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STUDIES ON THICKENED TAILINGS DEPOSITION IN FLUME TESTS USING THE CFD METHOD

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Abstract

CFD simulations of laboratory flume tests on thickened tailings were conducted to highlight the factors that may influence the slopes of final profiles achieved in such flumes. The numerical model was first validated against the analytical solution of a sheet of Bingham fluid on a flat plane at flow stoppage. It was then used to investigate the influence of several factors on the average slopes of the final profiles. The results show that an increase of volume, energy, flume width or base angle reduces the resulting slope angle. Moreover the yield stress of the fluid generally has more influence on the final profiles than the viscosity. In addition, the viscosity tends to have less influence on the formation of the final profiles if the inertial effects are relatively weak. Finally, two dimensionless parameters were proposed to establish the relationship between the average slope, rheological properties and geometrical parameters for planar deposition of thickened tailings in both S-R and discharge flume tests. These results provide a better understanding of the deposition of thickened tailings in the field. The agreement between simulation results and laboratory observations in the literature gives confidence in the veracity of the computational results.

Keywords: Thickened tailings; Flume test; Beach slope; CFD modelling.

1. Introduction

The beach slope developed by tailings slurry after it is discharged into a tailings storage facility (TSF) is of great importance for the design and operation of a TSF in the mining industry (Vick 1990; Simms et al. 2011). Therefore the ability to accurately predict likely beach slopes is important for TSFs, and becomes even more crucial for thickened tailings disposal (also often referred to as a 'tailings stack' or 'central thickened discharge' scheme) because the beach slope of thickened tailings can determine the feasibility of the entire scheme in many such cases (Fitton 2007).

The discharge flume test, where the tailings are delivered through a pipe to the flume to form a beach profile, has been used for many years to simulate hydraulic deposition of conventional tailings. In such tests the particle size distribution is usually non-uniform along the beach as a result of sequential sedimentation during tailings deposition (Blight et al. 1985). The coherence between experimental results and field observations implies that discharge flume tests are at least sufficient to qualitatively simulate conventional tailings deposition (Blight et al. 1985; Kupper et al. 1992a).

Technical advancements in tailings thickening have enabled the growing implementation of thickened tailings disposal owing to its many inherent advantages over conventional tailings, which may include low water consumption, reduced risk of ground water contamination, significant safety improvement, easier rehabilitation, etc.

(Sofra and Boger 2002; Fourie 2009). Unlike conventional tailings, which typically have a low solids concentration and where larger particles settle out prior to finer ones during deposition and transportation, thickened tailings are in general more concentrated and non-segregating, with yield stresses in most cases ranging from 20 Pa to 50 Pa (Fitton and Seddon 2012). Notwithstanding successful simulation for conventional tailings deposition, flume tests have turned out to be problematic for direct beach slope prediction of thickened tailings. Beach slopes in the field are typically around 2% to 3% with 5% scarcely being achieved for thickened tailings (or paste) schemes implemented world-wide (Oxenford and Lord 2006; Williams et al. 2008). However the slopes for thickened tailings developed in flume tests tend to be unrealistically steeper than those in the field (Engman et al. 2004; Fourie and Gawu 2004; Meintjes 2004; Simms 2007; Fourie and Gawu 2010; Jewell 2012).

A significant amount of work has been carried out to explain the enormous disparity between beach slopes of thickened tailings in flume tests and field observations. Sofra and Boger (Sofra and Boger 2001; Sofra and Boger 2002) carried out a series of sudden-release (S-R) laboratory flume tests to identify the factors, including yield stress, viscosity, hydraulic head and slope of the underlying base, affecting the deposition behaviour of three types of thickened tailings (red mud, titanium dioxide suspension and hydrous sodium lithium magnesium silicate) with the yield stress varying from 17 to 210 Pa.

Simms (2007) used the theoretical solution for slow spreading flow (SSF) of Bingham fluid (Liu and Mei 1989) to explain the disparity in beach slopes of thickened tailings achieved in discharge flume tests and in the field. He found that the overall slope of the deposit is significantly influenced by both the scale of flow and the underlying topography. The influence of flume width and initial energy of tailings was not discussed. Henriquez et al. (2009) pursued the method proposed in Simms (2007) to study the dynamic flow behaviour and multilayer deposition of gold paste tailings. Both discharge and S-R flume tests were conducted in their work. They observed that the S-R flume tests yielded thinner and longer final profiles (It is noteworthy that the 'profile' and 'slope' are different: 'profile' is the shape of entire surface, from deposition point to end of flow, while slope is a linear value – usually fitted to some portion of the profile), especially with a larger volume of tailings used compared to the discharge flume tests, which was attributed to the significant and rapid release of potential energy. Additionally the effect of deposition rate, which may influence the final profile, was not studied in their work. Mizani et al. (2013) carried out discharge flume tests on high density gold tailings to investigate stack geometry of thickened tailings in the laboratory. They reported that the final profile was sensitive to flow rate when it was below 1.6 litres per minute (LPM) in the flume tests. It is worth noting that the SSF model which is derived within the framework of long-wave approximation, was used to describe the final profile of thickened tailings in laboratory flume tests by

several workers (Simms 2007; Henriquez and Simms 2009; Mizani et al. 2013), which may be inappropriate as the assumption of long-wave approximation typically cannot be fulfilled in a laboratory flume test (Gao and Fourie 2015b).

Fourie and Gawu (Fourie and Gawu 2010) developed a mathematical model taking the friction of flume sidewalls into account to explain the much steeper slope angles yielded by flume tests for thickened tailings. The model reveals that the slope of thickened tailings in a flume test will decrease with an increasing flume width. Moreover, they found that the slope decreases with an increase of the volume of thickened tailings. However, the inertial effects which may influence the profiles in flume tests were not discussed.

Compared to the large number of experimental and theoretical studies in the literature, very little work has been conducted using computational fluid dynamics (CFD) to investigate flume tests for the deposition of thickened tailings. To develop a clear understanding of the flow behaviour of thickened tailings, CFD simulations for both S-R and discharge flume tests were performed to assess the influence of several factors, including tailings volume, energy, yield stress, viscosity, flume width and base angle on the final profiles achieved in the two types of flume test. It is emphasised that yield stress ranges roughly from 20 to 60 Pa in actual thickened tailings disposal operations (Fitton and Seddon 2012). Therefore we only focused on the yield stresses (see **Table 4**)

within this range in the present work.

Similar to the argument made by Kupper et al. (1992b) who used a laboratory flume test to study hydraulic fill, we emphasise that the flume test is not a scaled version of any prototype but a fundamental test which provides a cheap and easy way to understand the physical phenomenon of thickened tailings flow. It is stressed that the intention of this work is not to compare CFD models with experimental data from flumes, or directly predict the beach slope in the field using laboratory flume tests. Rather, it is to highlight the factors that influence the flow of thickened tailings and provide useful insight into the deposition of thickened tailings, particularly in laboratory flume tests.

It is noted that the flume width in planar simulations for both S-R and discharge flume test are effectively considered as infinite. Therefore both S-R flume tests and discharge flume tests with 2D geometries are different from the laboratory flume test where typically a narrow flume is used. In other words, the 2D flume test simulations do not take the sidewall friction into account.

In this paper the numerical model (CFD) is described first, followed by its validation. Thereafter the influence of energy, volume, yield stress, viscosity, flume width and base angle on the final profile achieved in flume tests is investigated using CFD simulations. After that, a dimensionless parameter ($\varepsilon \cdot B \cdot F_r^2 / R_e$) is proposed to establish a relationship between the average slope, rheological properties and geometrical parameters for

planar deposition of thickened tailings. Finally some conclusions are reached.

