SUPPLEMENTARY MATERIALS

**Close-range Photogrammetry with Spherical Panoramas for Mapping Spatial
Location and Measuring Diameters of Trees under Forest Canopies**

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Correction for Optical Fork Effects

Fig. S1. Optical fork effects and a correction method adapted from Grosenbaugh (1963).

As shown in Figure S1, optical fork effects occur when two lines of sight (LoS) originating from the camera O (OA and OD) intersect two opposite edges of a tree stem (A and D) and form a chord (AD) within the tree stem cross-section. Since AD does not pass through C the center of the tree stem, AD is not the diameter. To further complicate the matter, when the two LoS intersect the tree stem at a tilt angle, both form an elliptical plane within the tree stem (Grosenbaugh, 1963). Thus, a correction method for an estimated diameter from any photogrammetry method and at any tree height is needed to obtain the true diameter.

The following correction method is adapted from Grosenbaugh (1963). When viewed from camera O towards a tree stem at a height h , two LoS (OA'' and OD'') form a triangle $\triangle OA''D''$ with a fork angle α'' . Let d'' be the uncorrected tree diameter (DIA_{uncor}) estimated from one of the photogrammetry methods ($A''D''$). Since OA'' and OD'' cut an elliptical cross-section in the tree stem, Grosenbaugh (1963) suggested orthogonal projection of the elliptical cross-section to a circular cross-section on a horizontal plane to the camera O to correct d'' for the optical fork effects. As a result, triangle $\triangle OA''D''$ is orthogonally projected to triangle $\triangle OAD$ by a dihedral angle ϕ , and the projected triangle $\triangle OAD$ has a fork angle α , where $\alpha'' \neq \alpha$. Let R'' be the actual distance from camera O to point B'' and R be the actual distance from camera O to point B. With OB'' and OB as angle bisectors of α'' and α , respectively, R is related to R'' as (Grosenbaugh, 1963),

$$R = R'' \cos \phi = \left[\frac{d''/2}{\tan(\alpha''/2)} \right] \cdot \cos \phi$$

One could also imagine that the elliptical and the circular planes intersect at the coincident lines $A''D''$ and AD at the dihedral angle ϕ . As a result, $d'' = d$ and, half of the fork angle α of the triangle $\triangle OAD$ is,

$$\alpha/2 = \arctan\left(\frac{d''/2}{R}\right) = \arctan\left(\frac{d''/2}{\frac{d''/2}{\tan(\alpha''/2)} \cdot \cos \phi}\right) = \arctan\left(\frac{\tan(\alpha''/2)}{\cos \phi}\right)$$

Since the projected circular cross-section is on a horizontal plane to the camera O, the triangles ΔOAC and ΔABC are right angled triangles. With the trigonometric identities and the trigonometry of a right angled triangle, the radius r of the projected circular cross-section is (AC),

$$r = \frac{d''/2}{\sin(\pi/2 - \alpha/2)} = \frac{d''/2}{\cos(\alpha/2)} = \frac{d''/2}{\cos\left(\arctan\left(\frac{\tan(\alpha''/2)}{\cos \phi}\right)\right)}$$

Thus, the estimated DBH and Dh corrected for optical fork effects are (DIA, cm)

$$DIA = \frac{1}{\cos\left(\arctan\left(\frac{\tan(\alpha''/2)}{\cos \phi}\right)\right)} \cdot DIA_{uncor}$$

The left term of right-hand side of this equation is the correction factor. If a tree diameter is estimated at a height h that is on a horizontal plane to camera O, the correction factor reduces to $1/\cos(\alpha''/2)$, which agrees with Grosenbaugh (1963; pg. 3). The fork angle α'' and the dihedral angle ϕ are extracted from the panorama. The fork angle α'' is estimated as $\alpha'' = P_{DIA} \cdot (2\pi/L)$ with P_{DIA} defined in the main text. The dihedral angle ϕ is estimated as $\phi = |H/2 - Y_{DIA}| \cdot (\pi/H)$ with Y_{DIA} defined as the Y-pixel coordinate of a point located on the tree diameter in the panorama (e.g., Y_U of Point U in Figure S4b).

Fig. S2. Correction factor for uncorrected tree diameter DIA_{uncor} for different combinations of fork angle α'' and dihedral angle ϕ .

