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Gearbox fault diagnosis via generalized velocity synchronous Fourier transform and order analysis

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Abstract

Gearbox plays an essential role in many industrial equipment. Fault detection for gearboxes is very important yet extremely difficult due to the volatile working conditions which lead to non-stationary vibration signals. Order tracking is considered as a classic and effective technique for non-stationary vibration analysis and fault diagnosis of rotating machinery. To date, many order tracking methods that do not require a tachometer have been proposed, such as the resampling-based methods. However, most of them are complex and often introduce interpolation errors. To avoid such difficulties, a simple yet effective method is proposed in this paper. This method employs the generalized demodulation approach to extract a component whose frequency is proportional to the instantaneous shaft rotational frequency from the vibration signal, followed by demodulating the extracted component to recover the instantaneous shaft rotational phase. With such information the order spectrum can be directly obtained via the velocity synchronous discrete Fourier transform and finally the fault can be diagnosed by order spectrum analysis. The effectiveness of this method is validated by both simulated and lab experimental vibration signals of gearbox under time-varying rotational speed conditions.

Keywords

Fault diagnosis; gearbox; Nonstationary signal; Order tracking; Generalized demodulation

1 Introduction

Gearboxes are widely used in industrial equipment as they can transmit relatively large power in a very compact structure. The harsh working conditions, e.g., heavy load, dust, corrosion, make the gearboxes particularly prone to damage, which may lead to catastrophic losses. Therefore, the health monitoring of gearboxes is very important for industrial equipment.

The presence of a local fault in gear can lead to amplitude and phase modulations of the vibration signal, which in turn generates peaks at sidebands of meshing frequency associated with fault characteristic frequency in the spectrum (Dalpiaz et al 2000; McFadden 1986; Randall 1982). The increment in the number and amplitude of such sidebands generally indicates a fault condition. For this reason, frequency-based analytical tools such as Fourier transform and demodulation techniques are widely used in the fault diagnosis of rotational machinery. Based on the spectral patterns of the faulty gearbox, various frequency-based methods, such as sideband analysis and demodulation techniques, have been proposed for gearbox fault diagnosis (McFadden 1986; Wang 2001).

Although the frequency-based methods can detect gearbox faults with some successes, most of those methods are based on the assumption of stationary signal, i.e., the signal is collected under the condition of constant running speed and external load. However, due to the frequent changes in external load and driving speed, the vibration signals of gearboxes are usually non-stationary. In recent years, time-frequency analysis methods were proposed for fault diagnosis of gearbox under non-stationary conditions (Feng and Chen 2018; Feng et al 2015, 2016a; Feng and Liang 2014; Feng et al 2016b). Such methods have shown that the diagnosis can be realized by detecting the time-varying sidebands of meshing frequency. However, identifying the time-varying frequency components in 3D time-frequency plane is complex and it requires the user to have prior good knowledge about signal structure. This drawback limits its effectiveness.

Computed order tracking (COT) gives insights into the complex structure of the non-stationary vibration signals of rotating machinery collected under varying speed conditions (Fyfe and Munck 1997). The essence of this method is converting the non-stationary time domain signal into the stationary angular domain signal by resampling the vibration signal at a

constant shaft angular increment and then a traditional frequency based method could become applicable. However, the COT method has two disadvantages. The first one is that it requires additional phase markers to provide the information of the instantaneous shaft rotational phase (ISRP), which increases the cost and causes problems of machine adjustment. The second disadvantage is reflected by the inherently incorporated error into the result of COT. This is because the resampling is attained via polynomial interpolations, however vibration signals are produced by cyclic phenomena and thus are not polynomial but sinusoidal. To solve the aforementioned disadvantages, Bonnardot et al (2005) and Combet and Gelman (2007) proposed to estimate ISRP by demodulating the band-pass filtered signal around gear meshing frequency. Nevertheless, such methods are primarily applicable to the condition with limited speed fluctuation, because in large speed fluctuation conditions, the frequency band of the concerned component will overlap with those of other components and cannot be separated by traditional band-pass filters. Zhao et al (2013a) proposed to estimate the ISRP by demodulating the component extracted by Kalman filtering. The potential difficulties in the implementation of this method include its complexity due to the complex structure and data equations of Kalman filter. Furthermore, although the methods mentioned above can obtain the order spectrum for fault diagnosis from the signal with some successes, most of the current studies focus on extracting ISRP information from the signal, which thus overcomes the first difficulty. However, the second one remains to be addressed.