2. Numerical model and validation

A commercially available CFD code, ANSYS FLUENT was used to perform the simulations. Considering the low flow velocity in a flume test, both thickened tailings and air in the work are assumed to be incompressible, therefore the continuity and momentum equations are simplified as:

$$\nabla \cdot (\vec{v}) = 0 \quad (1)$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho \nabla \cdot (\vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\bar{\bar{\tau}}) + \rho \vec{g} \quad (2)$$

where $\vec{v}$ is the velocity field, ρ is the density, p is the pressure, $\vec{g}$ is the gravitational acceleration and $\bar{\bar{\tau}}$ is the stress tensor, which is given by:

$$\bar{\bar{\tau}} = \mu(\nabla \vec{v} + \nabla \vec{v}^T) \quad (3)$$

where μ is the shear viscosity of the fluid.

Although other constitutive models for rheology are being explored for tailings (Mizani and Simms 2016; Mizani et al. 2017), the thickened tailings in this work is modelled using the Bingham constitutive law (Bingham 1916):

$$\eta = \begin{cases} \mu_0 + \frac{\tau_y}{\dot{\gamma}}, & \tau \geq \tau_y \\ \infty, & \tau < \tau_y \end{cases} \quad (4)$$

where η is the apparent viscosity, μ_0 is the plastic viscosity, τ_y is the yield stress of Bingham fluid, and $\dot{\gamma}$ is the shear rate. It is noted that the plastic viscosity is also

referred to as viscosity in the present work.

The ideal Bingham model (Eq. (4)) is discontinuous at the yield point, which results in difficulties for computations. Firstly, velocity components and thus shear rates are always affected by numerical noise therefore it is impossible to differentiate the flow region ($\dot{\gamma} > 0$) from the rigid region ($\dot{\gamma} = 0$). Moreover, the stresses will not be determined in the rigid region (Tanner and Milthorpe 1983; Odonovan and Tanner 1984). To avoid these difficulties, Tanner and Milthorpe (1983) developed the bi-viscosity model, which introduced a critical shear rate, $(\dot{\gamma}_c)$, below which the fluid becomes very viscous, acting like a rigid solid. A modified bi-viscosity model has been adopted by the commercial code ANSYS FLUENT (ANSYS 2013) and has been widely used to simulate yield stress fluids (Jeon et al. 2010; Craig et al. 2013; Filali et al. 2013; Gao and Fourie 2015b; Tang et al. 2016). The apparent viscosity of the modified bi-viscosity model in ANSYS FLUENT is given by the following relation (ANSYS 2013b):

$$\eta = \begin{cases} \mu_0 + \frac{\tau_y}{\dot{\gamma}}, & \dot{\gamma} \geq \dot{\gamma}_c \\ \mu_0 + \frac{\tau_y(2 - \dot{\gamma}/\dot{\gamma}_c)}{\dot{\gamma}_c}, & \dot{\gamma} < \dot{\gamma}_c \end{cases} \quad (5)$$

where $\dot{\gamma}_c$ is the critical shear rate.

The critical shear rate $(\dot{\gamma}_c)$ should be as small as possible to reproduce the flow behaviour of a Bingham fluid. However excessively small $\dot{\gamma}_c$ may lead to numerical instability. Gao and Fourie (2015a) reported that the proper critical shear rate $(\dot{\gamma}_c)$ is

primarily dependent on the yield stress of Bingham fluid and a value of 0.005 s^{-1} for $\dot{\gamma}_c$ is appropriate for the yield stress ranging from approximately 18 to 60 Pa. As discussed in Section 1, the yield stress (see **Table 4**) of interest in this work is roughly within this range. Therefore the critical shear rate of 0.005 s^{-1} was employed in the simulations.

The surface tension effect was neglected in this work as a result of the high yield stresses of interest. Roussel et al. (2005) studied the surface tension effects of several pastes in mini-cone tests. They found that the surface tension effects can be safely neglected if the spread is smaller than 0.35 m, which is corresponding to a yield stress of approximately 1.5 Pa. In the present work, the yield stress ranges from 18.61 to 62.62 Pa, as shown in **Table 4**, which is much higher than 1.5 Pa. Therefore neglecting the surface tension in the simulations is justified.

The interface between air and thickened tailings was tracked by using the coupled level-set and volume of fluid (VOF) model provided in ANSYS FLUENT (ANSYS 2013a). The coupled level-set and VOF model which overcomes the deficiencies of the level-set method and VOF method (Hirt and Nichols 1981; Osher and Sethian 1988; Sussman and Puckett 2000; Olsson et al. 2007) is specifically designed for two-phase flows without mass transfer. In a two-phase system, the level-set function $\varphi(x, t)$ which is also known as the signed distance function or oriented

distance function, is defined as the distance of a given point x from the interface (Osher and Fedkiw 2001):

$$\varphi(x, t) = \begin{cases} +|d| & \text{if } x \in \text{the primary phase} \\ 0 & \text{if } x \in \Gamma \\ -|d| & \text{if } x \in \text{the secondary phase} \end{cases} \quad (6)$$

where d is the distance from the interface of the two phases and Γ is the interface, given by:

$$\Gamma = \{x | \varphi(x, t) = 0\} \quad (7)$$

The evolution of the level-set function is given in a similar fashion as the VOF model (Hirt and Nichols 1981; Sussman and Puckett 2000):

$$\frac{\partial \varphi}{\partial t} + \nabla \cdot (\vec{v} \varphi) = 0 \quad (8)$$

Detailed information on the coupled level-set function and VOF method can be found elsewhere (Sussman and Puckett 2000; Sussman 2003).

The numerical model in ANSYS FLUENT for flume test simulations of Bingham fluid has been discussed previously (Gao and Fourie 2015b). Velocity profiles of Bingham fluid in a circular pipe from both an analytical solution and the CFD simulation were compared to validate the Bingham model (Gao and Fourie 2015a).

Further validation of the model for flume test simulations is carried out in the present work, by comparing the final profile of the thickened tailings achieved in a planar simulation of flume test with the analytical solution for a thin layer of Bingham fluid on a flat plane (Liu and Mei 1989, 1990):

$$\frac{dy}{dx} = -\frac{\tau_y}{\rho g y} \quad (9)$$

where y is the elevation of the free surface at a particular x . It is emphasised that Eq.(9) is based on static equilibrium at flow cessation of a Bingham fluid with extensional stresses and surface tension neglected. Given the material properties (i.e. ρ and τ_y) and a single point (e.g. flow distance at the end of the final profile) on the final profile, the entire profile of a Bingham fluid on a flat plane can be obtained by solving Eq.(9).

A 2D planar simulation of a discharge flume test was performed to obtain the final profile of a Bingham fluid on a horizontal base. The same geometry, mesh and setup that were used in Section 3.1.2 were employed here with a fluid discharge time of 16 s. For the modified bi-viscosity model, the apparent viscosity will be null if the shear rate vanishes, as suggested by Eq.(5). Therefore the yield stress fluid cannot completely stop in the simulations (Babaoglu and Simms 2017). However, it is acceptable to assume that the flow of a yield stress fluid has stopped if the interface moves slowly enough (Tanner and Milthorpe 1983; Beverly and Tanner 1992; Tregger et al. 2008; Gao and Fourie 2015a).

