Dimensional analysis of pore-water pressure response in a vegetated infinite slope

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Dimensional analysis of pore-water pressure response in a vegetated infinite slope

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Abstract

Pore-water pressure (PWP) induced by root water uptake has been usually investigated by individual physical quantities. Limited dimensional analysis has been available for investigating PWP response in vegetated slope. In this study, dimensional analysis was conducted to explore dimensionless numbers controlling PWP distributions in vegetated slope. Three dimensionless numbers governing unsaturated seepage were proposed, including capillary effect number ($CN$; describing the relative importance of water flow driven by PWP gradient over that driven by gravity), root water uptake number ($RN$; representing effects of root water uptake) and water transfer-storage ratio ($WR$; ratio of water transfer to water storage rate). Dimensionless relationships were further proposed to estimate PWP and root influence zone in vegetated slope. Then analytical parametric studies were conducted to study effects of $RN$, $CN$ and $WR$ on PWP distributions. Thereafter, the proposed relationships were validated by published field and centrifuge tests. During drying period, effects of root water uptake on PWP and root influence zone become more significant as $CN$ decreases or $RN$ increases. During wetting period, the larger the $WR$, the deeper the wetting front moves and more reduction of negative PWP occurs. The proposed dimensionless relationships can determine PWP and root influence zone in vegetated soil reasonably.
Key words: Dimensional analysis; dimensionless number; pore-water pressure; vegetation; infinite slope

Introduction

Root water uptake affects soil hydrology and stability of civil engineering infrastructures, including slopes, landfill covers and embankments (Liu et al. 2016; Smethurst et al. 2006, 2015; Sinnathamby et al. 2013). Root water uptake reduces pore-water pressure (PWP) inside and outside root zone, and its influence zone could be up to about five times of the root depth (Ni et al. 2018). The reduction in PWP by root water uptake results in lower water permeability but higher shear strength (Simon and Collison 2002; Gonzalez-Ollauri and Mickovski 2014; Indraratna et al. 2006). As a result, rainfall infiltration would be reduced, while shallow slope stability (i.e. up to depth of 2 m) could be enhanced (Simon and Collison 2002; Lu and Godt 2013; Fell et al. 2005).

Although the influence of hydraulic parameters of soil on water infiltration and PWP distributions in bare ground/slope have been widely studied (Green and Ampt 1911; Zhan and Ng 2004; Kasim et al. 1998), the controlling parameters of PWP distributions in vegetated slope are not fully understood. Zhan and Ng (2004) carried out analytical analysis and found that PWP distributions are mainly governed by both $q\alpha / k_s$ and $k_s / \alpha$, rather than the individual value of each parameter (where $q$ is rainfall intensity, $\alpha$ is desaturation coefficient describing the rate of decrease in water content as negative PWP (i.e., matric suction) increases, $k_s$ is saturated permeability). However,
root water uptake was ignored in their study. Numerical simulations were conducted to investigate variations in PWP distributions due to root water uptake in a vegetated slope by Nyambayo and Potts (2010), who found that the maximum negative PWP decreases as root depth increases for a given amount of root water uptake. Fatahi et al. (2009) conducted comprehensive numerical parametric studies of the individual parameters affecting PWP distributions induced by root water uptake, including transpiration rate, soil saturated permeability, etc. Nevertheless, only flat vegetated ground was considered (i.e., ignoring the effect of the slope angle). More recently, Ng et al. (2015) derived analytical solutions for calculating PWP in an infinite vegetated slope. All the above mentioned valuable work only partially investigated the individual physical quantities affecting PWP response in the vegetated soil, while rare study has been conducted on the combined effects of relevant parameters. On one hand, the combined parameters could help reduce the groups of independent variables, the number of which is frequently large due to the highly nonlinearity and variability of hydraulic properties of soil and root. On the other hand, the combined parameters in terms of dimensionless number could be useful for designing physical model (e.g., centrifuge test), as they are not affected by the specific value of individual physical quantity and are scale-independent. Dimensional analysis provides an effective method to investigate the combined effects of different parameters by developing dimensionless numbers, which could help to reduce the number of independent variables and to design dimensionally valid models of many kinds (Butterfield 1999). However, very few dimensional analysis and dimensionless numbers have been available for investigating PWP response induced by root water uptake in vegetated slope.

Controlling parameters are usually explored either by using analytical solutions or by conducting numerical simulations. Since the governing equation of unsaturated seepage is highly non-linear,
analytical solutions of PWP distributions can only be derived under certain assumptions and specified boundary and initial conditions (Philip 1957; Srivastava and Yeh 1991; Zhan et al. 2013). However, analytical solutions have unique advantages over numerical simulations. For example, analytical solutions can provide insights on selection of controlling parameters of PWP distributions. This is particularly useful when combined with dimensional analysis, due to the explicit relationships among different parameters provided by analytical solutions. Analytical solutions have been derived to investigate PWP distributions induced by root water uptake (Raats 1974; Yuan and Lu 2005; Ng et al. 2015). In this study, the analytical solutions derived by Ng et al. (2015) were selected for dimensional analysis. Note that the solutions consider almost all the key parameters affecting PWP distributions in vegetated slope, including root depth, evapotranspiration rate, soil hydraulic properties (e.g., unsaturated water permeability), and antecedent and subsequent rainfall/evaporation, etc.

This study aims to conduct dimensional analysis to investigate controlling parameters of PWP response in vegetated slope. Dimensionless numbers governing PWP response in vegetated slope were proposed. Then dimensionless relationships were proposed for estimating PWP and root influence zone in vegetated slope based on the proposed dimensionless numbers. Thereafter, parametric studies were performed to investigate the influences of these dimensionless numbers on PWP distributions in vegetated slope. Finally, the proposed dimensionless relationships were verified by field and centrifuge tests.
Dimensional analysis of PWP response in a vegetated infinite slope

