Spectrum, Power, and Storage Management in Device-to-Device Networks

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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Abstract

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Device-to-device (D2D) communication allows direct data transmission among devices, which has been considered a cost-effective means to improve system performance in future networks. Potentially, D2D communication may reduce data transmission delay, increase network throughput, expand network coverage, and enhance network robustness. In spite of these promising benefits, the management of D2D wireless resources, such as spectrum, power, and storage, remains a challenging problem.

In this thesis, we focus on the spectrum, power, and storage management in D2D networks. First, we focus on optimizing the transmission probabilities of D2D pairs, where the D2D pairs share spectrum resources in a random access manner. We aim at maximizing $\alpha$-fairness in D2D networks with exact topology information or with only spatial statistics. Then, we consider power control among D2D pairs in D2D networks with delayed feedback of the network state information via an on-line learning approach. We aim at maximizing the long-term averaged sum-rate of D2D pairs with performance guarantee for cellular communication. Finally, we study probabilistic caching in D2D networks in order to leverage file storage in devices. We jointly optimize spectrum allocation between cellular communication and D2D communication and file caching probabilities to maximize the logarithm utility.
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Chapter 1

Introduction

To meet the increasing demand for better system performance in future wireless communication systems, substantial recent technological advancements have been made to support direct transmission between transmitter-receiver pairs in proximity, such as device-to-device (D2D) communication in 5G and the Internet-of-Things (IoT) environment [1]. D2D communication weakens the role of a base station (BS) in data transmission, where the BS may serve as a relay between devices in the conventional cellular communication. On the other hand, the BS plays an important role in the control of D2D communication, such as device paring, and transmission power coordination.

Generally speaking, D2D communication refers to any kind of direct communicate among devices, for instance, communication between two cell phones, and vehicle-to-vehicle communication, as shown in Fig. 1.1. D2D communication can potentially bring many benefits. For instance, D2D communication may reduce data transmission delay between devices since the transmission hop between a D2D transmitter and a BS may not be required. In addition, system throughput may be improved by frequency reuse between cellular communication and D2D communication with proper power coordination among the D2D pairs. Also, D2D communication may increase network coverage with some devices serving as relays between BSs and cell-edge devices. Furthermore, D2D communication may increase network robustness. For instance, a device may communicate with another device via D2D communication even when the associated BS fails.

D2D communication may be operated in two modes, i.e., overlay mode and underlay mode. In the overlay mode, D2D communication and cellular communication use orthogonal sub-bands for easier interference control. In the underlay mode, D2D communication reuses spectrum with cellular communication. In this mode, power control of D2D pairs needs to be carefully treated in order to limit the interference to cellular
communication, since D2D communication is usually considered as an “add-on” function, and is expected to have little impact on cellular communication.

1.1 Motivation

Despite the appealing advantages of D2D communication, many challenges remain. In the following, we describe some of the challenges that motive the research in this thesis.

1.1.1 D2D Random Access Networks

It is expected that there will be 50 billion connected devices by 2020 [2]. With such a huge number of devices, spectrum management in a centralized manner may be a prohibitive task. Therefore, spectrum sharing in a random access manner has been considered in future D2D networks [3]. One important problem in random access networks is to optimize the transmit probabilities of D2D pairs to maximize network-wide utility [4–24]. Based on the knowledge of network topology, two important scenarios have been considered in existing works. Some of them consider small-scale networks and assume that the exact network topology can be easily acquired. Other works consider networks comprising a large number of D2D pairs, where acquiring the exact topology information is a prohibitive task, and thus assume that only the statistics of the network topology is known.

Among the works based on the known network topology, some are focused on loosely
coordinated scheduling. For example, the authors of [4, 5] propose maximizing the α-fair utility via a coordinate descent (CD) technique. A main advantage of CD-based iterative methods is that only one transmitter-receiver pair updates its transmit decision in each iteration. However, coordination among devices can be easier in the future network architecture. For instance, the in-band transmission among D2D equipment in 5G networks [25] may be coordinated by a BS. Thus, an interesting problem in random access networks is to develop efficient methods that tradeoff coordination complexity for a faster convergence rate.

Among the works based on the statistics of network topology, the spatial Aloha model [7] is a commonly considered network model. Each transmitter in the network randomly transmits following the slotted Aloha medium access control (MAC) protocol. Both the Poisson point process (PPP) and Poisson bipolar process (PBP) are commonly used to model the location of transmitters and receivers in spatial Aloha networks. In the PBP model, the transmitters form a PPP, and each transmitter is paired with a dedicated receiver at some distance away. Optimizing the transmission probability in spatial Aloha network is a challenging problem, often with a non-convex objective function due to signal interference. In [7,13,26–30], this problem is studied where all transmitters are assumed to use the same transmission probability if the exact location of nodes is unknown. This model is suitable only when the network is uniform. In many practical scenarios, the transmitters may have different powers and the D2D distance may be different for different pairs, so that the transmitters should use different transmit probabilities.

Furthermore, most random access networks in existing works assume single-rate communication either based on a single received SIR threshold [7,13,26–30], such that the data rate is log(1 + Th) if the received SIR is above some threshold Th, and is zero otherwise; or based on the “protocol model” [4,14,16–19] wherein the data rate is some fixed term if the nearby transmitters do not transmit. In terms of physical implementation, both cases correspond to the usage of only a single modulation-coding scheme at the transmitter. Such a model simplifies mathematical analysis but has limited application in more sophisticated multi-rate systems.

Hence, one of the challenges in the spectrum management in D2D networks is to efficiently optimize transmission probabilities of D2D pairs to maximize network utility in multi-rate D2D random access networks for the cases with the exact network topology information and with only the spatial statistics. This problem is studied in Chapters 3 and 4.
1.1.2 Power Control in D2D Networks

In the underlay mode, D2D pairs reuse spectrum with cellular user equipments (CUEs) in order to improve system spectrum efficiency. However, this introduces more interference to the cellular communication between BSs and CUEs. Hence, one important problem in D2D communication is to coordinate the transmission power among D2D pairs to improve D2D transmission rate while maintaining satisfactory performance for cellular communication.

Power control schemes have been proposed in different D2D communication scenarios. One common assumption in these works is that the instantaneous network state information (NSI) is known by network coordinators. However, there is unavoidable delay in NSI feedback in practical systems. Thus, the delayed NSI received by the coordinators may be inconsistent with, or even independent of, the instantaneous NSI.

Some works have proposed special channel models to deal with the delayed NSI, for instance, using Markov chains. However, there are several disadvantages in methods that are based on specific channel models: 1) These simplified channel models may fail to capture some important characteristics in real channels in wireless systems. 2) These methods require statistics information regarding the channel model, which may not be easily obtainable.

Hence, one challenge in the power management in D2D networks with delayed NSI feedback is to design power control schemes in D2D networks with delayed NSI, which are robust to different types of channels, without the requirement for prior statistics of the channels. We address this challenge in Chapter 5.

1.1.3 Probabilistic Caching in D2D Networks

Cache-enabled D2D networking, where the DTxs store files popularly requested by the DRxs, has the potential to dramatically reduce the traffic load on the wireless backhaul [31] and [32]. In a cache-enabled D2D network, a DRx may request a file in two modes. In the direct communication mode, it fetches the file from a BS, while in the D2D communication mode, it does so from a nearby DTx that has cached that file. Not only does the D2D mode offloads traffic away from the BSs and wireless backhaul, it also improves the file transfer data rate due to the shorter distance between neighboring devices. However, different from BSs, the DTxs often have limited storage capacity for caching. It is important to consider how the DTxs should judiciously choose the files they cache to effectively serve the DRx requests.

Various cache placement strategies for cache-enabled D2D networks have been pro-
posed. Some of these strategies require the knowledge of network topology or channel state information (CSI) [33–39]. For instance, in the often adopted protocol model [34–36], a D2D transmission link between a pair of devices can be activated if and only if the distance between the pair is less than a threshold, and there is no other nearby activated link. The strategies developed under this model require the DTxs to acquire the exact network topology or CSI prior to caching decisions, which often is a challenging task, especially for large-scale networks. Therefore, probabilistic caching strategies based on random network topology have attracted significant attention since they require less CSI [40–50]. In these works, DTxs randomly cache files according to some designed caching probabilities, which are computed based on spatial statistics of the network, such as the densities of DTx and DRx and the file requesting probabilities.

The strategy of simply caching the most popular files is not necessarily optimal in probabilistic caching design, as shown for instance in [40–50], because of the interfering nature of wireless channels, the sharing of limited resources among devices, and peculiarities of specific optimization objectives. In addition, fairness has scarcely been studied in existing works on optimal caching.

Another challenge in D2D networks is spectrum allocation between direct and D2D communication. In the common overlay mode in the licensed band [1], direct communication and D2D communication are assigned orthogonal sub-bands to eliminate their mutual interference. Spectrum allocation has been studied in [51, 52]. However, to our best knowledge, no existing work has addressed the problem of joint spectrum allocation and probabilistic caching design in D2D networks.

Hence, one of the challenges in the storage management in D2D networks is to jointly optimize spectrum allocation and probabilistic caching to maximize the logarithm utility. We consider this challenge in Chapter 6.

1.2 Summary of Contributions

In this thesis, we focus on the resource management in D2D networks as mentioned above. We summarize our contributions as follows.

1.2.1 \(\alpha\)-Fair Utility Maximization in Multi-Rate Random Access Networks

In Chapter 3, we study the problem of optimizing transmission probabilities of D2D pairs in order to maximize the network-wide \(\alpha\)-fair utility. We consider a more general
random access network where D2D pairs can transmit with different rates. We propose a computationally-efficient iterative algorithm termed Pair Separation Random Transmission (PSRT) based on a lower bound for the $\alpha$-fair utility. In PSRT, all D2D pairs' transmission probabilities are updated separately via solving one-dimensional convex optimization problems.

1.2.2 $\alpha$-Fair Utility Maximization in Multi-Tier Multi-Rate Spatial Aloha Networks

In Chapter 4, we further consider the maximization of $\alpha$-fair utility in a generalized spatial Aloha network consisting of multiple tiers of D2D pairs each forming a Poisson bipolar process. Multi-rate communication between the D2D pairs is facilitated by multiple received SIR thresholds. We aim to optimize the transmission probability of each tier. This results in a complex non-convex optimization problem due to intra-tier and cross-tier interference. We propose a solution termed Minorize-Maximization with Tier Separation (MMTS), through designing an iterative sequence of lower-bound problems that can be decomposed into tier-separable one-dimensional convex optimization problems and solved efficiently. Specific solutions are derived for the cases $0 \leq \alpha < 1$, $\alpha = 1$, and $\alpha > 1$.

1.2.3 On-line Power Control in D2D Networks with Delayed NSI

In Chapter 5, we consider power control in a D2D network with delayed NSI. We aim to maximize the long-term averaged sum rate of all D2D pairs without knowing the instantaneous NSI or its statics. We first recast the problem into per-time slot problems via a convexification technique and the OGD method. We then propose a method termed On-line Power Control for D2D with Full NSI Feedback (OPCD-FNF), where an quadratic optimization problem is solved in each time slot. We present a performance bound for OPCD-FNF. Furthermore, we consider the scenario where a limited number of D2D pairs feedback its local NSI, and propose a method termed On-line Power Control for D2D with Partial NSI Feedback (OPCD-PNF), where several D2D pairs are randomly selected to feedback its NSI in each time slot. In addition, we give a performance bound for OPCD-PNF.
1.2.4 Joint Spectrum Allocation and Probabilistic Caching in D2D Networks

In Chapter 6, we consider a cache-enabled D2D network where a DRx can fetch the file from either a BS or a DTx caching the file. Each DTx has storage capacity that allows it to cache at most $K$ files. We focus on probabilistic caching in DTxs, aiming to jointly optimize spectrum allocation between direct and D2D communication and the DTx file caching probabilities, to maximize the DRx coverage-rate logarithm utility. We first show an interesting result that, when given fixed spectrum allocation, the intuitive strategy of caching top $K$ popular files is not always optimal. Yet, when spectrum allocation is jointly optimized with caching, an optimal caching strategy can be established, where the DTxs either cache no file or cache the top $K$ popular files depending on the relative density of BSs, DTxs, and DRxs.

1.3 Notations

$\|\cdot\|$ is $l_2$ norm, and $1(.)$ is indicator function. $\nabla f(x)$ is the gradient of $f(x)$. $\mathbb{P}(A)$ is the probability of event $A$, and $\mathbb{E}(.)$ is the expectation operator. Let $|B|$ be the cardinality of set $B$. 
Chapter 2

Background

In Chapter 1, we briefly introduce D2D networks, the motivation of the studied problems, and the contribution of the thesis. In this chapter, we further discuss $\alpha$-fair utility in network utility maximization (NUM), and the mathematical tools of minorize-maximization and on-line convex optimization, which are the main mathematical tools used in this thesis.

2.1 $\alpha$-Fair Utility Maximization

$\alpha$-fair utility is initially introduced in [53], and is widely adopted in NUM problem. As an example, limited spectrum resources are allocated to $N$ users. Let $\mathbf{x} = [x_1, \cdots, x_N]$ be the vector of the amount of resources allocated to different users, and the rate achieved by user $i$ is denoted by $R_i(\mathbf{x})$. Then the $\alpha$-fair utility of user $i$ is defined as

$$
U_i = \begin{cases} 
(R_i)^{1-\alpha} & \text{if } \alpha \neq 1, \\
\frac{1}{1-\alpha} & \text{if } \alpha = 1,
\end{cases}
$$

where $\alpha \geq 0$.

Thus, the $\alpha$-fair utility based NUM is given by

$$
\max_{\mathbf{x}} \sum_{i=1}^{N} U_i \quad \text{subject to} \quad \mathbf{x} \in \mathcal{X},
$$

where $\mathcal{X}$ is the feasible set of resource allocation vectors. When $0 \leq \alpha < 1$, the $\alpha$-fair utility ($U_i$) is non-negative. When $\alpha > 1$, the $\alpha$-fair utility ($U_i$) is non-positive, and the
Chapter 2. Background

Problem is equivalent to

\[ \min_x \sum_{i=1}^{N} -U_i \]  

s.t. \( x \in \mathcal{X} \).

We can interpret \(-U_i\) as a loss of user \( i \), and the new problem is to minimize the total loss of the system.

The advantage of \( \alpha \)-fair utility is the flexibility in adjusting the network fairness by tuning the value of \( \alpha \). Roughly speaking, when \( \alpha \) becomes larger, the system becomes more fair in the sense that the difference of rates achieved by different users becomes smaller. Furthermore, when \( \alpha \) are chosen as some specific values, NUM degrades to some well-known utility maximization problems.

When \( \alpha = 0 \), NUM degrades to the following sum-rate maximization problem:

\[ \max_x \sum_{i=1}^{N} R_i \]  

s.t. \( x \in \mathcal{X} \).

In the sum-rate maximization problem, fairness is ignored among users. In general, users with good transmission conditions (e.g., high rate per resource) are allocated more resources, while users with bad transmission conditions (e.g., low rate per resource) are allocated less resources.

When \( \alpha = 1 \), NUM degrades to the following proportional-fair utility maximization problem:

\[ \max_x \sum_{i=1}^{N} \log R_i \]  

s.t. \( x \in \mathcal{X} \).

Proportional-fair utility is adopted in CDMA and LTE systems to ensure some trade-off between system throughput and user fairness.

When \( \alpha = \infty \), the NUM problem degraded to the following max-min problem

\[ \max_x \min R_i \]  

s.t. \( x \in \mathcal{X} \).
In practical systems, the value of $\alpha$ may be chosen by network operators to reach a certain trade-off between system throughput and user fairness.

### 2.2 Minorize-Maximization

Minorize-maximization (MM) method [54] is an important tool to solve non-linear optimization problems in many areas with the advantage of the avoidance of matrix inversion, separation in optimization variables, and etc. It should be noted that MM itself is not an algorithm but a framework. The real challenge in MM is to find an appropriate lower bound for the objective function.

Consider the following general optimization problem:

$$\max_x U(x)$$

s.t. $x \in \mathcal{X}$,

where $\mathcal{X}$ is a convex set. Note that $U(x)$ is not necessarily concave in $x$, and thus the problem is not necessarily convex.

The general MM approach to solve this problem is as follows:

**Algorithm 1** MM framework

1: Pick a point $x^0$ from the feasible set $\mathcal{X}$;
2: $t = 0$;
3: repeat
4: $t = t + 1$,
5: Let $x^t = \arg \max_x U_{LB}(x, x^{t-1})$,
6: until convergence

In Algorithm 1, if $U_{LB}(x, x^{t-1})$ satisfies the following two properties:

- $U(x) \geq U_{LB}(x, x^{t-1}), \ \forall x, x^{t-1} \in \mathcal{X}$,
- $U(x^{t-1}) = U_{LB}(x^{t-1}, x^{t-1}), \ \forall x^{t-1} \in \mathcal{X}$,

then the objective is guaranteed to increase in each iteration until some convergence condition is met. Hence, the challenge in the MM framework is to find $U_{LB}(x, x^{t-1})$ satisfying the aforementioned properties.

MM method has been applied to solve many problems in wireless communication networks. For instance, the authors of [55] propose a power control method using the
Taylor expansion as lower bound, and the authors of [56] propose a new power control scheme based on a new lower bound for the objective.

2.3 On-line Convex Optimization

The general framework of On-line Convex Optimization (OCO) is as follows.

Online Convex Optimization Framework

1: Input: Convex feasible set $\mathcal{X}$, $T$
2: for $t = 1$ to $T$
3: Decision maker chooses a solution, $x^*_t \in \mathcal{X}$, for iteration $t$,
4: After choosing $x^*_t$, the decision maker observes loss function $f_t(x_t)$ for iteration $t$, and suffers loss $f_t(x^*_t)$.
5: end for

In the OCO framework, we consider the following convex optimization problem with varying objectives:

$$\mathcal{P}_{\text{OCO}} : \min_{\{x_t\}} \sum_{t=1}^{T} f_t(x_t)$$
\[ s.t. \ x_t \in \mathcal{X}, \ \forall t, \]  

where $\mathcal{X}$ is a convex set, and $f_t(x)$ is convex in $x$ for all $t$. For any $x, y \in \mathcal{X}$ and $t$, we assume that $\|x - y\|$, and $\|\nabla f_t(x)\|$ are bounded.

This problem becomes challenging in the online environment, i.e., in iteration $t$, the decision maker only knows about historical objectives, $\{f_r(x_r)\}_{r=1}^{t-1}$. In other words, there is a delay in revealing the instantaneous objective function, $f_t(x_t)$. In iteration $t$, the decision maker needs to find a $x^*_t$ in $\mathcal{X}$, without knowing the objective function $f_t(x_t)$.

On-line Gradient descent (OGD) [57] has been proposed to solve this problem. The proposed OGD method is that in time slot $t$, $x^*_t$ is obtained by solving the following convex optimization problem:

$$\min_{x} \|x - (x_{t-1}^* - \eta_{t-1} \nabla f_{t-1}(x_{t-1}^*))\|_2$$
\[ s.t. \ x \in \mathcal{K}, \]  

(2.11)
In OGD method, if we set $\eta_t = \frac{\delta}{\sqrt{t}}$, we have the following performance guarantee given by

$$\sum_{t=1}^{T} f_t(x_t^*) - \min_{x \in \mathcal{X}} \sum_{t=1}^{T} f_t(x) \leq O(\sqrt{T}),$$

where the left-hand side is the difference between the objective achieved by OGD and any strategy that is fixed in each iteration. OGD guarantees that the long-term averaged performance of OGD is as good as any fixed strategy when the number of iterations is sufficiently large.

**Remark.** Due to the lack of the knowledge of the instantaneous objective function, it is possible that the solution produced by OGD is the worst solution, i.e., the maximizer of the instantaneous objective in each iteration, i.e.,

$$x_t^* = \arg \max_{x_t \in \mathcal{X}} f_t(x_t),$$

for all $t$. Hence, in the current OCO framework, it is not meaningful to consider bounding

$$\sum_{t=1}^{T} f_t(x_t^*) - \sum_{t=1}^{T} \min_{x_t \in \mathcal{X}} f_t(x_t).$$
Chapter 3

α-Fair Utility Maximization in Multi-Rate D2D Random Access Networks

In Chapter 3, we aim at optimizing the transmit probabilities in a D2D random access network operating in the overlay mode to maximize the network-wide α-fair utility, which is discussed in Chapter 2. We consider a transmission model with multiple SINR thresholds that correspond to multiple transmission rates. Our main contributions are summarized as follows:

- We consider a general random access network and formulate the problem of network-wide α-fairness maximization, based on a closed-form derivation of the average throughput for the case of multiple SINR thresholds. We allow arbitrary non-negative α values to facilitate different levels of fairness among the D2D pairs. The formulated problem is non-convex in general.

- We propose a computationally efficient method, termed Pair Separation Random Transmission (PSRT), to address the formulated non-convex problem. PSRT is designed in the general framework of Minorize-Maximization (MM), where we iteratively solve tight lower-bound problems developed separately for the cases $0 \leq \alpha < 1$, $\alpha = 1$, and $\alpha > 1$. We show that PSRT is guaranteed to converge to the objective value of a KKT point of the original optimization problem.

- We show that, in each iteration of PSRT, for all cases of α, the lower-bound problems can be decomposed into separable one-dimensional sub-problems, each corresponding to optimizing the transmission probability of one D2D pair. Unlike the CD
approach, these sub-problems can be solved simultaneously. Thus, this decompo-
station allows substantial improvement computational efficiency without increasing
the number of iterations for convergence. Furthermore, it also enables distributed
implementation of PSRT.

- We compare PSRT to alternatives under different network settings, and fairness
objectives. Our simulation results demonstrate that PSRT is superior in both
performance and convergence rate. We observe that a moderate number of SINR
thresholds is sufficient for optimal system operation. Furthermore, in distributed
implementation, utility close to the case of global information exchange can be
achieved by limited information exchange among subsets of D2D pairs.

3.1 Related Work

There are many existing works on utility maximization in random access networks. In [6–
13], optimal transmit probabilities are studied, where the locations of the D2D pairs are
modeled by a spatial random process, commonly the Poisson point process. In contrast,
works in [4,5,14–24] consider the locations of the D2D pairs, or the channel gain among
the D2D pairs, as deterministic without assuming any spatial model. In this work, we
focus on the latter scenario.

The works in [14–22] adopt the protocol model, while [4,5,23,24] adopt the interference
model with a single SINR threshold. Both models are single-rate models, i.e., a D2D pair
transmits with a fixed given rate when the transmission is successful. In our work, we
consider multiple SINR thresholds, so that a D2D pair transmits at multiple rates based
on the SINR. As shown in Section 3.4, considering multiple SINR thresholds can lead to
substantially higher throughput.

Furthermore, among the works based on the single-rate interference model, general
$\alpha$-fair utility maximization is considered in [4] and [5]. The objective of [23] comprises
both $\alpha$-fair utility and the cost of transmission power, while [24] considers only sum rate
maximization, which corresponds to the case $\alpha = 0$. In addition, the methods proposed
in [4,5,23] are CD based, taking advantage of the fact that the optimization problems
are convex in each optimization variable separately. However, CD-based methods may
converge slowly since they allow only one D2D pair to update the transmission probability
in each iteration. Furthermore, a linear programming method is proposed in [24], but
it is only applicable to the case of sum-rate objective and the single-rate transmission
model.
Chapter 3. α-Fair Utility Maximization in Random Access Networks

![System Model Diagram](image)

Figure 3.1: System model

Table 3.1: Table of Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of D2D pairs</td>
</tr>
<tr>
<td>$P_i$</td>
<td>transmission power of Tx $i$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>transmission probability of Tx $i$</td>
</tr>
<tr>
<td>$p_{i,\text{min}}$</td>
<td>Minimum transmission probability of Tx $i$</td>
</tr>
<tr>
<td>$p_{i,\text{max}}$</td>
<td>Maximum transmission probability of Tx $i$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Pathloss exponent</td>
</tr>
<tr>
<td>$d_{ij}$</td>
<td>Distance between Rx $i$ and Tx $j$</td>
</tr>
<tr>
<td>$h_{ij}$</td>
<td>Small-scale power fading term between Rx $i$ and Tx $j$</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of SINR thresholds to modulate the received signal</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Fairness index in the utility function</td>
</tr>
</tbody>
</table>

Our problem of optimizing the transmit probabilities for α-fair utility maximization under a multi-rate model has not been considered in prior work. Even though we will show later in Section 3.4 that we can develop a CD-based solution for the multi-rate case, our simulation results demonstrate that the proposed PSRT algorithm is superior in both utility performance and convergence.

3.2 System Model and Problem Formulation

We first describe a random access network with multiple SINR thresholds, and further derive the expected throughput of D2D pairs in closed form. We then formulate the problem of maximizing network-wide α-fair utility based on the derived average throughput. Table 3.1 summarizes the important notations use throughout this chapter.
3.2.1 Random Access Network

As illustrated in Fig. 3.1, we consider a general random access network comprising $N$ D2D pairs denoted by $\mathcal{N} = \{1, \ldots, N\}$. Each D2D pair $i$ consists one transmitter (Tx $i$) and one receiver (Rx $i$). The distance between Rx $i$ and Tx $j$ is $d_{ij}$. In particular, we call $d_{ii}$ the D2D distance of D2D pair $i$. All D2D pairs under consideration reuse a given channel in a random access manner. For convenience, we normalize the bandwidth of the channel to one.

Time is slotted. In each time slot, Tx $i$ transmits to Rx $i$ with probability $p_i$. When Tx $i$ transmit, it uses power $P_i$. Then, the received power of Rx $i$ from Tx $j$ is given by

$$P_{ij} = e_j P_j h_{ij} d_{ij}^{-\gamma},$$

where $e_j$ is the event that Tx $j$ transmits, $h_{ij}$ represents channel power gain with small-scale fading, which is assumed to be exponential with mean $\frac{1}{\mu}$, and $\gamma$ is the path loss exponent. We assume that small-scale fading is i.i.d. over different time slots and among different links.

When Tx $i$ transmits (i.e., $e_i = 1$), the SINR at Rx $i$ is given by

$$\text{SINR}_i = \frac{P_i h_{ii}}{\sum_{j=1, j\neq i}^{N} e_j P_j h_{ij} d_{ij}^{-\gamma} + \sigma^2},$$

where $\sigma^2$ is the noise power.

3.2.2 SINR Thresholds and Multi-Rate Reception

We assume that an Rx adopts $L$ different SINR thresholds to demodulate the received signal, corresponding to $L$ modulation and coding schemes and hence transmission rates. The sets of these SINR thresholds and corresponding transmission rates are $\mathcal{T} = \{T_1, \cdots, T_L\}$ and $\mathcal{R} = \{r_1, \cdots, r_L\}$. Without loss of generality, we assume that $T_i < T_j$ and $r_i < r_j$ if $i < j$. For mathematical convenience, we add another auxiliary threshold and an auxiliary rate, $T_0 = 0$ and $r_0 = 0$, to $\mathcal{T}$ and $\mathcal{R}$ respectively. Specifically, $r_i$ is the transmission rate when a D2D pair’s SINR is between $T_i$ and $T_{i+1}$, and $r_L$ is the transmission rate when the D2D pair’s SINR is no less than $T_L$. Therefore, given the SINR of D2D pair $i$, denoted by $\text{SINR}_i$, its throughput, $R_i$, is given by

$$R_i = \sum_{l=0}^{L} 1(T_l \leq \text{SINR}_i < T_{l+1}) r_l + 1(\text{SINR}_i \geq T_L) r_L,$$
where \( \mathbf{1}(\cdot) \) is the indicator function.

Furthermore, the average throughput of D2D pair \( i \) is given by

\[
\bar{R}_i = \mathbb{E}_{e_i, \{e_j\}_{j \neq i}, \{h_{ij}\}}[R_i]
= \mathbb{E}_{\{e_j\}_{j \neq i}}[R_i | e_i = 1] \mathbb{P}(e_i = 1) + 0 \times \mathbb{P}(e_i = 0)
= p_i \left\{ \sum_{l=1}^{L-1} \mathbb{P}(T_l \leq \text{SINR}_i < T_{l+1} | e_i = 1) r_l + \mathbb{P}(	ext{SINR}_i \geq T_L | e_i = 1) r_L \right\}
= p_i \sum_{l=1}^{L} (r_l - r_{l-1}) \mathbb{P}(	ext{SINR}_i \geq T_l | e_i = 1)
= p_i \sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i} \left( 1 - p_j \frac{1}{1 + b_{ijl}} \right),
\]

where \( a_{il} = (r_l - r_{l-1}) \exp \left( -\mu \frac{T_l d_{il}^\gamma \sigma^2}{P_i} \right), \ b_{ijl} = \frac{P_j d_{ijl}^\gamma}{r_l d_{il}^\gamma}, \) and the derivation of \( \mathbb{P}(	ext{SINR}_i \geq T_l | e_i = 1) \) is given in Appendix 3.6.1.

### 3.2.3 \( \alpha \)-Fair Utility Maximization

Using the above derived average throughput of D2D pairs, the \( \alpha \)-fair utility of D2D pair \( i \), \( U_i(p) \), is given by

\[
U_i(p) = \begin{cases} 
(\bar{R}_i)^{1-\alpha} / (1-\alpha) & \text{if } \alpha \neq 1, \\
\log(\bar{R}_i) & \text{if } \alpha = 1,
\end{cases}
\]

where \( p = [p_1, p_2, \ldots, p_N] \). And the objective of the network-wide \( \alpha \)-fair utility is given by

\[
U(p) = \sum_{i=1}^{N} U_i(p).
\]

In this work, we aim at optimizing the transmit probabilities, \( p \), in order to maximize the network-wide \( \alpha \)-fair utility \( U(p) \). Therefore, the optimization problem is formulated as

\[
P : \quad \max_p U(p) \quad \text{(3.1)}
\]

s.t. \( p_{i,\min} \leq p_i \leq p_{i,\max}, 1 \leq i \leq N, \) \quad \text{(3.2)}
where $p_{i,\text{max}}$ and $p_{i,\text{min}}$ are the maximum and minimum transmission probability of Tx $i$. We denote by $\Psi$ the set of feasible $p$ in optimization problem $P$.

In problem $P$, the fairness level in the random access network can be adjusted by tuning the value of $\alpha$. For instance, the maximization of sum throughput can be formulated by setting $\alpha = 0$. When $\alpha = 1$, problem $P$ is equivalent to maximizing the celebrated proportional-fair utility. Furthermore, when $\alpha \to \infty$, maximizing $U(p)$ results in the max-min fairness among D2D pairs.

In general, the objective $U(p)$ is not concave in $p$, so problem $P$ is non-convex and challenging to solve. Next, we present a computationally efficient PSRT algorithm for this problem.

### 3.3 Pair Separation Random Transmission

Central to the design of PSRT is our derivation of three tight lower bounds on $U_i(p)$ for the cases $0 \leq \alpha < 1$, $\alpha = 1$, and $\alpha > 1$, which are then utilized within the MM optimization framework. One important feature of these lower bounds is that they are separable in the optimization variables, $p$. This contributes to the decomposition of the original optimization problem into low-complexity subproblems.