Fortunately, the velocity synchronous discrete Fourier transform (VSDFT) recently proposed by Borghesani et al (2014) can effectively address the second issue. The VSDFT is defined as a type of Fourier transform with a kernel whose frequency varies with the instantaneous shaft rotational frequency (ISRF). By taking advantage of this properly designed kernel, the signal can be transformed from time domain to order domain directly. As the VSDFT does not require resampling, interpolation errors will not be introduced. It is also

proved that the VSDFT is more computationally efficient than the COT methods in the diagnosis applications where amplitudes at a few significant harmonic and sideband components are of interest (Borghesani et al 2014). However, this method also requires a very accurate tachometer to provide the ISRP information, like the COT method.

To resolve the two aforementioned difficulties present in the COT method, we propose a combined generalized demodulation (GD) and VSDFT method to transform the signal into the order domain. This method is named the generalized velocity synchronous Fourier transform (GVSFT). In the proposed method, the information of the ISRP can be recovered by the GD from the raw vibration signal with simple vector multiplication, band-pass filtering and demodulation. Then the signal can be transformed to the order domain directly via the VSDFT for analysis. In this way, both of the two aforementioned difficulties present in the COT method can be resolved.

The rest of the paper is structured as follows: The forthcoming section presents the proposed GVSFT method. Later sections examine the effectiveness of the proposed method using simulation and experimental vibration signals respectively. The conclusions are drawn in the last section.

2 Generalized velocity synchronous Fourier transform

In this section, we first briefly review the GD method for signal decomposition and the VSDFT for order tracking. The GVSFT method is then presented to transform the signal into order domain without a tachometer.

2.1 Overview of the generalized demodulation method

Generalized demodulation defines a demodulation function according to a specified instantaneous phase. By multiplying the signal with it, the time-variant frequency of the

interested component can be mapped into a constant one, thus avoiding frequency overlap with other components (Olhede and Walden 2005). This property makes it possible to separate any component of interest from the non-stationary multi-component signal. (Cheng et al 2009; Feng et al 2015; Feng et al 2011; Zhao et al 2013b)

The GD is derived from generalized Fourier transform. For an arbitrary mono-component analytic signal $x(t) = A(t) \exp[i\theta(t)]$, the GFT is given by

$$X_G(f) = \int_{-\infty}^{\infty} x(t) \exp\{-i2\pi[ft + s(t)]\} dt \quad (1)$$

where $s(t)$ is a real-valued function depending on time only and specifies the evolutionary phase behavior of the signal. Note this is the same as applying the standard Fourier transform to the signal $y(t) = x(t) \exp[-i2\pi s(t)]$.

The inverse GST is given by

$$x(t) = \int_{-\infty}^{\infty} X_G(f) \exp\{i2\pi[ft + s(t)]\} df \quad (2)$$

Hence, if $X_G(f) = \delta(f - f_0)$, then $x(t) = \exp[i2\pi s(t)] \int_{-\infty}^{\infty} X_G(f) \exp(i2\pi f t) df = \exp\{i2\pi[f_0 t + s(t)]\}$. It means that a signal with time-varying instantaneous frequency (IF) $f_0 + \mathfrak{F}(t)$ is mapped to the new signal with a constant IF f_0 . Therefore, if we wish to map a signal with, for example, the time varying frequency specified by $\mathfrak{F}(t) = f_0 + \mathfrak{F}(t)$ into a new signal with constant frequency f_0 , we simply need to create a new signal $y(t) = x(t) \exp[-i2\pi s(t)]$ satisfying $s(t) = \int (\mathfrak{F}(t) - f_0) dt$, where $\mathfrak{F}(t)$ can be estimated via conventional time–frequency analysis.

2.2 Overview of the velocity synchronous discrete Fourier transform

The velocity synchronous discrete Fourier transform (VSDFT) (Borghesani et al 2014) is an effective and efficient order tracking technique and is briefly reviewed in the following.

