Estimation of the air permeability function from the Soil-Water Characteristic Curve

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Estimation of the air permeability function from the Soil-Water

Characteristic Curve

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Abstract:

The multiphase flow (including liquid flow and air flow) in unsaturated soil is related to many engineering problems such as contaminant transport, rainwater infiltration and soil-water evaporation. It is proven that water flow in unsaturated soil can be estimated using the concept of pore-size distribution function. Many models have been proposed to estimate the water flow or water permeability function, k_w , from soil-water characteristic curve (SWCC). On the other hand, a limited model has been proposed to estimate the air flow or air permeability function, k_a , from SWCC. Most of the models used for the estimation of the air permeability functions are empirical

and they are dependent on the empirical parameters. In this paper, the relative air coefficient of permeability was estimated using the concept of pore-size distribution function. In the method proposed in this paper, there was no empirical parameters adopted and the estimation results purely depended on the soil-water characteristic curve. The proposed method was verified against experimental data from published literature.

Key words: Soil-water characteristic curve, pore-size distribution function, air permeability function, estimation

1. Introduction

The prediction of water/air flow under transient conditions is important in both soil science and civil engineering. As the moisture movement in unsaturated soil can be in the form of liquid or vapor, the water/air permeability function plays an important role in the estimation of moisture movement in the unsaturated soil.

Multi-fluid flow processes are governed by geometrical pore-space characteristics (Tuli et al. 2005). Due to inherent complexity and heterogeneity of soil, it is difficult to determine the water/air flow within soil pores under unsaturated condition. Therefore, most multi-phase flow models adopt the empirical fitting parameters. There have been many models proposed by different researchers for the estimation of the air permeability. Bate et al. (2005) categorized these estimation models into two groups: (i) power function models from Irmay (1954), Wyllie (1962), Falta et al (1989), Delage et al. (1998) and Chen et al. (1999) and (ii) based on SWCC model such as Stylianou and DeVantier (1995). In this paper, a new model to estimate air permeability using the statistical method was proposed.

Zhai and Rahardjo (2015) and Zhai et al. (2017) derived the equations to estimate the

water permeability function from SWCC based on the concept of pore-size distribution function. In this paper, the concept of pore-size distribution function was also adopted for the estimation of the air permeability function.

The objectives of this paper are to develop a new method to estimate the relative air permeability function using the information from soil-water characteristic curve. In the proposed equations, the relative air permeability function can be defined using SWCC fitting parameters. Experimental data from literatures were utilized to verify the proposed equations in this paper. The procedure to improve the accuracy of the estimation results is also discussed and recommended in this paper.

2. Literature review

Tang et al. (2011) established the relationship between soil air permeability (k_a) and air-filled porosity (v_a/v), void ratio (e), degree of saturation (S), and vertical stress (σ_v) of French soil. The results of experimental work from Tang et al. (2011) showed that k_a decreases with the increase in S . By using the scaling scheme which is based on the similar media concept, Wang et al. (2014) estimated k_a by considering the relationship between k_a and soil moisture. The work from Wang et al. (2017) showed that k_a for a soil mixed with larger aggregates was higher than that of the soil mixed with smaller aggregates because the soil mixed with larger aggregates create larger pores. He et al. (2017) also showed that k_a for Téguline clay increased with the increase in drying-wetting cycles due to the presence of micro-cracks. In summary, the work from previous researchers show that air permeability is strongly related to pore-size distribution in soil. The variation in k_a can be estimated from the variation in pore-size distribution function in soil.

