Multivariate probability distribution for some intact rock properties

Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2018-0175.R1
Manuscript Type:	Article
Date Submitted by the Author:	01-Sep-2018
Complete List of Authors:	Ching, Jianye; National Taiwan University, Phoon, Kok-Kwang; National University of Singapore, Department of Civil & Environmental Engineering Li, Kuang-Hao; National Taiwan University, Weng, Meng-Chia; National Chiao Tung University,
Keyword:	intact rock properties, multivariate probability distribution, correlations, Bayesian analysis, transformation models
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

SCHOLARONE™
Manuscripts

Submitted to Canadian Geotechnical Journal

Multivariate probability distribution for some intact rock properties

Jianye Ching¹, Kok-Kwang Phoon², Kuang-Hao Li³, and Meng-Chia Weng⁴

ABSTRACT

A multivariate probability distribution model for nine parameters of intact rocks, including unit weight (γ), porosity (n), L-type Schmidt hammer hardness (R_L), Shore scleroscope hardness (S_h), Brazilian tensile strength (σ_{bt}), point load strength index (I_{s50}), uniaxial compressive strength (σ_c), Young's modulus (E), and P-wave velocity (V_p), is constructed based on the ROCK/9/4069 database that was compiled by the authors. It is shown that the multivariate probability distribution captures the correlation behaviors in the database among the nine parameters. This multivariate distribution model serves as a prior distribution model in the Bayesian analysis and can be updated into the posterior distribution of the design intact rock parameter when multivariate site-specific information is available. In this paper, the parameters for the posterior distribution of the design intact rock parameter are summarized into user-friendly tables so that engineers do not need to conduct the actual Bayesian analysis. Caution should be taken in extrapolating the results of this paper to cases that are not covered by ROCK/9/4069, because the resulting posterior distribution can be misleading.

¹ (Corresponding author) Professor, Dept of Civil Engineering, National Taiwan University, Taiwan. Email: jyching@gmail.com. Tel: +886-2-33664328.

² Professor, Dept of Civil and Environmental Engineering, National University of Singapore, Singapore.

³ Graduate Student, Dept of Civil Engineering, National Taiwan University, Taiwan.

⁴ Professor, Dept of Civil Engineering, National Chiao Tung University, Taiwan.

Submitted to Canadian Geotechnical Journal

17 Key words: intact rock properties; multivariate probability distribution; correlations; Bayesian analysis;
18 transformation models.

19 INTRODUCTION

20 A global multivariate database for nine intact rock properties has been compiled from 184 studies by
21 Ching et al. (2018). This database is named ROCK/9/4069. The nine intact rock properties include unit
22 weight (γ), porosity (n), L-type Schmidt hammer hardness (R_L), Shore scleroscope hardness (S_h),
23 Brazilian tensile strength (σ_{bt}), point load strength index (I_{s50}), uniaxial compressive strength (σ_c),
24 Young's modulus (E), and P-wave velocity (V_p). They are denoted by ($Y_1, Y_2, \dots, Y_9$) in this paper:

$$\begin{aligned}
 25 \quad Y_1 &= \gamma & (\gamma \text{ in kN/m}^3) \\
 26 \quad Y_2 &= n & (n \text{ in } \%) \\
 27 \quad Y_3 &= R_L \\
 28 \quad Y_4 &= S_h & (1) \\
 29 \quad Y_5 &= \ln(\sigma_{bt}) & (\sigma_{bt} \text{ in MPa}) \\
 30 \quad Y_6 &= \ln(I_{s50}) & (I_{s50} \text{ in MPa}) \\
 31 \quad Y_7 &= \ln(\sigma_c) & (\sigma_c \text{ in MPa}) \\
 32 \quad Y_8 &= \ln(E) & (E \text{ in GPa}) \\
 33 \quad Y_9 &= \ln(V_p) & (V_p \text{ in km/s})
 \end{aligned}$$

34 Dimensionless parameters rarely appear in the rock mechanics literature, and this norm in the literature
35 is followed in this paper. The natural logarithm is taken for σ_{bt} , I_{s50} , σ_c , E , and V_p because these
36 properties cover relatively wide ranges, across more than one order of magnitude. ROCK/9/4069 is a
37 valuable database because it summarizes the correlation behaviors among ($Y_1, Y_2, \dots, Y_9$) found in the
38 literature. The purpose of this study is to construct the multivariate probability density function (PDF),
39 denoted by $f(y_1, y_2, \dots, y_9)$, based on ROCK/9/4069. In the literature, multivariate soil databases have
40 been compiled for clays (Ching and Phoon 2012a, 2013, 2014a; Ching et al. 2014a; D'Ignazio et al.

Submitted to Canadian Geotechnical Journal

2016), sands (Ching et al. 2017a), and rocks (Prakoso 2002; Aladejare and Wang 2017; Kim and Hunt 2017; Ching et al. 2018), and multivariate probability distribution models have been constructed successfully from some of these databases.

The multivariate PDF $f(y_1, y_2, \dots, y_9)$ is useful because it serves as the PDF that summarizes the “generic” correlation structure between rock properties. This generic PDF can be combined with local dataset to derive a hybrid PDF in Ching and Phoon (2018). This generic PDF can also serve as a prior PDF in Bayesian analysis. Feng and Jimenez (2014, 2015), Ng et al. (2015), and Wang and Aladejare (2015, 2016) have demonstrated the application of the prior PDF in Bayesian analysis. The prior PDF summarizes generic (non-site-specific) prior information, and this prior PDF can be updated by site-specific data into the “posterior” PDF using Bayesian analysis. For instance, consider a site with investigation data for $(Y_1, Y_2, Y_6, Y_9) = [\gamma, n, \ln(I_{s50}), \ln(V_p)]$, yet $Y_7 = \ln(\sigma_c)$ is unknown and to be estimated. This circumstance can happen when uniaxial compressive strength test is not available (e.g., due to budget limitation) whereas cheaper tests (such as index tests, point load strength tests, and P-wave velocity tests) are available. According to Bayes’ rule, the posterior PDF $f(y_7|y_1, y_2, y_6, y_9)$ can be derived from $f(y_1, y_2, \dots, y_9)$:

$$f(y_7|y_1, y_2, y_6, y_9) = \frac{f(y_1, y_2, y_6, y_7, y_9)}{f(y_1, y_2, y_6, y_9)} = \frac{\int \int \int \int f(y_1, y_2, \dots, y_9) dy_3 dy_4 dy_5 dy_8}{\int \int \int \int f(y_1, y_2, \dots, y_9) dy_3 dy_4 dy_5 dy_7 dy_8} \quad (1.)$$

The posterior PDF $f(y_7|y_1, y_2, y_6, y_9)$ has the following advantages compared with deterministic transformation models, e.g., a deterministic equation that predict σ_c based on I_{s50} :

Submitted to Canadian Geotechnical Journal

1. Unlike deterministic transformation models that only provide point estimates, the posterior PDF provides the probability distribution for the design parameter of interest. Both point estimate (e.g., median) and confidence interval (e.g., 95% confidence interval) can be derived from the posterior PDF. The confidence interval quantifies the transformation uncertainty (Phoon and Kulhawy 1999). When statistical methods are used, Eurocode 7 (EC7) (EN 1997-1 2004) recommends the characteristic value to be the 5% fractile, which can be readily derived from the posterior PDF.
 2. The posterior PDF allows more than one input variable (e.g., γ , n , I_{s50} , and V_p), unlike existing transformation models typically allowing a single input variable (see Tables 1 and 2 in Ching et al. 2018 for some existing transformation models with a single input). Nonetheless, a design parameter is typically correlated with more than one laboratory and/or field test index. It is prudent and arguably more cost-effective to incorporate all multivariate data sources to infer the design parameter. More information typically results in reduced transformation uncertainty in the posterior PDF. This uncertainty reduction will be demonstrated later in this paper. Furthermore, the posterior PDF allows more than one output variable. In an example to be demonstrated later, the bivariate posterior PDF for σ_c and E will be updated. Note that the 95% confidence domain for (σ_c, E) is not the same as combining the 95% confidence interval for σ_c and E separately. This 95% confidence interval requires the bivariate PDF for (σ_c, E) .
- When the above two advantages are combined, the posterior PDF can provide incentive for more detailed site investigation and increase the value of site data in making design decisions. This is

Submitted to Canadian Geotechnical Journal

illustrated in Figure 1. Consider two scenarios: (a) much information (γ , n , I_{s50} , V_p) is available, so the resulting posterior PDF is $f(\sigma_c|\gamma, n, I_{s50}, V_p)$; (b) only γ information is available, so the resulting posterior PDF is $f(\sigma_c|\gamma)$. In Bayesian analysis, $f(\sigma_c|\gamma, n, I_{s50}, V_p)$ is typically narrower than $f(\sigma_c|\gamma)$ because scenario (a) contains more information. Suppose the two posterior PDFs have about the same mean value. The 5% fractile for $f(\sigma_c|\gamma, n, I_{s50}, V_p)$, denoted by $\sigma_{c,k}^{(a)}$ in the figure, is larger than that for $f(\sigma_c|\gamma)$, denoted by $\sigma_{c,k}^{(b)}$. This suggests that a large characteristic value for σ_c can be used when there is much information (γ , n , I_{s50} , V_p), hence a more economic design outcome can be obtained. From this point of view, site investigation is an *investment* that may lead to savings in the final design, not merely a *cost* (Ching and Phoon 2012b; Ching et al. 2014b). In contrast, because $f(\sigma_c|\gamma)$ is wide, the resulting characteristic value for σ_c , denoted by $\sigma_{c,k}^{(b)}$ in Figure 1, is small, hence a conservative design outcome is obtained. As a result, insufficient site investigation is penalized by a conservative design. In other words, there is a linkage between site investigation efforts and final design savings under the Bayesian analysis, which is a distinctive feature in the geotechnical reliability-based design (RBD) (International Organization for Standardization 2015; Phoon and Retief 2015; Phoon et al. 2016). However, this linkage does not exist if deterministic transformation models are adopted, because they do not take transformation uncertainties into account for the σ_c estimates. The ability to monetize data is now more important than reliability-based design as it is the bedrock for Industry 4.0.

The purpose of this current paper is to construct a generic multivariate PDF $f(y_1, y_2, \dots, y_9)$ based on the ROCK/9/4069 database and then to conduct the subsequent Bayesian analysis for the

Submitted to Canadian Geotechnical Journal

development of the posterior PDF. The constructed multivariate PDF represents a prior distribution model. Given the site-specific information (e.g., γ , n , I_{s50} , and V_p), the prior PDF is updated into a site-specific posterior PDF that encapsulates all available information, i.e., site-specific data and global ROCK/9/4069. One practical contribution of this paper is to summarize useful posterior PDFs for design intact rock parameters (σ_c and E) using tables, as presented later in this paper. These posterior PDFs are probabilistic transformation models with multiple inputs. Practitioners can look up these tables to find the posterior PDF for the design parameter without conducting detailed Bayesian analysis. All posterior PDFs are univariate, although there are multiple inputs. Multivariate posterior PDFs are not covered in these tables, although an example with a bivariate posterior PDF will be presented to demonstrate that more general forms are possible.

Multivariate PDFs have been constructed for clays (Ching and Phoon 2012a, 2013, 2014b; Ching et al. 2014a) and sands (Ching et al. 2017b), but this is the first attempt to construct a multivariate PDF for *intact rock* properties. Furthermore, a novel method is proposed to detect outliers with respect to the calibration database. In principle, a multivariate PDF is only applicable within the range of conditions covered by a calibration database such as ROCK/9/4069. For the common bivariate case, it is easy to detect outliers when the regression line is used beyond the supporting data points (extrapolation). This is more difficult in higher dimensions, but extrapolation remains an important pitfall to guard against. The proposed method for detecting outliers is illustrated using actual cases with exceptionally low L-type Schmidt hammer hardness (R_L) values. This novel method is generic and applicable to

Submitted to Canadian Geotechnical Journal

116 calibration databases for soils and rocks.

