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New character of a laser channel for squeezed chaotic state *

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Abstract

We in this paper report a new character of laser channel, namely, during squeezed chaotic state (SCS) evolving in the laser channel, its density operator keeps form-invariant, only newly found two typical parameters evolve with time well-regulated, so they can well describe the feature of this physical process. The typical parameter is comparable to the variance of normal distribution.

keywords: laser channel, evolution law, squeezed chaotic state typical parameter

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1 Introduction

In quantum statistics theory and quantum optics theory physicists pay much attention to dissipation, diffusion and laser processes. A laser channel is described by the master equation^[1]

$$\frac{d\rho}{dt} = g(2a^\dagger \rho a - aa^\dagger \rho - \rho aa^\dagger) + \kappa(2a \rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \quad (1)$$

where $a, a^\dagger$ are photon annihilation operator and creation operator, $[a, a^\dagger] = 1$, κ and g represent the cavity loss and gain, respectively. In this paper we report

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a new character of laser channel laser: channel keeps the density operator of squeezed chaotic state (SCS) form-invariant, namely, when an initial SCS (a mixed state) enters into a laser channel, the output operator remains a SCS, only newly found two typical parameters evolve with time well-regulated.

Squeezing and squeezed states have been a hot topic since 1970^[2-8] because they have potential uses in optical communication and gravitational detection. Squeezed state exhibits lower noise in one quadrature at the expense of higher noise in another quadrature^[9]. Theoretically, a single-mode squeezed state is constructed by acting single-mode squeezing operator $S_1(\gamma) = \exp\left[\frac{\gamma}{2}(a^2 - a^{\dagger 2})\right]$ on the vacuum state. The SCS, whose initial density operator is

$$(1 - e^k) S_1^\dagger(\gamma) e^{ka^\dagger a} S_1(\gamma) = \rho_0, \quad (2)$$

is produced by operating squeezed operator on the chaotic light with the density operator ρ_c

$$(1 - e^k) e^{ka^\dagger a} = \rho_c, \quad k = -\frac{\omega \hbar}{KT}, \quad K \text{ is the Boltzman constant} \quad (3)$$

The physical background of SCS in a laser channel is like this: supposing that a beam of SCS light is prepared and stored in a cavity surrounded by a thermo reservoir, and there is also a pump emitting photons into the cavity. It will be interesting to know how SCS evolves in a laser channel which includes both dissipation and pumping. Based on the thermo-entangled state representation theory and using the technique of integration within an ordered product (IWOP) of operators we shall show that the laser channel keeps the density operator of squeezed chaotic state form-invariant, during SCS evolves in the channel, only two characteristic parameters evolves with time well-regulated.

Our paper is arranged as follows. In Sec. 2 we briefly review the operator-sum solution to master equation of laser channel by virtue of the thermo entangled state representation. In Sec. 3 and Sec. 4, we derive the normally-ordered and anti-normally-ordered form of the density operator of SCS respectively. Using them in Sec. 5 we derive the output state which turns out to be a new SCS with two typical parameters evolving with time well-regulated in the laser channel. In Sec. 6 we analyse physical meaning of the two typical parameters. In Sec. 7 we calculate the photon number in the output state which is determined by the two typical parameters.

2 Brief review of deducing the infinitive operator-sum solution to Eq. (1)

In order to expose our way of obtaining operator-sum solution of Eq. (1), we construct the two-mode entangled state^[10-12]

$$|\eta\rangle = \exp\left[-\frac{1}{2}|\eta|^2 + \eta a^\dagger - \eta^* \tilde{a}^\dagger + a^\dagger \tilde{a}^\dagger\right] |0\tilde{0}\rangle \quad (4)$$

where $\tilde{a}$ is a fictitious mode independent of the real mode a , $[\tilde{a}, \tilde{a}^\dagger] = 1$ and $\tilde{a}|\tilde{0}\rangle = 0$. Employing the IWOP technique^[13–16] we can prove the orthogonal-completeness relation

$$\langle \eta' | \eta \rangle = \pi \delta(\eta' - \eta) \delta(\eta'^* - \eta^*) \quad (5)$$