#### 3.3.1 Lower Bounds for $\alpha$-Fair Utility

When $0 \leq \alpha < 1$, the utility function, $U_i(p)$, has a special partially-convex property. We derive a tight lower bound based on this property in the following lemma.

**Lemma 1.** When $0 \leq \alpha < 1$, for all $p = [p_i]_{N \times 1}$, $p^{t-1} = [p_i^{t-1}]_{N \times 1} \in \Psi$, we have

$$U_i(p) \geq u_i^{t-1} \log(p_i) + \sum_{j=1,j\neq i}^{N} \sum_{l=1}^{L} v_{il}^{t-1} \log(1 - \frac{p_j}{1 + b_{ijl}}) + u_i^{t-1},$$

where

$$u_i^{t-1} = \left( \sum_{l=1}^{L} a_{il} p_i^{t-1} \prod_{j=1,j\neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{ijl}} \right) \right)^{1-\alpha}, \quad (3.3)$$

$$v_{il}^{t-1} = a_{il} \left( \sum_{l'=1}^{L} a_{il'} \prod_{j=1,j\neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{ijl'}} \right) \right)^{-\alpha} (p_i^{t-1})^{1-\alpha} \prod_{j=1,j\neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{ijl}} \right), \quad (3.4)$$
\[
    w^{t-1}_i = \frac{\left(\sum_{l=1}^{L} a_{il} P^{t-1}_l \prod_{j=1, j \neq i}^{N} \left(1 - \frac{p^{t-1}_{jl}}{1+b_{ijl}}\right)\right)^{1-\alpha}}{1-\alpha} - u^{t-1}_i \log(p^{t-1}_i)
    - \sum_{l=1}^{L} \sum_{j=1, j \neq i}^{N} v^{t-1}_{il} \log(1 - \frac{p^{t-1}_{jl}}{1+b_{ijl}}),
\]

and the equality holds if \( p = p^{t-1} \).

Proof. First, we show the following function \( f : \mathbb{R}^N \to \mathbb{R}, \)

\[
f(x) = \left(\frac{\sum_{i=1}^{N} s_i \exp(x_i)}{\beta}\right)^{\beta}\]

is convex in \( x = [x_1, \cdots, x_N] \), for \( s_i \geq 0 \) and \( \beta > 0 \).

Define function \( h : \mathbb{R} \to \mathbb{R} \) as

\[
h(y) = \frac{1}{\beta} \exp(\beta y),
\]

and function \( g : \mathbb{R}^N \to \mathbb{R} \) as

\[
g(x) = \log \left(\sum_{i=1}^{N} s_i \exp(x_i)\right).
\]

Then we can rewrite \( f(x) \) as

\[
f(x) = \frac{1}{\beta} \exp \left(\beta \log \left(\sum_{i=1}^{N} s_i \exp(x_i)\right)\right) = h(g(x)).
\]

Furthermore, \( h(y) \) is convex and nondecreasing w.r.t. \( y \), and \( g(x) \) is convex in \( x \). Hence, according to the convexity composition rule, function \( f(x) \) is convex in \( x \).

Based on the convexity of \( f(x) \), we have, \( \forall x^{t-1} \in \text{dom} f, \)

\[
f(x) \geq f(x^{t-1}) + \nabla f(x^{t-1})(x - x^{t-1}). \tag{3.5}
\]

Substituting \( f(x) = \frac{1}{\beta} \left(\sum_{i=1}^{N} s_i \exp(x_i)\right)^{\beta} \) into (3.5), we have

\[
\frac{1}{\beta} \left(\sum_{i=1}^{N} s_i \exp(x_i)\right)^{\beta} \geq \frac{1}{\beta} \left(\sum_{i=1}^{N} s_i \exp(x^{t-1}_i)\right)^{\beta}. \tag{3.6}
\]
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Then when \( \alpha \in [0, 1) \), we have

\[
\frac{\left( \sum_{l=1}^{L} a_{il}p_i \prod_{j=1,j\neq i}^{N} \left( 1 - \frac{p_j}{1+b_{ijl}} \right) \right)^{1-\alpha}}{1-\alpha} \geq u_{t-1}^t \log(p_i) + \sum_{j=1,j\neq i}^{N} \sum_{l=1}^{L} v_{il}^t \log(1 - \frac{p_j}{1 + b_{ijl}}) + w_{i}^t - \sum_{l=1}^{L} \sum_{j=1,j\neq i}^{N} g_{il}^t \log\left( 1 - \frac{p_j}{1 + b_{ijl}} \right) \quad (3.8)
\]

and the equality holds if \( \mathbf{p} = \mathbf{p}^{t-1} \).

Proof. The proof is similar to the proof of Lemma 1, and is omitted for brevity. \(\square\)
When $\alpha > 1$, We take advantage of a special partially-concave structure of $U_i(p)$, and develop the following lower bound.

**Lemma 3.** When $\alpha > 1$, for all $p = [p_i]_{N \times 1}$, $p_{t-1} = [p_{t-1}^t]_{N \times 1} \in \mathcal{P}$, we have

$$U_i(p) \geq \sum_{l=1}^{L} \frac{\mu_{il}^{t-1}}{(1 - \alpha)^{\zeta_{il}^{t-1}}} p_i \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} \frac{\mu_{jl}^{t-1}}{(1 - \alpha)^{\eta_{jl}^{t-1}}} \left(1 - \frac{p_j}{1 + b_{ijl}}\right)^{(1 - \alpha)^{\rho_{jl}^{t-1}}} p_{t-1}$$

where

$$\rho_{il}^{t-1} = \prod_{j' = 1, j' \neq i}^{N} \left(1 - \frac{p_{j'}^{t-1}}{1 + b_{ilj'}}\right),$$

$$\zeta_{il}^{t-1} = \frac{\log p_i^{t-1} + \log \rho_{il}^{t-1}}{\log p_i^{t-1}},$$

$$\eta_{jl}^{t-1} = \frac{\log p_j^{t-1} + \log \rho_{jl}^{t-1}}{\log \left(1 - \frac{p_j}{1 + b_{ijl}}\right)}$$

$$\mu_{il}^{t-1} = \frac{a_{il}^{(t-1)^{\alpha}}}{\left(\sum_{l' = 1}^{L} a_{il}^{t-1}\right)^{\alpha}}.$$

**Proof.** First, we note that function $f(x) = x^\beta$ is convex in $x$ when $\beta < 0$ and $x > 0$.

Let

$$\xi_{il}^{t-1} = \sum_{l' = 1}^{L} a_{il}^{t-1} p_{il} \prod_{j = 1, j \neq i}^{N} \left(1 - \frac{p_j^{t-1}}{1 + b_{ilj'}}\right),$$

$$\theta_{il}^{t-1} = p_i \prod_{j = 1, j \neq i}^{N} \left(1 - \frac{p_j^{t-1}}{1 + b_{ijl}}\right).$$

Then we have

$$a_{il} p_i \prod_{j = 1, j \neq i}^{N} \left(1 - \frac{p_j}{1 + b_{ijl}}\right) = a_{il} \theta_{il}^{t-1} \xi_{il}^{t-1} \prod_{j = 1, j \neq i}^{N} \left(1 - \frac{p_j}{1 + b_{ijl}}\right).$$

Note that $\sum_{l=1}^{L} \frac{a_{il} \theta_{il}^{t-1}}{\xi_{il}^{t-1}} = 1$.

Furthermore, we have

$$\left(\sum_{l=1}^{L} a_{il} p_i \prod_{j = 1, j \neq i}^{N} \left(1 - \frac{p_j}{1 + b_{ijl}}\right)\right)^{1 - \alpha}$$
Further decomposed into separable one-dimensional convex optimization problems. Based on the lower bounds above, we formulate lower-bound problems in order to facilitate the MM approach. Furthermore, we show that the lower-bound problems can be completed the proof.

Next, we propose the following inequality, \( \forall z_i > 0, z_i^{t-1} > 0: \)

\[
\prod_{i=1}^{N} z_i = \exp \left( \sum_{i=1}^{N} \log(z_i) \right) = \exp \left( \sum_{i=1}^{N} \log(z_i^{t-1}) \sum_{j'=1}^{N} \log(z_{j'}^{t-1}) \log(z_i) \right) = \exp \left( \frac{\sum_{i=1}^{N} \log(z_i^{t-1}) \sum_{j'=1}^{N} \log(z_{j'}^{t-1}) \log(z_i)}{\sum_{i'=1}^{N} \log(z_{i'}^{t-1})} \right) \tag{3.14}
\]

where the equality holds if \( z_i = z_i^{t-1} \). Based on (3.14), we have

\[
p_i^{1-\alpha} \prod_{j=1, j \neq i}^{N} \left( 1 - \frac{p_j}{1 + b_{ij}} \right)^{1-\alpha} \leq \frac{1}{\xi_{it}^{t-1}} p_i^{(1-\alpha)\xi_{it}^{t-1}} + \sum_{j=1, j \neq i}^{N} \frac{1}{\eta_{ijl}^{t-1}} \left( 1 - \frac{p_j}{1 + b_{ij}} \right)^{(1-\alpha)\eta_{ijl}^{t-1}} \tag{3.15}
\]

where \( \xi_{it}^{t-1} = \frac{\log(p_i^{t-1}) + \sum_{j'=1, j' \neq i}^{N} \log(1 - \frac{p_{j'}^{t-1}}{1 + b_{ij'}})}{\log(p_i^{t-1})} \) and \( \eta_{ijl}^{t-1} = \frac{\log(p_i^{t-1}) + \sum_{j'=1, j' \neq i}^{N} \log(1 - \frac{p_{j'}^{t-1}}{1 + b_{ij'}})}{\log(1 - \frac{p_i^{t-1}}{1 + b_{ijl}})} \).

Recall that \( 1 - \alpha < 0 \). Dividing the right-hand and left-hand sides of (3.15) by \( (1 - \alpha) \) completes the proof.

### 3.3.2 Separable Lower-Bound Problems in PSRT

Based on the lower bounds above, we formulate lower-bound problems in order to facilitate the MM approach. Furthermore, we show that the lower-bound problems can be further decomposed into separable one-dimensional convex optimization problems.
First, for all \( \mathbf{p} = [p_i]_{N \times 1}, \mathbf{p}^{t-1} = [p_i^{t-1}]_{N \times 1} \in \mathcal{P} \), we define

\[
U_{\text{LB},i}(p_i, \mathbf{p}^{t-1}) = \begin{cases} 
\tilde{U}_i(p_i, \mathbf{p}^{t-1}) + w_i^{t-1}, & \text{if } 0 \leq \alpha < 1, \\
\hat{U}_i(p_i, \mathbf{p}^{t-1}) + h_i^{t-1}, & \text{if } \alpha = 1, \\
\bar{U}_i(p_i, \mathbf{p}^{t-1}), & \text{if } \alpha > 1,
\end{cases}
\]

where

\[
\tilde{U}_i(p_i, \mathbf{p}^{t-1}) = u_i^{t-1} \log(p_i) + \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} v_{jl}^{t-1} \log\left(1 - \frac{p_i}{1 + b_{jil}}\right),
\]

\[
\hat{U}_i(p_i, \mathbf{p}^{t-1}) = \log(p_i) + \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} g_{jl}^{t-1} \log\left(1 - \frac{p_i}{1 + b_{jil}}\right),
\]

\[
\bar{U}_i(p_i, \mathbf{p}^{t-1}) = \sum_{l=1}^{L} \frac{\mu_{il}^{t-1}}{(1 - \alpha) \zeta_{il}^{t-1}} P_i^{(1-\alpha) \zeta_{il}^{t-1}} + \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} \frac{\mu_{jl}^{t-1}}{(1 - \alpha) \eta_{jil}^{t-1}} \left(1 - \frac{p_i}{1 + b_{jil}}\right)^{(1-\alpha) \eta_{jil}^{t-1}}.
\]

Furthermore, let us define

\[
U_{\text{LB}}(\mathbf{p}, \mathbf{p}^{t-1}) = \sum_{i=1}^{N} U_{\text{LB},i}(p_i, \mathbf{p}^{t-1}).
\]

(3.17)

Based on the inequalities in Lemma 1, Lemma 2, and Lemma 3, it is easy to verify that the objective of the optimization problem \( \mathcal{P} \) satisfies

\[
U(\mathbf{p}) \geq U_{\text{LB}}(\mathbf{p}, \mathbf{p}^{t-1}),
\]

for all \( \alpha \geq 0 \), and the equality holds when \( \mathbf{p} = \mathbf{p}^{t-1} \).

Hence, for any given \( \mathbf{p}^{t-1} \in \mathcal{P} \), we formulate a lower-bound problem as

\[
\mathcal{P}_{\text{LB}}^t : \max_{\mathbf{p}} U_{\text{LB}}(\mathbf{p}, \mathbf{p}^{t-1})
\]

\[
\text{s.t. } p_{i,\min} \leq p_i \leq p_{i,\max}, \ 1 \leq i \leq N.
\]

**Remark:** As shown in (3.17), the objective in the lower-bound problem, \( U_{\text{LB}}(\mathbf{p}, \mathbf{p}^{t-1}) \), can be decomposed into \( N \) separate terms, \( U_{\text{LB},i}(p_i, \mathbf{p}^{t-1}) \). For any given \( \mathbf{p}^t \), \( U_{\text{LB},i}(p_i, \mathbf{p}^{t-1}) \) is a function of a single variable \( p_i \), which is the transmission probability of D2D pair \( i \). This separation structure allows the decomposition of the lower-bound problem into \( N \)
separable one-dimensional optimization problems, given by, for \(1 \leq i \leq N\),

\[
\mathcal{P}_{\text{LB},i}^t : \max_{p_i} U_{\text{LB},i}(p_i; \mathbf{p}^{t-1})
\]

\[
s.t. ~ p_{i,\min} \leq p_i \leq p_{i,\max}.
\]

We observe that problem \(\mathcal{P}_{\text{LB},i}^t\) is convex in the following lemma.

**Lemma 4.** Problem \(\mathcal{P}_{\text{LB},i}^t\) is convex in \(p_i\), for all \(i\) and \(\alpha\).

**Proof.** The constraints are linear, so we only need to verify the concavity of the objective \(U_{\text{LB},i}(p_i; \mathbf{p}^{t-1})\) in \(p_i\).

For \(0 \leq \alpha < 1\), \(\log(p_i)\) is concave in \(p_i\) when \(p_i > 0\). Since composition with affine mapping preserves concavity, \(\log(1 - \frac{p_i}{1+b_{ijl}})\) is also concave. Also, \(u_i > 0\) and \(v_{jl} > 0\). Therefore, \(\tilde{U}_i(p_i, \mathbf{p}^{t-1})\) is concave in \(p_i\).

Similarly, for \(\alpha = 1\), \(\hat{U}_i(p_i, \mathbf{p}^{t-1})\) is concave in \(p_i\).

For \(\alpha > 1\), we have \(\mu_{ijl}^{t-1} < 0\), \(\zeta_{ijl}^{t-1} > 0\), and \(\eta_{ijl} > 0\). Function \(f(x) = x^\beta\) is convex in \(x\) when \(x > 0\) and \(\beta < 0\). Since composition with affine mapping preserves convexity, \(p_i^{(1-\alpha)\zeta_{ijl}^{t-1}}\) and \((1 - \frac{p_i}{1+b_{ijl}})^{(1-\alpha)\eta_{ijl}^{t-1}}\) are both convex in \(p_i\). Since \(\lambda_{il}^{t-1} < 0\) and \(\mu_{ijl}^{t-1} < 0\), \(\bar{U}_i(p_i, \mathbf{p}^{t-1})\) is concave in \(p_i\).

Thus, problem \(\mathcal{P}_{\text{LB},i}^t\) is a one-dimensional convex optimization problem, and it can be easily solved by many low-complexity methods, e.g., bi-section search.

### 3.3.3 PSRT Algorithm and Convergence

We now present the PSRT algorithm, which is based on iteratively solving the lower-bound problem \(\mathcal{P}_{\text{LB},i}^t\) until convergence. Specifically, we first choose the form of the objective in the lower-bound problem by (3.16) according to the value of \(\alpha\). In iteration \(t\), we compute the parameters in the lower-bound problem, \(\{u_i^{t-1}\}\) and \(\{v_{il}^{t-1}\}\), by (3.3) and (3.4) for \(0 \leq \alpha < 1\), \(\{g_{il}^{t-1}\}\) by (3.8) for \(\alpha = 1\), or \(\{\zeta_{il}^{t-1}\}\), \(\{\eta_{ijl}^{t-1}\}\), and \(\{\mu_{ijl}^{t-1}\}\) by (3.11), (3.12), and (3.13) for \(\alpha > 1\). Then we formulate and solve \(\mathcal{P}_{\text{LB},i}^t\) for \(1 \leq i \leq N\) via bi-section search. This procedure is repeated until the algorithm converges. In Algorithm 2, we present the pseudo code of PSRT.

In the following theorem, we summarize the convergence and performance of PSRT.

**Theorem 5.** The PSRT algorithm converges to the objective value of a KKT point of optimization problem \(\mathcal{P}\).

**Proof.** See Appendix 3.6.2.
Algorithm 2 \textit{PSRT} Algorithm for Solving $\mathcal{P}$

\textbf{Input:} $\{a_{il}\}, \{b_{jil}\}, \{p_{i,\text{min}}\}, \{p_{i,\text{max}}\}, \alpha, N, L$

\textbf{Output:} $\mathbf{p}^*$

1: Set $t = 0$, and randomly pick initial $\mathbf{p}^0$ in the feasible set;

2: repeat

3:   $t = t + 1$;

4:   Update $\{u_{it}^{t-1}\}, \{v_{it}^{t-1}\}$ by (3.3) and (3.4) if $0 \leq \alpha < 1$; update $\{g_{il}^{t-1}\}$ by (3.8) if $\alpha = 1$; or update $\{\xi_{il}^{t-1}\}, \{\eta_{jil}^{t-1}\}, \{\mu_{il}^{t-1}\}$ by (3.11), (3.12), and (3.13) if $\alpha > 1$.

5:   for $i \in \{1, \ldots, N\}$ do

6:     Set $p_{\text{lower}} = p_{i,\text{min}}, p_{\text{upper}} = p_{i,\text{max}}$.

7:     repeat

8:       Compute $p_{\text{mid}} = \frac{p_{\text{lower}} + p_{\text{upper}}}{2}$.

9:       If $0 \leq \alpha < 1$, compute $G = \frac{u_{it}^{t-1}}{p_{\text{mid}}} - \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} v_{jil}^{t-1} \frac{1}{1 + b_{jil} - p_{\text{mid}}}$;

10:      if $\alpha = 1$, compute $G = \frac{1}{p_{\text{mid}}} - \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} g_{jil}^{t-1} \frac{1}{1 + b_{jil} - p_{\text{mid}}}$;

11:      or if $\alpha > 1$, compute $G = \sum_{l=1}^{L} \mu_{il}^{t-1} P_{\text{mid}} (1 - \alpha) \eta_{jil}^{t-1} - \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} \mu_{il}^{t-1} \left(1 - \frac{p_{\text{mid}}}{1 + b_{jil}}\right)^{(1-\alpha)\eta_{jil}^{t-1}}$.

12:      if $G > 0$ then

13:         Set $p_{\text{lower}} = p_{\text{mid}}$.

14:      else

15:         Set $p_{\text{upper}} = p_{\text{mid}}$.

16:      end if

17:   until convergence

18:   Set $p_{t}^{i} = p_{\text{mid}}$.

19: end for

20: until convergence

21: $\mathbf{p}^* = \mathbf{p}^t$
3.3.4 Implementation of PSRT

Note that PSRT may be implemented in a centralized manner where some central node collects the required information and determines the transmit probabilities of all D2D pairs based on Algorithm 2. Alternatively, because of the separable structure as explained in Section 3.3.2, it may be implemented in the following distribution manner where each D2D pair determines its own transmission probability.

Initially, each D2D pair collects the channel information around itself. Specifically, D2D pair $i$ collects the values of $\{d_{ij}\}_{j=1,j\neq i}^{N}$, $\{d_{ji}\}_{j=1,j\neq i}^{N}$, and $d_{ii}$. Then each D2D pair $i$ sends a message containing $d_{ii}$ and $P_i$ to the other D2D pairs.

In the iterative stage, each D2D pair updates its transmission probability based on the received messages from other D2D pairs until convergence. Specifically, in iteration $t$, D2D pair $i$ computes $\{v_{il}^{t-1}\}_{l=1}^{L}$ by (3.4) if $0 \leq \alpha < 1$, $\{g_{il}^{t-1}\}_{l=1}^{L}$ by (3.8) if $\alpha = 1$, or $\{\rho_{il}^{t-1}\}_{l=1}^{L}$ by (3.10) if $\alpha > 1$, and sends these values to the other D2D pairs. Then D2D pair $i$ formulates and solves the one-dimensional optimization problems $P_{LB,i}^{t}$ with its received and stored information, following lines 5-16 in Algorithm 2. Finally, D2D pair $i$ sends the updated transmission probability to the other D2D pairs.

With the above information exchange between all D2D pairs, the distributed implementation gives the same solution as the centralized implementation. In Section 3.4, to reduce the information exchange overhead, we will further consider the scenario where each D2D pair only acquires the channel information and receive messages from a limited number of local D2D pairs. Simulation results suggest that exchanging information with a small percentage of the D2D pairs suffices to achieve $\alpha$-fair utility close to that of the centralized implementation.
Figure 3.3: Utility versus number of SINR thresholds

Table 3.2: Number of iterations versus number of D2D pairs when $\alpha = 0$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>No. of iterations</th>
<th>No. of pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>PSRT scheme</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>CDRT scheme</td>
<td></td>
<td>22</td>
</tr>
</tbody>
</table>

Table 3.3: Number of iterations versus number of D2D pairs when $\alpha = 1$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>No. of iterations</th>
<th>No. of pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>PSRT scheme</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>CDRT scheme</td>
<td></td>
<td>31</td>
</tr>
</tbody>
</table>

Table 3.4: Number of iterations versus number of D2D pairs when $\alpha = 2$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>No. of iterations</th>
<th>No. of pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>PSRT scheme</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td>CDRT scheme</td>
<td></td>
<td>28</td>
</tr>
</tbody>
</table>

Table 3.5: Number of iterations versus number of SINR thresholds when $\alpha = 0$

<table>
<thead>
<tr>
<th>Scheme</th>
<th>No. of iterations</th>
<th>No. of thresholds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>PSRT scheme</td>
<td></td>
<td>21</td>
</tr>
<tr>
<td>CDRT scheme</td>
<td></td>
<td>52</td>
</tr>
</tbody>
</table>
Table 3.6: Number of iterations versus number of SINR thresholds when $\alpha = 1$

<table>
<thead>
<tr>
<th>No. of iterations</th>
<th>No. of thresholds</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheme</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSRT scheme</td>
<td></td>
<td>2</td>
<td>4</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>CDRT scheme</td>
<td></td>
<td>37</td>
<td>49</td>
<td>66</td>
<td>71</td>
</tr>
</tbody>
</table>

Table 3.7: Number of iterations versus number of SINR thresholds when $\alpha = 2$

<table>
<thead>
<tr>
<th>No. of iterations</th>
<th>No. of thresholds</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scheme</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PSRT scheme</td>
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<td>20</td>
<td>20</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>CDRT scheme</td>
<td></td>
<td>50</td>
<td>48</td>
<td>44</td>
<td>44</td>
</tr>
</tbody>
</table>

Table 3.8: Impact of limited information exchange on PSRT Performance when $\alpha = 0$

<table>
<thead>
<tr>
<th>Radius of info. ex. (m)</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. n. of info. ex. pairs/D2D pair</td>
<td>7.7</td>
<td>15.7</td>
<td>25.3</td>
<td>35.4</td>
<td>45.5</td>
</tr>
<tr>
<td>N. of iterations</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>27</td>
<td>24</td>
</tr>
<tr>
<td>Utility loss. (%)</td>
<td>-60</td>
<td>-24</td>
<td>-6.1</td>
<td>-1.5</td>
<td>-0.43</td>
</tr>
</tbody>
</table>

Table 3.9: Impact of limited information exchange on PSRT Performance when $\alpha = 1$

<table>
<thead>
<tr>
<th>Radius of info. ex. (m)</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. n. of info. ex. pairs/D2D pair</td>
<td>7.7</td>
<td>15.7</td>
<td>25.3</td>
<td>35.4</td>
<td>45.5</td>
</tr>
<tr>
<td>N. of iterations</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Utility loss. (%)</td>
<td>-15</td>
<td>-9.6</td>
<td>-2.8</td>
<td>-0.85</td>
<td>-0.27</td>
</tr>
</tbody>
</table>

Table 3.10: Impact of limited information exchange on PSRT Performance when $\alpha = 2$

<table>
<thead>
<tr>
<th>Radius of info. ex. (m)</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. n. of info. ex. pairs/D2D pair</td>
<td>7.7</td>
<td>15.7</td>
<td>25.3</td>
<td>35.4</td>
<td>45.5</td>
</tr>
<tr>
<td>N. of iterations</td>
<td>20</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>20</td>
</tr>
<tr>
<td>Utility loss. (%)</td>
<td>-1100</td>
<td>-81</td>
<td>-21</td>
<td>-6.4</td>
<td>-2.7</td>
</tr>
</tbody>
</table>
3.4 Numerical Evaluation

In this section, we study the performance of PSRT. We compare it with the CD Random Transmission (CDRT) scheme, which is an extension of a CD method proposed in [4, 5] to our case of multiple SINR thresholds and described in Appendix 3.6.3, and an always transmit (AT) scheme, in which all D2D pairs always transmit. We randomly pick the same initial point for PSRT and CDRT. The convergence condition of these two schemes is that the norm of the difference of two solutions in consecutive iterations is less than $\epsilon = 10^{-2}$.

The simulation area is set as $250 \times 250$ m$^2$. The pathloss exponent $\gamma$ is 4. The power of noise is $-132$ dBm. The mean of the small-scale power fading term is normalized to 1. The maximum and minimum transmission probability of all D2D pairs are set as 1 and 0, respectively. The transmission power of all Txs is set as 1 mW, and the D2D distance is uniformly drawn between 25 m and 85 m. In the default setting, three received SINR thresholds are chosen from [58] and set as $\{1.1915, 4.4926, 16.9395\}$, which corresponds to the transmission rates $\{0.8770, 1.9141, 3.3223\}$ bit/s/Hz [59]. The default number of D2D pairs is 75. Each data point in the figures is averaged over 200 realizations.

3.4.1 Impact of Number of D2D pairs and SINR Thresholds

In Fig. 3.2, we study the impact of the number of D2D pairs on performance. We observe that the utility achieved by PSRT is substantially higher than that of the alternatives. It also shows that the utility of PSRT increases with the number of D2D pairs when $\alpha = 0$, and decreases with with the number of D2D pairs when $\alpha = 1$ and $\alpha = 2$. This is because when $\alpha = 0$, maximizing the utility becomes maximizing the sum throughput, which does not consider fairness among D2D pairs. With more D2D pairs, PSRT can choose D2D pairs with better transmission condition to assign higher transmit probabilities, which yields higher throughput. For $\alpha = 1$ and $\alpha = 2$, fairness is introduced in the utility. The transmit probabilities of D2D pairs with favorable transmission condition are lower in order to limit their interference to other D2D pairs. With more D2D pairs, there are more of those with unfavorable transmission condition, which leads to decreased fairness.

In Fig. 3.3, we study the impact of the number of SINR thresholds on performance. We select evenly spaced SINR thresholds from the set $\{0.2025, 0.4808, 1.1915, 4.4926, 16.9395, 38.7972, 96.1391\}$ [58], which corresponds to transmission rates $\{0.1523, 0.3770, 0.8770, 1.9141, 3.3223, 4.5234, 5.5547\}$ bit/s/Hz [59]. We observe that the utility of all schemes increase with the number of SINR thresholds. However, this increase diminishes quickly, suggesting that a moderate number of SINR thresholds is sufficient for optimal
system operation.

In Tables 3.3 and 3.4, we compare the time complexity of PSRT and CDRT. We observe that the number of iterations of PSRT is much smaller than that of CDRT, especially for the case of $\alpha = 1$, which suggests that PSRT has faster convergence rate than CDRT. Furthermore, Tables 3.2-3.7 suggest that the number of iterations in PSRT mostly depends only on the value of $\alpha$. The number of D2D pairs and the number of SINR thresholds have little impact.

3.4.2 Distributed Implementation with Limited Information

We study the impact of limited information exchange on the performance of PSRT under distributed implementation. We assume that a D2D pair only collects the information sent from D2D pairs that are less than $R_{mc}$ away from itself. In Tables 3.8-3.10, for different values of $\alpha$ and $R_{mc}$, we list the average number of information exchange pairs per D2D pair, the number of iterations in PSRT, and percentage utility loss compared with the case of global information exchange, i.e., $R_{mc} = \infty$.

As expected, with larger $R_{mc}$, a D2D pair receives information from more D2D pairs, and thus the system attains higher utility. However, we observe that to achieve utility close to the case of global information exchange, we require only moderate numbers of information exchange pairs, approximately 25, 25, and 35, respectively, for $\alpha = 0, 1$, and 2. In addition, we also observe that $R_{mc}$ has little impact on the required number of iteration until convergence.

