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Analysis of Surface Erosion of Cohesionless Soils using a 3D Coupled CFD-DEM Model

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Abstract

A fluid-solid interaction model has been implemented by coupling two numerical methods, Computational Fluid Dynamics (CFD) and Discrete Element Method (DEM), which captures the meso-scale behaviors of the fluid-solid system. The model is first validated by comparing the results of simulations with two types of experiments: free settling of a single sphere in water and formation of angle of repose of particles under water, which show its capability in modeling the behaviors of both particle phase and fluid phase. The verified model is then used to study factors affecting the soil erodibility, where case studies are designed for soil particles deposited inside a pipe and subsequently subjected to water flow induced surface erosion. Influencing factors for soil erodibility, including particle diameter and inter-particle bond, are analyzed. For cohesionless soils without bond strength, the critical shear stress is found to be linearly related to particle size; while for soils with bond strength, the simulation results show that inter-particle bond largely decelerates the erosion process and causes a much lower erosion rate. To further the understanding of soil surface erosion under turbulent flow, the k- ϵ turbulence model has been

successfully implemented for the fluid phase. Comparison between the laminar model and the turbulence model shows turbulence significantly accelerates the erosion process.

Keywords: fluid-particle interaction, CFD-DEM model, k- ϵ turbulence model, soil erosion, multiphase flow

1. Introduction

Interactions between fluid and particulate solid materials are very common yet complicated processes involved in multiple engineering fields such as geotechnical engineering, mining, mechanical engineering, manufacturing, and chemical engineering. In the field of geotechnical engineering, soil erosion possesses threats to the safety of a variety of geostructures including levee, earth dam, and bridge across waterway. Bridge scour, which is related to the soil erosion process under turbulent flow, is responsible for over 60% of bridge failures in the United States (Lagasse et al. 2006). The mechanism of soil erosion, however, still remains to be elucidated in light of soil structure and the characteristics of fluid field. Experiment remains the primary way to quantify the soil erosion process (Briaud et al. 2004; Cyril et al. 2010; Haghighi et al. 2013). The advantages are that it provides practical macroscopic design parameters. Computational simulation provides an alternative way to gain insight on the important soil erosion phenomenon, which involves the complicated interactions of fluid flow and soil particles.

In the recent years, with the progresses in advanced numerical method and computing power, various coupled fluid-solid numerical models have been developed to simulate the behaviors of complicated engineering processes involving the interactions of fluid and particulate materials.

Most of these models have their own advantages and limitations in certain aspects, e.g., accuracy, efficiency, or convergence rate. The general idea of these coupled models is to simultaneously solve the governing equations of fluid phase and solid particle phase via the defined coupling interaction coefficients. The holistic simulation provides an opportunity to generate insights on the physics underlying the soil erosion process.

The locally averaged variables are widely adopted to simplify the fluid field as a continuous field, regardless of the space occupied by particles (Anderson and Jackson 1967, 1969). This method bypasses the details of complicated fluid-solid interactions surrounding an individual solid particle by the use of a drag force model, which is accurate enough for most engineering applications. A similar transport equations can be used to describe the transport of solid particles based on volumetric fraction, and this coupled method is called Two Fluid Method (TFM). The performance of this method has been verified by experimental observations (Bouillard et al. 1989; Nguyen et al. 2009; Nguyen et al. 2012; Shah et al. 2013). An advantage of TFM is the easy implementation of different fluid models, especially for turbulent flows (Dadashi et al. 2014; Patro and Dash 2014). However this model tends to predict a converging flow for the solid phase (Chen and Wang 2014).

Another method for the solid phase is the Discrete Element Method (DEM) proposed by Cundall and Strack (Cundall and Strack 1979). Instead of generating a continuous material space, DEM uses a large amount of discrete particles to simulate the mechanical behaviors of granular materials. A model that couples locally averaged fluid field (Computational Fluid Dynamics (CFD)) and discrete particle field is a rational approach in describing the meso-scale behaviors of solid-fluid systems. This work was first introduced by Tsuji in his research on fluidized beds (Tsuji et al. 1992, 1993). Applications based on this method span across multiple fields from

chemical engineering to geotechnical engineering, including fluidized beds (Hoomans et al. 1996; Xu and Yu 1997; Kafui et al. 2002; Tomac and Gutierrez 2014; Guo et al. 2014; Wang et al. 2014; Saidi et al. 2015), particle mixing techniques (Limtrakul et al. 2003; Jovanovic et al. 2014), seepage flow in soils (Chen et al. 2011), soil particle sedimentation (Zhao et al. 2014), impact of wet debris flows (Zhao and Shan 2013, 2014), and dry debris splashing into fluid (Zhao et al. 2015). One potential benefit of the DEM model is the capability to quantify the microscale interaction force between clay particles, which is the determinant of clay's different erosion behaviors compared to sand. For clay aggregates of size in the submicron range, the capillary force, electrostatic force, and van der Waals force will be much stronger than the gravitational force, as is illustrated in Figure 1 (Santamarina 2001; Zhu et al. 2007).

Besides, a few new numerical methods are also proposed for the computational simulation of soil-fluid systems. Two promising methods are Lattice Boltzmann Method (LBM) and Direct Numerical Simulation (DNS). The coupled LBM-DEM model is able to provide a holistic description of the fluid-particle interactions of individual particles (Han 2012; Galindo-Torres, 2013; Shamy and Abdelhamid 2014). The DNS method features the advantages in describing fluid-solid interactions for small pore-scale models (Deen et al. 2014; Feng et al. 2014). It, however, requires extensive computational powers.