A discharge flume test simulation was conducted to investigate the sensitivity of average slope angle to the spreading speed of the front. The paste was discharged at the beginning of the simulation and stopped after 8 seconds. The average slope of the profile at different spreading speeds of the front was recorded with the flow time, which

is shown in **Figure 1**.

It is clear that there is minimal change in the average slope although the spreading speed of the front increased by 250% (from 0.2 to 0.5 mm/s). This confirms that it is acceptable to assume that the flow has ceased if the front spreading speed is lower than 0.5 mm/s. On the other hand, it is noteworthy that the flow time (hence the computation time) increased significantly from 8.6s to 25s when the spreading speed reduced from 0.5 to 0.2 mm/s. Consequently, it was considered a reasonable compromise to stop the simulations when the spreading speed of the front toe is slower than 0.3 mm/s with both accuracy and computing time taken into account. **Figure 2** illustrates the final profiles of the 2D flume test from both the analytical solution and simulation. The good agreement indicates that the numerical model in ANSYS FLUENT and the critical shear rate of 0.005 s^{-1} are capable of modelling the planar deposition of Bingham fluids.

3. Results and discussion

This section firstly investigates the influence of several factors, including fluid volume, energy, yield stress, viscosity, flume width and base angle, on the final profile achieved in flume tests. Then a dimensional analysis was performed to establish the relationship between average beach slope and several dimensionless parameters.

There are two types of flume tests used in the work: S-R flume test (see **Figure 3(a)**)

and discharge flume test (see **Figure 3(b)**). For the simulations of S-R flume tests, a certain amount of fluid is patched at one end of the flume at $t=0$, as shown in **Figure 3(a)**. Non-slip conditions were applied to all the walls of the flume except for the inlets and outlets. Care was taken to prevent the Bingham fluids from flowing out of flumes so that entire profiles could be achieved in both types of flume test. The yield stress, viscosity and density respectively of the three main materials used in this work were ((a) 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³; (b) 18.61 Pa, 0.32 Pa·s and 1315.0 kg/m³; (c) 50.1 Pa, 0.64 Pa·s and 1367.4 kg/m³) and were based on mixtures of kaolin and tap water. The determination of the material properties can be found elsewhere (Gao and Fourie 2015a). It is noted that there is no comparison between experimental results and numerical results in the present work.

The slope of the final profile was obtained by a linear fit of a certain percentage of the whole final profile. **Figure 4** shows the sketch for obtaining the slope of a convex profile achieved in a discharge flume test. To determine the part of the profile to be used for a linear fit, the influence distance (L') of the inlet discharge is first removed. Then 20% of the remainder ($L - L'$) at the front toe is also removed to reduce the influence of the sharp curvature at the front toe of the profile on the linear fit. Therefore the range used for a linear fit is (L', x), where x is given by:

$$x = 0.8L + 0.2L' \quad (10)$$

where L is the overall length of the final profile, as shown in **Figure 4**.

It is noted that the variation of influence distance (L') near the inlet may change the slope obtained by linear fitting of the same profile. Consequently the slopes presented in particular figures in the work used similar L' to enable the slopes to be rationally compared.

The isoline (2D simulations) or isosurface (3D simulations) of the fluid volume fraction at 0.5 is determined to be the interface between the two phases (Bingham fluid and air).

To guarantee the grid-independence of the 2D simulation results, square cells used to discretise the computational domain were refined from $5mm \times 5mm$, to $2mm \times 2mm$ and finally to $1mm \times 1mm$ for all the 2D simulations in the work. It was found that the $2mm \times 2mm$ cells are not only fine enough to ensure the grid-independence of results but also of higher efficiency in computation than the $1mm \times 1mm$. Thus the simulation results of a computational mesh consisting of $2mm \times 2mm$ cells are discussed in the work. In addition, mesh-independence investigations were performed for all the 3D simulations as well, as discussed in Section 3.4.

3.1 The influence of fluid volume on the slope in a flume test

2D simulations for both S-R and discharge flume tests with horizontal bases, as shown in **Figure 3**, were carried out to investigate the influence of tailings volume on

the slopes of the final profiles. Step-like flumes were employed to trim the computational domain thereby reducing the computing time. Contours of fluid volume fraction were saved during simulations and inspected after the simulations to ensure that the Bingham fluid did not contact the irrelevant walls of the steps.

3.1.1 The influence of volume on the slope in an S-R flume test

Using the flume bottom as the reference level, the potential energy of the Bingham fluid per unit width in **Figure 3(a)** is given by:

$$E_p = \frac{1}{2} \rho g a b^2 = \frac{1}{2} \rho g S b \quad (11)$$

where a and b are the length and height of the patched area respectively, S is the area of the patched material, which is given by $S = ab$.

Thus the potential energy per unit volume reads:

$$E'_p = \frac{1}{2} \rho g b \quad (12)$$

Eq.(12) indicates that the increase of the length of patched area (a) with a fixed height (b) in a S-R flume test does not vary the potential energy per unit volume of the patched fluid.

To facilitate the analysis of the influence of fluid volume on the achieved slope, it is preferable to ensure that the potential energy per unit volume is similar in each S-R flume test since the potential energy may influence the slope as well (see Section 3.2).

Accordingly the variation of volume of Bingham fluid in the four simulations reported in this section was realised by incrementally extending the length of patched area (a in **Figure 3(a)**) from 50 mm to 100 mm, and then 200 mm and finally 400 mm, with the height (b in **Figure 3(a)**) fixed at 130 mm.

illustrates the final profiles from S-R flume test and the prediction by Eq.(9). It is clear that the profile from the S-R flume test is thinner and longer than that of the prediction. Since there are no inertial effects in the prediction by Eq.(9), it is argued that the inertial effects in the S-R flume test were very high which 'pushed' the final profile to flow further, thus concealing the volume influence. It is noteworthy that the phenomenon that S-R flume tests tend to result in distorted profiles due to significant release of potential energy was also observed in the laboratory by Henriquez and Simms (2009).

To reduce the inertial effects, the viscosity of the Bingham fluid in the simulations of the S-R flume test was increased from 0.4 Pa·s to 10 Pa·s. Runs 7-10 in **Table 4** summarize the detailed information of the four simulations of S-R flume tests with different patched areas. The final profiles with a higher viscosity (10 Pa·s) tend to be convex while the profile in **Figure 5** from S-R flume test simulation with a lower viscosity (0.4 Pa·s) is initially concave. This difference indicates that the inertial effects in the simulations of S-R flume test on material of relatively high viscosity

were weak, which was conducive to the investigation of volume influence. It is clear that the slopes achieved in S-R flume test simulations (Runs 7-10) decrease with the increase of Bingham fluid volume, which indicates that the beach slope of Bingham fluid in a S-R flume test is flow-scale dependent.

3.1.2 The influence of volume on the slope in a discharge flume test

Four 2D simulations of the discharge flume test (Runs 1-4 in **Table 1**) were performed to investigate the volume influence on the slope. The inlet size (d in **Figure 3(b)**) was 20 mm with an inlet velocity of 0.1 m/s. The height of the discharging point (h in **Figure 3(b)**) was 100 mm. The fluid volume varied among the four simulations by employing different discharge times: 2 s, 4 s, 8 s and 16 s. The material properties used in the simulations were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³. Detailed information and simulation results of the four simulations are summarised by runs 1-4 in **Table 1**. It is clear that the average slopes (BS_a) of the final profiles from discharge flume test simulations decrease with the increase of fluid volume, which was observed in laboratory discharge flume tests by several workers (Henriquez and Simms 2009; Fourie and Gawu 2010; Mizani et al. 2013).