$$\int \frac{d^2\eta}{\pi} |\eta\rangle \langle \eta| = 1, \quad (6)$$

so $|\eta\rangle$ is qualified to be a representation. Acting the both sides of Eq. (1) on the state $|I\rangle \equiv |\eta = 0\rangle$, and denoting $|\rho\rangle \equiv \rho|I\rangle$, by using the replacement property

$$a|I\rangle = \tilde{a}^\dagger|I\rangle, \quad \tilde{a}|I\rangle = a^\dagger|I\rangle \quad (7)$$

we can reform Eq. (1) as

$$\frac{d}{dt}|\rho\rangle = [g(2a^\dagger\rho a - aa^\dagger\rho - \rho aa^\dagger) + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)]|I\rangle \quad (8)$$

the solution to Eq. (8) is

$$|\rho(t)\rangle = \exp\{gt(2a^\dagger\tilde{a}^\dagger - aa^\dagger - \tilde{a}\tilde{a}^\dagger) + \kappa t(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a})\}|\rho_0\rangle \quad (9)$$

where $|\rho_0\rangle \equiv \rho_0|I\rangle$, ρ_0 is the initial density operator. Then we have the infinitive-sum form solution (the details are shown in Ref. [17])

$$\begin{aligned} \rho(t) &= \sum_{l,j=0}^{\infty} T_3 \frac{\kappa^l g^j T_1^{l+j}}{l!j!T_2^{2j}} e^{a^\dagger a \ln T_2} a^{\dagger j} a^l \rho_0 a^{\dagger l} a^j e^{a^\dagger a \ln T_2} \\ &= \sum_{l,j=0}^{\infty} M_{l,j} \rho_0 M_{l,j}^\dagger \end{aligned} \quad (10)$$

where

$$\begin{aligned} T_1 &= \frac{1 - e^{-2(\kappa-g)t}}{\kappa - ge^{-2t(\kappa-g)}}, \quad T_2 = \frac{(\kappa - g)e^{-(\kappa-g)t}}{\kappa - ge^{-2t(\kappa-g)}}, \\ T_3 &= \frac{\kappa - g}{\kappa - ge^{-2t(\kappa-g)}} = 1 - gT_1, \\ T_2^2 &= (1 - gT_1)(1 - \kappa T_1) \end{aligned} \quad (11)$$

and

$$M_{l,j} = \sqrt{\frac{T_3 \kappa^l g^j}{l!j!T_2^{2j}}} T_1^{l+j} e^{a^\dagger a \ln T_2} a^{\dagger j} a^l \quad (12)$$

We can prove

$$\sum_{l,j=0}^{\infty} M_{l,j}^\dagger M_{l,j} = 1 \quad (13)$$

Thus, for any given initial state ρ_0 , the density operator $\rho(t)$ can be directly calculated from Eq. (10).

3 The normally-ordered form of ρ_0

We should convert ρ_0 of SCS into its anti-normal ordering so that the summation over l as shown in Eq. (10) can be proceeded, since within the anti-normal ordering symbol (denoted by $\dot{\cdot}$) the creation operator and the annihilation operators can be permuted. For this aim, we first express ρ_0 in the Fock representation firstly^[18]

$$\rho_0 = (1 - e^k) \sum_{n=0}^{\infty} e^{kn} S^{-1}(\gamma) |n\rangle \langle n| S(\gamma) \quad (14)$$

where $|n\rangle$ is the Fock state, $a^\dagger a |n\rangle = n |n\rangle$. Then using the coordinate representation of squeezing operators^[19]

$$S^{-1}(\gamma) = \exp \left[-\frac{\gamma}{2} (a^2 - a^{\dagger 2}) \right] = \sqrt{\epsilon} \int_{-\infty}^{+\infty} dx |\epsilon x\rangle \langle x|, \quad \epsilon = e^\gamma \quad (15)$$

here $|x\rangle$ is the coordinate eigenstate

$$|x\rangle = \pi^{-\frac{1}{4}} \exp \left[-\frac{x^2}{2} + \sqrt{2} x a^\dagger - \frac{a^{\dagger 2}}{2} \right] |0\rangle \quad (16)$$

with the overlap

$$\langle x | n \rangle = \frac{1}{\sqrt{\sqrt{\pi} 2^n n!}} e^{-\frac{x^2}{2}} H_n(x) \quad (17)$$