In this paper, a coupled CFD-DEM model has been formulated and implemented to describe the mechanical interactions between soils and fluid. To study the particle responses in turbulent flow, a k - ϵ turbulence model is first implemented for the fluid phase, considering its viability in soil erosion analysis in the TFM model (Sumer 2007). The model is verified with experiments using two benchmark cases, i.e., simulating the free settling of a single particle in water and simulating the angle of repose of glass beads deposited under water. The validated model is used

to study the erosion behavior of soils under laminar and turbulent flows inside a pipe, with relatively simple flow boundary conditions. Influencing factors of soil erodibility, including particle diameter and inter-particle bond strength, are analyzed. The soil erosion process under turbulent flow is compared with that under laminar flow, exhibiting quite different erosion behaviors.

2. Methodology and mathematical formulations

The basic mathematical formulations describing the physics involved in the coupled CFD-DEM model are summarized in this section. The fluid phase and solid phase are solved separately with different governing equations by the use of CFD and DEM respectively, and data related to fluid-particle interaction (such as porosity and interaction force) are exchanged at each coupling time step.

2.1. Governing equations for fluid phase

The locally averaged Navier-Stokes equations are used for the fluid phase. To consider the influence of the solid phase, both the mass and momentum conservation laws are averaged based on porosity (Anderson and Jackson 1967). The interaction force acting on the fluid is averaged as a volume force based on the volume of mesh elements. The governing equations for mass and momentum conservation are shown in Eqs. 1-2:

$$\frac{\partial n\rho}{\partial t} + (\nabla \cdot n\rho\mathbf{u}) = 0 \quad (1)$$

$$\frac{\partial n\rho\mathbf{u}}{\partial t} + (\nabla \cdot n\rho\mathbf{u}\mathbf{u}) = -n\nabla p + \nabla \cdot n\mathbf{K} + n\rho\mathbf{g} + \mathbf{f}^p \quad (2)$$

where n is porosity of the solid phase; t is the time; $\mathbf{u}$ is fluid velocity vector; ρ is the density of fluid; p is the fluid pressure; $\mathbf{K}$ is stress tensor of the fluid, and $\mathbf{K} = \mu(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)$ for laminar flow; μ is the fluid dynamic viscosity; $\mathbf{g}$ is the gravitational acceleration with g_z equaling to -9.80 m/s^2 ; $\mathbf{f}^p$ is fluid-particle interaction force.

As a quick note, the locally averaged momentum equation (Eq. 2) may have different forms based on different physical assumptions. Eq. 2 adopted in this paper is based on the assumption that the fluid pressure drop is shared by fluid phase as well as solid phase (Gidaspow 1994). A few previous studies assumed the pressure drop is shared only by fluid phase (i.e., Feng and Yu 2004), which lead to a slightly different equation format. Numerical comparison by the authors (Guo and Yu 2017) shows that different equation forms lead to around 20% differences in the particle drag force.

The laminar flow is solved with Newtonian fluid assumptions. For turbulent flows, turbulence models are required for the closure of the equation system. Here the k - ϵ turbulence model is implemented for the turbulent flow simulation, and the Reynolds Averaged Navier-Stokes (RANS) equations are used for the mass and momentum conservations (same forms as Eqs. 1-2). The model also introduces two additional variables: turbulent kinetic energy k and dissipation rate of turbulence energy ϵ . The turbulent viscosity μ_T is defined by Eq. 3. The transport equations for k and ϵ are shown in Eqs. 4-5 (Launder and Spalding 1973).

$$\mu_T = \rho C_\mu \frac{k^2}{\epsilon} \quad (3)$$

$$\rho \frac{\partial k}{\partial t} + \rho(\mathbf{u} \cdot \nabla)k = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_k} \right) \nabla k \right] + P_k - \rho\epsilon \quad (4)$$

$$\rho \frac{\partial \varepsilon}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \varepsilon = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_\varepsilon} \right) \nabla \varepsilon \right] + C_{\varepsilon 1} \frac{\varepsilon}{k} P_k - C_{\varepsilon 2} \rho \frac{\varepsilon^2}{k} \quad (5)$$

The production term of kinetic energy, P_k , is defined as:

$$P_k = \mu_T [\nabla \mathbf{u} : (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)] - \frac{2}{3} \rho k \nabla \cdot \mathbf{u} \quad (6)$$

where σ_k , σ_ε , C_μ , $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are model constants which can be determined from experiments.

The stress tensor for the turbulence model is given by Eq. 7.

$$\mathbf{K} = (\mu + \mu_T)(\nabla \mathbf{u} + (\nabla \mathbf{u})^T) - \frac{2}{3} \rho k \mathbf{I} \quad (7)$$

where μ , μ_T are dynamic and turbulent viscosity; $\mathbf{u}$ is the Reynolds averaged fluid velocity; ρ is the fluid density; $\mathbf{I}$ is the identity matrix.