Brooks and Corey (1964) proposed an empirical equation to correlate the air

permeability and the degree of saturation. van Genuchten (1980) proposed a simple equation for the description of SWCC and this simple equation allows the derivation of the relative hydraulic conductivity based on the model of Mualem (1976). Mualem (1976)'s hydraulic conductivity model, as illustrated in Equation (1), had been modified by Chen et al. (1999), Tuli et al. (2005), and Bate et al. (2005) for the estimation of the air permeability.

$$k_{rw} = \frac{k_w}{k_s} = S_w^{0.5} \left[\frac{\int_0^{S_w} \frac{dS_w}{\psi}}{\int_0^1 \frac{dS_w}{\psi}} \right]^2 \quad \text{----- (1)}$$

where, k_{rw} = relative hydraulic conductivity, k_w = unsaturated hydraulic conductivity, k_s = saturated hydraulic conductivity, S_w = degree of saturation, ψ = matric suction.

The term of $S_w^{0.5}$ in Equation (1) is adopted for the correction due to tortuosity and pore connectivity. Clennell (1997) carried out a comprehensive review of the tortuosity concept for a range of different flow and transport processes in porous media. Vervoort and Cattle (2003) recommended that effect on the water/air permeability due to tortuosity and pores connectivity could be quantified using $S_w^{l_w}$ and $S_a^{l_a}$ (where, S_w = water degree of saturation, S_a = air degree of saturation, l_w = tortuosity-connectivity parameter for water and l_a = tortuosity-connectivity parameter for air), respectively. On the other hand, Vervoort and Cattle (2003) found that the hydraulic conductivity and tortuosity parameters were strongly related to porosity, pore-size distribution and mean pore size. Tuli and Hopmans (2004) observed that both pore geometry and size distribution were the main factors determining the functional relations between degree of fluid saturation and water hydraulic conductivity and air conductivity. In this paper, pore-size distribution function which

consists of information about pore geometry and pore size distribution was adopted for the estimation of air permeability function. The effect of tortuosity and connectivity was not considered (i.e., the tortuosity-connectivity parameter, $l_c = 0$) in this study.

Childs and Collis-George (1950) were one of the pioneers who proposed the statistical method to estimate the permeability function from SWCC. Childs and Collis-George (1950) treated the pores in soil as a series of tubes and adopted the concept of “cutting and random rejoining” to determine the statistical probability of the connectivity of two tubes. Consequently, the relative coefficient of permeability was calculated from a ratio between the equivalent cross sectional area of water flow in unsaturated state to the equivalent cross sectional area in saturated state. Childs and Collis-George (1950)’s model was further extended by Marshall (1958) and Kunze et al. (1968), Mualem (1976), Fredlund et al. (1994), Zhai and Rahardjo (2015), and Zhai et al. (2017). Zhai and Rahardjo (2015) pointed out that only SWCC in the form of degree of saturation was analogous to the pore-size distribution function. Zhai and Rahardjo (2015) also recommended that the permeability function to be estimated from the SWCC in the form of degree of saturation. Childs and Collis-George (1950) developed their model (or statistical model) by studying the water phase within soil specimen. The statistical method for the estimation of unsaturated hydraulic conductivity is calculated from the capacity of the section which allows water to pass through at different suction conditions. Marshall (1958) and Zhai and Rahardjo (2015) demonstrated that property of fluid affected the saturated hydraulic conductivity of soil but it did not affect the relative hydraulic conductivity (as the parameters on the property of fluid are canceled during the derivation of relative hydraulic conductivity) of soil. In this case, the statistical method can also be applied to estimate the air

permeability function. The assumptions adopted for the estimation of water permeability are also applicable for the estimation of the air permeability.