where $H_n(x)$ is the Hermite polynomial, we derive

$$\begin{aligned} S^{-1}(\gamma) |n\rangle &= \sqrt{\epsilon} \int_{-\infty}^{+\infty} dx |\epsilon x\rangle \langle x | n \rangle \\ &= \int_{-\infty}^{+\infty} \frac{\sqrt{\epsilon} dx}{\sqrt{\pi 2^n n!}} \exp \left[-\frac{x^2}{2} (1 + \epsilon^2) + \sqrt{2} x \epsilon a^\dagger - \frac{a^{\dagger 2}}{2} \right] |0\rangle H_n(x) \\ &= \frac{\sqrt{\sec h \gamma}}{\sqrt{2^n n!}} (\tanh \gamma)^{n/2} H_n \left(\frac{a^\dagger}{\sqrt{2 \sinh(2\gamma)}} \right) \exp \left(\frac{\tanh \gamma}{2} a^{\dagger 2} \right) |0\rangle \end{aligned} \quad (18)$$

where in the last step we have used the integration formula^[20]

$$\int_{-\infty}^{+\infty} dx e^{-\frac{(x-y)^2}{2d}} H_n(x) = \sqrt{2\pi d} (1 - 2d)^{n/2} H_n \left[y (1 - 2d)^{-1/2} \right] \quad (19)$$

By substituting Eq. (18) into Eq. (14) and using $|0\rangle \langle 0| =: e^{-a^\dagger a} :$ as well as the generating formula of the Hermite polynomials^[21]

$$\sum \frac{t^n}{2^n n!} H_n(x) H_n(y) = \frac{1}{\sqrt{1-t^2}} \exp \left[\frac{2ytx - t^2(x^2 + y^2)}{1-t^2} \right] \quad (20)$$

we obtain

$$\begin{aligned} \rho_0 &= \frac{1-e^k}{\sqrt{\cosh^2 \gamma - e^{2k} \sinh^2 \gamma}} \exp \left[\frac{-a^{\dagger 2} (e^{2k} - 1) \tanh \gamma}{2(1 - e^{2k} \tanh^2 \gamma)} \right] \\ &\times \exp \left[a^\dagger a \left(\frac{e^k}{\cosh^2 \gamma - e^{2k} \sinh^2 \gamma} - 1 \right) \right] : \\ &\times \exp \left[\frac{-a^2 (e^{2k} - 1) \tanh \gamma}{2(1 - e^{2k} \tanh^2 \gamma)} \right] \end{aligned} \quad (21)$$

By noticing that the average photon number of the chaotic state $\rho_c = (1 - e^k) e^{ka^\dagger a}$ is

$$\bar{n} = \frac{1}{e^{-k} - 1} = \frac{1}{e^{\frac{\omega \hbar}{KT}} - 1}, \quad k = -\frac{\omega \hbar}{KT}, \quad (22)$$

and $2\bar{n} + 1 = \frac{e^{-k} + 1}{e^{-k} - 1}$, we introduce two parameters

$$\begin{aligned} \mu &= \frac{1}{(2\bar{n} + 1) e^{-2\gamma} + 1}, \\ \nu &= \frac{1}{(2\bar{n} + 1) e^{2\gamma} + 1} \end{aligned} \quad (23)$$

Using Eq. (22) we see

$$\begin{aligned} \mu - \nu &= \frac{2 \sinh 2\gamma}{(2\bar{n} + 1) + 2 \cosh 2\gamma + \frac{1}{2\bar{n} + 1}} \\ &= -\frac{(e^{2k} - 1) \tanh \gamma}{1 - e^{2k} \tanh^2 \gamma} \end{aligned} \quad (24)$$

thus Eq. (21) becomes to

$$\rho_0 = 2\sqrt{\mu\nu} : \exp \left[\frac{1}{2} (\mu - \nu) (a^{\dagger 2} + a^2) - (\mu + \nu) a^\dagger a \right] : \quad (25)$$

