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**3D stochastic assimilation of gravity data in Lalor volcanogenic
massive sulphide, Manitoba, Canada**

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Abstract

We propose a new numerical workflow based on stochastic data integration where we merge a conceptual geological model, the drillhole geophysical and geological logs as well as surface geophysical data to compute a unified numerical model of a VMS deposit. The first step of the workflow consists in building a 3D numerical conceptual model of the geology. This conceptual model as well as geological logs are then used to generate multiple equiprobable scenarios of the geology by means of multiple point simulation (MPS). The MPS method studies high-order statistics in the space of

33 a numerical conceptual model making it possible to reproduce complex
34 geological structures. We then use conventional conditional sequential Gaussian
35 simulation, which is a method based on a node-by-node sequential process to
36 stochastically populate the geological grid with densities. For this purpose we
37 use available density logs to simulate multiple equiprobable spatial distributions
38 of the density at high spatial resolution within each geological unit separately.
39 The stochastic high-resolution density models are iteratively combined by the
40 gradual deformation method in order to minimize the difference between
41 measured Bouguer anomaly data and the data computed on the combined
42 realizations of density. Application of the proposed method to the Lalor deposit,
43 a volcanogenic massive sulphide deposit in Manitoba, Canada, produces a
44 density model that honours the geology of the deposit and the Bouguer
45 anomaly data. This unified model has the advantage to include all the available
46 information (geological and density logs and surface geophysics) at scales
47 appropriate for mining applications.

48

49 **Keywords:** Stochastic inversion, Data assimilation, Gravity data inversion,
50 Multiple-point simulation

Draft

51 Introduction

52 In order to understand the geometry and the spatial distribution of the
53 physical properties of a deposit, many types of data must be collected,
54 processed and interpreted. This amount of diverse data raises the problem of
55 numerical data integration. In deposit characterization, the available data are the
56 geology and geochemistry estimated from diamond drillholes and assays,
57 geophysical logs and surface geophysics. In particular, gravity surveys have
58 been used in different mineral exploration contexts (Paterson and Reeves,
59 1985; Li and Oldenburg, 1998). The inversion of gravity data has proven to be
60 an efficient tool for mineral exploration for its capability to image density
61 contrasts related to the presence of ore. Originally, Bouguer anomaly maps
62 were used for the detection of large positive anomalies related to the presence
63 of large deposits. Then, inversion algorithms were built in order to recover the
64 geometry of deeper deposits in three dimensions in more complex geological
65 environments. The conventional inversion approach uses a deterministic least-
66 squares algorithm providing, by design, a single smooth and low-resolution 3D

67 model of the spatial variability of the density in the ground (Francis, 2006;
68 Sams and Saussus, 2010). The downside is that the least-squares approach
69 oversamples the mean density while undersampling the extreme low and high
70 values that are the most important in resources characterization. As such, this
71 deterministic model can only be used for exploration and, in addition, does not
72 have sufficient resolution for further use in mine design. To overcome this
73 problem, stochastic approaches seek the posterior probability density function of
74 variables, which is the probability of the event happening conditional to all the
75 background information and allows for recovering the whole density distribution
76 and not only its mean. In other words, stochastic approaches produce
77 equiprobable numerical realizations (i.e., outcome of a stochastic process) of
78 the spatial distribution of a physical property while respecting constraints on
79 their statistics. In many cases, stochastic inversion methods (Franklin 1970;
80 Mosegaard and Tarantola 1995; Eidsvik et al. 2004; Tarantola 2005) and their
81 application on geophysical data (Haas and Dubrule 1994; Torres-Verdin et al.
82 1999; Asli et al. 2000; Chasseriau and Chouteau 2003; Gloaguen et al. 2005;

83 Giroux et al. 2007; Shamsipour et al. 2010, 2011, 2012) have been useful to
84 deal with the limitations of conventional inversion (i.e., the poor reproduction of
85 the statistics of physical parameters, Shamsipour et al. 2013). Furthermore,
86 when a plurality of geophysical measurements are available, joint-inversion
87 allows for more than one parameter at a time to be considered (Gyulai et al.
88 2013) which, in theory, reduces the number of solutions and can improve the
89 spatial resolution. Joint inversion, however, requires the cross-gradient
90 computation which may result in spurious correlations and lead to a huge
91 Jacobian matrix. Joint inversions are indeed difficult to implement unless the
92 two variables have the same resolution and are linearly related. Another
93 approach, based on structural similarity, allows avoiding these limitations (Haber
94 and Oldenburg 1997) but the computational cost and the control on the
95 geological plausibility is still cumbersome. Here, we have developed a nested
96 stochastic approach to reconcile the geological conceptual model, the log data
97 and multiple surface geophysical data in order to yield a consistent high-
98 resolution model.

