Canadian Journal of Physics

Analysis of the Fusion Cross Sections for $^{16,18,20}\text{O} + ^{12}\text{C}$ Systems at Low Energies

Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2018-0665.R1
Manuscript Type:	Article
Date Submitted by the Author:	22-Nov-2018
Complete List of Authors:	Kocak, G.; Erciyes Universitesi, Physics
Keyword:	Optical model, nucleon-nucleon double folding, fusion cross-section, neutron-rich reactions, neutron stars
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

SCHOLARONE™
Manuscripts

Analysis of the Fusion Cross Sections for $^{16,18,20}\text{O}+^{12}\text{C}$ Systems at Low Energies

G. Kocak

*Department of Physics, Science Faculty,
Erciyes University, 38039, Kayseri, Turkey*

(Dated: December 6, 2018)

Abstract

Fusion cross sections data for the $^{16,18,20}\text{O}+^{12}\text{C}$ systems at energies near and below the Coulomb barrier are studied within the framework of the optical model. In order to examine these reactions, the microscopic nucleon-nucleon double folding potentials for real and imaginary parts is performed. To make a comprehensive analysis of the effect of neutron rich systems, we used three different reactions from stable to unstable systems. The microscopic nucleon-nucleon double folding potentials show very good agreement for $^{16,18,20}\text{O}+^{12}\text{C}$ systems with a very weak imaginary potential. Also, we have obtained better agreement by using fully-microscopic nucleon-nucleon double folding potentials with the $^{20}\text{O}+^{12}\text{C}$ systems experimental data than in previous works but especially at low energies we have still some problems to reproduce the $^{20}\text{O}+^{12}\text{C}$ systems fusion cross section data.

PACS numbers: 24.10.Ht; 24.50.+g; 25.70.Jj; 26.60.Gj

Keywords: Optical model; nucleon-nucleon double folding; fusion cross-section; neutron-rich reactions.

I. INTRODUCTION

Fusion reactions with stable as well as unstable systems at energies near and below the Coulomb barrier have attracted great attention both experimentally and theoretically for numerous systems to understand the origin of elements. For this purpose, the nucleosynthesis in the stellar region or in the early universe era is an area of great interest to scientists. Also, neutron stars are a unique environment for nuclear reactions in a broad range. The outer crust of accreting neutron stars is a fascinating research area the fusion reactions by neutron-rich nuclei [1–8]. The results of fusion reactions with neutron-rich systems have become very important for the astrophysical environment and it has been suggested that the fusion between neutron-rich nuclei could be a heat source for thermonuclear $^{12}\text{C}+^{12}\text{C}$ reactions and produce an x-ray superbust [2]. However, experimental data or theoretical studies up to now have been insufficient.

Recently, the enhancement of fusion probability for neutron-rich nuclei has been reported as compared to stable systems at sub-barrier energies [6, 8, 9]. A few years ago Rudolph *et al.* [8] measured the $^{20}\text{O}+^{12}\text{C}$ systems fusion data for the first time by using the evaporation residues technique and reported significant underprediction between the experimental data and theoretical results which were obtained from statistical decay code model. In an another attempt [10], the microscopic density-constrained time-dependent Hartree-Fock (DC-TDHF) theory was used to explain the $^{20}\text{O}+^{12}\text{C}$ neutron-rich system. However, the DC-TDHF model fails to explain $^{20}\text{O}+^{12}\text{C}$ reaction at sub-barrier energies and the authors suggested that the neutron transfer channels are become much more important for neutron-rich systems at sub-barrier energies. Also, the fusion cross section for the $^{18}\text{O}+^{12}\text{C}$ system has been examined with a DC-TDHF model [11]. They concluded that the larger cross sections at low energies come from larger tunneling probabilities. This may be associated with smaller and narrower barrier than with the theoretical predictions.