Review of an analytical model for PWP response in a vegetated infinite slope

The analytical model developed by Ng et al. (2015) was adopted and is briefly introduced. A vegetated infinite slope of angle $\beta$ is shown in Fig. 1. Infiltration/transpiration is considered at the slope surface, while the water table is located at the bottom of the slope and is assumed to be fixed in position. During rainfall, the model ignores the transpiration rate due to high relative humidity. Moreover, surface runoff is ignored, i.e., the applied infiltration should be calculated as the difference between rainfall and surface runoff. During drying, the applied transpiration rate should be calculated/measured separately. The unsaturated seepage could be simplified as a one-dimensional flow perpendicular to the slope surface by assuming that iso-bars are parallel to the slope surface. Thus, the governing equation of unsaturated seepage in this vegetated infinite slope could be expressed as follows:

$$\frac{\partial^2 k}{\partial z^2} + \alpha \cos \beta \frac{\partial k}{\partial z} - \alpha T_p g(z') H(z - L'_i) = \frac{\alpha (\theta'_s - \theta'_r)}{k_s} \frac{\partial k}{\partial t}$$  \hspace{1cm} (1)

where $k$ represents the water permeability of soil ($m \text{ s}^{-1}$); $z'$ is the perpendicular distance to the slope bottom ($m$; see Fig. 1); $\alpha$ is the desaturation coefficient of soil ($m^{-1}$; details are shown later in Eq.(4)); $\beta$ is the slope angle (degree); $T_p$ represents the transpiration rate ($m \text{ s}^{-1}$); $L'_i$ is the thickness of the domain outside root zone ($m$; Fig. 1); $\theta'_s$ and $\theta'_r$ are the residual and saturated volumetric water contents (dimensionless), respectively; $k_s$ represents the saturated water permeability of soil ($m \text{ s}^{-1}$); $t$ is time ($s$); and $H(z - L'_i)$ (dimensionless) is expressed as follows...
\begin{equation}
H(z' - L_1) = \begin{cases} 
1 & L_1 \leq z' \leq (L_1 + L_2) \\
0 & 0 \leq z' \leq L_1 
\end{cases}
\text{inside root zone}
\text{outside root zone}
\end{equation}

where \( L_2 \) is the thickness of root zone (\( m; \text{Fig. 1} \)).

\( g(z') \) describes the effect of root architecture (\( m^{-1} \)). In this study, the uniform root architecture is considered due to its simplicity and is defined as follows:

\begin{equation}
g(z') = \frac{1}{L_2}
\end{equation}

In practice, the uniform root can be found for a grass species named vetiver, which is frequently used in most parts of Asia (e.g., China, Thailand and Malaysia) for ecological restoration and enhancement of man-made slope stability (Liu 2017). It is noted that the PWP distribution is affected by different root architectures (Ng et al. 2015), including an exponential root distribution (where the root density decreases exponentially with depth) and a triangular distribution (a linear decrease with depth). According to the analytical studies conducted by Ng et al. (2015), the PWP distributions between exponential and triangular root distributions are similar, both induce larger negative PWP by about 16 kPa than the uniform root for a given transpiration rate (i.e., 6.6 mm/day) during drying period. Due to the assumption of uniform root architecture and hence constant transpiration rate with depth, effects of root architecture can not be considered for this study.

The water permeability function and soil water retention curve (SWRC) are described respectively, as follows (Gardner 1958):
{\[ k = k_s \exp(\alpha \psi) \]}

{\[ \theta_w = \theta_r + (\theta_s - \theta_r)\exp(\alpha \psi) \]}

where $\psi$ is PWP head ($m$). It is noted that when the soil is near saturated, Eqs. (4) and (5) may not be very applicable, due to the two equations usually underestimate volumetric water content and unsaturated water permeability, respectively (Zhan et al. 2013). However, they have been adopted for deriving analytical solutions for unsaturated seepage analysis due to mathematical simplicity (Gardner 1958; Srivastave and Yeh 1991; Yuan and Lu 2005; Zhan et al. 2013). In addition, the hysteresis effect during wetting and drying cycles is ignored, thus a constant $\alpha$ is adopted for both drying and wetting. Moreover, it has been recognized that the soil hydraulic properties could be altered by the presence of roots in the soil (Glendinning et al. 2009; Li and Ghodrati 1994; Khalilzad et al. 2014; Jotisankasa and Sirirattanachat 2017; Vergani and Graf 2016). However, there are no conclusive findings, since some research reported a reduction in the saturated permeability ($k_s$), while some showed the opposite. According to numerical simulations of Ni et al. (2018), the root-induced reduction in $k_s$ could help preserve suction in the vegetated slope after rainfall, while a larger $k_s$ results in a lower suction due to more rainfall infiltration. Due to the assumption of negligible effect of root on soil hydraulic properties, the variation in the PWP distribution caused by root-induced alteration of soil hydraulic properties can not be considered by this study. For the above-mentioned analytical model (Eqs. (1)-(5)), Ng et al. (2015) validated the derived analytical solutions against numerical simulations, which were conducted by a commercial finite element software named COMSOL (COMSOL 4.3b 2013).
In order to develop dimensionless numbers, the following dimensionless variables are defined:

\[ \frac{z}{L} \quad (6) \]

\[ k_r = \frac{k}{k_s} \quad (7) \]

\[ r = \frac{L_r}{L} \quad (8) \]

where \( L \) is the thickness of the slope (i.e., \( L = L_1 + L_2 \); see Fig. 1; unit: \( m \)); \( \vec{z} \) is the normalized \( z \) by \( L \) (dimensionless); \( k_r \) is the relative permeability (dimensionless); and \( r \) is the root depth ratio (dimensionless).

Substituting Eqs. (6), (7) and (8) into Eq. (1) and rearranging yields the normalized governing equation of water flow in the vegetated infinite slope with the uniform root:

\[ \frac{1}{\alpha \cos \beta L} \frac{\partial^2 k_r}{\partial z^2} + \frac{\partial k_r}{\partial z} - \frac{T_p}{k_s \cos \beta} \frac{1}{r} H(z' - L_1) = \frac{(\theta_s - \theta_r) L}{k_s \cos \beta} \frac{\partial k_r}{\partial t} \quad (9) \]

On the left handside of Eq. (9), the first and second terms are related to water flow driven by PWP gradient and gravity, respectively, while the third term is related to root water uptake.
Based on Eq. (9), three dimensionless numbers can be proposed as follows:

**Capillary effect number (CN)**

Capillary effect number (CN) is defined as follows:

\[
CN = \frac{1}{\alpha \cos \beta L}
\]  

(10)

CN describes the relative importance of water flow driven by PWP gradient \( \frac{1}{\alpha \cos \beta L} \frac{\partial^2 k_r}{\partial z^2} \) in Eq. (9) over that driven by gravity \( \frac{\partial k_r}{\partial z} \) in Eq. (9). The relative importance of water flow driven by PWP gradient becomes more significant with CN. For a fixed \( L \), CN is affected by the desaturation coefficient \( \alpha \) and the slope angle \( \beta \) (Eq. (10)). Generally, the greater the sand content, the larger the value of \( \alpha \). Correspondingly, the unsaturated water permeability decreases more quickly as the negative PWP increases (Zhan and Ng 2004). According to Philip (1969), the typical value of \( \alpha \) is about 1 m\(^{-1}\), and the range of values 0.2-5 m\(^{-1}\) covers most soils. The range of 0.1 to 1 m\(^{-1}\) covers most of the published \( \alpha \) values (Morrison and Szecsody 1985). Hence, for a given water table depth (\( L \) in Fig. 1) and a slope angle \( \beta \), \( CN = \frac{1}{\alpha \cos \beta L} \) ranges from \( \frac{1}{L \cos \beta} \) to \( \frac{10}{L \cos \beta} \), considering \( 0.1 \leq \alpha \leq 1 \).