117 **DATABASE ROCK/9/4069**

118 The ROCK/9/4069 database (Ching et al. 2018) consists of 4069 intact rock data points from 184
119 studies. This database is dominated by igneous and sedimentary rocks (27.5% and 59.4%, respectively).

120 The remaining (about 13.1%) data points are metamorphic rocks. The majority of the data in
121 ROCK/9/4069 are sedimentary rocks, the most common rocks exposed on the earth surface (Taylor and

122 McLennan 1985; Wilkinson et al. 2008). As a result, ROCK/9/4069 is more relevant to engineering
123 projects, which usually take place within less than 100 m of the ground surface. For example,
124 geotechnical geophysical surveys generally do not extend beyond this depth (Anderson et al. 2008).

125 The purpose of ROCK/9/4069 is not to represent the earth crust, which is mostly formed by igneous
126 and metamorphic rocks. In ROCK/9/4069, about 14% of the data points are for weathered rocks, and
127 about 4% are foliated metamorphic rocks (e.g., argillite, gneiss, orthogneiss, phyllite, paragneiss, schist,
128 and slate). The moisture content of the majority of intact rocks in the database is in their natural state.

129 There is no data point for saturated rocks. The geographical regions cover 44 countries/regions. The
130 properties of the data in ROCK/9/4069 cover a wide range of unit weight (γ) (15 to 35 kN/m³), porosity

131 (n) (0.01 to 55%), uniaxial compressive strength (σ_c) (0.7 to 380 MPa), Young's modulus (E) (0.03 to
132 120 GPa), and P-wave velocity (V_p) (0.4 to 8 km/sec). ROCK/9/4069 is considered to be "global" in
133 the sense that it has coverage wider than most existing transformation models (Ching et al. 2018).

134 Some data in ROCK/9/4069 have low strengths and may arguably resemble soils. These low strength

Submitted to Canadian Geotechnical Journal

cases are still classified by International Society for Rock Mechanics (1981) as rocks rather than soils: intact rocks with UCS between 0.25 MPa and 5 MPa are classified as “extremely weak rocks” to “very weak rocks” according to International Society for Rock Mechanics (1981). Kulhawy et al. (1991) is a mini-classic in this debate.

ROCK/9/4069 consists of nine parameters. They are categorized into four groups:

1. Index properties: unit weight (γ), porosity (n), L-type Schmidt hammer hardness (R_L), and Shore scleroscope hardness (S_h). For N-type Schmidt hammer hardness (R_N), R_N can be converted to R_L using the following empirical equation (Aydin and Basu 2005):

$$R_N = 1.0646R_L + 6.3673 \quad (2)$$

2. Strengths: Brazilian tensile strength (σ_{bt}), point load strength index (I_{s50}), and uniaxial compressive strength (σ_c), where I_{s50} is the point load strength index corrected to a standard diameter of 50 mm (Franklin 1985).

3. Stiffness: Young’s modulus (E). Although different definitions of E (e.g., tangent modulus at 50% of the peak strength, secant modulus at 50% of the peak strength, and average modulus for the linear portion of stress-strain curve) produce different values, Ching et al. (2018) found that these different definitions do not produce distinctive populations in the context of transformation models, that is, distinct data clusters do not appear when different definitions of E are plotted against other rock parameters (refer to Fig. 5 in Ching et al. 2018).

4. Dynamic property: P-wave velocity (V_p).

Submitted to Canadian Geotechnical Journal

154 The statistics of the nine parameters are summarized in Table 4 by Ching et al. (2018). This table is
155 reproduced as Table 1 in this paper. Two additional columns are added in Table 1 for 1% and 99%
156 percentiles. It will be clear later that the posterior PDF for the design parameter (σ_c or E) may not be
157 meaningful if the site-specific data (e.g., γ , n , I_{s50} , and V_p) fall beyond the 1% and 99% percentiles. To
158 keep notations concise, the nine parameters (or natural logarithm transform) are denoted by Y_1 , Y_2 , ...,
159 and Y_9 as shown in Eq. (1).

160 **Factors affecting intact rock parameters**

161 Intact rock parameters such as uniaxial compressive strength (σ_c) and Young's modulus (E) are clearly
162 influenced by factors such as rock class (igneous, sedimentary, and metamorphic), degree of
163 weathering, and degree of foliation. However, it is important to distinguish between factors affecting an
164 *individual* parameter such as σ_c and factors affecting a *relationship* between two parameters such as σ_c
165 and E . As an illustration, it is known that σ_c for sedimentary rocks is lower than that for igneous rocks
166 on the average. This does not imply that the relationship between σ_c and E is distinct for different rock
167 class, that is, there are three separate relationships for igneous, sedimentary, and metamorphic rocks.
168 The reason is that a lower σ_c is associated with a lower E for sedimentary rocks while a higher σ_c is
169 associated with a higher E for igneous rocks. There is no reason why the relationship cannot be the
170 same for both rocks, with the lower part of the relationship occupied by data points from sedimentary
171 rocks and the higher part of the relationship occupied by data points from igneous rocks.

172 Ching et al. (2018) found that there is no strong evidence indicating that the transformation

Submitted to Canadian Geotechnical Journal

relationships among the nine parameters depend on the rock class, degree of weathering, and degree of foliation. Prakoso (2002) also observed that several transformation relationships do not show distinct difference among rock classes. Kulhawy and Prakoso (2003) further showed that the decrease in intact rock strength and stiffness with increasing degree of weathering is relatively consistent. This suggests that the strength-stiffness transformation relationships may be relatively independent of weathering degree. This does not imply that the rock class, weathering, and foliation are not important on the individual rock parameters as noted above. For instance, Prakoso (2002) commented that the coefficient of variation of σ_{bt} , I_s , σ_c , and E_{t50} tends to increase with the weathering grade. The site-level statistics for an individual parameter in ROCK/9/4069 have been presented in Ching et al. (2018) as a function of the rock class (igneous, sedimentary, and metamorphic) – there are differences in say the mean values of σ_c .

In addition, it is important to distinguish between: (1) strong conclusions derived from a series of experiments that is deliberately designed to discern the influence of one factor while keeping other factors as constant as practicably possible (e.g. study the effect of weathering for the same rock type in one site) and (2) weaker statistical conclusions based on comparisons between populations where the underlying factors are varying simultaneously to different degrees. The key distinguishing feature here is that the former reduces variabilities to a minimum (hence strong conclusions) while the latter must deal with the irreducible and fairly significant variabilities intrinsic to these populations. It is well known in statistical hypothesis testing that two populations (typically modeled as two random variables)

Submitted to Canadian Geotechnical Journal

192 can be statistically the “same” (or more rigorously, no evidence to reject null hypothesis that the two
193 variable are the “same”), but the two variables can be distinct if we have sufficient evidence to examine
194 them at the detailed site level where variabilities are very much reduced. All the conclusions presented
195 in this study are drawn from ROCK/9/4069, which contains only sufficient information for population
196 level statistical analysis (not detailed site-specific analysis). Hence, it is not surprising that the
197 transformation uncertainty for two or more parameters (scatter about the regression line) is significant
198 in this study. The effect of the rock class, degree of weathering, and degree of foliation cannot be
199 discern within the scatter exhibited by ROCK/9/4069, implying they are not *statistically* significant.
200 This important observation has been previously discussed with reference to Fig. 6 and 7 in Ching et al.
201 (2018). As mentioned above, this does not preclude the possibility that they can be significant for a
202 high quality site-specific database where variabilities are significantly reduced.

203 With the above considerations in mind, all data points are combined together to form the
204 ROCK/9/4069 database for the subsequent statistical analysis. By doing so, we assume that the
205 transformation relationships do not depend on the rock class, degree of weathering, and degree of
206 foliation. In short, although it is desirable to develop a multivariate PDF for each rock class (igneous,
207 sedimentary, or metamorphic), the quality of ROCK/9/4069 (for example, there are insufficient data for
208 metamorphic rocks) at most allows us to characterize a single PDF. The penalty is that this generic
209 PDF will be more uncertain than site-specific PDFs that may be developed in the future as functions of
210 rock class, degree of weathering, and degree of foliation when such high quality site-specific databases

Submitted to Canadian Geotechnical Journal

are made available. Regardless of whether the resulting PDFs are generic or differentiated, the approach adopted in this paper is empirically grounded and statistically rigorous as it responds consistently to the quality of the database.

MULTIVARIATE PROBABILITY DENSITY FUNCTION

The amount of information is limited in ROCK/9/4069, so only the marginal PDFs and correlations can be reliably estimated. The translation model (Liu and Der Kiureghian 1986; Li et al. 2012) is adopted to transform the parameters into standard normal variables ($X_1, X_2, \dots, X_9$) and construct the underlying multivariate normal PDF $f(x_1, x_2, \dots, x_9)$:

$$f(\underline{x}) = |\mathbf{C}|^{-\frac{1}{2}} (2\pi)^{-\frac{9}{2}} \exp\left[-\frac{1}{2} \underline{x}^T \mathbf{C}^{-1} \underline{x}\right] \quad (2.)$$

where $\underline{x} = (x_1, x_2, \dots, x_9)^T$ is a standard normal variable vector; “T” is the vector/matrix transpose; $\mathbf{C}$ is the correlation matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & \delta_{12} & \delta_{13} & L & \delta_{19} \\ & 1 & \delta_{23} & L & \delta_{29} \\ & & 1 & M & \\ & & & O & \delta_{89} \\ \text{Sym.} & & & & 1 \end{bmatrix} \quad (3.)$$

where δ_{ij} is the Pearson product-moment correlation coefficient between X_i and X_j . This model has been adopted in past studies (Ching and Phoon 2012a, 2013, 2014b; Ching et al. 2014a; Ching et al. 2017b), where they adopted the Johnson family of distributions (Johnson 1949) as the marginal PDF model for analytical simplicity. The well-known lognormal distribution is a member of this family. With this Johnson-based translation model, they showed that physically meaningful results can be

Submitted to Canadian Geotechnical Journal

obtained. The same Johnson-based translation model is adopted in this study.