In this study, to investigate the effect of the neutron-rich systems on the reaction observables we used three different reactions from stable to unstable systems and made a comprehensive analysis by using a realistic microscopic nucleon-nucleon Double Folding (NN-DF) model for the $^{16,18,20}\text{O}+^{12}\text{C}$ systems. In our analysis, we used only the direct part of the microscopic potential. It could also be made using both the direct part and the exchange part as given in Ref. [12–14].

Firstly, we begin with the well-known $^{16}\text{O}+^{12}\text{C}$ systems. We aim to explain the fusion cross sections and angular distribution for the $^{16}\text{O}+^{12}\text{C}$ system by using microscopic NN-DF model within the framework of the optical model, simultaneously. As a next step, we tried to explain the $^{18}\text{O}+^{12}\text{C}$ systems and obtained very good agreement between the theoretical and experimental results. Finally, we have analyzed $^{20}\text{O}+^{12}\text{C}$ system to understand the behavior of unstable systems. In our previous work [15], we presented our preliminary results by using semi-microscopic NN-DF. In order to examine this reaction, the microscopic nucleon-nucleon double folding potentials with Woods-Saxon type imaginary parts is used. Unfortunately, we could not explain the $^{20}\text{O}+^{12}\text{C}$ experimental data. In this study, we have performed the fully-microscopic NN-DF approach with a weak shallow imaginary potential and obtained very good results for the $^{20}\text{O}+^{12}\text{C}$ experimental data especially at low energy region with very simple and effective way.

In the next section, we introduce our optical model and our microscopic potentials were used to analyze the experimental data of the $^{16,18,20}\text{O}+^{12}\text{C}$ system. Then, we show the results of these analyses in Section III. Section IV is devoted to our summary and conclusion.

II. THE OPTICAL MODEL

In the optical model, the total potential $V_{\text{total}}(r)$ consists of

$$V_{\text{total}}(r) = V_{\text{Nuclear}}(r) + V_{\text{Coulomb}}(r) + V_{\text{Centrifugal}}(r) \quad (1)$$

Coulomb and centrifugal potentials are well-known. The Coulomb potential [16] due to a charge $Z_P e$ interacting with a charge $Z_T e$ distributed uniformly over a sphere of radius R_c is given by

$$V_{\text{Coulomb}}(r) = \frac{1}{4\pi\epsilon_0} \frac{Z_P Z_T e^2}{r}, \quad r \geq R_c, \quad (2)$$

$$= \frac{1}{4\pi\epsilon_0} \frac{Z_P Z_T e^2}{2R_c} \left(3 - \frac{r^2}{R_c^2} \right), \quad r < R_c, \quad (3)$$

where R_c is the Coulomb radius, and Z_P and Z_T denote the charges of the projectile P and the target nuclei T , respectively. The centrifugal potential is

$$V_{\text{Centrifugal}}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2} \quad (4)$$

where μ is the reduced mass of the colliding pair.

In order to make a comparative study, we use three different reactions by using the NN-DF for the real part of the optical model potential. We provide the details of nuclear potentials in the following sections.

A. Nucleon-Nucleon Double Folding Potential (NN-DF)

In the optical model, the nuclear potential consists of a real and imaginary part. In order to obtain the real part of the nuclear potential, we use the NN-DF potential evaluated by using the nuclear matter distributions for projectile ρ_P and target ρ_T nuclei with an effective NN interaction potential (ν_{nn}) is given by

$$V_{DF}(\vec{r}) = \int \int \rho_P(\vec{r}_1) \rho_T(\vec{r}_2) \nu_{nn}(|\vec{r} + \vec{r}_2 - \vec{r}_1|) d\vec{r}_1 d\vec{r}_2 \quad (5)$$

In our NN-DF calculations, we used the Gaussian form for the ^{12}C and ^{16}O ground state matter density distributions,

$$\rho_M(r) = \rho_0(1 + wr^2) \exp(-\beta r^2), \quad (6)$$

The parameters of the density distributions are shown in Table I. The density distributions data for the ^{18}O and ^{20}O nuclei were taken from the Hartree-Fock-Bogolubov method based on the BSk2 Skyrme force in Ref. [19].

