the sample is sheared at a constant volume, the critical state friction angle $(\phi_{cv})_{ds}$ is mobilized. This parameter represents the minimum shear strength the soil can display. An appropriate evaluation of $(\phi_{cv})_{ds}$ as a fundamental soil parameter constitutes an important step towards a robust development/validation of any strength-dilatancy formulation. In direct shear tests, the critical state shearing resistance can be obtained through several methods using the results of single or multiple tests. Among the five different methods available in the literature and presented in Table 2, methods c and d using multiple tests were suggested by many researchers (e.g., Bolton 1986; Jewell 1989; Pedley 1990; Simoni and Houlsby 2006) to evaluate the $(\phi_{cv})_{ds}$ values, since they minimize the influence of errors compared to the single test methods. In addition, direct shear test can produce meaningful data at peak state and using the measurements taken at this point are expected to be more reliable than at large strains as required for method a. Following the procedures of methods c and d, $(\phi_{cv})_{ds}$ was determined for each distribution from a series of tests at different combination of relative densities and normal pressures.

The variations of $(\phi_{cv})_{ds}$ with D_{50} are presented in Fig. 7c. Fig. 7c shows that the maximum difference between $(\phi_{cv})_{ds}$ values obtained from methods c and d are less than 0.5 degrees, and both the methods show the independency of $(\phi_{cv})_{ds}$ to the particle size distribution elements, D_{50} and C_u . This observation is in agreement with previous studies, which illustrated the dependency of $(\phi_{cv})_{ds}$ only on the mineral constituting the particles (e.g., Wijewickreme 1986; Negussey et al. 1988).

Conversion of Direct Shear Strength Parameters to Plane Strain Shear Strength Parameters

Following the assumptions discussed in previous sections referring to the reliability of boundary measurements in symmetrical direct shear tests to interpret the shearing resistance of granular materials, the direct shear measured boundary stresses and the Mohr's circle of strain increments can be combined to determine the plane strain frictional shearing resistance of granular material ϕ_{ps} from the geometry of Mohr's circle of stress (Davis 1968). Jewell and Wroth (1987) developed a relationships between plane strain friction angle ϕ_{ps} , direct shear friction angle ϕ_{ds} , and dilation angle ψ by introducing Davis's (1968) equation and Rowe's (1962) stress-dilatancy relationship. An improved version of this framework is given by Lings and Dietz (2004). Jewell and Wroth (1987) suggested that the direct shear angle of friction calculated from

the stress ratio at failure on the horizontal plane is smaller than the plane strain angle of friction, and these two angles of friction can be linked through the theoretical relation:

$$\sin\phi_{ps} = \frac{\tan\phi_{ds}}{\cos\psi(1 + \tan\psi \tan\phi_{ds})} \quad (6)$$

In addition, the plane strain friction angle at the critical state $(\phi_{cv})_{ps}$ can be obtained for zero dilation angle from:

$$\tan(\phi'_{cv})_{ds} = \sin(\phi'_{cv})_{ps} \quad (7)$$

However, some researchers (e.g., Arthur and Assadi 1977; Oda 1981; Tatsuoka et al. 1986; Bolton 1987; Arthur and Dunstan 1988) pointed out that other than the factors (a)-(d) which mentioned in previous sections, a parameter expressing the anisotropy in strength and deformation characteristics also has to be taken into account while comparing the strength parameters obtained from the direct shear and plain strain compression tests. Following the significant work of Bolton (1986), the necessity of considering the strength anisotropy in deriving empirical relationships between ψ and ϕ_{ps} in Eq. 1 (Tatsuoka et al. 1986; Bolton 1987), and also between ϕ_{ds} and ϕ_{ps} in Eq. 6 (Tatsuoka et al. 1986; Arthur and Dunstan 1988) were extensively discussed for various cohesionless soils.