Overall both results from the S-R flume test and discharge flume test suggest that the slope of Bingham fluid declines with an increase of the volume of deposited material. This scale-dependency feature of Bingham fluid is very important as it may prevent us

from directly extrapolating the slope obtained using small-scale deposition tests to the field (Mizani et al. 2013). Although this finding, as with some others in this study, is perhaps intuitively obvious, many flume studies have been conducted without accounting for the effects of variables such as flow volume. Quantification of the influence of these variables, as is done in this paper, should result in more considered flume tests in future.

3.2 The influence of energy on the slope in a flume test

To investigate the energy influence on the final slope of Bingham fluid, 2D simulations of both discharge and S-R flume tests were carried out.

For the discharge flume test simulations, the geometry employed was the same as that in Section 3.1.2. The inlet velocities for the three simulations were 0.2 m/s, 0.4 m/s and 0.8 m/s respectively. In Section 3.1 it was demonstrated that the volume of fluid has an influence on the slope of the final profile achieved in a flume test, therefore the same fluid volume was employed for the three simulations by varying the discharge time. **Figure 6** shows the final profiles from the 2D planar simulations of the flume tests with the same fluid volume discharged at different velocities and the corresponding slopes. As can be seen clearly from **Figure 6(b)**, the slope of the final profile decreases with the increment of the discharging velocity. This was observed in laboratory flume tests by Mizani et al. (2013).

Three 2D simulations of S-R flume tests (Run 1, 5 and 6 in **Table 4**) with the same patched area of $32,000 \text{ mm}^2$ were carried out to investigate the influence of the initial potential energy on the final profiles. Since the patched area (S) was fixed, the variation of the height (b) of the patched area results in different potential energy per unit width, according to Eq.(11). The patched areas for the three simulations were $80 \text{ mm} \times 400 \text{ mm}$, $160 \text{ mm} \times 200 \text{ mm}$ and $320 \text{ mm} \times 100 \text{ mm}$ (Height $\times$ Length). As shown by Run 1, 5 and 6 in **Table 4**, with an increase of the height of the patched area, the distance of flow (L) extends and the slope of the final profile decreases. It indicates that the sudden release of fluid with higher potential energy introduces more inertial effects, thereby leading to extension of flow distance and reduction of slope.

Simulation results for both discharge and S-R flume tests reveal that the increase of energy reduces the slope of the final profile, which was observed in laboratory flume tests as well (Sofra and Boger 2002; Mizani et al. 2013).

3.3 The influence of yield stress and viscosity on the slope in a flume test

Twelve 2D simulations in four groups were carried out to study the influence of yield stress and viscosity on the slope achieved in both S-R flume tests and discharge flume tests. Detailed information of the simulations is listed in **Table 2**.

As shown in **Table 2**, Group 1 and Group 2 simulate discharge flume tests with inlet velocities of 0.1 m/s and 0.4 m/s respectively and Group 3 and Group 4 are for S-R flume tests with rectangular patched shapes ($46\text{mm} \times 300\text{mm}$ and $69\text{mm} \times 200\text{mm}$). Each group contains three simulations designated as Case 0, Case 1 and Case 2. Case 1 and Case 2 have a 25% higher yield stress and a 25% higher viscosity, respectively compared to Case 0 which is the base case. The six simulations for discharge flume tests have the same volume of fluid, as is the case for the six simulations for S-R flume tests. The slopes obtained from the simulations listed in **Table 2** are illustrated in **Figure 7**. **Figure 8** summarizes the slope variations from the base cases (Case 0 in each group) induced by the increase of yield stress and viscosity.

It can be concluded from the comparison of slopes between Group 1 and Group 2, and Group 3 and Group 4 that higher energy resulted in a smaller slope for both kinds of flume tests, as shown in **Figure 7**. Moreover in each group the smallest slopes are always from the base cases (Case 0) rather than the cases with higher yield stresses or viscosities, which indicates that the slope of the Bingham fluid was influenced by both yield stress and viscosity.

Although both yield stress and viscosity influenced the slope, the 25% increase of yield stress resulted in a much greater variation of slope from the base case than the 25% increased viscosity, as demonstrated in **Figure 8**, suggesting that yield stress

plays a more dominant role in the formation of the beach profile of thickened tailings in the flume tests, and presumably also with field deposition.

To understand the influence of viscosity in different scenarios with respect to inertial effects, the ratio (R_{vy}) of slope variations caused by increased viscosity and increased yield stress in each group is used to depict the relative importance of viscosity and yield stress:

$$R_{vy} = \frac{\Delta S_v}{\Delta S_y} \quad (13)$$

where ΔS_v and ΔS_y are the variations of slope resulting from increased viscosity (Case 2) and yield stress (Case1), compared to Case 0 in the same group.

For example, the variations of slope caused by viscosity and yield stress increase in Group 1 is 2.8% and 13.3%, as shown in **Figure 8**. Thus the ratio of the slope variations between Case 2 and Case 1 in Group 1 is 0.21 (2.8%/13.3%). Following this method, the ratio of slope variations by increased viscosity (Case2) and yield stress (Case 1) for the other groups can be calculated and are summarized in **Figure 9**.

As can be seen in **Figure 9**, all ratios for both discharge and S-R flume tests are less than unity which suggests that the viscosity plays a less important role in the slope formation than the yield stress. Furthermore, the ratios for scenarios where the energy is relatively high (Group 2 and Group 4) are larger than the corresponding scenarios of low energy (Group 1 and Group 3) for both types of flume test. The results suggest

that the importance of viscosity in the final profile formation of a Bingham fluid increases with the increase of inertial effects during the deposition process, although it is less important than the yield stress generally. On the other hand, we may infer that the influence of viscosity can be negligible if the inertia effects are sufficiently low during the deposition of Bingham fluid (Gao and Fourie 2015b). Considering the energy release in a dam-break is significant and quick, the inertial effects are expected to be strong. Therefore the viscosity could be an important factor in a real dam-break.

3.4 The influence of flume width on the slope in a flume test

3D simulations of both S-R and discharge flume tests with three different flume widths (100 mm, 200 mm and 400 mm) were carried out to investigate the influence of flume width on the final profiles achieved in flume tests. **Figure 10** illustrates the geometries of the 3D simulations of S-R and discharge flume tests. Nonslip wall conditions were applied to sidewalls and bottoms of the flume.

To make the simulation results for discharge flume tests with different flume widths comparable, the width of the inlet of the discharge flume test was identical to the flume width with an inlet length at 60 mm, as shown in **Figure 10** (a). Furthermore the three simulations of discharge flume tests shared the same inlet velocity (0.5 m/s), discharge time (1 s) and material properties. With these measures, both flow rate and fluid volume per unit width of the discharge flume tests with different flume widths

are equivalent. Therefore the difference in the final profiles of the three simulations, if any, should be caused by the flume width.

The initial shapes of the fluid in the S-R flume test simulations were rectangular cuboids of which the height and length were fixed at 100 mm and 200 mm respectively with widths consistent with the flume widths, as shown in **Figure 10** (b). Thus both the volume of fluid per unit width and the potential energy of fluid per unit volume (see Eq.(12)) were identical in the three S-R flume test simulations, thereby facilitating the analysis of the flume width influence.

Regular hexahedron elements were used to discretise the 3D geometries and care was taken to reduce the difference in sides of the elements to improve the convergence rate and accuracy of the simulations. Mesh-independence investigations were performed for all the 3D simulations in the present work but only the discharge flume test with a width of 200 mm, as an example, is presented for the sake of conciseness.