4 The anti-normally-order form of ρ_0

After substituting ρ_0 into Eq. (10) we find that in order to perform summation over l , we must convert Eq. (25) into anti-normally ordered form, so we use the formula^[22]

$$\rho_0 = \int \frac{d^2 z}{\pi} : \langle -z | \rho_0 | z \rangle \exp \left(|z|^2 + z^* a - z a^\dagger + a^\dagger a \right) :, \quad (26)$$

here $|z\rangle$ is a coherent state,

$$|z\rangle = \exp\left[\frac{-|z|^2}{2} + za^\dagger\right] |0\rangle, \quad \langle -z|z\rangle = \exp\left(-2|z|^2\right). \quad (27)$$

Substituting Eq. (25) into Eq. (26) we obtain the anti-normal ordering form of ρ_0

$$\begin{aligned} \rho_0 &= 2\sqrt{\mu\nu} \int \frac{d^2z}{\pi} \exp\left[-|z|^2 - \frac{\nu}{2}(z - z^*)^2 + \frac{\mu}{2}(z + z^*)^2 + z^*a - za^\dagger + a^\dagger a\right] : \\ &= 2\sqrt{\mu\nu} \int \frac{d^2z}{\pi} \exp\left[(\mu + \nu - 1)|z|^2 - a^\dagger z + az^* + \frac{1}{2}(\mu - \nu)(z^2 + z^{*2}) + a^\dagger a\right] : \\ &= 2\sqrt{\frac{\mu\nu}{W}} \exp\left[\frac{\mu - \nu}{2W}(a^{\dagger 2} + a^2) + \frac{4\mu\nu - \mu - \nu}{W}a^\dagger a\right] : \end{aligned} \quad (28)$$

where

$$W = (2\mu - 1)(2\nu - 1) \quad (29)$$

5 Evolution law of the SCS in laser channel

Substituting Eq. (28) into the right-hand side of Eq. (10) we can perform summation over l within antinormally ordering

$$\begin{aligned} &\sum_{l=0}^{\infty} \frac{\kappa^l T_1^l}{l!} a^l \left\{ 2\sqrt{\frac{\mu\nu}{W}} \exp\left[\frac{\mu - \nu}{2W}(a^{\dagger 2} + a^2) + \frac{4\mu\nu - \mu - \nu}{W}a^\dagger a\right] : \right\} a^{\dagger l} \\ &= 2\sqrt{\frac{\mu\nu}{W}} \exp(\kappa T_1 a a^\dagger) \exp\left[\frac{\mu - \nu}{2W}(a^{\dagger 2} + a^2) + \frac{4\mu\nu - \mu - \nu}{W}a^\dagger a\right] : \equiv \mathfrak{A}, \end{aligned} \quad (30)$$

Further, in order to perform summation over j as Eq. (10) needs, we must converting Eq. (30) into normally ordered form, using the coherent state representation^[23]

$$1 = \int \frac{d^2z}{\pi} |z\rangle \langle z| = \int \frac{d^2z}{\pi} : \exp[-|z|^2 + za^\dagger + z^*a - a^\dagger a] : , \quad (31)$$

we have

$$\begin{aligned}
\mathfrak{A} &= 2\sqrt{\frac{\mu\nu}{W}} \int \frac{d^2z}{\pi} \exp \left[\frac{\mu-\nu}{2W} (a^{\dagger 2} + a^2) + \left(\frac{4\mu\nu - \mu - \nu}{W} + \kappa T_1 \right) a^{\dagger} a \right] : |z\rangle \langle z| \\
&= 2\sqrt{\frac{\mu\nu}{W}} \int \frac{d^2z}{\pi} \\
&\times \exp \left[\frac{(\mu-\nu)}{2W} (z^{*2} + z^2) + \left(\frac{4\mu\nu - \mu - \nu}{W} + \kappa T_1 \right) |z|^2 \right] : e^{-|z|^2 + za^{\dagger} + z^*a - a^{\dagger}a} : \\
&= 2\sqrt{\frac{\mu\nu}{W}} \int \frac{d^2z}{\pi} \\
&\times : \exp \left[\left(\frac{4\mu\nu - \mu - \nu}{W} + \kappa T_1 - 1 \right) |z|^2 + za^{\dagger} + z^*a + \frac{\mu-\nu}{2W} (z^{*2} + z^2) - a^{\dagger}a \right] : .
\end{aligned} \tag{32}$$