Figure S2 suggests an exponential relationship between the dihedral angle ϕ and the correction factor. For example, for a tree at 1 m away from plot center, fork angles α'' of 15, 30, 45, and 60° corresponds to uncorrected tree diameter DIA_{uncor} of approximately 25, 50, 75, and 100 cm. In Figure S2, correction factor increases significantly with larger fork angles α'' for a given dihedral angle ϕ . This implies that when DIA_{uncor} is small, correction is very little even for a diameter high up on the tree bole (correction factor = 1.071 when $\phi = 70^\circ$ for $\alpha'' = 15^\circ$). On the other hand, for a very large DIA_{uncor} , the corrected diameter could be double of the DIA_{uncor} at a very steep viewing angle (correction factor = 1.962 when $\phi = 70^\circ$ for $\alpha'' = 60^\circ$). In summary, correction for optical fork effects is necessary for trees with large upper stem diameters at upper part of the tree stem.

Parameters Extracted from a Spherical Panorama

The following figure is a spherical panorama from Plot 1 in the Heshe *Cunninghamia lanceolata* stand.

Fig. S3. Example spherical panorama of a forest stand with example features and pixel information extracted from it.

Basic features and pixel information for Tree 1 and Tree 5 extracted from QGIS (<http://qgis.org>).

Features	Pixels	Description
L	22348	Length of the panorama in pixels
H	11174	Height of the panorama in pixels
T1P1	(2206,5854)	X and Y-pixel coordinate for left edge of DBH of Tree 1
T1P2	(2312,5856)	X and Y-pixel coordinate for right edge of DBH of Tree 1
T5P1	(6002,6047)	X and Y-pixel coordinate for left edge of DBH of Tree 5
T5P2	(6375,6048)	X and Y-pixel coordinate for right edge of DBH of Tree 5
T5P3	(6155,7829)	X and Y-pixel coordinate for middle of tree base of Tree 5
T5P4	(5984,5543)	X and Y-pixel coordinate for left edge of target of Tree 5
T5P5	(6349,5546)	X and Y-pixel coordinate for right edge of target of Tree 5
T5P6	(6017,4929)	X and Y-pixel coordinate for left edge of D2 of Tree 5
T5P7	(6369,4913)	X and Y-pixel coordinate for right edge of D2 of Tree 5
T5P8	(5988,3501)	X and Y-pixel coordinate for left edge of D3 of Tree 5
T5P9	(6356,3489)	X and Y-pixel coordinate for right edge of D3 of Tree 5

Using Tree 5 as an example, the following parameters required by the three photogrammetry methods are calculated from the feature information presented in the above table:

- Spherical Panorama
 1. Equator = $H/2 = 11174/2 = 5587$ pixel
- Azimuth (AZ)
 1. $P_{1k} = |(X_{T1P1} + X_{T1P2})/2 - (X_{T5P1} + X_{T5P2})/2| = |2259 - 6189| = 3930$ pixel
 2. $RS = 360^\circ/L = 360^\circ/22348 = 0.0161^\circ/\text{pixel}$
- Trigonometry Principle (TRIGO)
 1. $\theta_B = Y_{T5P3} \times (\pi/H) = 7829 \times (\pi/11174) = 2.2011$ radian
 2. $DS_{TRIGO} = M/(|Y_{T5P3} - H/2|) = 130/(|7829 - 5587|) = 0.0580$ cm/pixel
 3. $P_{DBH} = X_{T5P2} - X_{T5P1} = 6375 - 6002 = 373$ pixel
 4. $P_{D2} = X_{T5P7} - X_{T5P6} = 6369 - 6017 = 352$ pixel
 5. $P_{D3} = X_{T5P9} - X_{T5P8} = 6356 - 5988 = 368$ pixel
- TRIGO Corrected for Tree Slope (TRIGOSLP)
 1. $\theta_S = -4.1^\circ \times (\pi/180^\circ) = -0.0716$ radian
 2. $\theta_C = \theta_B + \theta_S - \pi/2 = 2.2011 - 0.0716 - 1.5708 = 0.5587$ radian
 3. $P_S = \theta_S \times (H/\pi) = -0.0716 \times (11174/\pi) = -255$ pixel
 4. $DS_{TRIGOSLP} = M/(|Y_{T5P3} - H/2 + P_S|) = 130/(|7829 - 5587 - 255|) = 0.0645$ cm/pixel
- Pinhole Camera Model (PINHOLE)
 1. $f = 20$ mm
 2. $W = 21$ cm
 3. $I_R = 24/3840 = 0.00625$ mm/pixel
 4. $P_W = X_{T5P5} - X_{T5P4} = 6349 - 5984 = 365$
 5. $DS_{PINHOLE} = W/P_W = 21/365 = 0.0575$ cm/pixel
- Optical Fork Effects for D2 as an example
 1. $\alpha'' = P_{D2} \times (2\pi/L) = 0.0990$ radian
 2. $\phi = |H/2 - (Y_{T5P7} + Y_{T5P6})/2| \times (\pi/H) = |5587 - (4913 + 4929)/2| \times (\pi/11174) = 0.1872$ radian