**Water transfer-storage ratio (WR)**

Water transfer-storage ratio (WR) is defined as follows:
characteristic water flow driven by gravity component perpendicular to the slope
characteristic rate of water storage change across \( L \)

In Eq. (11), \( k_s \cos \beta \) represents the saturated water flow driven by gravity component perpendicular to slope, which is chosen as the characteristic water flow, while \( \frac{(\theta_s - \theta_r)L}{t} \) denotes the characteristic rate of water storage change across \( L \). \( \theta_s - \theta_r \) represents the maximum water storage capacity of a given soil. Generally, the larger the pore sizes, the larger the value of \( \theta_s - \theta_r \) (Vanapalli et al. 1998).

Root water uptake number (RN)

Root water uptake number (RN) is defined as follows:

\[
RN = \frac{T_p}{k_s \cos \beta} = \frac{rate \ of \ root \ water \ uptake}{characteristic \ water \ flow \ due \ to \ gravity \ component \ perpendicular \ to \ slope}
\]

RN describes the relative importance of root water uptake (i.e., \( T_p \)) over water flow driven by gravity component perpendicular to slope. The larger the RN, the more significant influence of root water uptake. For a given slope angle \( \beta \), \( k_s \) ranges from \( 10^{-3} \) m/s (sand) to \( 10^{-9} \) m/s (clay),
while $T_r$ can range from 0 mm/day (rainy period) and 8 mm/day (sunny period; Leung and Ng 2013). Hence, for most soils, $RN$ ranges from 0 to $\frac{90}{\cos \beta}$ for a given slope angle of $\beta$.

It is noted that the combined parameters in terms of dimensionless numbers could be arranged in other alternative forms. For instance, the effects of slope angle (i.e., $\cos \beta$) would be isolated from $RN$ and $CN$ as an individual dimensionless parameter.

**Estimating variations in PWP due to root water uptake based on dimensionless numbers**

For the vegetated infinite slope (Fig. 1), the negative PWP at the surface is the largest at drying steady state (Ng et al. 2015), when the negative PWPs enable a time independent flow from the (fixed position) water table given a certain imposed root water uptake. Based on the proposed dimensionless numbers, it is possible to estimate the negative PWP by using the dimensionless numbers (i.e., $CN$ and $RN$) at drying steady state.

The steady-state PWP head ($\psi |_{z=L}$; unit: m) at slope surface can be calculated by using the analytical solutions for the uniform root architecture derived by Ng et al. (2015):

$$\psi |_{z=L} = \frac{\ln k_r |_{z=L} }{\alpha} \quad (k_r |_{z=L} \leq 1) \quad (13)$$

In Eq. (13), $k_r |_{z=L}$ represents the relative permeability at slope surface (dimensionless) and is expressed as follows:
where \( q_0 \) represents rainfall (positive value) or evaporation (negative value) at slope surface (m/s), which is ignored in this study (i.e., \( q_0 = 0 \) m/s) in order to isolate influence of root water uptake on PWP distributions; and \( k_{r,\text{veg}} \mid z = L \) represents the relative permeability affected by root water uptake at slope surface (dimensionless) and is expressed as follows:

\[
k_{r} \mid z = L = \exp\left(\frac{-1}{CN}\right) + \frac{q_0 \left[ \exp\left(\frac{-1}{CN}\right) - 1 \right]}{k_s} + k_{r,\text{veg}} \mid z = L \tag{14}
\]

On the righthand side of Eq. (14), the first, second and third terms describe variations in relative permeability due to the effects of water table, rainfall (or evaporation) and root water uptake, respectively.

Based on Eq. (14), the PWP induced by root water uptake is affected by water table (i.e., \( \exp\left(\frac{-1}{CN}\right) \)). Hence, the PWP in vegetated slope is normalized by that induced by water table (i.e., hydrostatic PWP). A normalized PWP (\( \psi' \)) is proposed as follows:

\[
\bar{\psi'} = \frac{\psi_r}{\psi_s} \tag{16}
\]
where $\psi_r$ is the PWP head induced by root water uptake at a particular depth $z'$ (m); and $\psi_b$ is the hydrostatic PWP head in an infinite slope at a particular depth $z'$ (m), which is determined as follows:

$$\psi_b = z' \cos \beta$$ \hspace{1cm} (17)

Based on Eq. (15), effects of root water uptake are mainly affected by $RN$ and $CN$. A new dimensionless number $\eta$, representing a ratio of the effect of the root water uptake and that of the water table, is proposed as follows (details of derivation can be found in Appendix):

$$\eta = RN \frac{\exp(CN^{-1})}{1 + CN^{-1} \times r}$$ \hspace{1cm} (18)

As shown in Eq. (18), $\eta$ considers the combined effects of $RN$, $CN$ and $r$ on $\psi$. The dimensionless relationship between $\eta$ and $\eta$ is later established based on the calculated results using Eq. (13). Thereafter, PWP induced by root water uptake in vegetated slope could be estimated (see details below).

Estimating the influence zone of root water uptake based on dimensionless numbers

Root water uptake affects PWP distributions inside and outside root zone. Based on Ng et al. (2015), the PWP distributions induced by root water uptake for the uniform root architecture outside root zone can be expressed for steady state condition, as follows:

$$\psi = \frac{1}{\alpha} \ln \{ \exp \left( - \frac{z}{CN} \right) + \frac{q_0 \left[ \exp \left( - \frac{z}{CN} \right) - 1 \right]}{k_s} + RN \left[ \exp \left( - \frac{z}{CN} \right) - 1 \right] \}$$ \hspace{1cm} (19)
On the right-hand side of Eq. (19), the first, second and third terms describe the effects of water table, rainfall/evaporation and root water uptake on PWP distributions, respectively. Based on the third term in Eq. (19), the PWP induced by root water uptake outside root zone at a particular depth \( \bar{z} \) depends on \( RN \) and \( CN \).