2.2. Governing equations for particle phase

The solid particles are governed by Newton's Laws of Motion. Modeling complex-shaped particle is challenging and computationally demanding. Therefore, for simplicity, soils are treated as spherical particles in this study. Cundall's soft particle model (1979) is used and the contact models implemented in this study include a linear spring model, a frictional model for sliding, and a viscous damping model (see Figure 2). To simplify the contact model, the critical viscous damping ratio is set as 0.0 in this paper except for the simulation case of repose angle formation. The conservations of momentum and angular momentum for each particle follow Eqs. 8-9.

$$m_i \frac{d\mathbf{U}_i^p}{dt} = \sum_{j=1}^{n_i^c} \mathbf{F}_{ij}^c + \mathbf{F}_i^f + \mathbf{F}_i^g \quad (8)$$

$$I_i \frac{d\mathbf{w}_i}{dt} = \sum_{j=1}^{n_i^c} \mathbf{M}_{ij} \quad (9)$$

where m_i is the mass of the particle i ; $\mathbf{U}_i^p$ is the translational velocity of particle i ; t is the time; n_i^c is the number of total contacts of particle i ; $\mathbf{F}_{ij}^c$ is the contact force acting on particle i from particle j ; $\mathbf{F}_i^f$ is the fluid interaction force acting on particle i , which has the same magnitude but opposite direction to $\mathbf{f}^p$; $\mathbf{F}_i^g$ is the gravitational force acting on particle i ; I_i is the moment of inertia of particle i ; $\mathbf{w}_i$ is the angular velocity of particle i ; $\mathbf{M}_{ij}$ is the torque applied from particle j to particle i , which equals to the contact force times the radius of particle i .

2.3. Fluid-Solid interaction force

Accurate modeling for the fluid-solid interaction is a key component for the reliability of fluid-particle simulation. This is a complicated issue as different forces, including buoyant force (or pressure gradient force), drag force, virtual mass force, lift force, and capillary force, can all play important roles under different situations. This study assumes the soil particles are initially submerged under water and maintained a fully saturated condition. The buoyant force and drag force play the dominant role under such condition and therefore the other terms are neglected in this study. The coupling framework is implemented so that other types of fluid-particle interaction forces can be easily added into the framework. In this study, the model for drag force of Wen and Yu (1966) is considered (Eq. 10).

$$\mathbf{F}_{d,i}^f = \frac{1}{8} C_D \pi \rho_f d_p^2 |\mathbf{u} - \mathbf{U}| (\mathbf{u} - \mathbf{U}) \quad (10)$$

where $\mathbf{F}_{d,i}^f$ is the drag force acting on particle i ; C_D is the drag coefficient; ρ_f is the fluid density;

d_p is the particle diameter; $\mathbf{u}, \mathbf{U}$ are the fluid and particle velocities. The drag coefficient is determined by Eq. 11.

$$C_D = \frac{24}{Re} (1 + 0.15 Re^{0.687}) \quad \text{for } Re \leq 1000$$

$$C_D = 0.44 \quad \text{for } Re > 1000 \quad (11)$$

where Re is the particle Reynolds number which equals to $\rho_f |\mathbf{u} - \mathbf{U}| d_p n / \mu$.

The hydrostatic buoyant force is given by Eq. 12.

$$\mathbf{F}_{b,i}^f = \frac{1}{6} \pi d_p^3 \rho_f \mathbf{g} \quad (12)$$

where $\mathbf{F}_{b,i}^f$ is the hydrostatic buoyant force of particle i ; d_p is the particle diameter; ρ_f is the fluid density; $\mathbf{g}$ is the gravitational acceleration.

3. Implementation of the coupled fluid-particle model

The coupled CFD-DEM framework is implemented by developing customized code under Matlab environment, which links the fluid and particle modules provided by two commercial software, COMSOL for the CFD simulation and PFC3D for the particle flow simulation. The Matlab code provides the main control for data extraction, process manipulation, and information exchange between CFD and DEM modules. The flow chart of implementing the coupled CFD-DEM simulation is shown in Figure 3. For each coupling time step, the CFD and DEM models operate separately with their respective initial conditions. Particle coordinates and porosity will be extracted from the results of the DEM simulation and transmitted to the CFD model. The drag force and buoyant force from the results of CFD simulation are extracted and passed to the DEM model. These input will drive the CFD and DEM to calculate the phase behaviors at the

next time step.

A sensitivity study has been conducted on the size of coupling time step, which determines how frequent the CFD and DEM modules exchange data. Based on this, the data exchange between DEM and CFD is set to occur at every 10^{-3} second (or 1 ms), which ensures the accuracy of simulation while avoiding excessive computational time due to frequent data exchange between modules. The time steps of the DEM model and the CFD model are determined separately by PFC3D and COMSOL in consideration of convergence. The time step in DEM simulation is much smaller (typically in the μs range).

4. Validation and application of the numerical model of fluid-solid system

The developed simulation framework is first validated by comparing to two common experimental scenarios that involve interactions of solid particles with fluid, i.e., free settling of a single particle in water and deposition of particles under water. Comparison between theoretical prediction and experimental result serves to validate the performance of the coupled CFD-DEM simulation framework.

4.1. Simulation of free settling of a single particle in water

In this simulation case, a single spherical particle is assumed to be released into a water tank and allowed to settle until reaching a constant velocity, which is typically referred as the free settling velocity. The free settling velocity has been studied by theories such as the Stokes' law, and the mathematical solution for the free settling velocity of a spherical particle can be provided. The

comparison between numerical simulation and mathematical solution helps to verify the correct implementation of CFD-DEM framework.

Schematic of the simulation case is shown in Figure 4. In the CFD model, liquid water is assumed to fill up a rectangular tank with a dimension of 0.1m (length) $\times$ 0.1m (width) $\times$ 0.6m (depth). The particle component includes a single spherical particle of 1.0 mm in diameter, which is generated at the top middle of the water tank and subsequently released (Figure 4). The particle is assumed to have a density of 2600 kg/m³, and the fluid is assumed to be water with a density of 1000 kg/m³ and a dynamic viscosity of 0.001 Pa·s. The coupled CFD-DEM model simulates the process of the particle movement when settling under water. The simulated result of the velocity of the falling particle is shown in Figure 5, which indicates that once released the particle falling speed increases until reaching a steady value.