To examine the efficiency of Eq. 6 for anisotropic cohesionless soils, Tatsuoka et al. in (Arthur and Dunstan 1988) compared shearing parameters of Tayoura sand specimens with bedding at $\delta = 0^\circ$ - 90° (where δ is the angle of σ_1 direction from the bedding plane) in plane strain and simple shear conditions. Their results showed that the shear strength close to minimum is attained in both conditions, where the horizontal plane is the bedding plane of the specimen. In addition, for a given σ_3 , δ , $b=(\sigma_2-\sigma_3)/(\sigma_1-\sigma_3)$ values, the ratio between the measured plane strain to direct shear friction angles is of the order of 1.2 as suggested by Jewell and Wroth (1987). They showed that the theoretical relationships (Eqs. (6) – (7)) between the angle of friction in direct shear ϕ_{ds} and plane strain ϕ_{ps} conditions work well for angle δ smaller than 40° , while ϕ_{ps} is underestimated for angles δ greater than 40° , and up to 90° .

To evaluate if the proposed equations can be used in the current study to determine the friction angles in plane strain conditions from direct shear tests results, the angle of the σ_1 direction from the bedding plane in direct shear tests was quantified by using the data registered during the numerical simulations in the micro-scale (Roubtsova et al. 2011, 2015; Amirpour Harehdasht

et al. 2017b). In addition, referring to the work by Bathurst and Rothenburg (1990), one may use the second-order form of Fourier series expansion to approximate the directional distribution of micro quantities of fabric anisotropy, force anisotropy, and the angle between the principal fabric direction and the applied loading direction. The particle-scale data obtained from discrete element simulations of direct shear tests (Amirpour Harehdasht et al. 2017b) showed that under normal stresses of 50, 100 and 200 kPa, the distribution functions of these micro quantities appear to well represent the trends of the computed data of Bathurst and Rothenburg (1990) in all stages. So that, the principal force and fabric directions became coaxial with the loading direction at peak stress state, deviated 35° – 40° from the horizontal bedding plane of the sample. In other words, the strength parameters obtained by the symmetrical direct shear test can be accurately used to obtain the theoretical plane strain friction angles in which the σ_1 direction with respect to the bedding plane δ is less than 40° , similar to that of the direct shear sample at failure. An example of calculating the direct shear strength parameters of basalt beads and converting them to their counterparts in plane strain condition are presented in Fig. 8 for distribution C₁ to C₅. Similar procedures were followed for pérignonka and Eastmain sands.

Evaluating the Contribution of Particle Size Distribution to Bolton's Flow Rule

In this Section, a systematic investigation has been done to examine if the maximum shear resistance of granular materials can be described by a unique function of maximum dilation or dilatancy index, regardless of particle characteristics. As a first step and in a way similar to that presented in Fig. 8, the shear strength parameters of each symmetrical direct shear test was converted to its plane strain equivalent using Eqs. (6) – (7), assuming δ angles less than 40° for both conditions. Thereafter, the converted ϕ_{ps}^{meas} values of all tests made up of basalt beads and sands were plotted in Fig. 9 versus the ϕ_{ps}^{pred} values estimated using Eqs. (1) – (3). As suggested by Bolton (1986) for quartz sands, the Q parameter equal to 10 were considered for Pérignonka and Eastmain sands. Comparing the yield stresses $\overline{\sigma}_0$ of basalt and quartz sand made up of rounded particles, which are both around 10 MPa (Althufi and Coop 2011), suggested using the Q parameter equal to 10 for tested basalt bead distributions. It worth to mention that 25% of the tested samples showed contractive behavior at high normal stresses and low relative densities. Similar procedure was applied on symmetrical direct shear tests results of material made up of particles with different angularities collected from literature, and the outcomes

were also presented in Fig. 9. The δ angle was verified for collected data where Eqs. (6) – (7) were applied. The scatter of data in Fig. 9 are somewhat large confirming the imperfection of Bolton's equations for an accurate evaluation of ϕ_{ps} based on the constant proportion of dilatancy. Figs. 9a and 9c show that most of the calculated ϕ_{ps} values using Eq. (1) fall within around 23% deviation from the measured values. The corresponding data using Eq. (3) (Figs. 9b and 9d) shows lesser degree of agreement between the calculated and the measured ϕ_{ps} values with even up to 45% of deviation.