The detailed steps for the estimation of the Johnson parameters for ($Y_1, Y_2, \dots, Y_9$) can be found in Phoon and Ching (2013) and Ching and Phoon (2015). The estimation results, including the distribution type and the parameters (a_X, b_X, a_Y, b_Y), are summarized in Table 2, and the resulting Johnson PDFs are plotted in Figure 2. With the Johnson marginal PDFs of ($Y_1, Y_2, \dots, Y_9$) estimated, the rock parameters ($Y_1, Y_2, \dots, Y_9$) can be transformed into standard normal random variables ($X_1, X_2, \dots, X_9$) using the following equations:

$$\begin{aligned}
 X_1 &= 0.714 + 0.834 \cdot \ln \left(\frac{\gamma - 26.550}{1.128} + \sqrt{1 + \left(\frac{\gamma - 26.550}{1.128} \right)^2} \right) & X_2 &= 1.214 + 0.588 \cdot \ln \left(\frac{n + 0.006}{55.587 - n} \right) & X_3 &= 0.024 + 2.003 \cdot \ln \left(\frac{R_L + 9.089}{101.611 - R_L} \right) \\
 X_4 &= -39.265 + 5.283 \cdot \ln \left(\frac{S_h + 63.237}{0.129} + \sqrt{1 + \left(\frac{S_h + 63.237}{0.129} \right)^2} \right) & X_5 &= 2.305 + 2.035 \cdot \ln \left(\frac{\ln(\sigma_{bt}) - 3.131}{0.809} + \sqrt{1 + \left(\frac{\ln(\sigma_{bt}) - 3.131}{0.809} \right)^2} \right) \\
 X_6 &= 1.738 + 2.037 \cdot \ln \left(\frac{\ln(I_{ss0}) - 2.329}{1.073} + \sqrt{1 + \left(\frac{\ln(I_{ss0}) - 2.329}{1.073} \right)^2} \right) & X_7 &= 2.052 + 1.848 \cdot \ln \left(\frac{\ln(\sigma_c) - 5.390}{0.951} + \sqrt{1 + \left(\frac{\ln(\sigma_c) - 5.390}{0.951} \right)^2} \right) \\
 X_8 &= -1.488 + 1.011 \cdot \ln \left(\frac{\ln(E) + 5.772}{4.851 - \ln(E)} \right) & X_9 &= -10.420 + 2.021 \cdot \ln \left(\frac{\ln(V_p) + 147.913}{2.256 - \ln(V_p)} \right)
 \end{aligned} \quad (4.)$$

Correlations among ($Y_1, Y_2, \dots, Y_9$)

This section provides the correlation statistics computed based on bivariate data (Y_i, Y_j) in ROCK/9/4069. Figure 3 presents the scatter plots between all possible combinations of (Y_i, Y_j). Table 3 shows that bivariate data points are abundant. Three correlation estimates are considered: Pearson product-moment correlation coefficients (ρ_{ij}), Spearman rank correlation coefficients (r_{ij}), and Kendall's tau rank correlation coefficients (τ_{ij}). Ching et al. (2018) found that correlation behaviors among rock parameters observed in different local databases may deviate from one another significantly. To avoid a correlation estimate to be dominated by certain local databases, a re-sampling

Submitted to *Canadian Geotechnical Journal*

strategy is adopted, as explained in the following using the S_h - I_{s50} bivariate pair as an example. There are 94 bivariate data points from seven previous studies (seven local databases) for the S_h - I_{s50} relationship. Among the seven local databases, two of them contain about 57% of the data points. If the 94 data points are directly adopted to obtain the S_h - I_{s50} correlation estimate, this estimate will probably be dominated by the two local databases. This is not desirable because the current study aims to construct a *generic* multivariate PDF $f(y_1, y_2, \dots, y_9)$. The following bootstrap (Efron and Tibshirani 1993) procedure is adopted to re-sample 94 data points:

1. Randomly pick one local database (with replacement) from the seven local databases.
2. Suppose there are N S_h - I_{s50} data points in the randomly picked local database. Randomly pick one S_h - I_{s50} data point (with replacement) from the N data points.
3. Repeat Steps 1-2 for 94 times to obtain 94 bootstrap S_h - I_{s50} samples.

By adopting the above procedure, the 94 bootstrap samples will be uniformly distributed over the seven local databases. Based on these 94 bootstrap samples, a bootstrap correlation estimate can be calculated. This procedure is repeated for N_b times to obtain N_b bootstrap correlation estimates. Table 4 presents the median values for $N_b = 10,000$ bootstrap correlation estimates. The parentheses show the 95% bootstrap confidence intervals that reflect the statistical uncertainties.

Pearson product-moment correlations among ($X_1, X_2, \dots, X_9$)

The correlation estimates in Table 4 are for correlations among ($Y_1, Y_2, \dots, Y_9$). However, the multivariate normal distribution model requires the Pearson correlations among ($X_1, X_2, \dots, X_9$),

Submitted to Canadian Geotechnical Journal

namely δ_{ij} in the correlation matrix in Eq. (3.). Ching et al. (2016) showed that the following equation proposed by Hotelling and Pabst (1936) is fairly robust for estimating δ_{ij} :

$$\delta_{ij} = 2 \sin\left(\frac{\pi}{6} \times r_{ij}\right) \quad (5.)$$

The 10,000 bootstrap samples of r_{ij} obtained in a previous section are converted to 10,000 bootstrap samples of δ_{ij} through Eq. (5.). Table 5 shows the resulting median and 95% confidence interval for δ_{ij} . Ching and Phoon (2014b) proposed a rejection sampling method to obtain a positive definite $\mathbf{C}$. Table 6 lists the resulting $\mathbf{C}$ matrix.

Simulation

The simulation of the rock properties ($Y_1, Y_2, \dots, Y_9$) based on the translation model is simple. Let $\underline{\mathbf{U}}$ be a 9×1 vector of independent standard normal random variables and $\mathbf{L}$ be the lower triangular Cholesky factor satisfying $\mathbf{C} = \mathbf{L} \times \mathbf{L}^T$. ($X_1, X_2, \dots, X_9$) can be simulated as $\underline{\mathbf{X}} = \mathbf{L} \times \underline{\mathbf{U}}$, and then ($Y_1, Y_2, \dots, Y_9$) can be simulated based on the inverse transformation of Eq. (4.). Details are given elsewhere (Ching and Phoon 2015). Figure 4 shows the simulated data versus actual data points in ROCK/9/4069. General consistency between simulated and actual data points is evident.

APPLICATION OF THE MULTIVARIATE PDF – BAYESIAN UPDATING

The multivariate PDF constructed above can be treated as the “prior” PDF in the Bayesian analysis. The prior PDF can be updated PDF into the “posterior” PDF. Phoon and Ching (2013), Ching et al. (2014a), and Ching and Phoon (2015) showed that the posterior PDF is still Johnson. This is a significant computational advantage. Moreover, after the updating the Johnson distribution type and ($a_Y,$

Submitted to *Canadian Geotechnical Journal*

282 b_Y) remain unchanged, and only (a_X, b_X) change. The detailed steps for the Bayesian updating have
 283 been illustrated in Ching and Phoon (2014b). Let us denote the updated (a_X, b_X) by (a'_X, b'_X) .

284 To illustrate the Bayesian updating, consider an example involving updating $Y_7 = \ln(\sigma_c)$ for an
 285 intact rock case in Kahraman et al. (2004) with the following information: $\gamma = 25.21 \text{ kN/m}^3$, $n = 0.93\%$,
 286 $I_{s50} = 7.1 \text{ MPa}$, $V_p = 6 \text{ km/s}$. As a result, $Y_1 = \gamma = 25.21$, $Y_2 = n = 0.93$, $Y_6 = \ln(I_{s50}) = \ln(7.1) = 1.960$,
 287 and $Y_9 = \ln(V_p) = \ln(6) = 1.792$. Based on Eq. (4.), $X_1 = -0.126$, $X_2 = -1.177$, $X_6 = 1.051$, and $X_9 =$
 288 1.251 . For the posterior PDF of $Y_7 = \ln(\sigma_c)$, the distribution type Johnson SU, $a_{Y7} = 0.951$, and $b_{Y7} =$
 289 5.390 (see Table 5) remain unchanged. From Table 7, it is clear that $a'_{X7} = 3.553$ and $b'_{X7} = 3.946 +$
 290 $0.318X_1 + 0.505X_2 - 1.236X_6 - 0.357X_9 = 1.563$. Moreover, the Johnson SU posterior PDF of Y_7 has
 291 the following expression:

$$292 \quad f'(y_7) = \frac{a'_{X7}}{a_{Y7}\sqrt{2\pi}} \cdot \frac{1}{\sqrt{1 + [(y_7 - b_{Y7})/a_{Y7}]^2}} \cdot \exp\left(-0.5 \left[b'_{X7} + a'_{X7} \sinh^{-1}\left(\frac{y_7 - b_{Y7}}{a_{Y7}}\right) \right]^2\right) \quad (6.)$$

293 where f' denotes the posterior PDF; the inverse hyperbolic function $\sinh^{-1}(x) = \ln[x + (1+x^2)^{0.5}]$. The
 294 posterior p-fractile for Y_7 has the following expression:

$$295 \quad y_{7,p} = b_{Y7} + \frac{a_{Y7}}{2} \left[\exp\left(\frac{\Phi^{-1}(p) - b'_{X7}}{a'_{X7}}\right) - \exp\left(-\frac{\Phi^{-1}(p) - b'_{X7}}{a'_{X7}}\right) \right] \quad (7.)$$

296 where Φ^{-1} is the inverse of the cumulative density function for standard normal variable. The median
 297 value of Y_7 is its 0.5-fractiles ($p = 0.5$ in Eq. 7.), and its 95% confidence interval is the interval formed
 298 by 0.025- and 0.975-fractiles ($p = 0.025$ and 0.975 in Eq. 7.). The posterior PDF for $Y_7 = \ln(\sigma_c)$ is
 299 shown in Figure 5a with the posterior median and 95% confidence interval indicated in the figure. The

Submitted to Canadian Geotechnical Journal

actual measured value is $\sigma_c = 175$ MPa (Kahraman et al. 2004) is also indicated in the figure, which is assumed to be unknown during the Bayesian updating. The posterior PDF is significantly narrower than the prior PDF, i.e., the transformation uncertainty for σ_c is greatly reduced when $(\gamma, n, I_{s50}, V_p)$ information is available. Yet, the actual σ_c value still lies within the 95% confidence interval (Figure 5a).

If the purpose is to update $Y_8 = \ln(E)$, the posterior PDF is still Johnson SB, and $a_{Y8} = 10.623$ and $b_{Y8} = -5.772$ (see Table 5) remain unchanged. From Table 8, it is clear that $a'_{X8} = 1.640$ and $b'_{X8} = -2.414 - 0.527X_1 - 0.345X_2 - 0.635X_6 - 0.585X_9 = -3.342$. Moreover, the Johnson SB posterior PDF of Y_8 has the following expression:

$$f'(y_8) = \frac{a'_{X8}}{\sqrt{2\pi}} \cdot \frac{a_{Y8}}{(y_8 - b_{Y8})(a_{Y8} + b_{Y8} - y_8)} \cdot \exp\left(-0.5 \left[b'_{X8} + a'_{X8} \ln\left(\frac{y_8 - b_{Y8}}{a_{Y8} + b_{Y8} - y_8}\right) \right]^2\right) \quad (8.)$$

The posterior p-fractile for Y_8 has the following expression:

$$y_{8,p} = b_{Y8} + a_{Y8} \left[1 + \exp\left(-\frac{\Phi^{-1}(p) - b'_{X8}}{a'_{X8}}\right) \right]^{-1} \quad (9.)$$

The posterior PDF for $Y_8 = \ln(E)$ is shown in Figure 5b with the posterior median and 95% confidence interval indicated in the figure. The actual measured value is $E = 70.47$ GPa (Kahraman et al. 2004) is also indicated in the figure.

It is also possible to evaluate the bivariate posterior PDF for $Y_7 = \ln(\sigma_c)$ and $Y_8 = \ln(E)$. The detailed procedure can be found in Ching and Phoon (2014b). Figure 6 shows the bivariate posterior PDF. It is clear that the dashed rectangular boxes constituted by combining the conventional 95%

Submitted to Canadian Geotechnical Journal

confidence intervals shown in Figure 5 would not capture 95% of the probability mass in Figure 6. In other words, although we can say that the probability of the actual value of σ_c or E falling within the respective 95% confidence interval in Figure 5 is 0.95, we cannot say that the probability of a pair of (σ_c, E) falling within the rectangular box is 0.95. Hence, the dashed boxes are not meaningful in the probabilistic sense. The reason why a conventional 95% confidence interval cannot extend beyond a single parameter is because it does not consider correlation between σ_c and E (e.g., Figure 6a).