In Eq. (19), the first term on the righthand side is positive, while the third term is negative since root water uptake decreases PWP. Hence, by comparing the negative value of the third term to the first term, the following dimensionless number is defined for estimating the influence depth of root water uptake:

\[
\xi = \frac{T_p}{k_s \cos \beta} \frac{1 - \exp(-\alpha z' \cos \beta)}{\exp(-\alpha z' \cos \beta)}
\]

\[
= RN \left[ \exp \left( -\frac{z}{CN} \right) - 1 \right]
\]

Based on Eq. (20), effects of root water uptake become less prominent as \( \xi \) decreases along depth. The valve value of \( \xi \) is defined as \( \xi_{valv} \) (dimensionless), below which the effects of root water uptake are negligible. Based on Eq. (20), \( \xi \) could be approximated as proportional to the ratio of \( RN \) to \( CN \). Hence, \( \xi_{valv} \) for a given value of \( RN \) and \( CN \) may be determined as follows:

\[
\xi_{valv} = \frac{\xi_{valv}^0 \times RN_{valv}^0 \times CN}{RN \times CN_{valv}^0}
\]

where \( \xi_{valv}^0 \) is the reference valve value of \( \xi \) for a given value of \( RN_{valv}^0 \) and \( CN_{valv}^0 \) (dimensionless). The determination of \( \xi_{valv}^0 \) is given later.
According to Eq. (20), \( \bar{z} \) corresponding to \( \xi_{valv} \) can be determined as follows:

\[
\bar{z} \left|_{\xi_{valv}} = CN \ln(1 + \frac{\xi_{valv}}{RN}) \right. 
\]

(22)

Hence, when \( \bar{z} \leq \bar{z} \left|_{\xi_{valv}} \right. \), the effect of root water uptake is negligible.

Based on Eq. (22), the normalized influence depth of root water uptake by root depth can be determined as follows:

\[
RIF = \frac{1 - \bar{z}}{r} = \frac{1 - CN \ln(1 + \frac{\xi_{valv}}{RN})}{r} 
\]

(23)

**Introductory analysis for parametric study**

**Steady state**

With the defined dimensionless numbers (Eqs. (10) to (12)), the normalized governing equation (Eq. (9)) at steady state is rearranged as follows:

\[
CN \frac{\partial^2 k_r}{\partial z^2} + \frac{\partial k_r}{\partial z} - RN \frac{1}{r} H(z' - L_1) = 0 
\]

(24)

Hence, for a given root architecture (i.e., the uniform root) and root depth ratio (i.e., \( r \)), the variations in steady-state PWP distributions due to root water uptake depend on \( RN \) and \( CN \) only.
**Transient state**

With the defined dimensionless numbers, the normalized governing equation (Eq. (9)) in vegetated slope with the uniform root architecture at transient state can be rewritten as:

\[
CN \frac{\partial^2 k_r}{\partial z^2} + \frac{\partial k_r}{\partial z} - RN \frac{1}{r} H(z' - L_s) = \frac{\partial k_r}{\partial WR}
\]  

(25)

It can be deduced from Eq. (25) that, for a given root architecture (i.e., the uniform root) and a root depth ratio \( r \), the PWP distributions in the vegetated slope at the transient state depend on \( CN \), \( RN \), and \( WR \).

**Analytical scheme of parametric study**

An infinite vegetated slope is shown in Fig. 1. The slope angle is 40° and the uniform root is considered. The vertical depth of root is 0.5 m (\( L_s \)). The water table is located at the bottom of the slope (i.e., \( H_0 = 5 \) m). Table 1 shows the parametric analyses. Since \( CN \) is affected by the desaturation coefficient \( \alpha \) and the slope angle \( \beta \) for a fixed slope thickness \( L' \) (Eq. (10)), the first and second series of analytical experiments aim to investigate the effects of \( CN \), which is varied by controlling the desaturation coefficient \( \alpha \) and the slope angle \( \beta \) respectively. The third and fourth series of analytical experiments aim to study the effects of \( RN \), which is controlled by \( k_s \) and \( T_p \) respectively. The fifth series of analytical experiment is designed for studying effects of \( WR \) on PWP response under rainfall at both transient and steady state with varied \( CN \) (0.7 and 0.3 in series 5 in Table 1).
Analysis of the calculated results

In the following discussion, the calculated results of the parametric study are presented in terms of the normalized PWP by hydrostatic pressure. The related PWP values can be found in Figs. S1-S4 in the supplementary document.

Effects of CN on PWP distributions

Fig. 2(a) shows the normalized PWP profiles in the vegetated slope with CN controlled by the desaturation coefficient \( \alpha \) (series 1 in Table 1). The effects of root water uptake on normalized PWP becomes more significant for a smaller CN (i.e., \( \alpha \) increases). This is attributed to that for a constant \( k_s \), unsaturated water permeability becomes lower for a larger \( \alpha \) at a given negative PWP. For a given amount of root water uptake (i.e., the same \( RN \)), according to Darcy’s law, the lower the water permeability, the larger hydraulic gradient is needed to generate the same water flux equaling root water uptake at steady state. Hence the soil with a larger \( \alpha \) has a larger negative PWP.

Fig. 2(b) shows the effects of CN, controlled by the slope angle \( \beta \) (series 2 in Table 1), on the normalized PWP distributions in the vegetated slope. The normalized PWP reduces as the slope angle increases, since the overall water flux outside root zone is upward and equals the applied root water uptake (i.e., \( T_p \)) based on mass balance. As the water flux caused by gravity (i.e.,\[
\alpha \cos \beta \frac{\partial k}{\partial z}
\] in Eq. (1)) is downward and increases as the slope angle decreases, a larger PWP gradient is needed to generate the same overall upward water flux following Darcy’s law. Although the case of the slope angle of 60° has the largest \( RN \) of 0.04 (see series 2 in Table 1), its negative
PWP is the lowest. It is because $CN$ for the slope angle of 60° is the largest, leading to more significant effects of water flow driven by PWP gradient ($CN \frac{\partial^2 k}{\partial z^2}$ in Eq. (24)) than that of root water uptake (i.e., $RN \frac{1}{r} H \left(z - L \right)$ in Eq. (24)). As a result, the negative PWP reduces.