Employing the integral formula^[20]

$$\begin{aligned}
&\int \frac{d^2z}{\pi} \exp \left(\zeta |z|^2 + \xi z + \eta z^* + w' z^2 + \delta' z^{*2} \right) \\
&= \frac{1}{\sqrt{\zeta^2 - 4w'\delta'}} \exp \left[\frac{-\zeta\xi\eta + \xi^2\delta' + \eta^2w'}{\zeta^2 - 4w'\delta'} \right],
\end{aligned} \tag{33}$$

we obtain

$$\begin{aligned}
\mathfrak{A} &= 2\sqrt{\frac{\mu\nu}{W}} \frac{1}{\sqrt{\left(\frac{1}{2\mu-1} + \kappa T_1 \right) \left(\frac{1}{2\nu-1} + \kappa T_1 \right)}} \\
&\times : \exp \left[\frac{-\left(\frac{4\mu\nu - \mu - \nu}{W} + \kappa T_1 - 1 \right) a^{\dagger} a + \frac{\mu-\nu}{2W} (a^2 + a^{\dagger 2})}{\left(\frac{1}{2\mu-1} + \kappa T_1 \right) \left(\frac{1}{2\nu-1} + \kappa T_1 \right)} - a^{\dagger} a \right] : ,
\end{aligned} \tag{34}$$

Letting

$$m = \frac{1}{2\mu-1}, \quad n = \frac{1}{2\nu-1}, \tag{35}$$

and

$$X = (m + \kappa T_1)(n + \kappa T_1) \tag{36}$$

then

$$\mathfrak{A} = 2\sqrt{\frac{mn\mu\nu}{X}} : \exp \left[\frac{-\left(\frac{m+n}{2} + \kappa T_1 \right) a^{\dagger} a + \frac{n-m}{4} (a^2 + a^{\dagger 2})}{X} - a^{\dagger} a \right] : , \tag{37}$$

Substituting Eq. (37) into Eq. (10), and performing the summation over j within normal ordering where $a^\dagger$ and a can be permuted, we have

$$\begin{aligned}
 \rho(t) &= 2T_3 \sqrt{\frac{mn\mu\nu}{X}} e^{a^\dagger a \ln T_2} \\
 &\times \sum_{j=0}^{\infty} \frac{g^j T_1^j}{j! T_2^{2j}} : a^{\dagger j} \exp \left[\frac{\frac{n-m}{4} (a^2 + a^{\dagger 2}) - (\frac{m+n}{2} + \kappa T_1) a^\dagger a}{X} - a^\dagger a \right] a^j : e^{a^\dagger a \ln T_2} \\
 &= 2T_3 \sqrt{\frac{mn\mu\nu}{X}} e^{a^\dagger a \ln T_2} \exp \left[\frac{(n-m) a^{\dagger 2}}{4X} \right] \\
 &\times : \exp \left\{ \left[e^{\ln \left(\frac{gT_1}{T_2^2} - \frac{m+n+2\kappa T_1}{2X} \right)} - 1 \right] a^\dagger a \right\} : \exp \left[\frac{(n-m) a^2}{4X} \right] e^{a^\dagger a \ln T_2} \\
 &= 2T_3 \sqrt{\frac{mn\mu\nu}{X}} e^{a^\dagger a \ln T_2} \exp \left[\frac{(n-m) a^{\dagger 2}}{4X} \right] \\
 &\times \exp \left[a^\dagger a \ln \left(\frac{gT_1}{T_2^2} - \frac{m+n+2\kappa T_1}{2X} \right) \right] \exp \left[\frac{(n-m) a^2}{4X} \right] e^{a^\dagger a \ln T_2}, \quad (38)
 \end{aligned}$$

where in the last step we have used $: \exp [(e^\lambda - 1) a^\dagger a] : = e^{\lambda a^\dagger a}$.

