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On the Use of Fredlund Gas/Fluid Compressibility Relationship to Model Medium-Dense Gassy Sand Behavior

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Abstract:

This paper presents the influence of gas bubbles trapped in the soil mass on the stress-strain-strength response of medium-dense sands. A hypoplastic constitutive sand model enhanced with the intergranular strain concept was coupled with the Fredlund gas/fluid compressibility relationship to capture gassy soil behavior. Boundary value element representations in a finite element platform of oedometer and saturated drained and undrained triaxial compression tests are performed for the calibration of soil parameters. For the numerical simulation of gassy soil behavior, pore fluid compressibility is modified to account for the presence of free gas in the pore fluid. The gassy soil mechanical response is studied by using only one set of parameters determined from the saturated soil response. The testing bed for this evaluation is a laboratory experimental program conducted on sands retrieved from the Oakridge Sanitary Landfill located in South Carolina, USA. The hypoplasticity sand model specialized with the Fredlund relationship reproduces reasonably well the stress-strain-strength response of these sands for a wide range of loading conditions and at a reasonable level of testing for the calibration of constitutive parameters. It is found that a slight reduction in the degree of saturation significantly decreased the undrained shear strength of the soil and caused changes in volume (i.e., drained-like behavior).

Author Keywords: unsaturated soils, gassy soils, gas/fluid compressibility, hypoplasticity, numerical modeling.

Introduction

The mechanical behavior of gassy soils has been traditionally studied by the offshore geotechnical community. Recent studies in soil improvement using blast densification techniques indicated that gas released during blasting can remain trapped in the soil mass for several years (Okamura et al. 2006; Yegian et al. 2007; Okamura et al. 2011). The presence of gas in marine sediments and blast-improved sand deposits greatly affects the material properties and mechanical behavior (Rad et al. 1994; Christian and Cranston 1997; Grozic et al. 1999). A gassy soil is a type of unsaturated soil composed by a structure of solid particles saturated with a pore fluid containing gas, either in a dissolved or free-state form, and the gas phase is not connected to the atmosphere (Sobkowicz and Morgenstern 1984; Nageswaran 1983). Rad and Lunne (1994) and Rad et al. (1994) stated that gas in a free-state can form only when the pore fluid is fully gas saturated. Wheeler (1986) described that the structure of a gassy soil varies with the relative dimensions of the gas bubbles and soil particles. For coarse-grained materials, the bubbles are smaller than the particle grain size and remain occluded in the pore fluid without alteration of the particle arrangement. The opposite happens when bubbles are larger than soil particles; gas bubbles generate large voids changing the soil skeleton. This paper proposes a numerical strategy to account for the presence of gas in the pore fluid to capture the behavior of medium-dense gassy sands. This is accomplished by modifying compressibility of the pore fluid in the numerical platform.

Low suction stresses are developed in granular materials containing little to no fines with degrees of saturation between 75 and 100% (Fredlund and Xing 1994; Kim and Hwang 2003; Gitirana and Fredlund 2004; Lu et al. 2009; Likos et al. 2010). These suction stresses are not usually considered in practice because gas is discretely present in a water phase that is still continuous. Therefore, the Terzaghi's effective stress concept for saturated soils is still valid for gassy sands and can be used to determine the state of stresses in a soil mass with or without the presence of occluded gas bubbles (Mihalache and Buscarnera 2016; Finno et al. 2017). The presence of gas in the soil matrix changes the mechanical behavior of sands in terms of stiffness and strength. The liquefaction resistance of loose sands is improved and the peak monotonic shear strength of dense sands is affected (Rad et al. 1994; Grozic et al.

1999; Vega-Posada 2012; Gallant 2014). The presence of free gas in the void space reduces the degree of saturation and increases the pore fluid compressibility. These occluded gas bubbles have the potential to affect the material response in terms of strength and volume changes, and pore water pressure generation. In this paper, the compressibility of the pore fluid is increased by varying the concentration of free gas in the pore fluid, thus, numerically capturing the behavior of gassy soils.

The main objective of this paper is the evaluation of the hypoplastic model for sands (Von Wolffersdorff 1996) enhanced with the intergranular strain concept (Niemunis and Herle 1997) to capture gassy soil behavior by using the Fredlund gas/fluid compressibility relationship (Fredlund 1976). The numerical analyses are developed using the finite element program *PLAXIS 2D*, modifying the compressibility of the pore fluid to account for the presence of free gas in the soil structure and study its effects on strength, pore pressure generation, and volume change. This is accomplished using only one set of parameters in the numerical analysis. The laboratory testing program presented by Vega-Posada (2012) for medium-dense sands retrieved from the Oakridge Sanitary Landfill located in South Carolina, USA is used as the testing bed of the proposed numerical strategy to capture gassy soil behavior.