Figure 11 summarises the three final profiles of the 3D simulations using different numbers of elements for the discharge flume test with the flume width at 200 mm. The corresponding slopes and the slope variations from the base case (Case 3: 423680 elements) are listed in **Table 3**. As shown in **Figure 11**, the final profiles from simulations of different numbers of elements are very similar. Further analysis suggests that, compared to the big difference in slopes between Case 1 and Case 3

(14.78%), the difference between Case 2 and Case 3 is relatively minor (1.74%), although the number of elements of Case 3 is eight times more than that of Case 2, as shown in **Table 3**. Accordingly the mesh resolution of Case 2 is considered sufficiently high with both accuracy and computing time taken into account.

Figure 12 and **Figure 13** show the final profiles and the corresponding slopes from the simulations of discharge and S-R flume tests with flume widths of 100 mm, 200 mm and 400 mm. It is evident that the slopes of the final profiles in both types of flume tests decrease with increasing flume width, which was demonstrated theoretically by Fourie and Gawu (2010). This is a very useful result which may partially account for the unrealistic, steeper slopes achieved in laboratory flume tests for thickened tailings compared with those observed in the field. Moreover it suggests that the analytical solution (Eq.(9)) which was initially developed for sheet flow of Bingham fluid is not suitable for laboratory flume tests on thickened tailings, as discussed by Gao and Fourie (2015b). In addition, it is worth noting that Fourie and Gawu (2010) investigated the effects of flume width variation on the average slope angle of deposited gold and copper tailings with the findings that the angle of slope is practically independent of the flume width at values of flume width larger than 1600 mm.

3.5 The influence of base angles on the final profiles of thickened tailings

To investigate the influence of the base angle on the final profiles of thickened tailings, simulations (2D) of both S-R and discharge flume tests with different base angles (0, 1%, 2%, and 3%) were conducted. The volume of fluid was identical for all the simulations in this section. The inlet size (d in **Figure 3(b)**), the height of the inlet (h in **Figure 3(b)**) and the inlet velocity was the same as those in Section 3.1.2. For the simulations of S-R flume tests, the initial shape of the fluid patched in the flumes is 80 mm (b in **Figure 3(a)**) $\times$ 400 mm (a in **Figure 3(a)**).

Figure 14 shows the final profiles and the corresponding slopes from the 2D simulations of S-R flume tests with four different base angles and **Figure 15** shows results for discharge flume tests. It is evident that the final profiles tend to be longer and flatter with increasing base angles. More interestingly, the slope of the profile seems to be a linear function of the base angle for both types of flume tests, as shown in **Figure 14 (b)** and **Figure 15 (b)**. Sofra and Boger (Sofra and Boger 2002) carried out S-R flume tests with different base angles on Laponite slurries of yield stress ranging from 17 to 110 Pa. A linear relation between base angles and slopes of final profiles was observed, which confirms the CFD simulations. In addition, the impact of base angle on the resulting slope implies that the beach slope of thickened tailings in the field is likely to be influenced by the underlying topography, as reported by

other researchers (Simms 2007; Mizani et al. 2013).

3.6 Dimensional analysis

To develop a better understanding of the planar (2D) deposition of thickened tailings, dimensional analyses were performed to find out the relationship between the relevant parameters and the average beach slopes. The relationship obtained from small-scale deposition may provide useful insight into the deposition of thickened tailings in the field.

A dimensional analysis using the Buckingham π theorem (Buckingham 1914) was performed by Sofra and Boger (Sofra 2001; Sofra and Boger 2002) to determine the important dimensionless parameters for thickened tailings deposition based on laboratory S-R flume tests using different density suspensions. Three relevant dimensionless groups were identified: dimensionless yield stress ($\frac{\tau_y}{\rho v^2}$), Reynolds number (R_e) and Froude number (F_r):

$$R_e = \frac{\rho v W}{\mu} \quad (14)$$

$$F_r = \frac{v^2}{gW} \quad (15)$$

where W is the flume width and v is the instantaneous initial suspension velocity, which was calculated by assuming that all of the potential energy of the fluid in the reservoir is instantaneously converted to kinetic energy:

$$\rho g h = \frac{1}{2} \rho v^2 \quad (16)$$

Therefore:

$$v = \sqrt{2gh} \quad (17)$$

where h is the initial height of the fluid in the reservoir plus the height of the reservoir above the horizontal.

They also reported an appropriate form of the relationship between these groups and the average beach slope (BS_a):

$$BS_a = f_1 \left(\frac{\tau_y F_r}{\rho v^2 R_e} \right) = f_1 \left(\frac{\mu_0 \tau_y}{\rho^2 W^2 g v} \right) \quad (18)$$

For the 2D planar simulations of flume tests carried out in the present work, the flume width (W) is deemed to be infinite, which renders Eqs.(14)-(15) inapplicable due to the presence of flume width in these equations. Moreover, although the assumption that all of the potential energy of the fluid in the reservoir is converted to kinetic energy is valid in the work of Sofra and Boger (Sofra 2001; Sofra and Boger 2002) as a result of the relatively low viscosity of the suspensions (0.0058–0.0382 Pa·s) they used, the assumption does not hold for the materials with very high viscosities, such as the runs 7-10 in **Table 4** where the viscosity is 10 Pa·s. This is because the high viscosity results in significant frictional losses, thereby voiding Eq.(17). Additionally, both tailings volume and base angle, which influence the average beach slope were not taken into account in their dimensional analysis (Sofra and Boger 2001; Sofra and Boger 2002).

Figure 16 shows the variation in the average beach slope with the yield stress, according to the planar simulations of both S-R and discharge flume tests performed in the present work. There are several different slopes for one yield stress, clearly indicating that the yield stress is not the only parameter influencing the slope of thickened tailings.

To find out the important dimensionless groups for planar thickened tailings deposition, a dimensional analysis was conducted for both S-R and discharge flume tests in the present work (see Appendices A and B).

3.6.1 Dimensional analysis for S-R flume tests

For the S-R flume tests, the average beach slope is given by (see Appendix A):

$$BS_a = f\left(\theta, \frac{\tau_y}{\sqrt{A}\rho g}, \frac{b}{\sqrt{A}}, \frac{\mu_0}{A^{0.75}\rho\sqrt{g}}\right) \quad (19)$$

where A is the volume of thickened tailings ($A=ab$), b is the height of the patched area (see **Figure 3(a)**) and θ is the average base angle.

An appropriate combination of the dimensionless groups in Eq. (19) is determined based on the data of S-R flume tests listed in **Table 4**:

$$BS_a = f_1\left(\frac{\tau_y - \rho g H \sin\theta}{\sqrt{A}\rho g} \times \left(\frac{\mu_0}{A^{0.75}\rho\sqrt{g}}\right)^{0.3} \div \frac{b}{\sqrt{A}}\right) = f_1\left[\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin\theta)}{\rho^{1.3}g^{1.15}A^{0.225}b}\right] \quad (20)$$

where H is the characteristic thickness of thickened tailings deposited on an inclined plane and is determined to be 0.038m for both S-R and discharge flume tests based on the simulation results.

Application of the combined dimensionless group in Eq. (20) to the data from S-R flume tests in **Figure 17** shows that there is a clear linear relation between the average beach slope and the proposed combination of relevant parameters:

$$BS_a = 793.04 \frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g^{1.15} A^{0.225} b} \quad (21)$$

The high coefficient of determination ($R^2 \approx 0.99$) indicates that the statistical model (Eq. (21)) fits the data very well. Moreover, the relevant parameters for the process of S-R flume tests have been included by the proposed dimensionless group.