At the first look, it is likely that the potential source of scatters in Fig. 9 may be attributed to the fact that Eqs. (1) – (3) were developed neglecting the explicit consideration of the soil grain characteristics. To examine the sensitivity of Eqs. (1) – (3) to particle size and grading, the percentage error (%Error) in predicting the value of ϕ_{ps} using Eqs. (1) – (3) was calculated for each test as the percentage deviation of ϕ_{ps}^{pred} from ϕ_{ps}^{meas} , $[(\phi_{ps}^{meas} - \phi_{ps}^{pred}) / \phi_{ps}^{meas}] \times 100$. The average percentage prediction errors for all tests on a given particle-size distribution were then calculated for three different group of ϕ_{ps}^{pred} values: (1) (Ave-%Error)_{All} represents the average percentage prediction errors of all tests performed on each particle-size distribution; (2) (Ave-%Error)_{< ϕ_{Ave}} and (3) (Ave-%Error)_{> ϕ_{Ave}} are the averages percentage errors for ϕ_{ps} values which are respectively smaller and larger than $(\phi_{ps})_{Average}$, where $(\phi_{ps})_{Average}$ is the average of all ϕ_{ps} values obtained for a given particle-size distribution. These three introduced (Ave-%Error) values were calculated for all distributions and presented in Fig. 10.

The comparison of (Ave-%Error) values in Fig. 10 clearly demonstrates the effect of particle-size distribution on the deviation between measured and predicted ϕ_{ps} values using Eqs. (1) – (3). The Scrutiny of the error bars plotted in Fig. 10a reveals the increasing trend of (Ave-%Error) values with the increase of D_{50} for all samples made up of rounded and angular particles. Besides, Fig. 10a shows the increase of (Ave-%Error) values with the increase of ϕ_{ps} for a given particle-size distribution. The difference between (Ave-%Error)_{> ϕ_{Ave}} and (Ave-%Error)_{< ϕ_{Ave}} is affected by the particle-size distribution, so that their deviation increases more than 8% and 5%, respectively for basalt beads and angular sands from series 1 ($D_{50} = 106 \mu m$) to series 5 ($D_{50} = 1000 \mu m$). Comparing the (Ave-%Error) values for each series of particle-size distribution in Fig. 10a, indicates that the (Ave-%Error) values decreases with increasing angularity of the grains so that for a given particle-size distribution, the (Ave-%Error) values for sands composed of angular

grains are smaller than those of basalt beads. However, as mentioned above the sensitivity of (Ave-%Error) values to particle-size effect still exist for angular material.

In Fig 10b, the (Ave-%Error) values predicted by Eq. (3) have been supplemented. In general, the (Ave-%Error) values presented in Fig 10b are found to be larger than those predicted by Eq. (1), but similarly having the increasing trend with the increase of D_{50} . It is apparent from Fig 10b that the (Ave-%Error) values using Eq.(3) are more pronounced for basalt beads than angular sands. Note that the maximum variation of (Ave-%Error) values due to the increase of D_{50} is roughly 3.5% for angular sands.

These observations clearly demonstrate the importance of the grain-size characteristics while predicting ϕ_{ps} values with Bolton's Equations. Thus, an appropriate revising of the constant coefficient in Eqs (8) – (9) is needed based on various particle-size distribution elements to quantify their effects in the correlation between the maximum shear resistance and dilation.

