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Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2018-0153.R2
Manuscript Type:	Note
Date Submitted by the Author:	27-Aug-2018
Complete List of Authors:	Khan, Qasim; National University of Singapore Faculty of Engineering Subramanian, Sathya; National University of Singapore Faculty of Engineering Wong, Dawn; National University of Singapore Faculty of Engineering Ku, Taeseo; National University of Singapore, Dept. of Civil & Environmental Engineering
Keyword:	Bender Elements, Shear Waves, Cemented Clay, Near Field Effect
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

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Bender elements in stiff cemented clay: shear wave velocity (V_s) correction by applying wavelength considerations

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ABSTRACT: For the quality control of cement mixing in clays, small strain shear stiffness G_{max} is now being increasingly used due to enhanced repeatability in shear wave velocity (V_s) measurements. These stiff cemented clays have higher resonant frequencies that require the use of higher input frequencies in bender element testing for reliable V_s measurements. However, the practical requirements for suitable signals (with minimal near field effects and wave reflections) can be often difficult to implement. To facilitate such V_s measurements, the current study proposes a methodology that can correct V_s values corresponding to lower wave propagation distance to wavelength ratios (L_w/λ) to more reliable values of V_s at reference L_w/λ criterion suggested in previous studies (e.g., 2, 3.33 and 4). Two clay types are mixed with Ordinary Portland Cement and various mix ratios are utilized in order to cover a wider range of soil stiffness. Based on the collected database, it is found that the resulting fitting functions enable us to reasonably estimate the stabilized V_s values corresponding to the suggested L_w/λ criterion regardless of the nature of input sine signal.

Keywords: Bender Elements, Shear Waves, Cemented Clay, Near Field Effects

INTRODUCTION

It is well established that small strain shear stiffness (G_{max}) is a fundamental parameter for static and dynamic analyses of soil (Burland 1989; Clayton 2011; Ku et al. 2013; Richart et al. 1970). Especially with respect to bender element, the low cost, small size, minimal sample disturbance and relative ease of installation into various laboratory equipment for soil testing are the primary reasons for its popularity in measurement of G_{max} (Gu et al. 2013; Seng and Tanaka 2012). Consequently, bender elements have been installed within triaxial (Fioravante 2000; Howie and Styler 2013; Jovičić and Coop 1998; Kuwano et al. 2000; Leong et al. 2009; Pennington et al. 1997; Pennington et al. 2001), oedometers (Dyvik and Olsen 1991; Kang et al. 2014; Sukolrat 2007; Zeng and Grolewski 2005) and even resonant columns (Ferreira et al. 2007; Gu et al. 2013; Youn et al. 2008). By measuring the travel time Δt of the shear wave propagating between the source and receiver elements, V_s is found by using $L_{tt}/\Delta t$, where L_{tt} is the distance between the tips of the bender elements. After obtaining V_s and using the bulk density ρ of soil, the shear modulus is simply calculated: $G_{max} = \rho V_s^2$.

As cement stabilization is a commonly adopted method for ground improvement in Singapore, the measurement of V_s for stiff artificially cemented clays is also becoming increasingly popular. The non-destructive nature of bender elements is utilized in monitoring the hardening (due to hydration) of cement mixed clays for curing times ranging from few hours to up till 90 days (Flores et al. 2009; Kang et al. 2017). As opposed to larger variation observed in unconfined compressive strength q_u measurements, better repeatability is observed in measuring G_{max} as the coefficient of variation in V_s values is found to be less than 15% (Asaka and Abe 2011; Puppala et al. 2006). Consequently, further studies have also been carried out investigating the parameters

influencing the strength gain of treated clay soil such as cement content, porosity and curing time (Consoli et al. 2011; Fatahi et al. 2013; Lu et al. 2012; Piriyaikul 2013). These studies have devised various correlations between q_u and G_{max} for establishing the nondestructive quality control of cement mixing. Due to the larger resonant frequency of stiffer materials, the input frequency for the generation of the shear waves has to be increased (Lee and Santamarina 2005). In general, the working input frequency range for various soil types is between 1 and 30 kHz but it has to be increased further in case of stiff cemented clays (Wang et al. 2016). Applying this working frequency range reduces uncertainties in the received shear wave signals due to the presence of near field effects and wave reflections.