3.6.2 Dimensional analysis for discharge flume tests

For the discharge flume tests, the relation between the average beach slope and the dimensionless groups are (see Appendix B):

$$BS_a = g(\theta, \frac{A\tau_y}{\rho Q^2}, \frac{\mu_0}{\rho Q}, \frac{A^{1.5}g}{Q^2}) \quad (22)$$

where Q is the discharge flow rate in a discharge flume test.

Further investigation suggests that the appropriate combination of the dimensionless groups for discharge flume tests listed in **Table 1** is as follows:

$$BS_a = g_1 \left(\frac{A(\tau_y - \rho g H \sin \theta)}{\rho Q^2} \times \left(\frac{\mu_0}{\rho Q} \right)^{0.3} \div \frac{A^{1.5}g}{Q^2} \right) = g_1 \left[\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g A^{0.5} Q^{0.3}} \right] \quad (23)$$

Figure 18 illustrates the linear relationship between the average beach slope and the proposed combination of relevant parameters in Eq. (23) for the discharge flume tests:

$$BS_a = 403.442 \frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g A^{0.5} Q^{0.3}} + 1.926 \quad (24)$$

The high coefficient of determination ($R^2 \approx 0.99$) indicates that the linear function (Eq. (24)) can be used to predict the average beach slope of thickened tailings in the planar discharge flume tests with high accuracy. Additionally, it suggests that the proposed dimensionless group in Eq. (23) involves the relevant parameters for discharge flume tests.

3.6.3 Discussions of the proposed model

Theoretically, the proposed model (Eqs. 21 and 24) has the potential to predict the average beach slope in the field, similar to the “master profile” proposed by Blight et al. (Blight et al. 1985; Blight 1994), which has been successfully used to model the beach slope of conventional tailings. However, there are still several issues that need to be addressed before the proposed model can be confidently applied to field applications. First, the flow of thickened tailings on the tailings beach is complex (Chryss et al. 2006; Fitton 2006), and lateral spreading is likely to occur (Sofra and Boger 2002). It is necessary to investigate lateral spreading considering that the proposed method is based on planar simulations. Moreover, it is not easy to determine the rheological parameters in the field, such as the yield stress, viscosity and density, because of the variation of thickener output (Fourie and Gawu 2010; Chryss et al. 2012). In addition, although the value of 0.038m for characteristic thickness of the deposited tailings, which is necessary for the beach slope prediction with a non-zero

base angle, works very well for the flume tests, further investigations into this value for the field deposition of thickened tailings are required. Notwithstanding these issues, the proposed model provides good insights into the factors influencing the beach formation of thickened tailings and offers some interesting possibilities for beach slope prediction of thickened tailings.

4. Conclusions

Based on the present work, several conclusions may be reached regarding flume tests on thickened tailings:

1. The average beach slope of thickened tailings achieved in a planar flume can be predicted using the proposed models (Eqs.21 and 24). The models have the potential to predict the average beach slope of thickened tailings in the field, although further investigations are required. It should be emphasised that direct extrapolation of results from flume tests to prediction of field beach slopes is not currently possible, but flume tests are extremely valuable for calibration of numerical models.
2. The slope of the final profile of a yield-stress fluid decreases with an increasing fluid volume, which indicates that the beach slope of thickened tailings is flow-scale dependent. In other words, it is not advisable to use small-scale tests for direct extrapolation to field applications.

3. Higher energy at discharge results in longer and flatter final profiles of thickened tailings. This illustrates the potential benefit of reducing deposition flowrates, e.g. by increasing the number of spigots discharging tailings at any time. It also points to the need to prevent flow from two (or more) adjacent spigots converging, as this would result in higher energy flow and thus lower beach slopes.
4. Generally, the yield stress of thickened tailings has more influence on the slope than does viscosity. The importance of viscosity for the final profile formation of thickened tailings increases with an increase of inertial effects during the deposition process. Therefore, it is inferred that the influence of viscosity can be negligible if the inertia effects are sufficiently low in the deposition of thickened tailings. Conversely, viscosity may be the dominant variable in dam-break simulations.
5. The flume width has a significant influence on the slope of thickened tailings in a laboratory flume test. A smaller flume width increases the slope of thickened tailings in a flume. This is an important result as it demonstrates the inadvisability of using slopes achieved in laboratory flume tests to directly extrapolate to beach slopes in field deposition.
6. An increasing base angle results in the reduction of the resulting slope, which suggests that the beach slope of thickened tailings is influenced by base topography in the field.

7. The agreement between simulation results and laboratory observations in the literature gives confidence in the veracity of the computational results. Consequently, CFD simulations provide a potentially effective method to investigate the flow problem of thickened tailings.
8. Quantification of the influence of the factors, as is studied in this paper, should result in more considered flume tests in future.

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Figure Captions

Figure 1 Effect of front spreading speed on predicted average slope angle. The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³

Figure 2 Comparison of analytical and simulated final profiles in a 2D flume test. The parameters of the material used were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 3 Sketch of geometries for 2D planar flume tests. (a) S-R flume test (b) Discharge flume test. The shaded area in panel (a) is the fluid patched in the flume at $t=0$. The empty area is occupied by air for both flumes. The walls, except for the inlets and outlets, are treated as non-slip walls.

Figure 4 Sketch illustrating how the linear slope of a profile in a flume test was obtained. x is given by $0.8L + 0.2L'$.

Figure 5 Final profiles of the prediction by Eq.(9) and the 2D planar simulation of S-R flume test with a patched area of 130 mm(Height) $\times$ 100 mm(Length). The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 6 Final profiles from 2D simulations of discharge flume tests with different inlet velocities and the corresponding slopes. The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 7 Slopes from simulations of discharge flume tests (a) and S-R flume tests (b)

for yield stress and viscosity influence investigation. Detailed information of the simulations is listed in **Table 2**.

Figure 8 Slope variations from Case 0 caused by 25% increase of yield stress and viscosity in simulations of discharge flume tests (a) and S-R flume tests (b). Detailed information of the simulations is listed in **Table 2**.

Figure 9 Ratio of slope variations between Case 2 and Case 1 for simulations of discharge flume tests (a) and S-R flume tests (b). The Bingham fluids employed in Case 2 and Case 1 have higher viscosity and yield stress respectively than Case 0, as shown in **Table 2**.

Figure 10 Sketches for the 3D simulations of the discharge flume test with a flume-wide inlet (a) and the S-R flume test at the initial state (b). The shaded volume in panel (b) is the initial volume patched in the flume at $t=0$. Non-slip conditions are applied to the walls except for the outlets and inlets.

Figure 11 Final profiles from 3D simulations of different number of elements for the discharge flume test with flume width at 200 mm.

Figure 12 Final profiles (a) and the corresponding slopes (b) from discharge flume test simulations (3D) of different flume widths. The inlet velocity was 0.5 m/s with a discharging time of 1 s. The input material properties were: 18.61 Pa, 0.32 Pa·s and

1315.0 kg/m³.

Figure 13 Final profiles (a) and the corresponding slopes (b) from S-R flume test simulations (3D) of different flume widths. The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 14 Final profiles (a) and the corresponding slopes (b) from simulations (2D) of S-R flume test with different base angles. The material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 15 Final profiles (a) and the corresponding slopes (b) from simulations (2D) of discharge flume test with different base angles. The material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 16 The average beach slope vs. yield stress from the planar simulation results of both S-R and discharge flume tests.