The Michigan 3 Yukawa (M3Y-NN) effective interaction form is taken as

$$\nu_{nn}(r) = 7999 \frac{\exp(-4r)}{4r} - 2134 \frac{\exp(-2.5r)}{2.5r} + J_{00}(E) \delta(r) (\text{MeV}), \quad (7)$$

where $J_{00}(E)$ is the exchange term and changes with energy expressed as

$$J_{00}(E) = 276[1 - 0.005E/A_p] \text{MeV fm}^3 \quad (8)$$

The NN-DF potential is related to the real part of the nuclear potential as $V_{\text{Real}}(r) = N_R V_{DF}(r)$, where N_R is a normalization factor. Therefore, the nuclear potential consists of a real and imaginary part, as in Equation 9. The imaginary part of the optical potential in the microscopic approach could be taken within the high energy approximation or proportionally the real part of the nuclear potential as can be seen in Ref. [20, 21]. In this study, we have

chosen the imaginary part of the nuclear potential as the proportional to the real part of the nuclear potential.

$$V_{\text{Nuclear}}(r) = V_{DF}(r)(N_R + iN_I). \quad (9)$$

where N_I is a normalization factor for the imaginary part of the potential.

The parameters of the real and imaginary potentials are shown in Table II. The Coulomb potential [22, 23] is distributed uniformly over a sphere of radius with $R_C = r_C(A_P^{1/3} + A_T^{1/3})$ with $r_C = 1.2$ fm. The code DFPOT [24] was used for the microscopic DF potential calculation and the code FRESCO [25] was used to obtain all reaction observables.

B. Fusion Cross Sections

The fusion reactions are very important for stellar energy production and nucleosynthesis and provide new insights into reaction dynamics and nuclear structure. The optical model and coupled-channels methods have been widely used for the fusion cross-section calculations. Fusion cross sections are usually calculated either using an incoming wave boundary condition (IWBC) or using an imaginary potential. In our study, fusion cross sections were obtained from FRESCO [25] by using the barrier penetration model (BPM). In BPM, fusion cross section is given by

$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_{\ell}(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_{\ell}(E), \quad T_{\ell}(E) = (1 - |S_{\ell}|^2) \quad (10)$$

where $T_{\ell}(E)$ is the quantum-mechanical transmission probability through the potential barrier. $T_{\ell}(E)$ is obtained by numerical integration of the Schrödinger equation as follows,

$$\frac{d^2 u_{\ell}(r)}{dr^2} + \frac{2\mu}{\hbar^2} [E - V_{\text{total}}(r)] u_{\ell}(r) = 0 \quad (11)$$

where μ is the reduced mass of the projectile and target system and the total potential $V_{\text{total}}(r)$ is defined as in Equation 1. The results of NN-DF are discussed in the following section.

III. RESULTS

In order to make a comprehensive analysis for the effect of the neutron-rich systems we used three different reactions which change from stable to unstable systems for the $^{16,18,20}\text{O}+^{12}\text{C}$ reactions.

A. $^{16}\text{O}+^{12}\text{C}$ System

To test our results realibility, we first used the well known $^{16}\text{O}+^{12}\text{C}$ reaction. We simultaneously analyzed the elastic scattering angular distributions and fusion cross section calculations by using the same parameters. The parameters used for the real and imaginary part of the nuclear potential are shown in Table II. To explore the elastic scattering angular distribution of the $^{16}\text{O}+^{12}\text{C}$ system, we used the experimental data from Ref. [26] at energies $E_{cm} = 10.01$ MeV and 11.04 MeV. The results of the elastic scattering angular distributions and fusion cross section for calculations are shown in Fig. 1 and Fig 2, respectively. As can be seen In Fig. 1, The NN-DF potential produces the oscillatory structure of the elastic scattering angular distributions data with correct phases and magnitudes. We also investigated the fusion cross section data by using the same parameters. In Fig. 2, there are several different experimental data [8, 10, 28] and, as one can see from the figure, our results show very good agreement with the experimental data. One of the important point in this study, best results were obtained using very weak imaginary potentials for all calculations. Also, Morais *et. al.* [26] used the similar weak imaginary potential parameters for $^{16}\text{O}+^{12}\text{C}$ elastic scattering angular distribution data. In Ref. [27], the best fit to the elastic scattering and fusion cross section data is obtained with a very weak surface transparent imaginary potential for $^{16}\text{O}+^{16}\text{O}$ system. As a result, our NN-DF potential could describe the broad features of the fusion and the elastic scattering angular distribution data simultaneously.