More uncertainties could be attributed to the dimensional limitations associated with the cylindrical specimens used commonly for shear wave velocity measurements (Holman and Finno 2005). In order to minimize the influence of such uncertainties and improve the reliability for measuring Δt , the wavepath to wavelength (L_{tr}/λ) ratio has been suggested as a suitable criterion (Sanchez-Salinero et al. 1986). This study aims to propose a function that can reliably estimate the accurate V_s values for stiff cemented clays. The methodology is subsequently investigated based on the results of unconfined bender element testing of cement mixed clay. Depending on the cement content, water content and curing time, a range of input frequencies are utilized to consider the varying stiffness values of all mix ratios in this study. Consequently, it is demonstrated that the study successfully establishes the reliability of the proposed function in correcting V_s values obtained at lower L_{tr}/λ ratios to more accurate V_s values that correspond to standardized L_{tr}/λ ratios of 2, 3.33 and 4 (Leong et al. 2005; Sanchez-Salinero et al. 1986).

METHODOLOGY

Background and problem statement

Although the use of bender elements is a popular technique in small strain stiffness measurements, the uncertainties associated with their application remain a major concern in the interpretation of the obtained results. The major source of uncertainty in interpreting the results is the detection of shear wave arrival time (Viggiani and Atkinson 1995). Based on the findings of Yamashita et al. (2009), who summarized the results of shear moduli from 23 laboratories from around the world, the most commonly adopted method in determining Δt is the start to start or first arrival method. It is also one of the earliest methods used to determine Δt . However, the presence of near field components and wave reflections can adversely affect the arriving signal leading to errors in travel time interpretation. Cruse and Rizzo (1968) and Sanchez-Salinero et al. (1986) analytically predicted the presence of near field components in shear waves traveling through a medium. Experimental observations further confirmed the presence of near field effects (Brignoli et al. 1996; Leong et al. 2005). Arroyo et al. (2003) described the near field component in the arriving shear waves as a “distorted and transposed replica of the input signal”. The near field component rapidly decays with increasing number of wavelengths between the source and receiver (Brignoli et al. 1996). Therefore, a criterion based on wavepath to wavelength (L_{tr}/λ) ratio has been suggested by many authors in attempting to minimize the influence of near field effects on the arrival of shear wave signals. Sanchez-Salinero et al. (1986) initially suggested this ratio to be between 2 and 4. The upper limit was given to prevent signal attenuation due to damping. Leong et al. (2005) prescribed using a ratio of 3.33 or greater as a criterion. However, strictly adhering to these limits can become tricky by using bender elements for stiffer materials due to the requirement of high input frequencies for such samples. This is

because higher input frequencies may exceed the capacity of the bender elements and may also result in producing output signals demonstrating a weak response (Kumar and Madhusudhan 2010; Leong et al. 2009; Pennington et al. 2001). By using higher input frequencies, overshooting can occur due to which the receiving element does not respond to the input signal (Jovičić et al. 1996).

The L_{tt}/λ ratio may also be increased by using larger samples that provide longer travel distance for the shear wave. However, in case of using cylindrical specimens for obtaining V_s values, reflected waves from the walls of the specimen increase the uncertainties in detecting the shear wave arrival. Cylindrical specimens can act as waveguides due to the wave reflections from their finite boundaries (Holman and Finno 2005). This can induce axisymmetric flexural waves that are dispersive (contain frequency dependent group velocities V_g) in nature, which can result in the delay of shear wave arrival (Achenbach 1973). Moreover, a slenderness ratio of 2 is commonly utilized for preparing specimens for triaxial testing (Arroyo et al. 2006). Abiding by a slenderness ratio of 2 as well as other dimensional requirements suggested by (Sawangsurriya 2012) for addressing the influence of wave reflections can pose practical challenges. Longer samples also may result in lower signal to noise ratio which can cause weaker output signals (Leong et al. 2005). These challenges have led to various other techniques being suggested in the time domain for replacing the first arrival method. By using the time difference between characteristic peaks or troughs in input and output signals, Δt can also be determined. However, signal attenuation and material damping cause the output signals to be no longer identical to input signals which reduces the reliability of this technique (Arulnathan et al. 1998; Khan et al. 2017; Vianna da Fonseca et al. 2009). Cross correlation is an automated technique that utilizes a function which measures the degree of similarity between input and output signals. The time

