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Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2018-0061.R2
Manuscript Type:	Article
Date Submitted by the Author:	11-Jul-2018
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Keyword:	Bianchi type V, $f(R,T)$ gravity, perfect fluid, Hybrid exponential law, Equation of state
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

LRS Bianchi Type V perfect fluid cosmological model in $f(R, T)$ theory

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Abstract

This article deals with the study of Bianchi type V cosmological model filled with perfect fluid in the context of $f(R, T)$ gravity. R and T represents the Ricci scalar and trace of the energy-momentum tensor respectively. The modified Einstein's field equations are solved under the consideration of hybrid exponential law (HEL) for two cases of $f(R, T)$. The presented models are shear free.

PACS number:04.50kd.

Keywords: Bianchi type V , $f(R, T)$ gravity, perfect fluid

1 Introduction

In recent times, there has been a lot of interest in the accelerating expansion of the universe. There are many questions rising in the human mind about accelerating of the universe. Why the universe is expanding at an increasing rate and spreading its contents over greater dimensions of space? Still, uncertainty persists in getting the answer of expanding of universe. In the last two decades, the study of cosmic string has attracted the attention of cosmologists and astrophysicists as they believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies. These strings possess stress energy and are coupled to the gravitational fields. since the observational data are more concrete than the assumption data. In recent years there are many observational data like CMB (anisotropy of the Cosmic Microwave Background) [1–3], SNeIa (supernova type Ia) [4–8], LSS (large scale structure) [9–11], BAO (baryon acoustic oscillations) [12] and WL (weak lensing) [13] have suggested that the current cosmic expansion leads to the late time acceleration of the universe and the existence of dark matter. There are two major approaches for the universe to be strictly homogeneous and isotropic i.e. (i) to introduce dark energy within general relativity (ii) to modify gravity for large distances. Einstein, first gave the concept of dark energy by introducing the cosmological constant. However the Einstein's constant is now thought that it may be a suitable candidate for dark energy.

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Carmeli [14] predicted for the first time in 1996 that universe expansion was accelerating. Later, the supernova experiments explained the nature of dark energy and the accelerated expansion of the universe. The researchers are working in the field of dark energy. They have proposed many theoretical models based on quintessence, phantom, k-essence, tachyon, chaplygin gas, f-essence, g-essence, etc. In view of some observational data the universe is constitutes of 4% ordinary matter, 20% DM, 76% DE. From the above data it is concluded that the major part of the universe is filled with DE which denotes that there is some common relation between theoretical model and observational data. Also it provides evidence for the accelerating expansion of the universe. It is assumed that, the dynamical changes of the Universe is due to the exotic type of fluid. In literature usually dubbed as “dark energy”. The dark energy holds large negative pressure. The researchers have focused on how the dark energy can be replaced by another theory. According to the cosmological models, universe exists either with the incorporation DM or the unification of early time inflation and late time acceleration which is an alternative to the dark energy.

There are some modifications of GR such as curvature, singularity, transition phase of the universe from deceleration to acceleration, unification of early time inflation and late time acceleration. Kanti et al. [15] have proposed that if we took higher order curvature then there did not exist cosmological singularity. The above problem can be overcome by the two possible ways of modifying GR. They are (i) by incorporating an exotic dark energy form on the matter side of the field equations in GR to provide an extra repulsive force (pressure) (ii) by modifying the geometric part of the field equation (i.e. modifying the Hilbert action) by replacing with a more general functional form of the Ricci scalar R and/or by coupling some matter field in to it.