B. $^{18}\text{O}+^{12}\text{C}$ System

As a next step, to understand the behavior of our potential model we analyzed the $^{18}\text{O}+^{12}\text{C}$ system fusion cross sections at near and below the Coulomb barrier energies. Unfortunately, there is no experimental elastic scattering data for $^{18,20}\text{O}+^{12}\text{C}$ reactions to test our potential set. Steinbach *et al.* [11] measured the $^{18}\text{O}+^{12}\text{C}$ system fusion cross sections

in the sub-barrier region by using the evaporation residue technique. Also, they used the density-constrained time-dependent Hartree-Fock (DC-TDHF) model to examine this system and reported that the larger cross sections at low energies depend on larger tunneling probabilities. Our theoretical results of the $^{18}\text{O}+^{12}\text{C}$ system are shown in Fig. 3. We made a very small change to the normalization factor for real and imaginary potentials. The realistic NN-DF model explained perfectly the experimental data not only above the barrier but also at the low energy region. The phase and magnitude of fusion cross section data could be reproduced well by using the NN-DF model. The results showed that the NN-DF model works well for the $^{16,18}\text{O}+^{12}\text{C}$ systems.

C. $^{20}\text{O}+^{12}\text{C}$ System

In the final stage, we examined the $^{20}\text{O}+^{12}\text{C}$ system. In our previous work [15], we presented the our first results by using semi-microscopic NN-DF. To analyse this reaction, the microscopic nucleon-nucleon double folding potentials with Woods-Saxon type imaginary parts is used. Unfortunately, we were unable to explain the fusion cross-section data for the $^{20}\text{O}+^{12}\text{C}$ system. A significant underprediction appeared between experimental data and our results, especially at energies lower than 9 MeV. Also, other theoretical studies like the statistical decay code, TDHF and DC-TDHF [8, 10] could not explain the experimental data in these energy regions. In order to explain this discrepancy in Ref. [15], we used the effects of the long range part of the potential. In this work, we presented the fully-microscopic approach without any additional potential. In this approach, the real and imaginary potential parts of the nuclear potential are derived from the NN-DF model. We have shown our theoretical results for the $^{20}\text{O}+^{12}\text{C}$ system in Fig. 4 and compared our results with the DC-TDHF and DC-TDHF, CP calculations. As can be seen from the Fig. 4, our results produce the oscillatory structure of the fusion cross section data with correct phases and magnitudes. In this calculation, we have obtained very reasonable explanation with very effective way by using microscopic potential for the real and imaginary part of the nuclear potential. However, we have still problems to understand the magnitude difference and oscillatory structure behaviour at nearly 9 MeV. Also, we have showed that the real and imaginary potentials depths for the for the $^{16,18,20}\text{O}+^{12}\text{C}$ reactions, as can be seen from Fig. 5. As expected, the normalization factor for the real part (N_R) is showed increase and

the normalization factor for the imaginary part (N_I) is decreased with the different oxygen isotopes. Also, we observe that the depth of the imaginary potential has a significant impact on fusion cross section data in the neutron-rich system and it decreased nearly half with increasing neutron number. The best results have been obtained by using normalization factor $N_I=0.006$ for $^{20}\text{O}+^{12}\text{C}$ system.