The settling particle drives the surrounding water to move downward. Assume the driving force on fluid is relatively small and can be neglected, then the settling velocity can be described by Eq. 13. Under steady state, the derivative term on the left hand side equals to zero, so that the final free settling velocity can be calculated. The magnitude of free settling velocity of the particle determined from the analytical solution is consistent with the results of CFD-DEM simulation (see Figure 5).

$$\frac{\pi}{6} d_p^3 \rho_p \frac{dv_z}{dt} = \frac{\pi}{6} d_p^3 (\rho_p - \rho_f) g - C_D \frac{\pi}{4} d_p^2 \rho_f \frac{v_z^2}{2} \quad (13)$$

where d_p is the particle diameter; ρ_p is the particle density; v_z is the particle velocity in vertical direction; ρ_f is the fluid density; g is the gravitational acceleration and equals to 9.80 m/s²; C_D is the drag coefficient determined from Eq. 11.

4.2 Simulation of angle of repose of particles deposited in water

The numerical model described above involves the interaction of a single particle with water, which serves to verify the accuracy of the CFD-DEM framework. The simulation example of the formation of angle of repose of glass beads deposited under water is developed to further validate the coupled CFD-DEM model, particularly when a large number of particles are involved.

The angle of repose is an inherent parameter of granular materials, which is highly related to the internal angle of shearing resistance of the bulk granular material. Experiments in documented literatures as well as that conducted by the authors show that angle of repose of granular materials tends to be smaller when deposited under water than deposited in the air (Figure 6). In the experiment, 2.0 mm-diameter glass beads are assembled in the hopper just above the water surface and then released. The hopper has an end-diameter of 1.0 cm at the bottom and a height of 5.0 cm with a cone angle of 45° . The cylindrical glass container has a diameter of 10.0 cm and is filled with water to a height of 3.0 cm. Five tests are conducted for the wet and dry conditions separately in order to prove the repeatability of the experiment. In the wet condition, the angle of repose of glass beads is $24.4^\circ \pm 2.9^\circ$, while in the dry condition, it increases to $29.0^\circ \pm 1.9^\circ$.

The experiment is strictly replicated with the CFD-DEM model. Dimensions of the container and particles are kept exactly the same as the experiment (Figure 7). The 2.0 mm-diameter spherical particles are generated in a hopper located above the surface of water level and then released. Due to the relatively small disturbance, the water field is assumed to be laminar and therefore only laminar flow equations are used in the CFD model. Viscous damping model is

used to describe the energy dissipation of particles, and the critical viscous damping ratio is set to 0.1 in the simulation. The parameters of particles and fluids used in the simulation are given in Table 1. In total 4,500 sand particles are generated and deposited into water to form a pile. Totally 5.0 seconds of the deposition process are simulated.

Figure 7 shows the particle-water velocity field one second after the deposition begins. As can be seen, the deposition of particles into water produces water wave from its central surface similar as the experimental observations. Particles are damped by the drag force applied from the fluid, and form a stable slope under water. The shape of the final pile at the end of deposition ($t = 5.0$ s) is shown in Figure 8 for both deposition under water (Figure 8a) and without water (Figure 8b). The angle of repose is 21° for the particles deposited in the water, and increases to 25° when deposited without the water.

Both experiment and simulation predict a higher repose angle deposited in the air. The simulated angle of repose tends to be smaller compared with laboratory experiment. This difference mainly results from the lack of consideration of particle rolling resistance in the CFD-DEM model. For granular materials submerged under fluid, rolling resistance mainly attributes from two parts: viscous dissipation at contacts and damping effect from fluid. Laboratory experiments on the slumping of granular column under dry condition show the final geometry of granular assembly depends largely on the rolling motion of particles (Lajeunesse et al. 2005; Lube et al. 2005). In some related studies, additional rolling resistance models are proposed to simulate the rolling motion of granules with/without water and achieve desired results (Zhou et al. 1999; Utili and Nova 2008; Zhao and Shan 2013; Utili et al. 2015). Further understanding of particle rolling damping under fluid is still needed. However for validation purpose, this case proves the capability of the developed CFD-DEM framework in simulating fluid-particle interaction

problems.

5. Factors affecting erodibility of soils inside a pipe flow

The previous examples validate the performance of the coupled CFD-DEM simulation framework. The framework is then utilized to simulate soil erosion behavior subjected to water flow inside a pipe. The soil erodibility parameters, including critical shear stress and erosion rate, are analyzed under both macroscale and mesoscale.

5.1. Model description

In this model, soil particles (2.0 mm in diameter) are first deposited under gravity inside a pipe with a length of 0.6 m and a diameter of 0.1 m, as illustrated in Figure 9(a). Then the pipe is assumed to be filled with water and subjected to water flow with a constant inlet pressure of 200 Pa, which erodes the soil deposit. Soil grains are idealized as spherical particles for computational efficiency. However the irregularly shaped sand grains can be simulated in PFC3D through particle clumps. The viscous damping of particles is not considered in this case. Non-slip boundary is used for the pipe wall ($\mathbf{u} = 0$). Parameters of particles and fluid are listed in Table 2. As a quick note, the number of particles and their diameter have important effects on the computing time, which are factors considered in setting up the numerical model. Overall this coupled CFD-DEM model consists of 10829 tetrahedral elements in the CFD model and 9375 particles in the DEM model. Simulation for soil erosion is conducted for five seconds, which took about four days by use of a desktop computer with a 3.40 GHz processor. An example

schema of simulated flow field is shown in Figure 9(b).

