**Lane-based signal optimization with left turn prohibition in urban road networks**

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                          | Liu, Huijun; Technische Universität Braunschweig, Institute of Communication Technology  
                          | Friedrich, Bernhard; Technische Universität Braunschweig, Institute of Transportation and Urban Engineering |
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Lane-based signal optimization with left turn prohibition in urban road networks

Qinrui Tang
Institute of Transportation and Urban Engineering,
Technische Universität Braunschweig, Braunschweig, Germany.
Email: q.tang@tu-braunschweig.de

Huijun Liu
Institute of Communication Technology,
Technische Universität Braunschweig, Braunschweig, Germany
Email: ansleliu@gmail.com

Bernhard Friedrich
Institute of Transportation and Urban Engineering,
Technische Universität Braunschweig, Braunschweig, Germany.
Email: friedrich@tu-braunschweig.de

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1 Corresponding author. Current affiliation: Institute of Transportation Systems, German Aerospace Center, Berlin, Germany. Tel: +49 30 67055 9626. Fax:+49 30 67055 291. Email: qinrui.tang@dlr.de
Abstract

Left turns may generate efficiency problems which can be solved by appropriately prohibiting left turns. The goal of this paper is to propose a method for the purpose of minimizing total travel times in urban road networks by prohibiting left turns. With left turn prohibition, the signal timing plan is optimized with the lane-based method because the method can adequately handle both signal timing optimization and lane assignment. The total travel time is calculated with link flows and link travel time being estimated with signal settings. In numerical examples, prohibiting left turns reduces the total travel time of car traffic in road networks. As the left turn prohibition results can handle the randomness in the network, these results provide potential implications for congestion management.

Key words: Left turn prohibition, Lane-based signal optimization, Turning restriction, Stochastic user equilibrium, Left turn treatment
1 Introduction

Left turn movements may have negative impacts on the efficiency of signalized urban networks if they were not well accommodated. Permitted left turns are one of the most hazardous movements because they are permitted to access intersections at the same time as oncoming vehicles. To avoid conflicts, left turns can be protected in single phases; however, protected left turns reduce effective green time. As permitted and protected left turns limit capacities at critical intersections, we investigate left turn prohibition in urban road networks to achieve better use of capacity.

Prohibited left turn flows should be carefully accommodated. One of the solutions is to construct unconventional intersections such as U-turns (Lu, Dissanayake, Xu and Williams 2001; Lu, Dissanayake, Zhou and Yang 2001; Bared and Kaisar 2002; Chowdhury, Derov and Tan 2003; Liu et al. 2007; Leng, Zhao and Zhang 2009; Zhao et al. 2016) in the middle of roads and mid-block left turns (Bonneson and McCoy 1981; Chowdhury, Derov and Tan 2005) so the left turns at intersections can be avoided. Although the goal of constructing unconventional intersections is to improve the performance of intersections, the goal may not always be achieved. For example, Roudsari, Kaufman and Nirula (2007) found that mid-block left turns may have safety problems. Moreover, unconventional intersections require more space so that it is more difficult to implement them in urban road networks. Thus, it is a more feasible solution that directly prohibiting left turns at conventional intersections to redistribute left turning vehicles within the network.

Directly prohibiting left turns may improve the efficiency of urban road networks, more
specifically, reduce delays at intersections (Hajbabaie, Medina and Benekohal 2010; Yu and Prevedouros 2012). Previous researchers (Long et al. 2010; Long, Szeto and Huang 2014; Foulds et al. 2014; Zhao and Yang 2014; Zhao et al. 2015) who aimed to solve turning restriction problems, find that left turns are most frequently prohibited in their research. To solve the restriction problem, Long et al. (2010); Long, Szeto and Huang (2014) and Foulds et al. (2014) treated it as the pure network design problem with one decision referring to turning prohibition by formulating a bi-level programming whose lower level is a traffic assignment model. The traffic assignment model provides a solid base for detour route choice after turnings are prohibited, but signal timing and lane assignment, which are necessary to be addressed, are not integrated with the traffic assignment models. Zhao et al. (2015) consider both signal timing and lane assignment by applying lane-based signal optimization model (Wong and Wong 2003) whereas their dynamical scenario requires the assumption that vehicles go to the next intersection and take a U-turn for forecasting detour paths, which is reasonable but not always realistic.

A suitable solution to achieve the mentioned issues is to propose a method combining traffic assignment model and signal optimization so that detour path forecasting and signal optimization can be integrated into one model. The combination of the traffic assignment and the signal optimization has been studied by Yang and Yagar (1995), Maher, Zhang and Van Vliet (2001) and Cascetta, Gallo and Montella (2006) in the congested networks. However, the stage-based signal optimization method is not flexible enough in the left turn prohibition problem because the lane markings in the stage-based method were fixed, which is against the fact that lane markings change after left turn prohibition. Lane-based signal optimization
method can handle the change of lane assignment. The lane-based method was developed by Wong and Wong (2003) and was extended from the group-based method (Improta and Cantarella 1984; Silcock 1997). Unlike the group-based method that the lane markings are given, the main improvement of the lane-based method is that it takes lane assignments as decision variables so it can deal with the changeable traffic flow distribution. However, after the lane-based optimization, the saturation flows of links need to be adjusted because the number of lanes for a movement significantly influences the saturation flows of the movement. This is the difference between the stage-based method and the lane-based method when a signal optimization method is combined with traffic assignment models, but this difference is rarely figured out in previous research.

