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Tripartite entanglement and Non-locality in three-qubit Greenberger-Horne-Zeilinger states with bit-flip noise

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M. Ardehali [Phys. Rev. A 46, 5375 (1992)] has derived a Bell-type inequality for a n -particle system in a Greenberger-Horne-Zeilinger (GHZ) state. This inequality helps relax the detection efficiencies required for closing the detection loophole in Bell tests. V. Coffman [Phys. Rev. A 61, 052306 (2000)] has given a simplified form of tripartite entanglement of three-qubit pure states. We use theory and experiment to investigate both results using three-qubit GHZ states with different intensities of bit-flip noise. The purpose of our work is to relate the Ardehali inequality and the tripartite concurrence of the tripartite GHZ states in a noisy environment. Our results show that the experimental values are quite consistent with theoretical predictions for three-qubit GHZ states in a bit-flip noisy environment.

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I. INTRODUCTION

Non-locality is one of the most surprising and important characteristics of quantum physics. The assumption of local realism (LR) led Einstein, Podolsky, and Rosen (EPR) to argue that the theory of quantum mechanics (QM) is not a complete theory [1]. Because of the significance of the Bell inequality and the Clauser, Horne, Shimony, and Holt (CHSH) inequality [2], several experiments based on the polarization-entangled photons [3–5] and other entangled systems [6, 7] have verified the theoretical predictions of QM. Ardehali has derived an Bell-type inequality for an n -particle GHZ state [8], so that now we can describe the non-local property of quantum states more conveniently. Furthermore, Bell-type inequalities, when used to refute the local realism model, are important not only to fundamental research, but also to quantum information processing, for example, proving the security of quantum cryptography [9, 10] and reducing communication complexity [11]. The security of quantum cryptography decreases because of the non-ideal entangled system used. Fortunately, Ekert has shown that one can estimate the security of quantum channel by testing Bell's theorem [12].

In the recent years, many efforts have been made to prepare multi-qubit entangled states [13, 14] and to derive different inequalities for refuting the local realism description [15, 16]. But there is almost not further research on the relation between robustness and the entanglement of a multi-bit entangled system with noise. As we all know, in realistic applications pure entangled states become mixed states due to influence of environment noise, which will lead to failure of the information processing [17, 18]. Violations of the Bell inequalities provide a method to characterize the robustness of the entangled states against noise [19]. Thus, the experimental investigations of the robustness of Bell's inequality of multi-particle entanglement states have an important physical significance and application value [20]. Another significant work is that of Coffman [21] who has derived a simplified form of tripartite entanglement of the three-qubit pure state, namely

concurrence. However, whether it is suitable for mixed states with noise needs further theoretical and experimental investigation.

In this paper, we demonstrate a test of the tripartite entanglement introduced by Coffman and investigate the Ardehali inequality for three particle GHZ states with bit-flip noise. We have calculated theoretically the Ardehali inequality and concurrence of GHZ states and measured the expectation values of the Ardehali operator and tripartite entanglement in three photon GHZ states with different noise levels. Using quantum-state tomography, we reconstructed the detailed density matrices of the states and obtained the tripartite entanglement of the GHZ states. The experimental results are quite consistent with the theoretical predictions. Furthermore, we display the unique relationship between tripartite entanglement and tripartite non-locality in GHZ states. Our results also provide an experimental criterion to estimate the safety of the quantum channel in a noisy environment.

II. THEORETICAL DESCRIPTION OF ARDEHALI INEQUALITY AND TRIPARTITE ENTANGLEMENT

Mermin [22] derived an n -particle Bell inequality, and Ardehali [8] derived an optimized Bell inequality for an n -particle GHZ state. Consider an n -particle GHZ state $|\Phi\rangle = (|H \dots H_i \dots H_n\rangle + |V \dots V_i \dots V_n\rangle) / \sqrt{2}$, where H_i and V_i are the horizontal and vertical polarization of the i^{th} particle. According to the local realism assumption, the nonlocal character of the state $|\Phi\rangle$ can be characterized using operator A defined as $A = A_1 + A_2$, where