Summary of tested material and testing procedures

The tested sand was collected at depths between 7.5 and 13 m from the Oakridge Waste Sanitary Landfill site, located in Dorchester, SC. The water table level at the site is located at a depth of 1.0 m. A blast densification program was implemented to increase soil resistance to liquefaction. The tested material presented angular particle shapes and was classified as a poorly-graded sand (SP) with a fine content of 7% by mass. The coefficient of uniformity and curvature, (i.e., C_u and C_c) were 1.63 and 1.03, respectively. The sand had maximum void ratio, e_{max} , of 1.09, minimum void ratio, e_{min} , of 0.58, and specific gravity of solids, G_s , of 2.66.

Saturated and gassy soil triaxial specimens were prepared by water pluviation to target void ratios between 0.703 and 0.841 representing medium-dense sands. The samples were anisotropically reconsolidated to a target effective mean normal stress, p' , of 100 kPa and sheared under drained and

globally undrained conditions. To create gassy specimens, CO₂ – saturated water was flushed by gravity through the sample to replace the de-aired water. Then, the backpressure was slowly reduced to force the CO₂ to come out of solution and form occluded bubbles. The triaxial tests were conducted on a dynamic triaxial testing system (DYNTTS), manufactured by *GDS Instruments Ltd.*, and equipped with a Hong Kong University of Science and Technology (HKUST) inner cell and a bonded high-air-entry porous disk (HAEPD). The total sample volume changes were measured with the HKUST inner cell (Ng et al. 2002) and a differential pressure transducer. Details regarding the capabilities of the triaxial device and preparation procedure of the gassy specimens can be found in Vega-Posada (2012) and Vega-Posada et al. (2014).

Hypoplasticity sand model, parameter definitions, and identification procedures

In this paper, the hypoplasticity sand model (HS model) developed by Von Wolffersdorff (1996) and the intergranular strain concept proposed by Niemunis and Herle (1997) are employed to model the behavior of saturated and gassy sands. The HS model uses a constitutive relation developed on the framework of hypoplasticity principles and critical state soil mechanics to reproduce the nonlinear stress-strain behavior of granular materials (Kolymbas 1985). The hypoplasticity concept, as opposed to the other conventional elasto-plasticity based models, does not consider complex definitions of yield surface, flow rule, hardening rule or plastic potential function in its constitutive formulation and does not separate the strains into elastic and plastic components. The HS model is defined by a single tensorial equation to represent directional triaxial stress probes and stiffness degradation of the material associated with the stress level and void ratio. The equation, in its most general form, is expressed as:

$$(1) \quad \dot{\mathbf{T}} = \mathbf{F}(\mathbf{T}, e, \mathbf{D})$$

where $\dot{\mathbf{T}}$ represents the stress rate tensor as a function of the current void ratio, e , the actual stress tensor, $\mathbf{T}$, and the deformation tensor, $\mathbf{D}$. The void ratio is determined from $(1 + e)tr\mathbf{D}$.

Later, Gudehus (1996) formulated a general constitutive equation where the void ratio represented an additional state variable to describe state changes in the granular material. This equation accounted for

properties such as barotropy (i.e., pressure dependency), pycnotropy (i.e., density dependency), argotropy (i.e., velocity dependency), unit invariance, and separability to properly reproduce cohesionless soil behavior within a wide range of densities, pressures, and deformations. Bauer (1996) proposed a factorized representation of the general equation given by Gudehus (1996). This form allowed the inclusion of new relations to describe various components of the model such as the critical state surface, the compression behavior, and the barotropy and pycnotropy interaction. Von Wolfferdorff (1996), based on the general constitutive equation proposed by Gudehus (1996) and Bauer (1996), improved the mathematical formulation of the hypoplastic model by using a new tensorial function based on the Drucker/Prager model and the Matsuoka/Nakai yielding criterion. As a result, the asymptotic behavior of granular materials was described. The modified equation is expressed as:

$$(2) \quad \dot{\mathbf{T}} = f_b f_e \frac{1}{\text{tr}(\hat{\mathbf{T}}^2)} \left\{ F^2 \mathbf{D} + a^2 \text{tr}(\hat{\mathbf{T}} \mathbf{D}) \hat{\mathbf{T}} + f_d a F (\hat{\mathbf{T}} + \hat{\mathbf{T}}^*) \|\mathbf{D}\| \right\}$$

where $\hat{\mathbf{T}} = \mathbf{T} / \text{tr} \mathbf{T}$ denotes the granulate stress ratio tensor; $\hat{\mathbf{T}}^* = \hat{\mathbf{T}} - (\mathbf{1}/3)\mathbf{I}$ is the deviatoric of $\hat{\mathbf{T}}$; $\mathbf{I}$ is the unit tensor; a and F are scalars; and f_d , f_e and f_b are pycnotropy and barotropy factors.