We aim to propose a model to solve the left turn prohibition problem with the goal of minimization of total travel time in the fixed time period. The model combines the lane-based signal optimization method and a stochastic user equilibrium (SUE) model, which is one of the traffic assignment models. The lane-based method is developed from the method of Wong and Wong (2003) but adjusted to the left turn prohibition problem. Signal settings, being defined by cycle length, start and duration of greens, and lane assignment, are locally optimized for each intersection. The saturation flow adjustment for permitted left turns and the lanes sharing movements is treated before calculating the total travel time. By calculating the total travel time in networks with different situations of left turn prohibition, the left turn prohibition solution with the minimum total travel time is determined.

The remainder of this paper is organized as follows. The method of left turn prohibition problem is proposed in Section 2, followed by algorithms in Section 3. The numerical examples
are given by testing the proposed model with the network of Hanover South City, Germany in Section 4 and the best left turn prohibition solution handling multiple random seeds in the SUMO simulation toll is analyzed through the sensitivity analysis. Finally, this paper is discussed in Section 5 and concluded in Section 6, respectively.

2 Methodology

The proposed model starts by prohibiting feasible left turns (See Figure 1). Left turns are selected to be prohibited with a genetic algorithm. The left turns meeting the constraint, which refers to the connectivity between origins and destinations, are feasible prohibited left turns. Once a left turn is prohibited, the travel time of the left turn is infinite and the traffic demand distribution in the network changes accordingly.

The SUE model describes how the traffic demand is distributed according to the different perceived travel time. The travel time estimation is thereby solved with the travel time based on BPR (Bureau of Public Roads) function and the signal timing for different purposes. The SUE with the travel time based on BPR function is applied just after left turn prohibition. The BPR function is the function of link flows with given free flow travel times and link capacities. In this paper, the capacity in the BPR function is assigned as saturation flows because the signal timings are unavailable yet. The saturation flows of through movements, right turns and protected left turns are given according to HCM (2000).

With the link flows generated from the SUE with the travel time based on the BPR function, the signal settings are optimized. Then the SUE with the travel time based on signal timing is applied so that the left turn prohibition and corresponding signal settings can be evaluated.
The travel time based on signal timing is the sum of free flow travel time and link delay.

These processes repeat until all selected feasible prohibited left turns are tested. The left
turn prohibition solution with minimum total travel time is recorded as the best solution.

2.1 Left turn prohibition

The objective of the left turn prohibition problem is to minimize the total travel time of
the network which is defined as the sum of travel times of all vehicles from each origin to
each destination. The left turn prohibition indicator, which indicates whether a left turn is
prohibited, must meet the constraint that vehicles can come from all origins to all destinations
even if left turns were prohibited. Thus, left turn prohibition solutions have to ensure that the
path set for each OD pair is not empty; otherwise, the left turn prohibition solutions are not
feasible. The objective function is expressed as:

\[
\min TT = \sum_{a \in E/L} q_a t_a(q_a, \eta) + \sum_{a \in L} (1 - \gamma_a) q_a t_a(q_a, \eta).
\] (1)

subject to

\[
K_{od} \neq \emptyset, \forall o \in O, d \in D.
\] (2)

where

\( TT \) total travel time (h);

\( E \) set of links;

\( L \) set of left turn links;
2.2 Lane-based signal optimization

In the left turn prohibition problem, to make use of the prohibited left turn lanes, lane assignments should be adjusted accordingly when left turns are deleted from signal groups (See the example in Figure 2). Thus, lane-based optimization is the most suitable method in this paper.

Figure 2. (a) An example of lane assignment before left turn prohibition;
(b) An example of lane assignment after left turn prohibition

2.2.1 Intersection notations

Denote $z$ as the intersection index and $z = 1, ..., N_Z$ where $N_Z$ is the number of intersections in the network. At intersection $z$, there are $N_{A,z}$ number of arms and for each arm the index is $i$. 
In each arm, maximum three directions can be assigned to different lanes. The directions are in a set \( \{ LT, TH, RT \} \) where \( LT \) is left turn; \( TH \) is through movement and \( RT \) is right turn. The direction index is \( j \), so a movement is represented as \( (i, j) \) which means a direction \( j \) from arm \( i \). Please note that, the movement \( (i, LT) \) is the same as link \( a \) where \( a \in L \). The lane index is \( k \). \( k = 1, ..., N_{L,z,i} \) where \( N_{L,z,i} \) is the number of lanes in arm \( i \) at intersection \( z \). The intersection is represented in Figure 3 and the decision variables are summarized in Table 1.