For a given time domain vibration signal $x(t)$, its order spectrum $X(\Omega)$ is the angular counterpart of the common frequency spectrum and obtained by Fourier transform of the angular domain signal $x(\theta_{\text{shaft}})$, expressed by

$$X(\Omega) = \int_{-\infty}^{\infty} x(\theta_{\text{shaft}}) \exp(-i\Omega\theta_{\text{shaft}}) d\theta_{\text{shaft}} \quad (3)$$

where θ_{shaft} is the ISRP and Ω is the vector of orders for the representation of the order spectrum.

A change of variable in the domain of integration leads to:

$$X(\Omega) = \int_{-\infty}^{\infty} x(\theta_{\text{shaft}}(t)) \exp[-i\Omega\theta_{\text{shaft}}(t)] \frac{d\theta_{\text{shaft}}}{dt} dt \quad (4)$$

The expression is then simplified in the following:

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) \omega_{\text{shaft}}(t) \exp[-i\Omega\theta_{\text{shaft}}(t)] dt \quad (5)$$

where $\omega_{\text{shaft}}(t) = d\theta_{\text{shaft}}(t)/dt$ is the instantaneous shaft angular frequency (ISAF). This expression is the continuous form of the VSDFT. It can be seen that the VSDFT is capable of transforming the signal from the time domain to the order domain directly according to the ISRP.

2.3 Scheme of the generalized velocity synchronous Fourier transform

As discussed in the previous subsection, the ISRP information is required in order to transform the signal into the order domain using the VSDFT. Phase demodulation technique

can extract phase information from the signal; however common demodulation technique has a mono-component requirement. Since real world signals usually consist of multiple components, thus demodulation technique cannot process such signals directly. As such, we propose a GD approach to separate a component whose IF is proportional to shaft rotational frequency, in this way the mono-component requirement of a demodulation technique is satisfied. The ISRP can then be recovered by demodulating the extracted component. According to the recovered ISRP, the signal can be transformed to the order domain directly via the VSDFT.

The vibration signal acquired from a gearbox is mainly composed of the meshing components, its harmonics components and sidebands components. The instantaneous frequencies of these components are all synchronized thus proportional to the ISRF. The multi-components vibration signal $x(t)$ can be represented as

$$x(t) = \sum_{k=1}^K x_k(t) = \sum_{k=1}^K A_k(t) \cos \left[2\pi \int_{-\infty}^{\infty} f_k(t) dt + \theta_k \right] \quad (6)$$

where K is the number of the components, $A_k(t)$ is the real-valued instantaneous amplitude envelope, θ_k is the initial phase of the k th component and $f_k(t)$ is the IF of the k th component. Assuming the phase shifting of the signal components introduced by the transmission path effect is negligible, the IF components can be seen as proportional to the ISRF with a ratio p_k , expressed by

$$f_k(t) = p_k f_{\text{shaft}}(t) \quad (7)$$

where $f_{\text{shaft}}(t)$ is the ISRF.

The procedure to transform the signal to order domain using the proposed GVSFT method is summarized as follows.

Step. 1. Estimate the IF of the component of interest. As discussed previously, in order to perform GD, the estimated IF of a certain component is required. Here we assume that the IF of the component of interest is $\hat{f}_I(t)$, where $I(1 \leq I \leq K)$ is the ordinal number of the component of interest. The IF of the component of interest can be roughly estimated based on the ridges of the time-frequency representation (TFR) obtained by the short-time Fourier transform (STFT) (Feng et al 2011; Urbanek et al 2013; Wang et al 2014). The STFT of the signal is represented by

$$X(t, f) = \int_{t=-\infty}^{t=+\infty} x(\tau) \lambda_{(w)}(\tau - t) \exp(-j2\pi f \tau) d\tau \quad (8)$$