Implementing the Particle Size Parameters into Eqs (1) – (3), Over a Wide Range of Stresses and Densities

The above discussion showed that when employing the constant proportion of dilatancy or dilatancy index in Bolton's Equations, the ϕ_{ps} values are over- or under-predicted. In addition, a hypothesis is established based on the initial investigation of ϕ_{ps} values obtained from Eqs. (6) – (7), using symmetrical direct shear data in that disregarding the particle characteristics in Bolton's Equations with constant coefficients might be the source of scattering and over-prediction of ϕ_{ps} values. Similar to Fig. 8, the plane strain data for the three tested materials, obtained from Eqs. (6) – (7), can be alternatively plotted as a correlation between the $(\phi_{ps}-\phi_{cv})$ and maximum dilation angle, ψ . By fitting the points to linear regression lines for each D_{50} , it can be seen that the results of the tests on each series, regardless of C_u , were consistent in terms of $(\phi_{ps}-\phi_{cv})$ and ψ (Fig. 11). However, the range of the obtained b values is not coincident with the Bolton's equation $b=0.8$ for plane strain condition, and is observed to decrease as a function of the increase in D_{50} , for materials made up of rounded to angular particles. Moreover for series 2 to 4, the regression lines for the $(\phi_{ps}-\phi_{cv})$ - ψ data of particle size distribution curves with equal D_{50} but different C_u fall together, showing the C_u -independence of $(\phi_{ps}-\phi_{cv})$ values for $1.5 < C_u < 5$. An example of C_u -independence of $(\phi_{ps}-\phi_{cv})$ - ψ data of series 3 of basalt beads is presented in

Fig. 11b. This observation is in agreement with Amirpour Harehdasht et al. (2017a), where the range of the obtained b values in triaxial condition was not coincident with the constant $b=0.33$ in Vaid and Sasitharan' (1992) equation, and is observed to decrease as a function of the increase in D_{50} , independent of C_u variation, for materials made up of rounded to angular particles.

To clarify the causes of the drop in coefficient b with D_{50} , Amirpour Harehdasht et al. (2017a) examined the sensitivity of each side of these equations to D_{50} over a wide range of stresses and densities during triaxial compression tests, using the state parameter proposed by Been and Jefferies (1985), define as $(e-e_{cr})$. This parameter combines the influence of void ratio and the stress level with reference to a steady state and allows the quantification of many aspects of granular material behavior using a single parameter. Amirpour Harehdasht et al. (2017a) showed, for a given state parameter $(e-e_{cr})$, that $(\phi_{ps}-\phi_{cv})$ decreases by more than ψ as D_{50} increases, causing a significant drop in the proportion b of the contribution of dilation to peak shear resistance. As shown in Fig. 12a, for distributions with $63 \mu\text{m} < D_{50} < 2000 \mu\text{m}$, the correlation between $(\phi_{ps}-\phi_{cv})$ and ψ can be described as Eq. (8), where parameter b varies with D_{50} as:

$$\phi_{ps} - \phi_{cv} = b \psi, \quad b = c_1 (D_{50})^{-c_2} \quad (8)$$

The decrease of b value with increasing D_{50} is described by a power function (Eq. (8)), with constant $c_1=1.60$ and $c_2=0.16$ for material made up of rounded particles. These constants decrease to $c_1=1.28$ and $c_2=0.09$ with increasing the particle angularity. The parameters c_1 , c_2 , and b are calculated from Eq. (8) for each series considered and summarized in columns 6-8 of Table 1.