5.2. Definition of equivalent meso-scale shear stress for soil particles

The current soil erosion model mainly consists of two parts: erosion criteria model and erosion rate model. Parameters used in erosion criteria models include critical shear stress (Briaud et al. 2001), critical shear velocity (Blacka et al. 2003), and Shield parameter (Wei et al. 2014). The critical shear stress is one of the most commonly used parameters in the research community. For most experiments, the magnitude of shear stress is deduced through fluid velocity and surface roughness. However it cannot be easily defined for granular materials since soil particles are directly subjected to external forces and in most cases they are strong enough to resist breakdown. Detachment and entrainment are two major modes for soil erosion. To compare the macro-scale shear stress with the particle-scale drag force, the following definition is proposed:

$$\tau = \frac{F_{\text{ext}}}{\alpha \cdot A} \quad (14)$$

where τ is the defined shear stress for soil particle; F_{ext} is the external forces of the particle, and equals to the drag force (excluding the buoyant force); α is ratio of the effective area over the maximum cross section of a particle; A is the maximum cross section area, for sphere particles $A = \pi r^2$. Choice of α is highly related to soil porosity. The experimental studies conducted by Briaud (2001) on the critical shear stress of sands show that α value is around 6. Here we choose $\alpha = 6.0$ in the subsequent analyses.

For the pipe flow, shear stress on the pipe wall can be estimated from the Moody chart through Eq. 15 (Briaud et al. 2001).

$$\tau' = \frac{1}{8} f \rho v^2 \quad (15)$$

where τ' is shear stress estimated through Moody chart; f is the friction factor obtained from Moody chart; ρ is the density of water; v is the mean flow velocity. The average height of roughness element is defined as half of the particle diameter.

The average shear stresses on the soil surface defined by Eq. 14 and Eq. 15 are compared with different inlet pressures (Figure 10). These two definitions predict quite similar results especially for low Reynold number flows. Shear stress distribution inside the sandpile is plotted in Figure 11, and areas with large shear stress is typically corresponding to areas with large erosion rate.

5.3. Influence of grain size on soil erodibility

For cohesionless soils, erosion resistance is mainly determined by their physical properties, including specific gravity, particle size, and porosity. The influence of grain size on soil's critical shear stress is investigated here by a series of simulation cases with different particle diameters, ranging from 2.0 mm, 2.5 mm, to 3.0 mm respectively. Inlet pressure of the pipe flow is set to 500 Pa, and other boundary conditions are kept the same. The number of soil particles in the three cases was adjusted to ensure the total mass of soils remains the same.

The distribution of drag force (excluding buoyant force) as well as velocity of soil grains after five seconds erosion are presented in Figure 12 & 13. Increase of particle size leads to an increase of drag force (from Eq. 10). However because of their high inertia, average particle velocity of 3.0 mm particles is the lowest (Figure 13). It should be noted that the average particle velocity is obtained by dividing the sum of particle velocities by the total particle

number. Due to the larger denominator, the average particle velocity of the 2.0 mm-diameter particles is slightly smaller than that of the 2.5 mm model. The velocity distribution of 2.0 mm particles is more uniform due to more collisions and energy transmissions. From Figure 13, about 80% of the soil particles have velocities ranging from 10^{-5} m/s to 10^{-3} m/s.

Since only five seconds erosions have been simulated and the final soil profile after erosion cannot be drawn directly, the particle velocity is used as an indicator for erosion, which classifies soil particles into eroded soils and uneroded soils, as is shown in Figure 14. The threshold velocity is defined to be 1.0×10^{-4} m/s for 2.0 mm particles considering the average particle displacement during the five seconds period, and increases to 1.25×10^{-4} m/s and 1.5×10^{-4} m/s for 2.5 mm and 3.0 mm particles in order to maintain a constant ratio of velocity to particle size. The critical shear stress is defined as the average shear stress of particles (by Eq. 14) under the threshold conditions. The results show there is a linear relationship between critical shear stress and grain size (Figure 15), which is consistent with the experimental observations by prior studies on erosion of sandy soils such as Briaud et al. (2001).

5.4. Influence of grain bond on soil erodibility

The erosion behaviors of sand grains with different particle bonds are simulated through a simple contact-bond model in order to see the potential benefits of sand cementation in soil erosion control. This contact-bond model is supported by PFC3D and can be envisioned as a glue joining two particles in both normal and shear directions. The bond strength is a constant value and is independent from the normal/shear contact force. Compared with some ductile or force dependent bond model (Utili and Nova 2008; Jiang et al. 2011, 2013), this contact-bond model

features a fragile failure under shearing. The contact law can be visualized from Figure 16. As is shown in Figure 16a, if the tensile normal contact force reaches the normal bond strength, the bond breaks and both the normal and shear contact forces are set to zero. If the magnitude of shear contact force reaches the shear bond strength (Figure 16b), the bond breaks and the slip model is activated (Itasca Group 2014).

The bond strengths of particles in the normal and shear directions are assumed to be equal in the following simulations for simplicity, and the magnitude of bond strength is scaled by particle weight. The scaled coefficient of particle bond (SC) is a dimensionless factor defined in Eq. 16, i.e., a scaled coefficient of 2.0 represents normal and tangential bond strengths two times of particle weight.

$$SC^{n/s} = \frac{F_c^{n/s}}{G_p} \quad (16)$$

where $SC^{n/s}$ is a dimensionless scaled coefficient of the normal/shear bond strength; $F_c^{n/s}$ is the normal/shear bond strength, in N ; G_p is the particle weight, in N , $G_p = 3/4\pi r_p^3 \rho_p g_z$, in which r_p and ρ_p are particle radius and particle density, g_z is gravitational acceleration.