3.5 Summary

In this work, we have considered the optimal transmission probabilities of different D2D pairs to maximize network-wide $\alpha$-fair utility. We propose the PSRT algorithm to solve the formulated problem with the grantee of convergence to a KKT point. In PSRT, separable one-dimensional convex optimization problems are iteratively solved to update the transmission probabilities of different D2D pairs. Numerical results demonstrates our proposed algorithm is superior to other alternatives. It should be noted that our proposed method can also be easily applied to solve the problems of similar structure with minor modifications, such as the problems in [4,5].
3.6 Appendices

3.6.1 Derivation of $\mathbb{P}(\text{SINR}_i \geq T_l | e_i = 1)$

$$
\mathbb{P}(\text{SINR}_i \geq T_l | e_i = 1) = \mathbb{P} \left( \sum_{j=1, j \neq i}^{M} \frac{P_j h_{ij}}{d_{ij}^\gamma} e_j \frac{P_j h_{ij}}{d_{ij}^\gamma} + \sigma^2 \geq T_l \right)
$$

$$
= \mathbb{E}_{\{h_{ij}, e_j\} | j \neq i} \left[ \mathbb{P} \left( \sum_{j=1, j \neq i}^{N} \frac{P_j h_{ij}}{d_{ij}^\gamma} e_j \frac{P_j h_{ij}}{d_{ij}^\gamma} + \sigma^2 \geq T_l \right) \right]
$$

$$
= \mathbb{E}_{\{h_{ij}, e_j\} | j \neq i} \left[ \mathbb{P} \left( h_{ii} \geq T_l \frac{d_{ij}^\gamma}{P_i} \left( \sum_{j=1, j \neq i}^{N} e_j \frac{P_j h_{ij}}{d_{ij}^\gamma} + \sigma^2 \right) \right) \right]
$$

$$
= \mathbb{E}_{\{h_{ij}, e_j\} | j \neq i} \left[ a_{il} \exp \left\{ -\mu T_l d_{ii}^\gamma \frac{\sum_{j=1, j \neq i}^{N} e_j \frac{P_j h_{ij}}{d_{ij}^\gamma}}{P_i} \right\} \right]
$$

$$
\overset{(a)}{=} a_{il} \prod_{j=1, j \neq i} a_{il} \mathbb{E}_{h_{ij}, e_j} \left[ \exp \left\{ -\mu T_l d_{ii}^\gamma \frac{\sum_{j=1, j \neq i}^{N} e_j \frac{P_j h_{ij}}{d_{ij}^\gamma}}{P_i} \right\} \right]
$$

$$
\overset{(b)}{=} a_{il} \prod_{j=1, j \neq i} \mathbb{E}_{h_{ij}} \left[ 1 - p_j + p_j \exp \left\{ -\mu T_l d_{ii}^\gamma \frac{P_j h_{ij}}{P_i d_{ij}^\gamma} \right\} \right]
$$

$$
\overset{(c)}{=} a_{il} \prod_{j=1, j \neq i} \mathbb{E}_{h_{ij}} \left[ 1 - p_j + p_j \exp \left\{ -\mu T_l d_{ii}^\gamma \frac{P_j h_{ij}}{P_i d_{ij}^\gamma} \right\} \right]
$$

$$
\overset{(d)}{=} a_{il} \prod_{j=1, j \neq i} \left( 1 - p_j + \frac{p_j}{1 + \frac{1}{b_{ijl}}} \right)
$$

where $a_{il} = \exp \left( -\mu T_l d_{ii}^\gamma \frac{a^2}{P_i} \right)$, and $b_{ijl} = \frac{P_j d_{ij}^\gamma}{T_l P_i d_{ii}^\gamma}$. (a) is based on the assumption that $h_{ii}$ is exponentially distributed with mean $1/\mu$. (b) is from the independence among $\{h_{ij}\}$. (c) is from the distribution of $e_i$ and the independence between $e_i$ and $p_i$. (d) is derived from the distribution of $h_{ij}$. 


3.6.2 Proof of Theorem 5

First, we note that the feasible set is bounded. Hence, the sequence of points generated by PSRT, \( \{p^1, p^2, \ldots \} \) is bounded, and has at least one limit point [60].

In order to prove Theorem 5, we need to further verify the following two conditions of PSRT based on [61].

**Condition 1:** \( \forall p, p^{t-1} \in \mathcal{P}; \ U(p) \geq U_{LB}(p, p^{t-1}) \), and the equality holds when \( p = p^{t-1} \).

**Condition 2:** \( \forall p, p^{t-1} \in \mathcal{P} \), for \( 1 \leq k \leq N \), \( \frac{\partial U(p)}{\partial p_k} |_{p=p^{t-1}} = \frac{\partial U_{LB}(p, p^{t-1})}{\partial p_k} |_{p=p^{t-1}} \).

Condition 1 has been verified in Section 3.3.2. Next we verify that Condition 2 is satisfied.

According to the definition of \( U(p) \), we have

\[
U(p) = \begin{cases} 
- \sum_{i=1}^{N} \left( \frac{p_i \sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i}^{N} \left(1 - \frac{p_j}{1 + b_{ijl}}\right)}{1 - \alpha} \right) & \text{if } \alpha \neq 1, \\
- \sum_{i=1}^{N} \log \left( p_i \sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i}^{N} \left(1 - \frac{p_j}{1 + b_{ijl}}\right) \right) & \text{if } \alpha = 1,
\end{cases}
\]

The partial derivative of \( U(p) \) at \( p = p^{t-1} \) is

\[
\frac{\partial U}{\partial p_k} |_{p=p^{t-1}} = \sum_{i=1, i \neq k}^{N} p_i^{t-1 - \alpha} \left( \sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i, j \neq k}^{N} \left(1 - \frac{p_j^{t-1}}{1 + b_{ijl}}\right) \right)^{-\alpha} \left( \sum_{l=1}^{L} \frac{a_{il}}{1 + b_{kl}} \prod_{j=1, j \neq i, j \neq k}^{N} \left(1 - \frac{p_j^{t-1}}{1 + b_{ijl}}\right) \right) - p_k^{t-1 - \alpha} \left( \sum_{i=1}^{N} a_{ikl} \prod_{j=1, j \neq k}^{N} \left(1 - \frac{p_j^{t-1}}{1 + b_{kj}}\right) \right)^{1-\alpha}.
\]

We now derive the partial derivative of \( U_{LB}(p, p^{t-1}) \) at \( p = p^{t-1} \) to verify Condition 2 for different \( \alpha \) values.

When \( 0 \leq \alpha < 1 \), \( U_{LB}(p, p^{t-1}) \) is

\[
U_{LB}(p, p^{t-1}) = - \sum_{i=1}^{N} u_i^{t-1} \log(p_i) + \sum_{l=1}^{L} \sum_{j=1, j \neq i}^{N} v_{il}^{t-1} \log(1 - \frac{p_j}{1 + b_{ijl}}) + w_i^{t-1}.
\]
Then
\[
\frac{\partial U_{LB}}{\partial p_k}|_{p=p^{t-1}} = \sum_{i=1}^{N} \sum_{l=1}^{L} \frac{v_l^{t-1}}{1 + b_{ilk} - p_l^{t-1}} - \frac{u_k^{t-1}}{p_k^{t-1}}.
\] (3.18)

And we have
\[
\frac{u_k^{t-1}}{p_k^{t-1}} = \frac{1}{p_k^{t-1}} \left( \sum_{l=1}^{L} a_{kl} p_l^{t-1} \prod_{j=1, j \neq k}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{kjl}} \right) \right)^{1-\alpha}
\] (3.19)

and
\[
\sum_{l=1}^{L} \frac{v_l^{t-1}}{1 + b_{ilk} - p_l^{t-1}} = \sum_{l=1}^{L} a_{il} \left( \sum_{l'=1}^{L} a_{il'} p_l^{t-1} \prod_{j=1, j \neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{iji'}} \right) \right)^{-\alpha}
\]
\[
= p_i^{t-1-\alpha} \left( \sum_{l'=1}^{L} a_{il'} p_l^{t-1} \prod_{j=1, j \neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{iji'}} \right) \right)^{-\alpha}
\]
\[
\sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{iji'}} \right) \frac{1}{1 + b_{ilk} - p_l^{t-1}}
\]
\[
= p_i^{t-1-\alpha} \left( \sum_{l'=1}^{L} a_{il'} p_l^{t-1} \prod_{j=1, j \neq i}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{iji'}} \right) \right)^{-\alpha}
\]
\[
\sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i, j \neq k}^{N} \left( 1 - \frac{p_j^{t-1}}{1 + b_{iji'}} \right) .
\] (3.20)

Substituting (3.19) and (3.20) into (3.18), we conclude that when \(0 \leq \alpha < 1\),
\[
\frac{\partial U}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial U_{LB}}{\partial p_k}|_{p=p^{t-1}}, \forall k.
\]

Similar to the case \(0 \leq \alpha < 1\), we can show that when \(\alpha = 1\),
\[
\frac{\partial U}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial U_{LB}}{\partial p_k}|_{p=p^{t-1}}, \forall k.
\]
When $\alpha > 1$, $U_{LB}(p, p^{t-1})$ is

$$U_{LB}(p, p^{t-1}) = -\sum_{i=1}^{N} \sum_{l=1}^{L} \lambda_{il}^{t-1} p_i^{(1-\alpha)} \zeta_{il}^{t-1} - \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \sum_{l=1}^{L} \mu_{ijl}^{t-1} \left(1 - \frac{p_j}{1 + b_{ijl}}\right)^{(1-\alpha) \eta_{ijl}^{t-1}}.$$  

Then

$$\left. \frac{\partial U_{LB}}{\partial p_k} \right|_{p=p^{t-1}} = -\sum_{l=1}^{L} (1-\alpha) \zeta_{kl}^{t-1} \lambda_{kl}^{t-1} P_i^{(1-\alpha) \zeta_{kl}^{t-1}} + \sum_{j=1, j \neq k}^{N} (1-\alpha) \zeta_{kl}^{t-1} (1-\alpha) \zeta_{kl}^{t-1} \left(1 - \frac{p_k}{1 + b_{jkl}}\right)^{(1-\alpha) \eta_{jkl}^{t-1}}.$$  

(3.21)

And we have

$$\sum_{l=1}^{L} (1-\alpha) \zeta_{kl}^{t-1} \lambda_{kl}^{t-1} P_i^{(1-\alpha) \zeta_{kl}^{t-1}}$$

$$= \sum_{l=1}^{L} \frac{a_{kl}}{(1-\alpha) \zeta_{kl}^{t-1} \left(\sum_{l'=1}^{L} a_{kl'} \prod_{j=1, j \neq k}^{N} (1 - \frac{p_{l'}^{t-1}}{1 + b_{kjl'}})\right)}$$

$$= \sum_{l=1}^{L} \frac{a_{kl}}{(1-\alpha) \zeta_{kl}^{t-1} \left(\sum_{l'=1}^{L} a_{kl'} \prod_{j=1, j \neq k}^{N} (1 - \frac{p_{l'}^{t-1}}{1 + b_{kjl'}})\right)} \left(1 - \frac{p_k^{t-1}}{1 + b_{kjl'}}\right)^{1-\alpha}$$

$$= p_k^{t-1-\alpha} \left(\sum_{l=1}^{L} a_{kl} \prod_{j=1, j \neq k}^{N} \left(1 - \frac{p_j^{t-1}}{1 + b_{kjl}}\right)\right)^{1-\alpha},$$  

(3.22)

and

$$\sum_{l=1}^{L} (1-\alpha) \mu_{jkl}^{t-1} \eta_{jkl}^{t-1} \left(1 - \frac{p_k}{1 + b_{jkl}}\right)^{(1-\alpha) \eta_{jkl}^{t-1}}.$$
\[ \sum_{l=1}^{L} \frac{a_{jl}}{1 + b_{jkl}} \prod_{k' = 1, k' \neq j}^{\prod_{l' = 1}^{L} \left( 1 - \frac{p_{j', l'}^{t-1}}{1 + b_{j', l'}} \right)^{\alpha} \left( \sum_{l'=1}^{L} a_{j', l'} \prod_{j' = 1}^{N} \left( 1 - \frac{p_{j', l'}^{t-1}}{1 + b_{j', l'}} \right) \right)}{\left( 1 - \frac{p_{j, l}^{t-1}}{1 + b_{jkl}} \right)}^{(1-\alpha) \log\left( \frac{1}{1 - p_{j, l}^{t-1}} \right) - 1} \]

Substituting (3.23) and (3.22) into (3.21), we conclude that when \( \alpha > 1 \),
\[ \frac{\partial U}{\partial p_k} \bigg|_{p=p^{t-1}} = \frac{\partial U_{LB}}{\partial p_k} \bigg|_{p=p^{t-1}, \forall k}. \]

### 3.6.3 Description of CDRT

CDRT is an extension of the CD method previously proposed for the case of a single SINR threshold [4, 5] to our model with multiple SINR thresholds.

First, we rewrite the objective \( U(p) \) in the following form:
\[
U(p_i, p_{-i}) = \begin{cases} 
\frac{(d_i p_i)^{1-\alpha}}{1 - \alpha} + \sum_{k=1, k \neq i}^{N} \frac{(e_{ik} - o_{ik} p_i)^{1-\alpha}}{1 - \alpha} & \text{if } \alpha \neq 1, \\
\log(d_i p_i) + \sum_{k=1, k \neq i}^{N} \log(e_{ik} - o_{ik} p_i) & \text{if } \alpha = 1,
\end{cases}
\]

for all \( i \in \mathcal{N} \), where \( p_{-i} = [p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N] \), and
\[
c_{kil} = \prod_{j=1, j \neq k, j \neq i}^{L} \left( 1 - \frac{p_{j}}{1 + b_{kjl}} \right),
\]
\[
d_i = \sum_{l=1}^{L} a_{il} \prod_{j=1, j \neq i}^{L} \left( 1 - \frac{p_{j}}{1 + b_{ijl}} \right),
\]
This allows us to observe that, even though $U(p)$ is not concave in $p$, it is concave in each $p_i$, as stated in the lemma below.

**Lemma 6.** Given arbitrary $p_{-i}$, $U(p)$ is concave in $p_i$ for $1 \leq i \leq N$.

**Proof.** $f(x) = \frac{x^{1-\alpha}}{1-\alpha}$ is concave in $x$ when $\alpha \in [0,1) \cup (1, +\infty)$ and $x \geq 0$. And $f(x) = \log(x)$ is concave in $x$ when $x > 0$. Since composition with an affine mapping preserves concavity, we conclude that $U(p_i, p_{-i})$ (i.e., $U(p)$) is concave in $p_i$. \qed

With fixed $p_{-i}$, optimization problem $\mathcal{P}$ is degraded to

$$\mathcal{P}_{CD}^i \max_{p_i} U(p_i, p_{-i})$$

s.t. $p_{i,\min} \leq p_i \leq p_{i,\max}$.

This is a one-dimensional convex optimization problem, and the optimization variable $p_i$ is the transmission probability of D2D pair $i$. This property is similar to that in the single-threshold scenario in [4,5].

In CDRT, similar to the schemes in [4,5], we update the transmission probabilities of D2D pairs iteratively until convergence. Specifically, in a round-robin manner, in each iteration, one D2D pair is chosen and its transmission probability is updated by solving the corresponding optimization problem $\mathcal{P}_{CD}^i$. Convergence is guaranteed because the objective increases in each iteration [4,5].
Chapter 4

α-Fair Utility Maximization in Multi-Tier Multi-Rate Spatial Aloha Networks

In Chapter 3, we study the utility maximization in D2D random access networks, where we assume that the locations of D2D pairs are known. However, in some networks, acquiring such information may be a prohibitive task. Hence, it is important to consider network-wide utility maximization without the knowledge of their locations. In this chapter, we model an interference-limited D2D network operating in the overlay mode as a generalized spatial Aloha network consisting of multiple tiers of D2D pairs each forming a Poisson bipolar process. Each tier of the network is defined by the power of the transmitters and the D2D distance. We consider both intra-tier and inter-tier interference. We also accommodate multi-rate communication through multiple received SIR thresholds. We aim to optimize the transmission probability of each tier, to maximize a general α-fair utility function.

Our main contributions are as follows:

- We first derive a closed-form expression of the average throughput of D2D pairs in each tier, which takes into account the random location of multi-tier interferers and multiple received SIR thresholds. This is then used in the formulation of an optimization problem to maximize the network-wide spatial α-fair utility, which is generally non-convex.

- We propose a computationally efficient iterative algorithm, which is termed Minorize-Maximization with Tier Separation (MMTS), to address this optimization problem. By exploring the partial-convexity and partial-concavity of the objective function
when $0 \leq \alpha \leq 1$ and $\alpha > 1$, respectively, we develop special lower bounds to the $\alpha$-fair objective, which we dynamically update in each iteration through solving an optimization sub-problem. Furthermore, the lower bounds are designed so that these sub-problems can be decomposed into one-dimensional convex optimization problems that are separable according to D2D tiers, which drastically reduces the computational complexity.

- We show that MMTS converges to the objective value of a KKT point of the optimization problem. We further provide various sufficient conditions under which the KKT point is the global optimum. Numerical evaluation results demonstrate that MMTS is near optimal over a wide range of parameter settings, and it substantially outperforms existing alternatives.

4.1 Relate Work

There has been a large amount of research into ad hoc, device-to-device, or direct-transmission networks that employ the Aloha MAC protocol [4, 7–12, 14, 16–19, 26–30, 62, 63]. Among them, [4, 14, 16–19] consider a fixed transmitter/receiver topology, [7–12] use the PPP model, [26, 29, 30, 62, 63] use the PBP model, while [27, 28] use the PBP model with partial topology information. The networks in the latter three groups are commonly termed spatial Aloha networks. In this section, we briefly review works in optimizing transmission probabilities in spatial aloha networks.

Several studies consider the optimization of transmission probability in spatial Aloha [7, 26–30]. However, in these works all transmitters are assumed to use the same transmission probability when the exact location of nodes is unknown. In our work, we design different transmission probabilities for different tiers of the network based on T-R distance and transmission power. Our numerical results show that this can lead to substantial performance improvement.

Furthermore, all of [7, 26–30] use a simple single-rate communication model based on a single received SIR threshold, with [26] further considering the Shannon-rate upper bound, while in this work we allow multiple SIR thresholds for multi-rate communication between each T-R pair. Finally, the performance objectives of these works are narrow: [7, 26, 29, 30] focus on the sum throughput, while [27, 28] concern the log utility. All of these objectives are special cases of the general $\alpha$-fair utility in this work.
Table 4.1: Table of Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of tiers</td>
</tr>
<tr>
<td>$\lambda_n$</td>
<td>Intensity of PBP of transmitters in tier $n$</td>
</tr>
<tr>
<td>$R_n$</td>
<td>D2D distance in tier $n$</td>
</tr>
<tr>
<td>$P_n$</td>
<td>Transmission power of transmitter in tier $n$</td>
</tr>
<tr>
<td>$p_n$</td>
<td>Transmission probability of transmitter in tier $n$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Pathloss exponent</td>
</tr>
<tr>
<td>$L$</td>
<td>Number of SIR thresholds to modulate the received signal</td>
</tr>
<tr>
<td>$T_l$</td>
<td>$l$th SIR threshold</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Fairness index in the utility function</td>
</tr>
</tbody>
</table>

4.2 System Model and Problem Formulation

In this section, we describe multi-tier, multi-rate spatial aloha network. We further consider the fairness in the network, and formulate the problem of maximizing the $\alpha$-fair utility of the network. Important notations throughout this chapter are summarized in Table 4.1.

4.2.1 Multi-tier Spatial Aloha Network

Consider a spatial random network in two-dimensional Euclidean space consisting of multiple simple D2D pairs with fixed transmission power, such as some sensor nodes, communicating over a shared channel as illustrated in Fig.4.1. The D2D pairs are differentiated into $N$ tiers, defined by their transmission power and D2D distance. Each tier independently forms a PBP, i.e., the tier $n$ transmitters form a PPP with intensity $\lambda_n$, denoted by $\Phi^t_n$, and each transmitter is associated with a receiver that is uniformly randomly located on a circle of fixed radius $R_n$ centered at the transmitter. We observe that by this definition of the PBP, the tier $n$ receivers also form a PPP with intensity $\lambda_n$, since they are i.i.d. marks of $\Phi^t_n$.

Due to the large number of D2D pairs in the network, we assume that each pair only acquires its own D2D distance, transmission power, and spatial statistics of the interfering D2D pairs. The D2D pairs employ the slotted Aloha MAC protocol due to the lack of topology information [7]. The transmission probability of each tier $n$ transmitter is denoted by $p_n$. Then, the independent thinning property of a PPP implies that the active transmitters in tier $n$ is a PPP with intensity $p_n\lambda_n$. We denote by $P_n$ the fixed transmission power of active tier $n$ transmitters.

Note that this $N$-tier spatial Aloha network model is general. For example, if in
practice the power of transmitters in a tier becomes unequal, this tier can be further split into multiple tiers such that the above definition of a tier is upheld. The total number of tiers can be set large enough based on the required precision of system modeling and analysis. In this work, our analysis proceeds assuming that the tiers are already given.

4.2.2 Multiple SIR Thresholds and Average Throughput

The received power of the receiver located at \( y \) from the transmitter located at \( x \) is given by

\[
P_{xy} = \frac{P_x h_{xy}}{|x - y|^{\gamma}}, \tag{4.1}
\]

where \( P_x \) is the transmission power, \( h_{xy} \) is the channel power gain under Rayleigh fading, and \( \gamma \) is the path loss exponent where \( \gamma > 2 \). We assume that \( h_{xy} \) is i.i.d. with unit mean and independent of \( \Phi_t^n \) for all \( n \).

Because of spatial stationarity, we may focus on an arbitrary D2D pair in tier \( n \), termed the \textit{typical pair}. We further assume the transmitter and the receiver in the typical pair are situated at \( x_{n0} \) and origin 0, respectively. Then, the SIR of the receiver in the typical pair is given by

\[
\text{SIR}_n = \frac{P_n h_{x_{n0}0}}{\sum_{k=1,k\neq n} \sum_{x \in \Phi_t^k} \frac{P_k h_{x0}}{|x|^{\gamma}} + \sum_{x \in \Phi_t^k \setminus \{x_{n0}\}} \frac{P_k h_{x0}}{|x|^{\gamma}}}. 
\]

We assume the system is interference limited. The receivers use \( L \) SIR thresholds to demodulate the received signal, denoted by \( \mathcal{T} = \{T_1, \cdots, T_L\} \). Without loss of generality, we assume that \( T_i < T_j \) if \( i < j \). For mathematical convenience, we add \( T_0 = 0 \) to \( \mathcal{T} \). For \( 0 \leq l < L \), if the SIR is between \( T_l \) and \( T_{l+1} \), then the D2D rate is \( r_l \), where \( r_0 = 0 \); if the SIR is no smaller than \( T_L \), then the D2D rate is \( r_L \). Since \( T_l > T_{l-1} \), we have \( r_l > r_{l-1} \). In the above, we have normalized the channel bandwidth to one.
The average throughput of the typical pair in tier \( n \) is given by
\[
\begin{align*}
  r_n &= p_n \left[ \sum_{l=1}^{L-1} \mathbb{P}(T_l \leq \text{SIR}_n \leq T_{l+1}) r_l + \mathbb{P}(\text{SIR}_n \geq T_L) r_L \right] \\
  &= p_n \sum_{l=1}^{L} a_l \mathbb{P}(\text{SIR}_n > T_l),
\end{align*}
\]
where \( a_l = r_l - r_{l-1} \).

### 4.2.3 Problem Statement

Similar to \([4,16,19]\), we define the spatial \( \alpha \)-fair utility as
\[
U(p) = \begin{cases} 
\sum_{n=1}^{N} \frac{(\lambda_n r_n)^{1-\alpha}}{1-\alpha} & \text{if } \alpha \neq 1, \\
\sum_{n=1}^{N} \log(\lambda_n r_n) & \text{if } \alpha = 1,
\end{cases}
\]
where \( p = [p_n]_{N \times 1} \), and formulate our main optimization problem as
\[
P: \max_p U(p) \quad \text{s.t.} \quad p_{n,\min} \leq p_n \leq p_{n,\max}, \quad 1 \leq n \leq N,
\]
where \( p_{n,\max} \) and \( p_{n,\min} \) are the maximum and minimum transmission probability of transmitters in tier \( i \). Furthermore, we define \( \mathcal{P} \) as the feasible set of problem \( P \).

Thus, we can adjust the fairness among different tiers by tuning the value of \( \alpha \). It should be noted that, since the D2D pairs in each tier have the same D2D distance, transmission power, and interference statistics, they have the same expected average throughput. Therefore, although the objective of Problem \( P \) is formulated as a sum of utility over tiers, tuning the value of \( \alpha \) can also adjust the fairness among individual D2D pairs. Generally, when \( \alpha \) is set larger, maximizing the \( \alpha \)-fair utility allows D2D pairs in unfavorable transmission conditions (e.g., long D2D distance or lower transmission power) to obtain more throughput, and thus the system becomes more fair. Specifically, when \( \alpha = 0 \), \( U(p) \) degrades to the sum throughput over all D2D pairs; when \( \alpha = 1 \), maximizing \( U(p) \) leads to the celebrated proportional fairness; and when \( \alpha \to \infty \), maximizing \( U(p) \) leads to max-min fairness.

The challenges of problem \( P \) are two-fold. First, the average throughput of commu-
communication pairs in different tiers (i.e., \( r_n \)) needs to be derived for the multi-tier, multi-rate scenario. Second, \( \mathcal{P} \) is non-convex in most cases because of the non-concavity of its objective function. Thus, conventional convex solvers do not apply.

### 4.3 Average Throughput Derivation

In this section, we derive a closed-form expression of the average throughput of the typical D2D pair. The probability of the event, that the received SIR of the receiver in the typical D2D pair is greater than \( T \), denoted by \( \text{SIR}_n \), is given by

\[
\mathbb{P}(\text{SIR}_n > T) = \mathbb{P}\left( \frac{h_{x_{n0}}}{P_n} > \frac{TIR_n}{P_n} \right) \equiv \mathbb{E}_I \left[ \exp\left( -\frac{TIR_n}{P_n} \right) \right], \tag{4.6}
\]

where \( I \) is the sum intra-tier and inter-tier interference received by the receiver in the typical D2D pair, given by

\[
I = \sum_{k=1, k \neq n}^{N} \sum_{x \in \Phi_k} P_k h_{x|0} + \sum_{x \in \Phi_n \setminus \{x_{n0}\}} P_n h_{x|0},
\]

and \( (a) \) is based on the distribution of \( h_{x_{n0}} \) and its independence of \( I \).

As shown in Appendix 4.8.1, the Laplace transform of \( I \) is given by

\[
\mathbb{L}_I(s) = \prod_{j=1}^{N} \exp\left( -p_j \lambda_j \pi (sP_j)^{\frac{2}{\gamma}} \Gamma(1 - \frac{2}{\gamma}) \Gamma(1 + \frac{2}{\gamma}) \right). \tag{4.7}
\]

Substituting (4.7) into (4.6), we have

\[
\mathbb{P}(\text{SIR}_n > T) = \exp\left( -\sum_{j=1}^{N} p_j \lambda_j R_n^2 \left( \frac{P_j}{P_n} \right)^{2/\gamma} C \right), \tag{4.8}
\]

where \( C = \pi T^2 \Gamma(1 - \frac{2}{\gamma}) \Gamma(1 + \frac{2}{\gamma}) \).

Substituting (8) into (2) and simplifying, we find the average throughput of the typical D2D pair:

\[
r_n = p_n \sum_{l=1}^{L} a_l \exp\left( -m_{nl} \sum_{j=1}^{N} p_j \lambda_j P'_j \right), \tag{4.9}
\]

where \( C_l = \pi T^2 \Gamma(1 - \frac{2}{\gamma}) \Gamma(1 + \frac{2}{\gamma}) \), \( P' = P_n^{2/\gamma} \), and \( m_{nl} = \frac{R_n}{P_n} \). Note that even though \( C_l \) contains the gamma function, it is a positive constant, so the right-hand side of (4.9) is in closed-form with respect to \( p \). This contributes to the tier-separability and efficiency of MMTS.
4.4 Minorize-Maximization with Tier Separation

In this section, we present MMTS to solve problem $\mathcal{P}$. We first develop three lower bounds for objective (4.4), for $0 \leq \alpha < 1$, $\alpha = 1$, and $\alpha > 1$. Then, we develop lower bound problems for these three cases, and show how they can be decomposed to multiple one-dimensional convex optimization problems that are separable according to the D2D tiers, which can be solved either in closed-form or otherwise efficiently. Finally, we explain how these tier-separable solutions can be employed in a recursive MM framework to address the original problem $\mathcal{P}$.

4.4.1 Lower Bound Problem for $0 \leq \alpha < 1$

Though objective (4.4) is not concave, it has a special partially-convex structure when $0 \leq \alpha < 1$. We take advantage of this special structure and develop a lower bound as stated in Lemma 6.

**Lemma 7.** When $0 \leq \alpha < 1$, for all $\mathbf{p} = [p_n]_{N \times 1}, \mathbf{p}^{t-1} = [p_n^{t-1}]_{N \times 1} \in \mathcal{P}$,

$$
\left( \lambda_n p_n \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j) \right)^{1-\alpha} \geq -d_n^{t-1} \sum_{j=1}^{N} P_j' \lambda_j (p_j - p_j^{t-1}) + e_n^{t-1} \log \frac{p_n}{p_n^{t-1}} + f_n^{t-1},
$$

(4.10)

where

$$
d_n^{t-1} = \frac{(\lambda_n p_n^{t-1})^{1-\alpha} \sum_{l=1}^{L} a_l m_{nl} \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j^{t-1})}{\left( \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j^{t-1}) \right)^{\alpha}},
$$

$$
e_n^{t-1} = \left( \lambda_n p_n^{t-1} \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j^{t-1}) \right)^{1-\alpha},
$$

$$
f_n^{t-1} = \frac{(\lambda_n p_n^{t-1} \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j^{t-1}) )^{1-\alpha}}{1 - \alpha},
$$

and the equality holds if $\mathbf{p} = \mathbf{p}^{t-1}$.

**Proof.** See Appendix 4.8.2. \hfill \Box

From Lemma 7, a lower bound of objective (4.4) is

$$
\hat{U}(\mathbf{p}, \mathbf{p}^{t-1}) = \sum_{n=1}^{N} \left( -\sum_{j=1}^{N} d_j^{t-1} P_j' \lambda_n (p_n - p_n^{t-1}) + e_n^{t-1} \log \frac{p_n}{p_n^{t-1}} + f_n^{t-1} \right),
$$
for all $p, p^{t-1} \in \mathfrak{P}$, and $\tilde{U}(p, p^{t-1}) = U(p)$ when $p = p^{t-1}$. Therefore, when $0 \leq \alpha < 1$, for any given $p^{t-1} \in \mathfrak{P}$, we may consider the following optimization problem that lower bounds $\mathcal{P}$.

$$\mathcal{P}^{1}_\text{LB} : \max_p \tilde{U}(p, p^{t-1})$$

subject to

$$p_{n,\min} \leq p_n \leq p_{n,\max}, \quad 1 \leq n \leq N.$$  

Remark: Note that the right-hand side (RHS) of (4.10) has been designed to be separable with respect to the optimization variables $\{p_n\}$. This enables our decomposition of $\mathcal{P}^{1}_{\text{LB}}$ into $N$ separable one-dimensional convex optimization problem according to the D2D tiers. The $n$th problem, for $1 \leq n \leq N$, is given by

$$\mathcal{P}^{1, n}_{\text{LB}} : \max_{p_n} \tilde{U}_n(p_n)$$

subject to

$$p_{n,\min} \leq p_n \leq p_{n,\max}.$$  

where

$$\tilde{U}_n(p_n) = -\sum_{j=1}^{N} d_j^{t-1} P'_{n} \lambda_n (p_n - p_n^{t-1}) + e_n^{t-1} \log \left( \frac{P_n}{P_n^{t-1}} \right) + f_n^{t-1}.$$  

Problem $\mathcal{P}^{1, n}_{\text{LB}}$ has a closed-form optimal solution as stated in Lemma 8, whose proof is given in Appendix 4.8.3.

**Lemma 8.** Problem $\mathcal{P}^{1, n}_{\text{LB}}$ is convex in $p_n$, and its optimal solution is

$$\tilde{p}_n = \left[ \frac{e_n^{t-1}}{P_n' \lambda_n \sum_{j=1}^{N} d_j^{t-1}} \right]_{p_{n,\min}}^{p_{n,\max}},$$  

where $[x]_{x_{\min}}^{x_{\max}} = \min \{ \max \{ x, x_{\min} \}, x_{\max} \}$.