![Figure 3 about here.]

Figure 3. Intersection notations

Table 1. Decision variables of the lane-based method

![Table 1 about here.]

2.2.2 Objective function

The lane-based optimization problem is formulated as a mixed integer linear programming by maximizing the reserved capacity for each intersection (See Eq. 3). The reserved capacity was defined by Allsop (1972) as the multiplier indicating the maximum flows assigned to a lane. If the reserved capacity is more than 1, the intersection is not congested; if the reserved capacity is less than 1, otherwise. The reserved capacity for each intersection could be different. With given traffic flows and a conflict matrix for each intersection, the lane-based method could optimize signal timing, signal sequence, and lane assignment.

\[
\max \mu_z. \tag{3}
\]

where \( z = 1, ..., N_Z \) is the intersection index.
2.2.3 Constraints

**Left turn prohibition** If the left turn on the arm \( i \) is prohibited, the left turn cannot be permitted on any lanes of the arm \( i \). That means, if \( \gamma_{z,i} = 1 \), or the same as \( \gamma_a = 1, a \in L \), \( \delta_{z,i,LT,k} \) must be 0. This constraint is applied to the arms with left turns.

\[
\forall z = 1, ..., N_Z; i = 1, ..., N_{A,z}; j = LT; k = 1, ..., N_{L,z,i}
\]

\[
0 \leq \delta_{z,i,j,k} \leq \gamma_{z,i} - 1.
\]  \( (4) \)

where \( \delta_{z,i,j,k} \) is the lane permission indicator:

\[
\delta_{z,i,j,k} = \begin{cases} 
1, & \text{movement } (i, j) \text{ is permitted on lane } k \\
0, & \text{otherwise}
\end{cases}
\]  \( (5) \)

**Minimum permitted movement** At least one movement must be permitted on a lane because with idle lanes the road capacity is not fully made use of. The number of movements on the lane \( k \) is expressed as the sum of \( \delta_{z,i,j,k} \) by turning direction \( j \). The minimum number of permitted movements is 1 (Wong and Wong 2003).

\[
\forall z = 1, ..., N_Z; i = 1, ..., N_{A,z}; k = 1, ..., N_{L,z,i},
\]

\[
\sum_{j \in M} \delta_{z,i,j,k} \geq 1.
\]  \( (6) \)

**Maximum permitted lanes** For the safety reasons, the number of permitted lanes for each movement \( (i, j) \), is no more than the number of exit lanes of the movement. Otherwise, vehicles will conflict with each other when merging into one lane. The sum of \( \delta_{z,i,j,k} \) by lane \( k \) is the number of lanes occupied by movement \( (i, j) \). This number must be less than the number of exit lanes \( N_{E,z,i,j} \) (Wong and Wong 2003).
\[ \forall z = 1, \ldots, N_z; i = 1, \ldots, N_{A,z}; j \in M, \]
\[ \sum_{k=1}^{N_{L,z,i}} \delta_{z,i,j,k} \leq N_{E,z,i,j}. \]  \hfill (7)

**Conflict elimination on adjacent lanes**  For purpose of eliminating the internal conflicts on adjacent lanes, if a through movement is permitted on a lane, right turns will be not permitted on the left lanes of the through movement; if a left turn is permitted on a lane, through movements and right turns will be not permitted on the left lanes of the right turn. (See Figure 4).

Denote \( M' \) is the subset of \( M \). For \( j \in M \), if \( j = LT \), \( M' = \{LT, TH, RT\} \); if \( j = TH \), \( M' = \{TH, RT\} \); if \( j = RT \), \( M' = \{RT\} \). For \( m \in M' \), if movement \((i, m)\) is permitted on the lane \( k \), movement \((i, j)\) cannot be permitted on the lane \( k + 1 \) (Wong and Wong 2003).

\[ \forall z = 1, \ldots, N_z; i = 1, \ldots, N_{A,z}; j \in M; m \in M'; k = 1, \ldots, N_{L,z,i} - 1, \]
\[ \delta_{z,i,j,k+1} \leq 1 - \delta_{z,i,m,k}. \]  \hfill (8)

[Figure 4 about here.]

**Maximum amount of traffic increase**  According to the definition of the reserved capacity, the maximum amount of traffic flow of a movement, is the product of reserved capacity \( \mu_z \) and link flow \( q_{z,i,j}' \) from the SUE model with BPR function. The maximum amount is equal to the sum of traffic flows of movement \((i, j)\) being assigned to all lanes on arm \( i \) (Wong and Wong 2003).
\[ \forall z = 1, \ldots, N_Z; i = 1, \ldots, N_{A,z}; j \in M, \]
\[ \mu_{z,i,j} = \sum_{k=1}^{N_{L,z,i}} f_{z,i,j,k}. \]  
(9)

where \( f_{z,i,j,k} \) is the assigned flow of movement \((i, j)\) on lane \(k\). Assigned flow is the theoretical maximum flow, and accompanies with the reserved capacity.