where w is window length, τ and f denote time and frequency respectively. Restricted by the frame of linear transform, the STFT has limited time-frequency resolution. To have a better frequency resolution, the window length should be relatively long (Iatsenko et al 2015). However, as the STFT employs a set of bases with fixed frequencies and the signal frequencies are time-varying, the STFT will have smear effects and for a long window, the Taylor reminder will be large, which may lead to large error of the TFR (Yu and Zhou 2016). Therefore, one needs to take a tradeoff between accuracy and resolution. It is suggested to firstly set a relatively short window. If the strong components cannot be detected from the TFR, then we increase the window length until the TFR is clear enough for coarse frequency estimation. The component of interest can be extracted from the TFR using a local maxima search algorithm (Urbanek et al 2013), represented by

$$\hat{f}_I(t) = \underset{f}{\text{Argmax}} |x(t, f)|, \text{ for } f \in \Delta f_I, \quad (9)$$

$$\Delta f_I \subset \{ \hat{f}_I(t - d\tau) - \delta, \hat{f}_I(t - d\tau) + \delta \}, \quad (10)$$

where δ is the given frequency tolerance for maxima detection. For $t=0$, $f_{\max}$ should be given by the user. It should be noted that the STFT has limited time-frequency resolution, and thus the accuracy of the estimated IF is low. To improve the accuracy, the IF estimation will be refined by the GD subsequently as provided in steps. 2-6.

Step. 2. Create the analytic form $y(t)$ of the signal using the Hilbert transform. The Hilbert transform of the signal can be approximated by the corresponding quadrature part (Feldman 1997), expressed by

$$H(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \approx \sum_{k=1}^K A_k(t) \sin \left[2\pi \int_{-\infty}^{\infty} f_k(t) dt + \theta_k \right] \quad (11)$$

where $H(t)$ is the Hilbert transform of $x(t)$. The analytic signal is then expressed as

$$y(t) = x(t) + iH(t) = \exp \left\{ i \left[2\pi \int_{-\infty}^{\infty} f_k(t) dt + \theta_k \right] \right\} \quad (12)$$

Step. 3. Construct the GD function according to the preliminary estimated instantaneous frequency $\hat{f}_I(t)$ and the target frequency f_0 . Since the resolutions of the signal and the preliminary estimated instantaneous frequency $\hat{f}_I(t)$ are different, the resolution of $\hat{f}_I(t)$ is firstly improved by polynomial fitting (because the resolution of $\hat{f}_I(t)$ is lower). The GD function is expressed by

$$v(t) = \exp \left\{ i 2\pi \left[f_0 t - \int_{-\infty}^{\infty} \hat{f}_I(t) dt \right] \right\} \quad (13)$$

Forward map the analytic signal $y(t)$ to the new signal $y(t)$ by multiplying the GD mapping function $v(t)$, resulting a forward mapped signal

$$z(t) = y(t)v(t) = \sum_{k=1}^K A_k(t) \exp \left\{ i2\pi \left[\int_{-\infty}^{\infty} f_k(t) dt - \int_{-\infty}^{\infty} \hat{f}_I(t) dt + f_0 t \right] + i\theta_k \right\}. \quad (14)$$

For this forward mapped signal, the IF of the k th component is $f_k(t) - \hat{f}_I(t) + f_0$. It means that all the components' IFs are respectively mapped to the new frequencies equaling to their original frequencies $f_k(t)$ plus the IF of GD function $f_v(t) = -\hat{f}_I(t) + f_0$. Specially, the component's IF of interest is thus $f_I(t) = f_I(t) - \hat{f}_I(t) + f_0 \approx f_0$, showing that the time-varying frequency of the interested component is mapped to a new frequency which is almost constant and will not likely overlap other components' frequency bands. It can be seen the frequency of the forward mapped signal is around f_0 . To avoid the interferences of negative frequencies and frequency aliasing, the frequency of the forward mapped signal should be in range $(0, f_s/2)$, where f_s is the sampling frequency. For this reason, setting the target frequency f_0 in the range $(f_s/8, 3f_s/8)$ is suggested.

Step. 4. Separate the component $z_I(t)$ by filtering the forward mapped signal $z(t)$. The center frequency of filter is equal to the target frequency f_0 and the bandwidth is equal to the minimum frequency difference between the IF of its adjacent components. The filter signal $z_I(t)$ is expressed by

$$z_I(t) = A_I(t) \exp \left\{ i2\pi \left[\int_{-\infty}^{\infty} f_I(t) dt - \int_{-\infty}^{\infty} \hat{f}_I(t) dt + f_0 t \right] + i\theta_I \right\} \quad (15)$$

One can see that this resulted component $z_I(t)$ is a mono-component and only contains the forward mapped component of interest.