Since $\mathcal{P}^{1}_{\text{LB}}$ is separable into convex problems, itself is also a convex problem. This facilitates our analysis of the convergence of MMTS as described in Section 4.5.

### 4.4.2 Lower Bound Problem for $\alpha = 1$

When $\alpha = 1$, the objective of $\mathcal{P}$ is

$$U(p) = \sum_{n=1}^{N} \log(\lambda_n p_n) + \log(\sum_{l=1}^{L} a_l \exp(-m_n l \sum_{j=1}^{N} P_j' \lambda_j p_j)).$$
We note that \( \log(\sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} x_j)) \), for all \( l \) and \( a_l > 0 \), is convex in \( x = [x_j]_{N \times 1} \) based on the convexity of the LogSumExp function [64]. Hence, for all \( \mathbf{p} = [p_n]_{N \times 1}, \mathbf{p}^{t-1} = [p_n^{t-1}]_{N \times 1} \in \mathcal{P} \), we have

\[
\log(\sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j^{t-1} \lambda_j p_j)) \geq -g^{t-1}_n \sum_{j=1}^{N} P_j^{t-1} \lambda_j (p_j - p_n^{t-1}) + h^{t-1}_n, \tag{4.17}
\]

where

\[
g^{t-1}_n = \frac{\sum_{l=1}^{L} a_l m_{nl} \exp(-m_{nl} \sum_{j=1}^{N} P_j^{t-1} \lambda_j p_n^{t-1})}{\sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j^{t-1} \lambda_j p_n^{t-1})},
\]

\[
h^{t-1}_n = \log \left( \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j^{t-1} \lambda_j p_n^{t-1}) \right),
\tag{4.18}
\]

and the equality holds if \( \mathbf{p} = \mathbf{p}^{t-1} \).

Hence, a lower bound of objective (4.4) is given by

\[
\hat{U}(\mathbf{p}, \mathbf{p}^{t-1}) = -\sum_{j=1}^{N} \sum_{n=1}^{N} g^{t-1}_j P_j^{t-1} \lambda_n (p_n - p_n^{t-1}) + \sum_{n=1}^{N} \log(\lambda_n p_n) + \sum_{n=1}^{N} h^{t-1}_n, \tag{4.19}
\]

for all \( \mathbf{p}, \mathbf{p}^{t-1} \in \mathcal{P} \), and \( \hat{U}(\mathbf{p}, \mathbf{p}^{t-1}) = U(\mathbf{p}) \) when \( \mathbf{p} = \mathbf{p}^{t-1} \). Therefore, when \( \alpha = 1 \), for any given \( \mathbf{p}^{t-1} \in \mathcal{P} \), we may consider the following optimization problem that lower bounds \( \mathcal{P} \).

\[
\mathcal{P}^2_{LB} : \quad \max_{\mathbf{p}} \hat{U}(\mathbf{p}, \mathbf{p}^{t-1}) \tag{4.19}
\]

\[
\text{s.t.} \quad p_{n,\min} \leq p_n \leq p_{n,\max}, 1 \leq n \leq N. \tag{4.20}
\]

Similarly to the case \( 1 \leq \alpha < 1 \), the RHS of (4.17) has been designed to be separable with respect to the optimization variables \( \{p_n\} \). We can decompose \( \mathcal{P}^2_{LB} \) into \( N \) separable one-dimensional convex optimization problem according to the D2D tiers. The \( n \)th problem, for \( 1 \leq n \leq N \), is given by

\[
\mathcal{P}^{2,n}_{LB} : \quad \max_{p_n} \hat{U}_n(p_n) \tag{4.21}
\]

\[
\text{s.t.} \quad p_{n,\min} \leq p_n \leq p_{n,\max}, \tag{4.22}
\]

where

\[
\hat{U}_n(p_n) = -\sum_{j=1}^{N} g^{t-1}_j P_j^{t-1} \lambda_n (p_n - p_n^{t-1}) + \log(\lambda_n p_n) + h^{t-1}_n.
\]
Problem $\mathcal{P}_{LB}^{2,n}$ has a closed-form optimal solution as stated in Lemma 9.

**Lemma 9.** Problem $\mathcal{P}_{LB}^{2,n}$ is convex in $p_n$, and its optimal solution is

$$\hat{p}_n^* = \left[ \frac{1}{P_n\lambda_n \sum_{j=1}^{N} g_{j}^{l-1}} \right]_{p_{n,\text{min}}}^{p_{n,\text{max}}}.$$  \hspace{1cm} (4.23)

### 4.4.3 Lower Bound Problem for $\alpha > 1$

When $\alpha > 1$, we take advantage of a special *partially-concave* structure of objective (4.4) and develop the following lower bound as stated in Lemma 10. For notational convenience, we define $\alpha' = (N + 1)(1 - \alpha)$.

**Lemma 10.** When $\alpha > 1$, for all $p = [p_n]_{N \times 1}, p^{t-1} = [p^{t-1}_n]_{N \times 1} \in \mathcal{P}$,

$$\left( \frac{\lambda_n p_n \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j)}{1 - \alpha} \right)^{1-\alpha} \geq \frac{r_{n}^{t-1} p_n^{t-1}}{\alpha'} + \sum_{j=1}^{N} \sum_{l=1}^{L} s_{njl}^{t-1} \exp(-\alpha m_{nl} P_j' \lambda_j p_j),$$

where

$$r_{n}^{t-1} = \lambda_n^{1-\alpha} p_n^{t-1N(\alpha-1)} \left( \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P_j' \lambda_j p_j^{t-1}) \right)^{1-\alpha},$$

$$s_{njl}^{t-1} = \frac{a_l \exp(-m_{nl} \sum_{k=1}^{N} P_k' \lambda_k p_k^{t-1} - \alpha P_j' \lambda_j p_j^{t-1})}{(\lambda_n p_n^{t-1})^{\alpha-1} \left( \sum_{l'=1}^{L} a_{l'} \exp(-m_{nl'} \sum_{k=1}^{N} P_k' \lambda_k p_k^{t-1}) \right)^{\alpha}}.$$ \hspace{1cm} (4.25)

and the equality holds if $p = p^{t-1}$.

**Proof.** See Appendix 4.8.4. \hfill \Box

From Lemma 10, a lower bound of objective (4.4) is

$$\bar{U}(p, p^{t-1}) = \sum_{n=1}^{N} \left( \frac{r_{n}^{t-1} p_n^{t-1}}{\alpha'} + \sum_{j=1}^{N} \sum_{l=1}^{L} s_{njl}^{t-1} \frac{\exp(-\alpha m_{nl} P_j' \lambda_j p_j)}{\alpha'} \right),$$

for all $p, p^{t-1} \in \mathcal{P}$, and $\bar{U}(p, p^{t-1}) = \bar{U}(p)$ when $p = p^{t-1}$. Therefore, when $\alpha > 1$, for any given $p^{t-1} \in \mathcal{P}$, we may consider the following optimization problem that lower bounds $\mathcal{P}$.

$$\mathcal{P}_{LB}^{3} : \max_{p} \bar{U}(p, p^{t-1})$$

s.t. $p_{n,\text{min}} \leq p_n \leq p_{n,\text{max}}, \ 1 \leq i \leq N.$ \hspace{1cm} (4.26)
Again, similarly to the other cases of $\alpha$, the RHS of (4.24) has been designed to be separable with respect to the optimization variables $\{p_n\}$. We can decompose $P_{LB}^3$ into $N$ separable one-dimensional convex optimization problem according to the D2D tiers. The $n$th problem, for $1 \leq n \leq N$, is given by

$$P_{LB}^{3,n} : \max_{p_n} \bar{U}_n(p_n)$$

$$\text{s.t. } p_{n,\min} \leq p_n \leq p_{n,\max},$$

where

$$\bar{U}_n(p_n) = \frac{r_{t,n}^{t-1} P'_n}{\alpha'} + \sum_{j=1}^N \sum_{l=1}^L s_{jnl}^{t-1} \exp(-\alpha' m_{jl} P'_n \lambda_n p_n).$$

Problem $P_{LB}^{3,n}$ is convex as stated in Lemma 11.

**Lemma 11.** Optimization problem $P_{LB}^{3,n}$ is convex in $p_n$.

**Proof.** Recall that $\frac{r_{t,n}^{t-1}}{\alpha'} < 0$ and $\frac{s_{jnl}^{t-1}}{\alpha'} < 0$ when $\alpha > 1$. $f(x) = x^\beta$ is convex in $x$ when $x > 0$ and $\beta < 0$, and $f(x) = \exp(ax)$ is convex in $x$, for all $a, x \in \mathbb{R}$. Therefore, the objective $\bar{U}_n(p_n)$ is concave in $p_n$. Thus, the problem $P_{LB}^{3,n}$ is convex with linear constraints.

**Remark:** Though we cannot find a closed-form solution to $P_{LB}^{3,n}$, the global optimum can still be efficiently computed by methods such as bi-section search.

**4.4.4 MMTS Algorithm Description and Complexity**

Based on the above lower bounds, we design MMTS to solve problem $P$. The main idea of MMTS is to solve the lower bound problems iteratively until convergence. Specifically, for $0 \leq \alpha < 1$, in iteration $t$, we compute the parameters in the lower bound problem, $\{d_{n,\alpha}^{t-1}\}$ and $\{e_{n,\alpha}^{t-1}\}$, by (4.11), and obtain the transmission probability of each tier by (4.16). For $\alpha = 1$, in iteration $t$, we compute the parameters in the lower bound problem, $\{g_{n,\alpha}^{t-1}\}$, by (4.18), and obtain the transmission probability of each tier by (4.23). For $\alpha > 1$, in iteration $t$, we compute the parameters in the lower bound problem, $\{r_{n,\alpha}^{t-1}\}$ and $\{s_{n,\alpha}^{t-1}\}$, by (4.25), and obtain the transmission probability of each tier by solving optimization problem $P_{LB}^{3,n}$, for $1 \leq n \leq N$, via bi-section search. We thus iteratively update the transmission probability of each tier in this way until convergence.

The pseudo code of MMTS is presented in Algorithm 3. In each iteration of MMTS, the lower bound problem either has a straightforward closed-form solution for $0 \leq \alpha \leq 1$, or is otherwise separable into $N$ one-dimensional convex optimization problems, which
can be solved efficiently by bisected search. Thus the computational complexity in each iteration is low. Specifically, when $0 \leq \alpha < 1$, the main computation in each iteration is the calculation of parameters $\{d_{t-1}^n\}$ and $\{e_{t-1}^n\}$ related to the sub-problems, since the solution of problems in each iteration is in closed-form. The computational complexity of this is $O(NL)$, where $N$ is the number of tiers, and $L$ is the number of SIR thresholds. When $\alpha = 1$, similar to the $0 \leq \alpha < 1$ case, the main computation in each iteration is the calculation of parameters $\{g_{t-1}^n\}$, the computation complexity of which is also $O(NL)$. When $\alpha > 1$, the computation in each iteration consists of two parts, the computation of parameters $\{s_{t-1}^n\}$ and $\{r_{t-1}^n\}$ with computational complexity $O(N^2L)$, and the computation of gradient in bisection search to solve the sub-problems with computational complexity $O(N^2L)$. In summary, the computational complexity in each iteration is $O(NL)$ for $0 \leq \alpha \leq 1$ and $O(N^2L)$ for $\alpha > 1$.

Furthermore, such decomposition of the original problem into $N$ sub-problems allows distributed computation by each tier, separately from the other tiers. This can be implemented either by some elected representative for each tier, or by each individual node in each tier. In one possible distributed implementation scenario, a representative first collects the intensity of each tier. This information can be collected by some central node, such as a base station, and sent to each representative. Alternatively, the representative of each tier collects the local intensity information in its neighborhood, and exchanges such information among all representatives, so that they can estimate the intensity of all tiers. Then, in each iteration of MMTS, each representative solves the sub-problem for its own tier. Specifically, each representative runs lines 5-21 in Algorithm 1 and sends the updated transmission probability of its tier to the representatives in other tiers. The information exchange can be realized, for example, via the methods in [5, 65].

### 4.5 Convergence and Optimality of MMTS

In this section, we discuss the convergence and optimality of MMTS. First, as stated in Theorem 12, we observe that MMTS always converges to a KKT point of problem $\mathcal{P}$.

**Theorem 12.** MMTS converges to the objective value of a KKT point of optimization problem $\mathcal{P}$.

**Proof.** See Appendix 4.8.5.

A KKT point can be the global optimum, a local optimum, or even a saddle point.

\(^1\)The KKT conditions are necessary conditions for global or local optimality.
Algorithm 3 MMTS for Solving $\mathcal{P}$

**Input:** $\{T_l\}$, $\{r_l\}$, $\{P_n\}$, $\{R_n\}$, $\{\lambda_n\}$, $\{p_{n,\text{min}}\}$, $\{p_{n,\text{max}}\}$, $\alpha$, $\gamma$, $N$, $L$.

**Output:** $p$.

1. Compute $a_l = r_l - r_{l-1}$, $C_l = \pi T_l \frac{2}{\gamma} \Gamma(1 - \frac{2}{\gamma}) \Gamma(1 + \frac{2}{\gamma})$, $P'_n = P_n^{\frac{2}{\gamma}}$, $m_{nl} = \frac{R_n^2 C_l}{P'_n}$, for $1 \leq n \leq N$, $1 \leq l \leq L$, and $\alpha' = (N + 1)(1 - \alpha)$.

2. Pick initial point $p^0 = [p^0_n]_{N \times 1}$ in $\mathcal{Q}$, and set $t = 0$.

3. repeat
   4. $t = t + 1$.
   5. if $0 \leq \alpha < 1$ then
      6. Compute $\{d^{t-1}_n\}$ and $\{e^{t-1}_n\}$ by (3.21), and set $p^t_n = \left[ \frac{e^{t-1}_n}{p_{n,\text{max}}} \right]_{p_{n,\text{min}}}^{p_{n,\text{max}}}$, for $1 \leq n \leq N$.
   7. else if $\alpha = 1$ then
      8. Compute $\{g^{t-1}_n\}$ by (4.18), and set $p^t_n = \left[ \frac{1}{p_{n,\text{max}}} \right]_{p_{n,\text{min}}}^{p_{n,\text{max}}}$, for $1 \leq n \leq N$.
   9. else
      10. Compute $\{r^{t-1}_n\}$ and $\{s^{t-1}_nl\}$ by (4.25).
      11. for $n \in \{1, ..., N\}$ do
          12. $p_{\text{lower}} = p_{n,\text{min}}$ and $p_{\text{upper}} = p_{n,\text{max}}$.
          13. repeat
              14. $p_{\text{mid}} = (p_{\text{lower}} + p_{\text{upper}})/2$.
              $g = r^{t-1}_n p_{\text{mid}}^{\alpha'-1} - \sum_{j=1}^{N} \sum_{l=1}^{L} s^{t-1}_{nl} m_{jl} P'_n \lambda_n \exp(-\alpha' m_{jl} P'_n \lambda_n p_{\text{mid}})$.
              if $g > 0$ then
                  $p_{\text{lower}} = p_{\text{mid}}$.
              else
                  $p_{\text{upper}} = p_{\text{mid}}$.
              end if
          15. until convergence
      16. end for
      17. Set $p^t_n = p_{\text{mid}}$.
   18. end if
5. until convergence
23. end if
24. return $p^t = [p^t_n]_{N \times 1}$
In the following, we present some sufficient conditions under which MMTS converges to the global optimum.

Define \( q_{il,\text{max}} = \exp(-m_{il} \sum_{k=1}^{N} P_k' \lambda_k p_{k,\text{min}}) \), and \( q_{il,\text{min}} = \exp(-m_{il} \sum_{k=1}^{N} P_k' \lambda_k p_{k,\text{max}}) \).

Define \( a_{i,\text{max}} = \sum_{l=1}^{L} a_l q_{il,\text{max}}, a_{i,\text{min}} = \sum_{l=1}^{L} a_l q_{il,\text{min}}, \xi_{i,\text{max}} = \sum_{l=1}^{L} a_l m_{il} q_{il,\text{max}}, \xi_{i,\text{min}} = \sum_{l=1}^{L} a_l m_{il} q_{il,\text{min}}, \delta_{i,\text{max}} = \sum_{l=1}^{L} a_l m_{il}^2 q_{il,\text{max}}, \) and \( \psi_{i,\text{max}} = (\lambda_i p_{i,\text{max}} a_{i,\text{max}})^{1-\alpha} \). Further define,

\[
\omega_{i,\text{min}} = \sum_{j=1}^{N} \frac{(\lambda_j p_{j,\text{min}})^{1-\alpha} \xi_{j,\text{min}}^{\alpha}}{a_{j,\text{max}}^{\alpha}},
\]

\[
V_{ii} = \max\{ \frac{a_{i,\text{max}}^{1-\alpha}}{w_{i,\text{min}}^2 P_i' (\lambda_i p_{i,\text{min}})^{\alpha}} + \psi_{i,\text{max}} \sum_{j=1}^{N} \frac{(\lambda_j p_{j,\text{max}})^{1-\alpha} \delta_{j,\text{max}}^{\alpha}}{w_{j,\text{min}}^2 a_{j,\text{min}}^{\alpha}} - \frac{\alpha \xi_{i,\text{min}}^2}{\xi_{i,\text{max}}^{\alpha}} \},
\]

\[
V_{ij} = \max\{ \psi_{i,\text{max}} \sum_{j=1}^{N} \frac{(\lambda_j p_{j,\text{max}})^{1-\alpha} \delta_{j,\text{max}}^{\alpha}}{w_{j,\text{min}}^2 a_{j,\text{min}}^{\alpha}} - \frac{\alpha \xi_{i,\text{min}}^2}{\xi_{i,\text{max}}^{\alpha}} \}, \forall j \neq i.
\]

**Theorem 13.** For \( 0 \leq \alpha < 1 \), if

\[(1 - \alpha)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij}^2 < 1, \quad (430)\]

then MMTS converges to the global optimum of problem \( \mathcal{P} \).

**Proof.** See Appendix 4.8.6. \( \square \)

Based on Theorem 13, we have a corollary as stated in Corollary 14. First, we define \( q_{1,\text{max}} = \max_{i,l} q_{il,\text{max}}, q_{1,\text{min}} = \min_{i,l} q_{il,\text{min}}, q_{2,\text{max}} = \max_{i,l} m_{il} q_{il,\text{max}}, q_{2,\text{min}} = \min_{i,l} m_{il} q_{il,\text{min}}, q_{3,\text{min}} = q_{2,\text{min}} \sum_{j=1}^{N} \min\{1, \lambda_j p_{j,\text{min}}\}, \) and \( q_{4,\text{max}} = \max_{i,l} m_{il}^2 q_{il,\text{max}} \). We further define

\[
\tilde{\beta} = \sum_{i=1}^{N} \frac{1}{P_i^2} \min\{1, \lambda_i p_{i,\text{min}}\} + N \frac{\max\{1, \lambda_i p_{i,\text{max}}\} q_{2,\text{max}}}{q_{1,\text{min}} q_{3,\text{min}}},
\]

\[
\tilde{\beta} = N \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\max\{1, \lambda_j p_{j,\text{max}}\} \max\{1, \lambda_j p_{j,\max} p_{1,\text{max}}\} q_{4,\text{max}}}{q_{1,\text{min}} q_{3,\text{min}}^2}.
\]
\[ \hat{\beta} = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\rho_{2,\max} \max\{1, \lambda_j p_{j,\max} \rho_{1,\max}\}}{\rho_{1,\min} \rho_{3,\min} \sum_{i=1}^{2} P_i \min\{1, \lambda_i p_{i,\min}\}}. \]

**Corollary 14.** For \( 1 - \frac{1}{\hat{\beta}} < \alpha < 1 \), if
\[
\hat{\beta} = \frac{1}{(\hat{\beta} + (1 - \alpha)\hat{\beta})^{\frac{1}{2(1-\alpha)}}},
\]
then MMTS converges to the global optimum.

**Proof.** See Appendix 4.8.6.

Remark: Corollary 14 asserts that for \( 1 - \frac{1}{\hat{\beta}} < \alpha < 1 \), when the maximum transmission rate \( r_L \) is sufficiently small, MMTS is optimal. Furthermore, larger minimum transmission probability of each tier, i.e., \( p_{i,\min} \), for all \( i \), or smaller maximum transmission probability of each tier, i.e., \( p_{i,\max} \), for all \( i \), can increase the range of \( \alpha \) where MMTS is optimal.

For \( \alpha = 1 \), we have the following sufficient condition as stated in Theorem 15.

**Theorem 15.** For \( \alpha = 1 \), if
\[
T_L < \left( \frac{1}{N^2} + 1 \right)^{\hat{\beta}} T_1,
\]
then MMTS converges to the global optimum.

**Proof.** See Appendix 4.8.7.

Remark: Theorem 15 asserts that, for \( \alpha = 1 \), i.e., when proportional fairness is our optimization objective, if the maximum SIR threshold \( T_L \), and the minimum SIR threshold \( T_1 \), are sufficiently close, the number of tiers \( N \) is sufficiently small, or the pathloss exponent \( \gamma \) is sufficiently large, then MMTS is globally optimal.

For \( \alpha > 1 \), we have another sufficient condition as stated in Theorem 16. We define
\[
o_{i,\max}' = \max\{1, \lambda_i p_{i,\max} o_{i,\max}\}, \quad o_{i,\min}' = \min\{1, \lambda_i p_{i,\min} o_{i,\min}\}, \quad w_{\min}' = \sum_j m_j o_{j,\min}', \quad \text{and} \quad \psi_{i,\max}' = \max\{1, \lambda_i p_{i,\max} o_{i,\max}\}.
\]
We further define
\[
W_{ii} = \max\left\{ \frac{o_{i,\max}'}{w_{\min}' P_i \lambda_i p_{i,\min}}, o_{i,\max}', \sum_{j=1}^{N} \frac{m_j o_{j,\max}'}{w_{\min}'^2}, m_i o_{i,\max}', \frac{o_{i,\max}'}{w_{\min}'}, \frac{m_i o_{i,\max}'}{w_{\min}'^2 P_i \lambda_i p_{i,\min}} \right\},
\]
\[
W_{ij} = \max\left\{ \psi_{i,\max}', \sum_{j=1}^{N} \frac{m_j o_{j,\max}'}{w_{\min}'^2}, m_i o_{i,\max}', \frac{\psi_{i,\max}'}{w_{\min}'^2 P_j \lambda_j p_{j,\min}} \right\}, \quad \forall j \neq i.
\]
Theorem 16. For $\alpha > 1 - \frac{1}{M}$, where $M = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} W_{ij}^2}$, if $L = 1$, then MMTS converges to the global optimum.

Proof. See Appendix 4.8.8.

4.6 Numerical Performance Evaluation

In this section, we study the convergence of MMTS and the impact of different system parameters on the performance. We set the number of tiers $N = 10$. The D2D distance in these tiers are $\{15, 20, \cdots, 60\}$ m. The transmission power of each tier is randomly generated between 1 mW and 5 mW in a uniform manner. We choose five SIR thresholds $\{0.2025, 0.7494, 4.4926, 26.1397, 96.1391\}$ from [58], which corresponds to three transmission rates $\{0.1523, 0.6016, 1.9141, 3.9023, 5.5547\}$ bit/s/Hz in [59]. The default pathloss exponent $\gamma$ is 4. The maximum transmission probability, $p_{n,\text{max}}$, and minimum transmission probability, $p_{n,\text{min}}$, is set as 1 and $10^{-6}$, respectively, for $1 \leq n \leq 10$. We evaluate the performance of MMTS when the number of D2D pairs is 1000, 3000, 5000, 10000, and 30000 per cell and the cell size is 500m, as recommended by [66]. This corresponds to the sum intensity of all tiers, $\lambda$, is $1.3 \times 10^{-3}$, $3.6 \times 10^{-3}$, $6.5 \times 10^{-3}$, $1.3 \times 10^{-2}$, and $3.9 \times 10^{-2}$. We further added an extra smaller intensity setting of $0.65 \times 10^{-3}$ for the simulations of wider simulation settings. We uniformly generate the intensity of each tier. For the $\alpha$-fair utility, we consider the unit of the average throughput per unit area, with the unit bit/s/Hz/m$^2$.

We choose the initial point in MMTS in the following log-uniform manner. We uniformly draw the initial point, $y^0 = [y_i^0]_{N \times 1}$, from set $\mathcal{Y}' = \{y \in \mathbb{R}^{N} | \log_{10} p_{n,\text{min}} \leq y_n \leq \log_{10} p_{n,\text{max}}, 1 \leq n \leq N\}$. Then the initial point in MMTS is $p_i^0 = 10^{y_i}$, for $1 \leq i \leq N$. Based on our numerical observations, the KKT points of the optimization problem tend to be small. Hence, the selection of initial points in log-uniform manner allows us to select smaller initial points with higher probability, which can speed up the the rate of convergence to a KKT point. We run MMTS with multiple initial points, from which we pick the best one that leads to the maximum utility at convergence. Since we do not have prior knowledge about the location of the global optimum, by running MMTS with multiple initial points, we can increase the probability of MMTS converging to the global optimum.

We compare the performance of MMTS with 1 and 5 initial points to the following alternatives: 1) the optimum by exhaustive search method where we exhaustively search for the global optimum over the feasible set, 2) the equal transmission probability method
where we compute a single transmission probability for all transmitters, which is achieved via MMTS over a single tier in which the D2D distance is set to the average distance among all tiers, i.e., 37.5m, an approximation to the actual multi-tier network topology, and the transmission power is the average of the transmission power of all tiers, 3) the method in [26]. The method proposed in [26] is based on a single tier of D2D pairs and a single SIR threshold. We use it to compute the transmission probability after setting the SIR threshold to the average of all SIR thresholds, the transmission power to the average, and D2D distance to 37.5m. We then assign this probability to all D2D pairs. When we compute the utility of this method, we use multiple SIR thresholds and multiple tiers for fair comparison. Note that [26] further presents a method based on Shannon rate upper bound, which gives similar performance as that in the single SIR threshold, and is omitted for brevity.

4.6.1 Convergence of MMTS

We study the impact of number of tiers and $\alpha$ on the convergence of MMTS. For some given number of tiers, we select evenly spaced D2D distances between 15m and 60m. The convergence condition is that the relative difference of the objective value in consecutive iterations is less than $10^{-3}$.

In Fig. 4.2, we show the convergence behavior of MMTS in one realization. We observe that the objective increases in each iteration as expected in the MM framework. In addition, in this realization, MMTS meets the convergence condition in 11, 4, and 53 iterations, when $\alpha = 0$, $\alpha = 1$, and $\alpha = 2$, respectively. Table 4.2 further summarizes the average number of iterations when the convergence condition is met. We observe that for $\alpha = 0.5$ and $\alpha = 1$, the number of tiers has little impact on the number of iterations, while for other $\alpha$ values, the number of iterations increases almost linearly in the number of tiers. Furthermore, the simulation results suggest that the value of $\alpha$ has significant impact on the convergence behavior. MMTS converges faster when $\alpha$ is close to 1.

4.6.2 Impact of Different System Parameters

We study the impact of intensity, pathloss, and number of SIR thresholds on the utility of the four schemes, when $\alpha = 0$, $\alpha = 1$, and $\alpha = 2$. In Figs. 4.3-4.5, we see that under different $\alpha$ values, the utility of MMTS is very close to the optimal utility via exhaustive search. For the case $\alpha = 0$, we observe that running MMTS with multiple initial points can improve the utility, since it can pick a better KKT point compared with running MMTS with 1 initial point. For the cases $\alpha = 1$ and $\alpha = 2$, the utility
of MMTS is almost identical to that of the optimal utility via exhaustive search. The reason is that under the simulation settings, the optimization problem is highly likely to have a single KKT point, which is globally optimal by default. Even though the sufficient conditions of Theorems 13, 15, and 16 are not always satisfied, our numerical data indicate that MMTS often converges to a good KKT point over a wide range of parameter settings. Furthermore, MMTS substantially outperforms the method in [26] and the equal transmission probability method. This suggests the importance of assigning different transmitting probabilities to different types of D2D pairs in maximizing the \( \alpha \)-fair utility.

**Impact of Intensity**

In Figs. 4.3a-4.3c, we study the utility achieved by MMTS and the other schemes under different node intensity settings. These figures show that, when the intensity is sufficiently high, the utility of MMTS does not increase with respect to the intensity, i.e., the system is “saturated”. The transmission of a D2D pair introduces interference to other transmitting

Table 4.2: Number of iterations versus number of tiers

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th># of iterations</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0 )</td>
<td>10.3</td>
<td>14.2</td>
<td>17.3</td>
<td>19.1</td>
<td>20.8</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0.25 )</td>
<td>11.0</td>
<td>13.0</td>
<td>13.5</td>
<td>13.6</td>
<td>13.7</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0.5 )</td>
<td>9.8</td>
<td>8.9</td>
<td>8.7</td>
<td>8.5</td>
<td>8.5</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 0.75 )</td>
<td>6.6</td>
<td>6.5</td>
<td>6.5</td>
<td>6.6</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 1 )</td>
<td>4.1</td>
<td>4.0</td>
<td>4.0</td>
<td>4.2</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 1.25 )</td>
<td>12.8</td>
<td>15.9</td>
<td>18.6</td>
<td>21.7</td>
<td>24.5</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 1.5 )</td>
<td>21.3</td>
<td>28.8</td>
<td>36.2</td>
<td>43.4</td>
<td>50.3</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 1.75 )</td>
<td>31.0</td>
<td>44.7</td>
<td>58.9</td>
<td>71.8</td>
<td>84.0</td>
<td></td>
</tr>
<tr>
<td>( \alpha = 2 )</td>
<td>40.7</td>
<td>61.6</td>
<td>82.2</td>
<td>101.0</td>
<td>118.9</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.2: Convergence behavior
D2D pairs. When the intensity is sufficiently large, allowing more D2D pairs to transmit does not benefit the utility because of the introduced interference. In addition, we observe that the intensity where the system becomes saturated is larger when $\alpha = 0$ is larger than that when $\alpha = 1$ and $\alpha = 2$. The system is more fair when maximizing the utility with a larger $\alpha$ value. Generally, with larger $\alpha$, D2D pairs in favorable transmission conditions (e.g., shorter D2D distance or higher transmission power) receive lower throughput, while D2D pairs in unfavorable transmission conditions receive higher throughput. With a larger $\alpha$ value, in order to improve the throughput of the D2D pairs in unfavorable transmission conditions, the interference from other D2D pairs is reduced by decreasing the density of the transmitting D2D pairs. Therefore, the intensity where the system becomes saturated becomes smaller with a larger $\alpha$ value.