Three cases are simulated with scaled coefficient of 0.0, 1.0 and 2.0. The inlet pressure of the pipe flow is assumed to be 500 Pa to provide relatively large drag forces. Other model parameters remain the same as the previous simulations. The results show that there is no appreciable change in the profile of soil aggregates compared with the case without bond strength, indicating a much lower erosion rate.

The average particle drag forces of the three models are similar ($2.20 \sim 2.22 \times 10^{-5}$ N) at $t = 5.0$ s regardless of their different particle bond strengths. Figure 17 shows the frequency distribution of particle velocity of the three models at $t = 5.0$ s. The horizontal axis representing the

magnitude of particle velocity is plotted in the logarithmic scale, in order to cover a wide velocity range. The average velocity of particles is calculated by dividing the sum of all the particle velocities by the particle number. From Figure 17 as the bond strength increases from 0.0 to 2.0, the average velocity of soil particles decreases from 0.195 mm/s to 0.128 mm/s, which is an indication of the reduced rate of erosion. Because of the existence of particle bond, a longer time is required for particles to detach from each other and develop the kinetic velocity. Consequently, the threshold velocity concept cannot be used here to classify eroded or uneroded particles. Such behaviors will need to be identified from results of longer time simulations, which are consistent with the time-dependent erosion phenomena observed in cemented soils.

5.5. Influence of turbulence on soil erodibility

Turbulence is an important contribution factor for soil erosion. To further understand the erosion process under turbulent flow, the k- ϵ turbulence model is implemented into the fluid phase of the CFD-DEM model. The physical model of soils and flow field is similar as the previous cases, and a constant inlet pressure of 200 Pa is maintained. The parameters for the k- ϵ turbulence model are listed in Table 3.

Figure 18 plots the evolvments of average drag force and particle velocity in laminar and turbulent models. The drag force on particles under turbulent flow is much larger than that under laminar flow. The major reason is under turbulent flow, the boundary layer is much thinner and the fluid velocity is evenly distributed, leading to a much larger drag force. Consequently, about 5% of the particles are eroded away from their original positions under laminar flow conditions, while the number increases to more than 25% under turbulent flow conditions. As time

increases, soil particles accelerate and the drag force decreases due to the decrease of relative velocity. At 5.0 seconds, the average particle velocity of the turbulence model is about two orders of magnitude higher than that of the laminar model (Figure 18b). It should be noted that the calculated Reynolds number of the pipe flow model is around 32,000 at $t = 5.0$ seconds, which is far beyond the critical Reynolds number (around 2,200). The direct application of the laminar model to the soil erosion simulation may underestimate the erosion process significantly.

6. Conclusions

This paper describes the development and implementation of a customized CFD-DEM model to simulate the complicated soil-water systems. The model is first validated via two benchmark examples, i.e., the free settling of a single particle under water and the formation of angle of repose by the deposition of spherical particles under water. The results validate the performance of the coupled model in describing the particle-fluid interactions. The validated model is used to understand a few influencing factors on soil erodibility.

Simulation cases are designed where soil particles deposited inside a pipe are subjected to the flow induced surface erosion. Meso-scale shear stress is defined using external force divided by effective cross section area of the particles. The influencing factors on particle's erodibility, including particle size, inter-particle bond and turbulence, are analyzed using the CFD-DEM simulation. The threshold velocity is found to be able to classify eroded particles and uneroded particles; it however is found inapplicable for cemented soils due to a much lower erosion rate. For cohesionless particles without inter-particle bond, the critical shear stress increases linearly as the grain size increases from 2.0 mm to 3.0 mm; the results also show that the particle bond

significantly decelerates the erosion process. Both are consistent with experimental observations. The $k-\epsilon$ turbulence model is also implemented to understand soil particle detachment mechanism under turbulent flow conditions. Comparison between the results of erosion by laminar and turbulence models shows that turbulence significantly accelerates the erosion process. Particularly, the drag force and particle detachment rate can be one order of magnitude larger under turbulent flow condition than that under laminar flow condition.

This work demonstrates the potential of using advanced computational model to understand the critical factors affecting the soil erosion process. It is also realized that the interaction of fluid and soils involves very complicated processes. Further advancements are necessary in understanding soil physical-chemical characteristics and models of interaction between fluid and soils, as well as efficient computational algorithms tailored for such numerical studies.