Step. 5. Recover the component of interest by multiplying the filtered component $z_I(t)$ by a reverse mapping GD function $v^{-1}(t) = \exp\left\{i2\pi\left[-f_0t + \int_{-\infty}^{\infty} \hat{f}_I(t)dt\right]\right\}$, expressed by

$$\begin{aligned} y_I(t) &= z_I(t) v^{-1}(t) \\ &= A_I(t) \exp\left\{i2\pi\left[\int_{-\infty}^{\infty} f_I(t)dt - \int_{-\infty}^{\infty} \hat{f}_I(t)dt + f_0t\right] + i\theta_I\right\} \exp\left\{i2\pi\left[\int_{-\infty}^{\infty} \hat{f}_I(t)dt - f_0t\right]\right\} \\ &= A_I(t) \exp\left[i2\pi\int_{-\infty}^{\infty} f_I(t)dt + i\theta_I\right] \end{aligned} \quad (16)$$

which is still analytic. In this step, multiplying the reverse of the same GD function as used in step (3) is equivalent to recovering the original IF of the component of interest. As the recovered signal contains only one component, it satisfies the mono-component requirement by demodulation.

Step. 6. Demodulate the recovered mono-component signal to obtain the instantaneous phase and frequency of the component of interest, expressed by

$$\theta_I(t) = \text{unwrap}\left\{\arctan\frac{\text{real}[y_I(t)]}{\text{imag}[y_I(t)]}\right\} \quad (17)$$

where $\text{real}[y_I(t)]$ and $\text{imag}[y_I(t)]$ denote the real and imaginary parts of $y_I(t)$ respectively, and unwrap denotes the phase unwrapping operation.

The ISRP is obtained by normalizing the phase of the component of interest, expressed by

$$\theta_{\text{shaft}}(t) = \frac{\theta_I(t)}{p_I} \quad (18)$$

where p_I is the ratio of the IF of the component of interest to the ISRF.

The ISAF is obtained by taking derivative of the ISRP, which can be written as

$$\omega_{\text{shaft}}(t) = \frac{d\theta_{\text{shaft}}(t)}{dt}. \quad (19)$$

As in this step, the ISAF is obtained by demodulation, thus it does not have resolution problem of the STFT. In this way, the estimated frequency is refined.

Step. 7. Apply the VSDFT to obtain the order spectrum $X(\Omega)$ for fault diagnosis according to the ISRP $\theta_{\text{shaft}}(t)$ and the ISAF $\omega_{\text{shaft}}(t)$, expressed by

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) \omega_{\text{shaft}}(t) \exp[-i\Omega \theta_{\text{shaft}}(t)] dt. \quad (20)$$

Now as the order spectrum $X(\Omega)$ is obtained, we can diagnose which gear has fault (if any) by matching the spectral peaks with the theoretical sidebands orders associated with faults. Two positions of fault are considered, i.e., on the pinion or on the larger gear. As the faults cause modulations with the rotational frequency and its harmonics as modulating frequencies, and meshing frequency and its harmonics as carrier frequencies. The theoretical sidebands orders can be expressed as $mo_m \pm no_r$, where o_m denotes meshing order, o_r denotes rotational order of the faulty gear, m and n are positive integers. To facilitate the understanding of the proposed method, the steps of the proposed GVSFT method are presented in a flowchart as shown in Fig. 1.