**Impact of Pathloss Exponent**

In Figs. 4.4a-4.4c, the utility under different pathloss exponent settings is studied. We observe that the utility of MMTS increases with the increase of pathloss exponent $\gamma$. But the behavior of MMTS with respect to $\gamma$ is different under different $\alpha$ value. The
increase of pathloss exponent leads to the weakening of the useful signal, as well as decrease of interference from other D2D pairs. It has different impact on the tiers with different D2D distance and transmission power. From these figures, we see that the utility changes differently with respect to $\gamma$ under different fairness index $\alpha$ settings. In addition, we observe similar behavior in other alternative schemes.

### Impact of SIR Thresholds

In Figs. 4.5a-4.5c, we study the utility versus the number of SIR thresholds. Evenly spaced SIR thresholds from \{0.2025, 0.4808, 1.1915, 4.4926, 16.9395, 38.7972, 96.1391\} from [58], which corresponds to transmission rates \{0.1523, 0.377, 0.8770, 1.9141, 3.3223, 4.5234, 5.5547\} bit/s/Hz in [59]. We observe that MMTS increases with more thresholds. However, the increase of utility becomes flat when the number of SIR thresholds is sufficiently large. Numerical results such as these can provide design guidelines to system operators on the appropriate number of modulation-coding levels to balance the transmitter complexity and the system performance. Furthermore, in Fig. 4.5c, the utility of the equal transmission probability scheme decreases with more SIR thresholds when $\alpha = 2$, and the number of thresholds is larger than 3. In equal transmission probability scheme, we approximate the D2D distance of all the D2D pairs with the average when computing the optimal transmission probability. This approximation results in lower likelihood of successful transmission of D2D pairs with longer D2D distance and shorter transmission power when the system has high SIR thresholds. This increases unfairness, and thus decreases the utility. We observe that in Fig. 4.5a and Fig. 4.5b, the utility of the method in [26] slightly decreases when the number of SIR thresholds increase from 3 to 5. This is also due to the approximation of the multi-tier multi-rate system to the single-tier single rate system in [26].
4.7 Summary

We have studied the optimization of the transmit probabilities in a multi-tier, multi-rate spatial Aloha network with multiple received SIR thresholds with respect to spatial $\alpha$-fairness. For different ranges of $\alpha$, the proposed MMTS algorithm utilizes a sequence of iteratively updated lower bound problems, which in turn are decomposed into tier-separable one-dimensional convex problems. The convergence to a KKT point is always guaranteed, and several sufficient conditions for global optimality are given. In numerical evaluation, we present the convergence behavior of MMTS and find that MMTS converges faster when $\alpha$ is close to 1. We further study the impact of different system parameters, including the intensity, the pathloss, and the number of SIR thresholds. Our simulation results suggest that MMTS is nearly optimal and has substantial advantage over prior solutions. This chapter has been published in [67].

4.8 Appendices

4.8.1 Derivation of Laplace Transform of $I$

The derivation of the Laplace transform of $I$ is as follows:

$$
\mathbb{L}(s) = \mathbb{E}[\exp(-sI)]
$$

$$
= \mathbb{E}_I \left[ \exp \left( -s \left( \sum_{k=1, k\neq n}^N \sum_{x \in \Phi^t_k} \frac{P_k h_{x0}}{|x|^\gamma} + \sum_{x \in \Phi^t_n \setminus \{x_n\}} \frac{P_n h_{x0}}{|x|^\gamma} \right) \right) \right]
$$

$$
= \prod_{k=1, k\neq n}^N \mathbb{E}_{\Phi^t_k \setminus \{h_{x0}\}} \left[ \exp \left( -s \sum_{x \in \Phi^t_k} \frac{P_k h_{x0}}{|x|^\gamma} \right) \right]
$$

$$
= \prod_{k=1, k\neq n}^N \mathbb{E}_{\Phi^t_k} \left[ \prod_{x \in \Phi^t_k} \mathbb{E}_{h_{x0}} \left[ \exp \left( -s \frac{P_k h_{x0}}{|x|^\gamma} \right) \right] \right]
$$

$$
= \prod_{k=1, k\neq n}^N \mathbb{E}_{\Phi^t_k} \left[ \prod_{x \in \Phi^t_k \setminus \{x_n\}} \mathbb{E}_{h_{x0}} \left[ \exp \left( -s \frac{P_n h_{x0}}{|x|^\gamma} \right) \right] \right]
$$

$$
= \prod_{k=1, k\neq n}^N \mathbb{E}_{\Phi^t_k} \left[ \prod_{x \in \Phi^t_k \setminus \{x_n\}} \frac{1}{1 + s \frac{P_k}{|x|^\gamma}} \right]
\prod_{x \in \Phi^t_n \setminus \{x_n\}} \frac{1}{1 + s \frac{P_n}{|x|^\gamma}}
$$
\[
\begin{align*}
&\overset{(c)}{=} \prod_{j=1}^N \exp \left( p_j \lambda_j \int_{\mathbb{R}^2} \left( 1 - \frac{1}{1 + s \frac{P_j}{\sigma^2}} \right) \, dx \right) \\
&\overset{(d)}{=} \prod_{j=1}^N \exp \left( -p_j \lambda_j 2\pi \int_0^{\infty} \left( 1 - \frac{1}{1 + s \frac{P_j}{\sigma^2}} \right) \, ddd \right) \\
&\overset{(e)}{=} \prod_{j=1}^N \exp \left( -p_j \lambda_j \pi (s P_j)^{\frac{2}{\gamma}} \Gamma(1 - \frac{2}{\gamma}) \Gamma(1 + \frac{2}{\gamma}) \right),
\end{align*}
\]

where (a) is based on the independence between \( \{h_{x_0}\} \) and \( \{\Phi_k\} \), (b) is from the distribution of \( \{h_{x_0}\} \), (c) is from the Slivnyak’s theorem and probability generating functional (PGFL) of homogeneous PPP, (d) is based on the transformation to polar coordinate, and (e) is from the manipulation of Gamma function.

### 4.8.2 Proof of Lemma 7

First, we show that function \( f(x) : \mathbb{R}^N \to \mathbb{R} \), given by

\[
 f(x) = \frac{\left( \sum_{n=1}^N \rho_n \exp(x_n) \right)^\beta}{\beta},
\]

is convex in \( x = [x_n]_{N \times 1} \in \mathbb{R}^N \), for \( \rho_n \geq 0, 1 \leq n \leq N \), and \( \beta > 0 \). We rewrite \( f(x) \) as

\[
 f(x) = \frac{1}{\beta} \exp \left( \beta \log \left( \sum_{n=1}^N \rho_n \exp(x_n) \right) \right). 
\]

Then \( f(x) \) can be viewed as the composition of \( h(y) : \mathbb{R} \to \mathbb{R} \) and \( g(x) : \mathbb{R}^N \to \mathbb{R} \), i.e., \( f(x) = h(g(x)) \), where \( h(y) = \frac{1}{\beta} \exp(\beta y) \), and \( g(x) = \log \left( \sum_{n=1}^N \rho_n \exp(x_n) \right) \). Since \( h(y) \) is convex and nondecreasing in \( y \), and \( g(x) \) is convex in \( x \), according to the composition rule [64], function \( f(x) = h(g(x)) \) is convex in \( x \).

Due to the convexity of \( f(x) \), for all \( x \) and \( x^{t-1} \) in the domain of \( f \), we have \( f(x) \geq f(x^{t-1}) + \nabla f(x^{t-1})(x - x^{t-1}) \).

Hence

\[
\frac{\left( \sum_{n=1}^N \rho_n \exp(x_n) \right)^\beta}{\beta} \geq \frac{\left( \sum_{n=1}^N \rho_n \exp(x_{n}^{t-1}) \right)^\beta}{\beta} + \left( \sum_{j=1}^N \rho_j \exp(x_j^{t-1}) \right)^{\beta-1} \sum_{n=1}^N \rho_n \exp(x_{n}^{t-1})(x_n - x_{n}^{t-1}),
\]

and the equality holds when \( x = x^{t-1} \).

Since \( 0 \leq \alpha < 1 \), we have
\[
\left( \lambda_n p_n \sum_{l=1}^{L} a_l \exp(-m_{nl} \sum_{j=1}^{N} P'_j \lambda_j p_j) \right)^{1-\alpha} \geq \lambda_n^{1-\alpha} \left( \sum_{l=1}^{L} a_l \exp(\log p_n - m_{nl} \sum_{j=1}^{N} P'_j \lambda_j p_j) \right)^{1-\alpha} \]

\[
(\text{a}) 
\lambda_n^{1-\alpha} \left( \sum_{l=1}^{L} a_l \exp(\log p_n - m_{nl} \sum_{j=1}^{N} P'_j \lambda_j p_j) \right)^{1-\alpha}
\]

\[
\geq -d_n^{t-1} \sum_{j=1}^{N} P'_j \lambda_j (p_j - p_j^{t-1}) + e_n^{t-1} \log \frac{p_n}{p_n^{t-1}} + f_n^{t-1},
\]

where (a) results from \( p_n = \log(\exp(p_n)) \), (b) is from (4.31), and the equality holds if \( p = p^{t-1} \).

### 4.8.3 Proof of Lemma 8

Recall that \( e_n^{t-1} \geq 0 \) for all \( 1 \leq n \leq N \). Hence, \( e_n^{t-1} \log p_n \) and \( d_j^{t-1} P'_j \lambda_n p_n \) are both concave in \( p_n \). Thus, \( \tilde{U}_n(p_n) \) is concave in \( p_n \), which makes the optimization problem \( P_{1,n} \) convex. The Lagrangian is

\[
\mathcal{L}(p_n, \mu_1, \mu_2) = -\tilde{U}_n(p_n) + \mu_1(p_n - p_{n,\max}) + \mu_2(-p_n + p_{n,\min}).
\]

KKT conditions are

\[
\frac{\partial \mathcal{L}}{\partial p_n} \bigg|_{p_n=\tilde{p}_n} = -\frac{e_n^{t-1}}{\tilde{p}_n^*} + P'_n \lambda_n \sum_{j=1}^{N} d_j^{t-1} + \mu_1 - \mu_2 = 0,
\]

\[
\mu_1(\tilde{p}_n - p_{n,\min}) = 0,
\]

\[
\mu_2(p_{n,\max} - \tilde{p}_n^*) = 0,
\]

\[
p_{n,\min} \leq \tilde{p}_n^* \leq p_{n,\max},
\]

\[
\mu_1 \geq 0, \mu_2 \geq 0.
\]

If \( \tilde{p}_n^* \neq p_{n,\min} \) and \( \tilde{p}_n^* \neq p_{n,\max} \), then \( \tilde{p}_n = \frac{e_n^{t-1}}{P'_n \lambda_n \sum_{j} d_j^{t-1}} \) based on (4.33). If \( \tilde{p}_n^* = p_{n,\min} \), then \( \mu_2 = 0 \). Substitute \( \mu_1 \geq 0, \mu_2 = 0 \), and \( \tilde{p}_n^* = p_{n,\min} \) to (4.33), we have \( p_{n,\min} \leq \frac{e_n^{t-1}}{P'_n \lambda_n \sum_{j} d_j^{t-1}} \). Similarly, if \( \tilde{p}_n^* = p_{n,\max} \), we have \( p_{n,\max} \geq \frac{e_n^{t-1}}{P'_n \lambda_n \sum_{j} d_j^{t-1}} \). In conclusion,

\[
\tilde{p}_n^* = \left[ \frac{e_n^{t-1}}{P'_n \lambda_n \sum_{j} d_j^{t-1}} \right] p_{n,\max}.
\]
4.8.4 Proof of Lemma 10

Define \( x_n = P_n' \lambda_n p_n, x_n^{t-1} = P_n' \lambda_n p_n^{t-1}, \nu_n^{t-1} = \sum_{j=1}^{N} a_t' \exp(-m_{nt} \sum_{j=1}^{N} x_j^{t}), \) and \( \theta_{nt}^{t-1} = \exp(-m_{nt} \sum_{j=1}^{N} x_j^{t-1}), \) for all \( n, l. \) Then we have

\[
a_t x_n \exp(-m_{nt} \sum_{j=1}^{N} x_j) = \frac{a_t' \nu_n^{t-1}}{\nu_n^{t-1}} x_n \exp(-m_{nt} \sum_{j=1}^{N} x_j).
\]

Note that \( \sum_{l=1}^{L} \frac{a_t' \nu_n^{t-1}}{\nu_n^{t-1}} = 1. \)

Hence, we have

\[
\left( \sum_{l=1}^{L} a_t x_n \exp(-m_{nt} \sum_{j=1}^{N} x_j) \right)^{1-\alpha} = \left( \sum_{l=1}^{L} \frac{a_t' \nu_n^{t-1}}{\nu_n^{t-1}} x_n \exp(-m_{nt} \sum_{j=1}^{N} x_j) \right)^{1-\alpha}
\]

\[
\leq \sum_{l=1}^{L} a_t \left( \frac{\nu_n^{t-1}}{\nu_n^{t-1}} \right)^\alpha x_n^{1-\alpha} \prod_{j=1}^{N} \exp(-m_{nt}(1-\alpha)x_j),
\]

where \( (a) \) is from Jensen’s inequality and the fact that function \( f(x) = x^\beta \) is convex in \( x \) when \( \beta < 0 \) and \( x > 0. \) Note that equality holds when \( x = x^{t-1}, \) where \( x = [x_n]_{N \times 1}, \) and \( x^{t-1} = [x_n^{t-1}]_{N \times 1}. \)

Next, note that for all \( z_n > 0, z_n^{t-1} > 0, \) we have \( \prod_{n=1}^{N} z_n \leq \sum_{n=1}^{N} \frac{1}{N} \prod_{j=1, j \neq n}^{N} z_n^{t-1} N z_n, \) and the equality holds if \( z_n = z_n^{t-1}, \) for all \( n. \) This can be easily proved by the inequality of arithmetic and geometric means. Hence, we have

\[
x_n^{1-\alpha} \prod_{j=1}^{N} \exp(-m_{nt}(1-\alpha)x_j) \leq \frac{\zeta_{nt}^{t-1} x_n^{\alpha'}}{N+1} + \sum_{j=1}^{N} \phi_{ntj}^{t-1} \exp(-\alpha' m_{nt} x_j) \tag{4.38}
\]

where

\[
\zeta_{nt}^{t-1} = \prod_{k=1}^{N} \frac{\exp(-(1-\alpha)m_{nt} x_k^{t-1})}{x_k^{t-1(1-\alpha)N}} x_n^{t-1},
\]

\[
\phi_{ntj}^{t-1} = \frac{x_n^{t-1(1-\alpha)N} \prod_{k=1, k \neq j}^{N} \exp(-(1-\alpha)m_{nt} x_k^{t-1})}{\exp(-(1-\alpha)N m_{nt} x_j^{t-1})}.
\]

Recall that \( 1 - \alpha < 0. \) From (4.38), we have
Chapter 4. \(\alpha\)-Fair Utility Maximization in Spatial Aloha Networks

\[
\left( \sum_{l=1}^{L} a_l x_n \exp\left( -m_{nl} \sum_{j=1}^{N} x_j \right) \right)^{1-\alpha} \ \frac{1}{1 - \alpha} \\
\geq \sum_{i=1}^{L} a_i \left( \frac{\theta_{nl}^{t-1}}{\mu_{nl}^t} \right)^{\alpha} \left( \frac{c_{nl}^{t-1} x_n}{\alpha t} + \sum_{j=1}^{N} \frac{\theta_{njl}^{t-1} \exp(-\alpha t m_{jl} x_j)}{\alpha t} \right),
\]

where the equality holds if \(x = x^{t-1}\).

Substituting \(x_n = P'_n \lambda_n p_i\), and \(x^{t-1}_n = P'_n \lambda_n p^{t-1}_n\) into (4.39) completes the proof.

### 4.8.5 Proof of Theorem 12

First, we note that the feasible set is bounded. Hence, the sequence of points generated by MMTS, \(\{p_1, p_2, \cdots\}\) is bounded, and has at least one limit point [60].

We need to further verify the following two conditions of our proposed lower bounds to show that MMTS converges to a KKT point [61].

**Condition 1**: \(\forall p, p^{t-1} \in \mathcal{P};\) for \(0 \leq \alpha < 1, U(p) \geq \tilde{U}(p, p^{t-1});\) for \(\alpha = 1, U(p) \geq \tilde{U}(p, p^{t-1});\) and for \(\alpha > 1, U(p) \geq \tilde{U}(p, p^{t-1}).\) Furthermore, in all cases above, the equalities hold when \(p = p^{t-1}\).

This condition has been verified in section 4.4.

**Condition 2**: \(\forall p, p^{t-1} \in \mathcal{P};\) for \(0 \leq \alpha < 1\) and \(1 \leq k \leq N, \frac{\partial U(p)}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}};\) for \(\alpha = 1\) and \(1 \leq k \leq N, \frac{\partial U(p)}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}};\) and for \(\alpha > 1\) and \(1 \leq k \leq N, \frac{\partial U(p)}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}}.

To show that this condition is satisfied, we first note that

\[
\frac{\partial U(p)}{\partial p_k} = \lambda_k^{1-\alpha} p_k^{1-\alpha} \exp \left( -m_{il} \sum_{j=1}^{N} p_j \lambda_j P'_j \right) - \\
\sum_{i=1}^{N} (\lambda_i p_i)^{1-\alpha} P'_k \sum_{l=1}^{L} a_l \exp \left( -m_{il} \sum_{j=1}^{N} p_j \lambda_j P'_j \right). \\
\]

For \(0 \leq \alpha < 1,\)

\[
\frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}} = e_k' p^{t-1} - \sum_{i=1}^{N} a_i P'_i \lambda_k. \\
\]

Plugging (4.11) into (4.40), we can verify that \(\frac{\partial U(p)}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}},\) for \(1 \leq k \leq N.\)

Similarly, we can verify that \(\frac{\partial U(p)}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}},\) for \(\alpha = 1\) and \(1 \leq k \leq N,\) and \(\frac{\partial U(p)}{\partial p_k}|_{p=p^{t-1}} = \frac{\partial \tilde{U}(p, p^{t-1})}{\partial p_k}|_{p=p^{t-1}},\) for \(\alpha > 1\) and \(1 \leq k \leq N.\)
4.8.6 Proof of Theorem 13

First, we present a useful lemma.

**Lemma 17.** [68] Consider an iterative algorithm,

\[ x(t + 1) = T(x(t)), t = 0, 1, \ldots, \]

where mapping \( T : \mathcal{X} \to \mathcal{X} \), and \( \mathcal{X} \) is a closed subset of \( \mathbb{R}^N \). If \( T \) satisfies

\[ |T(x) - T(y)| \leq \sigma |x - y|, \forall x, y \in \mathcal{X} \]

where \( |\cdot| \) is some norm and \( \sigma \) is a constant in \([0, 1]\), then the mapping \( T \) has unique fixed point \( x^* \). In addition, for every initial point \( x^0 \in \mathcal{X} \), the sequence generated by \( x(t + 1) = T(x(t)) \) converges to \( x^* \).

For notational convenience, we let \( x_i = P'_i \lambda_i p_i, \ x_t = P'_i \lambda_t p_i, \ x_{i, \text{max}} = P'_i \lambda_t p_{i, \text{max}}, \ x_{i, \text{min}} = P'_i \lambda_t p_{i, \text{min}}, \ 1 \leq i \leq N \), and \( \mathcal{X} = \{ x \in \mathbb{R}^N | x_{i, \text{min}} \leq x_i \leq x_{i, \text{max}}, 1 \leq i \leq N \} \).

We further define a vector-valued function \( f(x) \), where

\[ f_i(x) = \left( \frac{\sum_{l=1}^{L} a_l x_{l, \text{exp}}(-m_d \sum_{k=1}^{N} x_k)}{P'_i} \right)^{1-\alpha}, \forall i. \tag{4.41} \]

The Jacobian matrix of \( f(x) \) is defined as \( J = \left[ \frac{\partial f_i}{\partial x_j} \right]_{N \times N} \).

Furthermore, we use the following mapping

\[ x_{i+1} = \left[ f_i(x') \right]_{x_{i, \text{max}}}^{x_{i, \text{min}}}. \]

Let \( o_i = \sum_{l=1}^{L} a_l \exp(-m_d \sum_{k=1}^{N} x_k), \xi_i = \sum_{l=1}^{L} a_l m_d \exp(-m_d \sum_{k=1}^{N} x_k) \), and \( \delta_i = \sum_{l=1}^{L} a_l m_d^2 \exp(-m_d \sum_{k=1}^{N} x_k) \).

The numerator and denominator of \( f_i(x) \) is given by \( \psi_i(x) = \frac{x_i^{1-\alpha} o_i^{1-\alpha}}{P'_i^{1-\alpha}}, \) and \( \omega(x) = \sum_{j' \neq i} \frac{x_{j'}^{1-\alpha} \xi_{j'}}{P'_{j'}^{1-\alpha} o_{j'}^{1-\alpha}} \) respectively. After simple derivation, the partial derivatives of \( \psi_i(x) \) and \( \omega(x) \) are give by

\[ \frac{\partial \psi_i}{\partial x_i} = \frac{(1-\alpha)(o_i - x_i \xi_i)}{P'_i^{1-\alpha} x_i^{\alpha} o_i^{\alpha}}, \quad \frac{\partial \psi_i}{\partial x_j} = \frac{(1-\alpha) x_i^{1-\alpha} \xi_i}{P'_i^{1-\alpha} o_i^{\alpha}}, \quad \forall j \neq i, \]
and
\[
\frac{\partial \omega}{\partial x_j} = \sum_{j'} x_j^{1-\alpha} P_{j'}^{1-\alpha} \left( \frac{\alpha \xi_j^2}{o_{j'}} - \delta_{j'} \right) + \frac{(1 - \alpha) x_j^{-\alpha} \xi_j}{P_{j'}^{1-\alpha} o_{j'}^{\alpha}}, \forall j.
\]

Then we have
\[
\frac{\partial f_i}{\partial x_i} = \frac{\partial \psi_i}{\partial x_i} \omega - \frac{\partial \omega}{\partial x_i} \psi_i = \frac{(1 - \alpha) o_i}{w P_i^{1-\alpha} x_i o_i^{\alpha}} + \psi_i \sum_{j'} x_j^{1-\alpha} \left( \frac{P_{j'}^{1-\alpha} \xi_j}{w^{2} P_{j'}^{1-\alpha} o_{j'}^{\alpha}} \left( \delta_{j'} - \frac{\alpha \xi_j^2}{o_{j'}^{\alpha}} \right) \right)
\]
\[
- \left( \frac{(1 - \alpha) x_i \xi_i}{w P_i^{1-\alpha} x_i o_i^{\alpha}} + \psi_i \left( \frac{(1 - \alpha) x_i^{-\alpha} \xi_j}{w^{2} P_i^{1-\alpha} o_i^{\alpha}} \right) \right), \forall j \neq i.
\]

Based on the Cauchy-Schwarz inequality, it is easy to show
\[
\delta_j o_j \geq \xi_j^2, \forall j.
\]

Then since \(\alpha \leq 1\), we have \(\delta_j \geq \alpha \xi_j^2/o_j\). Then it is obvious that
\[
\left| \frac{\partial f_i}{\partial x_i} \right| \leq (1 - \alpha)V_{ii}, \left| \frac{\partial f_i}{\partial x_j} \right| \leq (1 - \alpha)V_{ij}
\]

Based on the assumption in Theorem 13, \(|J(x)| \leq \sqrt{(1 - \alpha)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij}^2} < 1\). Hence, for all \(x^1, x^2 \in X\), we have
\[
|f(x^1) - f(x^2)| \leq |J(\tilde{x})||x^1 - x^2| \leq \sqrt{(1 - \alpha)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij}^2|x^1 - x^2|},
\]
where \(\tilde{x}\) is a convex combination of \(x^1\) and \(x^2\). Based on Lemma 17, the proposed algorithm converges to the unique fixed point, which must be the global optimum. Recall
that \( a_t = r_t - r_{t-1} \) and \( r_0 = 0 \). Therefore, \( \sum_{t=1}^{L} a_t = r_L \). For notional convenience, we define \( a = r_L \). It is easy to prove that \( o_{i, \max} \leq a \varrho_{1, \max} \), \( o_{i, \min} \geq a \varrho_{1, \min} \), \( \xi_{i, \max} \leq a \varrho_{2, \max} \), \( \xi_{i, \min} \geq a \varrho_{2, \min} \), and \( \delta_{i, \max} \leq a \varrho_{4, \max} \) for all \( 1 \leq i \leq N \).

Furthermore,

\[
\omega_{\min} = \sum_{j} \left( \frac{\lambda_j p_{j, \min}}{\max_{r_j} a} \right)^{1-\alpha} \xi_{j, \min} \geq \sum_{j} \min \left\{ 1, \frac{\lambda_j p_{j, \min}}{\max_{r_j} a} \right\} \varrho_{2, \min} \geq \varrho_{3, \min} \geq \frac{1}{\varrho_{3, \min} \varrho_{1, \max}}.
\]

where (a) is is from the fact that \( \varrho_{d, \max} < 1 \) for all \( 1 \leq i \leq N \) and \( 1 \leq l \leq L \).

In addition, we have

\[
\psi_{i, \max} = (\lambda_i p_{i, \max} o_{i, \max})^{1-\alpha} \leq (\lambda_i p_{i, \max} \varphi_{1, \max})^{1-\alpha} a^{1-\alpha} \leq \max \{ 1, \lambda_i p_{i, \max} \varphi_{1, \max} \} a^{1-\alpha}
\]

Therefore,

\[
\frac{o_{i, \max}^{1-\alpha}}{w_{\min} P_i (\lambda_i p_{i, \min})^{\alpha}} \leq \frac{a^{1-\alpha} \varphi_{1, \max}^{1-\alpha}}{\varrho_{3, \min} \varrho_{1, \max}^{\alpha}} \leq \frac{1}{\varrho_{3, \min} \varrho_{1, \max} P_i \min \{ 1, \lambda_i p_{i, \min} \}}.
\]

Similarly, we have

\[
\frac{(\lambda_i p_{i, \max})^{1-\alpha} \delta_{i, \max}}{w_{\min}^2 o_{i, \min}^{\alpha}} \leq \frac{\max \{ 1, \lambda_i p_{i, \max} \} \varrho_{4, \max} a^{1-\alpha}}{\varrho_{1, \min} \varrho_{3, \min}^2},
\]

\[
\frac{(\lambda_i p_{i, \max})^{1-\alpha} \xi_{i, \max}}{w_{\min} o_{i, \min}^{\alpha}} \leq \frac{\max \{ 1, \lambda_i p_{i, \max} \} \varrho_{2, \max}}{\varrho_{1, \min} \varrho_{3, \min}},
\]

\[
\frac{(\lambda_i p_{i, \min})^{1-\alpha} \xi_{i, \max}}{w_{\min}^2 P_i o_{i, \min}^{\alpha}} \leq \frac{\varrho_{2, \max}}{\varrho_{1, \min} \varrho_{3, \min}^2 P_i \min \{ 1, \lambda_i p_{i, \min} \}} a^{1-\alpha}.
\]

After simple derivation, we have

\[
(1 - \alpha) \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij} \leq (1 - \alpha) \tilde{\beta} + (\tilde{\beta} + (1 - \alpha) \tilde{\beta}) a^{2(1-\alpha)}.
\]

Recall that \( a = r_L \) and \( 1 - \frac{1}{\beta} < \alpha < 1 \). If \( r_L < \left( \frac{1-(1-\alpha)\tilde{\beta}}{\tilde{\beta} + (1-\alpha)\tilde{\beta}} \right)^{\frac{1}{2(1-\alpha)}} \), we have

\[
(1 - \alpha) \tilde{\beta} + (\tilde{\beta} + (1 - \alpha) \tilde{\beta}) a^{2(1-\alpha)} < 1,
\]
and
\[(1 - \alpha)^2 \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij}^2 \leq \left( (1 - \alpha) \sum_{i=1}^{N} \sum_{j=1}^{N} V_{ij} \right)^2 < 1.\]

Based on Theorem 13, MMTS converges to the global optimum.

### 4.8.7 Proof of Theorem 15

For \(\alpha = 1\), we have
\[
\frac{\partial f_i}{\partial x_j} = \frac{1}{w^2} \sum_{j'} \left( \frac{\delta_{j'}}{\sigma_{j'}} - \frac{\xi_{j'}}{\sigma_{j'}} \right). \tag{4.44}
\]

Furthermore, \(\frac{\delta_{j'}}{\sigma_{j'}} \leq m_{j'L}^2\) and \(\frac{\xi_{j'}}{\sigma_{j'}} \geq m_{j'1}^2\) for all \(j'\). In addition, \(\omega = \sum_{j'} \frac{\xi_{j'}}{\sigma_{j'}} \geq \sum_{j'} m_{j'1}^2\).

Therefore
\[
\frac{\partial f_i}{\partial x_j} \leq \frac{1}{\left( \sum_{j'} m_{j'1} \right)^2} \sum_{j'} \left( m_{j'L}^2 - m_{j'1}^2 \right). \tag{4.45}
\]

Similar to the proof of Theorem 13, the sufficient condition for global optimality is
\[
\frac{1}{\left( \sum_{j} m_{j1} \right)^2} \sum_{j} \left( m_{jL}^2 - m_{j1}^2 \right) < \frac{1}{N^2}. \tag{4.45}
\]

We relax the sufficient condition (4.45) as
\[
\sum_{j} \left( m_{jL}^2 - m_{j1}^2 \right) < \frac{1}{N^2} \sum_{j} m_{j1}^2, \tag{4.46}
\]

based on the fact that \(\sum_{j} m_{j1}^2 \leq \left( \sum_{j} m_{j1} \right)^2\).

Plugging \(m_{il} = \frac{R_i^2}{P_i} \pi T_i \Gamma(1 - \frac{2}{\gamma}) \Gamma(1 + \frac{2}{\gamma})\) into (4.46), we have
\[
T_L < \left( \frac{1}{N^2} + 1 \right)^{\frac{2}{\gamma}} T_1. \tag{4.47}
\]

### 4.8.8 Proof of Theorem 16

For notational convenience, we let \(m_i = m_{i1}\) for all \(i\).
**Case 1**: $\alpha \in (1 - \frac{1}{M}, 1)$.