To further illustrate the uncertainty reduction with increasing information, Figure 7 shows how the posterior PDFs change with increasing information for this particular case from Kahraman et al. (2004). It is clear that the posterior PDFs become narrower when more information is available, yet the actual values of σ_c and E still lie within the 95% confidence intervals. This suggests that it is preferable to have more information, because the actual values of σ_c and E will fall into narrower confidence intervals. However, if the future case to be predicted is from a different population (e.g., a rare rock with an exceptionally high/low parameter value that is not within the range in ROCK/9/4069) or its parameters are contaminated by large measurement errors, the above conclusion (more information is preferable) may not apply. It may be counter-productive to include these out-of-range or contaminated parameters in the prediction of σ_c and E . An outlier detection method is proposed in this study to identify cases that are not from the same population as the calibration database.

PERFORMANCE OF THE MULTIVARIATE DISTRIBUTION MODEL

This section examines the performance of the multivariate distribution model against new cases in Kiliç

Submitted to Canadian Geotechnical Journal

and Teymen (2008) that lies outside ROCK/9/4069. The actual values of σ_c and E for these cases are assumed to be unknown during the Bayesian updating process. After the updating, these actual values are compared with the updated median and 95% confidence interval. Nineteen intact rock cases collected in Kiliç and Teymen (2008) are investigated. These data points are summarized in Table 9. They are all 42×85 mm intact rock samples collected from Southern Anatolia, Turkey. There are five tuff cases with low R_L values (grey entries in Table 9), four of them are lower than the 1% percentile of R_L in Table 1 and five are lower than the 5% percentile. This suggests that there is fairly limited data in ROCK/9/4069 with such low R_L , hence for these cases the prediction based on R_L may involve extrapolation.

Outlier detection method

A future case is called an “outlier” if it is not within the coverage of the calibration database. Such a future design case may invoke extrapolation, which is not desirable for the prediction of σ_c and E . In the multivariate standard normal space ($\underline{X}$ space), the coverage domain of the calibration database can be visualized as a multivariate ellipsoid. Figure 8a illustrates such an ellipsoid in two dimensions for ease of visualization. The vast majority of the calibration data points are within this ellipsoid. $\underline{X}$ can be further converted into independent standard normal $\underline{U}$ using the relationship $\underline{U} = \underline{L}^{-1} \times \underline{X}$, where $\underline{L}$ is the lower triangular Cholesky factor for the correlation matrix given in Table 6. The ellipsoid in Figure 8a becomes a sphere (or a circle in two dimensions) in Figure 8b. Consider a future design case $\underline{U} = (U_1, U_2, \dots, U_n)$. If the distance between the future case and the origin is large, i.e., $U_1^2 + U_2^2 + \dots + U_n^2$ is large,

Submitted to Canadian Geotechnical Journal

the future design case may be an outlier with respect to the calibration database. Note that $U_1^2 + U_2^2 + \dots + U_n^2$ is a chi-square random variable with n degrees of freedom. As a result, the P-value for the future design case can be computed as

$$P - \text{value} = P(\chi_n^2 > U_1^2 + U_2^2 + \dots + U_n^2) \quad (10.)$$

where χ_n^2 is a chi-square random variable with n degrees of freedom. In Matlab, this can be computed using $P - \text{value} = 1 - \text{chi2cdf}(U_1^2 + U_2^2 + \dots + U_n^2, n)$. A small P-value indicates that the future design case is likely to be an outlier with respect to the calibration database, i.e., the prediction for this future design case may involve extrapolation. This P-value will be referred to as the “outlier P-value”. Figure 8 illustrates the contours for several outlier P-value thresholds. A typical threshold is 0.05, although other thresholds can be adopted. If $\underline{X}$ is incomplete (e.g., only four entries X_1, X_2, X_6 , and X_9 are known), $\underline{L}$ is then the lower triangular Cholesky factor for the (4×4) sub-correlation matrix. The P-value can be computed by the Matlab command $1 - \text{chi2cdf}(U_1^2 + U_2^2 + U_6^2 + U_9^2, 4)$, where 4 is the degrees of freedom. Clearly, the above proposed outlier detection method is purely statistical. It is meant to alert the engineer that extrapolation beyond the calibration database may be involved so that he/she could exercise judgment to scrutinize the analysis results more carefully.

The above outlier detection method is adopted for all 19 cases in Table 9. The tenth column in the table shows the outlier P-values. The five tuff cases from Kiliç and Teymen (2008) with low R_L are with relatively low P-values. In particular, there are two cases with P-values less than 0.05. The prediction results for these two cases are not presented, because the results may involve extrapolation,

Submitted to Canadian Geotechnical Journal

375 so there are only 17 remaining cases. Figure 9 shows the posterior median values and 95% confidence
376 intervals for σ_c for the 17 cases based on various information. If the vertical bar covers the 1:1 line
377 (dashed lines), the 95% confidence interval covers the actual value of σ_c . It is evident that the width of
378 the confidence interval shrinks with increasing information from Figure 9a to Figure 9c, yet the vertical
379 bars still cover the 1:1 line. Figure 10 shows the results for E based on various information. The similar
380 conclusion is found.

381 CONCLUSIONS

382 This paper constructs the multivariate probability density function (PDF) for nine intact rock
383 parameters based on the ROCK/9/4069 database (Ching et al. 2018) to capture the correlation
384 behaviors in ROCK/9/4069. The inevitable variabilities arising from compilation of global data does
385 not allow more differentiated PDFs to be constructed as a function of potential factors such as rock
386 class, degree of weathering, and degree of foliation. This limitation is fundamental in the statistical
387 sense and manifested clearly in the form of data clusters differentiated by rock class/degree of
388 weathering/degree of foliation overlapping when two or more intact rock parameters are plotted
389 together. It is important *not* to confuse with the effect of rock class/weathering/foliation on *individual*
390 parameters. This effect is clearly discernible even within ROCK/9/4069 when statistics of individual
391 parameters for say different rock classes are compared. The conclusion (which is statistically rigorous)
392 is that there is no evidence to support the hypothesis that the class/weathering/foliation affects
393 transformation models involving two or more parameters. This *statistical* conclusion does not preclude

Submitted to Canadian Geotechnical Journal

394 the possibility that differentiated PDFs can be constructed for a high quality site-specific database
395 where variabilities are significantly reduced.

396 The generic multivariate PDF presented in this paper summarizes the prior information in
397 ROCK/9/4069. With site-specific information, the prior PDF can be updated into the posterior
398 distribution by Bayesian analysis. This paper further summarizes posterior PDF parameters using
399 user-friendly tables (Tables 7 and 8). Practitioners can interrogate these tables to determine the
400 posterior PDFs for the design intact rock parameter without conducting detailed Bayesian analysis.
401 Posterior median and 95% confidence interval can be derived from the posterior PDF. With the
402 multivariate PDF, multivariate site specific information can be combined to reduce the uncertainty in
403 the design intact rock parameter so that the site investigation effort can be linked to final design
404 savings.

405 A crucial assumption is that the site at hand is covered by the database that is adopted to construct
406 the multivariate distribution model. For cases that are not covered by the database (e.g., intact rocks
407 with very small R_L), the posterior PDF can be misleading, because the actual value of the design intact
408 rock parameter can easily fall outside the 95% confidence interval. This is to be expected and merely
409 re-affirms the importance of applying transformation models in a sensible way that is consistent with
410 geological and rock mechanics principles. In short, it is always important to guard against extrapolation,
411 regardless of the methodology. A new statistical method is proposed in this study to alert the engineer
412 that a particular set of input parameters is outside the range of conditions covered by the calibration

Submitted to Canadian Geotechnical Journal

413 database and the results produced by the proposed multivariate distribution model need careful
414 scrutiny.

415 REFERENCES

- 416 Anderson, N., Croxton, N., Hoover, R., and Sirles, P. 2008. Geophysical Methods Commonly
417 Employed for Geotechnical Site Characterization, Transportation Research Circular E-C130,
418 Transportation Research Board, Washington D.C., USA.
- 419 Aladejare, A.E., and Wang, Y. 2017. Evaluation of rock property variability. *Georisk: Assessment and*
420 *Management of Risk for Engineered Systems and Geohazards*, 11(1): 22-41.
- 421 Aydin, A., and Basu, A. 2005. The Schmidt hammer in rock material characterization. *Engineering*
422 *Geology*, 81(1): 1-14.
- 423 Ching, J., and Phoon, K.K. 2012a. Modeling parameters of structured clays as a multivariate normal
424 distribution. *Canadian Geotechnical Journal*, 49(5): 522-545.
- 425 Ching, J., and Phoon, K.K. 2012b. Value of geotechnical site investigation in reliability-based design.
426 *Advances in Structural Engineering*, 15(11): 1935-1945.
- 427 Ching, J., and Phoon, K.K. 2013. Multivariate distribution for undrained shear strengths under various
428 test procedures. *Canadian Geotechnical Journal*, 50(9): 907-923.
- 429 Ching, J., and Phoon, K.K. 2014a. Transformations and correlations among some parameters of clays –
430 the global database. *Canadian Geotechnical Journal*, 51(6): 663-685.
- 431 Ching, J., and Phoon, K.K. 2014b. Correlations among some clay parameters – the multivariate
432 distribution. *Canadian Geotechnical Journal*, 51(6): 686-704.
- 433 Ching, J., Phoon, K.K., and Chen, C.H. 2014a. Modeling CPTU parameters of clays as a multivariate
434 normal distribution. *Canadian Geotechnical Journal*, 51(1): 77-91.
- 435 Ching, J., Phoon, K.K., and Yu, J.W. 2014b. Linking site investigation efforts to final design savings
436 with simplified reliability-based design methods. *ASCE Journal of Geotechnical and*
437 *Geoenvironmental Engineering*, 140(3): 04013032.
- 438 Ching, J., and Phoon, K.K. 2015. Constructing multivariate distributions for soil parameters. Chap. 1 in
439 *Risk and Reliability in Geotechnical Engineering* (Eds.: K.K. Phoon and J. Ching). Taylor &
440 Francis.
- 441 Ching, J., Phoon, K.K., and Li, D.Q. 2016. Robust estimation of correlation coefficients in the
442 multivariate normal framework. *Structural Safety*, 63: 21-32.
- 443 Ching, J., Li, K.H., Phoon, K.K., and Weng, M.C. 2018. Generic transformation models for some intact
444 rock properties. *Canadian Geotechnical Journal*, in press.
- 445 Ching, J., Lin, G.H., Chen, J.R., and Phoon, K.K. 2017a. Transformation models for effective friction
446 angle and relative density calibrated based on a multivariate database of coarse-grained soils.