The scalars a and F determine the critical state surface in the stress space. The term a is dependent only on the critical friction angle, φ_c , while F is a function of $\hat{\mathbf{T}}^*$. The scalars a and F are given by:

$$(3) \quad a = \frac{\sqrt{3} (3 - \sin \varphi_c)}{2\sqrt{2} \sin \varphi_c}$$

$$(4) \quad F = \sqrt{\frac{1}{8} \tan^2 \Psi + \frac{2 - \tan^2 \Psi}{2 + \sqrt{2} \tan \Psi \cos 3\vartheta}} - \frac{1}{2\sqrt{2}} \tan \Psi$$

where $\tan \Psi$ and $\cos 3\vartheta$ are invariants in the principal stress space and are given by (Von Wolfferdorff 1996) as:

$$(5) \quad \tan \Psi = \sqrt{3} \|\hat{\mathbf{T}}^*\| \quad \text{and} \quad \cos 3\vartheta = -\sqrt{6} \frac{\text{tr}(\hat{\mathbf{T}}^{*3})}{[\text{tr}(\hat{\mathbf{T}}^{*2})]^{\frac{3}{2}}}$$

The pycnotropy factors (f_d and f_e) control the transition to the critical state line, the peak friction angle and the influence of the void ratio on the incremental stiffness. The barotropy factor (f_b) takes into account the influence of stress level on the soil properties. They are given by:

$$(6) \quad f_d = \left(\frac{e - e_d}{e_c - e_d} \right)^\alpha$$

$$(7) \quad f_e = \left(\frac{e_c}{e} \right)^\beta$$

$$(8) \quad f_b = \frac{h_s(1 + e_i)}{n} \left(\frac{e_{i0}}{e_{c0}} \right)^\beta \left(-\frac{tr \mathbf{T}}{h_s} \right)^{1-n} \left[3 + a^2 - \sqrt{3}a \left(\frac{e_{i0} - e_{d0}}{e_{c0} - e_{d0}} \right)^a \right]^{-1}$$

where α and β are material parameters; h_s and n are model parameters; e_i , e_d and e_c are the upper limit, lower limit and critical void ratios, respectively; and p_s is the mean pressure with $tr \mathbf{T} = -3p_s$. The characteristic void ratios are related to p_s by the following expression:

$$(9) \quad \frac{e_i}{e_{i0}} = \frac{e_c}{e_{c0}} = \frac{e_d}{e_{d0}} = \exp \left[- \left(\frac{-tr \mathbf{T}}{h_s} \right)^n \right]$$

Niemunis and Herle (1997) extended the hypoplastic model to surpass shortcomings in the simulation of cyclic loading paths by adding a small-strain stiffness formulation, known as the intergranular strain concept. This extension improved the performance of the basic hypoplasticity model to capture stiffness changes after stress or strain path reversals. The intergranular strain concept considers the deformation of an interface layer between soil particles.

The HS model evaluated in this paper was fully implemented in the finite element program *PLAXIS 2D* (Gudehus et al. 2008). Table 1 presents a description of the input parameters for the HS model and the adopted values to represent the mechanical behavior of medium-dense sands from the Oakridge landfill. The table also lists a summary of how each parameter was obtained.

Parameters for the basic hypoplasticity model

The basic hypoplasticity model, as proposed by Von Wolfferdorff (1996), is controlled by eight parameters to model granular material behavior over a wide range of stresses and densities. The

identification procedure of material parameters, as described by Herle and Gudehus (1999), can be easily conducted from granulometric properties of sand, index tests, and standard laboratory element-scale tests.

The critical state friction angle of the soil, $\varphi_c = 31^\circ$, was determined from the position of the critical state line (CSL) as reported by Vega-Posada (2012). Its determination employed five strain-controlled anisotropically consolidated undrained triaxial compression tests conducted on reconstituted saturated loose sand specimens. The granular hardness, h_s , used as a reference pressure, influences the stiffness of the material while the exponent, n , relates the sensitivity of a granular skeleton to changes of pressure. Herle and Gudehus (1999) suggested the use of compression tests on loose samples to determine these parameters. Isotropic triaxial compression and oedometer tests can also be considered for this purpose, which are routinely performed in geotechnical laboratory testing programs. Parameters h_s and n were determined by best fitting standard oedometer test results. In oedometric conditions, h_s and n control the slope and shape of the compression curve, respectively. In this work, the values adopted for h_s and n are 200 MPa and 0.28, respectively. Herle and Gudehus (1999) and Anaraki (2008) reported the agreement between the model parameters e_{do} and e_{co} and the laboratory values e_{min} and e_{max} . Hence, these parameters were determined from index tests following ASTM D4254 and ASTM D4253, respectively. The remaining void ratio e_{io} can be established as the maximum void ratio of a simple grain skeleton, which is reached during isotropic consolidation of a grain suspension in a gravity-free space. Herle and Gudehus (1999) proposed correlations to determine this value which are used in this work. The void ratios e_{do} , e_{co} , and e_{io} were 0.58, 1.096, and 1.315, respectively.