Figure 17 Average beach slope as a function of $\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g^{1.15} A^{0.225} b}$ for S-R flume tests.

Figure 18 Average beach slope as a function of $\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g A^{0.5} Q^{0.3}}$ for discharge flume tests.

Appendix A. Dimensional analysis of S-R flume tests

The variables that govern the flow of thickened tailings in an S-R flume test are listed in **Table A1**.

Table A1. Variables for dimensional analysis of the 2D S-R flume test.

Variables	Symbol	Units	Dimensions
Average beach slope	BS_a	(°)	$M^0 L^0 T^0$
Base angle	θ	(°)	$M^0 L^0 T^0$
Yield stress	τ_y	Pa	$M^1 L^{-1} T^{-2}$
Volume of tailings	A	m^2	$M^0 L^2 T^0$
Height of patched area	b	m	$M^0 L^1 T^0$
Plastic viscosity	μ_0	Pa·s	$M^1 L^{-1} T^{-1}$
Density	ρ	kg/m^3	$M^1 L^{-3} T^0$
Gravitational acceleration	g	m/s^2	$M^0 L^1 T^{-2}$

We choose A , ρ and g as the repeating variables, which can be justified by:

$$\begin{vmatrix} 0 & 2 & 0 \\ 1 & -3 & 0 \\ 0 & 1 & -2 \end{vmatrix} = -2 \neq 0 \quad (A1)$$

Therefore the Π terms are:

$$\Pi_1 = \frac{BS_a}{A^{a_1} \rho^{b_1} g^{c_1}} \quad (A2)$$

$$\Pi_2 = \frac{\theta}{A^{a_2} \rho^{b_2} g^{c_2}} \quad (A3)$$

$$\Pi_3 = \frac{\tau_y}{A^{a_3} \rho^{b_3} g^{c_3}} \quad (A4)$$

$$\Pi_4 = \frac{b}{A^{a_4} \rho^{b_4} g^{c_4}} \quad (A5)$$

$$\Pi_5 = \frac{\mu_0}{A^{a_5} \rho^{b_5} g^{c_5}} \quad (A6)$$

Since the Π terms are dimensionless, it is possible to determine the indices (a_i , b_i , c_i)

in Eqs.A2 to A6. Thus we have:

$$\Pi_1 = BS_a \quad (A7)$$

$$\Pi_2 = \theta \quad (A8)$$

$$\Pi_3 = \frac{\tau_y}{\sqrt{A}\rho g} \quad (A9)$$

$$\Pi_4 = \frac{b}{\sqrt{A}} \quad (A10)$$

$$\Pi_5 = \frac{\mu_0}{A^{0.75}\rho\sqrt{g}} \quad (A11)$$

The relation between the Π terms can be expressed as:

$$BS_a = f\left(\theta, \frac{\tau_y}{\sqrt{A}\rho g}, \frac{b}{\sqrt{A}}, \frac{\mu_0}{A^{0.75}\rho\sqrt{g}}\right) \quad (A12)$$

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Appendix B. Dimensional analysis of discharge flume tests

The variables that govern the flow of thickened tailings in a discharge flume test are listed in **Table B1**.

Table B1. Variables for dimensional analysis of the discharge flume test.

Variables	Symbol	Units	Dimensions
Average beach slope	BS_a	(°)	$M^0 L^0 T^0$
Base angle	θ	(°)	$M^0 L^0 T^0$
Yield stress	τ_y	Pa	$M^1 L^{-1} T^{-2}$
Volume of tailings	A	m^2	$M^0 L^2 T^0$
Flow rate	Q	m^2/s	$M^0 L^2 T^{-1}$
Plastic viscosity	μ_0	Pa·s	$M^1 L^{-1} T^{-1}$
Density	ρ	kg/m^3	$M^1 L^{-3} T^0$
Gravitational acceleration	g	m/s^2	$M^0 L^1 T^{-2}$

Tailings volume A , discharge flow rate Q and density ρ are chosen as the repeating variables, which can be justified by:

$$\begin{vmatrix} 0 & 2 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 0 \end{vmatrix} = -2 \neq 0 \quad (B1)$$

Therefore the Π terms are:

$$\Pi_1 = \frac{BS_a}{A^{a_1} Q^{b_1} \rho^{c_1}} \quad (B2)$$

$$\Pi_2 = \frac{\theta}{A^{a_2} Q^{b_2} \rho^{c_2}} \quad (B3)$$

$$\Pi_3 = \frac{\tau_y}{A^{a_3} Q^{b_3} \rho^{c_3}} \quad (B4)$$

$$\Pi_4 = \frac{\mu_0}{A^{a_4} Q^{b_4} \rho^{c_4}} \quad (B5)$$

$$\Pi_5 = \frac{g}{A^{a_5} Q^{b_5} \rho^{c_5}} \quad (B6)$$

The indices (a_i, b_i, c_i) may be determined by equating the coefficients in Eqs.B2 to B6. Thus we have:

$$\Pi_1 = BS_a \quad (B7)$$

$$\Pi_2 = \theta \quad (B8)$$

$$\Pi_3 = \frac{A\tau_y}{\rho Q^2} \quad (B9)$$

$$\Pi_3 = \frac{\mu_0}{\rho Q} \quad (B10)$$

$$\Pi_4 = \frac{A^{1.5}g}{Q^2} \quad (B11)$$

The average beach slope can be expressed as:

$$BS_a = g(\theta, \frac{A\tau_y}{\rho Q^2}, \frac{\mu_0}{\rho Q}, \frac{A^{1.5}g}{Q^2}) \quad (B12)$$

Figure 1 Effect of front spreading speed on predicted average slope angle. The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³

Figure 2 Comparison of analytical and simulated final profiles in a 2D flume test. The parameters of the material used were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 3 Sketch of geometries for 2D planar flume tests. (a) S-R flume test (b) Discharge flume test. The shaded area in panel (a) is the fluid patched in the flume at $t=0$. The empty area is occupied by air for both flumes. The walls, except for the inlets and outlets, are treated as non-slip walls.

Figure 4 Sketch illustrating how the linear slope of a profile in a flume test was obtained. x is given by $0.8L + 0.2L'$.

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Figure 5 Final profiles of the prediction by Eq.(9) and the 2D planar simulation of S-R flume test with a patched area of 130 mm(Height) $\times$ 100 mm(Length). The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 6 Final profiles from 2D simulations of discharge flume tests with different inlet velocities and the corresponding slopes. The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 7 Slopes from simulations of discharge flume tests (a) and S-R flume tests (b) for yield stress and viscosity influence investigation. Detailed information of the simulations is listed in **Table 2**.

Figure 8 Slope variations from Case 0 caused by 25% increase of yield stress and viscosity in simulations of discharge flume tests (a) and S-R flume tests (b). Detailed information of the simulations is listed in **Table 2**.

Figure 9 Ratio of slope variations between Case 2 and Case 1 for simulations of discharge flume tests (a) and S-R flume tests (b). The Bingham fluids employed in Case 2 and Case 1 have higher viscosity and yield stress respectively than Case 0, as shown in **Table 2**.

Figure 10 Sketches for the 3D simulations of the discharge flume test with a flume-wide inlet (a) and the S-R flume test at the initial state (b). The shaded volume in panel (b) is the initial volume patched in the flume at $t=0$. Non-slip conditions are applied to the walls except for the outlets and inlets.