Submitted to Canadian Geotechnical Journal

- 447 Canadian Geotechnical Journal, 54(4): 481-501.
- 448 Ching, J., Lin, G.H., Phoon, K.K., and Chen, J.R. 2017b. Correlations among some parameters of
449 coarse-grained soils – the multivariate probability distribution model. Canadian Geotechnical
450 Journal, 54(9): 1203-1220.
- 451 Ching, J., and Phoon, K.K. 2018. Constructing site-specific probabilistic transformation model by
452 Bayesian machine learning, ASCE Journal of Engineering Mechanics, in press.
- 453 Conover, W.J. 1999. Practical Nonparametric Statistics. 3rd edition, John Wiley & Sons, Inc., New
454 York.
- 455 D'Ignazio, M., Phoon, K.K., Tan, S.A., and Lansivaara, T. 2016. Correlations for undrained shear
456 strength of Finnish soft clays. Canadian Geotechnical Journal, 53(10): 1628-1645.
- 457 Efron, B., and Tibshirani, R. 1993. An Introduction to the Bootstrap. Boca Raton, FL. Chapman and
458 Hall/CRC.
- 459 EN 1997-1. 2004. Eurocode 7: Geotechnical design — Part 1: General rules. European Committee for
460 Standardization (CEN), Brussels, Belgium.
- 461 Fang, K., Kotz, S., and Ng, K.W. 1990. Symmetric Multivariate and Related Distribution. Chapman &
462 Hall, London.
- 463 Feng, X., and Jimenez, R. 2014. Bayesian prediction of elastic modulus of intact rocks using their
464 uniaxial compressive strength. Engineering Geology, 173: 32-40.
- 465 Feng, X., and Jimenez, R. 2015. Estimation of deformation modulus of rock masses based on Bayesian
466 model selection and Bayesian updating approach. Engineering Geology, 199: 19-27.
- 467 Franklin, J.A. 1985. Suggested method for determining point load strength. International Journal of
468 Rock Mechanics and Mining Sciences & Geomechanics Abstracts, 22(2): 51-60.
- 469 Güney, A., Altındag, R., Yavuz, H., and Saraç, S. 2005. Evaluation of the relationships between
470 Schmidt hardness rebound number and other (engineering) properties of rocks. Proc. 19th
471 International Mining Congress and Fair of Turkey, Turkey.
- 472 Hotelling, H., and Pabst, M.R. 1936. Rank correlation and tests of significance involving no
473 assumption of normality. American Mathematical Statistics, 7: 29-43.
- 474 International Organization for Standardization. 2015. General Principles on Reliability of Structures.
475 ISO2394:2015, Geneva. http://www.iso.org/iso/catalogue_detail.htm?csnumber=58036.
- 476 International Society for Rock Mechanics. 1981. Rock Characterization, Testing, and Monitoring:
477 ISRM Suggested Methods. Brown, E. T. (Editor). Pergamon Press, Oxford.
- 478 Johnson, N.L. 1949. Systems of frequency curves generated by methods of translation. Biometrika, 36:
479 149-176.
- 480 Kahraman, S., Fener, M., and Gunaydin, O. 2004. Predicting the sawability of carbonate rocks using
481 multiple curvilinear regression analysis. International Journal of Rock Mechanics and Mining
482 Sciences, 41(7): 1123-1131.
- 483 Kılıç, A., and Teymen, A. 2008. Determination of mechanical properties of rocks using simple

Submitted to Canadian Geotechnical Journal

- 484 methods. Bulletin of Engineering Geology and the Environment, 67(2): 237-244.
- 485 Kim, E., and Hunt, R. 2017. A public website of rock mechanics database from Earth Mechanics
486 Institute (EMI) at Colorado School of Mines (CSM). Rock Mechanics and Rock Engineering,
487 50(12): 3245-3252.
- 488 Kulhawy F.H., and Prakoso, W.A. 2003. Variability of rock index properties. In Soil and Rock
489 America 2003, 2765-2770.
- 490 Kulhawy, F.H., Trautmann, C.H., and O'Rourke, T.D. 1991. The soil-rock boundary: what is it and
491 where is it? Detection of and Construction at the Soil/Rock Interface (GSP28), ASCE. New York,
492 1-15.
- 493 Li, D.Q., Wu, S.B., Zhou, C.B., and Phoon, K.K. 2012. Performance of translation approach for
494 modeling correlated non-normal variables. Structural Safety, 39: 52-61.
- 495 Liu, P.L., and Der Kiureghian, A. 1986. Multivariate distribution models with prescribed marginals and
496 covariances. Probabilistic Engineering Mechanics, 1(2): 105-112.
- 497 Ng, I.T., Yuen, K.V., and Lau, C.H. 2015. Predictive model for uniaxial compressive strength for
498 Grade III granitic rocks from Macao. Engineering Geology, 199: 28-37.
- 499 Phoon, K.K., and Kulhawy, F.H. 1999. Evaluation of geotechnical property variability. Canadian
500 Geotechnical Journal, 36(4): 625-639.
- 501 Phoon, K.K., and Ching, J. 2013. Multivariate model for soil parameters based on Johnson distributions,
502 Foundation Engineering in the Face of Uncertainty, Geotechnical Special Publication honoring
503 Professor F. H. Kulhawy, 337-353.
- 504 Phoon, K.K., and Retief, J.V. 2015. ISO2394:2015 Annex D (Reliability of Geotechnical Structures).
505 Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 9(3):
506 125-127.
- 507 Phoon, K.K., Retief, J.V., Ching, J., Dithinde, M., Schweckendiek, T., Wang, Y., and Zhang, L. 2016.
508 Some Observations on ISO2394:2015 Annex D (Reliability of Geotechnical Structures).
509 Structural Safety, 62: 24-33.
- 510 Prakoso, W.A. 2002. Reliability-based Design of Foundations on Rock Masses for Transmission Line
511 and Similar Structures. Ph.D. Dissertation, Cornell University, Ithaca, NY.
- 512 Slifker, J.F., and Shapiro, S.S. 1980. The Johnson system: selection and parameter estimation.
513 Technometrics, 22(2): 239-246.
- 514 Taylor, S.R., and McLennan, S.M. 1985. The Continental Crust: Its Composition and Evolution: An
515 Examination of the Geochemical Record Preserved in Sedimentary Rocks. Blackwell, Oxford.
- 516 Wang, Y., and Aladejare, A.E. 2015. Selection of site-specific regression model for characterization of
517 uniaxial compressive strength of rock. International Journal of Rock Mechanics and Mining
518 Sciences, 75: 73-81.
- 519 Wang, Y., and Aladejare, A.E. 2016. Bayesian characterization of correlation between uniaxial
520 compressive strength and Young's modulus of rock. International Journal of Rock Mechanics and

Submitted to Canadian Geotechnical Journal

521 Mining Sciences, 85: 10-19.
522 Wilkinson, B.H., McElroy, B.J., Kesler, S.E., Peters, S.E., and Rothman, E.D. 2008. Global geologic
523 maps are tectonic speedometers – rates of rock cycling from area-age frequencies. Geological
524 Society of America Bulletin, 121: 760-779.

525

526

527

528

529

Draft

FIGURE CAPTIONS

Figure 1 Posterior PDFs for σ_c based on different site investigation scenarios.

Figure 2 Empirical histograms constructed from Y data and the fitted Johnson distributions.

Figure 3 Scatter plots between the intact rock parameters, Y_i and Y_j .

Figure 4 Comparison between simulated and actual data in ROCK/9/4069.

Figure 5. Prior and posterior PDFs for (a) σ_c and (b) E (arrow indicates actual values).

Figure 6. Bivariate PDFs for σ_c and E: (a) prior; (b) conditioning on $(\gamma, n, I_{s50}, V_p)$.

Figure 7. Posterior PDFs obtained by conditioning on increasing information: (a) σ_c ; (b) E.

Figure 8. Illustration for the coverage domain of the calibration database in two dimensions: (a) X space; (b) U space.

Figure 9. Bayesian updating results for σ_c by conditioning on (a) n, R_L , and S_h ; (b) n, R_L, S_h , and V_p ; and (c) $n, R_L, S_h, V_p, \sigma_{bt}$, and I_{s50} .

Figure 10. Bayesian updating results for E by conditioning on (a) n, R_L , and S_h ; (b) n, R_L, S_h , and V_p ; and (c) $n, R_L, S_h, V_p, \sigma_{bt}$, and I_{s50} .

<https://mc06.manuscriptcentral.com/cgj-pubs>

Figure 1 Posterior PDFs for σ_c based on different site investigation scenarios.

Figure 2 Empirical histograms constructed from Y data and the fitted Johnson distributions.

<https://mc06.manuscriptcentral.com/cgj-pubs>

Figure 3 Scatter plots between the intact rock parameters, Y_i and Y_j .

Figure 4 Comparison between simulated and actual data in ROCK/9/4069.

<https://mc06.manuscriptcentral.com/cgj-pubs>

Figure 5. Prior and posterior PDFs for (a) σ_c and (b) E (arrow indicates actual values).

<https://mc06.manuscriptcentral.com/cgj-pubs>

Figure 6. Bivariate PDFs for σ_c and E : (a) prior; (b) conditioning on $(\gamma, n, I_{s50}, V_p)$.

Figure 7. Posterior PDFs obtained by conditioning on increasing information: (a) σ_c ; (b) E .

Figure 8. Illustration for the coverage domain of the calibration database in two dimensions: (a) X space; (b) U space.

<https://mc06.manuscriptcentral.com/cgj-pubs>

Figure 9. Bayesian updating results for σ_c by conditioning on (a) n, R_L , and S_h ; (b) n, R_L, S_h , and V_p ; and (c) $n, R_L, S_h, V_p, \sigma_{bt}$, and I_{s50} .

Figure 10. Bayesian updating results for E by conditioning on (a) n , R_L , and S_h ; (b) n , R_L , S_h , and V_p ; and (c) n , R_L , S_h , V_p , σ_{bt} , and I_{s50} .

Table 1 Selected statistics of the nine parameters in ROCK/9/4069 (from Ching et al. 2018).

Parameter	N (Igneous + Sedimentary + Metamorphic)	Mean	COV	Min	Max	Percentiles			
						1%	5%	95%	99%
γ (kN/m ³)	1288 (238+879+171)	24.6	0.12	15.0	34.7	17.1	18.2	28.7	30.9
n (%)	1371 (289+957+125)	11.0	1.02	0.01	55.0	0.16	0.31	35.0	44.1
R _L	812 (356+390+66)	41.4	0.29	8.1	81.6	15.6	21.0	61.7	65.0
S _h	333 (62+233+38)	47.6	0.44	8.4	100.0	8.9	14.1	88.0	96.0
σ_{bt} (MPa)	854 (196+512+146)	8.2	0.61	0.07	34.4	0.49	1.1	18.1	21.3
I _{s50} (MPa)	1303 (420+659+224)	4.2	0.69	0.05	17.4	0.18	0.66	10.0	13.6
σ_c (MPa)	3226 (857+1971+398)	72.3	0.77	0.68	379.0	1.5	5.0	182.0	253.7
E (GPa)	1495 (477+895+123)	26.5	0.99	0.03	116.3	0.06	0.34	77.9	101.9
V _p (km/s)	1858 (619+989+250)	4.0	0.39	0.44	8.0	0.63	1.3	6.3	7.4

Draft

Table 2. Distribution type and distribution parameters for $(Y_1, Y_2, \dots, Y_9)$.

Random variable	Rock parameter	Distribution type	Distribution parameters				p-value
			a_X	b_X	a_Y	b_Y	
Y_1	γ	SU	0.834	0.714	1.128	26.550	3.30e-06
Y_2	n	SB	0.588	1.214	55.593	-0.006	0.07
Y_3	R_L	SB	2.003	0.024	101.611	-9.089	0.64
Y_4	S_h	SU	5.283	-39.265	0.129	-63.237	0.52
Y_5	$\ln(\sigma_{bt})$	SU	2.035	2.305	0.809	3.131	0.07
Y_6	$\ln(I_{s50})$	SU	2.037	1.738	1.073	2.329	0.16
Y_7	$\ln(\sigma_c)$	SU	1.848	2.052	0.951	5.390	0.10
Y_8	$\ln(E)$	SB	1.011	-1.488	10.623	-5.772	0.13
Y_9	$\ln(V_p)$	SB	2.021	-10.420	150.169	-147.913	6.17e-05

Remark: (SU, SB, SL) are the Johnson distribution types, whereas (a_X, b_X, a_Y, b_Y) are the Johnson parameters. The details can be found in Phoon and Ching (2013) and Ching and Phoon (2015).