As explained above, the air permeability can be estimated from SWCC using the statistical method based on the pore-size distribution function concept. The mathematical equations proposed by Fredlund and Xing (1994), as illustrated in Equation (2) and van Genuchten (1980), as illustrated in Equation (3), are commonly used as for best fitting SWCC. In this paper, both equations are adopted for the estimation of air permeability.

$$\theta = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} = \left[1 - \frac{\ln \left(1 + \frac{\psi}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right] \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad \text{----- (2)}$$

where:

C_r = input value, can be a roughly estimated value for residual suction, $C_r = 1500$ kPa for most cases as suggested by Fredlund and Xing (1994) and Zhai and Rahardjo (2012a, 2012b); θ = volumetric water content; ψ = matric suction; e = Euler's number and a , n and m = fitting parameters.

$$\Theta = \left[\frac{1}{1 + (ah)^b} \right]^c \quad \text{----- (3)}$$

where:

$\Theta = \frac{\theta - \theta_r}{\theta_s - \theta_r}$; θ = volumetric water content; θ_s = saturated volumetric water content;

θ_r = a value for rough estimation of residual volumetric water content, should be treated as a fitting parameter as recommended by Zhai et al. (2017); h = pressure head and a , b and c = fitting parameters.

Triaxial permeameter used for the measurement of water permeability function could also be used to measure air permeability (Fredlund and Rahardjo, 1993; Fredlund et al.

2012). It was firstly developed by Matyas (1967), Barden et al (1969), and Barden and Pavlakis (1971). The triaxial permeameter has the advantage of being able to measure the water and air coefficients of permeability on the same soil specimen. In principle, air permeabilities are measured at different water contents with respect to different matric suctions of soil (Fredlund and Rahardjo, 1993, Fredlund et al. 2012). The flexible wall triaxial permeameter system consists mainly of a permeameter cell, pressure and flushing lines, water flow and air flow measuring systems, and a total volume change measuring device. The set up the triaxial permeameter for air permeability test is illustrated in Figure 1. The details of the measurement procedure can be referred to Fredlund and Rahardjo (1993), Fredlund et al. (2012) and Samangan et al. (2003).

3. Theory

The statistical model for the estimation of air permeability function is proposed in this section. The assumptions adopted in the statistical model are explained and the application of the statistical model for the estimation of water and air permeability functions are then discussed. In addition, the equations for the estimation of air permeability function using fitting parameters a , n and m in Fredlund and Xing (1994)'s equation and a , b and c in van Genuchten (1980) equation are derived.

3.1 Assumptions for the statistical model

The statistical model makes five major assumptions: (a) pores in soil can be considered a set of randomly distributed interconnected pores characterized by a pore radius, r , and its density, $f(r)$, and the density, $f(r)$, is the same for any cross-section; (b) the Poiseuille equation is applicable; (c) SWCC is analogous to the pore-size

distribution function; (d) the pore-size distribution function $f(r)$ is constant with change in matric suction; and (e) the water cannot flow through the dry pores.

Vanapalli et al. (1996) observed that air phase might not be continuous in the boundary effect zone and the water phase might not be continuous in the residual zone as illustrated in Figure 2. Once the water/ air phase becomes discontinuous, the Poiseuille's law is not applicable any more. In other words, the statistical model for the estimation of water permeability is only applicable in the boundary effect zone and the transient zone. On the other hand, the statistical model for the estimation of air permeability is only applicable in the transition zone and the residual zone.

It is also observed from the shrinkage curve, that significant soil volume changes occur before the air-entry value. In this case, the pore-size distribution function is not constant and assumption (d) is not applicable any more. Therefore, the statistical model is not applicable to estimate the water/air permeability in the boundary effect zone if the soil volume change is significant.

In other words, the statistical model is applicable to estimate the air permeability in the transition and residual zones. Therefore, the air-entry value is adopted as the minimum suction for the estimation of air permeability in this paper.

3.2 Equivalent effective area for the air permeability of dry soil

Assume that two pores are connected on section A as illustrated in Figure 3 and air flows from the left to right. In scenario (i), the pores on the both sides of section A are dry pores so that the air can flow cross the section and the permeability of air is dependent on the smaller size of the pore. On the other hand, in scenario (ii), the pore in the right-hand side of the section A is wet pore which blocks the air flow across the section and the air permeability is equal to zero.