These results illustrate that, the negative PWP induced by root water uptake becomes larger for a smaller $CN$ (i.e., by increasing $\alpha$ or decreasing slope angle $\beta$) with a given $RN$. In practice, $\alpha$ becomes larger for a given type of soil with more uniform distribution of grain size (Zhan and Ng 2004). Moreover, influence of root water uptake on PWP distributions and thus slope stability is more prominent in gentle slope, such as sloping landfill cover, where the maximum slope angle is 18° based on design guideline (CAE 2000; Feng et al. 2017).

**Effects of RN on PWP distributions**

Fig. 3 shows the influence of $RN$, controlled by the saturated permeability (series 3 in Table 1), on the steady-state PWP distributions in the vegetated slope. Generally, a larger negative PWP induced by root water uptake is observed for a larger $RN$ (i.e., a lower $k_s$). For the case with the largest $RN$ of 0.3, the negative PWP at the surface of vegetated slope is more than twice the hydrostatic PWP. In contrast, for the case with $RN$ of 0.03, the negative PWP induced by vegetation is slightly larger than hydrostatic PWP. When $RN$ further reduces to 0.003, effects of root water uptake are almost negligible. The reason is that under a given root water uptake (i.e., the same $T_p$), a larger hydraulic gradient is needed to induce the same water flux for a lower water permeability.
Fig. 4(a) shows that for a given $CN$, the value of $RN$ (controlled by $T_p/k_s$; series 4a in Table 1) determines the PWP distributions induced by root water uptake. As expected, both the negative PWP and the influence zone of root water uptake increase as $RN$ increases from 0.01 to 0.1. For the two cases with different values of $T_p$ and $k_s$, but the same value of $T_p/k_s$ (the cases with identical $RN$ of 0.1, but varied $k_s$ of $0.2 \times 10^{-6}$ m/s and $2 \times 10^{-6}$ m/s respectively), the negative PWP distributions are the same. This is expected since these two cases have the same $CN$ (i.e., the same $\alpha$) and $RN$, leading to the same governing equation (Eq. (24)). Hence the calculated results are the same.

Fig. 4(b) shows the effects of $RN$ (controlled by $T_p/k_s$; series 4b in Table 1) on the PWP distributions with $CN$ of 0.5. Similar trend can be observed as that in Fig. 4(a). But the induced negative PWPs are larger than that in Fig. 4(a) for each case with the same $RN$. This is due to a reduction of water flow driven by PWP gradient (i.e., $CN \frac{\partial^2 k_s}{\partial z^2}$ in Eq. (24)) as $CN$ decreases. As a result, a larger PWP gradient is needed to compensate the same amount of root water uptake at a given $RN$, leading to a larger negative PWP.

These results suggest that soils with a relative small $k_s$ would help amplify the impact of vegetation on PWP distribution for a given root architecture (i.e., uniform root) and root depth. In practice, increasing the degree of compaction (DOC) would decreases $k_s$, leading to larger
negative PWP and thus enhanced slope stability. However, the vegetation growth would be constrained as DOC increases (Unger and Kaspar 1994; Kozlowski 1999). Hence, the engineers are supposed to strike a balance between these two opposing effects to select a proper DOC in man-made slope to exploit the benefits of vegetation on enhancing shallow slope stability.

**Effects of WR on PWP distributions**

Figs. 5(a) and 5(b) shows effects of WR on PWP distributions during rainfall with a constant CN of 0.7 (cases 1a to 2d in series 5 in Table 1) for (θ_s−θ_r) of 0.2 and 0.4, respectively. The initial PWP distributions for (θ_s−θ_r) of 0.2 (Fig. 5(a)) and 0.4 (Fig. 5(b)) are the same as expected, since the initial conditions are based on PWP profiles at drying steady state and are independent of (θ_s−θ_r) (see Eq. (24)). For the case with (θ_s−θ_r) of 0.2 after 12 hours of rainfall (Fig. 5(a)) and that with (θ_s−θ_r) of 0.4 after 24 hours of rainfall (Fig. 5(b)), WR is the same as 0.09. The PWP profiles for these two cases are identical, which is expected due to the same CN, RN (i.e., RN=0 during rainfall) and WR, leading to the same governing equation (see Eq. (25)). After 24 hours of rainfall, compared with the case with (θ_s−θ_r) of 0.4 (Fig. 5(b)), the one with (θ_s−θ_r) of 0.2 (Fig. 5(a)) has deeper wetting front and lower negative PWP. It is because the latter case has a lower water storage capacity (i.e., a lower value of (θ_s−θ_r)). Hence more water flow through soil compared with stored water, leading to deeper wetting front. These results are consistent with the Green and Ampt’s model (1911), from which it can be deduced that wetting front advances more quickly as water storage capacity decreases. After 300 hours of rainfall, WR reaches 2 (Fig. 5(a)) and 1 (Fig. 5(b)) for the cases with (θ_s−θ_r) of 0.2 and 0.4, respectively. The PWP profiles of these two cases are close to the wetting steady-state results. It should be noted that since steady-state PWP profile is
independent of water storage capacity (Eq. (24)), the wetting steady state lines for both \((\theta_s-\theta_r)\) of 0.2 and 0.4 are the same. It may conclude that when \(WR\) is larger than certain value (i.e., 1), the PWP profile nearly reaches the steady state, hence the PWP profile could be estimated by steady state analysis rather than transient analysis for saving time.

Figs. 5(c) and 5(d) shows the PWP response during rainfall with \(CN\) of 0.3 for \((\theta_s-\theta_r)\) of 0.2 and 0.4, respectively. With the same \(WR\), Fig. 5(c) shows shallower wetting front and larger negative PWP reduction, comparing with Fig. 5(a). It is attributed to that \(CN\) of Fig. 5(c) is lower than that of Fig. 5(a), indicating lower water flow driven by PWP gradient. Hence with the same \(WR\) and water storage capacity, more water is stored under the same rainfall infiltration with lower \(CN\). Correspondingly, a larger reduction of negative PWP and a shallower wetting front can be found. Similar trend can be observed by comparing Fig. 5(d) with Fig. 5(b). These results illustrate that effects of \(WR\) on PWP response during transient state mainly depends on \(CN\). Under a given \(CN\), the wetting front advances deeper and the normalized PWP reduces more, for a larger \(WR\) during rainfall. As \(CN\) reduces with a given \(WR\), the wetting front moves shallower, but the reduction in negative PWP becomes larger due to reduced water transfer in soils. Moreover, the wetting steady state line is governed by \(CN\). The PWP profile at wetting steady state become steeper for a smaller \(CN\). This is due to water flow in the soil is the same as the applied rainfall infiltration at the wetting steady state. As \(CN\) decreases, the influence of PWP gradient on water flow becomes less prominent (Eq. (24)), leading to more significant effects of water flow driven by gravity. Hence, PWP gradient reduces (i.e., PWP profile becomes steeper).
Dimensionless relationships for estimating PWP and root influence zone in vegetated slope