99

100 The objective of this study is to present an inversion algorithm for 3D-stochastic
101 assimilation of gravity data taking into account explicitly the geological
102 conceptual model as well as the spatial relationship between physical rock-
103 properties. The proposed approach consists of sequentially integrating the
104 variables from the coarsest to the finest resolution. This approach makes it
105 possible to test geological and geophysical models by means of forward
106 modeling of geophysical data and evaluating the differences between the
107 measured and computed data.

108

109 **Geological context**

110 The study is carried out on the Lalor volcanogenic massive sulphide
111 (VMS) deposit, which is located near Snow Lake, Manitoba (Canada, Fig.1). It
112 sits approximately 700 km north of Winnipeg and lies within the Proterozoic
113 Flin Flon-Snow Lake belt. The lateral extent of the deposit is approximately 900
114 m in the north-south direction and 700 m in the east-west direction. The

115 mineralization is found between 570 m to 1170 m below surface. The deposit
116 was discovered by HudBay Minerals in 2007 and is by far one of the largest
117 metal deposits within the Flin Flon Greenstone Belt. Several studies have been
118 carried out to study the deposit in detail (Bailes et al. 2013; Bellefleur et al.
119 2015; Caté et al. 2015; Duff et al. 2015; Schetselaar and Shamsipour 2015;
120 Caté 2016; Duff 2016; Schetselaar et al. 2017; Caté et al. 2018).

121 Through an extensive drilling program, (2007-2012), three distinct mineralization
122 zones were mapped: (1) zinc, (2) gold and (3) gold-copper zones (Duff et al.
123 2015; Bellefleur et al. 2015).

124 Figure 2 shows a vertical section of the deposit including the mineralized
125 zones. Among the 25 Mt of the reserve and the resource, the zinc rich zones
126 are the largest (15 Mt) and the shallowest and they consist of near-massive
127 sulphide mineralization. The gold and gold-copper zones, that include 8.8 Mt of
128 the reserves and the resources at 4.6 g/t Au, are disseminated and are
129 composed of stringers of sulphide mineralization. The gold-copper zone consists
130 of semi-massive and stockwork chalcopyrite and pyrrhotite. The average grade

of Cu for this zone is 4.64% (Carter et al., 2012; Bellefleur et al. 2015; Caté et al. 2015).

Geological Model

Among the geological studies, the work by Caté et al. (2015) proposes a geological numerical model constructed by merging a geological conceptual model and drillhole information. The complementary data used for creating the model include extensive drillhole descriptions and geochemical data from Hudbay Minerals as well as data collected by Caté et al. (2015). The model was built using a classification machine-learning algorithm to distinguish volcanic units and alteration types. For our case study, we have modified the size of the model to cover a volume of $974 \times 1929 \times 1663 \text{ m}^3$ (Fig. 3). Massive sulphide ore lenses are located in the Lalor Powderhouse dacite unit and the Upper Moore mafic unit, which are felsic and mafic volcanoclastic units respectively. In total the modified model includes 9 geological units (Caté 2016).

147 **Well logs and geophysical data**

148 Contrary to the practice in the petroleum industry, only a few studies
149 have tackled the problem of defining petrophysical relations in mineral
150 exploration. One of the main reasons is the very high number of drillholes
151 available in mining projects compared to the oil and gas ones. Also, the
152 geological settings are more complex in the mining environment rendering the
153 application of petrophysical relations non-trivial. At the Lalor deposit, only 17
154 boreholes were logged with geophysical tools to provide collocated information
155 on the host lithology, the orebodies and geophysical parameters. In this study,
156 13 boreholes are selected since they are located within the grid of our
157 geological model. This type of data is key to translate physical-properties into
158 geological properties and vice versa.