Define the soil porosity is n , pore size density corresponding to the pore with radius of r is $f(r)$. If the soil is completely dry, there are no wet pores. As a result, the equivalent effective radius for the air permeability of dry soil is dependent on the probability of connectivity of pores and it can be expressed in Figure 4 as follows:

By connecting the pores (from r_1 to r_n) on the left hand side to the pores (also from r_1 to r_n) on the right hand side as illustrated in Figure 4, it can be observed that the

coefficient of term r_i^2 is $\left(\sum_{j=1}^i f(r_j)\right)^2 - \left(\sum_{j=1}^{i-1} f(r_j)\right)^2$. It is noted that $\sum_{j=1}^i f(r_j) = S(\psi_1) -$

$S(\psi_i)$, and $\sum_{j=1}^{i-1} f(r_j) = S(\psi_1) - S(\psi_{i-1})$. The sum of the effective areas for air permeability

from pores r_1 to r_n results in the total effective area for the air permeability. As explained in Section 3.1, air-entry value (AEV) is adopted as the minimum suction for the estimation of the air permeability as illustrated in Equation (4).

$$r_t^2 = n^2 \left(\sum_{i=r(AEV)}^N \left[[S(\psi_{AEV}) - S(\psi_i)]^2 - [S(\psi_{AEV}) - S(\psi_{i-1})]^2 \right] r_i^2 \right) \quad \text{----- (4)}$$

where, r_N = smallest radius of pore in the soil; $r_{(AEV)}$ = radius of pore corresponding to the air-entry value based on the capillary law; ψ_i = matric suction; $S(\psi_i)$ = degree of saturation corresponding to suction of ψ_i ; and r_i = radius of pore corresponding to the matric suction of ψ_i based on the capillary law.

3.3 Equivalent effective area for the air permeability of unsaturated soil

If the soil is unsaturated, wet pores exist and scenario (ii) can occur. The number of wet pores is dependent on the suction value where lower suction results in more wet pores in the soil. Any wet pore produces the impervious section. The equivalent effective radius for the air permeability of unsaturated soil is dependent on the

probability of connections of saturated pores. If the suction value in the soil is ψ_m , all pores with a radius less than r_m (i.e. $r_m = 2T/\psi_m$) are wet pores, while pores with a radius greater than r_m are dry pores. The radii of dry pores are named r_1, r_2 to r_m , while the radii of wet pores are named r_{m+1}, r_{m+2} to r_n . As explained in Figure 3 (b), any wet pore can make the section to be impermeable for air as indicated by the last four combinations in Figure 5 that have a zero effective area. In this case, the effective area for air permeability can be calculated for different scenarios as illustrated in Figure 5. Summary of the effective area of the air permeability from pores r_1 to r_n results in the effective area for the air permeability as illustrated in Equation (5)

$$r_t^2 = n^2 \left(\sum_{i=r(AEV)}^m [S(\psi_{AEV}) - S(\psi_i)]^2 - [S(\psi_{AEV}) - S(\psi_{i-1})]^2 \right) r_i^2 \quad \text{----- (5)}$$

where, r_m = equivalent pore size corresponding to matric suction of ψ_m based on the capillary law.

3.4 Air coefficient of permeability for unsaturated soil

The ratio of the effective area for air permeability of unsaturated soil to the effective area for air permeability of completed dry soil represents the relative air coefficient of permeability. In this case, the relative air coefficient of permeability can be calculated by dividing Equation (5) with Equation (4) as follows:

$$k_{ra} = \frac{\left(\sum_{i=r(AEV)}^m [S(\psi_{AEV}) - S(\psi_i)]^2 - [S(\psi_{AEV}) - S(\psi_{i-1})]^2 \right) r_i^2}{\left(\sum_{i=r(AEV)}^N [S(\psi_{AEV}) - S(\psi_i)]^2 - [S(\psi_{AEV}) - S(\psi_{i-1})]^2 \right) r_i^2} \quad \text{----- (6)}$$

where, k_{ra} = relative air coefficient of permeability; ψ_m = suction state in the soil;