**Assigned flow of non-permitted lanes** If a movement \((i, j)\) is not permitted on lane \(k\), the assigned flow \( f_{z,i,j,k} \) must be 0. If \( \delta_{z,i,j,k} = 0 \), the values of \( f_{z,i,j,k} \) is forced to be 0.

\[ \forall z = 1, \ldots, N_Z; i = 1, \ldots, N_{A,z}; j \in M; k = 1, \ldots, N_{L,z,i}, \]
\[ 0 \leq f_{z,i,j,k} \leq H \delta_{z,i,j,k}. \]  
(10)

where \( H \) is an arbitrarily large positive constant.

**Identical flow factor of adjacent lanes being occupied by a movement** Flow factor is the ratio of the lane assigned flow in the lane saturation flow. Denote \( b_{z,i,k} \) as flow factor of lane \(k\) on arm \(i\).

\[ \forall z = 1, \ldots, N_Z; i = 1, \ldots, N_{A,z}; k = 1, \ldots, N_{L,z,i}, \]
\[ b_{z,i,k} = \sum_{j \in M} \frac{f_{z,i,j,k}}{s_{z,i,k}}. \]  
(11)

where \( s_{z,i,k} \) is the saturation flow of lane \(k\) being calculated with Eq. (31), but using \( f_{z,i,j,k} \) as inputs.

If a movement occupies multiple lanes, the flow factor of each lane must be the same. Drivers are not willing to be on the lane with longer waiting time. Thus, the lane permission indicators should manage to equalize the flow factor of adjacent lanes. If movement \((i, j)\) is permitted on
both lane $k$ and $k+1$, i.e. $\delta_{z,i,j,k} = 1, \delta_{z,i,j,k+1} = 1$, $b_{z,i,k}$ is equal to $b_{z,i,k+1}$ (Wong and Wong 2003).

$$\forall z = 1, \ldots, N_Z; i = 1, \ldots, N_{A,z}; k = 1, \ldots, N_{L,z,i},$$

$$-H(2 - \delta_{z,i,j,k} - \delta_{z,i,j,k+1}) \leq b_{z,i,k} - b_{z,i,k+1} \leq H(2 - \delta_{z,i,j,k} - \delta_{z,i,j,k+1}).$$  \hfill (12)

### Maximum acceptable degree of saturation

A degree of saturation should be no more than the maximum acceptable degree of saturation. As the degree of saturation is relevant to green split, this constraint can ensure that the degree of saturation is in an acceptable level and adjust the green durations. Denote $\rho_{z,i,k}$ as degree of saturation on lane $k$.

$$\forall z = 1, \ldots, N_Z; i = 1, \ldots, N_{A,z}; k = 1, \ldots, N_{L,z,i},$$

$$\rho_{z,i,k} = \frac{b_{z,i,k}}{\Phi_{z,i,k} + e_{z}} \leq \rho_{\text{max},i,k}.$$  \hfill (13)

where $\Phi_{z,i,k}$ is the green split of lane $k$; $\xi_z$ is the inverse of cycle length; $e$ is the difference between actual green time and effective green time; $\rho_{\text{max},i,k}$ is the maximum acceptable degree of saturation (Wong and Wong 2003).

### Cycle length, start of green and duration of green

Cycle length $c_z = 1/\xi_z$ should be in the interval $[c_{\text{min}}, c_{\text{max}}]$, where $c_{\text{min}}$ and $c_{\text{max}}$ are the minimum and the maximum cycle length, respectively. Start of green and green split are also ratios smaller than 1 (Wong and Wong 2003). If a left turn is prohibited, the green split is 0 (Eq. (17)).

$$\forall z = 1, \ldots, N_Z; i = 1, \ldots, N_{A,z}; j \in M,$$

$$1/c_{\text{max}} \leq \xi_z \leq 1/c_{\text{min}}.$$  \hfill (14)
0 ≤ θ_{z,i,j} + φ_{z,i,j} ≤ 1. \quad (15)

∀z = 1, ..., N_Z; i = 1, ..., N_{A,z}; j \in \{TH, RT\},

\[ g_{\min,z,i,j} \xi_{z} \leq \phi_{z,i,j} \leq 1. \quad (16) \]

∀z = 1, ..., N_Z; i = 1, ..., N_{A,z}; j \in \{LT\},

\[ (1 - \gamma_{z,i})g_{\min,z,i,j} \xi_{z} \leq \phi_{z,i,j} \leq 1 - \gamma_{z,i}. \quad (17) \]

where \( \theta_{z,j} \) is the start of green of movement \((i, j)\); \( g_{\min,z,i,j} \) is the minimum green time.