159 In this study, the bulk density was measured using a gamma-gamma tool. The
160 spatial sampling interval was 0.20 m. In addition, rock types along all boreholes
161 were provided by geologists. Figure 4 shows the natural gamma and density
162 logs along with the rock types for borehole DUB183. The density log for this

drillhole shows a range from 2.56 to 4.91 g/cm³. The log shows an increase in density values at approximately 720 m, which corresponds to the alternating mafic and felsic rock units.

The downhole gravity data were acquired by the Geological Survey of Canada along five drillholes (DUB-202, DUB-279, DUB-280, DUB-282 and DUB-287), intersecting the Lalor VMS. The survey was conducted to improve the 3D gravity inversion and to combine surface and borehole gravity measurements. The data were collected using an irregular sample spacing varying from 10 m to 100 m. Downhole measurements were carried out using GRAVILOG instrument (Nind et al. 2007). A total of 155 gravity measurements were acquired. For the purpose of quality control 39% of the readings were repeated that showed the repletion error to be less than 6.5 µGal. For the Bouguer correction the density value of 2.86 g/cm³ was used (unpubl. results: Borehole Gravity Survey, Lalor deposit project, ABITIBI Geophysics, 2014; Schetselaar and Shamsipour 2015). The surface gravity data were acquired, processed and distributed by Hudbay minerals. The raw data were interpolated on a grid

spacing of 200 m by 450 m. The raw gravity data were corrected for temporal drift, latitude, free air, Bouguer slab and terrain. The regional anomaly is obtained by upward continuation. Removing the regional trend from the Bouguer anomaly results in the residual anomaly (Fig. 5).

Methodology

Figure 6 shows the workflow of the stochastic inversion method proposed in the present study. The 3D conceptual geological model of the Lalor deposit is built by the geologist (Caté et al., 2015). The modified model (Fig. 3) is used as the training image to generate 100 different 3D stochastic geological scenarios using the multiple-point simulation (MPS) algorithm (Strebelle 2002a), while at the same time respecting the geological borehole data, and the conceptual model (step 2 in Fig. 6). The MPS technique is explained in more details in the following section. Each geological unit is then populated by density values from their intrinsic petrophysical relationships (step 3 in Fig. 6). Since local scale borehole measurements provide geophysical data of high-

195 resolution, it is challenging to integrate them with measurements from the
196 regional scale, low-resolution surface and airborne geophysical methods.
197 However, since this is a forward modeling step, there is no problem of
198 compatibility between resolution and scale of different measurements. The
199 locally simulated models may be inadequate with regards to the geophysical
200 data. Therefore, the final step is to combine the different petrophysical models
201 to minimize the difference between the calculated and measured data (step 4
202 in Fig. 6). In the following sections, we will describe in details each step of the
203 workflow.

205 **Multiple-point simulation (MPS)**

206 The MPS technique is commonly used in oil and gas geological
207 modelling to simulate complex geological structures (Guardiano and Srivastava
208 1993; Journel and Zhang 2006). It was first proposed and implemented by
209 Guardiano and Srivastava (1993) to overcome the limitations of conventional
210 geostatistical simulations based on the study of the variogram of the lithology

211 measured along drillholes (two-point statistics). Specifically, two-points algorithms
212 (like indicator kriging, indicator simulations, ...) can only reproduce simple
213 geology in space (for example uniformly distributed lenses with similar shapes)
214 and fail to mimic more complex geology (Journel 1993; Caers and Journel
215 1998; Strebelle 2002a, 2002b; Caers and Zhang 2004). MPS uses a training
216 image (TI), which is a 3D conceptual geological model representing the *a priori*
217 spatial structure of the geology on which high-order (or multiple-point) statistics
218 are computed. By multiple point statistics we mean taking into account the
219 correlation between three or more spatial locations rather than only computing
220 the variogram, which calculates the dissimilarity of a variable between only two
221 locations. The training image depicts only the conceptual geological structures
222 of the simulated phenomenon. This implies spatial continuity similar to the
223 actual phenomenon but does not provide precise spatial information (Arpat and
224 Caers 2007; Maharaja 2008). As described in Arpat and Caers (2007), the
225 dimensions of these training images are different from the study area. They

could be obtained from the available data such as outcrops or from an unconstrained realization of an object-based simulations method.