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References

- Anderson, T.B., and Jackson, R. 1967. A fluid mechanical description of fluidized beds: equations of motion. *Industrial & Engineering Chemistry Fundamentals* **6**(4): 527-539.
- Anderson, T.B., and Jackson, R. 1969. A fluid mechanical description of fluidized beds: comparison of theory and experiment. *Industrial & Engineering Chemistry Fundamentals* **8**(1): 137-144.
- Blacka, K.S., Peppeb, O.C., and Gustc, G. 2003. Erodibility of pelagic carbonate ooze in the northeast Atlantic. *Journal of Experimental Marine Biology and Ecology* **285-286**: 143-163.
- Bouillard, J.X., Lyczkowski, R.W., and Gidaspow, D. 1989. Porosity distributions in a fluidized bed with an immersed obstacle. *American Institute of Chemical Engineers Journal* **35**(6): 908-922.
- Briaud, J.L., Ting, F.C.K., Chen, H.C., Cao, Y., Han, S.W., and Kwak, K.W. 2001. Erosion function apparatus for scour rate predictions. *Journal of Geotechnical and Geoenvironmental Engineering* **127**(2): 105-113.
- Briaud, J.L., Chen, H.C., Li, Y., and Nurtjahyo, P. 2004. SRICOS-EFA method for complex piers in fine-grained soils. *Journal of Geotechnical and Geoenvironmental Engineering* **130**(11): 1180-1191.
- Chen, F., Drumm, E.C., and Guiochon, G. 2011. Coupled discrete element and finite volume solution of two classical soil mechanics problems. *Computers and Geotechnics* **38**(5): 638-647.
- Chen, X., and Wang, J. 2014. A comparison of two-fluid model, dense discrete particle model and CFD-DEM method for modeling impinging gas-solid flows. *Powder Technology* **254**: 94-102.
- Cundall, P.A., and Strack, O.D.L. 1979. A discrete numerical model for granular assemblies. *Geotechnique* **29**(1): 47-65.
- Cyril, G., Yves-Henri, F., Remi, B., and Chia-Chun, H. 2010. Contact erosion at the interface between granular coarse soil and various base soils under tangential flow condition. *Journal of Geotechnical and Geoenvironmental Engineering* **136**(5): 741-750.
- Dadashi, A., Zhu, J., and Zhang, C. 2014. A computational fluid dynamics study on the flow field in a liquid-solid circulating fluidized bed riser. *Powder Technology* **260**: 52-58.
- Deen, N.G., Peters, E., Padding, J.T., Kuipers, J.A.M. 2014. Review of Direct Numerical

- Simulation of fluid-particle mass, momentum and heat transfer in dense gas-solid flows. *Chemical Engineering Science* **116**, 710-724.
- Feng, Y.Q., and Yu, A.B. 2004. Assessment of model formulations in the discrete particle simulation of gas-solid flow. *Industrial & Engineering Chemistry Research* **43**: 8378-8390.
- Feng, Z.G., Ponton, M., Michaelides, E.E., and Mao, S.L. 2014. Using the Direct Numerical Simulation to compute the slip boundary condition of the solid phase in two-fluid model simulations. *Powder Technology* **265**: 88-97.
- Galindo-Torres, S.A. 2013. A coupled discrete element Lattice Boltzmann Method for the simulation of fluid-solid interaction with particles of general shapes. *Computer Methods in Applied Mechanics and Engineering* **265**: 107-119.
- Gidaspow, D. 1994. *Multiphase Flow and Fluidization: Continuum and Kinetic Theory Descriptions*. Academic Press.
- Guo, L., Morita, K., and Tobita, Y. 2014. Numerical simulation of gas-solid fluidized beds by coupling a fluid-dynamics model with the Discrete Element Method. *Annals of Nuclear Energy* **72**: 31-38.
- Guo, Y., and Yu, X. 2017. Comparison of the implementation of three common types of coupled CFD-DEM model for simulating soil surface erosion. *International Journal of Multiphase Flow* **91C**: 89-100.
- Haghighi, I., Chevalier, C., Duc, M., Guédon, S., and Reiffsteck, P. 2013. Improvement of hole erosion test and results on reference soils. *Journal of Geotechnical and Geoenvironmental Engineering* **139**(2): 330-339.
- Han, Y. 2012. Construction of a LBM-DEM coupling system and its applications in modeling fluid particle interaction in porous media flow. University of Minnesota.
- Hoomans, B., Kuipers, J., Briels, W., and Van Swaaij, W. 1996. Discrete particle simulation of bubble and slug formation in a two-dimensional gas-fluidised bed: a hard-sphere approach. *Chemical Engineering Science* **51**(1): 99-118.
- Itasca Consulting Group. 2014. *Particle Flow Code in 3 Dimensions (PFC3D) Manual*.
- Jiang, M., Yan, H., Zhu, H., and Utili, S. 2011. Modeling shear behavior and strain localization in cemented sands by two-dimensional distinct element method analyses. *Computers and Geotechnics* **38**: 14-29.
- Jiang, M., Zhang, W., Sun, Y., and Utili, S. 2013. An Investigation on loose cemented granular

- materials via DEM analyses. *Granular Matter* **15**: 65-84.
- Jovanovic, A., Pezo, M., Pezo, L., and Levic, L. 2014. DEM/CFD analysis of granular flow in static mixers. *Powder Technology* **266**: 240-248.
- Kafui, K., Thornton, C., and Adams, M. 2002. Discrete particle-continuum fluid modelling of gas-solid fluidised beds. *Chemical Engineering Science* **57**: 2395-2410.
- Lagasse, P.F., Clopper, P.E., Zevenbergen, L.W., and Girard, L.G. 2006. Countermeasures to protect bridge piers from scour. Transportation Research Board National Research Council, Washington, D.C.
- Lajeunesse, E., Monnier, J.B., and Homsy, G.M. 2005. Granular slumping on a horizontal surface. *Physics of Fluids* **17**(10): 103302.
- Launder, B.E., and Spalding, D.B. 1974. The numerical computation of turbulent flows. *Computer Methods in Applied Mechanics and Engineering* **3**(2): 269-289.
- Limtrakul, S., Chalermwattanatai, A., Unggurawirote, K., Tsuji, Y., Kawaguchi, T., and Tanthapanichakoon, W. 2003. Discrete particle simulation of solids motion in a gas-solid fluidized bed. *Chemical Engineering Science* **58**: 915-921.
- Lube, G., Huppert, H.E., Sparks, R.S., and Freundt, A. 2005. Collapses of two-dimensional granular columns. *Physical review. E, Statistical, nonlinear, and soft matter physics* **72**(4): 041301.
- Nguyen, D.H., Levy, F., Bang, D.P.V., Guillou, S., Nguyen, K.D., and Chauchat, J. 2012. Simulation of dredged sediment releases into homogeneous water using a two-phase model. *Advances in Water Resources* **48**: 102-112.
- Nguyen, K.D., Guillou, S., Chauchat, J., and Barbry, N. 2009. A two-phase numerical model for suspended-sediment transport in estuaries. *Advances in Water Resources* **32**(8): 1187-1196.
- Patro, P., Dash, S.K. 2014. Two-fluid modeling of turbulent particle-gas suspensions in vertical pipes. *Powder Technology* **264**: 320-331.
- Saidi, M., Basirat Tabrizi, H., Grace, J.R., and Lim, C.J. 2015. Hydrodynamic investigation of gas-solid flow in rectangular spout-fluid bed using CFD-DEM modeling. *Powder Technology* **284**: 355-364.
- Santamarina, J.C. 2001. Soil behavior at the microscale: particle forces. *Proceedings of the Symposium on Soil Behavior and Soft Ground Construction*.
- Shah, S., Ritvanen, J., Hyppanen, T., and Kallio, S. 2013. Wall effects on space averaged two-