$$A_1 = (\sigma_x^1 \sigma_x^2 \sigma_x^3 \dots \sigma_x^{(n-1)} - \sigma_y^1 \sigma_y^2 \sigma_x^3 \dots \sigma_x^{(n-1)} + \sigma_y^1 \sigma_y^2 \sigma_y^3 \sigma_x^4 \sigma_x^5 \dots \sigma_x^{(n-1)} - \dots + \dots)(\sigma_a^n - \sigma_b^n), \quad (1)$$

and

$$A_2 = (-\sigma_y^1 \sigma_x^2 \dots \sigma_x^{(n-1)} + \sigma_y^1 \sigma_y^2 \sigma_y^3 \sigma_x^4 \dots \sigma_x^{(n-1)} - \sigma_y^1 \dots \sigma_y^5 \sigma_x^6 \dots \sigma_x^{(n-1)} + \dots - \dots)(\sigma_a^n + \sigma_b^n). \quad (2)$$

here σ_a and σ_b are defined as $\sigma_a = (\sigma_x + \sigma_y)/\sqrt{2}$ and $\sigma_b = (\sigma_x - \sigma_y)/\sqrt{2}$, respectively. Using the generalized Clauser-Horne-Shimony-Holt lemma [23], the upper bound of the operator A is obtained by assuming LR

$$A \leq \begin{cases} 2^{n/2}, & n(\text{even}) \\ 2^{(n+1)/2}, & n(\text{odd}). \end{cases} \quad (3)$$

In comparison, the expected value of A according to the standard rules of QM is calculated as

$$\langle \Phi | A | \Phi \rangle = 2^{n-1/2}. \quad (4)$$

It can be seen that the theoretical quantum value exceeds the limit imposed by the premise of EPR by an exponentially large amount of $2^{(n-2)/2}$ for n odd and $2^{(n-1)/2}$ for n even.

For the three-qubit GHZ states, the Ardehali operator can be simplified to:

$$\begin{aligned} \mathcal{A} = & (\sigma_{x1}\sigma_{x2}\sigma_{a3} + \sigma_{x1}\sigma_{x2}\sigma_{b3} - \sigma_{y1}\sigma_{y2}\sigma_{a3} \\ & - \sigma_{y1}\sigma_{y2}\sigma_{b3} - \sigma_{x1}\sigma_{y2}\sigma_{a3} + \sigma_{x1}\sigma_{y2}\sigma_{b3} \\ & - \sigma_{y1}\sigma_{x2}\sigma_{a3} + \sigma_{y1}\sigma_{x2}\sigma_{b3})/\sqrt{2} \end{aligned} \quad (5)$$

here σ_x, σ_y are the Pauli operators, which are experimentally measured in $+/-$ and R/L bases, respectively, with $|+\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$, $|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle)$, and $|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle)$. The theory of QM and LR will predict completely different results for the Ardehali inequality

$$\begin{aligned} |\langle \mathcal{A} \rangle_{LR}| & \leq 4 \\ |\langle \mathcal{A} \rangle_{QM}| & \leq 4\sqrt{2}. \end{aligned} \quad (6)$$

The expectation value of $\mathcal{A}$ predicted by any LR cannot be larger than 4. The expectation value derived from QM is not larger than $4\sqrt{2}$. Span from 4 to $4\sqrt{2}$ is the violation of QM against LR.

Now, we consider the tripartite entanglement introduced by Coffman [21]. Assuming that there are three entangled qubits, denoted 1, 2, and 3, a measure of genuine tripartite entanglement is defined as

$$\tau = C_{1(23)}^2 - C_{12}^2 - C_{13}^2 \quad (7)$$

$C_{1(23)}$ measures the entanglement between qubit 1 and the joint state of qubits 2 and 3. The concurrences C_{12} and C_{13} quantify the bipartite entanglement between qubits 1 and 2, and 1 and 3, respectively. The three-tangle is invariant under permutation of the indices 1, 2, 3 and is bounded between 0 (for separable states) and 1 (for the maximally entangled GHZ states).