3 Simulation and demonstration

In this section, we examine the performance of the proposed method using a numerical synthetic signal. We generate the signal to simulate a scenario of a faulty pinion and a speed-down state using the faulty gearbox vibration model provided in McFadden (1986). Without loss of generality, we consider two harmonics of the gear meshing frequency $f_m(t)$ and two

harmonics of the rotational frequency of pinion $f_p(t)$. The frequency modulations caused by gear faults are not considered here, because they are relatively weak compared with the frequency modulations caused by running speed variations. The vibration signal can be written as follows

$$x(t) = \left\{ 1 + \sum_{n=1}^2 A_n \cos \left[2\pi n \int f_p(t) dt + \varphi_n \right] \right\} \sum_{k=1}^2 B_k \cos \left[2\pi k \int f_m(t) dt + \theta_k \right] + n(t) \quad (21)$$

AM by pinion fault
Gear meshing

where $A_n=0.5$ and $B_k=1$, the initial phrases $\varphi_n=\theta_k=0$, and a Gaussian white noise $n(t)$ is added to the signal such that the noise-to-signal ratio (SNR) is 0 dB to mimic the background noise. Assuming the faulty pinion has 8 teeth, the gear meshing frequency is eight times of the rotational frequency of pinion, expressed by $f_m(t)=8f_p(t)$. Taking the shaft rotational frequency of the pinion as the reference frequency, the meshing order o_m and the pinion rotational order o_p are 8 and 1 respectively. The signal length is five seconds and the sampling rate is 1600 Hz. To simulate the speed-varying state, the pinion rotational frequency is set as $f_p(t)=-0.1t^2-1.7t+18$.

Fig. 2(A-C) show the signal waveform, spectrum and the pinion rotational frequency respectively. Due to the time-varying nature of the rotational frequency, the Fourier spectrum does not provide sufficient information about the fault. To reveal the time-varying features of the signal, the signal TFR is shown in Fig. 2(D). It can be seen that there are two dominant components in the signal. According to the pinion rotational speed, these two components correspond to the gear meshing frequency $f_m(t)$ and its second harmonic. To extract the ISRP information from the signal using demodulation technique, the mono-component requirement must be satisfied. However the mono-component signal cannot be obtained by band-pass filtering, as the two constituent components share the same frequency band (234 Hz to 300 Hz).

To resolve this difficulty and reveal the fault features, the proposed GVSFT method is applied to the simulated signal. The meshing frequency is selected as the reference component since it is energy-dominant in the TFR and proportional to shaft rotational frequency. The preliminary meshing frequency is first estimated according to the TFR ridge as shown in Fig. 3(A). Due to the measurement noise and the smearing problem of the STFT, the accuracy of this IF estimation is low and the maximum IF estimation error is up to 4% as shown in Fig. 3(B). To reduce this error, the fine-IF estimation by the GD is carried out subsequently. Using the estimated instantaneous meshing frequency and the target frequency (200 Hz), the corresponding GD forward mapping vector is constructed. Multiplying the analytical version of the raw signal by this vector, the meshing frequency is de-curved and mapped to the target constant frequency of 200 Hz as shown in Fig. 3(C) and now it no longer overlaps with its second harmonic in the frequency band. Therefore, filtering the meshing harmonic component from the vibration signal is now possible using a band-pass filter whose center frequency is the target constant frequency (200 Hz) and the filtered signal is shown in Fig. 3(D). The IF of the filtered signal is recovered by multiplying the reverse mapping GD vector, as shown in Fig. 3(E) and the obtained signal only contains the component of interest thus the mono-component requirement of demodulation technique is satisfied. By demodulating the recovered mono-component and normalizing the result, the ISRP is obtained as shown in Fig. 3(F). The ISAF is obtained by taking derivative of the ISRP as shown in Fig. 3(G). To compare with the accuracy of the preliminary IF estimation obtained by the traditional TFA, the error of ISAF estimation is given in Fig. 3(H). It can be seen that the estimation error has been significantly reduced. More specifically, the maximum estimation error has been reduced to 0.5%.

In the following, the order spectrum is obtained by the VSDFT according to the ISRP and ISAF as shown in Fig. 4(A) where the order spectrum clearly reveals the meshing frequency ko_m where $k=1,2$ and its sidebands $ko_m \pm mo_p$, where $m=1,2$. This finding is consistent with the

theoretical expectation from the simulated signal (Eq. (21)). As these sideband orders exactly correspond to the pinion rotational order, we can conclude that the pinion has a fault.