In Guardiano and Srivastava's (1993) approach a new scan of the complete training image is required at each simulated node, which made it computationally inefficient (Wu et al. 2008). During the two last decades several MPS algorithms were developed to improve the computational performance of the original algorithm. These algorithms however, propose different simulation methods, termed *snestim* (Strebelle 2002a), *filtersim* (Wu et al. 2008; Zhang et al. 2006), *simpat* (Arpat and Caers 2007), and *hosim* (Mustapha and Dimitrakopoulos 2010). All of them are based on stationary training images. The stationarity of the training images is based on the principle of similarity and repetition of patterns in space. Different descriptions of the stationarity are given in literature but a simple definition would be the zones that have approximately the same mean, variance and/or other statistical moments (Deutsch and Journel 1998; Chiles and Delfiner 1999; Arpat and Caers 2007). Caers and Zhang (2004) describe the stationarity as having the data originating

242 from one population (a zone of stationarity) and being able to pool these data
243 to calculate the bivariate and multiple point statistics.

244 In the original MPS algorithm, the stationarity of the training image cannot be
245 respected when geological heterogeneity exists. To overcome this problem,
246 Arpat and Caers (2007) proposed the *simpat* algorithm in which the simulation
247 method of the original algorithm is modified by adding other *a priori* information
248 in the form of simple geometric rules, such as rotation and dilatation. de Vries
249 et al. (2009) proposed another solution by dividing the training image into
250 stationary subzones. In our case, we used the *impala* algorithm, which is an
251 improved version of the Chugunova and Hu (2008) algorithm. The latter applies
252 auxiliary constraints to distinguish patterns by storing the values of one or
253 several secondary variables in the search tree, which is a dynamic data
254 structure consisting of an ensemble of nodes corresponding to particular
255 patterns to store the multiple point statistics from the training image. To
256 improve its computational efficiency, Straubhaar et al. (2011) proposed the
257 *impala* algorithm, an improved parallel MPS algorithm using a “list approach”,

where the search tree is replaced by a list which not only makes the simulation less random-access memory (RAM) demanding but also deals with non-stationary training images. The list contains the data events and their occurrence for each geological unit. The advantage of this parallelization approach is that rather than simulating several realizations at the same time (Vargas et al. 2008; Mariethoz et al. 2010), it tends to parallelize the computation of the conditional probability density function at every node, speeding up this step which is the most time consuming one (Straubhaar et al. 2011).

Simulation of geological units with MPS

After building a 3D conceptual model, the second step of our workflow consists in computing a set of 100 realizations from the initial geological model used as the training image (TI) of the study area and the geological units identified on drill cores in a volume of 974 m × 1929 m × 1663 m. The TI contains nine lithological units with different densities. As mentioned in the

previous section *impala* preserves all known structural constraints. The objective is to simulate the geological units of the generated scenarios in such a way that the statistics and the patterns of the models are similar to those of the TI and conditional to drillhole geological unit data. Figure 7B shows a selection of slices of these 3D realizations for the initial 3D TI shown in Figure 7A. The grid used here is the same as the geological model and the same training image is used for all the units.

Petrophysical simulations

In the third step of our workflow, each geological unit is populated by density values with respect to petrophysical relationships, and to the geological model of the deposit. To this end, a conventional conditional sequential Gaussian simulation (CSGS) (Deutsch and Journel 1998; Doyen 2007) method is used to populate the stochastic realizations from the geological model with the petrophysical data, density in our case. To simulate continuous random variables with CSGS, we need to compute and model their experimental

variogram within each unit. This can be achieved by using well log data, for instance the gamma-gamma log.

The CSGS algorithm starts by choosing a random path through all the nodes of the simulation grid. Then it assigns a value of the simulated random variable to the first node of the grid, taking into account the conditioning data (well log data) located in the vicinity of the starting node. Under the assumption of multi-Gaussianity, the values of the conditional mean and the conditional variance of the first node are estimated using simple kriging (Deutsch and Journel 1998; Doyen 2007). A value is randomly selected within the conditional normal distribution and this simulated value is added to the list of conditioning data for the subsequent nodes on the random path. The simulation is terminated when all the nodes of the grid are visited once.