The exponent parameter α controls the evolution of the soil behavior between peak and critical stress. It is estimated using the results of dense samples tested under drained triaxial conditions. Herle and Gudehus (1999) suggest that α remains in a range between 0.05 and 0.30. In this work, α equal to 0.25 was found as the optimized value that minimized the error of computed and observed soil responses of drained triaxial stress probes. The constant β represents the change of stiffness at current density and can be estimated by comparing the stiffness modulus ratio of dense specimens in an oedometer test at

different void ratios. β is independent of granulometric properties; 1.4 is typically used for natural sands (Herle and Gudehus 1999).

Parameters for intergranular strains

The intergranular strain concept introduced five additional parameters into the hypoplastic framework. The parameters R , m_R , and m_T have physical meaning and can be obtained from laboratory measurements. The remaining parameters χ and β_R are constants that are correlated to the rate of strain accumulation, such that the constitutive equation reasonably reproduces hysteretic cycles of the stress-strain curve. The parameter R represents the strain range in which the incremental stiffness remains constant (i.e., size of the elastic range). The maximum value of this parameter can be found from stress-strain curves obtained from static or dynamic testing with strain path reversals. According to Kudella and Reul (2002), R varies between 1×10^{-7} for very large soil grains to 1×10^{-3} for very small soil grains. Previous investigations in the use of hypoplasticity for cohesionless soils suggested a value of 1×10^{-4} (Niemunis and Herle 1997; Meier 2009; Pham 2009; Tsegaye et al. 2010), which is the value adopted in this work.

The evolution of soil stiffness is adjusted with m_R and m_T . These model parameters are stiffness multipliers for 180° and 90° strain reversals, respectively. To determine these parameters, Niemunis and Herle (1997) recommended comparative tests at the same state but with different recent deformation histories. For granular materials, m_R and m_T are typically taken as 5 and 2, respectively (Niemunis and Herle 1997; Niemunis 2003; Meier 2009; Pham 2009; Tsegaye et al. 2010; Gallant 2014). The remaining parameters, χ and β_R , influence the stiffness degradation characteristics of the material. An increase in the value of χ , enhances the effect of m_R and m_T and results in larger stiffness. Low values of β_R tend to overestimate the stiffness at large stress levels. Niemunis and Herle (1997) reported for granular soils that χ is approximately equal to or larger than 1, while β_R can vary between 0.05 and 4. The final values adopted in this research are listed in Table 1, which ultimately represent optimized and best-fit values that minimized errors between computed and observed soil responses of oedometer and saturated drained

triaxial compression tests of medium-dense sands (Vergara 2017). This single set of HS model parameters will be used alongside a pore fluid compressibility parameter computed using the Fredlund gas/fluid compressibility equation to study the mechanical response of medium-dense gassy sands. It is expected that the HS model, at a reasonable level of testing for its calibration, adequately represents the stress-strain-strength response of these gassy sands for a wide range of loading conditions.

Evaluation of model capabilities for saturated conditions

The performance of the HS model is demonstrated by comparing the results of numerical simulations and experimental responses of saturated medium-dense sands, subjected to oedometer and anisotropically consolidated drained and undrained triaxial compression tests. The results are presented using only one set of parameters. The numerical analyses were conducted using boundary value representations of oedometer and triaxial compression tests in the finite element software *PLAXIS 2D*. The ability of the model to reproduce oedometric and triaxial responses at different initial void ratios is evaluated first under saturated conditions. Then, the same parameters are used to evaluate the ability of the numerical model to reproduce triaxial stress probes for the gassy sands when the pore fluid compressibility is modified using the Fredlund gas/fluid compressibility equation.

Oedometer tests

A boundary value element representation of an oedometer test was performed in *PLAXIS 2D* under axisymmetric conditions [see Fig.1(a)]. A rectangular numerical model with a width of 1 m and a height of 2 m was employed. The bottom cluster (1 × 1 m) represented the soil specimen and was modeled as a weightless material so that the model dimensions do not influence the results. The top cluster is used as a dummy layer to load the soil sample. Vertical stresses were applied by varying the unit weight of the dummy layer. This layer also eliminated the problem of a zero initial stress state, which is not allowed in hypoplasticity (Gudehus et al. 2008). To have a uniform strain and stress field, the dummy layer was modeled using a linear elastic material response with a Young's modulus of 1,000 kPa. Left and right model boundaries were restrained in the x-direction (i.e., $u_x=0$) while the bottom boundary was restrained

in the y -direction (i.e., $u_y=0$) to simulate one-dimensional constrained conditions. The numerical results were obtained using stress-controlled conditions with incremental loads that varied from 6.4 kPa to 3,280 kPa. An unload-reload cycle was also included in the loading sequence. Detailed calculation phases employed in *PLAXIS 2D* to simulate the oedometer tests are included in the supplementary material.