In literature, many researchers worked on different modified theories of gravity. The theories are $f(R)$ gravity theory [16–20], $f(T)$ gravity theory, where T is the torsion [21], $f(G)$ gravity theory [22, 23] and $f(R, T)$ gravity theory. In this article, we are interested in $f(R, T)$ gravity theory because it is the generalization of the $f(R)$ gravity. Basic review of $f(R)$ gravity is presented by different authors in [24–27] and also discussed stability of its different models [28]. Nojiri et al. [29] presented the idea of coupling between curvature and matter and also they investigated the explicit and implicit couplings in $f(R)$ gravity. Recently Harko et al. [30] presented a gravitational theory involving both curvature as well as matter components. This theory is known as $f(R, T)$ gravity (T denotes trace of the energy momentum tensor). Sharif et al. [31] examined an equilibrium description of thermodynamics which is possible in such a modified theory of gravity. The horizon entropy is constructed from the first law of thermodynamics corresponding to the Friedman equations. They explore the GSLT (generalized second law of thermodynamics) and find out the necessary condition for its validity in $f(R, T)$ gravity.

Adhav [32] has studied the LRS Bianchi type cosmological model in $f(R, T)$ gravity with perfect fluid. Reddy et al. [33] have investigated the negative constant deceleration parameter in Bianchi type III cosmological model. They have also [34] investigated the Bianchi type III dark energy model is derived in presence of perfect fluid using special law of variation for Hubbles parameter. Chaubey et al. [35] explored Bianchi type III, V, VI_0 and VI_h cosmological models by using linearly varying deceleration parameter. Yadav [36] constructed Bianchi type V string cosmological model with power law expansion in the $f(R, T)$ theory. Mishra et al. [37] have analysed Bianchi type VI_h cosmological model in presence

of perfect fluid in $f(R, T)$ gravity. They have also [38] studied Kaluza-Klein dark energy model in the form of wet dark fluid in same theory. Sahoo et al. [39] constructed an axially symmetric cosmological model in $f(R, T)$ theory in the presence of a perfect fluid source. Ahmed et al. [40] have investigated Bianchi type V cosmological model for a specific choice of $f(R, T) = f_1(R) + f_2(T)$. The cosmological constant Λ is expressed as a function of trace energy-momentum-tensor T and they dubbed a $\Lambda(T)$ gravity in it. Singh et al. [41] have investigated Bianchi type III cosmological model in the frame work of $f(R, T)$ gravity. Moraes [42] has analysed the $f(R, T)$ gravity by allowing the speed of light to vary. Sofuoglu [43] has investigated the homogeneous shear-free, rotating and expanding Bianchi type IX universe in the presence of perfect fluid in $f(R, T)$ theory of gravity. Sahoo et al. [44] have presented the big rip singularity for LRS Bianchi type I cosmological model in $f(R, T)$ gravity. Recently, Sahoo [45] and Sahoo et al. [46] have discussed various string cosmological model in $f(R, T)$ gravity.

In our earlier work [47], we have obtained some accelerating models in the framework of $f(R, T)$ theory of gravity for an anisotropic Bianchi type III universe. In that paper, we were able to integrate the modified gravity field equations by a novel method with the assumption of a power law function for a directional scale factor. In our another work [48], we have obtained accelerating cosmological models in a modified gravity theory dubbed as $f(R, T)$ gravity with the backdrop of an anisotropic Bianchi type III universe and Ricci scalar re-placed in the Einstein-Hilbert action of GR. The models are constructed for two different ways of modifying the Einstein-Hilbert action. Exact solutions of the field equations are obtained by a novel method of integration. We have explored the behaviour of the cosmic transit from decelerated phase of expansion to an accelerated phase to get the dynamical features of the universe. Within the formalism of the work, it is found that the modification of the Einstein-Hilbert action does not affect the scale factor. However the dynamics of the effective dark energy equation of state is significantly affected. In that work, we have taken linear directional scale factor leading to the generation of a logarithmic scale factor. The universe is believed to have undergone a transition from a decelerating phase to an accelerating phase. The redshift at which this transition occurred is termed here as the cosmic transit redshift. We have explored the cosmic transit redshift to constrain the model parameters. The present investigation is on the LRS Bianchi type V metric with the particular choice of functions of $f(R, T)$. This paper is organized as follows: the basic formalism of $f(R, T)$ gravity from Einstein- Hilbert action principle for LRS Bianchi type V metric are derived in the presence of perfect fluid in sect. 2. The metric and exact solutions of the field equations are studied using hybrid exponential law (HEL) for both the functions of $f(R, T)$ gravity in sects. 3 and 4 respectively. Some important physical and kinematical properties of the model are discussed in sect.5. In sect. 6, the discussion and conclusions of work are presented.