Density realizations using CSGS

For each geological unit of the 100 geological models simulated at the MPS step, five different and equiprobable density distributions are assigned

306 generating overall 500 density models. The spatial variability of simulated
307 densities is maintained by the geological models obtained by MPS whereas
308 their means and their standard deviations are constrained from well-log data.
309 Before using the conditioning density values for the simulation of the density
310 models, densities are classified based on the nine existing geological units in
311 the geological model and then the vertical variograms are modelled. Figure 8
312 shows the fit of experimental variograms by theoretical models in four regions
313 of the study area including the Lalor Powderhouse dacite and the Upper Moore
314 mafic units. Variogram parameters for borehole density data are presented in
315 Table 1.

316
317 The CSGS algorithm is applied separately for each geological unit in order to
318 effectively reproduce the corresponding density values and related statistics for
319 each type of unit. During the simulation, the realizations in each of the regions
320 are conditioned by the corresponding logged density, which were previously
321 classified. Figure 9A shows the first 3D density realization of the populated

model and Figure 9B presents vertical sections of 6 different 3D CSGS density realizations. The simulations are all at very high resolution (resolution of the logs) but none fits perfectly the raw gravity data. As presented in the next section, to deal with this problem an optimization step is applied on the realizations in order to combine the 500 realizations with the objective to minimize the difference between measured Bouguer anomaly and the one computed on the combined realizations.

Gravimetry forward modeling

In order to calculate the potential field caused by a density distribution in the subsurface, forward modeling is undertaken. The geophysical response of a given geophysical model is computed based on Equation (1).

$$G(m) = d \quad \text{Equation (1)}$$

where m represents a geophysical model characterized by physical parameters, which is obtained from the model combination step, d is the data set being

336 calculated and the operator G is a function relating m and d in geometric
337 terms.

338 For gravity modelling the subsurface was discretized using rectangular prisms.
339 The resulting grid is the same as geological grid of MPS. Several analytical
340 formulations for rectangular prism gravity response can be found in Li and
341 Chouteau (1998). In our case, we used the Haáz formulation (Haáz 1953),
342 which is valid for all observation points at any location. Calculating the gravity
343 response is based on the principle of superposition. The gravimetric response
344 of a body discretized into rectangular prisms is calculated by summing the
345 contribution of each of these prisms (Li and Chouteau 1998). In our case, the
346 3D subsurface volume of the ground is discretized into a grid with a cell-size
347 of 19.8 m × 39.4 m × 33.9 m along the x, y and z-axis respectively.

348

349 **Model combination using gradual deformation method**

350 Several variations of the gradual deformation method, first proposed by
351 Roggero and Hu (1998) have been proposed and tested (Hu 2000, 2002; Hu et

al. 2001; Le Ravalec and Mouche 2012). The method gradually combines two or more independent Gaussian realizations of the density ($y_1(r)$ and $y_2(r)$ in Equation 2) through a linear combination and computes the gravity response on the new combined density model $y(r)$. The combination in Equation 2 results in a Gaussian random model preserving the same mean and covariance as the two initial models (Feller 2008). The corresponding weights in the combinations allow the gradual modification of the density models between the poles formed by the original density models and prevent geologically unrealistic models.

$$y(r) = y_1 \cos(r) + y_2 \sin(r) \quad \text{Equation (2)}$$

In applying the gravimetry forward modeling on the new density model $y(r)$, we can vary or optimize r using line search algorithm in order to minimize the fit between measured and computed Bouguer anomaly. The deformation parameter r varies in the interval $[-\pi, \pi]$ within the iterative process of gradual deformation shown in Figure 10. In our study, we used a modified version of the gradual deformation method that combines conditional realizations and it is called gradual conditioning (GC) (Hu 2002). In this case, the GC density $y(r)$ is built

367 from combining three randomly selected conditional density realizations using
368 Equation (3) in order to ensure that the combined model $y(r)$ is also conditional
369 to the measured density along boreholes.

$$y(r) = \alpha_1 y_1 + \alpha_2 y_2 + \alpha_3 y_3 \qquad \text{Equation (3)}$$

370 The constraint on the weights $(\alpha_1, \alpha_2, \alpha_3)$ in the GC requires that the sum and
371 the sum of the square of the weights be equal to one. Equation (4) shows how
372 the weights (α_i) are calculated for gradual conditioning:

$$\begin{aligned} \alpha_1 &= \frac{1}{3} + \frac{2}{3} \cos r \\ \alpha_2 &= \frac{1}{3} + \frac{2}{3} \sin \left(-\frac{\pi}{6} + r \right) \\ \alpha_3 &= \frac{1}{3} + \frac{2}{3} \sin \left(-\frac{\pi}{6} - r \right) \end{aligned} \qquad \text{Equation (4)}$$

373 where r belongs to $[-\pi, \pi]$.