- fluid model equations for simulations of gas-solid flows in risers. *Chemical Engineering Science* **89**: 206-215.
- Shamy, U.E., and Abdelhamid, Y. 2014. Modeling granular soils liquefaction using coupled Lattice Boltzmann Method and Discrete Element Method. *Soil Dynamics and Earthquake Engineering* **67**: 119-132.
- Shan, T., and Zhao, J. 2014. A coupled CFD-DEM analysis of granular flow impacting on a water reservoir. *Acta Mechanica* **225**(8): 2449-2470.
- Sumer, B.M. 2007. Mathematical modelling of scour: a review. *Journal of Hydraulic Research*, **45**(6): 723-735.
- Tomac, I., and Gutierrez, M. 2014. Fluid lubrication effects on particle flow and transport in a channel. *International Journal of Multiphase Flow* **65**: 143-156.
- Tsuji, Y., Tanaka, T., and Ishida, T. 1992. Lagrangian numerical simulation of plug flow of cohesionless particles in a horizontal pipe. *Powder Technology* **71**: 239-250.
- Tsuji, Y., Kawaguchi, T., and Tanaka, T. 1993. Discrete particle simulation of two dimensional fluidized bed. *Powder Technology* **77**: 79-87.
- Utili, S., and Nova, R. 2008. DEM analysis of bonded granular geomaterials. *International Journal for Numerical and Analytical Methods in Geomechanics* **32**: 1997-2031.
- Utili, S., Zhao, T., and Houlsby, G.T. 2015. 3D DEM investigation of granular column collapse: Evaluation of debris motion and its destructive power. *Engineering Geology* **186**: 3-16.
- Wang, X., Wu, X., Lei, F., Lei, J., and Xiao, Y. 2014. 3D full-loop simulation and experimental verification of gas-solid flow hydrodynamics in a dense circulating fluidized bed. *Particuology* **16**: 218-226.
- Wei, G., Brethour, J., Grünzner, M., and Burnham, J. 2014. Sedimentation scour model. *Flow Science Report* 03-14.
- Wen, C.Y., and Yu, Y.H. 1966. Mechanics of fluidization. *Chemical Engineering Progress Symposium Series* **62**: 100-111.
- Xu, B., and Yu, A. 1997. Numerical simulation of the gas-solid flow in a fluidized bed by combining discrete particle method with computational fluid dynamics. *Chemical Engineering Science* **52**(16): 2785-2809.
- Zhao, T., Houlsby, G.T., and Utili, S. 2014. Investigation of granular batch sedimentation via DEM-CFD coupling. *Granular Matter* **16**(6): 921-932.

- Zhao, T., Utili, S., and Crosta, G.B. 2015. Rockslide and impulse wave modelling in the vajont reservoir by DEM-CFD analyses. *Rock Mechanics and Rock Engineering* **49**(6): 2437-2456.
- Zhao, J., and Shan, T. 2013. Coupled CFD-DEM simulation of fluid-particle interaction in geomechanics. *Powder Technology* **239**: 248-258.
- Zhou, Y.C., Wright, B.D., Yang, R.Y., Xu, B.H., and Yu, A.B. 1999. Rolling friction in the dynamic simulation of sandpile formation. *Physica A* **269**: 536-553.
- Zhu, H.P., Zhou, Z.Y., Yang, R.Y., and Yu, A.B. 2007. Discrete particle simulation of particulate systems: theoretical developments. *Chemical Engineering Science* **62**(13): 3378-3396.

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Table 1 Parameters used in repose angle test.

Parameter	Value
Particle diameter	2 mm
Particle density	2600 kg/m ³
Normal stiffness	1.0E8 N/m
Shear stiffness	1.0E8 N/m
Particle friction coefficient	0.4
Fluid density	1000 kg/m ³
Fluid dynamic viscosity	0.001 Pa*s
DEM time step size	1.0E-7 s
CFD time step size	5.0E-4 s
Time step for coupling	1.0E-3 s
Total time simulated	5.0 s

Table 2 Parameters used in erosion model.

Parameter	Value
Particle diameter	2.0 mm
Particle density	2600 kg/m ³
Normal stiffness	1.0E8 N/m
Shear stiffness	1.0E8 N/m
Particle friction coefficient	0.4
Fluid density	1000 kg/m ³
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Total time simulated	5.0 s

Table 3 Model constants for $k - \varepsilon$ turbulence model.

Constant	Value
C_μ	0.09
$C_{\varepsilon 1}$	1.44
$C_{\varepsilon 2}$	1.92
σ_k	1.0
σ_ε	1.3

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