If the air coefficient of permeability $k_a(\psi_a)$, with respect to a suction of ψ_a , is known, then the air coefficient of permeability $k_a(\psi_{a+i})$, with respect to a suction of ψ_{a+i} , can be calculated using equation (7):

$$k_{ra} = \frac{\left(\sum_{i=r(AEV)}^m \left[S(\psi_{AEV}) - S(\psi_i) \right]^2 - \left[S(\psi_{AEV}) - S(\psi_{i-1}) \right]^2 \right) / \psi_i^2}{\left(\sum_{i=r(AEV)}^N \left[S(\psi_{AEV}) - S(\psi_i) \right]^2 - \left[S(\psi_{AEV}) - S(\psi_{i-1}) \right]^2 \right) / \psi_i^2} \quad \text{----- (7)}$$

Substituting Equation (2) and (3) into Equation (7), equation for the estimation of air permeability function using Fredlund and Xing (1994)'s equation and van Genuchten (1980)'s equations can be obtained as follows:

$$k_{ra} = \frac{\sum_{i=r(AEV)}^m \left[\left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{\psi_m}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_i)}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a} \right)^n \right] \right\}^m} \right]^2 - \left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{\psi_m}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_i)}{\left\{ \ln \left[e + \left(\frac{\psi_{i-1}}{a} \right)^n \right] \right\}^m} \right]^2 \right] \psi_i^{-2}}{\sum_{i=r(AEV)}^N \left[\left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{AEV}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_i)}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a} \right)^n \right] \right\}^m} \right]^2 - \left[\frac{C(\psi_m)}{\left\{ \ln \left[e + \left(\frac{AEV}{a} \right)^n \right] \right\}^m} - \frac{C(\psi_i)}{\left\{ \ln \left[e + \left(\frac{\psi_{i-1}}{a} \right)^n \right] \right\}^m} \right]^2 \right] \psi_i^{-2}} \quad \text{----- (8)}$$

where, $r_{(AEV)}$ = radius of pore corresponding to the air-entry value (AEV).

$$k_{ra} = \frac{\sum_{i=r(AEV)}^m \left[\left[\frac{1}{1+(ah_m)^b} \right]^c - \left[\frac{1}{1+(ah_i)^b} \right]^c \right]^2 h_i^{-2}}{\sum_{i=r(AEV)}^N \left[\left[\frac{1}{1+(ah_{AEV})^b} \right]^c - \left[\frac{1}{1+(ah_i)^b} \right]^c \right]^2 h_i^{-2}} \quad \text{----- (9)}$$

where, h_{AVE} = air-entry value (AEV).

4. Applications and discussions

Due to the limited data with both SWCC and air permeability available in the literature, only five sets of data were used for verification of the proposed equation. As a disturbed soil is expected to have more significant soil volume change than an undisturbed soil, only the undisturbed soils and compacted soils, which have insignificant soil volume change, were selected in this study to avoid the effect of soil volume change on the air permeability. These five data sets are for natural soil (#131, #132 and #137) from Tuli et al. (2005), Singapore residual soil from Samangan et al. (2003) and Japanese soil from Moldrup et al. (2003). As no index property of these soils was reported from the literature, only the experimental results on SWCC and air permeability are illustrated in Figures 6 and 7.

It is noted that air permeability of soil can be expressed either in terms of air coefficient of permeability, k_a , or intrinsic air permeability, K_a . The relationship between k_a and K_a can be illustrated in Equation (10) as follows (Fredlund and

Rahardjo 1993):

$$K_a = \frac{k_a \mu}{\rho g} \quad \text{----- (10)}$$

where K_a = intrinsic air permeability (m^2); k_a = air coefficient of permeability (m/s); μ = dynamic viscosity of the fluid ($\text{kg}/(\text{m.s})$); ρ = density of fluid (kg/m^3); g = acceleration due to gravity, (m/s^2).