Figures

Fig. S4. (a) Dimension of a spherical panorama in $L \times H$ and $2\pi \times \pi$ with L being the length of the panorama in pixels and H being the height of the panorama in pixels, and point F has a pixel coordinate of (X_F, Y_F) . (b) A spherical panorama depicting the first sample tree ($T1$) and the k -th sample tree (Tk) and data extracted for the photogrammetry methods. P_{1k} is the image distance between the middle of $T1$ and Tk in pixels, P_{DIA} is the width of tree stem of Tk in pixels for estimating its diameter at that height, B is the point at the base of Tk with a pixel coordinate of (X_B, Y_B) , and U is the point at the left edge of tree stem of Tk with a pixel coordinate of (X_U, Y_U) .

Fig. S5. Measured azimuth against estimated azimuth. Blue solid line depicts fitted circular regression for predicting measured azimuth from estimated azimuth. The ρ indicates fraction of variance in measured azimuth by estimated azimuth and is analogous to R^2 , the correlation of determination in linear regression. The fitted circular model was presented. AZ_{meas}^* was measured azimuth, AZ_{est}^* was estimated azimuth from Equation (1), and * denotes that both azimuths were expressed as γ the angular coordinate of the polar coordinate system. The AZ_{meas}^* could be easily back transformed to the azimuth defined in this study.

Fig. S6. Percent error against observed horizontal distance for (a) trigonometry principle (TRIGO), (b) TRIGO corrected for tree slope (TRIGOSLP), and (c) pinhole camera model (PINHOLE). Each point depicts a tallied tree. Black points depict tallied trees with slope measurement. Gray points depict tallied trees without slope measurement. Thick red solid lines depict fitted locally weighted polynomial (LOESS) regression of percent errors against observations.

Fig. S7. Spatial error for (a) trigonometry principle (TRIGO), (b) TRIGO corrected for slope (TRIGOSLP), and (c) pinhole camera model (PINHOLE). Each point depicts a tallied tree located at its observed X-Y coordinate marked by a “+” sign. The center of each panel is plot center (0, 0). The size of a circle and its grey scale corresponds to spatial error of a tallied tree with larger and darker circle depicting larger spatial error, and vice versa.

Fig. S8. Percent error against observed DBH and upper stem diameters at 2 and 3 m (D2 and D3) for (a,d,g) trigonometry principle (TRIGO), (b,e,h) TRIGO corrected for tree slope (TRIGOSLP), and (c,f,i) pinhole camera model (PINHOLE). Each point depicts a tallied tree. Black points depict tallied trees with slope measurement. Gray points depict tallied trees without slope measurement. Thick red solid lines depict fitted locally weighted polynomial (LOESS) regression of error against observations.

Fig. S9. Percent error of estimated DBH and upper stem diameters at 2 and 3 m (D2 and D3) against observed horizontal distance for (a,d,g) trigonometry principle (TRIGO), (b,e,h) TRIGO corrected for tree slope (TRIGOSLP), and (c,f,i) pinhole camera model (PINHOLE). Each point depicts a tallied tree. Black points depict tallied trees with slope measurement. Gray points depict tallied trees without slope measurement. Thick red solid lines depict fitted locally weighted polynomial (LOESS) regression of error against observed horizontal distance.

Fig. S10. Observed tree slope against observed azimuth for all tallied trees with slope measurement. Each point depicts a tallied tree.

Fig. S11. Observed DBH against observed horizontal distance for each sample tree. Each point depicts a tallied tree.