2 Basic formalism of $f(R, T)$ gravity:

The $f(R, T)$ gravity field equations are obtained by varying the action

$$S = \frac{1}{16\pi} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x \quad (1)$$

with respect to metric tensor g_{ij} as

$$f(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + (g_{ij}\square - \nabla_i \nabla_j)f(R, T) = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\Theta_{ij} \quad (2)$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{\alpha\beta}} \quad (3)$$

Here $f(R, T)$ is an arbitrary function of Ricci scalar R , the trace T of stress-energy tensor T_{ij} of the matter field. L_m is the matter Lagrangian density. $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$, $\square \equiv \nabla^i \nabla_i$, ∇_i being the covariant derivative.

The $f(R, T)$ gravity field equations can be explained as [47, 48].

$$f_R(R, T) \left(R_{ij} - \frac{1}{3}Rg_{ij} \right) + \frac{1}{6}f(R, T)g_{ij} = (8\pi - f_T(R, T)) \left(T_{ij} - \frac{1}{3}Tg_{ij} \right) - f_T(R, T) \left(\Theta_{ij} - \frac{1}{3}\Theta g_{ij} \right) + \nabla_i \nabla_j f_R(R, T) \quad (4)$$

where $\Theta_{ij} = g^{ij}\Theta_{ij}$.

For a perfect fluid stress energy tensor for matter Lagrangian is given by $T_{ij} = (\rho + p)u_i u_j - pg_{ij}$ where $u^i = (0, 0, 0, 1)$ is the four velocity vector in co-moving coordinate system satisfying $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$. Here ρ and p denote the energy density and pressure of the fluid respectively. Since there is no unique choice for matter Lagrangian, we assume $L_m = -p$ so that $\Theta_{ij} = -2T_{ij} - pg_{ij}$.

It is noted that the field equations of $f(R, T)$ theory of gravity depends on the physical nature of the matter field through tensor Θ_{ij} . Hence in the case of $f(R, T)$ gravity depending on the nature of the matter source, we obtain several theoretical models corresponding to different matter contributions for $f(R, T)$ gravity are possible. It is fore sightly seen from eq.(4) the physical properties of the matter field can be studies by the field equations of $f(R, T)$ theory of gravity. So one can construct different types of cosmological models taking different possible choices of the functional $f(R, T)$. we have followed the two different possible choices of the functional $f(R, T)$ of Herko et al. [30].

In this paper we have consider two types of models: $f(R, T) = R + 2f(T)$ with $f(T) = \mu T$ and $f(R, T) = f_1(R) + f_2(T)$ with linear functions $f_1(R) = \mu R$ and $f_2(T) = \mu T$.

Now using the above value of Θ_{ij}, T_{ij} in the eq.(4) we obtain the field equation of $f(R, T)$ gravity, with the special choice of $f(R, T)$ Herko et al. [30] given by $f(R, T) = R + 2f(T)$ as (for a detail derivation of the field equation one can refer to Herko et al. [30]).

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij} \quad (5)$$

The field equations of above two specific cases by the eq.(5) are reduced to

$$R_{ij} - \frac{1}{2}Rg_{ij} = (8\pi + 2\mu)T_{ij} + [\mu(\rho - p)]g_{ij} \quad (6)$$

$$R_{ij} - \frac{1}{2}Rg_{ij} = \left(\frac{8\pi + \mu}{\mu} \right) T_{ij} + \left[\frac{\rho - p}{2} \right] g_{ij} \quad (7)$$

3 Metric and Exact solution for $f(R, T) = R + 2f(T)$

We consider LRS Bianchi type V metric of the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} [dy^2 + dz^2] \quad (8)$$

where A and B are functions of t only.