**Identical signal settings of movements on shared lanes**  When multiple movements share one lane, the signal settings of these movements are identical, to avoid internal conflicts on the lane. Thus, constraint (18) and (19) ensure the identical signal settings on shared lanes, and identical signal settings of one movement on multiple lanes (Wong and Wong 2003).

∀z = 1, ..., N_Z; i = 1, ..., N_{A,z}; j \in M; k = 1, ..., N_{L,z,i},

\[ -H(1 - \delta_{z,i,j,k}) \leq \Theta_{z,i,k} - \theta_{z,i,j} \leq H(1 - \delta_{z,i,j,k}). \quad (18) \]

\[ -H(1 - \delta_{z,i,j,k}) \leq \Phi_{z,i,k} - \phi_{z,i,j} \leq H(1 - \delta_{z,i,j,k}). \quad (19) \]

where \( \Theta_{z,i,k} \) is the start of green for lane \( k \); \( \Phi_{z,i,k} \) is the green split for lane \( k \).

**Signal sequence**  If two movements conflict with each other, they cannot be in the same green durations. Thus, one movement can be either the predecessor of another movement or the successor of another. The successor indicators \( \Omega_{z,i,j,l,m} \) indicate whether a movement is the
successor of other movements and this constraint is only applicable for movements conflict with each other. Constraint (22) says that two movements cannot be the successors of each other at the same time. Constraint (23) further limits the search space of the feasible area: when movement \((i, j)\) is the predecessor of movement \((l, m)\) and movement \((l, m)\) is the predecessor of movement \((u, v)\), movement \((i, j)\) must be the predecessor of movement \((u, v)\).

Denote \(\psi_{z,i,j,l,m}\) as conflict state indicators,

\[
\psi_{z,i,j,l,m} = \begin{cases} 
1, & \text{movement } (i, j) \text{ conflicts with the movement } (l, m) \\
0, & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (20)

\[
\Omega_{z,i,j,l,m} = \begin{cases} 
1, & \text{movement } (i, j) \text{ is the successor of movement } (l, m) \\
0, & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (21)

If \(\psi_{z,i,j,l,m} = 1\), following equations hold:

\[
\Omega_{z,i,j,l,m} + \Omega_{z,l,m,u,v} = 1.  \hspace{1cm} (22)
\]

\[
\forall u = 1, ..., N_{A,z}, v \in M, \text{ If } \psi_{z,i,j,l,m} = 1, \psi_{z,l,m,u,v} = 1 \text{ and } \psi_{z,i,j,u,v} = 1,
\]

\[
\Omega_{z,i,j,l,m} + \Omega_{z,l,m,u,v} - 1 \leq \Omega_{z,i,j,u,v} \leq \Omega_{z,i,j,l,m} + \Omega_{z,l,m,u,v}.  \hspace{1cm} (23)
\]

**Start of green for successor movements**  The start of green for successor movements must be later than the end of predecessor movements plus the intergreen time. It is necessary to have intergreen time between two conflict movements due to safety reasons; otherwise, the vehicles passing the stop line may conflict with the vehicles from conflict movements.

\[
\forall z = 1, ..., N_Z; i = 1, ..., N_{A,z}; j \in M; l = 1, ..., N_{A,z}; m \in M,
\]

\[
\theta_{z,i,j} + \phi_{z,i,j} + \omega_{z,i,j,l,m} \xi_z \leq \theta_{z,l,m} + \Omega_{z,i,j,l,m}.  \hspace{1cm} (24)
\]

\[
\text{https://mc06.manuscriptcentral.com/cjce-pubs}
\]
2.3 Stochastic user equilibrium

The demand model and the supply model are the main components of the SUE. The demand model is to estimate link flows and the supply model is to estimate link travel times. Initially, drivers choose routes according to the travel cost of empty networks, resulting in different link flows, while link flows also influence link travel costs. Drivers repeat their choices in the signalized network until any route change would not cause higher travel costs for all vehicles, which is the equilibrium state.

The link flows are estimated with the product of the probabilities of a path being selected and OD demands (Tang 2016). The probabilities are calculated using a logit model. The link travel times are estimated as either BPR function or the sum of free flow travel time and link delay.

2.3.1 Estimating link travel time with BPR function

The traffic demands in the signal setting optimization are obtained from the link flows of the SUE model with the travel time based on BPR function. Although the network has original signal settings before left turn prohibition, the original signal settings are inapplicable because they change after left turn prohibition. Thus, we assume that signal settings are unavailable and the link travel times change like highways.

\[ t_a(q_a) = t_{0,a} \left(1 + \alpha \left(\frac{q_a}{Q_a}\right)^\beta\right). \]  

(25)

where

\[ a \in E \quad \text{link index}; \]
2.3.2 Estimating link travel time with signal settings

Link travel times with signal settings is estimated as the sum of free flow travel time and link delay.

\[ t_a(q_a, \eta) = t_{0,a} + d_a(q_a, \eta). \] (26)