corresponding to maximum cross correlation is generally chosen to be Δt . However, the complex nature of the soil-bender system results in output signals to be of a different nature than the input signals leading to reduced reliability of the this technique as well (Santamarina and Fam 1997). Moreover, the cross correlation can only provide more reliable estimates if near-field effects do not predominate the arriving signal (Wang et al. 2007).

Proposed method

In this study, the unconfined bender element testing of cement mixed soft clay has been conducted by using sine waves with a range of input frequencies varying from 2-35 kHz. By using a wide range of input frequencies for determining the V_s of cement mixed clays, the L_{tt}/λ ratio ranges from around 1 and up to 14 depending on the stiffness of the mix ratio. Applying first arrival (zero crossing) as the arrival point of shear wave, the results for cement mixed kaolin of the particular mix ratios are given in Figure 1. The notation (a-b-c) utilized in the Figure 1 is defined as follows; a is the ratio of total water to the total solids (cement + soil) in the mixture, b is the cement content based on total soil and c is the soil type. As illustrated in Figure 1, the V_s values tend to increase with the L_{tt}/λ ratio and gradually stabilize after a certain threshold value of L_{tt}/λ (between 2 and 4). The shear wave signals for the mix ratio '0.78-30-Kaolin' plotted in Figure 2 further illustrate the stabilization of first arrival points (zero crossing and first bump) with L_{tt}/λ . This is the general trend that has also been observed in several studies for various soil types including lime treated plastic silt, Onsøy clay, pure kaolin clay, Toyoura sand and residual soil (Bonal et al. 2012; Leong et al. 2009; Leong et al. 2005; Wang et al. 2016; Wang et al. 2007). This stabilization behavior is indicative of the near field influence becoming gradually less pronounced with increasing number of wavelengths between the source and receiver elements. Moreover, at higher input frequencies, the compression waves (P waves) are found to

be more distinguishable from shear waves (S waves) which makes the travel time measurement more reliable (Wang et al. 2016). For the given samples in Figure 1 together with the entire set of mix ratios used in this study, the stabilization behavior of V_s is used as the basis for proposing a function that can correct V_s values from lower L_{tt}/λ ratios to standard L_{tt}/λ ratios. The conceptual illustration for the proposed function is given in Figure 3. The difference between the reference L_{tt}/λ value (denoted as $(L_{tt}/\lambda)_{ref}$) and L_{tt}/λ values lower than $(L_{tt}/\lambda)_{ref}$ is determined and V_s values are normalized by $(V_s)_{ref}$ (the V_s value at reference $(L_{tt}/\lambda)_{ref}$) (Figure 3a). By plotting the difference between L_{tt}/λ values against the normalized V_s values, the resulting function attempts to predict adjusted V_s based on reference $(L_{tt}/\lambda)_{ref}$. These reference values of $(L_{tt}/\lambda)_{ref}$ are chosen to be 2, 3.33 and 4, based on the suggestions in previous studies (Leong et al. 2005; Sanchez-Salinero et al. 1986). Therefore, if minimizing near field influence and completely distinguishing P and S waves are challenging due to practical limitations for measuring the stiffness of cement mixed soft clays, the proposed function can work as a robust alternative approach in the reliable estimation of V_s by giving a closer to $(V_s)_{ref}$ (Figure 3b).