In order to illustrate the advantage of the proposed GVSFT method, a comparison study is conducted. Fig. 4(B) shows the order spectrum generated by the VSDFT according to the IF estimation obtained by the traditional TFA. It can be seen that this order spectrum suffers from smearing problem caused by the inaccurate IF estimation, i.e., the amplitudes of the components decrease significantly and the component energy leaks to their surrounding bands. The sidebands of $2\omega_m$ are not very clear due to the spectral smearing. In contrast, the proposed GVSFT as shown in Fig. 4(A) can effectively reveal the sidebands associated with fault. The above analysis clearly demonstrates the effectiveness of the GVSFT in generating a precise order spectrum and extracting the fault features from a non-stationary gearbox vibration signal.

The computational efficiency of a method is important for online applications, so the efficiency of the GVSFT is evaluated here. We use MATLAB® R2014a to test the GVSFT. The computer configuration is provided as follows: Intel® i7-3615QM 2.3GHz CPU, 256 GB flash storage and 8 GB of DDR3 RAM. We generate ten synthetic signals whose lengths are set as integer seconds from 1 s to 10 s using the model given by Eq. (21). The sampling frequency and SNR are unchanged (i.e., 1600 Hz and 0 dB respectively). The frequencies of signals are properly set so that all the frequencies are continuous and lower than half of the sampling frequency to avoid frequency aliasing. The processing time by the GVSFT on these signals is shown in Fig. 5. It can be seen that as the signal length rises from 1 s to 10 s, the processing time rises approximately from 0.2 s to 1.1 s. This should be acceptable for most online applications.

4 Experimental tests

4.1 Experimental setup

In this section, we demonstrate the effectiveness of the proposed method by analyzing the vibration signal collected from an experimental gearbox test rig at the University of Ottawa lab. The test rig includes an AC drive motor for driving the gearbox, a fixed-shaft gearbox and a brake for loading as shown in Fig. 6. A pair of meshing gears are in the gearbox and the parameters are listed in Table 1.

Two experiments are conducted, i.e., baseline and spall damage. The spall fault is created on a pinion tooth shown in Fig. 7. The experimental data, sampled at a rate of 40 kHz, are collected by the two accelerometers mounted on the side of the casing of the gearbox. These two accelerometers collect the vibration along vertical direction and horizontal direction respectively. In this paper, only the signal measuring the vibration along vertical direction is used. To validate the effectiveness of proposed method in diagnosing gearbox faults under non-stationary conditions, we collect the vibration signals during a speed up and down processes, with the motor rotational frequency approximately increasing from 20 Hz to 30 Hz and then decreasing from 30 Hz to 20 Hz. A load of 2.1 Nm is applied to the output shaft of the gearbox by the brake. A tachometer is mounted on the shaft connected to the motor and pinion to measure its rotational speed. In this section, this shaft is selected as the reference shaft and accordingly the order means multiples of the instantaneous rotational frequency of the drive motor $f_d(t)$. The characteristic orders of the gearbox are calculated according to the configuration of the gearbox using the formulas provided in McFadden (1986), as listed in

Table 2.

4.2 Baseline case

Fig. 8(A-C) show the normal gearbox vibration signal waveform, the drive motor speed and Fourier spectrum of vibration signal, respectively. From Fig. 8 (B), it can be seen that the speed of the drive motor is between 20 Hz and 35 Hz, and accordingly the meshing frequency of gearbox is between 320 Hz and 560 Hz. Since the gear fault information is mainly reflected by the sidebands around the gear meshing frequency and its harmonics, we focus on the frequency band of 0.5 times the meshing frequency and 1.5 times of the meshing frequency (i.e., 160 Hz to 840 Hz). The signal Fourier spectrum, shown in Fig. 8 (C), suffers from frequency smearing problem due to the time-varying rotating frequency so that we could not identify the meshing frequencies of the gearbox and their sidebands. Fig. 8(D) shows the time-frequency representation of the normal signal and the gear meshing frequency f_{mesh} is detected according the relationship of meshing frequency and shaft rotational frequency. The GVSFT method is applied on the signal with the gear meshing frequency as the component of interest and the obtained order spectrum is illustrated in Fig. 9, where the meshing order o_{mesh1} is clearly discriminated. In this figure, two sidebands order, $o_m - o_p$ and $o_m + 2o_p$, can be discriminated in the spectrum. These two components are related to the fault of pinion, however the amplitudes of them are relatively low, which could be resulted from the inevitable gear manufacturing errors. Therefore this does not indicate that the gear is faulty.