For $L = 1$, we have $\delta_i = m_i^2 o_i$ and $\xi_i = m_i o_i$. Similar to the proof of Theorem 13, we have

$$\left| \frac{\partial f_i}{\partial x_i} \right| = (1 - \alpha) \left| \frac{o_i^{1-\alpha}}{w P_i^{1-\alpha} x_i^{\alpha}} + \psi_i \sum_{j'} \frac{m_j^2 x_j^{1-\alpha} o_j^{1-\alpha}}{w P_j^{1-\alpha}} \right. $$

$$\left. - \left( \frac{x_i \xi_i}{w P_i^{1-\alpha} x_i^{\alpha} o_i^{\alpha}} + \psi_i \frac{x_i^{-\alpha} \xi_i}{w P_i^{1-\alpha} x_i^{-\alpha} o_i^{-\alpha}} \right) \right| \leq (1 - \alpha) W_{ii}, \hspace{1cm} (4.48)$$

and

$$\left| \frac{\partial f_i}{\partial x_j} \right| = (1 - \alpha) \left| \psi_i \sum_{j'} \frac{m_j^2 x_j^{1-\alpha} o_j^{1-\alpha}}{w P_j^{1-\alpha}} \right. $$

$$\left. - \left( \frac{x_j \xi_j}{w P_j^{1-\alpha} x_j^{\alpha} o_j^{\alpha}} + \psi_i \frac{x_j^{-\alpha} \xi_j}{w P_j^{1-\alpha} x_j^{-\alpha} o_j^{-\alpha}} \right) \right| \leq (1 - \alpha) W_{ij}, \forall j \neq i. \hspace{1cm} (4.49)$$

Since $\alpha > 1 - \frac{1}{M}$, $|\mathbf{J}| \leq (1 - \alpha) M < 1$. Following the same approach in Theorem 13, the proposed algorithm converges to the global optimum.

Based on the following inequality

$$\sum_i m_i \frac{\hat{\omega}_i^{\prime \text{max}}}{w^{\prime \text{min}}} = \sum_i m_i \frac{\hat{\omega}_i^{\prime \text{max}}}{w^{\prime \text{min}}} \geq 1,$$

we can further show that $M > N$ after simple manipulation of $W_{ij}$.

**Case 2**: $\alpha \geq 1$.

For $\alpha = 1$, since $L = 1$, the objective degrades to

$$U(\mathbf{p}) = \sum_{i=1}^{N} \log(\lambda_i p_i) + \log(a_1 \exp(-m_{i1} \sum_{j=1}^{N} P_j^\prime \lambda_j p_j)).$$

Obviously, $U(\mathbf{p})$ is concave in $\mathbf{p}$.

For $\alpha > 1$, since $L = 1$, the objective degrades to

$$U(\mathbf{p}) = \sum_{i=1}^{N} \left( \frac{\lambda_i p_i a_1 \exp \left( -m_{i1} \sum_{j=1}^{N} P_j^\prime \lambda_j p_j \right) }{1 - \alpha} \right)^{1-\alpha}. $$
Let
\[ g'_i(x) = \frac{(x_i \exp(-m_i \sum_{j=1}^{N} x_j))^{1-\alpha}}{(P'_i)^{1-\alpha}(1-\alpha)}, \]
where \( x \in \mathbb{R}_n^+ \).

Then we have
\[ g'_i(x) = \frac{(\exp(\log(x_i) - m_i \sum_{j=1}^{N} x_j))^{1-\alpha}}{(P'_i)^{1-\alpha}(1-\alpha)} = \frac{\exp((1-\alpha) \log(x_i) - (1-\alpha)m_i \sum_{j=1}^{N} x_j))}{(P'_i)^{1-\alpha}(1-\alpha)} \]
(4.50)
where \( h'_i(x) = (1-\alpha) \log(x_i) - (1-\alpha)m_i \sum_{j=1}^{N} x_j \). For \( \alpha > 1 \), \( h'_i(x) \) is convex in \( x \). Furthermore, \( \exp(x) \) is convex in \( x \) and non-decreasing with respect to \( x \). Hence, \( \exp(h'_i(x)) \) is convex in \( x \) according to the composition rule. Therefore \( g'_i(x) \) is concave in \( x \). We also have \( U(p) = \sum_{i=1}^{N} g'_i(p) \), where \( g'_i(p) = g'_i(x)|_{x_i=P'_i \lambda, p_i} \). Therefore, we conclude that \( U(p) \) is concave in \( p \).

Hence, for \( \alpha \geq 1 \) and \( L = 1 \), problem \( P \) is a convex optimization problem. Furthermore, it is easy to verify that Slater’s condition is satisfied. Thus, KKT conditions are sufficient for global optimality. Combining this with Theorem 12, we see that MMTS converges to the global optimum.
Chapter 5

On-line Power Control in D2D Networks with Delayed NSI

In the previous chapters, we aim to optimize the transmission probabilities of the D2D pairs to maximize the $\alpha$-fair utility. In this chapter, we consider maximizing the weighted sum-rate of the D2D pairs with delayed NSI feedback in a D2D network operating in the underlay mode. We consider the uplink power coordination among the D2D pairs, and impose the maximum interference constraint on the transmission power of all D2D pairs in order to guarantee the performance of cellular communication. Our main contributions are summarized as follows:

- We formulate the long-term averaged weighted sum-rate problem with the maximum interference constraint. We propose a method termed OPCD-FNF by recasting the original problem into per-time slot problems via a convexification technique and the on-line gradient method. In addition, we provide a computationally-efficient method to solve the per-time slot problem.

- We further study the scenario of limited NSI feedback, where only a limited number of D2D pairs are allowed to send their local NSI to the coordinator. We propose a method termed OPCD-PNF. In OPCD-PNF, D2D pairs are randomly selected to send their local NSI to the power control coordinator. We show that the expected performance of OPCD-PNF is the same as that of OPCD-FNF.

- We compare OPCD-FNF and OPCD-PNF with other alternatives. The simulation results validate that our proposed schemes are superior than the alternatives in a wide range of system parameter settings.
Note that in this chapter, we focus on the scenarios where the instantaneous NSI and its statistics are difficult to obtain.

5.1 Related Work

In this section, we briefly summarize existing works on power control in D2D networks and resource scheduling with delayed NSI feedback.

5.1.1 Power Control in D2D Networks

There are many existing works that consider power control in D2D networks. One category of such works imposes a restriction that a channel used by a CUE can be shared by at most one D2D pair. For example, a simple binary power control method is proposed in [69] to maximize the throughput utility in D2D communication, where all D2D pairs, CUEs, and BSs are equipped with single antennas. The authors of [70] further consider the sum of the logarithm rate utility and the transmission power cost. The authors of [71] aim at maximizing the energy efficiency of the D2D pairs with quality-of-service guarantee for D2D pairs and CUEs. The authors of [72] further consider sum-rate maximization in the case of multiple antennas under the constraint of maximum interference to other nodes.

References [73–77] consider power control in D2D networks where multiple D2D pairs are allowed to reuse one channel. The authors of [73] jointly optimize the transmission power and sub-channel allocation to maximize the sum rates of all D2D pairs, while the authors of [75] focus on the objective of maximizing the energy efficiency. The authors of [74] aim at minimizing the sum transmission power of all D2D pair with quality-of-service guarantee for both the D2D pairs and the CUE. Joint user association and power control to maximize the weighted sum rates is studied in [76]. Furthermore, the authors of [77, 78] consider maximizing the ergodic sum-rate with probabilistic outage and long-term averaged power constraints.

These work all assume that the instantaneous NSI is available when the transmission power of D2D pairs are determined. Hence, their proposed methods are not applicable to D2D networks with delayed NSI feedback.

5.1.2 Resource Scheduling with Delayed NSI

Delayed feedback of NSI or channel state information (CSI) has been considered in other settings of wireless networking. In [79–83], the authors adopt a Gauss-Markov channel
state prediction model, $h_{\text{ins}} = \rho h_{\text{delayed}} + \sqrt{1 - \rho^2} w$, to predict the instantaneous channel state $h_{\text{ins}}$ according to the delayed channel state $h_{\text{delayed}}$, where $\rho$ is the correlation coefficient, and $w$ is a circularly symmetric complex Gaussian random variable. Other works [84, 85] model the channel states as a finite-state Markov chain to predict the instantaneous channel state. In these works, their proposed schemes depend on (1) specific channel models, which may overly simplify the real channel; and (2) the statistics information of channels, which may be difficult to obtain in some cases. Our proposed schemes do not require such assumptions, nor any prior information about its statistics.

Another common approach to exploit the delayed NSI feedback is on-line convex learning, i.e., recasting the problem as an on-line convex optimization (OCO) problem. The advantage of on-line convex learning is that neither an specific assumption on the channel nor any prior information regarding its statistics is required. OCO has been applied to power control problems in wireless networks with delayed NSI. For instance, [86] studies the problem of maximizing a single user’s utility in a MIMO network. The authors of [87] further consider maximizing the energy efficiency in a MIMO-OFDM system. The authors of [88] considers power control with long-term averaged power constraint in a point-to-point MIMO network. Furthermore, the authors of [89] consider maximizing an utility in transmission power with transmission energy harvested from the environment. All these algorithms treat the interference from other UEs as noise, and they do not consider the coordination of transmission power to further improve system performance. Furthermore, they do not consider the scenario where only limited NSI is available. In our work, we consider power coordination among D2D pairs to improve system performance and limit their interference to cellular communications. We further consider the scenario of limited NSI feedback.

5.2 System Model and Problem Formulation

In this section, we describe the system model, and formulate the problem of maximizing the weighted sum-rate of D2D pairs. We summarize important notations in Table 5.1.

5.2.1 System Model

We consider a D2D network as illustrated in Fig. 5.1, where multiple D2D pairs, denoted by $\mathcal{N} = \{1, \cdots, N\}$, share one channel with CUEs communicating with a BS. Each D2D pair consists of one D2D transmitter (DTx) and one D2D receiver (DRx). Without loss of generality, we normalize the bandwidth of the channel to one. We further assume that
Table 5.1: Table of Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of D2D pairs</td>
</tr>
<tr>
<td>$P_i(t)$</td>
<td>Transmission power of Tx $i$ in time slot $t$</td>
</tr>
<tr>
<td>$I_i(t)$</td>
<td>Sum power of external interference received by DRx $i$ in time slot $t$</td>
</tr>
<tr>
<td>$h_{ij}(t)$</td>
<td>Channel gain from DTx $j$ to DRx $i$</td>
</tr>
<tr>
<td>$P_{i,\text{min}}$</td>
<td>Minimum transmission power of DTx $i$</td>
</tr>
<tr>
<td>$P_{i,\text{max}}$</td>
<td>Maximum transmission power of DTx $i$</td>
</tr>
<tr>
<td>$G_i$</td>
<td>Mean channel gain from DTx $i$ to the BS</td>
</tr>
<tr>
<td>$I_{\text{C}}^\text{max}$</td>
<td>Maximum interference power received from D2D pairs for the BS</td>
</tr>
</tbody>
</table>

![System model](image)

Figure 5.1: System model

In this chapter, we focus on power control in the uplink. It can be easily extended to the downlink.

All D2D pairs are equipped with one antenna.

Time is slotted. The transmission power of DTx $i$ in time slot $t$ is $P_i(t)$. The channel gain from DTx $j$ to DRx $i$ in time slot $t$ is $h_{ij}(t)$. The channel gain from DTx $j$ to BS in time slot $t$ is $g_j(t)$.\footnote{In this chapter, we focus on power control in the uplink. It can be easily extended to the downlink.} We assume $\{g_i(t)\}$ are stationary random processes for all D2D pairs, and let $G_i = E[g_i(t)]$. In this network, we assume that all D2D pairs experience time-varying inter-cell interference, and interference from the CUEs in the same cell. Furthermore, we denote by $I_i(t)$ the power of external interference received by DRx $i$, which consists of the inter-cell interference and the interference from the CUEs reusing the same channel. Let $\mathcal{H}(t) = \{\{h_{ij}(t)|1 \leq i \leq N, 1 \leq j \leq N\}, \{I_i(t)|1 \leq i \leq N\}\}$ be the full network NSI in time slot $t$. And let $\mathcal{H}_i(t) = \{\{h_{ij}(t)|1 \leq j \leq N\}, I_i(t)\}$ be the local NSI of D2D pair $i$ in time slot $t$. Then, the data rate of D2D pair $i$ in time slot $t$
is given by
\[
R_{i,t}(P(t)) = \log \left( 1 + \Gamma \frac{P_i(t)h_{ii}(t)}{I_i(t) + P_N + \sum_{j=1,j\neq i} P_j(t)h_{ij}(t)} \right), \tag{5.1}
\]
where \(\Gamma\) accounts for the gap between the Shannon bound and realistic implementation, \(P(t) = [P_i(t)]_{1 \times N}\), and \(P_N\) is the power of noise. We further assume that there exist positive and finite bounds \(h_{\text{max}}\), \(I_{\text{min}}\), and \(I_{\text{max}}\), such that \(h_{ij}(t) < h_{\text{max}}\) and \(I_{\text{min}} < I_i(t) < I_{\text{max}}\) for any \(i, j,\) and \(t\).

We assume that there is a power control coordinator in the network. As an example, the coordinator may be the BS. The D2D pairs feed back their local NSI to the coordinator, which is used by the coordinator to compute the transmission power for all D2D pairs in each time slot.

We assume that there is a delay from the time when D2D pairs feed back the local NSI to the coordinator and the time when the D2D pairs receive the transmission power decision from the coordinator. For simplicity, we let the delay be one. This delay includes the delay for a DRx to feed back its NSI to the coordinator, the queuing and processing time for the computation of the transmission power at the coordinator, and the delay of the coordinator sending the transmission power decision to the DTxs. Thus, in time slot \(t\), DTxs transmit with power that is determined by the coordinator based on the historical NSI \(\mathcal{N}(0), \ldots, \mathcal{N}(t-1)\). Note that we assume the DTxs acquire the instantaneous channel state for adaptive transmission.

### 5.2.2 Problem Formulation

In this section, we focus on maximizing the weighted sum-rate of D2D pairs with the interference to the BS from all D2D pairs limited. Therefore, our optimization problem is formulated as

\[
\mathcal{P} : \max_{\{P(t)\}} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} w_i R_{i,t}(P(t)) \tag{5.2}
\]

s.t.  
\[
P_{i,\text{min}} \leq P_i(t) \leq P_{i,\text{max}}, \quad \forall i, t, \tag{5.3}
\]
\[
\sum_{i=1}^{N} G_i P_i(t) \leq I_{\text{max}}^C, \quad \forall t, \tag{5.4}
\]

where \(R_{i,t}(P(t))\) is given by (5.1), \(w_i\) is D2D pair \(i\)'s weight, and \(G_i\) is mean channel gain between DTx \(i\) and the BS. In constraint (5.3), the transmission power of D2D pair \(i\) is
bounded by $P_{i, \text{min}}$ and $P_{i, \text{max}}$. Constraint (5.4) restricts that the expected interference power received by the BS from all D2D pairs cannot exceed $I_{C, \text{max}}$. We denote by $\mathcal{P}$ the feasible set of problem $\mathcal{P}$.

The challenge of problem $\mathcal{P}$ is that the instantaneous NSI, i.e., $\mathfrak{H}(t)$, is not available to the coordinator in time slot $t$. In the proposed solution, we resort to the framework of OCO to solve problem $\mathcal{P}$. We first convexify the original problem and then recast the problem into per-time slot problems. In addition, we consider the scenario where only a limited number of D2D pairs feed their local NSI back to the coordinator.

## 5.3 On-line Power Control: Full NSI Feedback

In this section, we focus on the full NSI feedback scenario, where the coordinator collects the NSI of all D2D pairs.

### 5.3.1 Problem Convexification

The framework of OCO requires the convexity in optimization problem. However, with the presence of interference from other D2D pairs, the objective in (5.1) is not concave in the optimization variables. Hence, we propose the following method to convexify the problem.

$$R_{i,t}(\mathbf{P}(t)) = \log \left( 1 + \Gamma \frac{P_i(t)h_{ii}(t)}{I_i(t) + P_N + \sum_{j=1,j\neq i}^{N} P_j(t)h_{ij}(t)} \right)$$

$$= \log \left( \Gamma P_i(t)h_{ii}(t) + \sum_{j=1,j\neq i}^{N} P_j(t)h_{ij}(t) + I_i(t) + P_N \right) - \log(I_i(t) + P_N)$$

$$- \log \left( 1 + \frac{\sum_{j=1,j\neq i}^{N} P_j(t)h_{ij}(t)}{I_i(t) + P_N} \right)$$

$$\geq \log \left( \Gamma P_i(t)h_{ii}(t) + \sum_{j=1,j\neq i}^{N} P_j(t)h_{ij}(t) + I_i(t) + P_N \right) - \log(I_i(t) + P_N)$$

$$- \frac{\sum_{j=1,j\neq i}^{N} P_j(t)h_{ij}(t)}{I_i(t) + P_N},$$

where (a) is based on the inequality $\log(1 + x) \leq x$ when $x \geq 0$. 

We let
\[
\tilde{R}_{i,t}(\mathbf{P}(t)) = \log \left( \Gamma P_i(t) h_{ii}(t) + \sum_{j=1, j \neq i}^{N} h_{ij}(t) P_j(t) + I_i(t) + P_N \right) - \log(I_i(t) + P_N) - \frac{\sum_{j=1, j \neq i}^{N} h_{ij}(t) P_j(t)}{I_i(t) + P_N}.
\]

The concavity of \(\tilde{R}_{i,t}(\mathbf{P}(t))\) is shown in the following lemma.

**Lemma 18.** \(\tilde{R}_{i,t}(\mathbf{P}(t))\) is concave in \(\{P_i(t)\}\).

**Proof.** The term \(\log \left( \Gamma P_i(t) h_{ii}(t) + \sum_{j=1, j \neq i}^{N} h_{ij}(t) P_j(t) + I_i(t) + P_N \right)\) is concave in \(\{P_i(t)\}\) based on the concavity of the \(\log(.)\) function. The term \(\sum_{j=1, j \neq i}^{N} h_{ij}(t) P_j(t)\) is linear, and thus concave in \(\{P_i(t)\}\). Hence, \(\tilde{R}_{i,t}(t)\) is concave in \(\{P_i(t)\}\). \(\square\)

Thus, we now consider a convex lower bound of our original problem \(\mathcal{P}\) as follows:

\[
\hat{\mathcal{P}} : \max_{\mathbf{P}(t)} \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} \tilde{f}_{i,t}(\mathbf{P}(t))
\]

s.t. \(P_{i,\min} \leq P_i(t) \leq P_{i,\max}, \forall i \in \mathcal{N}, t \in \mathcal{T},\)

\[
\sum_{i=1}^{N} G_i P_i(t) \leq I_{\max}^{C}, \forall t \in \mathcal{T},
\]

where \(f_{i,t}(\mathbf{P}(t)) = w_i \tilde{R}_{i,t}(\mathbf{P}(t))\). Problem \(\hat{\mathcal{P}}\) is convex because \(\tilde{R}_{i,t}(\mathbf{P}(t))\) is concave and the constraints are linear.

### 5.3.2 Per-Time Slot Optimization Problem

We use the method of OGD to solve the optimization problem \(\hat{\mathcal{P}}\). In time slot \(t\), we define the following per-time slot optimization problem:

\[
\hat{\mathcal{P}}_{t} : \max_{\{P_i(t)\}} \sum_{i=1}^{N} \tilde{\theta}_i(t) P_i(t) - \frac{1}{\eta_t} \left( P_i(t) - \tilde{P}_i(t-1)^* \right)^2
\]

s.t. \(P_{i,\min} \leq P_i(t) \leq P_{i,\max}, \forall i \in \mathcal{N},\)

\[
\sum_{i=1}^{N} G_i P_i(t) \leq I_{\max}^{C},
\]
where

$$\tilde{\theta}_i(t) = \sum_{j=1}^{N} \frac{\partial f_{j,i-1}(P(t-1))}{\partial P_j(t-1)} \bigg|_{P(t-1)=\bar{P}(t-1)^*}^\ast,$$

and $\eta_t = \frac{\delta}{\sqrt{t}}$, where $\delta > 0$. Furthermore, we denote by $\bar{P}(t)^* = [\bar{P}_i(t)^*]$ the optimal solution to problem $\bar{P}_i$.

Problem $\bar{P}_i$ is convex, which can be solved by common convex solvers. However, we find a semi-closed-form solution to $\bar{P}_i$ as stated in Lemma 19, by exploiting the special structure of $\bar{P}_i$, which leads to a much more efficient method to solve this problem.

**Lemma 19.** The optimal solution to problem $\bar{P}_i$ has the form of

$$\bar{P}_i(t)^* = \left[ \frac{\beta_i - \lambda G_i}{2} \right]_{P_{i,\min}}^{P_{i,\max}}, \forall i,$$

where $\beta_i = \tilde{\theta}_i(t)\eta_t + 2\bar{P}_i(t-1)^*$, and $\lambda$ is some non-negative constant.

**Proof.** Given Lagrange multipliers $\lambda$, which corresponds to constraint (5.4), and $\mu_1^i$ and $\mu_2^i$, which correspond to constraint (5.3), the Lagrangian of problem $\bar{P}_i$ is

$$\mathcal{L}(\{P_i(t)\}, \lambda, \{\mu_1^i\}, \{\mu_2^i\}) = \sum_{i=1}^{N} (P_i(t)^2 - \beta_i P_i(t)) + \lambda \left( \sum_{i=1}^{N} G_i P_i(t) - I_{\text{max}}^C \right) + \sum_{i=1}^{N} \mu_1^i (P_i(t) - P_{i,\max}) + \mu_2^i (P_{i,\min} - P_i(t)) + \bar{P}_i(t-1)^*.$$

Thus, the KKT conditions are

$$2\bar{P}_i(t)^* - \beta_i + \lambda G_i + \mu_1^i - \mu_2^i = 0, \ \forall i,$$

$$\lambda \left( \sum_{i=1}^{N} G_i \bar{P}_i(t)^* - \bar{P}_{\text{max}}^C \right) = 0,$$

$$\mu_1^i (\bar{P}_i(t)^* - P_{i,\max}) = 0, \ \forall i,$$

$$\mu_2^i (P_{i,\min} - \bar{P}_i(t)^*) = 0, \ \forall i,$$

$$\lambda \geq 0,$$

$$\mu_1^i \geq 0, \mu_2^i \geq 0, \ \forall i.$$

If $P_{i,\min} < \bar{P}_i(t)^* < P_{i,\max}$, then $\mu_1^i = 0 \mu_2^i = 0$ according to (5.8) and (5.9). Hence, $\bar{P}_i(t)^* = \frac{\beta_i - \lambda G_i}{2}$ based on (5.6). This implies that $P_{i,\min} < \frac{\beta_i - \lambda G_i}{2} < P_{i,\max}$. 


If $P_i(t)^* = P_{i,\text{min}}$, then $\mu_i^1 = 0$ according to (5.8). Hence, $\mu_i^2 = 2P_{i,\text{min}} - \beta_i + \lambda G_i \geq 0$ based on (5.6) and (5.11). This implies that $\frac{\beta_i - \lambda G_i}{2} \leq P_{i,\text{min}}$.

Similarly, if $\tilde{P}_i(t)^* = P_{i,\text{max}}$, then $\frac{\beta_i - \lambda G_i}{2} \geq P_{i,\text{max}}$.

Hence, the optimal solution is

$$\tilde{P}_i(t)^* = \left[ \frac{\beta_i - \lambda G_i}{2} \right] P_{i,\text{max}} - P_{i,\text{min}}.$$ 

Furthermore, let $\lambda_i = \beta_i - \frac{2P_{i,\text{min}}}{G_i}$, and $\lambda_i = \beta_i - \frac{2P_{i,\text{max}}}{G_i}$. We have the following corollary.

**Corollary 20.** If $\lambda > \lambda_i$, then $\tilde{P}_i(t)^* = P_{i,\text{min}}$; if $\lambda < \lambda_i$, then $\tilde{P}_i(t)^* = P_{i,\text{max}}$; otherwise, $\tilde{P}_i(t)^* = \frac{\beta_i - \lambda G_i}{2}$.

**Remark.** Lemma 19 presents a semi-closed-form solution to problem $P_t^1$. Corollary 20 gives the optimal solution based on $\lambda$. In Algorithm 4, we propose a computationally-efficient method by searching for the optimal $\lambda$ based on Corollary 20.

### 5.3.3 On-line Power Control for D2D with Full NSI Feedback

The proposed On-line Power Control for D2D with Full NSI Feedback (OPCD-FNF) is described in Algorithm 4. Initially, the coordinator randomly selects one initial point, $\tilde{p}_i(0)^*$, for $1 \leq i \leq N$, from the feasible set. Then in each time slot, each D2D pair $i$ feeds back its local NSI to the coordinator. The coordinator solves the per-time slot problem, $\tilde{P}_t$, formulated by the received delayed NSI from all D2D pairs. Specifically, the coordinator runs a one-dimensional search method for $\lambda$, and obtains the transmission power of D2D pairs, $\tilde{P}(t)^*$, corresponding to the optimal solution to problem $\tilde{P}_t$. Finally, the coordinator sends $\tilde{P}(t)^*$ to the D2D pairs.

Based on the boundedness of $\{P_i(t)\}$, $\{h_{ij}(t)\}$, and $\{I_i(t)\}$, there exists constants $A$, and $B_1$ such that $\|P_1 - P_2\| \leq A$, and $\|\sum_{i=1}^N \nabla f_{i,t}(P)\| \leq B_1$ for any $P_1, P_2 \in \mathcal{P}$, and $t$. Then, we have the following performance guarantee for OPCD-FNF.

**Theorem 21.** Let $\{\tilde{P}(t)^*|1 \leq t \leq T\}$ be the solution produced by OPCD-FNF for any $T \geq 1$. We have

$$\sum_{t=1}^T \sum_{i=1}^N f_{i,t}(P^*) - \sum_{t=1}^T \sum_{i=1}^N w_{i,t} R_{i,t}(\tilde{P}(t)^*) \leq \frac{A^2}{\eta_{T+1}} + B_1^2 \sum_{t=1}^T \eta_{t+1}. \quad (5.12)$$

for any $P^* \in \mathcal{P}$. 
Algorithm 4 OPCD-FNF

Input: \(\{w_i\}, \{G_i\}, \eta, N, T\).

Coordinator: Coordinator randomly choose \(\tilde{p}_i(0)\) for \(1 \leq i \leq N\), from the feasible set.
for \(t = 1\) to \(T\) do

(D2D pairs): For all \(i\), D2D pair \(i\) collects its NSI, i.e., \(\{h_{ij}(t)|1 \leq j \leq N\}\) and \(I_i(t)\), and sends these information to the BS;

(Coordinator): Coordinator runs the following codes based on the delayed NSI received from D2D pairs;
Compute \(\{\tilde{\theta}_i(t)\}\) by (5.5),
Compute \(\{\beta_i\}, \{\Delta_i\}\), and \(\{\lambda_i\}\), where \(\beta_i = \tilde{\theta}_i(t)\eta + 2\tilde{P}_i(t-1)^*\), \(\lambda_i = \frac{\beta_i-2P_{i,\min}}{G_i}\), and \(\Delta_i = \frac{\lambda_i}{G_i}\).
if \(2P_{i,\min} \geq \beta_i\) for all \(i\) then
\(\tilde{P}_i(t)^* = P_{i,\min}\) for all \(i\);
else
Sort \(\{\lambda_i\}, \{\Delta_i\}\) in ascending order, and remove the negative numbers and duplicates. Denote by \(\{\lambda'_i\}\) the new sorted set, where \(\lambda'_i > \lambda'_j\) if \(i > j\). Let \(N' = |\{\lambda'_i\}|\).
for \(k = 2\) to \(N'\) do
Compute set \(N_i = \{i \in N : \lambda_i \leq \lambda'_{k-1}\}\), and set \(\tilde{P}_i(t)^* = P_{i,\min}\) for \(i \in N_i\).
Compute set \(N_i = \{i \in N : \lambda_i \geq \lambda'_k\}\), and set \(\tilde{P}_i(t)^* = P_{i,\max}\) for \(i \in N_i\).
Compute \(I_{i,\max}' = I_{i,\max} - \sum_{i \in N_i} G_i \tilde{P}_i(t)^*\).
Compute \(\lambda = \frac{\sum_{i \in N_i} \beta_i-2I_{i,\max}' G_i}{\sum_{i \in N'} G_i}\), where \(N' = N \setminus (N_i \cup N'_i)\).
if \(\lambda_{k-1} \leq \lambda \leq \lambda'_k\) then
Set \(\tilde{P}_i(t)^* = \frac{\beta_i-\lambda G_i}{2}\) for \(i \in N'\).
break.
end if
end for
Coordinator sends the power control decisions \(\{\tilde{P}_i(t)^*\}\) to the D2D pairs;
end for
Furthermore, if we pick $\eta_t$ such that $\eta_t = \frac{\delta}{\sqrt{t}}$ and $\delta > 0$, we have

$$\frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(P^*) - \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} w_i R_{i,t}(\tilde{P}(t)^*) \leq O\left(\frac{1}{\sqrt{T}}\right).$$

(5.13)

Proof. Based on the performance bound of OGD given in [57], we have

$$\sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(P^*) - \sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(\tilde{P}(t)^*) \leq \frac{A^2}{\eta_{t+1}} + B_2^2 \sum_{t=1}^{T} \eta_{t+1}.$$  

Furthermore, $f_{i,t}(P(t))$ is a lower bound of $w_i R_{i,t}(P(t))$. Therefore, we have

$$\sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(P^*) - \sum_{t=1}^{T} \sum_{i=1}^{N} w_i R_{i,t}(\tilde{P}(t)^*) \leq \frac{A^2}{\eta_{t+1}} + B_2^2 \sum_{t=1}^{T} \eta_{t+1}. \quad (5.14)$$

The asymptotic performance in (5.13) can be easily derived by analyzing the asymptotic performance of the right-hand side (RHS) of (5.14) using the inequality $\sum_{t=1}^{T} \frac{1}{\sqrt{t}} \leq 2 \sqrt{T}$.

Remark. Theorem 21 shows that the performance gap between OPCD-FNF and the performance of lower-bound problem $\tilde{P}$, with fixed transmission power of D2D pairs in every time slot, is sub-linear to $T$ with appropriate choice of $\eta_t$.

5.4 On-line Power Control: Partial NSI Feedback

In OPCD-FNF, the NSI of all D2D pairs is fed back to the coordinator. It potentially requires a large amount of NSI measurement and feedback to the coordinator. In this section, we focus on the scenario where only $K$, for $K \leq N$, D2D pairs send their local NSI to the coordinator.

5.4.1 Random Selection and Per-Time Slot Problem

Random D2D Selection

In each time slot, $K$ D2D pairs are uniformly randomly selected to feed back its local NSI. Let $\mathcal{I}_t$ be the collection of D2D pairs that feed back their NSI in time slot $t$. 
Chapter 5. On-line Power Control in D2D Networks

Per-Time Slot Problem

The transmission power of D2D pairs in time slot \( t \) is determined by the coordinator based on the delayed local NSI sent from D2D pairs in set \( I_{t-1} \). Then the coordinator formulates and solves the following per-time slot problem based on such delayed NSI.

\[
\bar{P}_t : \max_{\mathbf{P}(t)} \sum_{i=1}^{N} \bar{\theta}_i(t)P_i(t) - \frac{1}{\eta_t} (P_i(t) - \bar{P}_i(t-1)^*)^2
\]

s.t. \( P_{i,min} \leq P_i(t) \leq P_{i,max}, \forall i \in \mathcal{N} \),

\[
\sum_{i=1}^{N} G_i P_i(t) \leq I_{max}^C,
\]

where

\[
\bar{\theta}_i(t) = \frac{N}{K} \sum_{j \in I_{t-1}} \frac{\partial f_{j,t-1}(\mathbf{P}(t-1))}{\partial P_i(t-1)}|_{\mathbf{P}(t-1)=\bar{\mathbf{P}}(t-1)^*}.
\]

We denote by \( \bar{\mathbf{P}}(t)^* = [\bar{P}_i(t)^*] \) the optimal solution to problem \( \bar{P}_t \).