The field equations in (6), for the metric (8) takes the following form

$$2\frac{B''}{B} + \frac{B'^2}{B^2} - \frac{1}{A^2} = (8\pi + 3\mu)p - \mu\rho \quad (9)$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{A'B'}{AB} - \frac{1}{A^2} = (8\pi + 3\mu)p - \mu\rho \quad (10)$$

$$2\frac{A'B'}{AB} + \frac{B'^2}{B^2} - \frac{3}{A^2} = -(8\pi + 3\mu)\rho + \mu p \quad (11)$$

$$2\left(\frac{B'}{B} - \frac{A'}{A}\right) = 0 \quad (12)$$

Here the prime denotes ordinary differentiation with respect to t . On integrating equation (12) we get

$$A = k_1 B \quad (13)$$

With the help of (13), the system of equation [(9)-(11)] reduces to two equations as follows

$$2\frac{B''}{B} + \frac{B'^2}{B^2} - \frac{1}{k_1^2 B^2} = (8\pi + 3\mu)p - \mu\rho \quad (14)$$

$$3\frac{B'^2}{B^2} - \frac{3}{k_1^2 B^2} = -(8\pi + 3\mu)\rho + \mu p \quad (15)$$

Equations (14) and (15) leads to

$$\rho + p = \frac{1}{4\pi + \mu} \left[\frac{B''}{B} - \left(\frac{B'}{B}\right)^2 + \frac{1}{k_1^2 B^2} \right] \quad (16)$$

$$p - \rho = \frac{1}{4\pi + 2\mu} \left[\frac{B''}{B} + 2\left(\frac{B'}{B}\right)^2 - \frac{2}{k_1^2 B^2} \right] \quad (17)$$

From (16) and (17) we have

$$p = \frac{8\pi + 3\mu}{4(4\pi + \mu)(2\pi + \mu)} \frac{B''}{B} + \frac{\pi}{(4\pi + \mu)(2\pi + \mu)} \left(\frac{B'}{B}\right)^2 - \frac{\pi}{(4\pi + \mu)(2\pi + \mu)} \frac{1}{k_1^2 B^2} \quad (18)$$

With the help of (16), equation (17) leads to

$$\rho = \frac{\mu}{4(4\pi + \mu)(2\pi + \mu)} \frac{B''}{B} - \frac{2(3\pi + \mu)}{2(4\pi + \mu)(2\pi + \mu)} \left(\frac{B'}{B}\right)^2 + \frac{2(3\pi + \mu)}{2(4\pi + \mu)(2\pi + \mu)} \frac{1}{k_1^2 B^2} \quad (19)$$

The system of equations [(16)-(17)] has two equations and three unknowns. Thus to make the system complete we need one physically plausible relation. Also the physical quantities like pressure and energy density depends upon the metric potential B . Thus we consider the metric potential, as an increasing function of time, which is in the form [49]

$$B = kt^n e^{\beta t} \quad (20)$$

where β , n and $k > 0$ are constant quantities. This functional form of B is generally referred as hybrid exponential law (HEL). In this case, we have obtained the physical parameters as follows:

The pressure p and energy density ρ of the model can be expressed as

$$p = \frac{p_0}{t^2} + \frac{p_1}{t} + p_2 + \frac{p_3}{t^{2n} e^{2\beta t}}, \quad (21)$$

where $p_0 = \frac{3n^2(4\pi+\mu)-n(8\pi+3\mu)}{4(4\pi+\mu)(2\pi+\mu)}$, $p_1 = \frac{3n\beta}{2(2\pi+\mu)}$, $p_2 = \frac{3\beta^2}{4(2\pi+\mu)}$ and $p_3 = -\frac{\pi}{(4\pi+\mu)(2\pi+\mu)k^2 k_1^2}$.