*Dimensionless relationship of negative PWP induced by root water uptake*

For a given root architecture, the influence of root water uptake on PWP depends on $RN$, $CN$ and $r$ at steady state (Eq. (24)). The combined effects of $RN$, $CN$ and $r$ on PWP distributions may be described by $\eta$ (Eq. (18)). The effects of root water uptake on PWP are generally more significant for a larger $\eta$. For example, $\eta$ decreases with increasing $r$ (i.e., a deeper root depth) or $CN$, leading to less significant effects of root water uptake on PWP (see Fig. 2(a)). Meanwhile, as both $RN$ and $CN$ are affected by the slope angle $\beta$, $\eta$ becomes smaller for a larger slope angle $\beta$. Thereafter, the influence of root water uptake on PWP becomes less prominent (see Fig. 2(b)). Moreover, $\eta$ increases with $RN$ (i.e., either a larger $T_p$ or a smaller $k_s \cos \beta$), leading to more significant effects of root water uptake on PWP (see Fig. 4).

Fig. 6 shows the influence of $\eta$ on the calculated normalized PWP at the surface and root depth of the vegetated slope. The PWP profiles at drying steady state are adopted as the calculated results (i.e., Figs 2 to 5). Generally, the normalized negative PWP increases with $\eta$, due to more significant effect of root water uptake on PWP profiles. Quadratic functions are found to fit well relationships between $\eta$ and the normalized negative PWP. The fitted functions maybe useful for preliminary estimation of the negative PWP (i.e., based on Eq. (16): $\psi_r = \psi \psi_b$) induced by root water uptake with different weather conditions (i.e., $T_p$), soil properties and root characteristic (i.e., root depth).
Dimensionless relationship of influence zone of root water uptake

Both PWP profiles inside and outside root zone are affected by root water uptake. It would be useful to estimate root influence zone based on the proposed dimensionless relationship in Eq. (23). Fig. 7 (a) shows the PWP profiles for the uniform rooted slope with \( RN \) of 0.03 (the second and third cases in series 1 in Table 1). Note that hydrostatic pressure is the same for \( CN \) of 0.7 and 0.3 for bare soil, since the hydrostatic pressure at the depth of \( z \) is determined as \( \gamma_w z \cos \beta \) (where \( \gamma_w \) is the unit weight of water (\( N/m^3 \); Ng et al., 2015)). The root influence zone is determined as the depth where the difference of PWP between vegetated soil and bare soil (hydrostatic pressure) is more than 1kPa. For \( CN \) of 0.7 and 0.3, the root influence zone is determined as three and six times of root depth, respectively. Fig. 7(b) shows the determination of root influence zone by \( \xi \) (Eq. (20)). \( \xi \) increases near slope surface. Based on the root influence zone determined in Fig. 7(a), the valve value of \( \xi \) is determined as 0.05 (i.e., Eq. (21), \( \xi_{valv}^0 = 0.05, RN_{valv}^0 = 0.03, CN_{valv}^0 = 0.3 \)), below which effects of root water uptake is negligible. Hence, the root influence zone for a given \( RN \) and \( CN \) could be determined by Eq. (23).

Validation of the dimensionless relationships for estimating PWP and root influence zone

The proposed relationships in Fig. 6 for estimating negative PWP and the dimensionless relationship of influence zone of root water uptake (Eq. (23)) were validated by measurements of two field tests (Ni et al. 2018; Woon 2013) and two sets of centrifuge tests (Ng et al. 2016a; 2017). The selected field and centrifuge tests are the rare cases that provide all the necessary information.
for both plants and soils. The same soil type (completely decomposed granite (CDG), classified as silty sand) was adopted in all the selected cases. *Schefflera heptaphylla* and *Cynodon dactylon* were adopted in the field tests conducted by Ni et al. (2018) and Woon (2013), respectively. Both field tests consist of drying period (i.e., no rainfall). During testing, matric suction from depths of 0.01–0.5 m was monitored. The measured root architectures of *Schefflera heptaphylla* and *Cynodon dactylon* can be approximated as the parabolic root (i.e., the root density takes a parabolic form with depth, with a maxima between the ground surface and root tip)) and the uniform root, respectively. According to Ng et al. (2015), the PWP distributions induced by the parabolic and the uniform roots are similar, hence the uniform root was adopted for validation of the proposed relationships for root influence zone and PWP.

In the two sets of centrifuge tests by Ng et al. (2016a) and Ng et al. (2017), an artificial root system and real branch cuttings were used respectively, for simulating the influence of root water uptake on PWP in the centrifuge tests. In these two centrifuge tests, tap root was adopted, which consists of one primary vertical root with several branched secondary roots near its tip. Hence, it can be approximated as the uniform root (Ng et al. 2016a). Both centrifuge tests started with a simulation of transpiration until there was negligible change of negative PWP profiles along depth.

As for the input parameters in Eqs. (4) and (5), measured SWRCs reported by Ni et al. (2018) and Ng et al. (2016a; 2017) were adopted for the respective cases. The SWRC measured by Leung et al. (2015) was adopted for the selected case by Woon (2013). It is noted that these two cases used the same soil type (i.e., CDG) and similar DOC around 90%. Hence, the measured SWRC by
Leung et al. (2015) could be a reasonable approximation for the selected case by Woon (2013).

Fig. 8(a) shows the measured SWRCs of the soil used for the selected cases. By fitting the measured SWRCs based on Eq. (5), the parameters of $\theta_s$, $\theta_r$, and $\alpha$ were calibrated as shown in Table 2. By using these calibrated parameters and the measured $k_s$ reported in each selected case, unsaturated permeability functions were estimated by Eq. (4) and are shown in Fig. 8(b). The simulated transpiration rate of 1 mm/day as measured by Ng et al. (2016a) was used for the two centrifuge tests. In the field test reported by Ni et al. (2018), the measured transpiration rate was 4.8 mm/day. The transpiration rate for the field test of Woon (2013) was estimated as 3 mm/day, based on the measured evapotranspiration rate by Leung and Ng (2013). Note that Leung and Ng (2013) and Woon (2013) conducted field test in the similar period (i.e., June) and location (i.e., Hong Kong). Thus, the measured transpiration rate by Leung and Ng (2013) was adopted as a reasonable approximation. The measured slope angle, water table depth and root depth for each case are summarized in Table 2.