374
375 As shown in the next section, the norm of the objective function (here, the
376 misfit between measured and computed gravity) is an important parameter in
377 optimizing the model.

378

379 **Objective function**

380 The gradual deformation allows for multiple types of norms of the
 381 objective function. Here, we use the L_2 -norm (Equation 5) sensitive to extreme
 382 data values and L_1 -norm (Equation 6) sensitive to the median of the data, of
 383 the difference between observed and computed data at each location i of n
 384 total locations as the misfit indicator, where d_{obs} and $G(m)$ represent the
 385 measured and the calculated gravity data, respectively, and m is the density
 386 model.

$$||d - G(m)||_2 = \sqrt{\sum_{i=1}^m (d_{obs} - G(m)_i)^2} \quad \text{Equation (5)}$$

$$||d - G(m)||_1 = \sum_{i=1}^m |d_{obs} - G(m)_i| \quad \text{Equation (6)}$$

387

388 **Results**

As a reference we also computed a least-squares inversion of the observed gravity data. Here, we do not pretend to do a full and exhaustive comparison between least-squares and stochastic methods.

Least-squares gravity inversion: Lalor study case

Gravity inverse problem is an ill-posed problem, which implies non-uniqueness and instability of the solution (Menke 1984). Tikhonov regularization (Tikhonov and Arsenin 1977), commonly called least-squares regularized inversion, is generally used in geophysical inversion to stabilize the solution. Several studies have been carried out to overcome this limitation of the inverse problem (Barbosa and Silva 1994; Li and Oldenburg 2000; Lelièvre and Oldenburg 2009; Silva Dias et al. 2009). These studies also include the work of Green (1975) using the available geological or density data and a weighting matrix and the work of Li and Oldenburg (1998) using cell weighting inversely proportional to depth.

In this section the result of applying conjugate gradient least-squares gravity inversion on the dataset of the study area are briefly presented to show the necessity of the stochastic inversion method with the given dataset. The approach used to calculate the gravity response is based on that of Haáz (1953). In this case, the 3D subsurface volume is discretized into a regular grid of 31x31x16 cells along the x, y and z-axis respectively. The size of the cells within the grid determines the resolution of the model. The cell size is chosen in proportion to the distance between the gravity measurements. In the present inversion, a cell size of 100 meters in all dimensions was set for the density inversion grid, which was considered an appropriate choice as it offers reasonable computation time and does not aim to identify features too small to be resolved by the data, particularly at depth or away from the station locations. This cell size is comparable with the coarsest station spacing of the down-hole system.

In our case Tikhonov regularization parameter was used to overcome the instability of the solution and set by trials and errors to 0.1. In addition, density

421 values along boreholes were constrained to improve the solution; The 248
422 density measurements in drill holes, which are used here to constrain the
423 output model, are absolute density values, while the model values are density
424 contrasts relative to an unknown background value. Considering the volcanic
425 host rocks, a background value of 2.86 g/cm^3 is considered to be reasonable
426 for the inversion.

427
428 The decay of the gravity field is inversely proportional to the squared distance
429 between an anomalous mass and the gravimeter. The potential for detecting a
430 local anomalous mass is significantly enhanced in borehole gravity data
431 inversion in comparison to surface or airborne gravity surveys. Figure 11 shows
432 the density model after 35 iterations. Although the density is best located over
433 the deposit here, the output is still far from optimum and certainly not useful
434 for grade estimation as its resolution is too low and its distribution over
435 represents the mean while not reaching the extreme values measured along
436 boreholes.