$$\rho = \frac{\rho_0}{t^2} + \frac{\rho_1}{t} + \rho_2 + \frac{\rho_3}{t^{2n} e^{2\beta t}}, \quad (22)$$

where $\rho_0 = -\frac{[3n^2(4\pi+\mu)+\mu n]}{4(4\pi+\mu)(2\pi+\mu)}$, $\rho_1 = -\frac{3n\beta}{2(2\pi+\mu)}$, $\rho_2 = -\frac{3\beta^2}{4(2\pi+\mu)}$ and $\rho_3 = \frac{(3\pi+\mu)}{(4\pi+\mu)(2\pi+\mu)k^2 k_1^2}$. Consequently, the equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ becomes

$$\omega = \frac{t^{2n} e^{2\beta t} [p_0 + p_1 t + p_2 t^2] + p_3 t^2}{t^{2n} e^{2\beta t} [\rho_0 + \rho_1 t + \rho_2 t^2] + \rho_3 t^2} \quad (23)$$

The physical quantities of the model for polynomial expansion law and exponential expansion law can be obtained as the special case from hybrid exponential law.

For $k = 1$ & $n = 0$ and $k = 1$ & $\beta = 0$, the hybrid exponential law ($kt^n e^{\beta t}$) reduces to exponential expansion law ($e^{\beta t}$) and polynomial law (t^n) respectively. Thus hybrid exponential law is most generalized form of these two laws, which we have discussed in the above.

Figure 1: Variation of energy density against time for $\mu = 0.2$, $\beta = 0.5$, $k = 1$, $k_1 = 0.01$ different n .

Figure 2: Variation of energy density against redshift for $\mu = 0.2$, $k = 1$, $k_1 = 0.25$, $n = 0.6$ different β .

The unit of time is considered as Gy in all the plotting against time.

Figure 3: Variation of pressure against time for $\mu = 0.2, \beta = 0.5, k = 1, k_1 = 0.01$ different n .

Figure 4: Variation of pressure against redshift for $\mu = 0.2, k = 1, k_1 = 0.25, n = 0.6$ different β .

Figure 5: Variation of EoS parameter ω against time t for $\mu = 0.2, \beta = 0.5, k = 1, k_1 = 0.01$ different n .

Figure 6: Variation of EoS parameter ω against redshift for $\mu = 0.2, k = 1, k_1 = 0.25, n = 0.6$ different β .

4 Exact solution for $f(R, T) = f_1(R) + f_2(T)$

The field equations in (7), for the metric (8) takes the following form

$$2\frac{B''}{B} + \frac{B'^2}{B^2} - \frac{1}{A^2} = \left(\frac{8\pi + \mu}{\mu}\right)p - \left(\frac{\rho - p}{2}\right) \quad (24)$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{A'B'}{AB} - \frac{1}{A^2} = \left(\frac{8\pi + \mu}{\mu}\right)p - \left(\frac{\rho - p}{2}\right) \quad (25)$$

$$\frac{A''}{A} + \frac{B''}{B} + \frac{A'B'}{AB} - \frac{1}{A^2} = \left(\frac{8\pi + \mu}{\mu}\right)p - \left(\frac{\rho - p}{2}\right) \quad (26)$$

$$2\frac{A'B'}{AB} + \frac{B''}{B} - \frac{3}{A^2} = -\left(\frac{8\pi + \mu}{\mu}\right)\rho - \left(\frac{\rho - p}{2}\right) \quad (27)$$

$$2\left(\frac{B'}{B} - \frac{A'}{A}\right) = 0 \quad (28)$$

Here the prime denotes ordinary differentiation with respect to t . Following the same procedure as in the previous section, we have obtained the following physical quantities of the

model as follows:

The pressure p and energy density ρ of the model can be expressed as

$$p = \frac{p_0}{t^2} + \frac{p_1}{t} + p_2 + \frac{p_3}{t^{2n}e^{2\beta t}}, \quad (29)$$

where $p_0 = \frac{n^2(32\pi\mu+3\mu^2)-n(16\pi\mu+3\mu^2)}{2(4\pi+\mu)(8\pi+\mu)}$, $p_1 = \frac{n\beta(32\pi\mu+3\mu^2)}{(4\pi+\mu)(8\pi+\mu)}$, $p_2 = \frac{\beta^2(32\pi\mu+3\mu^2)}{2(4\pi+\mu)(8\pi+\mu)}$ and $p_3 = -\frac{4\pi\mu}{(4\pi+\mu)(8\pi+\mu)k^2k_1^2}$.

$$\rho = \frac{\rho_0}{t^2} + \frac{\rho_1}{t} + \rho_2 + \frac{\rho_3}{t^{2n}e^{2\beta t}}, \quad (30)$$

where $\rho_0 = -\frac{[3n^2\mu^2+24\pi\mu n^2+\mu^2n]}{2(4\pi+\mu)(8\pi+\mu)}$, $\rho_1 = -\frac{3n\beta(8\pi\mu+\mu^2)}{(4\pi+\mu)(8\pi+\mu)}$, $\rho_2 = -\frac{3\beta^2(8\pi\mu+\mu^2)}{2(4\pi+\mu)(8\pi+\mu)}$ and $\rho_3 = \frac{(12\pi\mu+2\mu^2)}{(4\pi+\mu)(8\pi+\mu)k^2k_1^2}$. Consequently, the equation of state (EoS) parameter $\omega = \frac{p}{\rho}$ becomes

$$\omega = \frac{t^{2n}e^{2\beta t} [p_0 + p_1t + p_2t^2] + p_3t^2}{t^{2n}e^{2\beta t} [\rho_0 + \rho_1t + \rho_2t^2] + \rho_3t^2} \quad (31)$$

In both the cases the metric (8) takes the form

$$ds^2 = dt^2 - k^2t^{2n}e^{2\beta t} [k_1^2dx^2 + e^{2x} (dy^2 + dz^2)] \quad (32)$$

Figure 7: Variation of energy density against time for $\mu = 0.2$, $\beta = 0.5$, $k = 1$, $k_1 = 0.01$ different n . Figure 8: Variation of energy density against redshift for $\mu = 0.2$, $k = 1$, $k_1 = 0.25$, $n = 0.6$ different β .

Figure 9: Variation of pressure against time for $\mu = 0.2$, $\beta = 0.5$, $k = 1$, $k_1 = 0.01$ different n . Figure 10: Variation of pressure against redshift for $\mu = 0.2$, $k = 1$, $k_1 = 0.25$, $n = 0.6$ different β .

Figure 11: Variation of EoS parameter ω against time t for $\mu = 0.2$, $\beta = 0.5$, $k = 1$, $k_1 = 0.01$ different n .

Figure 12: Variation of EoS parameter ω against redshift for $\mu = 0.2$, $k = 1$, $k_1 = 0.25$, $n = 0.6$ different β .

5 Physical and kinematical properties of the models

In cosmology, physical and kinematical properties of the model are important to study because through these parameters, we are able to describe the properties of the discussed models. Here we have discussed the physical properties of both the models as follows:

The mean Hubble parameter (H), scalar expansion (θ), shear scalar (σ^2) and anisotropic parameter (A_m) are given as

$$H = \frac{n}{t} + \beta \quad (33)$$

$$\theta = 3 \left(\frac{n}{t} + \beta \right) \quad (34)$$

$$\sigma^2 = 0 \quad (35)$$

$$A_m = 0 \quad (36)$$

The deceleration parameter (DP) q is obtained as

$$q = -1 + \frac{n}{(n + \beta t)^2} \quad (37)$$

Here we noticed that, the assumption of metric potential in the form (20) leads us to a variable deceleration parameter and our proposed universe models take a phase transition from deceleration to acceleration at $t = \frac{\sqrt{n-n}}{\beta}$. This restricts n in the interval $0 < n < 1$.