Fig. 9 shows the comparisons between the measured and the estimated PWP using the relationship between $\eta$ and the normalized PWP (Fig. 6). Generally, the estimated values of PWP are consistent with the measurements. The maximum difference between measured and estimated PWP is about 18 kPa, corresponding to the field test conducted by Woon (2013). This could be probably attributed to the ignorance of the effects of roots on soil hydraulic properties (i.e., water permeability; Ni et al. 2018; Liu et al. 2018). The analytical parametric studies carried out by Liu et al. (2018) show that PWP would be underestimated during drying period, when ignoring the reduction of water permeability due to the occupancy of soil pores by root. For the selected...
centrifuge test conducted by Ng et al. (2017), the PWP is underestimated by the proposed method by about 17 kPa. This is likely due to the steady state assumption made in the proposed relationships. Hence, the amount of root water uptake was overestimated, leading to an underestimation of PWP. As for the field test of Ni et al. (2018), the estimated PWP is larger than the measurement by about 10 kPa at the soil surface. This indicates that the amount of root water uptake near soil surface would be underestimated, probably due to the assumption of uniform root architecture and hence constant transpiration along root depth. The results demonstrate that the proposed relationships in Fig. 6 can predict PWP distributions in vegetated soil reasonably.

Fig. 10 shows the comparisons between the measured and the calculated influence depths of root water uptake using RIF (Eq. (23)). Generally, the measured and calculated results are consistent. The maximum difference between measured and calculated root influence depths is about 1 m, corresponding to the field test conducted by Woon (2013). This is probably due to the steady state assumption adopted in the proposed method, leading to deeper drying front propagation and hence larger root influence depth. The results illustrate that the proposed RIF (Eq. (23)) can predict root influence depth reasonably.

**Summary and conclusions**

Dimensional analysis of PWP distributions in a vegetated infinite slope was conducted. Three dimensionless numbers were proposed, including (i) capillary effect number \( CN \); Eq. (10), representing the relative importance of water flow driven by PWP gradient over that driven by gravity; (ii) water transfer-storage ratio \( WR \); Eq. (11), representing the relative importance of
water transfer over water storage at transient state; and (iii) root water uptake number ($RN$; Eq. (12)), describing the relative importance of root water uptake (i.e., $T_p$) over water flow. Then dimensionless relationships were proposed for estimating PWP and root influence zone in vegetated slope. Moreover, the effects of the newly proposed dimensionless numbers on PWP distributions in a vegetated infinite slope were investigated by conducting parametric studies. Finally, the proposed dimensionless relationships for estimating PWP and root influence zone were validated against published field and centrifuge tests.

At drying steady state, PWP profiles in a vegetated infinite slope are governed by $CN$ and $RN$ only for given boundary conditions. When $RN$ is less than 0.02, effects of root water uptake on PWP distributions might be ignored. The combined effects of $CN$ and $RN$ on PWP distributions and root influence zone could be described by $\eta$ (Eq. (18)) and $\xi$ (Eq. (20)), respectively. Generally, the negative PWP induced by root water uptake increases with $\eta$. As $\xi$ increases, the root influence zone becomes deeper.

At transient state, PWP response in a vegetated infinite slope depends on $CN$, $RN$ and $WR$. During transient wetting period (i.e., rainfall), effects of $RN$ are negligible due to high humidity and thus low transpiration. Under such case and given boundary conditions, PWP response depends on $WR$ and $CN$ only. For a given $CN$ and the same initial condition, the larger the $WR$, the deeper the wetting front moves and more reduction of negative PWP occurs. Generally, when $WR$ is larger than 1, PWP profile is close to the wetting steady-state results. Hence, the PWP may be predicted by the steady state simulation for saving time. For a given $WR$, as $CN$ decreases, the wetting front
moves shallower but more negative PWP reduction occurs, due to less water flow and more water
storage in soils.

Compared with measured field and centrifuge tests data, the proposed dimensionless relationships
are capable of determining PWP and root influence zone in vegetated soil reasonably. The steady
state assumption is made in the proposed relationships. It is noted that steady state conditions
seldom exist in the field, due to the variation in the climate condition. Nevertheless, the proposed
relationships could be useful (i) for estimating the potential range of negative PWP and depth of
root influence zone under drying condition in the field; (ii) for evaluating the time needed to reach
steady state condition; and (iii) for designing centrifuge tests where long periods of uniform
climate could be usually imposed. Hence, these proposed dimensionless relationships provide a
simple and valid method for estimating PWP distributions in vegetated slope.

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sincere gratitude to the two anonymous reviewers for their constructive comments and suggestions.
**Notation**

583  **CN**  Capillary effect number (*dimensionless*)

585  **CN**\(^0\)\(_{valv}\)  **CN** value corresponding to \(\xi^0_{valv}\) (*dimensionless*)

586  **DOC**  degree of compaction (%)  

587  **g(\(z\))**  the shape function describing root architecture (*m\(^{-1}\]*)

588  \(H(\(z' - L'\))\)  \(H(\(z' - L'\)) = \begin{cases} 1 & L'_1 \leq z' \leq (L'_1 + L'_2) \\ 0 & 0 \leq z' \leq L'_1 \end{cases}\)  inside root zone  outside root zone  (*dimensionless*)

589  **k**  the water permeability of soil (*m\( s\)\(^{-1}\))

590  **k\(_r\)**  the relative water permeability (*dimensionless*)

591  **k\(_s\)**  the saturated water permeability (*m\( s\)\(^{-1}\))

592  \(k|_{z=0}\)  the water permeability at the slope bottom (*m\( s\)\(^{-1}\))

593  \(k_{r}|_{z=L'}\)  the relative permeability at slope surface (*dimensionless*)

594  \(k_{r,veg}|_{z=L}\)  the relative permeability induced by root water uptake at slope surface (*dimensionless*)

596  \(L'_1\)  the thickness of the domain outside root zone (Fig. 1; *m*)