**Remark.** Problem \( \bar{P}_t \) has the same problem structure as problem \( \tilde{P}_t \). Hence, the method to solve problem \( \bar{P}_t \) can also be applied to problem \( \tilde{P}_t \) with minor modification.

5.4.2 On-line Power Control for D2D with Partial NSI Feedback

The proposed On-line Power Control for D2D with Partial NSI Feedback (OPCD-PNF) is described in Algorithm 5. Initially, the coordinator randomly selects an initial point, \( \bar{\mathbf{p}}_i(0)^* \), for \( 1 \leq i \leq N \), from the feasible set. Then in each time slot, \( K \) D2D pairs are randomly selected in a uniform manner, and feed its local NSI to the coordinator. Meanwhile, the coordinator solves the per-time slot problem, \( \bar{P}_t \), formulated by the most up-to-date delayed NSI it has received from each of the selected D2D pairs. Specifically, the coordinator runs a one-dimensional search for \( \lambda \), and obtains the transmission power, \( \bar{\mathbf{P}}(t)^* \), corresponding to the optimal solution to problem \( \bar{P}_t \). Finally, the coordinator sends \( \bar{\mathbf{P}}(t)^* \) to the D2D pairs.

5.4.3 Performance of OPCD-PNF

Let \( F_t(\mathbf{P}(t)) = \sum_{j=1}^{N} f_{j,t}(\mathbf{P}(t)) \), and \( \tilde{F}_t(\mathbf{P}(t)) = \frac{N}{K} \sum_{j \in I_t} f_{j,t}(\mathbf{P}(t)) \). The following conclusion is obvious.
Algorithm 5 OPCD-PNF

Input: \( \{w_i\}, \{G_i\}, \eta_t, K, N, T. \)

Coordinator: Coordinator randomly choose \( \{p_i(0)\} \) from the feasible set.

\[
\text{for } t = 1 \text{ to } T \text{ do}
\]

\( K \) D2D pairs are uniformly randomly selected to send the local NSI to the coordinator.

\((\text{D2D pairs }):\) For all \( i \), D2D pair \( i \) collects its NSI, i.e., \( \{h_{ij}(t)\}_{j=1}^N \) and \( I_i(t) \), and send these information to the BS;

\((\text{Coordinator}):\) Coordinator runs the following codes based on the delayed NSI received from D2D pairs;

Compute \( \{\bar{\theta}_i(t)\} \) by (5.15),

\[\bar{\lambda}_i = \frac{\beta_i - 2P_{i,\text{min}}}{G_i},\]

if \( 2P_{i,\text{min}} \geq \beta_i \) for all \( i \) then

\[P_i(t)^* = P_{i,\text{min}} \text{ for all } i;\]

else

Sort \( \{0, \{\lambda_i\}, \{\bar{\lambda}_i\}\} \) in ascending order, and remove the negative numbers and duplicates. Denote by \( \{\lambda_i'\} \) the new sorted set, where \( \lambda_i' > \lambda_j' \) if \( i > j \). And the cardinality is \( N' = |\{\lambda_i'\}|.\)

\[\text{for } k = 2 \text{ to } N' \text{ do}\]

Compute set \( \mathcal{N} = \{i \in N : \bar{\lambda}_i \leq \lambda_{k-1}'\} \), and set \( \bar{P}_i(t)^* = P_{i,\text{min}} \) for \( i \in \mathcal{N}. \)

Compute set \( \mathcal{N}' = \{i \in N : \bar{\lambda}_i \geq \lambda_k'\} \), and set \( \bar{P}_i(t)^* = P_{i,\text{max}} \) for \( i \in \mathcal{N}'. \)

Compute \( \lambda_{C_{\text{max}}}^C = I_{C_{\text{max}}}^C - \sum_{i \in \mathcal{N}' \cup \mathcal{N}} G_i \bar{P}_i(t)^*. \)

Compute \( \lambda = \frac{\sum_{i \in \mathcal{N}} \beta_i - 2\lambda_{C_{\text{max}}}^C}{\sum_{i \in \mathcal{N}} G_i} \), where \( \mathcal{N}' = N \setminus (\mathcal{N}' \cup \mathcal{N}) \).

if \( \lambda_{k-1}' \leq \lambda \leq \lambda_k' \) then

Set \( \bar{P}_i(t)^* = \frac{\beta_i - \lambda G_i}{2} \) for \( i \in \mathcal{N}' \).

break.

end if

end for

Coordinator sends the power control decisions \( \{\bar{P}_i(t)^*\} \) to the D2D pairs;

end for
Lemma 22. The solution of the OPCD-PNF algorithm is the same as the solution of running the OGD algorithm with loss function $\hat{F}_t(P(t))$.

Based on the boundedness of $\{P_i(t)\}$, $\{h_{ij}(t)\}$, and $\{I_i(t)\}$, there exists constants $A$, and $B_1$ such that $\|P_1 - P_2\| \leq A$, and $\|\sum_{i \in I_t} \nabla f_i(P)\| \leq B_2$ for any $P, P_1, P_2 \in \mathcal{P}$, $I_t$ and $t$. Then, using Lemma 22, we derive the following performance bound for OPCD-PNF.

Theorem 23. Let $\{\bar{P}(t)\}^{t \leq T}$ be the solution produced by OPCD-PNF for any $T \geq 1$. The expected performance guarantee of OPCD-PNF is given by

$$\sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(P^*) - E \left[ \sum_{t=1}^{T} \sum_{i=1}^{N} w_i R_{i,t}(P(t)^*) \right] \leq \frac{A^2}{\eta T} + B_1^2 \sum_{t=1}^{T} \eta_{t+1}. \quad (5.16)$$

for any $P^* \in \mathcal{P}$.

Furthermore, if we pick $\eta_t$ such that $\eta_t = \frac{\delta}{\sqrt{t}}$ and $\delta > 0$, we have

$$\frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(P^*) - \frac{1}{T} E \left[ \sum_{t=1}^{T} \sum_{i=1}^{N} w_i R_{i,t}(P(t)^*) \right] \leq O\left( \frac{1}{\sqrt{T}} \right). \quad (5.17)$$

Proof. Note that the random variables in OPCD-PNF are $\{I_\tau\}_{\tau=0}^{T}$.

Based on the rule of updating function $\hat{F}_t(P(t))$, we have

$$\hat{F}_t(P(t)) = \frac{N}{K} \sum_{j \in I_t} f_{j,t}(P(t)).$$

Also, based on Lemma 22, we have

$$\bar{P}(t)^* = A(\hat{F}_0(P(0)^*), \ldots, \hat{F}_{t-1}(P(t-1)^*)),$$

where $A(.)$ denotes the OGD algorithm. In other words, $\bar{P}(t)^*$ can be considered as functions of random variables $\{I_\tau\}_{\tau=0}^{t-1}$, given the loss functions $\{F_\tau(p(\tau))\}_{\tau=0}^{t-1}$.

Thus, we have

$$E \left[ \hat{F}_t(\bar{P}(t)^*) \right] = E_{I_0, \ldots, I_t} \left[ \hat{F}_t(\bar{P}(t)^*) \right]$$

$$\overset{(a)}{=} E_{I_0, \ldots, I_{t-1}} \left[ E_{I_t} \left[ \hat{F}_t(\bar{P}(t)^*) | I_0, \ldots, I_{t-1} \right] \right]$$

$$\overset{(b)}{=} E_{I_0, \ldots, I_{t-1}} \left[ E_{I_t} \left[ \hat{F}_t(\bar{P}(t)^*) | \bar{P}(t)^*, I_0, \ldots, I_{t-1} \right] \right]$$
where (a) is based on the fact that $\tilde{P}(t)^*$ is determined by $\{\mathcal{I}_t\}_{t=0}^{t-1}$, and (b) comes from the fact that $F_t(x)$ is determined by $\mathcal{I}_t$.

Furthermore, we have

$$E_{\mathcal{I}_t}\left[\hat{F}(\tilde{P}(t)^*)|\tilde{P}(t)^*, \mathcal{I}_0, \cdots, \mathcal{I}_{t-1}\right]$$

$$= E_{\mathcal{I}_t}\left[\hat{F}(\tilde{P}(t)^*)|\tilde{P}(t)^*\right]$$

$$= \sum_{i=1}^{N} \frac{N}{K} f_{i,t-1}(\tilde{P}(t)^*) \frac{K}{N}$$

$$= F_t(\tilde{P}(t)^*)$$

Hence,

$$E_{\mathcal{I}_0, \cdots, \mathcal{I}_t} \left[\hat{F}_t(\tilde{P}(t)^*)\right] = E_{\mathcal{I}_0, \cdots, \mathcal{I}_t} \left[F_t(\tilde{P}(t)^*)\right]$$

Similarity, we can show that for any fixed $\tilde{P}^* \in \mathfrak{P}$, we have

$$E_{\mathcal{I}_0, \cdots, \mathcal{I}_t} \left[\hat{F}_t(\tilde{P}(t)^*)\right] = E_{\mathcal{I}_0, \cdots, \mathcal{I}_t} \left[F_t(\tilde{P}(t)^*)\right]$$

Therefore,

$$E_{\mathcal{I}_0, \cdots, \mathcal{I}_T} \left[\sum_{t=1}^{T} F_t(\tilde{P}^*) - \sum_{t=1}^{T} F_t(\tilde{P}(t)^*)\right]$$

$$= E_{\mathcal{I}_0, \cdots, \mathcal{I}_T} \left[\sum_{t=1}^{T} \hat{F}_t(\tilde{P}^*) - \sum_{t=1}^{T} \hat{F}_t(\tilde{P}(t)^*)\right]$$

$$(a) \leq \frac{A^2}{\eta_{T+1}} + B_2^2 \sum_{t=1}^{T} \eta_{t+1}$$

where (a) is based on the performance bound of optimizing $\sum_{t=1}^{T} \hat{F}_t(p(t))$ via the OGD algorithm.

Then using a similar argument as in the proof of Theorem 21, we have

$$\sum_{t=1}^{T} \sum_{i=1}^{N} f_{i,t}(\tilde{P}^*) - \mathbb{E} \left[\sum_{t=1}^{T} \sum_{i=1}^{N} w_i R_{i,t}(\tilde{P}(t)^*)\right] \leq \frac{A^2}{\eta_{T+1}} + B_2^2 \sum_{t=1}^{T} \eta_{t+1}.$$ 

The asymptotic performance in (5.17) also can be shown similarly to how we show (5.13) in Theorem 21.
Remark. Theorem 23 suggests that the expected asymptotic performance of OPCD-PNF is the same as the deterministic asymptotic performance of OPCD-FNF.

5.5 Numerical Simulation

In this section, we evaluate the performance of the proposed OPCD-FNF and OPCD-PNF.

We consider a network centered at a BS with coverage radius 250 m. Five D2D pairs are randomly placed in the network. The DTx-to-DRx distance for each pair is 30 m. We set $\Gamma$ in (5.1) as 1. The maximum transmission power of D2D pairs is $10^{-3}$ W. The pathloss models for the link between D2D pairs and the link between DTx and BS are $128.1 + 36.7 \log(d)$ and $141 + 40 \log(d)$ respectively, where $d$ is the distance of the link. Furthermore, the small scale power fading term is i.i.d. among different links in each time slot, and follows exponential distribution with mean 1. The weight of each D2D pair is 1. The learning rate $\eta_t = 5 \times 10^{-9} \sqrt{t}$, where $t$ is the time slot index. The power of noise $P_N = 6 \times 10^{-17}$ W. The allowed maximum interference power received by the BS from the D2D pairs $I_{\text{max}}^C = 500P_N$. We generate time-varying interference $I_i(t)$ for D2D pair $i$ in time slot $t$ as follows:

$$I_i(t) = \begin{cases} I_{\text{low}} + I_{\text{vary}} \ast \text{unif}(0, 1) & \text{with prob. } a, \\ I_{\text{high}} + I_{\text{vary}} \ast \text{unif}(0, 1) & \text{with prob. } 1 - a, \end{cases}$$

where $I_{\text{low}} = 500P_N$, $I_{\text{high}} = 5 \times 10^5P_N$, $I_{\text{vary}} = 500P_N$, and $a = 0.5$ in their default settings, unless otherwise specified.

We compare our proposed schemes with the following two schemes: (1) Mean NSI scheme, where the problem is solved via a successive convexification method using the mean of channel gain and interference as instantaneous channel gain and interference; (2) Delayed NSI scheme, where the problem is solved via successive convexification method using using delayed channel gain and interference as instantaneous channel gain and interference.

Note that we evaluate the performance of OPCD-PNF in two scenarios where 3 and 1 D2D pairs feedback their NSI in each time slot. Each data point in the figures is averaged over 100 realizations and each realization is run for 500 time slots.

In Figs. 5.2-5.4, we compare the performance of the proposed schemes and alternative schemes. The relative performance gain is based on the Mean NSI scheme. We observe...
that OPCD-FNF outperforms the other two alternatives in most cases. In addition, the performance of 3 D2D pairs feeding back their NSI in OPCD-PNF is very close to that of OPCD-FNF, where all D2D pairs feedback their NSI. This suggests that randomly selecting several D2D pairs to feedback their NSI may maintain sufficiently satisfactory performance while reducing the amount of feedback to the coordinator.

In Fig. 5.2, we study the impact of the stability of the system via changing the probability of low interference, $a$. When $a$ is close to 1 or 0, the interference received by D2D pairs tends to be constant. We observe that OPCD-FNF achieves the largest relative performance gain when $a = 0.3$, and almost the same performance as Mean NSI scheme when $a = 0.9$. This suggests that the proposed schemes are more beneficial when the system is less stable.

In Fig. 5.3, we study the impact of the difference of the system states via changing the value of $I_{\text{high}}$. We observe that when $I_{\text{high}}$ is close to $I_{\text{low}}$, the performance of OPCD-FNF is close to that of Mean NSI. However, when $\frac{I_{\text{high}}}{I_{\text{low}}}$ increases, the relative performance gain of OPCD-FNF increases significantly. Specifically, when $\frac{I_{\text{high}}}{I_{\text{low}}} = 10^4$, the relative performance gain is 0.66. This suggests that the proposed schemes are beneficial when the system has vastly different states.

In Fig. 5.4, we study the impact of the allowed maximum interference power to the BS, $I_{\text{max}}$. We observe that the relative performance gain of OPCD-FNF is the largest when $I_{\text{max}} = 300P_N$, and decreases when $I_{\text{max}} > 300P_N$. This suggests that the proposed schemes are more beneficial when the system allows smaller interference to the BS. When $I_{\text{max}}$ is small, the feasible set of the optimization problem is small, and the proposed schemes and the mean NSI scheme tend to have close performance.

### 5.6 Summary

In this chapter, we study power control among D2D pairs with delayed NSI to maximize the long-term average weighted sum rates. The non-convex problem is recast as per-time slot quadratic convex problems via the proposed convexification technique, and the OGD method. We further propose OPCD-FNF, which leverages the per-time slot problem with linear complexity. Considering that OPCD-FNF requires the feedback of the NSI of all D2D pairs, we further propose OPCD-PNF to reduce the amount of feedback. We show that the expected performance guarantee of PCD-PNF is the same as that of PCD-FNF. Numerical simulations validate the benefit of our proposed schemes in comparison with existing alternatives.
Figure 5.2: Sum rates versus $\alpha$ when $\eta_t = \frac{5 \times 10^{-9}}{\sqrt{t}}$

Figure 5.3: Sum rates versus $\frac{I_{\text{high}}}{I_{\text{low}}}$ when $\eta_t = \frac{5 \times 10^{-9}}{\sqrt{t}}$
Figure 5.4: Sum rates versus $I_{\text{max}}^C$ when $\eta_t = \frac{5 \times 10^{-9}}{\sqrt{t}}$
Chapter 6

Joint Spectrum Allocation and Probabilistic Caching in D2D Networks

In the previous chapters, we study the resource management in D2D networks in terms of spectrum and transmission power. In this chapter, we explore an important application of D2D communication, where some devices may cache popular files in order to offload traffic from BSs and improve system performance. The D2D network considered operates in the overlay mode. Specifically, we jointly consider spectrum allocation and probabilistic caching in DTxs based on network spatial statistics. We recognize the practical constraint that the DTx storage is limited and assume that each DTx can cache at most $K$ files. We aim to maximize the DRx coverage-rate logarithm utility. Our main contributions are summarized as follows.

- We derive a closed-form expression for the DRx coverage-rate logarithm utility with consideration for interference. We use it as the objective to formulate the problem of jointly optimizing the spectrum allocation between direct and D2D communication and the caching probabilities.

- We study an intermediate sub-problem of optimizing the caching probabilities when given fixed spectrum allocation. We first show that this problem is convex. We further demonstrate through a numeric example that caching the top $K$ popular files is not always optimal in this case.

- We observe that the problem of jointly optimizing spectrum allocation and caching probabilities is non-convex in general. However, by exploiting the special structure
of this problem, we derive an optimal solution and identify how it varies drastically depending on the relative densities of BSs and DTxs. This solution has the interesting property that if the DTxs cache any file at all, they will cache the top $K$ popular files.

- Our simulation results demonstrate that the proposed solution can substantially improve the coverage-rate logarithm utility and fairness among DRxs. Furthermore, even though our solution is based on the coverage rate, additional simulation based on a Shannon rate model shows that it gives useful insights into network design with more general models.

## 6.1 Related Work

In this section, we summarize the related works on cache placement and spectrum allocation in wireless networks.

### 6.1.1 Cache Placement Only Schemes

Cache placement schemes for wireless networks have been proposed in [33–50]. None of them consider spectrum allocation. In our work, we tackle the non-trivial problem of joint spectrum allocation and caching design in D2D networks.

We further note that, even for the scenario where spectrum allocation is fixed, there are differences between our work and references [33–50]. The caching design in references [33–39] are based on the assumption that the exact network topology or CSI is known. In contrast, similar to [40–50], we consider probabilistic caching based on the spatial statistics of the network. Among probabilistic caching design [40–50], the authors of [40–43] assume that each DTx can cache only one file. In this work, we consider a more general D2D network where a DTx can cache multiple files. While [44–50] allow multiple cached files in DTxs. They consider the soft constraint that the sum of the caching probability of each file cannot exceed $K$. This soft constraint might lead to the case where some cache memory of DTxs is not used. In [50], the authors further propose an heuristic algorithm based on the solution to the problem with the soft constraint to make sure the cache of the DTxs will always be used. In our work, our derived optimal caching strategy is based on the original feasible set. In addition, none of [44–50] addresses fairness among the DRxs. In this chapter, we consider the logarithm utility, which strikes a balance between data rate and fairness. This objective, also known as
proportional fairness, is widely considered in wireless network design [90], but as far as we know has not been addressed in probabilistic caching design.

6.1.2 Spectrum Allocation in Multi-tier Networks

Spectrum allocation in D2D networks has been studied in [51] and [52]. The authors of [51] consider optimal spectrum allocation between direct and D2D communication to maximize the logarithm utility. The authors of [52] study the problem of joint mode selection and spectrum allocation between direct and D2D communication from a game theoretical point of view. Furthermore, similar studies have been conducted for multi-tier heterogeneous networks in [91] and [92]. None of these works consider caching design.

6.2 System Model and Problem Formulation

In this section, we first describe the cache-enabled D2D network. We then derive the coverage-rate logarithm utility. Finally, we formulate the joint spectrum allocation and probabilistic caching problem to maximize the coverage-rate logarithm utility. Important notations throughout this chapter are summarized in Table 6.1.

6.2.1 D2D Network

We consider a D2D network with BSs, DTxs, and DRxs, as shown in Fig. 6.1. Following the modeling convention in previous works (e.g., [40–50]), we assume that the locations of BSs, DTxs, and DRxs follow independent homogeneous Poisson point processes (PPPs) with intensities $\lambda_0$, $\lambda_1$, and $\mu$, respectively. We further assume that $\lambda_0 \ll \mu$ and $\lambda_1 \ll \mu$, which may represent for example the service scenario of file downloading where there are many more receivers than transmitters.

We focus on two modes of downlink communication in this network: direct, where the BSs communicate with D2D nodes directly; and D2D, where the DTxs communicate with the DRxs. We further assume the network operates in an overlay mode so that there is no interference between direct and D2D communication. The available spectrum is divided into two orthogonal sub-bands for direct and D2D communication. For simplicity, we normalize the bandwidth of the total spectrum, and denote by $\eta_0$ and $\eta_1$ the portion of spectrum allocated to direct and D2D communication, respectively. We further assume that each BS and DTx allocates spectrum equally to its associated DRxs.

The transmission power of the BS and DTx, denoted by $P_0$ and $P_1$ respectively, is fixed. Let $x$ and $y$ be the coordinates of one transmitter (BS or DTx) and one DRx
Table 6.1: Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Intensity of the DRx PPP</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Intensity of the BS PPP</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>Intensity of the DTx PPP</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>Spectrum allocated to direct communication</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>Spectrum allocated to D2D communication</td>
</tr>
<tr>
<td>$P_0$</td>
<td>Transmission power of a BS</td>
</tr>
<tr>
<td>$P_1$</td>
<td>Transmission power of a DTx</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Pathloss exponent</td>
</tr>
<tr>
<td>$K$</td>
<td>Maximum number of cached files in a DTx</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of files in the network</td>
</tr>
<tr>
<td>$\mathcal{N}$</td>
<td>Set of all files</td>
</tr>
<tr>
<td>$\mathbb{F}^K$</td>
<td>Collection of subsets of $\mathcal{N}$ with cardinality not greater than $K$</td>
</tr>
<tr>
<td>$\mathbb{F}^K_i$</td>
<td>Collection of subsets of $\mathbb{F}^K$ containing $i$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>Requesting probability of file $i$</td>
</tr>
<tr>
<td>$q_F$</td>
<td>Sum of requesting probabilities of files in set $\mathcal{F}$</td>
</tr>
<tr>
<td>$p_F$</td>
<td>Caching probability of file set $\mathcal{F}$</td>
</tr>
<tr>
<td>$p_i$</td>
<td>Sum of caching probabilities of file sets containing file $i$</td>
</tr>
<tr>
<td>$T$</td>
<td>SIR threshold</td>
</tr>
</tbody>
</table>

respectively. Then the received power of the DRx is given by

$$p_{x,y} = \frac{p_x h_{x,y}}{|x-y|^\gamma},$$

where $\gamma$ is the pathloss exponent, $p_x$ is the transmission power of the transmitter, and $h_{x,y}$ models the effect small-scale fading. We assume that $\gamma > 2$, and the small-scale fading is Rayleigh with unit mean.

6.2.2 File Requests and File Caching

We assume that there are $N$ files of equal size in the network, which are denoted by $\mathcal{N} = \{1, \cdots, N\}$. Let $\mathbb{F}^K$ be the collection of all sets of files with cardinality not greater than $K$, i.e., $\mathbb{F}^K = \{\mathcal{F} | \mathcal{F} \subseteq \mathcal{N}, |\mathcal{F}| \leq K\}$. Furthermore, we denote by $\mathbb{F}^K_i$ the collection of all sets of files in $\mathbb{F}^K$ that contains file $i$, i.e., $\mathbb{F}^K_i = \{\mathcal{F} \in \mathbb{F}^K | i \in \mathcal{F}\}$.

We assume that each DRx requests one file from set $\mathcal{N}$, and the probability that it requests file $i$ is $q_i$, such that $\sum_{i=1}^{N} q_i = 1$. Without loss of generality, we order the files, such that $q_i > q_j$ if $i < j$. In addition, for any $\mathcal{F} \subseteq \mathcal{N}$, we define $q_\mathcal{F} = \sum_{i \in \mathcal{F}} q_i$.

We assume that the BSs have all files (or have easy access to them), while the DTxs
have limited storage capacity, and can cache at most $K$ popular files. Similar to [40–50], we consider a probabilistic caching strategy in DTxs. Specifically, each DTx randomly caches a set $\mathcal{F}$ in $\mathbb{R}^K$ with probability $p_F$. Note that $\sum_{\mathcal{F} \subseteq \mathbb{R}^K} p_F = 1$. Furthermore, for any $i \in \mathcal{N}$, we define $p_i = \sum_{\mathcal{F} \subseteq \mathbb{R}^K} p_F$.

Furthermore, we consider the following DRx association rule. A DRx associates with a DTx caching its requested file if and only if its long-term averaged received power from this DTx is greater than that from other DTxs and all BSs. If no such DTx is found, the DRx associates with the nearest BS. Note that we assume that if a DRx associates with a BS, it fetches its required file from the BS directly without the assistance of DTxs.

### 6.2.3 Coverage Probability and Coverage Rate

We assume the system is interference-limited, so that we may ignore the impact of noise. Similar to the assumptions in [42, 44, 46, 91–93], we consider a single received signal to interference ratio (SIR) threshold, $T$, at each DRx to facilitate analytical tractability, so that when the received SIR of is no less than $T$, the received data rate is $\log(1 + T)$; and it is 0, otherwise. We will show in Section 6.6 that our design based on this rate model is still useful in a Shannon rate model.

We define the coverage probability $P_{\text{cover}}$ as the probability of the event that a DRx’s received SIR is no less than $T$. Without loss of generality, we shift the coordinates so that a DRx under consideration is located at 0, i.e., it is a typical DRx [94]. Then $P_{\text{cover}}$ is a function of the location of the DRx’s associated transmitter (BS or DTx), denoted...
by $x_{tx}$:

$$P_{\text{cover}} = P(\text{SIR} \geq T)$$

$$= P \left( \frac{P_{x_{tx}h_{tx,0}}}{|x_{tx}|^\gamma} \geq T \right)$$

$$= \exp \left( -TP_{x_{tx}}^{-1}|x_{tx}|^\gamma I_\Phi \right),$$

where $I_\Phi = \sum_{z \in \Phi} \frac{P_{h_z}h_z,0}{|z|^\gamma}$ is the sum interference received by the DRx, where $\Phi$ is the collection of locations of interfering transmitters. Equality (a) holds because of the assumption of Rayleigh fading with unit mean.

Then, the average data rate of the typical DRx is

$$R_{\text{cover}} = \beta P_{\text{cover}} \log(1 + T), \quad (6.1)$$

where $\beta$ is the amount of spectrum allocated to the DRx. In this work, we are interested in the coverage-rate logarithm utility [92] defined as

$$U = \mathbb{E}_{\Phi, x_{tx}, \{h_z,0\}_{z \in \Phi}, Q}[\log R_{\text{cover}}],$$

where $Q$ is the file requested by the DRx. This is motivated by the observation that the maximization of a sum-log utility function also maximizes the proportional fairness in a system [90].

### 6.2.4 Joint Spectrum Allocation and Probabilistic Caching

The problem of joint spectrum allocation and probabilistic caching to maximize the coverage-rate logarithm utility is formulated as

$$\mathcal{P} : \max_{\{p_F, \eta_0, \eta_1\}} \quad U \quad (6.3)$$

s.t. \quad \sum_{F \in \mathbb{P}^K \setminus \{\emptyset\}} p_F \leq 1, \quad (6.4)

$$p_F \geq 0, \quad \forall F \in \mathbb{P}^K, \quad (6.5)$$

$$\eta_0 + \eta_1 \leq 1, \quad (6.6)$$

$$\eta_0 \geq 0, \eta_1 \geq 0. \quad (6.7)$$
Here, (6.4) and (6.5) are standard probability constraints, while (6.6) and (6.7) are spectrum allocation constraints.

The challenges of optimization problem $P$ are two-folded. First, the objective $U$ is a complicated function of the decision variables. Second, the problem $P$ is non-convex when we jointly optimize the spectrum allocation and cache probabilities. Therefore, we will first derive a closed-form expression for $U$ in the next section, which will then be used to study the special structure of $P$ to find an optimal solution.

### 6.3 Derivation of Coverage-Rate Logarithm Utility

Let $E^0$ and $E_{F,i}^1$, receptively, be the events that the typical DRx located at $0$ associates with a BS or a DTx that caches a file set $F$, $F \in \mathcal{F}_i^K$. Then we decompose $U$ by expectation conditional on $Q$ as follows:

$$U = \sum_{i=1}^{N} q_i (A^0_i U^0_i + \sum_{F \in \mathcal{F}_i^K} A^1_{F,i} U^1_{F,i}),$$  

(6.8)

where $A^0_i \triangleq \mathbb{P}(E^0|Q = i)$; $A^1_{F,i} \triangleq \mathbb{P}(E^1_{F,i}|Q = i)$; $U^0_i$ is the coverage-rate logarithm utility given the event $E^0$ and the DRx requires file $i$, i.e., $U^0_i \triangleq \mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log R_{cover}|E^0, Q = i]$; and $U^1_{F,i}$ is the coverage-rate logarithm utility given the event $E^1_{F,i}$ and the DRx requires file $i$, i.e., $U^1_{F,i} \triangleq \mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log R_{cover}|E^1_{F,i}, Q = i]$. Note that a DRx never associates with a DTx without its requesting file. Hence, in (6.8), we exclude the events that a DRx associates with a DTx without its requesting file.

Using (6.1), we further decompose $U^0_i$ and $U^1_{F,i}$ as

$$U^0_i = \mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log \beta|E^0, Q = i] + \log[\log(1 + T)]$$

$$+ \mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log P_{cover}|E^0, Q = i],$$

$$U^1_{F,i} = \mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log \beta|E^1_{F,i}, Q = i] + \log[\log(1 + T)]$$

$$+ \mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log P_{cover}|E^1_{F,i}, Q = i].$$

In the rest of this section, we will derive closed-form expressions for the components of $U$ above. In Section 6.3.1, we find the association probabilities, $A^0_i$ and $A^1_{F,i}$. Then, in Section 6.3.2, we derive the following conditional expectations of the logarithm of coverage probability $P_{cover}$:

$$\mathbb{E}_{\Phi, x_{tx}, \{h_{z,0}|z \in \Phi\}}[\log P_{cover}|E^0, Q = i],$$  

(6.9)
\[ \mathbb{E}_{x, h, z \in \Phi} \log P_{\text{cover}} | E_{F, i}^1, Q = i]. \] (6.10)

Finally, in Section 6.3.3, we derive the following conditional expectations of the logarithm of the DRx’s allocated spectrum \( \beta \):

\[ \mathbb{E}_{x, h, z \in \Phi} \log | E_{F, i}^0, Q = i], \] (6.11)

\[ \mathbb{E}_{x, h, z \in \Phi} \log | E_{F, i}^1, Q = i]. \] (6.12)

For notational simplicity, we define

\[
\begin{cases}
\rho_0 \triangleq \frac{\lambda_0 P_0^{\frac{2}{\gamma}}}{\lambda_1 P_1^{\frac{2}{\gamma}} + \lambda_0 P_0^{\frac{2}{\gamma}}}, \\
\rho_1 \triangleq \frac{\lambda_1 P_1^{\frac{2}{\gamma}}}{\lambda_1 P_1^{\frac{2}{\gamma}} + \lambda_0 P_0^{\frac{2}{\gamma}}}.
\end{cases}
\]

### 6.3.1 Association Probabilities

Let \( P_{r,0} \) and \( P_{r,1} \) be the expected received power of the DRx from the nearest BS and DTx respectively, where the expectation is taken over small-scale fading. Note that they are still random variables since they depend on the random distance between the DRx and its associated transmitter, but they are independent of \( Q \). Furthermore, let \( C_F \) be the event that a DTx caches a file set \( F \).