As the BPR function is not well fit the travel time at signalized intersections (Skabardonis and Dowling 1997; Davis and Xiong 2007), a delay formula using in the saturated condition is proper to be applied at signalized intersections. In this paper, the delay formula follows Akcelik (1981) whose delay estimation can keep consistent with HCM (2000) but has simpler parameters. The link delay is estimated in Eq. (27) and Eq. (28) if a movement is permitted on lane \( k \). However, please note that in the delay formula, the queue is assumed as a vertical queue such that the queue length is not considered.

\[ d_{z,i,k}(q_{z,i,k}, \eta) = \frac{0.5 c_z (1 - g_{z,i,k}/c_z)^2}{1 - \min(1, x_{z,i,k}) g_{z,i,k}/c_z} + \min \left( 1, \frac{x_{z,i,k}}{x_{0,z,i,k}} \right) 900T \left[ (x_{z,i,k} - 1) + \sqrt{(x_{z,i,k} - 1)^2 + \frac{12(x_{z,i,k} - x_{0,z,i,k})}{Q_{z,i,k} T}} \right]. \] (27)

\[ d_a(q_a, \eta) = \begin{cases} d_{z,i,k}(q_{z,i,k}, \eta) & \text{if link } a \text{ on the lane } k \\ 0 & \text{otherwise} \end{cases}. \] (28)
where

\[ d_{z,i,k}(q_{z,i,k}, \eta) \] delay of the lane \( k \) in arm \( i \) at intersection \( z \) (s);

\[ q_{z,i,k} \] total traffic flow of the lane \( i \) (veh/h);

\[ c_z \] cycle length of intersection \( z \) (s);

\[ g_{z,i,k} \] green duration of the lane \( k \) (s);

\[ Q_{z,i,k} \] capacity of the lane \( k \) (veh/h);

\[ x_{z,i,k} = \frac{q_{z,i,k}}{Q_{z,i,k}} \] degree of saturation flow of the lane \( k \)

\[ T \] observation time period pre-determined as 0.25 (h);

\[ x_{0,z,i,k} = 0.67 + \frac{s_{z,i,k}g_{z,i,k}}{600} \] turning point where the delay dramatically increases;

\[ s_{z,i,k} \] saturation flow (veh/s);

\[ d_{a}(q_a, \eta) \] delay of the link \( a \) (s).

### 2.3.3 Saturation flow adjustment for permitted left turns and shared lanes

Saturation flows must be adjusted because the capacity \( Q_{z,i,k} \) and the turning point \( x_{0,z,i,k} \) are dependent on the saturation flow and then influence the delay estimation. It is necessary to adjust the saturation flow of permitted left turns and shared lanes, respectively.

The saturation flow of permitted left turns is adjusted by Akcelik (1981) who estimated the filtered saturation flow by gap acceptance theory and then adjusted the saturation flow of an exclusive lane by considering the unsaturated part of opposing through movement and the number of vehicles passing in amber time.

\[ \forall z = 1, \ldots, N_z; i = 1, \ldots, N_{A,z}; i' = 1, \ldots, N_{A,z} \text{ and arm } i' \text{ is the opposing arm of the arm } i, \]

\[ s_{z,i,\text{filtered}} = \frac{q_{z,i',TH} \exp(-q_{z,i',TH} l_c)}{1 - \exp(-q_{z,i',TH} l_f)} \] (29)
\[ s_{z,i,\text{permLT}} = s_{z,i,\text{filtered}}g_{z,i,u} + n_f \frac{s_{z,i,\text{permLT}}}{0.5g_{z,i,\text{permLT}}}. \]  

where

- \( s_{z,i,\text{filtered}} \) is the filtered saturation flow of permitted left turns (veh/s);
- \( q_{z,i,TH} \) is the opposing through movement flow (veh/s);
- \( l_c \) is the critical gap with a value 4.5s;
- \( l_f \) is the follow-up headway with value 2.5s;
- \( s_{z,i,\text{permLT}} \) is the saturation flow of permitted left turns (veh/s);
- \( s_{z,i',TH} \) is the saturation flow of through movement per lane (veh/s);
- \( g_{z,i,\text{permLT}} \) is the green duration of the left turn (s);
- \( g_{z,i,u} = \max\left(\frac{s_{z,i',TH}g_{z,i,\text{permLT}} - q_{z,i',TH}}{s_{z,i',TH} - q_{z,i',TH}}, 0\right) \) is the unsaturated part of the green period for the opposing through movement (s);
- \( c_z \) is the cycle length (s);
- \( n_f \) is the number of vehicles passing during the amber period and is predefined as 1.5 (veh) in Akcelik (1980); HCM (2000) unless field data are available.

The saturation flow of movements on the shared lane is weighted according to the flow of each movement (Akcelik 1981; HBS 2015). Lane assignment results determine whether movements share one lane.

\[ s_{z,i,k} = \frac{1}{\sum_{l=1}^{B} \sum_{m \in M} r_{z,l,m}/s_{z,m,j}}. \]  

where

- \( B \) is the number of movements sharing the lane;
- \( r_{z,l,m} \) is the ratio of flow of movement \((l, m)\) to the total flow on the lane;
\[ s_{z,m,j} \] saturation flow of movement \((l, m)\) (veh/h).