The intrinsic air permeability is commonly used in numerous disciplines. However, in geotechnical engineering, the air coefficient of permeability, k_a , is the most commonly used term (Fredlund and Rahardjo 1993). As the fluid properties are commonly considered to be constant during the flow process, the intrinsic air permeability can be converted into the air coefficient of permeability using Equation (10).

The experimental data from Samingan et al. (2003) were presented in terms of k_a while the original data from Moldrup et al. (2003) and Tuli et al. (2005) were presented in terms of K_a . Following the recommendation from Fredlund and Rahardjo (1993), k_a was used throughout the entire study presented in this paper. Bate et al. (2005) had converted Moldrup et al. (2003)'s data into the term of k_a . In this study, the experimental data from Tuli et al. (2005) were converted to k_a using Equation (10) and the properties of air at 20°C (i.e., $\mu = 18.21 \times 10^{-6} \text{ m}^2/\text{s}$, $\rho = 1.205 \text{ kg}/\text{m}^3$, $g = 9.81 \text{ m/s}^2$).

The Fredlund and Xing (1994)'s equation was used to best fit the experimental data and the fitting parameters are illustrated in Table 1. It is observed that Singapore residual soil has a bimodal shape, Satyanaga et al.'s (2013) equation was used to best fit the experimental data for Singapore residual soil. The best fitted SWCCs are illustrated in Figure 8. As soil volume change may occur in the boundary effect zone

(i.e., with the suction less than air-entry value), the assumption adopted for the estimation of air permeability may not be valid in the boundary effect zone. Therefore, the air-entry value was adopted as the minimum suction for the estimation of the air permeability function for these soils. The air-entry values (AEV) calculated using Zhai and Rahardjo (2012) and Zhai et al. (2017)'s method for these soils are illustrated in Table 2.

The fitting parameters in Table 1 were used for the estimation of the air permeability using Equation (8) and the comparison between the estimation results and experimental data for these soils is illustrated in Figure 9. The coefficients of determination R^2 of the estimated air permeability for these soils are illustrated in Table 3.

The estimation results shown in Table 3 and Figure 9 indicate that the proposed equation has a good performance in predicting the air permeability for these soils. Therefore, the equations derived in this paper (Equations (8) and (9)) can be used for the estimation of air permeability function for unsaturated soil

As explained in Section 3.1 and Equations (7) to (9), a minimum suction of air-entry value (AEV) rather a lower suction (e.g., 0.01 kPa) is adopted as the minimum suction for the estimation of air-permeability function. To test the accuracy of the estimation results from the statistical model with a minimum suction of 0.01 kPa, the air permeabilities of these soils were re-estimated and the estimation results are illustrated in Figure 10. The results illustrated in Figure 10 indicate that the accuracy of estimation was significantly decreased (0.01 kPa was adopted as the minimum suction for the estimation) as compared with the results shown in Figure 9 (AEV was

adopted as the minimum suction for the estimation). Therefore, it can be concluded that the AEV should be used as the minimum suction for the estimation of air permeability. In addition, ignoring the soil volume change can result in overestimation of the air coefficient of permeability. To overcome the overestimation, the tortuosity / connectivity parameter (e.g., $S_w^{0.5}$ in Equation 1) was introduced to make a correction. The additional parameter (tortuosity/connectivity parameter) can make the estimation results to be closer to the measurement data. However, this tortuosity/connectivity parameter is empirical and it varies much for different soils. Based on the study presented in this paper, the authors would like to recommend considering the effect of soil volume change (change in the pore-size distribution function) rather than using the empirical parameter to estimate the relative air coefficient of permeability.