A bit-flip noisy quantum channel (see Experimental results section) is simulated by using one half-wave plate (HWP) sandwiched with two quarter-wave plates (QWPs) [24] (see Fig.2(b)). The operation for qubit i with bit-flip probability λ , i.e., bit-flip noise, is given by [25]

$$f(\rho_i) = (1 - \lambda)\rho_i + \lambda X_i \rho_i X_i \quad (8)$$

FIG. 1: (Color online) Theoretical value of the Ardehali inequality (black, dashed, see left y-axis) and the tripartite entanglement (blue, solid, see right y-axis) versus different levels of bit flip noise in GHZ state. We assumed that the experimenter prepares 4000 instances of a GHZ state and chooses to measure either the 8 terms of the Ardehali inequality (each term with 1000 realizations of the states) or the 64 terms of the three-qubit states density matrix.

where λ denotes the intensity of bit-flip noise introduced in our experiment. For the three-qubits GHZ states with noise, we can plot both the Ardehali inequality and the three-tangle versus λ as shown in Fig. 1. It is obvious that the two curves evolve in a coordinated fashion, That is an important point we will validate experimentally.

In order to further characterize the generated states, we define $\mathcal{V}$ as the violation of the Ardehali inequality and the $\mathcal{V} = \langle \mathcal{A} \rangle - C_{lhv}$, here C_{lhv} is the up threshold of one inequality for local realism. Then the standard deviations of an entanglement test can be defined as [20]

$$S = \mathcal{V}/\mathcal{E} \quad (9)$$

where $\mathcal{E}$ is the statistical error of the experiment (see Table I of Experimental result section). It is clearly that $\mathcal{E}$ depends on the error model used. In this work the error represent one standard deviation deduced from propagated Poissonian counting statistics of the raw detection events. In any experiment S is a well characterized quantity and its notion is widely used in the paper, and the violation is expressed in terms of "standard deviations"

III. EXPERIMENTAL SETUP

A schematic drawing of the experimental set-up is shown in Fig. 2. Spontaneous down conversion is used to produce the expected three-photon states. The first step in the experiment is to follow our previous work [26] and produce polarization-entangled three-qubit GHZ. As shown in Fig.2(a), a mode-locked Ti:sapphire laser emits an infrared (IR) light beam with a central wavelength of 780nm, a pulse width of 100fs and a repetition of 80 MHz. After passing through a LiB_3O_5 (LBO)

crystal, the light pulse converts to an ultraviolet (UV) light pulse with a central wavelength at 390nm. Then the UV beam passes through five dichroic mirrors (DM) which are used to separate the mixed IR and UV components. Behind the five DMs, the UV light pencil is focused on a β -barium borate (BBO) crystal which prepares a pair of entangled photons ($|H_2H_3\rangle + |V_2V_3\rangle$)/ $\sqrt{2}$ in paths 2 and 3, while the transmitted IR light becomes a weak pseudo single photon source prepared in state ($|H_1\rangle + |V_1\rangle$)/ $\sqrt{2}$ in path 1, where H and V represent horizontal and vertical polarization respectively. Then, photon 1 is combined with photon 2 by a polarizing beam splitter (PBS_{12}). By carefully adjusting the delay time between path 1 and 2 to make sure that photon 1 and photon 2 arrive at the PBS_{12} simultaneously, we obtain three-qubit GHZ states $|\psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle)$. To get better fidelity of the output states, we lowered the average power of the UV laser to 90mw, and the two-photon coincidence count rate to $6 \times 10^3 s^{-1}$. The visibility of the two-photon entangled state is 97% in the H/V basis, and 95% in the $+/-$ basis. Using the maximum-likelihood technique we construct the density matrix of the state [27]. Using the estimated density matrix combined with the method of over-complete state tomography, we can calculate the fidelity of a GHZ state without added noise as $F = \langle \psi_{GHZ} | \rho | \psi_{GHZ} \rangle = 0.84 \pm 0.01$, where the intensity is 15Hz. Second, as shown in Fig. 2(b), the noisy quantum channels are engineered by one HWP sandwiched by two quarter-wave plates. The HWP is switched randomly between $+\theta$ and $-\theta$, and the quarter-wave plates are set at 0° with respect to the vertical direction. In this way, the noisy quantum channel can be generated with a noise intensity $\lambda = \sin^2(2\theta)$. Finally, using the detector setup we can obtain all the experimental data.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

Table 1 lists the experimental results for the Ardehali inequality and the tripartite entanglement for different noise levels λ . The experimental values of the Ardehali operator are measured directly and the tripartite entanglement is calculated from the density matrix that was reconstructed from the experimental data. Here we use the maximum likelihood technique to reconstruct the density matrix of the states. The values in Table I show that the experimental values, though a little smaller than theoretical results, are consistent with the theoretical values shown in Fig.1. There are two main reasons for the non-ideal experimental data. First, the multi-photon generation contributes to the main noise in the results due to the probabilistic character of the parametric down conversion sources. Second, imperfection in the linear optics elements such as the beam splitter makes the results non-ideal.