## 3 Algorithms

Left turn prohibition is solved using the genetic algorithm with constraints. The default configuration of JGAP (Meffert and Rotstan 2015), a genetic algorithm package in Java, is applied which refers to an elitist ranking selector cloning the top 90% of the user-specified population size with crossover rate 35% and mutation rate 1/12. The left turn combinations which cannot meet the constraint (2) are penalized with a static penalty function.

The lane-based optimization is solved with routine branch and bound algorithm by IBM ILOG Cplex. The algorithm to solve the SUE model is the Method of Successive Averages (MSA) (Sheffy 1985).

## 4 Numerical analysis

### 4.1 Test network

The test network is a simplified map of Hanover South City, Germany (Figure. 5). This network has 14 intersections, 16 origins and 16 destinations. In total there are 56 left turns. The free flow speed of all links is 50 km/h. Link length, the number of lanes for each link, the start node and end node of each link and OD matrix are provided at https://github.com/yemayet/HannoverSuedNetwork. As it is difficult to show signal timing plans of all intersections, intersection 1 is selected as an example.

The parameters \(\alpha, \beta\) in BPR function are 0.15 and 4, respectively. The minimum green
time is 5s. All intergreen times are 4s. The minimum cycle time is 60s and the maximum cycle
time is 90s. The default conflict matrices are extracted from network files of SUMO, which
is a microscopic simulation tool developed by the German Aerospace Center (DLR). In HCM
(2000), the saturation flow of through movement per lane, $s_{TH}$, is 1900veh/h; the saturation
flow of right turn per lane is $0.85s_{TH}$; the saturation flow of protected left turn per lane is
$0.95s_{TH}$; the saturation flow of permitted left turn is adjusted with Eq. (29) and Eq. (30).

In the genetic algorithm, the initial population size is 40 and the generation size is 50.
The convergence criterion of MSA is that the difference of average link flows between adjacent
iterations is less than $1.0 \times 10^{-4}$. Details of the convergence equation of MSA are in Sheffi
(1985).

[Figure 5 about here.]

Figure 5. Layout of Hanover South City, Germany drawing with SUMO

4.2 Results

The results of the network without left turn prohibition is the benchmark being compared
with the results of left turn prohibition. After running the test network, the minimum total
time travel is obtained in the 31st generation and the minimum value is kept until the genetic
algorithm ends (See Figure 6). Finally, 24 left turns are prohibited. Before left turn prohibition,
the total travel time is 301.7h whereas, after left turn prohibition, the minimal total travel time
is 258.0h with reduction of 14.5%.

With maximizing the reserved capacity for each intersection, the cycle length is determined
as 90 s for all intersections after left turn prohibition. The lane marking and signal timing plan
of the intersection 1 are shown in Figure 7. Note that "SG" in the figure means signal group.

Figure 6. Convergence of the genetic algorithm over generations

Figure 7. Lane marking and signal timing plane of intersection 1

4.3 Simulation study

The results obtained from the proposed model are evaluated with a SUMO simulation model. In a SUMO simulation model, vehicles always select the shortest route at the time they enter the network. This route choice behavior is natural in reality. Thus, with the SUMO simulation, it can be observed whether the left turn prohibition and corresponding signal settings can reduce the travel time. Please note that in a SUMO model, the average travel time is obtained, rather than the total travel time.

The configurations of the simulation model keep consistent as those of the proposed model. As the spill back may happen at some intersections, "time to teleport" is set as 3600s, which means if a vehicle is blocked at somewhere due to some reasons, the vehicle will be sent to the next segment after 3600s. The random seed is 42. The simulation ends when all vehicles leave the network.

After running the SUMO model, the average travel time before left turn prohibition is 1117.8s, and the average travel time after left turn prohibition is only 489.6s, which is 56.2% reduction. The left turn prohibition eliminates the spill backs in the network such that the average travel time dramatically decreases.
4.4 Sensitivity analysis

Although the left turn prohibition can be analytically solved, it is important to detect whether the decision can handle the randomness in the network; otherwise, the left turn prohibition decision is hard to be applied. Thus, we do a sensitivity analysis by using different random seeds on the SUMO model.

The random seeds are set in the range of $[42, 100]$ by every increase 2. The random seeds with different value indicate the combinations of randomness in a SUMO model. The randomness may include speed distribution, departure times of vehicles, arrival and departure distributions.

Figure 8 shows the variable average travel time with different random seeds. Although the average travel time varies, it is superior to the average travel time without left turn prohibition. The left turn prohibition result can handle the randomness of the network.

Figure 8. Variable average travel time with different random seeds

5 Discussion

This study could potentially lead to useful insights regarding congestion management. In the numerical examples, prohibiting left turns indeed reduces the total travel times of networks. The involvement of lane assignment in the signal setting optimization increases the accuracy of saturation adjustment and plays a role in precisely evaluating the effect of left turn prohibition.