The oedometer test results and numerical simulations using the HS model are presented in Figs. 1(b)-(i). The results of eight oedometer tests conducted on reconstituted samples of sand with void ratios varying between 0.712 and 1.046 are shown in the figure. The HS model adequately reproduces the observed primary loading during oedometer testing, particularly for the large void ratios. Some discrepancies are observed for denser samples with a general tendency to compute slightly stiffer soil responses. Larger differences are observed for the unload-reload stage of the tests, which is controlled by the intergranular strain parameters. In general, the HS model computes a softer response during the reloading stage.

Drained and undrained triaxial compression tests

Triaxial compression tests of the saturated sands were simulated using a similar boundary value representation as the one used for the oedometer tests. However, the left and bottom boundaries were defined as lines of symmetry while the right and top boundaries were free to move on both directions. The finite element model used to reproduce the triaxial compression tests is shown in Fig. 2. For the numerical simulations of the drained triaxial tests, the calculations were performed in two stages. In the first stage, the soil model was anisotropically consolidated to the estimated in situ stress conditions. A K_0 -value of 0.6 was used in the testing program performed by Vega-Posada (2012) and was adopted in this research to numerically simulate the initial conditions. In the second stage, the displacements were reset to zero and the sample was sheared under a constant prescribed displacement applied at the top of the sample (i.e., strain-controlled conditions). The confining stress was maintained constant throughout the shearing stage.

Detailed calculation phases employed in PLAXIS 2D to simulate the triaxial tests are included in the Supplementary material section.

The computed stress-strain responses of the drained reconstituted specimens of medium-dense sand are shown in Fig. 3 in relation to the observed laboratory test results. The figure shows deviatoric stress, q ($= \sigma'_a - \sigma'_r$, where σ'_a and σ'_r are the axial and radial effective stresses, respectively) versus axial strain, ϵ_a , responses and development of volume changes during the tests. The results are presented for samples reconstituted at four different target void ratios: 0.703, 0.715, 0.782, and 0.817. The computed response and shear strength for the medium-dense samples (i.e., void ratios of 0.782 and 0.817) agree well with the experimental results. This is because the experimental data was employed to calibrate the HS model parameters. For dense specimens (i.e., void ratios below 0.75), the numerical model shows a slightly stiffer response for axial strain levels larger than 3%. The shear-stress-strain response at low strain levels (i.e., below 2%) and the volumetric changes of the sand specimens (i.e., contractive then dilative behavior) were very well captured with the numerical model. This is because of the intergranular strain concept enhancement into the hypoplastic framework.

To simulate the undrained triaxial compression tests, the material was also sheared under strain-controlled conditions and maintaining the confining stress constant throughout the numerical simulation. Fig. 4 presents the computed and observed undrained triaxial compression responses for saturated sand specimens reconstituted to target void ratios of 0.706, 0.775, and 0.841. Slightly larger discrepancies between the measured and computed responses are observed with respect to those obtained for the drained triaxial stress probes. However, the HS model captures the general trend observed in the stress-strain and pore water pressure response of medium-dense sands. In terms of deviatoric stresses, the computed response is underpredicted for the denser samples (i.e., $e_0 = 0.706$ and 0.775). The numerical model generally captures the development of excess pore water pressures during the shearing stage.

Compressibility of the pore fluid: Fredlund equation

The presence of free gas in the pore fluid significantly increases its compressibility (i.e., decreases its bulk modulus). This affects the mechanical behavior of the soil skeleton, particularly under undrained conditions. This behavior can be explained by considering a system completely saturated where the soil particles have a coefficient of compressibility smaller than that of the pore fluid. For this case, under undrained conditions any decrease in total stress results in a decrease in pore water pressure. However, when a gas is included into the system, the reduction in pore water pressure results in the exsolution of gas, which increases the pore fluid compressibility by several orders of magnitude causing the majority of the decrease in total stress being transferred to the soil skeleton (Sobkowicz and Morgenstern 1984; Mabrouk 2012).

The presence of exsolved gases in the pore space and its effect on the geotechnical behavior of the soil has been modeled using finite element analyses by including a low bulk modulus to the mesh clusters representing the gassy soil zone (Mabrouk and Rowe 2011; Mabrouk 2012). In this paper, gassy soil is modeled having incompressible solid grains and a relatively compressible interparticle phase representing the pore fluid imbued with soluble gas. The model assumes the gas is present in the form of occluded bubbles within the pore fluid and surface tension of the bubble is ignored, hence gas bubble pressure is equal to fluid pore pressure. The compressibility of the gas-water phase was calculated based on Fredlund (1976) who used the classic concepts of mass conservation and Boyle's and Henry's laws to define the compressibility of miscible gas/fluid mixtures on the pore fluid phase of unsaturated soils. Fredlund (1976) equation has the following form:

$$(10) \quad \beta_m = S \cdot \beta_w + B_{aw} \frac{(1-S)}{u_a} + B_{aw} \frac{S \cdot h}{u_a}$$

where S is the initial degree of saturation, β_w is the water compressibility, B_{aw} is the pore pressure parameter, u_a is the absolute pore air pressure, and h is Henry's solubility constant. In Eq. (10), the first term accounts for the compressibility of the water; the second for the compressibility of the free gas; and the third for the amount of gas that can go into solution.