Then using operator identity^[24]

$$\begin{aligned}
 \exp(a^\dagger a \ln A) f(a^\dagger) \exp(-a^\dagger a \ln A) &= f(Aa^\dagger) \\
 \exp(a^\dagger a \ln A) f(a) \exp(-a^\dagger a \ln A) &= f(a/A)
 \end{aligned} \quad (39)$$

Finally, we obtain

$$\begin{aligned}
 \rho(t) &= 2T_3 \sqrt{\frac{mn\mu\nu}{X}} \exp \left[\frac{(n-m) T_2^2 a^{\dagger 2}}{4X} \right] \\
 &\times \exp \left\{ a^\dagger a \ln \left[gT_1 - \frac{(m+n+2\kappa T_1) T_2^2}{2X} \right] \right\} \exp \left[\frac{(n-m) T_2^2 a^2}{4X} \right] \\
 &= 2T_3 \sqrt{\frac{mn\mu\nu}{X}} \\
 &\times : \exp \left\{ a^\dagger a \left[gT_1 - \frac{(m+n+2\kappa T_1) T_2^2}{2X} - 1 \right] \right\} \exp \left[\frac{(n-m) T_2^2 (a^{\dagger 2} + a^2)}{4X} \right] : \quad (40)
 \end{aligned}$$

By noticing Eqs. (11), (35) and (36) we analyse the terms in (40),

$$\begin{aligned}
& gT_1 - \frac{(m+n+2\kappa T_1)T_2^2}{2X} - 1 \\
&= -\frac{1}{2} \left(\frac{1}{m+\kappa T_1} + \frac{1}{n+\kappa T_1} \right) T_3 (1-\kappa T_1) - T_3 \\
&= -\frac{1}{2} \left(\frac{1}{\frac{1}{2\mu-1} + \kappa T_1} + \frac{1}{\frac{1}{2\nu-1} + \kappa T_1} \right) T_3 (1-\kappa T_1) - T_3 \\
&= -\frac{1}{2} \left[\frac{(2\mu-1)(1-\kappa T_1)}{1+\kappa T_1(2\mu-1)} + 1 + \frac{(2\nu-1)(1-\kappa T_1)}{1+\kappa T_1(2\nu-1)} + 1 \right] T_3 \\
&= -\left[\frac{\mu T_3}{1+\kappa T_1(2\mu-1)} + \frac{\nu T_3}{1+\kappa T_1(2\nu-1)} \right] \quad (41)
\end{aligned}$$

and

$$\begin{aligned}
& \frac{(n-m)T_2^2}{4X} = \frac{(n-m)T_3(1-\kappa T_1)}{4(m+\kappa T_1)(n+\kappa T_1)} \\
&= \frac{1}{2} \left[\frac{\mu T_3}{1+\kappa T_1(2\mu-1)} - \frac{\nu T_3}{1+\kappa T_1(2\nu-1)} \right] \quad (42)
\end{aligned}$$

thus Eq. (38) becomes to

$$\begin{aligned}
\rho(t) &= 2 \frac{\sqrt{\mu T_3}}{\sqrt{1+\kappa T_1(2\mu-1)}} \frac{\sqrt{\nu T_3}}{\sqrt{1+\kappa T_1(2\nu-1)}} \\
&\times \exp \left\{ \frac{1}{2} \left[\frac{\mu T_3}{1+\kappa T_1(2\mu-1)} - \frac{\nu T_3}{1+\kappa T_1(2\nu-1)} \right] (a^{\dagger 2} + a^2) \right. \\
&\quad \left. - \left[\frac{\mu T_3}{1+\kappa T_1(2\mu-1)} + \frac{\nu T_3}{1+\kappa T_1(2\nu-1)} \right] a^{\dagger} a \right\} : \\
&= 2\sqrt{\mu'\nu'} \exp \left[\frac{1}{2} (\mu' - \nu') (a^{\dagger 2} - a^2) - (\mu' + \nu') a^{\dagger} a \right] : \quad (43)
\end{aligned}$$

where

$$\mu' = \frac{\mu T_3}{1+\kappa T_1(2\mu-1)}, \quad \nu' = \frac{\nu T_3}{1+\kappa T_1(2\nu-1)} \quad (44)$$

Comparing Eq. (43) with Eq. (25) we see the evolution law of the SCS in laser channel, that is: the evolution from ρ_0 to $\rho(t)$ is form-invariant, only the two typical parameters respectively evolves with time well-regulated

$$\mu \rightarrow \mu' = \frac{\mu T_3}{1+\kappa T_1(2\mu-1)}, \quad \nu \rightarrow \nu' = \frac{\nu T_3}{1+\kappa T_1(2\nu-1)} \quad (45)$$

where T_1, T_3 are the definite function of κ and g representing laser channel's loss and gain, respectively. The physical meaning of (μ, ν) is explained, so the evolution of SCS in the laser channel ascribed to the two typical parameters' evolution, this is the new character of a laser channel.