Figure 13: Variation of deceleration parameter q against time t for $\beta = 0.5$ and different n .

Figure 14: Variation of deceleration parameter q against redshift for $n = 0.6$ and different β .

Figure 15: Variation of deceleration parameter q against redshift for $\beta = 0.5$ and different n .

The state finder pair $\{r, s\}$ is defined and obtained as follows

$$r = \frac{\ddot{a}}{aH^3} = 1 - \frac{n(3n - 2 + 3\beta t)}{(n + \beta t)^3}, \quad (38)$$

where a is the scale factor.

$$s = \frac{r - 1}{3(q - 0.5)} = \frac{2n(3n - 2 + 3\beta t)}{3(3n^2 + 6n\beta t + 3\beta^2 t^2 - 2n)(n + \beta t)} \quad (39)$$

From equations (38) and (39), we have observed that, the state finder pair $\{r, s\}$ of our models reduce to Λ CDM model i.e. $\{r, s\} = \{1, 0\}$ only when $t = \frac{2-3n}{3\beta}$.

6 Discussion and Conclusions

In the presented manuscript, we have discussed about the Bianchi type-V cosmological model in $f(R, T)$ gravity. Further, we have obtained the modified field equations for the two classes of $f(R, T)$ gravity i.e. $f(R, T) = R + 2f(T)$ and $f(R, T) = f_1(R) + f_2(T)$. From the two modified field equations (6) and (7), it is noticed that they are resembles with each other for a particular choice of μ .

The profile of energy density, pressure and EoS parameter of model-I are presented in the Figure 1 to Figure 6 against time and redshift. The energy density profile indicates that, the energy density is positive valued and decreasing function of time whereas for smaller value of redshift (z), ρ is positive valued and increasing function of redshift (see Figure 1 and Figure 2). Similar qualitative behaviour is noticed for model-II of energy density profile (see Figure 7 and Figure 8).

The pressure profile shows that, the pressure is an increasing function of time and approaching towards zero with the evolution of time. For $z \in [-1, 1]$, we observed that pressure takes values from positive to negative with the evolution of redshift for different β (see Figure 3 and Figure 4) in model-I. Similar qualitative behaviour is noticed for model-II of pressure profile (see Figure 9 and Figure 10).

The EoS profile is presented in Figure 5 and Figure 6 indicates that, our model behaves like a phantom energy model in early universe as $\omega < -1$ for $t < t \sim 5.27$ and $\omega < 0$ ($-1.1 < \omega < -0.8$) for $t > t \sim 10.6$ for all values of n in model-I. Similar qualitative behaviour is noticed for model-II (see Figure 11 and Figure 12). In this case $\omega < -1$ for $t < t \sim 5.5$ and $\omega < 0$ ($-0.8 < \omega < -0.55$) for $t > t \sim 10.601$.

For the discussed models Hubble parameter and expansion scalar are decreasing function of time. Here $H \rightarrow \beta$ and $\theta \rightarrow 3\beta$ for $t \rightarrow \infty$. Equation (35) indicates that, our models are shear free. Figure 13, Figure 14 and Figure 15 shows the profile of deceleration parameter against time and redshift respectively for different values of n and β . In figure 14 we have taken the value of $n = 0.6$, $\beta = 0.25, 0.45, 0.65$ which coincide the observational data. The other values of n and β is not fit for our model. One can easily observed that, q is a decreasing function of time and $q \rightarrow -1$ when $t \rightarrow \infty$. Also it vary from negative valued to positive valued with the increase of redshift for different n and β , which follow the observational data. All the physical parameters of both the models are presented in terms of redshift through the relation $\frac{a}{a_0} = \frac{1}{1+z}$.

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