Hence, the consistency of the Ardehali inequality with the tripartite entanglement has been shown. Therefore, we have demonstrated experimentally the test of the Ardehali inequality and the tripartite entanglement introduced by Coffman in three-qubit GHZ states with noise. Moreover, from Table I we can see that when $\theta > \pm 5^\circ$, $C < 0.69$ correspondingly, and the Ardehali inequality will no longer violate the LR pre-

FIG. 2: (Color online) Schematic drawing of the experimental setup. a) The entangled photons are generated by pumping the UV laser beam on a BBO crystal and a pseudo-single photon source is prepared by attenuating the infrared light after passage through the dichroic mirror (DM). The average power in the experiment is 90mW and the average two-fold coincidence is $6 \times 10^3 s^{-1}$ in modes 2-3. Prism Δd is used to ensure that the input photons arrive at the PBS at the same time. Half-wave plate (HWP) is used to prepare the single photon state. To achieve good temporal and spatial overlap at PBS_{12} , every output is spectrally filtered ($\Delta_{FWHM} = 3nm$) and monitored by fiber coupled single-photon detectors. Throughout the experiment, the coincidence time window is set to be 5 ns, which ensures that accidental coincidence is negligible. b) The setup for engineering bit-flip noise. Noisy quantum channels are engineered by one HWP sandwiched by two quarter-wave plates (QWPs).

diction. These results provide us experimental criteria for estimating the safety of the quantum channel in a noisy environment. We note that, a good test of the correlation between non-locality and tripartite entanglement is a significant step toward a loophole-free Bell test [28, 29]. However, we would like to state that the generation of the GHZ state and the measurement of the Ardehali inequality implies that some qubits are carried by respective photon, therefore cannot be spatially separated. So our experiment cannot be used to close the locality loophole and detection loophole [30–32].

V. CONCLUSION

In summary, we investigated the Ardehali inequality and tripartite entanglement in three-qubit GHZ states with bit-flip noise and confirmed the theoretical prediction experimentally. The experimental results show that the non-locality and tripartite entanglement of GHZ states directly with the noise level. So our results provide additional support for the quantum theory in the on-going conflict between the theory of QM and LR remain. Moreover, we learn about the property of the three-qubit GHZ states from a new perspective. There are other interesting approaches available for subsequent work that characterize the entanglement and non-locality property, such as constructing witnesses [33] and other Ardehali-like Bell inequalities [34].

		Ardehali inequality			Tripartite entanglement	
θ	λ	$\mathcal{A}$	$\mathcal{E}(\mathcal{A})$	$\mathcal{S}(\mathcal{A})$	τ	$\mathcal{E}(\tau)$
$\pm 0^\circ$	0	5.41	0.037	38.1	0.95	0.011
$\pm 1^\circ$	0.0012	5.11	0.032	34.7	0.91	0.013
$\pm 2^\circ$	0.0049	4.69	0.033	20.9	0.85	0.012
$\pm 3^\circ$	0.0109	4.48	0.036	13.3	0.81	0.010
$\pm 4^\circ$	0.0194	4.19	0.031	6.1	0.74	0.012
$\pm 5^\circ$	0.0302	4.03	0.035	0.9	0.69	0.013
$\pm 6^\circ$	0.0432	3.88	0.034		0.65	0.011

TABLE I: Experimental values for the Ardehali inequality and the tripartite concurrence for different values of θ . $\mathcal{A}$, $\mathcal{E}(\mathcal{A})$ and $\mathcal{S}(\mathcal{A})$ represent the values of $\mathcal{A}$, $\mathcal{E}$ and $\mathcal{S}$ in testing the Ardehali inequality; τ and $\mathcal{E}(\tau)$ represent the values of tripartite entanglement and its $\mathcal{E}$. In this work the error represents the standard deviation deduced from propagated Poisson counting statistics of the raw detection events. Each setting $X_1 X_2 X_3$ in the Ardehali inequality is measured for 120 s, while each setting $H_1 H_2 H_3$ in the three-qubit states density matrix is measured for 60 s. The average total count number for inequality and concurrence are about 1000.

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