The ϕ_{ps} values predicted by Eq. (8) with the parameter $b(D_{50})$ are plotted versus the measured ones in Figs. 13a-13b. Since the deviation of the data from the line $\phi_{ps}^{\text{pred}} = \phi_{ps}^{\text{meas}}$ are small, averaged 0.77 ± 0.54 degrees for basalt beads and 1.12 ± 0.85 degrees for sands, the good prediction of the proposed correlation is confirmed. Note that the average variation of the predicted values from the measured ones is roughly 6% for both rounded beads and angular sands. Similar observation was also achieved for symmetrical direct shear data collected from literature. A comparison between the ϕ_{ps} values predicted by Bolton's Eq. (1) with its commonly used constant and those estimated by Eq. (8) using the coefficient b as a function of D_{50} shows

that, for particle-size distribution with large D_{50} of 1000 μm , Bolton's Eq. (1) predicts ϕ_{ps} of 1.14 ± 0.046 (rounded particles) and 1.11 ± 0.040 (angular particles) times larger than ϕ_{ps} values obtained from Eq. (8). For $D_{50}=600 \mu\text{m}$, these factors are reduced to 1.11 ± 0.046 and 1.09 ± 0.021 in rounded and angular particles, respectively. The deviation for $D_{50}=106 \mu\text{m}$ is further reduced by factors of 1.07 ± 0.012 and 0.095 ± 0.015 in rounded and angular particles, respectively. The increase of the deviation between ϕ_{ps} values obtained from Bolton's equation and those estimated using Eq. (8) with D_{50} is in agreement with observations illustrated in Fig. 10.

Similar to the Bolton's Eq. (3), a correlation of $(\phi_{ps}-\phi_{cv})$ with the dilatancy index I_R was developed. The functions were fitted to the data for each particle-size distribution of basalt bead and sands and presented in Figs. 14c-14d. The decrease of $(\phi_{ps}-\phi_{cv})$ with the increase of D_{50} for a given I_R value is also apparent in these figures. Figs. 14c-14d reveal that the coefficient c describing the density and pressure dependence of $(\phi_{ps}-\phi_{cv})$ values varies in the range of $2.51 < c < 5.59$ with respect to the particle-size and angularity. A new correlation between $(\phi_{ps}-\phi_{cv})$ and the relative dilatancy index I_R was developed for all tested series. In Fig. 12b, the parameter c is observed to decrease with the increase in D_{50} and cannot be considered as constant equal to 5 for plane strain condition as proposed by Bolton (1986). According to Fig. 12b, the parameter c can be expressed as a function of D_{50} as:

$$\phi_{ps} - \phi_{cv} = f(D_R, p') = c I_R, \quad c = c_3 (D_{50})^{-c_4} \quad (9)$$

This function was fitted to the data of the tested materials, resulting in constants $c_3=18.89$ and $c_4=0.30$ for materials made up of rounded particles. These constants decreased by increasing the angularity of the particles to $c_3=13.56$ and $c_4=0.21$ for materials with sub-rounded to sub-angular particles, and to $c_3=10.08$ and $c_4=0.13$ for materials with sub-angular to angular particles. The parameters c_3 , c_4 , and c are calculated from Eq. (9) for each series considered and summarized in columns 9-11 of Table 1. As apparent from Figs. 13c-13d the difference between the values of ϕ_{ps}^{pred} and ϕ_{ps}^{meas} decreased considerably if Eq. (9) is used (maximum error of 12% compared to 45% using Bolton's equation with constant c). However, the prediction of ϕ_{ps} by Eq. (9) is a relatively less accurate (averaged 1.14 ± 0.88 degrees for basalt beads and 1.67 ± 1.1 for sands) compared to the prediction of Eq. (8), especially for sands made up of angular particles.