Evaluation of model capabilities for unsaturated conditions (gassy sands)

The capability of the HS model to capture the mechanical response of reconstituted gassy sand specimens is presented. The model performance is tested using only one set of parameters for three different void ratios. The pore fluid compressibility is related to S and h via the Fredlund (1976) equation [Eq. (10)] and is computed in this paper for different gas-water mixtures. The results from the oedometer and triaxial testing program conducted by Vega-Posada (2012) were used to validate the applicability of the HS model coupled with the Fredlund gas/fluid compressibility equation to capture the behavior of medium-dense gassy sands. The main gases produced during blast densification projects are water vapor (H_2O), carbon dioxide (CO_2), nitrogen (N_2), and oxygen (O_2). The presence of these gases in the pore fluid are studied parametrically to determine how their presence affect the mechanical behavior of gassy sands. The pore fluid compressibility is evaluated by taking u_a equal to 2 atm (202.6 kPa), which corresponds to the atmospheric pressure plus the estimated hydrostatic pore pressure at the depth of interest; B_{aw} and β_w were taken equal to 1 and $4.7 \times 10^{-7} \text{ m}^2/\text{kN}$, respectively; and the constant, h , employed for different gas-water mixtures was taken as 0.83, 0.032, and 0.015, for CO_2 , O_2 , and N_2 , respectively (Sander 1999). The solubility coefficient of CO_2 is 25 and 55 times larger than the corresponding value for O_2 and N_2 , respectively. This explains why CO_2 is easily dissolved in the pore fluid increasing its compressibility. However, it does not contribute to lower the degree of saturation of the soil because it is unlikely to coexist in free form. Fig. 5(a) shows the variation of β_m as a function of S for gas-water mixtures including CO_2 , O_2 , and N_2 . The bulk modulus of the pore fluid, K_m , computed as the inverse of the β_m , is shown in Fig. 5(b). Low compressibility in the pore fluid phase suggests large K_m -values.

The behavior of gassy soil samples under triaxial compression loading was predicted using the same boundary value representation and calculation stages implemented for the saturated conditions. The sample was modeled in *PLAXIS 2D* as a material with a compressible pore fluid under undrained conditions. Most finite element simulations under undrained conditions assume incompressible soil

particles and pore water by assigning a large value to the Bulk modulus of water, K_w . In this paper, the compressibility of the pore fluid is manually set to match the estimated K_m of the gas-water mixture from Fig. 5. The comparison between the computed and experimental test results presented by Vega-Posada (2012) for three groups of samples with average initial void ratios, $e_{0(avg)}$, equal to 0.83, 0.79, and 0.70 are presented in Figs. 6 to 8. The results are presented in terms of deviatoric stresses, excess pore water pressures, and volumetric strains (negative means contraction and positive expansion or dilation) for three gas-water mixtures (i.e., N_2 , O_2 , and CO_2).

Fig. 6 presents the results for medium-dense gassy samples with $e_{0(avg)}$ of 0.83, $S = 83\%$ and 94% , and confinement stress levels, σ'_c , of 85.3 and 82.8 kPa, respectively. The figure shows that for $S = 83\%$, the computed deviatoric and volumetric responses reasonably match the experimental results for the three gas-water mixtures. Slightly larger discrepancies are observed for $S = 94\%$, especially for the excess pore water pressure response employing the CO_2 -water mixture. This is attributed to the large compressibility of the CO_2 pore fluid mixture in relation to the other two gases, causing a reduction in the generation of excess pore water pressures. Note also how well the HS model reproduces the dilative behavior observed in gassy sand samples, especially the transition point between contractive and dilative behavior. These observations are consistent for all the tests.