6 Physical meaning of the two typical parameters

In the above discussion we have revealed that the two typical parameters (μ, ν) embodies the evolution of SCS in the laser channel. We now endow their physical meaning. By introducing

$$Q = \frac{a + a^\dagger}{\sqrt{2}}, \quad P = \frac{a - a^\dagger}{\sqrt{2}i} \quad (46)$$

we can reform ρ_0 as

$$\rho_0 = 2\sqrt{\mu\nu} : \exp[-vQ^2 - \mu P^2] : \quad (47)$$

By comparing it with the 2-dimensional standard normal distribution expression

$$f = \frac{1}{\sigma_1\sigma_2} e^{-\frac{\tau_1^2}{2\sigma_1^2} - \frac{\tau_2^2}{2\sigma_2^2}} \quad (48)$$

where σ is the standard deviation, and σ^2 is the variance, we see that ρ_0 is a 2-dimensional normally-ordered normal distribution operator of Q^2 and P^2 , the correspondence is

$$\begin{aligned} \tau_1^2 &\rightarrow Q^2, \quad \tau_2^2 \rightarrow P^2 \\ 2\sigma_1^2 &\rightarrow 1/v, \quad 2\sigma_2^2 \rightarrow 1/\mu. \end{aligned} \quad (49)$$

i.e., the typical parameter is comparable to the variance of normal distribution, they can well describe the feature of the laser channel. Therefore, we conclude that a laser channel keeps the form of normally-ordered normal distribution operator invariant.

7 The photon number calculation relying on the two typical parameters

Now we calculate the photon number in the output state. According to Eq. (25), the photon number in the initial SCS is

$$\begin{aligned} \langle N \rangle_0 &\equiv \text{Tr}(\rho_0 a^\dagger a) = \text{Tr}(\rho_0 a a^\dagger) - 1 \\ &= 2\sqrt{\mu\nu} \text{Tr} \left\{ : a^\dagger a \exp \left[\frac{1}{2} (\mu - \nu) (a^{\dagger 2} + a^2) - (\mu + \nu) a^\dagger a \right] : \right\} - 1 \\ &= 2\sqrt{\mu\nu} \int \frac{d^2 z}{\pi} |z|^2 \exp \left[\frac{1}{2} (\mu - \nu) (z^{*2} + z^2) - (\mu + \nu) |z|^2 \right] - 1 \\ &= \frac{\mu + \nu}{4\mu\nu} - 1 \end{aligned} \quad (50)$$

which relies on the two typical parameters. Directly from the evolution law in Eq. (45) and Eq. (50) we immediately obtain the photon number in $\rho(t)$,

$$\begin{aligned}\langle N \rangle_t &= \frac{\mu' + \nu'}{4\mu'\nu'} - 1 = \frac{1}{4} \left(\frac{1}{\mu'} + \frac{1}{\nu'} \right) - 1 \\ &= \frac{1}{4} \left[\frac{\nu + \mu + \kappa T_1 (4\mu\nu - \mu - \nu)}{\mu\nu(1 - gT_1)} \right] - 1 \\ &= \frac{(\mu + \nu)}{4\mu\nu} e^{-2t(\kappa - g)} + \frac{g - \kappa e^{-2(\kappa - g)t}}{\kappa - g}\end{aligned}\quad (51)$$

Comparing with Eq. (50) we see

$$\langle N \rangle_t = (\langle N \rangle_0 + 1) e^{-2t(\kappa - g)} + \frac{g - \kappa e^{-2(\kappa - g)t}}{\kappa - g}\quad (52)$$

8 Conclusion

In summary, we have examined how a SCS evolves in a laser channel and found two physical quantities characteristic of this physical process, they are μ and ν . We conclude that a squeezed chaotic optical field evolves into a new squeezed chaotic state in the way of $\mu \rightarrow \mu' = \frac{\mu T_3}{1 + \kappa T_1(2\mu - 1)}$ and $\nu \rightarrow \nu' = \frac{\nu T_3}{1 + \kappa T_1(2\nu - 1)}$. We also see that the photon number of the initial SCS decays exponentially.

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