5. Conclusions

A new model for the estimation of air permeability function for the unsaturated soil is proposed in this paper. In the estimation, it is assumed there is insignificant soil volume change. The comparisons between the estimation results and measurement data from published literature show that the proposed equations provide good estimation of air permeability for different types of soil. The relative air permeability can be computed using the proposed equations in this paper via an electronic spreadsheet with SWCC fitting parameters (e.g., a , n and m in Fredlund and Xing's (1994) equation or a , b and c in van Genuchten's (1980) equation). The results of the comparison study in this paper recommend that air-entry value (AEV) should be used as the minimum suction for the estimation of air permeability function.

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Figure Captions:

Figure 1. Illustration of the set up of permeameter for air permeability test (from Samangan et al. (2003))

Figure 2. Illustration of the condition of water phase at different zones (from Vanapalli et al. 1996)

Figure 3. Illustration of different scenarios for air flow cross the section A.

Figure 4. Illustrated of effective area for air permeability when the soil is completely dry.

Figure 5. Illustrated of effective area for air permeability when the soil is under suction of ψ_m .

Figure 6. Illustration of five sets of SWCCs for the soils.

(a) SWCC for natural soil from Tuli et al. (2005)

(b) SWCC for Singapore residual soil from Samangan et al. (2003) and Japanese soil from Moldrup et al. (2003)

Figure 7. Experimental results for air permeability for the soils.

(a) Measured air permeability for natural soil from Tuli et al. (2005)

(b) Measured air permeability for Singapore natural soil and Japanese soil

Figure 8. Best fitted SWCCs for the soils

(a) Best fitted SWCCs for natural soils

(b) Best fitted SWCCs for Singapore residual soil and Japanese soil

Figure 9. Estimation results of air permeability for the soils

(a) Estimated air permeability for natural soil

(b) Estimated air permeability for Singapore residual soil and Japanese soil

Figure 10. Estimation results of air permeability for the soil using minimum suction of 0.01 kPa

- (a) Estimated results for natural soil using minimum suction of 0.01 kPa
- (b) Estimated results for Singapore residual soil and Japanese soil using minimum suction of 0.01 kPa

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Table Caption:

Table 1. Fitting parameters in Fredlund and Xing (1994)'s equation for the SWCCs of these soils

Table 2. Calculated air-entry values for the soils

Table 3. Coefficient of determinations R^2 for the estimation results for air permeability.

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Table 1. Fitting parameters in Fredlund and Xing (1994)’s equation for the SWCCs of these soils

Soils		SWCC model	Fitting parameters			C_r (kPa)	θ_s	R^2
			a (kPa)	n	m			
Undisturbed natural soil	#131	Unimodal	9.46	2.2	0.22	1500	0.40	98.15%
	#132	Unimodal	9.54	2.54	0.19	1500	0.41	99.98%
	#137	Unimodal	6.34	1.79	0.26	1500	0.41	99.87%
Japanese soil		Unimodal	2.32	5.05	0.18	1500	0.61	99.98%
Singapore residual soil		Bimodal	θ_{s1}	ψ_{a1}	ψ_{m1}	s_1	ψ_r	99.35%
			0.494	13	60	1.203	5982	
			θ_{s2}	ψ_{a2}	ψ_{m2}	s_2	θ_r	
			0.171	856.17	2154	1.503	0.02	

Table 2. Calculated air-entry values for the soils

Air-entry value	Undisturbed natural soil			Japanese soil	Singapore residual soil	
	#131	#132	#137		Subcurve 1	Subcurve 2
AEV (kPa)	6.49	6.94	3.92	1.80	20.85	292

Table 3. Coefficient of determinations R^2 for the estimation results for air permeability.

Coefficient of determination	Undisturbed natural soil			Japanese soil	Singapore residual soil
	#131	#132	#137		
R^2	96.31%	81.25%	92.52%	93.67%	90.73%

Figure 1.

Figure 2.

Figure 3.

Figure 4. **Figure 5.** **Figure 6.**

Figure 7.

Figure 8.

Figure 9.

Figure 10.