Similarly, Fig. 7 shows the comparison between experimental and computed responses for triaxial compression tests on gassy specimens with $e_{0(avg)}$ of 0.79, $S = 91\%$ and 95% , and σ'_c of 89.4 and 89.5 kPa, respectively. For the low S -values of 91% [see Fig. 7(a)], the model can reasonably reproduce the deviatoric stress, development of excess pore water pressures, and volumetric strains measured in the laboratory. For high S -values of 95% [see Fig. 7(b)], the computed responses for the three gas-water mixtures presented slightly lower values than the experimental results in terms of the deviatoric stress levels, development of excess pore pressure, and soil dilation. However, development of large excess pore pressure is underestimated when the CO_2 -water mixture is used. For strain levels above 8% , the CO_2 -water mixture model experiences a greater volumetric strain than the other two gas-water mixtures (i.e.,

N_2 and O_2). Development of large excess pore water pressure cannot be easily approximated with the numerical model using the highly compressible CO_2 -water mixture, which causes the gassy soil to mechanically respond and change in volume comparable to a drained condition. This is reflected by the near-zero development of the excess pore water pressures and significant changes in volume.

The computed and observed triaxial compression responses for gassy dense sand specimens with $e_{0(avg)}$ of 0.71 are presented in Fig. 8. Similarly, two different S -values equal to 91% and 98%, and σ'_c of 83.6 and 86.2 kPa, respectively, were employed. In general, the CO_2 -water mixture represents very well the observed response, except for u_e predictions at high S -values. Note also in the figure that for large values of axial strains (i.e., $\varepsilon_a > 6$ to 8%) the deviatoric stress level and excess pore water pressures are overestimated by the N_2 - and O_2 -water mixtures. Similar trends to those previously stated for gassy medium-dense sands are observed in the results presented in Fig. 8.

Discussion of results

For the medium-dense sands studied herein, the computed responses (i.e., deviatoric stress, excess pore pressure development, and volume changes) for the proposed gas-water mixtures match very well the experimental data, particularly for S -values lower than 90%. Larger discrepancies between computed and measured responses, especially in terms of u_e predictions, were observed at large strain levels (i.e., $\varepsilon_a > 6\%$) for near saturated conditions (i.e., $S \geq 95\%$). Recall that these computations were developed using only one set of parameters, calibrated from the saturated drained response of the same soil reconstituted at similar void ratios. The HS model, specialized with the Fredlund compressibility equation, can reasonably reproduce the mechanical response of medium-dense gassy sands measured in the laboratory.

Fig. 9 summarizes the shear strength measured at the maximum obliquity value from the laboratory tests and numerical simulations. Note that the shear strength of globally undrained gassy samples is bounded by those of saturated undrained and drained conditions. A slight reduction in the degree of saturation because of the presence of gas significantly decreases the undrained shear strength of the soil [e.g., $S = 98\%$ in Figs. 8(b) and 9] and causes the soil to change in volume and hence dilate [e.g., $S = 94\%$

in Fig. 6(b)], even under globally undrained conditions. The lower the degree of saturation, the smaller the undrained shear strength and excess pore water pressure developed during shear. These observations from the laboratory testing program are satisfactorily reproduced with the numerical model. In general, medium-dense gassy sand specimens exhibited shear strengths under globally undrained conditions that approached their drained values (Vega-Posada 2012), a feature that was also captured by the numerical model.

Summary and conclusions

The stress-strain-strength response, excess pore water pressure evolution, and soil volume changes of medium-dense saturated and gassy sands were modeled using a hypoplasticity sand model enhanced with the intergranular strain formulation and specialized with the Fredlund gas/fluid compressibility relationship. A single set of parameters for the hypoplasticity model was calibrated based on oedometer and drained triaxial tests under saturated conditions and using published correlations of its constitutive parameters. The model was used to study the triaxial compression response of soils having free gas in the pore fluid. The results were validated against advanced experimental results. From this study, the following conclusions can be drawn:

- The parameters of the basic HS sand model with the intergranular strain extension were calibrated with the results of conventional oedometer and triaxial compression tests. Numerical simulations of the oedometer tests indicated that the HS model reasonably reproduced the observed responses during primary loading, particularly for the medium-dense sands (i.e., larger void ratios). Some discrepancies were observed for the dense samples. The model had the tendency to compute slight stiffer responses than those observed in the tests. Larger differences were obtained during the unload-reload stage of the tests. In general, the HS model results tend to under predict the stiffness during the reloading stage. These discrepancies are attributed to limitations of the constitutive soil model.

- The numerical simulations, performed using a single set of parameters, for saturated drained triaxial compression tests indicated that the shear stress-strain response and the volumetric changes of the sand specimens (i.e., contractive then dilative behavior) were reasonably captured by the HS model enhanced with the intergranular strain formulation. For dense specimens, the numerical model predicted slightly stiffer responses than those observed in the tests. The shear stress-strain response at axial strain levels under 2% was reasonably computed with the numerical model for all the specimens. However, larger discrepancies between measured and computed responses were obtained for the samples tested under saturated undrained triaxial conditions.
- The HS model specialized with the Fredlund gas/fluid compressibility relationship reproduced reasonably well the observed stress-strain-strength response of medium-dense gassy sands for a wide range of stress states and loading conditions, at a reasonable computational cost and level of testing for the calibration of constitutive parameters. The computed gas-water mixtures matched reasonably well the experimental data for S -values lower than 90%. For near-saturated conditions (i.e., $S \geq 95\%$), larger discrepancies for u_e computations employing the CO_2 -water mixture were obtained. This is attributed to the large compressibility of the CO_2 pore fluid mixture in relation to the other two gases and to the fact that the compressibility of the pore fluid was used as an input in the finite element program that remained constant throughout the numerical analysis.
- In terms of shear strength, the model satisfactorily reproduced the main features of the mechanical response of gassy sands: a slight reduction in the degree of saturation because of the presence of gas significantly decreased the undrained shear strength of the soil and caused changes in volume (i.e., drained-like behavior).