Decreasing the b value with the increase of D_{50} is attributed to the formation of columnar structure of particles and large voids in the shear band, during shearing deformation. Many physical tests, as well as computer simulations, have been carried out to study this micro mechanism of deformation in recent years (e.g., Desrues et al. 1996; Oda and Kazama 1998; Otani et al. 2000; Batiste et al. 2004; Oda et al. 2004; Alshibli and Hasan 2008). According to these studies, in the strain-hardening process of granular soils, the main microstructural change is the setting up of columns parallel to the major principal stress direction, and generating the large voids between the arching actions of particles in the shear zone limits. According to some researches (e.g., Oda and Kazama 1998; Batiste et al. 2004; Oda et al. 2004; Alshibli and Hasan 2008), while the formation of these large voids is disproportionate to the particle characteristics and caused by the large dilatancy taking place inside a shear band, the frictional behavior of a granular sample highly depends on the number of grains within a given force chain surrounding the generated voids (e.g., Morgan 1999; Mair et al. 2002; Anthony and Marone 2005). Mair et al. (2002) and Anthony and Marone (2005) experiments suggested that depending on the size and shape of particles the number of the particles involved in the structure of a given size of columns would change, so that the sets of localized particle chains, or diffuse webs of fewer load-bearing particles can be formed in the shear zone. Therefore, the geometry of stress distribution throughout the shear zone would not be similar. In other words, for a given amount of generated void in shear zone, depending on the formation of chain forces, each particle would carry low to high local contact stress, and the frictional strength would be proportional to the number of constituent particles of columns in shear zone. The difference in columnar arrangement can also be observed in samples at a given initial state and with different particle characteristics. For instance, for dense samples with small D_{50} , small particles dominate the columnar structures, so that highly concentrated stress chains of large number of small particles with similar orientations are surrounded the voids in shear band (e.g., Morgan n.d., Oda and Konishi 1974; Oda and Kazama 1998; Alshibli and Hasan 2008). The slippage of the individual particle would not disturb the arrangements of formed column. These chains likely collapse under approximately the same shear displacement, corresponding to the maximum shear stress ratio, and failure subsequently occurs through the slip along a plane of particles accompanied by macroscopic stress drop (e.g., Morgan 1999; Mair et al. 2002). In dense samples with a larger D_{50} , arching actions that confine the voids are dominated by limited number of large particle pairs. When stresses built up, the particle arches can easily buckle and be reformed repeatedly

by the move of single particles, generating diffuse webs of load-bearing particles. So that, the slip zone was more scatter and the deformation in these samples is more erratic than in samples with a smaller D_{50} (e.g., Morgan 1999; Oda and Konishi 1974; Oda and Kazama 1998; Mair et al. 2002; Anthony and Marone 2005). The micro-scale configuration of these columns in terms of their size and length is manifested in parameter c_1 (decreased by the diffusion in columnar structure), which together with macro-scale measured parameter D_{50} , would led to the dimensionless parameter b in Eq. (8). In other words, the shear strength and dilation behavior of sample with uniform shear band with the thickness proportional to the particle sizes, where progressive failure is not likely to occur, are not affected by the scale effect of the shear box, but controlled with the micro mechanism defines by the extend of the arches surrounded the voids in the shear band. Considering all explanation above the difference between frictional behaviors of a granular samples due to the variation in the number of grains within force chains throughout the shear zone can be manifested in coefficient b and c in Eqs. (1) and (3), and This Equations can be rewritten as:

$$\phi_{ps} \propto N_{fc} = \frac{L_0 = f(shape, D_{50})}{D_{50}} \quad (10)$$

$$\phi_{ps} - \phi_{cv} = b \psi \rightarrow \phi_{ps} - \phi_{cv} = (a L_0 / D_{50})^{c_2} \psi \quad (11)$$

$$\phi_{ps} - \phi_{cv} = c I_R \rightarrow \phi_{ps} - \phi_{cv} = (d L_0 / D_{50})^{c_4} I_R \quad (12)$$

Where $N_{fc} = L_0 / D_{50}$ is average number of particles forming the force chains within the shear zone. a and d are constants that are different for materials made up of particles with different characteristics.

As stated before in this manuscript and also in Amirpour Harehdasht et al. (2017b), the shear strength and dilation behavior of sample were not affected by the scale effect of the symmetrical shear box. In such a case, the average length of force chains within the uniform shear zone as a function of D_{50} can be considered as a hypothetical quantity to produce a dimensionless ratio in Eqs. (11) and (12). This would greatly benefit from further numerical and experimental scrutiny to discover the role of any other probable dimensionless group of material properties, including particle size, to acquire a dimensional balance between two sides of the Eqs. (8) and (9), and this work provides comprehensive data to the geotechnical community to aid in that process.