EXPERIMENTAL DETAILS

Materials and sample preparation

For the purpose of this study, two types of soft clays (kaolin clay and Singapore Marine clay) were mixed with Ordinary Portland Cement (OPC). The physical properties of each clay type are given in Table 1. The details of mix ratios adapted in this study for each clay type are given in Table 2. The samples were prepared initially by adding enough water to achieve workability for further addition of cement. The mixture consisting of clay and water was then mixed in Hobart mixer for 10 minutes. Afterwards, cement was added and allowed to mix in the Hobart mixer for another 10 minutes. During the mixing, water was gradually added to achieve the target mix

ratio. The rate of mixing is 125 rpm and includes two rounds of mixing for 5 minutes. After the initial mixing of 5 minutes, the sample was mixed manually, ensuring that the walls of the mixer are scrubbed thoroughly for one minute. It was followed by mixing for additional 5 minutes. The mixing procedure is based on the Japanese Standard JGS 0821 (JGS (Japanese Geotechnical Society) 2017). Marine clay samples were prepared in PVC molds while kaolin clay samples were prepared in plastic molds (each having a diameter = 50mm and height = 100mm). The walls of each mold type were greased and tightly sealed from the top and bottom. The preparation was conducted by filling the molds with the sample mix and gradually tamping it in three layers to ensure minimal air voids. After the mold is filled, they are kept inside water at room temperature.

The frequency range used for each mix ratio is given in Table 2. The length and width of the benders are 13mm and 1mm respectively. The elements are fixed into a cylindrical tube using epoxy resin and the protrusion length is 2.5mm. For each given frequency, the travel time is determined by using first arrival (zero crossing) method for determining the value of V_s . In order to avoid damage due to the higher stiffness of the samples, a rectangular slot similar to the size of the bender is made at each end of the specimen. The benders are pressed into these slots and the shear wave signals are collected for the given frequency range. The travel distance has been taken to be between the tips of the benders and the tip to tip distance is 95mm (± 3 mm).

RESULTS

Figure 4 illustrates the plots obtained for the difference between lower L_{tt}/λ values and $(L_{tt}/\lambda)_{ref}$ versus the values of V_s obtained at lower L_{tt}/λ values normalized with $(V_s)_{ref}$. The values of V_s obtained at L_{tt}/λ lower than '1' have not been included in the plot. This is because choosing a value lower than 1 would imply that the complete shear wave has not even arrived at the

receiving element. Therefore, the signal would be heavily influenced by near field effects and wave interference due to reflections.

The notation for the description of mix ratios is similar to the one illustrated in Figure 1. For each chosen value of $(L_{tt}/\lambda)_{ref}$, a separate graph is plotted in Figure 4. Based on the chosen $(L_{tt}/\lambda)_{ref}$ value, the variation trend of V_s with L_{tt}/λ looks rather different. The general equation that is applied to express the relationship between normalized V_s with L_{tt}/λ difference is given in equation 1:

$$\frac{V_0}{(V_s)_{ref}} = 1 - \alpha \left[\left(\frac{L_{tt}}{\lambda} \right) - \left(\frac{L_{tt}}{\lambda} \right)_{ref} \right]^\beta \quad (1)$$

where α and β are the fitting parameters of the equation and V_0 is the value of V_s at lower L_{tt}/λ ratios. The values of α and β for each $(L_{tt}/\lambda)_{ref}$ are given in Table 3. For using $(L_{tt}/\lambda)_{ref}$ as 2, $\alpha = 0.032$ gives the best fit for equation 1. A linear relationship proves to be a better function in this case hence $\beta = 1$ (Figure 4a). Most of the samples tested in this study have high stiffness, hence the normalized values of V_s corresponding to L_{tt}/λ values below 2 are observed to be lesser than 1. The points in Figure 4a mostly correspond to samples cured for lesser days or with higher water content. These samples are the least stiff mix ratios in the database due to which more values of V_s are obtained at L_{tt}/λ values below 2. Those mix ratios with higher cement contents (30%, 50% and 100%) or lower water to solids ratio (0.9, 1.0 and 1.2) show increasingly non-linear relationship with a larger L_{tt}/λ difference. This illustrates the increased dependency of V_s on L_{tt}/λ making it crucial to choose appropriate $(L_{tt}/\lambda)_{ref}$ for the analysis. As the value of $(L_{tt}/\lambda)_{ref}$ is increased to 3.33 and 4, a power fitting function provides a better fitting for the non-linear reduction in the normalized V_s values with larger L_{tt}/λ differences (Figure 4b and 4c). By using $\alpha = 0.05$ and $\beta = 1.80$, the best fit for equation 1 is obtained when $(L_{tt}/\lambda)_{ref}$ is taken as 3.33. Moreover, $\alpha = 0.025$ and $\beta = 2.07$ provide the best fit in applying $(L_{tt}/\lambda)_{ref}$ as 4. In plotting the