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Figure Captions:

Figure 1. Oedometer test results and numerical simulations conducted with HS model: (a) finite element model; (b) $e_0 = 1.046$, $D_r = 9\%$; (c) $e_0 = 0.973$, $D_r = 23\%$; (d) $e_0 = 0.906$, $D_r = 36\%$; (e) $e_0 = 0.864$, $D_r = 44\%$; (f) $e_0 = 0.823$, $D_r = 52\%$; (g) $e_0 = 0.785$, $D_r = 60\%$; (h) $e_0 = 0.747$, $D_r = 67\%$; (i) $e_0 = 0.712$, $D_r = 74\%$.

Figure 2. Finite element model of a triaxial compression test: (a) consolidation stage; (b) shearing stage.

Figure 3. Computed and observed responses for saturated drained triaxial compression tests: (a) $e_0 = 0.703$, $D_r = 76\%$; (b) $e_0 = 0.715$, $D_r = 74\%$; (c) $e_0 = 0.782$, $D_r = 60\%$; (d) $e_0 = 0.817$, $D_r = 54\%$.

Figure 4. Computed and observed responses for saturated undrained triaxial compression tests: (a) $e_0 = 0.706$, $D_r = 75\%$; (b) $e_0 = 0.775$, $D_r = 62\%$; (c) $e_0 = 0.841$, $D_r = 49\%$.

Figure 5. Pore fluid compressibility and bulk modulus for different gas-water mixtures as a function of S .

Figure 6. Measured and computed responses for triaxial compression tests on reconstituted gassy samples with $e_{0(avg)} = 0.83$ ($D_{r(avg)} = 50\%$): (a) response for $S = 83\%$; (b) response for $S = 94\%$.

Figure 7. Measured and computed responses for triaxial compression tests on reconstituted gassy samples with $e_{0(avg)} = 0.79$ ($D_{r(avg)} = 60\%$): (a) response for $S = 91\%$; (b) response for $S = 95\%$.

Figure 8. Measured and computed responses for triaxial compression tests on reconstituted gassy samples with $e_{0(avg)} = 0.70$ ($D_{r(avg)} = 75\%$): (a) response for $S = 91\%$; (b) response for $S = 98\%$.

Figure 9. Shear strength measured at the maximum obliquity (σ'_1/σ'_3).

Table 1. Parameters for HS model with intergranular strain.

Number	Parameter	Description	Value	Source
1	φ_c	Critical state friction angle	31°	CSL ^a
2	h_s	Granular hardness	200 (MPa)	Oedometer tests
3	n	Exponent for pressure sensitive of a grain skeleton	0.28	Oedometer tests
4	e_{d0}	Minimum void ratio at zero pressure ($p_s = 0$)	0.58	Index test ASTM D4254
5	e_{c0}	Critical void ratio at zero pressure ($p_s = 0$)	1.096	Index test ASTM D4253
6	e_{i0}	Maximum void ratio at zero pressure ($p_s = 0$)	1.315	$e_{i0}/e_{max} \approx 1.2$
7	α	Exponent for the transition between peak and critical stresses	0.25	Drained TX tests
8	β	Exponent for stiffness dependency on pressure and density	1.4	Oedometer tests
9	R	Size of the elastic range	1×10^{-4}	Reference value ^b
10	m_R	Stiffness increase for 180° strain reversal	5	Reference value ^c
11	m_T	Stiffness increase for 90° strain reversal	2	Reference value ^c
12	χ	Material constant for the evolution of intergranular strains	1	Reference value ^d
13	β_R	Material constant representing stiffness degradation	0.4	Reference value ^d

Note: Parameters 1–8 correspond to the basic hypoplasticity model and 9–13 correspond to the intergranular strain concept; CSL= Critical State Line; TX= Triaxial.

^a Based on data presented by Vega-Posada (2012).

^b Recommended by Niemunis and Herle (1997), , Meier (2009), Pham (2009), and Tsegaye et al. (2010).

^c Recommended by Niemunis and Herle (1997), Niemunis (2003), Meier (2009), Pham (2009), Tsegaye et al. (2010) and Gallant (2014).

^d Recommended by Niemunis and Herle (1997).