We first find \( A_{F, i}^1 \).

\[ A_{F, i}^1 = \mathbb{P} [ P_{r,1} > P_{r,0}, C_F | Q = i] \]
\[ = \mathbb{P} [ C_F^1] \mathbb{P} [ P_{r,1} > P_{r,0}] \]
\[ = p_F \mathbb{P} [ P_{r,1} > P_{r,0}] \]
\[ = p_F \rho_1, \]

In the last equality above, the derivation for \( \mathbb{P} [ P_{r,1} > P_{r,0}] = \rho_1 \) is similar to that in [93], and is omitted for brevity.

Then we have

\[ A_i^0 = 1 - \sum_{F \in \Phi^K} A_{F, i}^1 \]
\[ = \frac{\lambda_0 P_0^{\frac{2}{\gamma}}}{\lambda_1 P_1^{\frac{2}{\gamma}} + \lambda_0 P_0^{\frac{2}{\gamma}}} + (1 - \sum_{F \in \Phi^K} p_F) \frac{\lambda_1 P_1^{\frac{2}{\gamma}}}{\lambda_1 P_1^{\frac{2}{\gamma}} + \lambda_0 P_0^{\frac{2}{\gamma}}} \]
6.3.2 Conditional Expectation of Log of Coverage Probability

Let $X^0_0$ and $X^1_{F,i}$, respectively, be the random distances between the typical DRx and its associated BS or DTx that caches a file set $F$, $F \in \mathcal{F}_i^K$. Let $f^0_i(d)$ and $f^1_{F,i}(d)$, respectively, be their PDFs conditioned on the event that the DRx requires file $i$. Then, we denote by $R_0$ and $R_1$ the random distances from the typical DRx to the nearest BS and DTx, respectively. From [93], the PDF of $R_j$, for $j = 0$ and 1, is given by

$$f_{R_j}(x) = 2\pi\lambda_j x \exp(-\lambda_j \pi x^2).$$

Furthermore, let $P_{r,0}(x_0)$ be the value of $P_{r,0}$, given that the distance from the DRx to the nearest BS is $x_0$. In addition, let $C^c_i$ be the event that a DTx does not cache any file sets that contains file $i$, so that $P(C^c_i) = 1 - p_i$.

We will first find the PDFs $f^0_i(d)$ and $f^1_{F,i}(d)$. We observe that

$$P[X^0_0 > d | Q = i] = P[R_0 > d | E^0, Q = i] = \frac{P[R_0 > d, E^0 | Q = i]}{A^0_i}.$$

In addition,

$$P[R_0 > d, E^0 | Q = i] = P[R_0 > d, P_{r,0}(R_0) > P_{r,1} | Q = i]$$

$$= P[R_0 > d, P_{r,0}(R_0) > P_{r,1}, C^c_i | Q = i]$$

$$+ (1 - p_i)P[R_0 > d, P_{r,0}(R_0) < P_{r,1} | Q = i]$$

$$= P[R_0 > d, P_{r,0}(R_0) > P_{r,1}]$$

$$+ (1 - p_i)P[R_0 > d, P_{r,0}(R_0) < P_{r,1}]. \quad (6.13)$$

The last equality above is because $R_0$ and $P_{r,0}(R_0)$ are independent of $Q$. Furthermore, we have

$$P[R_0 > d, P_{r,0}(R_0) > P_{r,1}]$$

$$= \int_d^\infty P[P_{r,0}(x) > P_{r,1}]f_{R_0}(x)dx$$
\[= \int_{d}^{\infty} \mathbb{P}[R_1 > \left( \frac{P_1}{P_0} \right)^{\frac{1}{\gamma}} x] f_{R_0}(x) dx\]
\[= 2\pi \lambda_0 \int_{d}^{\infty} \exp\left( -\pi \lambda_1 \left( \frac{P_1}{P_0} \right)^{\frac{1}{\gamma}} x^2 \right) \exp(-\pi \lambda_0 x^2) d dx\]
\[= 2\pi \lambda_0 \int_{d}^{\infty} \exp(-\pi \frac{\lambda_0}{\rho_0} x^2) x dx, \quad (6.14)\]

and

\[= \int_{d}^{\infty} \mathbb{P}[R_0 > d, P_{r,0}(R_0) < P_{r,1}] \]
\[= \int_{d}^{\infty} \mathbb{P}[P_{r,0}(x) < P_{r,1}] f_{R_0}(x) dx\]
\[= \int_{d}^{\infty} \mathbb{P}[R_1 < \left( \frac{P_1}{P_0} \right)^{\frac{1}{\gamma}} x] f_{R_0}(x) dx\]
\[= 2\pi \lambda_0 \int_{d}^{\infty} (1 - \exp\left( -\pi \lambda_1 \left( \frac{P_1}{P_0} \right)^{\frac{1}{\gamma}} x^2 \right)) \exp(-\pi \lambda_0 x^2) d dx\]
\[= 2\pi \lambda_0 \int_{d}^{\infty} \left( \exp(-\pi \lambda_0 x^2) - \exp(-\pi \frac{\lambda_0}{\rho_0} x^2) \right) d dx. \quad (6.15)\]

Substituting (6.14) and (6.15) into (6.13), we have

\[f_{i}^{0}(d) = \frac{d \mathbb{P}[X^0 < d | Q = i]}{dd}\]
\[= 2\pi \lambda_0 d \left[ p_i \exp(-\pi \frac{\lambda_0}{\rho_0} d^2) + (1 - p_i) \exp(-\pi \lambda_0 d^2) \right] A_i^0.\]

Similarly, we can show that

\[f_{i}^{1}(d) = \frac{2\pi \lambda_1}{A_{i,F,i}} dp_F \exp(-\pi \frac{\lambda_1}{\rho_1} d^2).\]

To use the above PDFs in deriving the conditional expectations in (6.9) and (6.10), we further condition them on the event that the distance between the DRx and its associated transmitter is \(|x_{tx}| = d\). We have

\[\mathbb{E}_{\Phi, x_{tx}, \{h_z \in \Phi\}}[\log(P_{\text{cover}}) | \|x_{tx}\| = d, E^0, Q = i] \]
\[= -\frac{2\pi T \lambda_0}{\gamma - 2} d^2, \quad (6.16)\]
\[\mathbb{E}_{\Phi, x_{tx}, \{h_z \in \Phi\}}[\log(P_{\text{cover}}) | \|x_{tx}\| = d, E^1_{F,i}, Q = i] \]
The derivation of the above results is similar to that in [95], and is omitted for brevity.

Deconditioning (6.16) and (6.17) on \( |x_{tx}| = d \), we have the following expressions for (6.9) and (6.10):

\[
E_{\Phi, x_{tx}, \{h_{x,0} | z \in \Phi\}}[\log(P_{\text{cover}}) | E_0, Q = i] = \int_0^\infty \left( -\frac{2\pi T \lambda_0}{\gamma - 2} d^2 \right) f_0^0(d) \, dd = -2T \frac{1 - (1 - \rho_0^2)p_i}{A_0^0(\gamma - 2)},
\]

\[
E_{\Phi, x_{tx}, \{h_{x,0} | z \in \Phi\}}[\log(P_{\text{cover}}) | E_1^1, Q = i] = \int_0^\infty \left( -\frac{2\pi T \lambda_1}{\gamma - 2} d^2 \right) f_{1^1}^i(d) \, dd = -2T \frac{\rho_1^2 p_F}{A_{F,i}^1(\gamma - 2)}.
\]

### 6.3.3 Conditional Expectation of Log of Allocated Spectrum

It is difficult to track (6.11) and (6.12) exactly. In [92], the authors use the following approximation:

\[
E_{\Phi, x_{tx}, \{h_{x,0} | z \in \Phi\}}[\log_\beta] \approx \log \frac{\eta}{M},
\]

where \( \eta \) is the total spectrum allocated to the transmitter, and \( M \) is the average number of devices associating with the transmitter. They show that the approximation in (6.18) is tight when there are many more receivers than transmitters. In this chapter, we resort to the same approximation method since we have assumed \( \lambda_0 \ll \mu \) and \( \lambda_1 \ll \mu \) as explained in Section 6.2.1.

The average number of DRxs associating with a BS is

\[
M^0 = \sum_{j=1}^N \frac{\mu q_j A_j^0}{\lambda_0} = \frac{\mu}{\lambda_0} (1 - \rho_1 \sum_{j=1}^N q_j p_j).
\]

Then, for \( \lambda_0 \ll \mu \), (6.11) is given by

\[
E_{\Phi, x_{tx}, \{h_{x,0} | z \in \Phi\}}[\log_\beta | E_0^0, Q = i] = \log \frac{\eta_0}{\lambda_0} (1 - \rho_1 \sum_{j=1}^N q_j p_j).\]

The average number of DRxs associating with a DTx that caches a file set \( F, F \in \mathbb{F}_i^K \), is

\[
M_{F,i} = \sum_{j \in F} \frac{\mu q_j A_{F,j}^1}{\lambda_1 p_F} = \sum_{j \in F} \frac{\mu q_j \rho_1}{\lambda_1} = \frac{\mu}{\lambda_1} q_F \rho_1.
\]
Then, for $\lambda_1 \ll \mu$, (6.12) is given by

$$E_{\Phi, x, \{h_x, a\}_{x \in \Phi}}[\log(\beta)]E_{F, i}^c, Q = i = \log \frac{\eta_1}{\lambda_1} q_F \rho_1.$$

### 6.4 Optimal Probabilistic Caching with Fixed Spectrum Allocation

As a step toward our ultimate goal of solving the joint optimization problem $P$, we first study optimal probabilistic caching with fixed spectrum allocation between direct and D2D communication, i.e., $\{\eta_0, \eta_1\}$. Then $P$ can be simplified to the following:

$$P_1: \max_{\{p_F\}} B \sum_{i=1}^N q_i p_i - \rho_1 \sum_{i=1}^N q_i \sum_{F \in \mathcal{F}_i^K} p_F \log(q_F)$$

$$- (1 - \rho_1 \sum_{i=1}^N q_i p_i) \log(1 - \rho_1 \sum_{i=1}^N q_i p_i)$$

s.t.

$$\sum_{F \in \mathcal{F}_i^K \setminus \{\emptyset\}} p_F \leq 1,$$

$$p_F \geq 0, \forall F \in \mathcal{F}_i^K,$$  

where $B = \rho_1 \left( \log \left( \frac{\eta_1 \lambda_1}{\eta_0 \lambda_0 \rho_1} \right) + \frac{4T \rho_0}{\gamma - 2} \right).$

Note that $p_i = \sum_{F \in \mathcal{F}_i^K} p_F$, as defined in Section 6.2.2, so $p_i$ is a linear combination of optimization variables $\{p_F\}$.

We have the following observation.

**Proposition 24.** Optimization problem $P_1$ is convex.

**Proof.** We first note that function $f(x) = -x \log(x)$ is concave in $x$, for $x > 0$. Then, since $\rho_1 \sum_{i=1}^N q_i p_i$ is a linear combination of $\{p_F\}$, the term $-(1 - \rho_1 \sum_{i=1}^N q_i p_i) \log(1 - \rho_1 \sum_{i=1}^N q_i p_i)$ is concave in $p_F$. Furthermore, $B \sum_{i=1}^N q_i p_i$ and $\rho_1 \sum_{i=1}^N q_i \sum_{s \in N^K_i} p_F \log(q_F)$ are linear in $p_F$. Hence, $P_1$ is convex. \qed

Therefore, we can solve $P_1$ efficiently and find optimal caching probabilities. Counter-intuitively, we find that under fixed spectrum allocation, caching the top $K$ popular files can be suboptimal. This is shown in the following example.

**Counter-intuitive example:** We consider a simple case where there are two files in the network. Each DTx can cache at most one file, i.e., $K = 1$. The requesting
probabilities of the first and the second files are 0.65 and 0.35 respectively. In addition, we set $\lambda_0 = \frac{1}{2500 \text{ m}^2}$, $\lambda_1 = \frac{5}{2500 \text{ m}^2}$, $\mu = \frac{30}{2500 \text{ m}^2}$, $T = 1$, $\frac{P_0}{P_1} = 2$, $\gamma = 6$, and $W = 20 \text{ MHz}$. In Fig. 6.2, we show the average logarithm utility of the DRxs versus the portion of spectrum allocated to D2D communication, $\eta_1$. This figure suggests that when $\eta_1 \leq 0.08$, caching the less popular file (file 2) is optimal; when $\eta_1 \geq 0.12$, caching the more popular file (file 1) is optimal; and when $0.08 \leq \eta_1 \leq 0.12$, a mixed random caching strategy based on the probabilities computed by solving problem $\mathcal{P}_1$ is optimal. The key observation is that simply caching the top $K$ popular files is not always optimal. In this example, caching the more popular file at the DTxs leads to more DRxs associating with the DTxs than the BS. When the portion of spectrum allocated to D2D communication is small, if many DRxs associate with DTxs, they do not receive sufficient data rate, leading to a low logarithm utility. Hence, in this case, it may be desirable to reduce the number of DRxs associating with DTxs, which implies caching the less popular file.

6.5 Optimal Joint Spectrum Allocation and Probabilistic Caching

In this section, with the freedom to optimize spectrum allocation as well as probabilistic caching, we solve the joint optimization problem $\mathcal{P}$. 
6.5.1 Optimal Spectrum Allocation Given Caching Probabilities

First, we find the optimal spectrum allocation given arbitrary file caching probabilities \{p_F\}. For any fixed \{p_F\}, problem \( P \) is reduced to

\[
P_2 : \max_{\{\eta_0, \eta_1\}} \left( 1 - \rho_1 \sum_{i=1}^{N} q_i p_i \right) \log \eta_0 + \rho_1 \sum_{i=1}^{N} q_i p_i \log \eta_1
\]

s.t.

\[
\eta_0 + \eta_1 \leq 1,
\]

\[
\eta_0 \geq 0, \eta_1 \geq 0.
\]

Clearly, problem \( P_2 \) is also convex. Using the Karush-Kuhn-Tucker (KKT) conditions, we obtain a closed-form solution in the following lemma.

**Lemma 25.** Given \{p_F\}, an optimal solution \{\eta_0, \eta_1\} to problem \( P_2 \) is

\[
\eta_0^* = 1 - \rho_1 \sum_{i=1}^{N} q_i p_i, \quad \eta_1^* = \rho_1 \sum_{i=1}^{N} q_i p_i.
\]  

\[ \text{(6.19)} \]

**Proof.** This result is based on standard analysis of KKT conditions. The details are omitted for brevity. \( \square \)

6.5.2 Joint Spectrum Allocation and Probabilistic Caching

Based on the optimal spectrum allocation derived above, we now find the optimal caching probabilities. Substituting (6.19) into (6.8), our objective becomes

\[
U' = -\frac{2T}{\gamma - 2} - \rho_1 \sum_{i=1}^{N} q_i p_i \sum_{F \in \mathcal{F}^K} p_F \log(p_F) - \log \left( \frac{\mu}{\lambda_0} \right) + \rho_1 \left( \sum_{i=1}^{N} \log \left( \sum_{i=1}^{N} q_i p_i \right) + \log \left[ \log(1 + T) \right] \right)
\]

\[ + \sum_{i=1}^{N} \left( \frac{4T}{\gamma - 2} \rho_0 \rho_1 + \rho_1 \log \left( \frac{\lambda_1}{\lambda_0} \right) \right) q_i p_i. \]

\[ \text{(6.20)} \]

Thus, problem \( P \) is recast as

\[
P_3 : \max_{\{p_F\}} U'
\]
s.t. \[ \sum_{\mathcal{F} \in \mathcal{F} \backslash \{\emptyset\}} p_{\mathcal{F}} \leq 1, \]
\[ p_{\mathcal{F}} \geq 0, \quad \forall \mathcal{F} \in \mathcal{F}^K. \]

We note that problem \( P_3 \) is not convex since \( U' \) is not concave. In fact, it is convex as stated in Lemma 26.

**Lemma 26.** \( U' \) is convex in \( \{p_{\mathcal{F}}\} \).

**Proof.** The proof is similar to that of Proposition 24. \( \square \)

Despite the non-concavity of \( U' \), the convexity of \( U' \) together with the form of the constraints yields a special problem structure. By exploiting this special structure, we are able to derive an optimal solution.

Consider the following optimization problem:

\[ \mathcal{P}_4: \max_x F(x) \]
\[ \text{s.t. } \sum_{i=1}^{M} x_i = t, \]
\[ x \succeq 0, \]

where \( t \geq 0, x = [x_i]_{M \times 1} \in \mathbb{R}^M \) and \( F : \mathbb{R}^M \to \mathbb{R} \) is a convex function w.r.t. \( x \).

**Lemma 27.** An optimal solution to \( \mathcal{P}_4 \) is where one element of \( x \) is \( t \) and the rest are 0.

**Proof.** See Appendix 6.8.1. \( \square \)

By applying Lemma 27, we derive an optimal solution to problem \( \mathcal{P} \) as follows.

**Proposition 28.** Problem \( \mathcal{P} \) has the following globally optimal solution:

- **Case 1:** If \( \frac{4T\lambda_0 \rho_0}{\gamma - 2} + \log \left( \frac{\lambda_1}{\lambda_0} \right) \leq 0, \)

\[
\begin{align*}
 p^*_0 &= 1, \\
 p^*_F &= 0, \quad \mathcal{F} \in \mathcal{F}^K \backslash \{\emptyset\}, \\
 \eta^*_0 &= 1, \\
 \eta^*_1 &= 0.
\end{align*}
\]
• Case 2: If $\frac{4T\lambda_0\rho_0}{\gamma-2} + \log \left( \frac{\lambda_1}{\lambda_0} \right) > 0$,

$$
\begin{align*}
& p_{\{1, \ldots, K\}}^* = 1, \\
& p_F^* = 0, \; F \in F^K \setminus \{\{1, \ldots, K\}\}, \\
& \eta_0^* = 1 - \rho_1 \sum_{i=1}^{K} q_i, \\
& \eta_1^* = \rho_1 \sum_{i=1}^{K} q_i.
\end{align*}
$$

Proof. From Lemma 27, we know that there exists one optimal solution $\{p_F\}$ to problem $P_3$, such that at most one element in $\{p_F\}$ is positive for $F \in F^K \setminus \{\emptyset\}$. This implies there exists some $F' \in F^K \setminus \{\emptyset\}$ such that $p_F = 0$ for $F \in F^K \setminus \{F', \emptyset\}$.

Then, by keeping only the components of $U'$ in (6.20) that correspond to $F'$, we have

$$
U' \triangleq - \log \left( \frac{\mu}{\lambda_0} \right) - \frac{2T}{\gamma-2} + \log[\log(1 + T)] + \rho_1 P_{F'}^r \left\{ p_{F'}^r \log(p_{F'}^r) + \left[ \frac{4T}{\gamma-2} \rho_0 + \log \left( \frac{\lambda_1}{\lambda_0} \right) \right] p_{F'}^r \right\}.
$$

Hence, optimization problem $P$ is simplified to

$$
\max_{F \in F^K \setminus \{\emptyset\}} \max_{p_F} U' \quad \text{s.t.} \quad 0 \leq p_F \leq 1.
$$

Note that the objective in (6.21) is convex in $p_F$. Similar to the proof of Lemma 27, we can show that its maximum is achieved at the boundary point, i.e., $p_F = 0$ or $p_F = 1$. We denote by $p_F^*$ the maximizer of $U'$ given $F$ in (6.21).

If $\frac{4T\lambda_0\rho_0}{\gamma-2} + \log \left( \frac{\lambda_1}{\lambda_0} \right) \leq 0$, comparing the objective (6.21) at these two boundary points, we see that $p_F^* = 0$, and otherwise $p_F^* = 1$.

If $\frac{4T}{\gamma-2} \rho_0 + \log \left( \frac{\lambda_1}{\lambda_0} \right) > 0$, we substitute $p_F^* = 1$ into (6.21) to see that problem (6.21) is equivalent to

$$
\max_{F \in F^K \setminus \{\emptyset\}} \rho_1 \left( \frac{4T}{\gamma-2} \rho_0 + \log \left( \frac{\lambda_1}{\lambda_0} \right) \right) q_F.
$$

Obviously, the optimal $F$, denoted by $F^*$, is $\{1, \cdots, K\}$ since $q_{\{1, \cdots, K\}}$ is the largest among $\{q_F\}$ for $F \in F^K \setminus \{\emptyset\}$.

In the following remarks, we consider two interesting special cases:
Remark 1: If \( \frac{\lambda_0}{\lambda_1} > \exp\left(\frac{4T}{\gamma - 2}\right) \), the condition in Case 1 is always satisfied. In this case the optimal strategy is to allocate the entire spectrum to direct communication, and not cache any file at the DTxs. This means that there is no benefit in DTx caching if the DTx density, \( \lambda_1 \), is not sufficiently high.

Remark 2: If \( \lambda_1 > \lambda_0 \), the condition in Case 2 is always satisfied. In this case caching the top \( K \) popular files with probability 1 is optimal. Recall that the example in Section 6.4 demonstrates that caching the top \( K \) popular files can be suboptimal under fixed spectrum allocation. However, when probabilistic caching is jointly considered with spectrum allocation, we see that caching the top \( K \) popular files is always optimal when the DTx density is higher than the BS density.

6.6 Numerical Evaluation

In this section, we evaluate the performance of the proposed scheme via simulation in Matlab. We also consider as alternatives an equal spectrum scheme, an equal caching probability scheme, and a no caching scheme.

In the equal spectrum scheme, the entire spectrum is divided into two equal sub-bands for direct and D2D communication, and the caching probabilities are found by solving problem \( \mathcal{P}_1 \). In the equal caching probability scheme, each DTx randomly caches a set of files in \( \{ \mathcal{F} \subseteq \mathcal{N} ||\mathcal{F}| = K \} \), and spectrum allocation is found by solving problem \( \mathcal{P}_2 \). In the no caching scheme, each DRx downloads the file from a BS.

The default system simulation parameters are in Table 6.2. The simulation results are averaged over 1000 spatial realizations. In each spatial realization, the data rate of each DRx is averaged over 500 time slots. Then we compute the average of all the DRxs’ logarithm utility on time-averaged data rate.

In Figs. 6.3 – 6.8, we study the impact of different system parameters on the average logarithm utility and the 5th percentile data rates. The logarithm utility, which corresponds to the celebrated proportional fairness criterion, is our optimization objective, while the 5th percentile data rates provide more direct visualization of the service performance to the disadvantaged network users. Furthermore, although we have used the widely adopted single-threshold model for analytical tractability, we further investigate the performance under a Shannon rate model, with rate computed as \( \log(1 + \text{SIR}) \). In all cases, we see that the Shannon rate model gives similar performance trends and comparison results, which demonstrates the usefulness of the proposed solution under a more general system model.
Table 6.2: Default simulation parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission power of BS $P_0$</td>
<td>40 W</td>
</tr>
<tr>
<td>Transmission power of DTx $P_1$</td>
<td>0.25 W</td>
</tr>
<tr>
<td>Intensity of the PPP of BS $\lambda_0$</td>
<td>$\frac{1}{2500m^2}$</td>
</tr>
<tr>
<td>Intensity of the PPP of DTx $\lambda_1$</td>
<td>$\frac{3}{2500m^2}$</td>
</tr>
<tr>
<td>Intensity of the PPP of DRx $\mu$</td>
<td>$\frac{100}{2500m^2}$</td>
</tr>
<tr>
<td>Pathloss exponent $\gamma$</td>
<td>4</td>
</tr>
<tr>
<td>Bandwidth of the entire spectrum</td>
<td>20 MHz</td>
</tr>
<tr>
<td>Number of files $N$</td>
<td>10</td>
</tr>
<tr>
<td>SIR threshold $T$</td>
<td>1</td>
</tr>
<tr>
<td>Maximum storage space of DTx $K$</td>
<td>3 Files</td>
</tr>
<tr>
<td>DRx’s file requesting probability</td>
<td>Zipf’s distribution, $q_i = \frac{i^\theta}{\sum_{j=1}^{N} j^\theta}$</td>
</tr>
<tr>
<td>Exponent of Zipf’s distribution $\theta$</td>
<td>1</td>
</tr>
</tbody>
</table>

6.6.1 Impact of Intensity Ratio between DTx and BS $\frac{\lambda_1}{\lambda_0}$

In Figs. 6.3 and 6.6, we fix the intensity of BS $\lambda_0 = \frac{1}{2500m^2}$, and change the intensity of DTxs $\lambda_1$. We see that the proposed scheme outperforms the other three schemes for all density ratios. In Fig. 6.6, we see that the 5th percentile data rates become stable even when the intensity of DTxs increases. This is because more DTxs introduce more interference to the network. When the intensity of DTxs is sufficiently large, the system becomes saturated.

6.6.2 Impact of Maximum File Storage $K$

In Figs. 6.4 and 6.7, we show the impact of $K$, the maximum number of files cached at a DTx. We see that as $K$ increases, the average logarithm utility gap between the proposed scheme and the other caching alternatives decreases, suggesting that even the naive schemes become nearly optimal when there is enough system resource. However, though the average logarithm utility gap between the proposed scheme and the equal spectrum scheme is small, the gap of 5th percentile data rates between the two schemes is large.
6.6.3 Impact of Exponent of Zipf’s Distribution $\theta$

In Figs. 6.5 and 6.8, we show the impact of the exponent of Zipf’s distribution $\theta$ (ref. Table 6.2). The average logarithm utility increases over $\theta$, since system with a more concentrated file request distribution benefit more from caching.

6.7 Summary

In this work, we study the problem of joint spectrum allocation and probabilistic caching to maximize the DRx coverage-rate logarithm utility. We find that with fixed spectrum allocation between direct and D2D communication, simply caching the most popular files can be sub-optimal. However, when we further consider jointly optimizing the spectrum allocation and caching, we find that there exist an optimal caching strategy that is non-probabilistic and bi-modal, where the DTxs either cache no file or cache the most popular files up to its storage capacity. We establish the conditions for these two cases. Furthermore, simulation results suggest that our solution developed based on the coverage rate model also provides useful insights for systems that follow the Shannon rate model.
6.8 Appendices

6.8.1 Proof of Lemma 27

Proof. Let \( e_i = [0, \ldots, 0, t, 0, \ldots, 0] \) for all \( 1 \leq i \leq M \). Then for any \( x = [x_i]_{M \times 1} \) satisfying \( \sum_{i=1}^{M} x_i = t \) and \( x \geq 0 \), we can rewrite \( x \) as \( x = \sum_{i}^{M} \frac{x_i}{t} e_i \).

Thus, we have

\[
F(x) = F\left(\sum_{i}^{M} \frac{x_i}{t} e_i\right) \overset{(a)}{\leq} \sum_{i}^{M} \frac{x_i}{t} F(e_i) \overset{(b)}{\leq} \max_{1 \leq i \leq M} F(e_i)
\]

where (a) is due to the convexity of function \( F(x) \), and (b) is from the constraints of \( x \). The equality holds when

\[
\begin{cases}
x_{i^*} = t, & \text{for } i^* = \arg \max_i F(e_i), \\
x_i = 0, & \text{for } i \neq i^*.
\end{cases}
\]

(6.23)
Figure 6.5: Average logarithm utility vs. $\theta$

Figure 6.6: 5th percentile data rates utility vs. $\frac{\lambda}{\lambda_0}$
Figure 6.7: 5th percentile data rates utility vs. $K$

Figure 6.8: 5th percentile data rates utility vs. $\theta$
Chapter 7

Conclusions and Future Works

7.1 Conclusions

This thesis is focused on the resource management in D2D networks. We aim at maximizing different utilities, including $\alpha$-fair utility, spatial logarithm utility, and weighed sum rate via managing resources in the dimensions of spectrum, power, and storage.

Specifically, in Chapter 3, we study the problem of optimizing transmission probabilities of D2D pairs in order to maximize the network-wide $\alpha$-fair utility. In Chapter 4, we further consider the maximization of $\alpha$-fair utility in a generalized spatial Aloha network consisting of multiple tiers of D2D pairs each forming a Poisson bipolar process. In Chapter 5, we consider power control in a D2D network with delayed NSI. We aim to maximize the long-term averaged sum-rate of all D2D pairs without knowing the instantaneous NSI nor its statics. In Chapter 6, we consider a cache-enabled D2D network where a DRx can fetch the file from either a BS or a DTx caching the file. We focus on probabilistic caching in DTxs, aiming to jointly optimize spectrum allocation between direct and D2D communication and the DTx file caching probabilities, to maximize the DRx coverage-rate logarithm utility.

7.2 Future Works

In this section, we discuss some possible research directions to extend this thesis.

In Chapter 3 and Chapter 4, we focus on maximizing the transmission probabilities of all D2D pairs with a single antenna. In future networks, devices are expected to be equipped with multiple antennas to further increase the throughput. Hence, it is important to consider the multi-antenna scenario. One of the challenges in the multi-antenna
scenario is the beamformer design. For instance, it is difficult to derive a closed-form throughput, and thus analyzing the optimal transmission probabilities can be difficult. In addition, we may consider jointly optimizing the transmission power in addition to the transmission probabilities in our optimization problem. In Chapter 3 and Chapter 4, we consider simple devices without the ability to adjust the transmission power. Consideration of transmission power optimization can further increase system utility.

In Chapter 5, we study power control on one channel among D2D pairs with delayed NSI. In some wireless systems, the spectrum may be further divided into multiple sub-channels, for instance, OFDMA systems. In addition, it is common to consider the sum power constraint on all sub-channels for each D2D pair in these systems. Hence, in future work, we may consider extending the proposed power control scheme to a new scheme that can be applied to networks with multiple channels. Furthermore, we may consider a long-term averaged power constraint besides the instantaneous power constraint for D2D pairs. In practical systems, long-term averaged power constraint is more meaningful than instantaneous constraint from the perspective of energy cost. The challenge is how to deal with the long-term averaged constraint, since the current OGD method that our proposed scheme relies on cannot accommodate long-term averaged constraints.

In Chapter 6, we consider probabilistic caching in D2D networks operating in the overlay mode. In the overlay mode, D2D communication and cellular communication use separate spectrums. However, D2D communication reusing the spectrum with cellular communication may further improve system performance. In future work, we may consider optimal probabilistic caching in the underlay mode. One of the challenges is to form a closed-form objective in the underlay mode.
Bibliography