Draft

Table 3 Numbers of data points with bivariate information (parenthesized numbers refer to breakdown into data number for igneous, sedimentary, and metamorphic rocks).

	$Y_2 = n$	$Y_3 = R_L$	$Y_4 = S_h$	$Y_5 = \ln(\sigma_{bt})$	$Y_6 = \ln(I_{s50})$	$Y_7 = \ln(\sigma_c)$	$Y_8 = \ln(E)$	$Y_9 = \ln(V_p)$
$Y_1 = \gamma$	469 (61+327+81)	171 (38+129+4)	111 (30+60+21)	279 (95+119+65)	283 (29+190+64)	963 (165+661+137)	495 (113+331+51)	867 (153+591+123)
$Y_2 = n$		190 (54+117+19)	91 (13+77+1)	320 (41+224+55)	443 (117+278+48)	911 (193+635+83)	607 (76+505+26)	637 (114+448+75)
$Y_3 = R_L$	Index properties		79 (30+40+9)	142 (46+64+32)	453 (220+199+34)	735 (319+360+56)	315 (134+163+18)	450 (267+150+33)
$Y_4 = S_h$				176 (27+123+26)	94 (3+89+2)	300 (41+225+34)	197 (31+144+22)	96 (36+45+15)
$Y_5 = \ln(\sigma_{bt})$				Strengths			349 (72+238+39)	267 (81+136+50)
$Y_6 = \ln(I_{s50})$							457 (152+273+32)	647 (205+310+132)
$Y_7 = \ln(\sigma_c)$							1405 (428+873+104)	1381 (422+772+187)
$Y_8 = \ln(E)$							Stiffness	
$Y_9 = \ln(V_p)$								Dynamic property

Table 4 Correlation coefficients, in the format of median (95% confidence interval) (1st row: Pearson ρ_{ij} ; 2nd row: Spearman r_{ij} ; 3rd row: Kendall τ_{ij}).

	$Y_1 = \gamma$	$Y_2 = n$	$Y_3 = R_L$	$Y_4 = S_h$	$Y_5 = \ln(\sigma_{bt})$	$Y_6 = \ln(I_{s50})$	$Y_7 = \ln(\sigma_c)$	$Y_8 = \ln(E)$	$Y_9 = \ln(V_p)$
$Y_1 = \gamma$		-0.80(-0.84~-0.75) -0.71(-0.76~-0.66) -0.53(-0.58~-0.48)	0.65(0.57~0.71) 0.62(0.53~0.69) 0.36(0.39~0.50)	0.78(0.71~0.84) 0.51(0.32~0.67) 0.36(0.21~0.49)	0.58(0.46~0.67) 0.52(0.42~0.61) 0.37(0.29~0.44)	0.49(0.40~0.57) 0.55(0.48~0.62) 0.38(0.32~0.43)	0.68(0.64~0.72) 0.50(0.44~0.55) 0.35(0.31~0.39)	0.76(0.71~0.79) 0.62(0.55~0.68) 0.45(0.40~0.51)	0.71(0.67~0.74) 0.67(0.62~0.71) 0.49(0.45~0.52)
$Y_2 = n$			-0.70(-0.74~-0.65) -0.79(-0.84~-0.73) -0.65(-0.67~-0.53)	-0.86(-0.91~-0.80) -0.79(-0.87~-0.67) -0.60(-0.69~-0.48)	-0.71(-0.77~-0.62) -0.57(-0.65~-0.48) -0.41(-0.48~-0.35)	-0.52(-0.58~-0.47) -0.57(-0.63~-0.50) -0.39(-0.44~-0.34)	-0.69(-0.72~-0.65) -0.63(-0.66~-0.58) -0.45(-0.48~-0.41)	-0.58(-0.64~-0.52) -0.46(-0.52~-0.39) -0.32(-0.36~-0.28)	-0.61(-0.66~-0.56) -0.62(-0.67~-0.57) -0.45(-0.49~-0.41)
$Y_3 = R_L$				0.88(0.83~0.92) 0.88(0.80~0.93) 0.64(0.59~0.80)	0.55(0.44~0.64) 0.51(0.37~0.64) 0.26(0.23~0.47)	0.71(0.66~0.75) 0.73(0.69~0.77) 0.50(0.48~0.58)	0.70(0.66~0.73) 0.67(0.63~0.71) 0.45(0.43~0.53)	0.70(0.64~0.75) 0.67(0.60~0.73) 0.43(0.41~0.53)	0.64(0.57~0.69) 0.54(0.45~0.62) 0.34(0.34~0.48)
$Y_4 = S_h$					0.76(0.70~0.81) 0.77(0.69~0.84) 0.60(0.52~0.67)	0.86(0.79~0.91) 0.82(0.72~0.89) 0.64(0.55~0.72)	0.80(0.76~0.83) 0.79(0.73~0.85) 0.62(0.56~0.67)	0.72(0.65~0.77) 0.62(0.51~0.72) 0.46(0.36~0.54)	0.74(0.67~0.80) 0.69(0.55~0.80) 0.51(0.40~0.61)
$Y_5 = \ln(\sigma_{bt})$						0.79(0.75~0.83) 0.72(0.65~0.77) 0.54(0.48~0.59)	0.81(0.78~0.84) 0.71(0.66~0.75) 0.53(0.49~0.57)	0.69(0.60~0.75) 0.50(0.40~0.58) 0.36(0.29~0.43)	0.69(0.60~0.75) 0.48(0.36~0.58) 0.34(0.25~0.42)
$Y_6 = \ln(I_{s50})$							0.75(0.72~0.78) 0.81(0.79~0.83) 0.62(0.60~0.64)	0.70(0.65~0.74) 0.66(0.60~0.71) 0.48(0.43~0.53)	0.74(0.71~0.77) 0.61(0.55~0.67) 0.44(0.39~0.49)
$Y_7 = \ln(\sigma_c)$						Strengths		0.82(0.79~0.84) 0.73(0.70~0.76) 0.55(0.52~0.58)	0.78(0.75~0.81) 0.63(0.59~0.67) 0.46(0.43~0.49)
$Y_8 = \ln(E)$								Stiffness	0.83(0.80~0.86) 0.68(0.63~0.72) 0.51(0.47~0.55)
$Y_9 = \ln(V_p)$									Dynamic property
									Symmetry

	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉
X ₁	1.00	-0.73 (-0.78~-0.67)	0.64 (0.55~0.71)	0.53 (0.33~0.69)	0.54 (0.43~0.63)	0.57 (0.49~0.64)	0.52 (0.46~0.57)	0.64 (0.57~0.70)	0.68 (0.64~0.72)
X ₂		1.00	-0.80 (-0.85~-0.74)	-0.81 (-0.88~-0.69)	-0.58 (-0.66~-0.50)	-0.58 (-0.64~-0.52)	-0.64 (-0.68~-0.60)	-0.47 (-0.53~-0.41)	-0.64 (-0.69~-0.59)
X ₃			1.00	0.89 (0.81~0.94)	0.53 (0.38~0.66)	0.75 (0.70~0.79)	0.69 (0.65~0.73)	0.69 (0.62~0.75)	0.56 (0.47~0.63)
X ₄		Index properties		1.00	0.78 (0.70~0.85)	0.83 (0.74~0.90)	0.81 (0.75~0.86)	0.64 (0.52~0.73)	0.71 (0.57~0.81)
X ₅					1.00	0.73 (0.67~0.79)	0.72 (0.68~0.76)	0.51 (0.42~0.60)	0.50 (0.38~0.60)
X ₆						1.00	0.82 (0.80~0.84)	0.68 (0.62~0.73)	0.63 (0.57~0.68)
X ₇					Strengths		1.00	0.75 (0.72~0.77)	0.65 (0.61~0.69)
X ₈								1.00 Stiffness	0.70 (0.65~0.74)
X ₉			Symmetry						1.00 Dynamic property

Table 6 Final C matrix.

	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉
X ₁	1.000	-0.725	0.637	0.598	0.521	0.568	0.516	0.638	0.680
X ₂		1.000	-0.800	-0.796	-0.588	-0.585	-0.637	-0.481	-0.638
X ₃			1.000	0.870	0.567	0.742	0.691	0.683	0.574
X ₄		Index properties		1.000	0.763	0.829	0.805	0.651	0.665
X ₅					1.000	0.739	0.723	0.510	0.507
X ₆						1.000	0.820	0.680	0.634
X ₇					Strengths		1.000	0.745	0.648
X ₈								1.000 Stiffness	0.694
X ₉	Symmetry								1.000 Dynamic property

Draft

Table 7 Updated Johnson parameters for $Y_7 = \ln(\sigma_c)$ under various combinations of information (Johnson distribution type is SU; $a_{Y7} = 0.951$ and $b_{Y7} = 5.390$ are fixed numbers).