Figure 11 Final profiles from 3D simulations of different number of elements for the discharge flume test with flume width at 200 mm.

Figure 12 Final profiles (a) and the corresponding slopes (b) from discharge flume test simulations (3D) of different flume widths. The inlet velocity was 0.5 m/s with a discharging time of 1 s. The input material properties were: 18.61 Pa, 0.32 Pa·s and 1315.0 kg/m³.

Figure 13 Final profiles (a) and the corresponding slopes (b) from S-R flume test simulations (3D) of different flume widths. The input material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 14 Final profiles (a) and the corresponding slopes (b) from simulations (2D) of S-R flume test with different base angles. The material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 15 Final profiles (a) and the corresponding slopes (b) from simulations (2D) of discharge flume test with different base angles. The material properties were: 33.28 Pa, 0.4 Pa·s and 1342.6 kg/m³.

Figure 16 The average beach slope vs. yield stress from the planar simulation results of both S-R and discharge flume tests.

Figure 17 Average beach slope as a function of $\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g^{1.15} A^{0.225} b}$ for S-R flume tests.

Figure 18 Average beach slope as a function of $\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin \theta)}{\rho^{1.3} g A^{0.5} Q^{0.3}}$ for discharge flume tests.

Table 1 Data from simulations of discharge flume tests.

Runs	$\tan\theta$	τ_y	μ_0	ρ	Q	A	H	L	$\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin\theta)}{\rho^{1.3} g A^{0.5} Q^{0.3}}$	BS _a
Units	-	Pa	Pa·s	kg/m ³	m ² /s	m ²	m	m	-	%
1	0	33.28	0.4	1342.6	0.002	0.004	-	0.1739	2.259E-02	10.91
2	0	33.28	0.4	1342.6	0.002	0.008	-	0.2833	1.597E-02	8.243
3	0	33.28	0.4	1342.6	0.002	0.016	-	0.4576	1.129E-02	6.421
4	0	33.28	0.4	1342.6	0.002	0.032	-	0.7436	7.987E-03	5.175
5	0	33.28	0.4	1342.6	0.004	0.032	-	0.7466	6.487E-03	4.88
6	0	33.28	0.4	1342.6	0.008	0.032	-	0.7691	5.269E-03	4.355
7	0	33.28	0.4	1342.6	0.016	0.032	-	0.8136	4.280E-03	3.217
8	0	18.61	0.32	1315	0.002	0.016	-	0.5563	6.069E-03	4.68
9	0	23.26	0.32	1315	0.002	0.016	-	0.5150	7.586E-03	5.298
10	0	18.61	0.4	1315	0.002	0.016	-	0.5550	6.489E-03	4.807
11	0	18.61	0.32	1315	0.008	0.016	-	0.6143	4.004E-03	3.543
12	0	23.26	0.32	1315	0.008	0.016	-	0.5686	5.005E-03	4.041
13	0	18.61	0.4	1315	0.008	0.016	-	0.6031	4.281E-03	3.685
14	0.01	33.28	0.4	1342.6	0.002	0.032	0.038	0.7781	6.688E-03	4.482
15	0.02	33.28	0.4	1342.6	0.002	0.032	0.038	0.8121	5.499E-03	3.78
16	0.03	33.28	0.4	1342.6	0.002	0.032	0.038	0.8512	4.427E-03	3.123
17	0	50.1	0.64	1367.4	0.008	0.016	-	0.4146	1.261E-02	7.007
18	0	62.62	0.64	1367.4	0.008	0.016	-	0.3887	1.576E-02	8.751
19	0	50.1	0.8	1367.4	0.008	0.016	-	0.4094	1.349E-02	7.356
20	0	50.1	0.64	1367.4	0.008	0.008	-	0.2621	1.784E-02	8.898
21	0	62.62	0.64	1367.4	0.008	0.008	-	0.2443	2.229E-02	11.04
22	0	50.1	0.8	1367.4	0.008	0.008	-	0.2575	1.907E-02	9.451

Table 2. Detailed information of flume test simulations for the investigation of yield stress and viscosity influence on final profiles

Groups	Cases	Yield stress(Pa)	Viscosity(Pa·s)	Energy conditions	Flume test
Group1	Case0	18.61	0.32	$v_{inlet} = 0.1m/s$	Discharge
	Case1	1.25×18.61	0.32		
	Case2	18.61	1.25×0.32		
Group2	Case0	18.61	0.32	$v_{inlet} = 0.4m/s$	
	Case1	1.25×18.61	0.32		
	Case2	18.61	1.25×0.32		
Group3	Case0	18.61	0.32	Patched shape:	S-R
	Case1	1.25×18.61	0.32	46mm $\times$ 300mm	
	Case2	18.61	1.25×0.32	(Height $\times$ Length)	
Group4	Case0	18.61	0.32	Patched shape:	
	Case1	1.25×18.61	0.32	69mm $\times$ 200mm	
	Case2	18.61	1.25×0.32	(Height $\times$ Length)	

Table 3 Comparison of the slopes from 3D simulations of different number of elements for the discharge flume test with the flume width at 200 mm.

Cases	Elements	Slope (%)	Slope variation from Case 3 (%)
Case 1	6620	1.646	14.78
Case 2	52960	1.459	1.74
Case 3	423680	1.426	-

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Table 4 Data from simulations of S-R flume tests.

Runs	$\tan\theta$	τ_y	μ_0	ρ	a	b	A	H	L	$\frac{\mu_0^{0.3}(\tau_y - \rho g H \sin\theta)}{\rho^{1.3} g^{1.15} A^{0.225} b}$	BS _a
Units	-	Pa	Pa·s	kg/m ³	m	m	m ²	m	m	-	%
1	0	33.28	0.4	1342.6	0.4	0.08	0.032	-	0.7970	4.264E-03	3.17
2	0.01	33.28	0.4	1342.6	0.4	0.08	0.032	0.038	0.8433	3.624E-03	2.65
3	0.02	33.28	0.4	1342.6	0.4	0.08	0.032	0.038	0.8924	3.055E-03	2.19
4	0.03	33.28	0.4	1342.6	0.4	0.08	0.032	0.038	0.9442	2.551E-03	1.79
5	0	33.28	0.4	1342.6	0.2	0.16	0.032	-	1.0436	2.132E-03	1.35
6	0	33.28	0.4	1342.6	0.1	0.32	0.032	-	1.3941	1.066E-03	0.756
7	0	33.28	10	1342.6	0.05	0.13	0.0065	-	0.2463	9.865E-03	9.045
8	0	33.28	10	1342.6	0.1	0.13	0.013	-	0.4155	8.440E-03	6.731
9	0	33.28	10	1342.6	0.2	0.13	0.026	-	0.6812	7.222E-03	5.224
10	0	33.28	10	1342.6	0.4	0.13	0.052	-	1.0865	6.179E-03	4.182
11	0	18.61	0.32	1315	0.3	0.046	0.0138	-	0.5199	4.814E-03	3.969
12	0	23.26	0.32	1315	0.3	0.046	0.0138	-	0.4769	6.018E-03	4.958
13	0	18.61	0.4	1315	0.3	0.046	0.0138	-	0.5146	5.148E-03	4.259
14	0	18.61	0.32	1315	0.2	0.069	0.0138	-	0.6035	3.210E-03	2.258
15	0	23.26	0.32	1315	0.2	0.069	0.0138	-	0.5535	4.012E-03	2.671
16	0	18.61	0.4	1315	0.2	0.069	0.0138	-	0.5857	3.432E-03	2.47