fitting functions, a few outliers indicated in each figure have been ignored. Figure 4a does not exhibit a scatter that can differentiate between the clay types used in this study. However, Figure 4b and Figure 4c do exhibit an upper and lower bound that depends on the type of clay mixed. In general, most of the values obtained using Singapore Marine clay are present along the lower bound of the fitting function while kaolin is closer to the upper bound. This indicates that the variation of V_s with L_{tt}/λ values may also have a dependency on the soil type. Therefore the fitting function is further adjusted in order to provide a range of values for V_s . The fitting parameters α and β have a range, indicated in Figure 4, that corresponds to the upper and lower bounds.

By expressing equation 1 in terms of $(V_s)_{ref}$ and $(L_{tt}/\lambda)_{ref}$, comparison plots with combined results obtained using kaolin and Singapore Marine clay are illustrated in Figure 5. These plots demonstrate the reliability of the proposed functions for correcting V_s at lower L_{tt}/λ values. As it can be seen in each figure, the scatter in the plot reduces significantly after the correction factor is applied in all three cases. When using $(L_{tt}/\lambda)_{ref}$ of '2', a relatively larger scatter still exists even after correction with the R^2 value being 0.93. However, power fitting functions by using $(L_{tt}/\lambda)_{ref}$ as 3.33 and 4 further reduce the scatter thus giving reliably close estimates with $(V_s)_{ref}$. This is further demonstrated by the corresponding higher R^2 values around 0.99 for all functions. These R^2 values are calculated for those functions with α and β values that provide the best fit in each case.

DISCUSSION

For determining the mixing quality of cement mixing, the non-destructive measurement of small strain stiffness through V_s is becoming a useful indicator in practice. In comparison to determining q_u , more repeatability would be achievable in V_s measurements by using bender

elements. However, stiff cemented clays require higher input frequencies for reliable testing using bender elements due to the high resonant frequencies of stiffer materials (Lee and Santamarina 2005). Moreover, near field influence and wave reflections need to be minimized in bender element testing. For stiff cemented clays, higher frequencies can provide suitable signals which are helpful in obtaining consistent V_s measurements (Wang et al. 2016). However abiding by these requirements can be often challenging in implementation. Therefore, the current study proposes empirical prediction function that can adjust V_s values obtained at lower L_{te}/λ values to more reliable V_s estimates at standard L_{te}/λ values suggested in previous studies. This function is based upon the stabilization behavior of V_s as the L_{te}/λ is increased up to and beyond the suggested values. Cement mixing was done with two clay types, kaolin and Singapore Marine clay, with varying cement content, water content and curing times. Testing numerous mix ratios enables the use of a comprehensive database that includes V_s measurements for a wider range of soil stiffness. In addition, a wide range of input frequencies (2-35 kHz) was investigated to account for the varying range of the stiffness. The suggested criteria in past studies of 2, 3.33 and 4 are chosen as the $(L_{te}/\lambda)_{ref}$. The following observations can be made after applying the proposed methodology:

- Soil samples with higher water contents and shorter curing times form most of the scatter points when $(L_{te}/\lambda)_{ref}$ is chosen to be 2. This illustrates that the V_s of few mix ratios stabilize around $(L_{te}/\lambda)_{ref}$ of 2. In this case, a linear relationship provides a good fit in describing the variation of V_s with L_{te}/λ .
- The variation of V_s with L_{te}/λ becomes increasingly non-linear especially for stiffer samples. Therefore, most of the collected data follow a power relationship which is better fitted as opposed to a linear function due to the non-linear decrease in V_s .