Information	Updated Johnson parameters	
	a'_{X7}	b'_{X7}
Single input		
γ	2.157	$2.396-0.602X_1$
n	2.398	$2.663+0.827X_2$
R_L	2.555	$2.837-0.955X_3$
S_h	3.114	$3.459-1.357X_4$
σ_{bt}	2.675	$2.971-1.047X_5$
I_{s50}	3.230	$3.587-1.434X_6$
V_P	2.427	$2.696-0.852X_9$
Two inputs		
γ, n	2.410	$2.677-0.149X_1+0.724X_2$
γ, R_L	2.579	$2.864-0.179X_1-0.850X_3$
γ, S_h	3.123	$3.468-0.092X_1-1.306X_4$
γ, σ_{bt}	2.753	$3.057-0.284X_1-0.929X_5$
γ, I_{s50}	3.249	$3.608-0.130X_1-1.368X_6$
γ, V_P	2.449	$2.720-0.185X_1-0.734X_9$
R_L, σ_{bt}	3.075	$3.415-0.688X_3-0.813X_5$
R_L, I_{s50}	3.306	$3.672-0.326X_3-1.226X_6$
S_h, σ_{bt}	3.248	$3.607-1.065X_4-0.458X_5$
S_h, I_{s50}	3.508	$3.896-0.759X_4-0.928X_6$
R_L, V_P	2.824	$3.136-0.726X_3-0.575X_9$
S_h, V_P	3.221	$3.578-1.168X_4-0.354X_9$
σ_{bt}, V_P	3.036	$3.372-0.872X_5-0.623X_9$
I_{s50}, V_P	3.375	$3.748-1.250X_6-0.392X_9$
Three inputs		
γ, n, R_L	2.608	$2.897-0.078X_1+0.286X_2-0.697X_3$
γ, n, S_h	3.127	$3.473-0.132X_1-0.104X_2-1.366X_4$
γ, n, σ_{bt}	2.892	$3.212-0.011X_1+0.500X_2-0.832X_5$
γ, n, I_{s50}	3.459	$3.842+0.184X_1+0.557X_2-1.314X_6$
γ, n, V_P	2.634	$2.925+0.125X_1+0.599X_2-0.627X_9$
γ, R_L, σ_{bt}	3.075	$3.415+0.006X_1-0.691X_3-0.815X_5$
γ, R_L, I_{s50}	3.307	$3.673-0.035X_1-0.309X_3-1.219X_6$
γ, R_L, V_P	2.837	$3.151+0.144X_1-0.782X_3-0.645X_9$
γ, S_h, σ_{bt}	3.250	$3.610-0.050X_1-1.042X_4-0.450X_5$
γ, S_h, I_{s50}	3.508	$3.896+0.001X_1-0.759X_4-0.929X_6$
γ, S_h, V_P	3.226	$3.583+0.078X_1-1.190X_4-0.394X_9$
$\gamma, \sigma_{bt}, I_{s50}$	3.396	$3.771-0.067X_1-0.459X_5-1.130X_6$
γ, σ_{bt}, V_P	3.039	$3.375+0.061X_1-0.888X_5-0.658X_9$
γ, I_{s50}, V_P	3.379	$3.752+0.066X_1-1.266X_6-0.427X_9$
Four inputs		
$\gamma, n, R_L, \sigma_{bt}$	3.081	$3.422+0.046X_1+0.121X_2-0.629X_3-0.802X_5$
γ, n, R_L, I_{s50}	3.460	$3.843+0.184X_1+0.584X_2+0.047X_3-1.334X_6$
γ, n, R_L, V_P	2.848	$3.163+0.194X_1+0.167X_2-0.695X_3-0.626X_9$
$\gamma, n, S_h, \sigma_{bt}$	3.251	$3.610-0.063X_1-0.034X_2-1.065X_4-0.447X_5$
γ, n, S_h, I_{s50}	3.539	$3.930+0.117X_1+0.272X_2-0.543X_4-1.028X_6$
γ, n, S_h, V_P	3.231	$3.589+0.037X_1-0.107X_2-1.252X_4-0.395X_9$
$\gamma, n, \sigma_{bt}, I_{s50}$	3.558	$3.952+0.197X_1+0.490X_2-0.373X_5-1.129X_6$
$\gamma, n, \sigma_{bt}, V_P$	3.132	$3.478+0.245X_1+0.396X_2-0.817X_5-0.599X_9$
γ, n, I_{s50}, V_P	3.553	$3.946+0.318X_1+0.505X_2-1.236X_6-0.357X_9$
$\gamma, R_L, \sigma_{bt}, I_{s50}$	3.470	$3.854+0.041X_1-0.342X_3-0.480X_5-0.956X_6$
$\gamma, R_L, \sigma_{bt}, V_P$	3.341	$3.711+0.303X_1-0.644X_3-0.792X_5-0.607X_9$
$\gamma, R_L, I_{s50}, V_P$	3.444	$3.825+0.169X_1-0.321X_3-1.111X_6-0.435X_9$
$\gamma, S_h, \sigma_{bt}, I_{s50}$	3.564	$3.958+0.020X_1-0.639X_4-0.290X_5-0.849X_6$
$\gamma, S_h, \sigma_{bt}, V_P$	3.388	$3.762+0.148X_1-0.885X_4-0.501X_5-0.447X_9$
$\gamma, S_h, I_{s50}, V_P$	3.590	$3.987+0.141X_1-0.686X_4-0.894X_6-0.333X_9$
Five inputs		
$\gamma, n, R_L, \sigma_{bt}, I_{s50}$	3.560	$3.953+0.197X_1+0.456X_2-0.058X_3-0.383X_5-1.100X_6$
$\gamma, n, R_L, \sigma_{bt}, V_P$	3.341	$3.711+0.306X_1+0.013X_2-0.638X_3-0.791X_5-0.605X_9$
$\gamma, n, R_L, I_{s50}, V_P$	3.553	$3.946+0.319X_1+0.495X_2-0.017X_3-1.228X_6-0.359X_9$
$\gamma, n, S_h, \sigma_{bt}, V_P$	3.388	$3.763+0.136X_1-0.029X_2-0.904X_4-0.498X_5-0.447X_9$
$\gamma, n, S_h, I_{s50}, V_P$	3.617	$4.017+0.246X_1+0.255X_2-0.484X_4-0.988X_6-0.325X_9$
$\gamma, n, \sigma_{bt}, I_{s50}, V_P$	3.676	$4.083+0.347X_1+0.425X_2-0.412X_5-1.025X_6-0.396X_9$
Six inputs		
$\gamma, n, R_L, S_h, \sigma_{bt}, I_{s50}$	3.605	$4.004+0.133X_1+0.329X_2+0.141X_3-0.495X_4-0.262X_5-0.984X_6$

$\gamma, n, R_L, S_h, \sigma_{bt}, V_P$	3.414	$3.792+0.196X_1-0.086X_2-0.299X_3-0.644X_4-0.578X_5-0.494X_9$
$\gamma, n, R_L, S_h, I_{s50}, V_P$	3.630	$4.032+0.225X_1+0.311X_2+0.191X_3-0.591X_4-1.020X_6-0.299X_9$
$\gamma, n, R_L, \sigma_{bt}, I_{s50}, V_P$	3.685	$4.093+0.354X_1+0.335X_2-0.146X_3-0.438X_5-0.946X_6-0.414X_9$
$\gamma, n, S_h, \sigma_{bt}, I_{s50}, V_P$	3.701	$4.110+0.297X_1+0.275X_2-0.312X_4-0.354X_5-0.895X_6-0.370X_9$

Note: ($X_1, X_2, X_3, X_4, X_5, X_6, X_9$) can be calculated from the measured ($\gamma, n, R_L, S_h, \sigma_{bt}, I_{s50}, V_P$) based on Eq. (4).

Draft

Table 8 Updated Johnson parameters for $Y_8 = \ln(E)$ under various combinations of information (Johnson distribution type is SB; $a_{Y8} = 10.623$ and $b_{Y8} = -5.772$ are fixed numbers).

Information	Updated Johnson parameters	
	a'_{X8}	b'_{X8}
Single input		
γ	1.312	-1.931-0.828X ₁
n	1.153	-1.697+0.549X ₂
R_L	1.383	-2.035-0.934X ₃
S_h	1.331	-1.959-0.857X ₄
σ_{bt}	1.175	-1.729-0.593X ₅
I_{s50}	1.378	-2.028-0.926X ₆
σ_c	1.514	-2.229-1.116X ₇
V_P	1.403	-2.065-0.963X ₉
Two inputs		
γ, n	1.313	-1.932-0.790X ₁ +0.053X ₂
γ, R_L	1.482	-2.182-0.501X ₁ -0.682X ₃
γ, S_h	1.458	-2.146-0.558X ₁ -0.605X ₄
γ, σ_{bt}	1.362	-2.005-0.688X ₁ -0.328X ₅
γ, I_{s50}	1.516	-2.231-0.557X ₁ -0.703X ₆
γ, σ_c	1.689	-2.486-0.577X ₁ -0.947X ₇
γ, V_P	1.478	-2.175-0.452X ₁ -0.707X ₉
R_L, σ_{bt}	1.413	-2.079-0.811X ₃ -0.253X ₅
R_L, I_{s50}	1.478	-2.175-0.580X ₃ -0.563X ₆
R_L, σ_c	1.615	-2.378-0.514X ₃ -0.835X ₇
R_L, V_P	1.602	-2.358-0.673X ₃ -0.714X ₉
S_h, σ_{bt}	1.331	-1.959-0.825X ₄ -0.042X ₅
S_h, I_{s50}	1.410	-2.075-0.389X ₄ -0.626X ₆
S_h, σ_c	1.527	-2.248-0.220X ₄ -0.948X ₇
S_h, V_P	1.499	-2.206-0.504X ₄ -0.694X ₉
σ_{bt}, I_{s50}	1.378	-2.028-0.023X ₅ -0.909X ₆
σ_{bt}, σ_c	1.517	-2.233+0.090X ₅ -1.183X ₇
σ_{bt}, V_P	1.451	-2.135-0.305X ₅ -0.841X ₉
I_{s50}, σ_c	1.539	-2.266-0.320X ₆ -0.872X ₇
σ_c, V_P	1.664	-2.449-0.838X ₇ -0.599X ₉
Three inputs		
γ, n, R_L	1.604	-2.362-0.824X ₁ -0.781X ₂ -1.184X ₃
γ, n, S_h	1.568	-2.308-0.895X ₁ -0.761X ₂ -1.080X ₄
γ, n, σ_{bt}	1.365	-2.010-0.749X ₁ -0.104X ₂ -0.359X ₅
γ, n, I_{s50}	1.531	-2.254-0.690X ₁ -0.219X ₂ -0.766X ₆
γ, n, σ_c	1.805	-2.656-0.946X ₁ -0.610X ₂ -1.231X ₇
γ, n, V_P	1.487	-2.189-0.548X ₁ -0.172X ₂ -0.758X ₉
γ, R_L, σ_{bt}	1.494	-2.199-0.462X ₁ -0.625X ₃ -0.158X ₅
γ, R_L, I_{s50}	1.561	-2.297-0.447X ₁ -0.398X ₃ -0.500X ₆
γ, R_L, σ_c	1.719	-2.530-0.476X ₁ -0.292X ₃ -0.819X ₇
γ, R_L, V_P	1.619	-2.383-0.220X ₁ -0.599X ₃ -0.618X ₉
γ, S_h, σ_{bt}	1.459	-2.147-0.563X ₁ -0.635X ₄ +0.042X ₅
γ, S_h, I_{s50}	1.525	-2.245-0.522X ₁ -0.209X ₄ -0.556X ₆
γ, S_h, σ_c	1.690	-2.488-0.594X ₁ +0.064X ₄ -0.991X ₇
γ, S_h, V_P	1.546	-2.275-0.357X ₁ -0.426X ₄ -0.535X ₉
$\gamma, \sigma_{bt}, \sigma_c$	1.724	-2.537-0.651X ₁ +0.305X ₅ -1.155X ₇
$\gamma, \sigma_{bt}, I_{s50}$	1.519	-2.236-0.574X ₁ +0.103X ₅ -0.772X ₆
γ, σ_{bt}, V_P	1.503	-2.213-0.386X ₁ -0.225X ₅ -0.656X ₉
$\gamma, I_{s50}, \sigma_c$	1.692	-2.490-0.557X ₁ -0.106X ₆ -0.873X ₇
γ, I_{s50}, V_P	1.601	-2.357-0.348X ₁ -0.556X ₆ -0.510X ₉
γ, σ_c, V_P	1.739	-2.559-0.417X ₁ -0.821X ₇ -0.378X ₉
Four inputs		
$\gamma, n, R_L, \sigma_{bt}$	1.642	-2.417-0.796X ₁ -0.874X ₂ -1.144X ₃ -0.279X ₅
γ, n, R_L, I_{s50}	1.676	-2.467-0.762X ₁ -0.744X ₂ -0.900X ₃ -0.462X ₆
γ, n, R_L, σ_c	2.039	-3.001-0.986X ₁ -1.216X ₂ -0.958X ₃ -1.107X ₇
γ, n, R_L, V_P	1.871	-2.754-0.579X ₁ -1.108X ₂ -1.290X ₃ -0.857X ₉
$\gamma, n, S_h, \sigma_{bt}$	1.573	-2.316-0.920X ₁ -0.785X ₂ -1.188X ₄ +0.131X ₅
γ, n, S_h, I_{s50}	1.596	-2.349-0.816X ₁ -0.635X ₂ -0.740X ₄ -0.368X ₆
γ, n, S_h, σ_c	1.840	-2.709-0.970X ₁ -0.829X ₂ -0.428X ₄ -1.040X ₇