437

438 **Stochastic inversion**

439 L_1 -norm function is less sensitive to data points that exhibit large
440 variability with respect to the measured data mentioned in surface geophysical
441 data. The minimum value of data misfit obtained from L_1 -norm after the gradual
442 deformation of all the simulated density models equals 0.3186 mGal. The RMS
443 error of L_1 -norm is slightly less than the error calculated for the L_2 -norm. The
444 density model obtained using L_1 -norm is illustrated in Figure 12B. As can be
445 seen, there is a clear correspondence between the lithological units in the
446 geological model (Fig. 12A) and the density contrasts in the density model (Fig.
447 12B), specifically the high-density zone corresponds to the Upper Moore mafic
448 unit. High-density values are also successfully retrieved at the orebody location
449 as shown in Figure 13. The high-density zone is less continuous compared to
450 the simulated realizations, which are more in accordance with the geology of
451 the mineralized lenses at Lalor (Schetselaar and Shamsipour, 2015). The
452 structural detail in the final image is controlled by the noise in the data as

modeled with the nugget effect in variogram. The cell size in the model and the resolution of the data contribute to the resolution of the final model.

Conclusions

Understanding the relationships between geology, rock physical properties and geophysical responses is very important in integrating geological and geophysical data. The purpose of this work was to study geological and geophysical data integration with the objective of building a geologically plausible density model. To this end, we proposed a new approach consisting of using the *a priori* geological model as a training image for the multiple point simulation step and application of conditional sequential Gaussian simulation. The proposed data integration method has the advantage of avoiding any matrix inversion, as is the case of conventional least-squares inversion methods. Thanks to the addition of all the available data (geology, borehole logs, surface geophysical data), the model obtained from stochastic inversion represents the complex geology of the deposit and the spatial variation of the

density contrasts caused by the orebody better than the conventional least-squares inversion. The stochastic inversion model is at very high resolution opening the door for a quantitative use of the geophysical models including density in the resources characterization.

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734 **Tables**

735 Table 1 Variogram parameters for borehole density data.

Draft

Figure captions

Fig. 1. Lalor VMS deposit, Snow Lake, Manitoba, Canada, modified from (Blakley 2008).

Fig. 2. Vertical section of the Lalor deposit showing Zn-rich, Au-rich and Cu-Au rich mineralized zones, modified from Bellefleur et al. (2015).

Fig. 3. Geological model of Lalor VMS area modified from Caté et al. (2015).

Fig. 4. Combined plots of natural gamma, density and lithofacies logs from borehole DUB183. The rock units are modified from unpublished results by Mira Geoscience (2014).

Fig. 5. Residual anomaly map of the Lalor deposit.

Fig. 6. Workflow of the proposed approach. Step 1: building the geological model. Step 2: applying multiple-point simulation (MPS) on the 3D initial geological model. Step 3: applying conditional sequential Gaussian simulation (CSGS) on a hundred 3D geological realizations (only six vertical sections from six different 3D realizations are shown). Step 4: performing gradual deformation (GD) on the five hundred 3D density realizations (only six vertical sections from six different 3D density realizations are shown) to build a single final density model.

Fig. 7. (A) 3D TI adapted from Caté et al. (2015), (B) Slices of the equiprobable simulated 3D realizations from the geological model using MPS.

Fig. 8. Vertical variogram models shown with red curves built for geological units A) Balloch basalt, B) Lalor Powderhouse dacite, C) Upper Moore mafic unit, D) Western Powderhouse dacite.

Fig. 9. (A) First 3D density realization of the populated model, (B) Vertical sections of 6 different 3D CSGS density realizations.

Fig. 10. Workflow of the gradual deformation step. y_i is the i^{th} petrophysical realization acquired from CSGS, randomly selected for gradual deformation, r is the deformation parameter and $y(r)$ is the deformed density realization from combining three realizations.

Fig. 11. Vertical sections of density model obtained from borehole Tikhonov regularized least-squares inversion.

Fig. 12. (A) 3D geological model, (B) 3D optimized density model using stochastic inversion for L_1 norm.

Fig. 13. Stochastic inversion density model vertical sections for L_1 norm, overlaid with the orebody location represented by purple stars.

Table 1 Variogram parameters for borehole density data.

Regions	Model	Nugget effect	Sill	Range (m)
Balloch basalt	Exponential	0.282	1.259	789.766
Lalor Powderhouse dacite	Spherical	0.346	0.885	0.516
	Exponential			18.11
Upper Moore mafic unit	Exponential	0.353	2.598	30.929
	Gaussian			436.655
Western Powderhouse dacite	Exponential	0.27	1	14.387
	Gaussian			32.419

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