- The results indicate that the variation of V_s with L_{tt}/λ is also soil dependent. Therefore, in the function, a range of values for the fitting parameters α and β are given in this study.
- In comparison plots with $(V_s)_{ref}$, applying the proposed function exhibits a significant reduction in the scatter especially by using the power relationship with $(L_{tt}/\lambda)_{ref}$ values of 3.33 and 4. These plots illustrate the increased reliability in using the proposed function for adjusting V_s obtained with L_{tt}/λ values below $(L_{tt}/\lambda)_{ref}$.

CONCLUSIONS

The V_s correction method proposed in this study are useful in ensuring more reliable estimates of V_s especially if obtaining suitable signals with minimal disturbances (near field influence, P and S wave interference and wave reflections) is practically challenging in bender element testing. As shown in the results, obtaining signals at lower L_{tt}/λ values can result in V_s values being much lower than the actual V_s values corresponding to $(L_{tt}/\lambda)_{ref}$. Note that the database employed for the proposed equation covers a frequency range between 2kHz and 35kHz. Based on the practical considerations of using cylindrical specimens with slenderness ratio of 2, the proposed function exhibits its robust applicability by adjusting V_s values for unconfined cement mixed clays specimens with a wide ranging stiffness (V_s ranging between 200m/s and 700m/s) due to the numerous mix ratios used in the study.

ACKNOWLEDGEMENT

The authors appreciate the financial support from Singapore Ministry of Education (MOE, Award Number: R-302-000-124-112).

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Figure 1. Variation of V_s with L_{tt}/λ .

Note: The notation for the legend is described as following: a - b - c where ' a ' is water to total solids (cement + soil) ratio, ' b ' is the cement percentage based on total soil, ' c ' is clay type.

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Table 1: Properties of clay types used in the study

Soil Properties	Singapore Marine Clay	Kaolin Clay
Specific Gravity G_s	2.62 - 2.69	2.60
Liquid Limit (%)	88	87
Plastic Limit (%)	41	40

Table 2: Soil mix design ratios

Soil Type	Cement Content (Percentage)	Water to Solids Ratio	Curing Time (Days)	Input frequencies (kHz)
Singapore Marine Clay	20	1.00	7	2,4,6,8,10,12,14,16,18,20,25,30
	30	1.00	7	2,4,6,8,10,12,14,16,18,20,25,30,35
	50	1.00	7	2,4,6,8,10,12,14,16,18,20,25,30,35
	50	1.33	7	2,4,6,8,10,12,14,16,18,20,25,30,35
	100	1.00	7	2,4,6,8,10,12,14,16,18,20,25,30,35
Kaolin Clay	30	0.78	3,7,14	2,5,7,10,12,15,17,20,25,30
	30	1.05	7,14,28	2,5,7,10,12,15,17,20,25,30
	30	1.39	7,14,28	2,5,7,10,12,15,17,20,25,30
	50	0.78	3,7,14,28,56	2,5,7,10,12,15,17,20,25,30
	50	1.05	3,14,28	2,5,7,10,12,15,17,20,25,30
	50	1.39	3	2,5,7,10,12,15,17,20,25,30

Table 3: Values for fitting parameters is equation 1 for each $(L/\lambda)_{ref}$

Proposed equation	$(L_{tt}/\lambda)_{ref}$	Fitting parameter ' α '	Fitting parameter ' β '
$\frac{V_0}{V_{ref}} = 1 - \alpha \left[\left(\frac{L_{tt}}{\lambda} \right) - \left(\frac{L_{tt}}{\lambda} \right)_{ref} \right]^\beta$	2	$0.024 < \alpha < 0.04$	1
	3.33	$0.035 < \alpha < 0.065$	$1.70 < \beta < 1.90$
	4	$0.020 < \alpha < 0.030$	$2.00 < \beta < 2.14$