Comparing the measured ϕ_{ps} data from the literature with ϕ_{ps} predictions by the proposed correlations

Similar to Fig. 1, Fig. 15 displays the comparison of ϕ_{ps} values predicted either by Eq. (8) or by Eq. (9) using the coefficients $b(D_{50})$ and $c(D_{50})$, with ϕ_{ps} data from the literature. The predictions of Eqs. (8) – (9) are shown in Fig. 15a. Considering the grain shape besides the D_{50} value for each tested material, most of the data points are plotted closely to the $\phi_{ps}^{pred} = \phi_{ps}^{meas}$ line with an average deviation of 1.00 ± 0.68 degrees. That implies that the measured and the predicted ϕ_{ps} values using these Equations coincide well. In addition, based on Figs. 1 and 15 it may be concluded that for materials with small D_{50} values, both Eqs. (1) and (8) deliver quite reasonable results, while for materials with large D_{50} , Eq. (8) works three times better.

The predictions of Eq. (9) are given in Fig. 15b. For gathered tested material, most of the data points are plotted close to the bisecting line with a maximum deviation of 3.65 degrees (averaged 1.57 ± 0.94 degrees). The average of ϕ_{ps} deviation from bisecting line is around 3.5 times smaller than when the original Bolton's equation was used in agreement with the observations in Fig. 10. However, limited ϕ_{ps} data with p' , and D_R information were available from literature and more data for angular sands with higher C_u values would be beneficial in future.

Conclusion

Comparing ϕ_{ps} values predicted using Bolton's strength–dilation formulations to the corresponding ϕ_{ps} data reported in the literature reveals that due to the fixed coefficients used in Bolton's equations the estimated ϕ_{ps} values are generally over-predicted, and the scatter between ϕ_{ps}^{meas} and ϕ_{ps}^{pred} is quite significant. 276 symmetrical direct shear test have been performed on three granular materials to describe the particle characteristics parameters dominating the contribution of dilatancy to the peak friction angle of 16 different particle size distributions. Applicability of Jewell and Worth's equation to convert ϕ_{ps} to ϕ_{ds} , reliability of boundary measurements, and uniformity of distribution of void ratio inside the physical symmetrical direct shear tests also discussed in detail.

It has been demonstrated that in the investigated range of $1.5 < C_u < 5$, and $106 \mu\text{m} < D_{50} < 1000 \mu\text{m}$, the maximum shear resistance of the tested materials ϕ_{ps} cannot be described by a unique function of maximum dilation or dilatancy index, and the coefficients b and c in Bolton's equations significantly decreases with increasing mean particle size D_{50} , and does not depend on the coefficient of uniformity C_u . The micro-scale mechanism controlling the columnar structure formation together with D_{50} was addressed as the main reason of the decrease of b and c parameters. Accordingly, the well-known Bolton's formulations have been developed to express the effect of particle-size distribution in the relationship between the friction angle and the dilation of granular materials at different relative densities and normal pressures.

In addition, an extension of the proposed correlations considering the influence of the particles angularity has been done by testing Péribonka and Eastmain sands made up of sub-rounded to angular particles. The proposed correlations predict quite well most of the ϕ_{ps} values tested in the laboratory, or reported in the literature for sands with a rounded to angular particle shape.

It should be noted that the soils considered in this study had linear particle-size distribution curves in the semi-logarithmic scale, and the data gathered from the literature correspond to standard sands having relatively uniform particle-size distribution curves. In the future, examining soils with more naturally shaped particle-size distribution curves, from poorly graded to well graded, would be beneficial. The influence of the fine particle content FC (%) in shear strength–dilation relationship can also be studied and integrated into the proposed correlations.

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