As a result, it seems that the microscopic nucleon-nucleon double folding potential provides a reliable explanation of the fusion cross section for the $^{16,18}\text{O}+^{12}\text{C}$ systems. Although, we have obtained better results than the other works, we still need to conduct further research to explain the neutron-rich $^{20}\text{O}+^{12}\text{C}$ system especially at low energies.

IV. SUMMARY AND CONCLUSIONS

In this paper, we aimed to explain the fusion cross section for the $^{16,18,20}\text{O}+^{12}\text{C}$ reactions within the framework of the optical model. For this purpose, we used the nucleon-nucleon double folding microscopic potentials to obtain reaction observables. In the first part of this paper, we present our results for the $^{16}\text{O}+^{12}\text{C}$ and $^{18}\text{O}+^{12}\text{C}$ reactions. To see validity of the fully microscopic NN-DF with a weak imaginary potential, we analyzed the elastic scattering angular distribution and fusion cross section data for $^{16}\text{O}+^{12}\text{C}$ system with the same parameters, simultaneously. It is clearly seen from Figures 1, 2 and 3 that the microscopic NN-DF potential predicts perfectly the reaction observables compared to other approaches and it produces the oscillatory structure of the fusion cross section data with correct phases and magnitudes. We would like to point out that small changes to the normalization constant, N_R , could improve the agreement further between the theoretical results and the experimental data.

We then analyzed the neutron-rich $^{20}\text{O}+^{12}\text{C}$ reaction, which is a very important reaction for the neutron star crust environment. Because of the neutron-rich structure of the ^{20}O nucleus, the NN-DF model does succeed in explaining the fusion cross section experimental data when compared the other works [8, 10] at the low energy region, especially. By using the NN-DF approximation, our theoretical results show very good agreement with the experimental data for the first time. One of the main results of our study is that the systems we analyzed are extremely sensitive to the imaginary potential at energies near and below barrier. We could not explain the reactions observables without a weak imaginary

potential. In conclusion, our results confirm that the the fully-microscopic NN-DF model is very reliable to explain neutron-rich systems and the imaginary potential has a significant impact on fusion cross section data but we should continue the research for the low energy region behavior.

Acknowledgments

This work was supported by the Erciyes University Scientific Research Projects Unit. Project Number:FBA-2014-5224.

-
- [1] J. M. Lattimer and M. Prakash, *Science* **304**, 536 (2004).
 - [2] C. J. Horowitz, H. Dussan and D. K. Berry, *Phys. Rev. C* **77**, 045807 (2008).
 - [3] C. J. Horowitz, O. L. Caballero and D. K. Berry, *Phys. Rev. E* **79**, 026103 (2009).
 - [4] C. J. Horowitz and D. K. Berry, *Phys. Rev. C* **79**, 065803 (2009).
 - [5] T. Strohmayer and L. Bildsten, *Compact X-ray Stellar Sources*, Cambridge University, Cambridge, (2006).
 - [6] W. Loveland *et al.*, *Phys. Rev. C* **74**, 064609 (2006).
 - [7] A. S. Umar, V. E. Oberacker and C. J. Horowitz, *Phys. Rev. C* **85**, 055801 (2012).
 - [8] M. J. Rudolph *et al.*, *Phys. Rev. C* **85**, 024605 (2012).
 - [9] J. F. Liang *et al.*, *Phys. Rev. C* **85**, 031601 (2012).
 - [10] R. T. deSouza, S. Hudan, V. E. Oberacker and A. S. Umar, *Phys. Rev. C* **88**, 014602 (2013).
 - [11] T. K. Steinbach *et al.*, *Phys. Rev. C* **90**, no. 4, 041603 (2014).
 - [12] D. T. Khoa and G. R. Satchler, *Nucl. Phys. A* **668**, 3 (2000).
 - [13] D. T. Khoa and H. S. Than, *Phys. Rev. C* **71**, 044601 (2005).
 - [14] E. V. Zemlyanaya, V. K. Lukyanov and K. V. Lukyanov, *Int. J. Mod. Phys. E* **20**, 2039 (2011).
 - [15] G. Kocak and R. Kabak, *J. Phys. Conf. Ser.* **590**, no. 1, 012048 (2015).
 - [16] G.R. Satchler, *Nucl. Phys.* **A409** (1983) 3c.
 - [17] M. El-Azab Farid and M.A. Hassanain, *Nucl. Phys. A* **678**, 39 (2000).
 - [18] M. El-Azab Farid, Z.M.M. Mahmoud and G.S. Hassan, *Nucl. Phys. A* **691**, 671 (2001).
 - [19] Reference Input Parameter Library (RIPL-3), <http://www-nds.iaea.org/RIPL-3/>.