597  \(L'_2\)  the thickness of root zone (Fig. 1; *m*)

598  \(L\)  the thickness of slope (i.e., \(L = L'_1 + L'_2\); *m*; see Fig. 1)
$q_0$ rainfall (positive value) or evaporation (negative value) at slope surface ($m/s$)

$RN$ Root water uptake number ($\text{dimensionless}$)

$RN_{valv}^0$ $RN$ value corresponding to $\xi_{valv}^0$ ($\text{dimensionless}$)

$RIF$ the normalized influence depth of root water uptake by root depth ($\text{dimensionless}$)

$r$ root depth ratio ($r = \frac{L_r}{L}$) ($\text{dimensionless}$)

$SWRC$ soil water retention curve

$T_p$ the transpiration rate ($m/s$)

$t$ time (s)

$WR$ Water transfer-storage ratio ($\text{dimensionless}$)

$z'$ the perpendicular distance to the slope bottom ($m$; see Fig. 1)

$z$ the normalized $z'$ by $L$ ($\text{dimensionless}$)

$z|_{\xi_{valv}}$ the normalized $z$ corresponding to $\xi_{valv}$ ($\text{dimensionless}$)

$\alpha$ the desaturation coefficient of soil ($m^{-1}$)

$\beta$ the slope angle (degree)

$\gamma_w$ the unit weight of water ($N/m^3$)
\[ \eta = RN \frac{\exp(CN^{-1})}{1 + CN^{-1} \times r} \]

\[ \theta_s \] the saturated volumetric water content (dimensionless)

\[ \theta_r \] the residual volumetric water content (dimensionless)

\[ \xi = \exp \left( \frac{-z}{CN} \right) - 1 \] (dimensionless)

\[ \xi_{\text{valv}} \] the valve value of \( \xi \) at \( RN \) and \( CN \) (dimensionless)

\[ \xi^0_{\text{valv}} \] the reference valve value of \( \xi \) at \( RN^0_{\text{valv}} \) and \( CN^0_{\text{valv}} \) (dimensionless)

\[ \psi \] PWP head (m)

\[ \psi \Big|_{z=L'} \] the steady-state PWP head at slope surface (dimensionless)

\[ \bar{\psi} \] the normalized PWP by hydrostatic pressure (dimensionless)

\[ \psi_r \] PWP head in vegetated slope (m)

\[ \psi_b \] hydrostatic PWP head in an infinite slope (m)

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Fitted function at slope surface

$$\bar{\psi} = 0.9942\eta^2 + 0.0448\eta + 1.0434$$

$R^2 = 0.991$

Fitted function at root depth

$$\bar{\psi} = 0.7219\eta^2 + 0.1346\eta + 1.0362$$

$R^2 = 0.9554$
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### Table 1 Summary of parametric studies

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Note: (a) The perpendicular depth of the slope ($L'$) is 3.8 m for different slope angles. (b) The initial condition before rainfall is obtained as pore-water pressure profiles at drying steady state with $RN=0.03$. During rainfall, root water uptake is ignored (i.e., $RN=0$). To simplify the problem, 40% of rainfall is assumed as surface runoff (Ng et al. 2015).
Table 2 Summary of input parameters for method validation

| Case                  | slope angle (degree) | water table depth (m) | root depth (m) | \(T_p\) (mm/day) | \(\alpha\) (m\(^{-1}\)) | \(k_s\) (m/s) | \(\theta_s\) | \(\theta_r\) |
|-----------------------|----------------------|-----------------------|----------------|-------------------|-----------------|--------------|--------------|
| Ng et al. (2016)a     | 45                   | 3                     | 0.75           | 1                 | 1               | 1.00×10\(^{-7}\) | 0.41         | 0.15         |
| Ng et al. (2017)b     | 40                   | 3.5                   | 1              | 1                 | 1               | 1.00×10\(^{-7}\) | 0.36         | 0.14         |
| Ni et al. (2018)c     | 0                    | 4.5                   | 0.2            | 4.8               | 0.7             | 1.22×10\(^{-6}\) | 0.36         | 0.15         |
| Woon (2013)d          | 0                    | 2.7                   | 0.07           | 3                 | 0.7             | 1.50×10\(^{-7}\) | 0.41         | 0.15         |

Note: (a) Based on test results of measured pore-water pressure along depth after simulated transpiration for slope models supported by tap root (uniform root).

(b) Based on measured pore-water pressure along depth after 1st drying by the simulated transpiration.

(c) Based on measured pore-water pressure along depth before ponding for the field studies reported by Ng et al. (2016b).

(d) Based on measured pore-water pressure profiles for grassed plots (G100-F) after 6 days of drying from 3-June to 9-June 2012 (see Fig. 4.5 in Woon (2013)).
Appendix A1

When $CN$ is small, $\frac{L_2}{\exp(\frac{1}{CN})}$ in Eq. (15) is much smaller than the other two terms. Hence Eq. (15) can be further simplified as:

$$k_{r,\text{veg}} \big|_{z=L} = \frac{RN}{L_2} \left[ -\frac{1}{\alpha \cos \beta} + \frac{1}{\alpha \cos \beta \exp\left(\frac{1}{CN} \times \frac{L_2}{L}\right)} \right]$$

$$= \frac{RN}{L_2} \left[ -\frac{1}{\alpha \cos \beta} + \frac{1}{\alpha \cos \beta \exp(\alpha \cos \beta L_2)} \right]$$  \hspace{1cm} (A1)

Using Taylor expansion

$$\frac{1}{\exp(\alpha \cos \beta L_2)} \approx \frac{1}{1 + \alpha \cos \beta L_2}$$  \hspace{1cm} (A2)

Substituting Eq. (A2) into Eq. (A1) yields:

$$k_{r,\text{veg}} \big|_{z=L} \approx \frac{RN}{L_2} \left[ -\frac{1}{\alpha \cos \beta} + \frac{1}{\alpha \cos \beta (1 + \alpha \cos \beta L_2)} \right]$$

$$= -\frac{RN}{(1 + \frac{r}{CN})}$$  \hspace{1cm} (A3)

The influence of root water uptake on PWP profiles may be evaluated by comparing with the effects of water table (i.e., $\exp(-CN^{-1})$ in Eq. (14)):
\[ \eta = RN \frac{1}{(1 + CN^{-1}r)} \exp(-CN^{-1}) \]
\[ = RN \frac{\exp(CN^{-1})}{(1 + CN^{-1}r)} \]