One reason causing this benefit is that prohibiting left turns reduces the number of conflict points at intersections. There are intergreen times between all conflict movements for the
safety reasons. After left turns are prohibited, the intergreen time between left turns and other movements is transferred as effective green time. All movements at that intersection could benefit from longer effective green time. The number of movements sharing lanes is not as many as that before left turn prohibition, so the lane assignment for movements is more flexible. For this reason, more movements can be assigned individually rather than colonially as signal groups. The signal timing is then more specifically adjusted. Hence, vehicles can access the intersection with shorter delays. The re-usage of the left turn lanes is another reason why prohibiting left turns saves total travel times. More lanes could be used for the rest movements, making the capacities of the movements increase. Due to the increase in the capacities, vehicles can faster pass the prohibited intersection.

Moreover, the left turn prohibition decision can handle the randomness in the network. It is because with the original OD matrix the network has enough capacities to handle changeable demands. The vehicles can make use of the capacities of the network by prohibiting left turns, which balances the use of capacities.

Although networks can benefit from left turn prohibition on the aspect of total travel time reduction, left turn prohibition is inapplicable if the number of lanes for the rest movements is more than the number of their exit lanes. Also if the number of exit lanes is larger, the lane-based method cannot use all exit lanes. This situation should be improved in the future.

6 Conclusions

This paper presents a method to minimize the total travel times in networks by prohibiting left turns. Prohibiting left turns reduces the total travel times in networks in acceptable levels.
The prohibition results can handle changeable demands through the sensitivity analysis. One oncoming research question on this topic is analyzing more influencing factors of left turn prohibition. The proposed model could also be improved by integrating left turn prohibition and signal setting optimization into one mathematical model but it requires more exploration on algorithm development.

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References


Lu, J., S. Dissanayake, H. Zhou and X. K. Yang. 2001. “Operational evaluation of right turns followed by u-turns as an alternative to direct left turns.” *the report based on "Methodology to quantify the effects of access management on roadway operation and safety"* III.


https://mc06.manuscriptcentral.com/cjce-pubs
Tang, Q. 2016. Lane-based optimization of signal timing including left turn prohibition in communication scenarios.


Prohibit feasible left turns

Run stochastic user equilibrium with the travel time based on BPR function

Optimize signal settings with the lane-based method

Run stochastic user equilibrium with the travel time based on signal timing

All selected feasible left turns?

Yes

Start

No

Start

Figure 1: Flow chart of the left turn prohibition method
Figure 2: (a) An example of lane assignment before left turn prohibition; (b) An example of lane assignment after left turn prohibition.
Figure 3: Intersection notations
Figure 4: Conflicts on adjacent lanes
Figure 5: Layout of Hanover South City, Germany drawing with SUMO
Figure 6: Convergence of the genetic algorithm over generations
(a) Lane marking

(b) Signal timing plan

Figure 7: Lane marking and signal timing plane of intersection 1
Figure 8: Variable average travel time with different random seeds
Table 1: Decision variables of the lane-based method

<table>
<thead>
<tr>
<th>Decision variables</th>
<th>Notations</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lane permission indicator</td>
<td>$\delta_{z,i,j,k}$</td>
<td>$\delta_{z,i,j,k} \in {0, 1}$</td>
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<tr>
<td>Successor indicator</td>
<td>$\Omega_{z,i,j,l,m}$</td>
<td>$\Omega_{z,i,j,l,m} \in {0, 1}$</td>
</tr>
<tr>
<td>Green split for lane</td>
<td>$\Phi_{z,i,k}$</td>
<td>$\Phi_{z,i,k} \in [0, g_{z,i,j,min}/c_z]$</td>
</tr>
<tr>
<td>Green split for movement</td>
<td>$\phi_{z,i,j}$</td>
<td>$\phi_{z,i,j} \in [0, g_{z,i,j,min}/c_z]$</td>
</tr>
<tr>
<td>Start of green for lane</td>
<td>$\Theta_{z,i,k}$</td>
<td>$\Theta_{z,i,k} \in [0, 1]$</td>
</tr>
<tr>
<td>Start of green for movement</td>
<td>$\theta_{z,i,j}$</td>
<td>$\theta_{z,i,j} \in [0, 1]$</td>
</tr>
<tr>
<td>Assigned flow (veh/h)</td>
<td>$f_{z,i,j}$</td>
<td>$f_{z,i,j} \in [0, \infty)$</td>
</tr>
<tr>
<td>Inverse of cycle length (s)</td>
<td>$\xi_z$</td>
<td>$\xi_z \in [1/c_{max}, 1/c_{min}]$</td>
</tr>
<tr>
<td>Reserved capacity</td>
<td>$\mu_z$</td>
<td>$\mu \in (0, \infty)$</td>
</tr>
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