- [20] V. K. Lukyanov, E. V. Zemlyanaya, K. V. Lukyanov, D. N. Kadrev, A. N. Antonov, M. K. Gaidarov and S. E. Massen, Phys. Rev. C **80**, 024609 (2009).
- [21] V. K. Lukyanov, D. N. Kadrev, E. V. Zemlyanaya, A. N. Antonov, K. V. Lukyanov and M. K. Gaidarov, Phys. Rev. C **82**, 024604 (2010).
- [22] G.R. Satchler, *Direct Nuclear Reactions* (Oxford University Press, Oxford 1983) and *Introduction to Nuclear Reactions* (The Macmillan Press Ltd, London 1980).
- [23] W. Greiner, J.Y. Park, W. Scheid, *Nuclear Molecules* (World-Scientific, Singapore 1995).
- [24] J. Cook, Comput. Phys. Commun. **25**, 125 (1982).
- [25] I. J. Thompson, Comput. Phys. Rept. **7**, 167 (1988).
- [26] M. C. Morais and R. Lichtenthler, Nucl. Phys. A **857**, 1 (2011).
- [27] G. Kocak, M. Karakoc, I. Boztosun and A. B. Balantekin, Phys. Rev. C **81**, 024615 (2010).
- [28] B. Cujec and C. A. Barnes, Nucl. Phys. A **266**, 461 (1976).

Nuclei	ρ_0 (fm^{-3})	w (fm^{-2})	β (fm^{-2})	$< r^2 >^{1/2}$ (fm)
^{12}C	0.1644	0.4988	0.3741	2.407
^{16}O	0.13173	0.6457	0.3228	2.640

TABLE I: The parameters of nuclear matter densities and RMS radii of the ^{12}C and ^{16}O [17, 18].

Reaction Type	N_R	N_I
$^{16}O+^{12}C$	1.1	0.035
$^{18}O+^{12}C$	1.2	0.017
$^{20}O+^{12}C$	1.46	0.006

TABLE II: The normalization constant parameters of the real and imaginary potentials of the optical model.

FIG. 1: (Color online) The results of the elastic angular distribution cross sections for the $^{16}\text{O}+^{12}\text{C}$ system; the solid line indicates our present results. The experimental data are taken from Ref. [26].

FIG. 2: (Color online) The results of the fusion cross sections for the $^{16}\text{O} + ^{12}\text{C}$ system; the solid line indicates our present results. The experimental data as given filled circles and squares are taken from Ref. [8] and as given diamonds and triangles are taken from Refs. [10] and [28], respectively.

FIG. 3: (Color online) The results of the fusion cross sections for the $^{18}\text{O}+^{12}\text{C}$ system; the solid line indicates the present results and filled circles the experimental data [11].

FIG. 4: (Color online) The fusion cross section results for the $^{20}\text{O}+^{12}\text{C}$ system obtained with (solid line) the NN-DF model and filled circles indicates that experimental data [8]. The DC-TDHF and DC-TDHF,CP results are taken from Ref. [10].

FIG. 5: (Color online) In the outer part of graph shows the real part of the total potential and the inner part of the figure indicates the imaginary potentials for $^{16,18,20}\text{O} + ^{12}\text{C}$ systems.