4.3 Detection of gear fault

Now we evaluate the proposed method for the diagnosis of the gear fault. Fig. 10 (A-C) show the vibration signal collected from faulty gearbox, the associated drive motor speed and the Fourier spectrum of vibration signal. From Fig. 10 (B), it can be seen that the rotational speed of the drive motor is between 20 Hz and 35 Hz again. Similar to the baseline case, we focus on the frequency band of 160-840 Hz, as this band can cover enough sidebands of meshing frequency that may contain fault features. Due to the time-varying speed, the signal

Fourier spectrum (Fig. 10 (C)) has smearing problem once again and no fault features such as the meshing frequencies and sidebands can be identified. The STFT is then applied to the vibration signal and the obtained time-frequency representation is shown in Fig. 10 (D). The proposed GVSFT method is applied with the meshing frequency as the component of interest once again to extract the fault features. The obtained order spectrum, illustrated in Fig. 11. Similar to the baseline case, the meshing order o_m also dominates the order spectrum. Apart from it, ten sidebands can be detected. These sidebands can all be expressed as $o_{me} \pm n o_p$ (n is a positive integer), which means they are associated with the fault of the pinion and these sidebands have pronounced values. This indicates that the pinion has a gear fault. This result is consistent with the setting of the experiment. The tests demonstrate the desired capability of the proposed method in fault detection of gearboxes.

5 Conclusions

In this paper, we proposed a GVSFT method to transform the signal into the order domain for fault diagnosis. With this method, a signal component whose instantaneous frequency is proportional to ISRF can be separated by the designed filter with a time-varying pass-band; next, the ISRP can be obtained via demodulating the extracted component and normalizing the result; and the order spectrum can then be directly obtained using a single-step transform according to the ISRP. By matching the spectral peaks with the fault-related theoretical sidebands order, the diagnosis decision of the gearbox can be made. Compared with the several existing order tracking methods, the advantages of the proposed approach include: (a) this method is flexible because the GD mapping vector can be arbitrarily designed with respect to time to follow the shaft speed change, and (b) it is simple yet effective as the ISRP can be recovered via the GD with simple vector multiplication, filtering and demodulation and with this recovered information the order spectrum can be obtained via a single-step transform. We

have validated the proposed GVSFT method using a numerically simulated signal and successfully diagnosed gear fault from the non-stationary vibration signals of a lab gearbox.

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Table 1. Configuration parameters of the gearbox

Gear	Number of gear teeth
Pinion (Drive)	16
Larger Gear (Driven)	48

Table 2. Characteristic orders of the gearbox

Order	Value
Pinion rotating	$o_p=1$
Larger gear rotating	$o_l=1/3$
Meshing	$o_m=16$

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Fig. 1. Flowchart of the proposed GVSFT approach

Fig. 2. Simulated case: (A) simulated signal, (B) frequency spectrum of simulated signal, (C) pinion rotational frequency and (D) TFR of simulated signal

Fig. 3. Scheme of the GD: (A) preliminary IF estimation based on the traditional TFA, (B) error of the IF estimation obtained by traditional TFA, (C) TFR of the forward mapped signal, (D) TFR of the filtered signal, (E) TFR of the recovered component of interest, (F) ISRP estimation, (G) ISAF estimation, and (H) error of the ISAF estimation

Fig. 4. Order spectrum of the simulated signal: (A) by the proposed GVSFT method and (B) by the VSDFT based on the traditional TFA

Fig. 5. Processing time

Fig. 6. Test rig

Fig. 7. Pinion tooth spall damage

Fig. 8. Healthy gearbox: (A) vibration signal, (B) motor speed, (C) frequency spectrum of vibration signal, (D) TFR of vibration signal

Fig. 9. Order spectrum of the baseline case obtained by the GVSFT

Fig. 10. Gearbox with fault: (A) vibration signal, (B) motor speed, (C) frequency spectrum of vibration signal, (D) TFR of vibration signal

Fig. 11. Order spectrum of the pinion fault case obtained by the GVSFT

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