γ, n, S_h, V_P	1.678	$-2.471-0.703X_1-0.813X_2-0.922X_4-0.580X_9$
$\gamma, n, \sigma_{bt}, I_{s50}$	1.532	$-2.255-0.691X_1-0.206X_2+0.059X_5-0.802X_6$
$\gamma, n, \sigma_{bt}, \sigma_c$	1.832	$-2.696-0.995X_1-0.587X_2+0.262X_5-1.400X_7$
$\gamma, n, \sigma_{bt}, V_P$	1.528	$-2.250-0.527X_1-0.293X_2-0.301X_5-0.725X_9$
$\gamma, n, I_{s50}, \sigma_c$	1.806	$-2.659-0.929X_1-0.607X_2-0.081X_6-1.172X_7$
γ, n, I_{s50}, V_P	1.640	$-2.414-0.527X_1-0.345X_2-0.635X_6-0.585X_9$
γ, n, σ_c, V_P	1.886	$-2.776-0.792X_1-0.685X_2-1.111X_7-0.472X_9$
$\gamma, R_L, \sigma_{bt}, I_{s50}$	1.564	$-2.302-0.463X_1-0.396X_3+0.094X_5-0.564X_6$
$\gamma, R_L, \sigma_{bt}, \sigma_c$	1.757	$-2.587-0.547X_1-0.307X_3+0.318X_5-1.030X_7$
$\gamma, R_L, \sigma_{bt}, V_P$	1.622	$-2.388-0.207X_1-0.572X_3-0.079X_5-0.604X_9$
$\gamma, R_L, I_{s50}, \sigma_c$	1.719	$-2.530-0.477X_1-0.296X_3+0.012X_6-0.826X_7$
$\gamma, R_L, I_{s50}, V_P$	1.654	$-2.435-0.227X_1-0.421X_3-0.338X_6-0.526X_9$
$\gamma, S_h, \sigma_{bt}, I_{s50}$	1.536	$-2.261-0.538X_1-0.299X_4+0.194X_5-0.623X_6$
$\gamma, S_h, \sigma_{bt}, \sigma_c$	1.726	$-2.540-0.635X_1-0.092X_4+0.335X_5-1.112X_7$
$\gamma, S_h, \sigma_{bt}, V_P$	1.546	$-2.276-0.358X_1-0.432X_4+0.007X_5-0.535X_9$
$\gamma, S_h, I_{s50}, \sigma_c$	1.696	$-2.496-0.581X_1+0.137X_4-0.167X_6-0.923X_7$
$\gamma, S_h, I_{s50}, V_P$	1.603	$-2.359-0.340X_1-0.084X_4-0.501X_6-0.495X_9$
$\gamma, \sigma_{bt}, I_{s50}, \sigma_c$	1.739	$-2.559-0.620X_1+0.374X_5-0.254X_6-1.022X_7$
$\gamma, \sigma_{bt}, \sigma_c, V_P$	1.770	$-2.606-0.492X_1+0.288X_5-1.023X_7-0.363X_9$
$\gamma, I_{s50}, \sigma_c, V_P$	1.740	$-2.561-0.406X_1-0.066X_6-0.777X_7-0.372X_9$
Five inputs		
$\gamma, n, R_L, \sigma_{bt}, I_{s50}$	1.679	$-2.471-0.761X_1-0.778X_2-0.925X_3-0.086X_5-0.402X_6$
$\gamma, n, R_L, \sigma_{bt}, \sigma_c$	2.070	$-3.046-1.038X_1-1.194X_2-0.959X_3+0.264X_5-1.279X_7$
$\gamma, n, R_L, \sigma_{bt}, V_P$	1.896	$-2.791-0.566X_1-1.172X_2-1.260X_3-0.213X_5-0.834X_9$
$\gamma, n, R_L, I_{s50}, \sigma_c$	2.079	$-3.060-1.080X_1-1.351X_2-1.150X_3+0.405X_6-1.374X_7$
$\gamma, n, R_L, I_{s50}, V_P$	1.886	$-2.776-0.571X_1-1.068X_2-1.156X_3-0.208X_6-0.792X_9$
$\gamma, n, R_L, \sigma_c, V_P$	2.194	$-3.230-0.798X_1-1.402X_2-1.087X_3-0.945X_7-0.621X_9$
$\gamma, n, S_h, \sigma_{bt}, I_{s50}$	1.611	$-2.371-0.843X_1-0.652X_2-0.858X_4+0.223X_5-0.440X_6$
$\gamma, n, S_h, \sigma_{bt}, \sigma_c$	1.914	$-2.818-1.076X_1-0.932X_2-0.708X_4+0.469X_5-1.219X_7$
$\gamma, n, S_h, \sigma_{bt}, V_P$	1.682	$-2.475-0.724X_1-0.831X_2-1.005X_4+0.099X_5-0.574X_9$
$\gamma, n, S_h, I_{s50}, \sigma_c$	1.846	$-2.717-1.012X_1-0.892X_2-0.540X_4+0.172X_6-1.113X_7$
$\gamma, n, S_h, I_{s50}, V_P$	1.696	$-2.496-0.652X_1-0.714X_2-0.667X_4-0.285X_6-0.552X_9$
$\gamma, n, S_h, \sigma_c, V_P$	1.913	$-2.816-0.822X_1-0.869X_2-0.366X_4-0.956X_7-0.445X_9$
$\gamma, n, \sigma_{bt}, I_{s50}, \sigma_c$	1.842	$-2.712-0.923X_1-0.575X_2+0.320X_5-0.210X_6-1.286X_7$
$\gamma, n, \sigma_{bt}, I_{s50}, V_P$	1.640	$-2.415-0.528X_1-0.339X_2+0.025X_5-0.650X_6-0.584X_9$
$\gamma, n, \sigma_{bt}, \sigma_c, V_P$	1.909	$-2.810-0.842X_1-0.662X_2+0.236X_5-1.268X_7-0.458X_9$
$\gamma, n, I_{s50}, \sigma_c, V_P$	1.886	$-2.776-0.787X_1-0.684X_2-0.028X_6-1.092X_7-0.470X_9$
$\gamma, R_L, \sigma_{bt}, I_{s50}, \sigma_c$	1.761	$-2.593-0.544X_1-0.269X_3+0.354X_5-0.140X_6-0.974X_7$
$\gamma, R_L, \sigma_{bt}, \sigma_c, V_P$	1.808	$-2.661-0.380X_1-0.321X_3+0.301X_5-0.889X_7-0.375X_9$
$\gamma, R_L, I_{s50}, \sigma_c, V_P$	1.773	$-2.610-0.308X_1-0.327X_3+0.067X_6-0.721X_7-0.395X_9$
$\gamma, S_h, \sigma_{bt}, \sigma_c, V_P$	1.770	$-2.606-0.492X_1-0.001X_4+0.288X_5-1.023X_7-0.363X_9$
$\gamma, S_h, I_{s50}, \sigma_c, V_P$	1.749	$-2.574-0.433X_1+0.208X_4-0.156X_6-0.848X_7-0.395X_9$
Six inputs		
$\gamma, n, R_L, \sigma_{bt}, I_{s50}, \sigma_c$	2.091	$-3.078-1.094X_1-1.307X_2-1.109X_3+0.177X_5+0.316X_6-1.431X_7$
$\gamma, n, R_L, \sigma_{bt}, I_{s50}, V_P$	1.898	$-2.793-0.565X_1-1.147X_2-1.216X_3-0.179X_5-0.076X_6-0.814X_9$
$\gamma, n, R_L, \sigma_{bt}, \sigma_c, V_P$	2.220	$-3.268-0.849X_1-1.380X_2-1.087X_3+0.232X_5-1.101X_7-0.607X_9$
$\gamma, n, R_L, I_{s50}, \sigma_c, V_P$	2.281	$-3.358-0.908X_1-1.629X_2-1.387X_3+0.585X_6-1.310X_7-0.714X_9$
$\gamma, n, S_h, \sigma_{bt}, I_{s50}, \sigma_c$	1.916	$-2.820-1.096X_1-0.963X_2-0.761X_4+0.460X_5+0.090X_6-1.254X_7$
$\gamma, n, S_h, \sigma_{bt}, \sigma_c, V_P$	1.974	$-2.906-0.933X_1-0.958X_2-0.625X_4+0.422X_5-1.127X_7-0.399X_9$
$\gamma, n, S_h, I_{s50}, \sigma_c, V_P$	1.921	$-2.828-0.869X_1-0.944X_2-0.498X_4+0.203X_6-1.041X_7-0.452X_9$
Seven inputs		
$\gamma, n, R_L, S_h, \sigma_{bt}, I_{s50}, \sigma_c$	2.093	$-3.080-1.110X_1-1.339X_2-1.061X_3-0.124X_4+0.207X_5+0.342X_6-1.420X_7$
$\gamma, n, R_L, S_h, \sigma_{bt}, I_{s50}, V_P$	1.898	$-2.793-0.562X_1-1.144X_2-1.224X_3+0.018X_4-0.184X_5-0.079X_6-0.817X_9$
$\gamma, n, R_L, S_h, \sigma_{bt}, \sigma_c, V_P$	2.236	$-3.292-0.794X_1-1.332X_2-1.287X_3+0.382X_4+0.117X_5-1.155X_7-0.670X_9$
$\gamma, n, R_L, S_h, I_{s50}, \sigma_c, V_P$	2.289	$-3.370-0.876X_1-1.564X_2-1.477X_3+0.240X_4+0.513X_6-1.348X_7-0.738X_9$
$\gamma, n, R_L, \sigma_{bt}, I_{s50}, \sigma_c, V_P$	2.284	$-3.362-0.917X_1-1.606X_2-1.366X_3+0.078X_5+0.543X_6-1.336X_7-0.703X_9$
$\gamma, n, S_h, \sigma_{bt}, I_{s50}, \sigma_c, V_P$	1.977	$-2.910-0.959X_1-1.002X_2-0.699X_4+0.409X_5+0.127X_6-1.175X_7-0.406X_9$

Note: ($X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_9$) can be calculated from the measured ($\gamma, n, R_L, S_h, \sigma_{bt}, I_{s50}, \sigma_c, V_P$) using Eq. (4).

Table 9. Intact rock data collected from Kiliç and Teymen (2008).

#	Rock type	Input parameters							Parameters to be estimated		
		γ (kN/m ³)	n (%)	R _L	S _h	σ_{bt} (MPa)	I _{s50} (MPa)	V _P (km/s)	Outlier P- value	σ_c (MPa)	E (GPa)
1	Diorite		0.40	52	82	19.0	8.6	6.75	0.32	239.0	28.9
2	Quartzite		0.44	53	82	19.7	7.3	5.75	0.42	210.0	23.9
3	Limestone		1.13	50	64	10.1	4.1	5.69	0.80	163.0	19.3
4	Sandstone		0.16	47	62	12.0	4.0	5.18	0.07	160.0	20.7
5	Granodiorite		1.18	48	69	11.8	3.9	4.76	0.60	143.0	22.0
6	Basalt		4.41	48	62	10.7	4.0	5.57	0.62	131.0	20.4
7	Limestone		0.35	50	52	10.3	3.0	6.00	0.14	127.0	19.7
8	Sandstone		2.32	43	55	10.3	3.3	4.75	0.89	122.0	18.1
9	Limestone		0.66	49	51	9.8	2.2	5.84	0.14	112.0	15.9
10	Sandstone		2.39	46	61	9.2	2.6	4.58	0.54	111.0	13.2
11	Marble		0.98	41	46	8.1	1.7	4.71	0.33	72.0	12.8
12	Trachyte		6.25	39	52	7.3	1.5	3.48	0.29	70.0	7.4
13	Travertine		4.17	37	48	7.1	1.7	4.50	0.57	62.0	13.0
14	Andesite		12.37	33	45	6.6	1.6	3.21	0.44	56.0	8.4
15	Tuff		19.35	18	28	3.4	1.2	3.12	0.15	37.0	6.1
16	Tuff		24.44	15	16	2.7	1.2	2.54	0.11	23.0	4.6
17	Tuff		24.21	14	14	2.2	1.1	2.37	0.06	18.0	3.4
18	Tuff		35.21	11	13	1.3	0.9	1.87	0.02*	6.8	2.1
19	Tuff		37.81	10	11	1.1	0.9	1.47	0.01*	6.2	2.0

* P-value less than 0.05 (extrapolation possible)