# A Finite Element Model to Study the Effect of Porosity Location on the Elastic Modulus of a Cantilever Beam

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A Finite Element Model to Study the Effect of Porosity Location on the Elastic Modulus of a Cantilever Beam

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Abstract: An investigation to determine the effect of porosity concentration and location on elastic modulus is performed. Due to advancements in testing methods, the manufacturing and testing of microbeams to obtain mechanical response is possible through the use of Focused Ion Beam technology. Meanwhile, rigorous analysis is required to enable accurate extraction of the elastic modulus from test data. First, one-dimensional investigation with beam theory, Euler-Bernoulli and Timoshenko, was performed to estimate the modulus based on load-deflection curve. Second, three-dimensional Finite Element (FE) model in Abaqus was developed to identify the effect of porosity concentration. Furthermore, the current work provided an accurate procedure to enable accurate extraction of the elastic modulus from load-deflection data. The use of macro-models such as beam theory and three-dimensional FE model enabled enhanced understanding of the effect of porosity on modulus.

Keywords

Porosity; Elastic modulus; Finite element analysis; Microcantilever; Beam theory; Focused ion beam
Introduction

Background

It is known that porosity affects the mechanical properties of metals. In many materials, increases in macroscale pore sizes have shown to decrease ultimate strength, yield stress, and fatigue life (Romanova, Kreimer, & Tumanov, 1974). However, due to advancements in manufacturing, pores in metals tend to be on the microscale instead of macroscale. This presents new concerns since the effect on a material’s mechanical properties due to this microporosity is unquantified (Hardin & Beckermann, 2007). This is critical because without determining the effect of microporosity on a material, the ability to predict behavior due to loading throughout its lifecycle is difficult. Throughout the life cycle of a structure, exposure to various environmental conditions, sometimes harsh, is possible. Due to these environmental conditions, it is possible that porosity can be increased in the material, as shown by Morrissey et al. (Morrissey, Handrigan, & Nakhla, 2019). As such, structures may be affected in various locations, and in differing intensities, depending on the exposure to the environment. Conversely, it was shown in Long et al. (Long, Feng, & Yao, 2017) through experiments that the process of annealing SAC305 solder healed microdefects to reduce the reduction in elastic modulus caused by porosity. As such, while a proper heat treatment can reduce the effect of porosity prior to environmental effects, it is still critical to quantify the effect of porosity and its location on a structure as evident from the results of Morrissey et al. (Morrissey et al., 2019).

Purpose of Study

It is known that porosity has an effect on elastic modulus. The work of Morrissey and Nakhla (Morrissey & Nakhla, 2018) presented a literature review on existing models
available in literature. These models, mostly empirical, describe the effect of porosity on elastic modulus. In their study, Morrissey and Nakhla (Morrissey & Nakhla, 2018) developed a two-dimensional Finite Element (FE) model that successfully captured the effect of porosity on elastic modulus in tension.

In the current work, a three-dimensional (3D) FE model is developed to investigate the effect of porosity on elastic modulus in bending. The effects of uniform distribution and concentrated zones of porosity along the beam length were investigated. All FE model results were compared to test data reported in literature. The FE model assumes uniform porosity distribution across the cross-section.

This paper provides detailed and thorough Finite Element Analysis of a microcantilever to provide guidelines for how an analysis of this particular experimental setup should be performed. The preparation of samples, performing of experiment, and value of the data are too costly to be misinterpreted; as such, guidelines must be developed. The model developed within highlighted the effect of substrate sizing, cross-sectional shape, aspect ratio, and Poisson’s effect. To delineate the effect of these factors, 3D FE models are developed in coupling with beam theories (Euler-Bernoulli and Timoshenko).

Procedures

Understanding Experimental Setup and Data

The first step in the current study was to examine experimental load versus deflection data for microcantilevers. For the current study, the work by Gong (Gong, 2015) was first analyzed to understand the correct beam theory that should be applied to determine elastic modulus, as well as to develop the 3D FE model. For example, Gong reported an elastic
modulus of 147 GPa obtained from his own testing; the experimental load-deflection curve from (Gong, 2015) can be seen in Figure 1. Meanwhile, his 3D FE model over predicts the modulus value by 38%. This contradiction is addressed within in terms of extracting elastic modulus from test data and accurate 3D FE modelling.

[insert Figure 1]

The microcantilevers were produced at the University of California, Berkeley (UCB) using a focused ion beam (FIB). Prior to the use of FIB, the entire slab was heat treated such that an average grain size of 8-10 µm was obtained. The FIB was used to cut three trenches using a 7-15 nA beam current – forming a U-shaped trench that had a width of 20-30 µm and a depth of 10 µm. Then using a 1-3 nA beam current, the outline of the beam was refined. Lastly, the sample was rotated 45°, both clockwise and counter-clockwise, around the longitudinal axis of the beam to allow for cutting of the triangular bottom of the beam. Figure 2 provides a schematic of the cross-section of test specimens. After the microcantilever beams were manufactured, a MicroMaterials nanoindenter was used to apply loading at the microcantilever beam tip and report beam tip displacement as a function of applied load. The load was applied with a displacement rate of 10 nm/s until fracture. Lastly, the depth of indentation into bulk material was removed from the measured deflection to ensure only the displacement due to bending is measured. A Scanning Electron Microscope (SEM) image of the microcantilever with the nanoindenter from (Gong, 2015) is shown in Figure 3.

[insert Figure 2]

[insert Figure 3]
Discussion of Beam Theories

To obtain the beam’s elastic modulus from load-deflection data, two beam theories are chosen: Euler-Bernoulli and Timoshenko. Expressions for tip deflection of a cantilever due to an applied tip load for Euler-Bernoulli and Timoshenko beam theories are shown in equations (1) and (2), respectively.

\[
\delta_{\text{Tip - EB}} = \frac{PL^3}{3EI}
\]

\[
\delta_{\text{Tip - Timoshenko}} = \frac{PL^3}{3EI} + \frac{PL}{\kappa AG}
\]

where, \(P\) is the applied load, \(L\) is the length of the moment arm, \(E\) is the elastic modulus, \(A\) and \(I\) are the area and second moment of area of the cross-section, and \(\kappa\) is the shear correction factor. Where \(G = E/2(1+\nu)\), for the Poisson’s ratio \(\nu\). The only difference between both expressions is the inclusion of the shear effect term in equation (2).

The assumptions for Euler-Bernoulli beam theory are: 1) normal cross-section remains normal, 2) plane cross-section remains plane, and 3) rigid cross-section. Whereas, Timoshenko beam theory releases the first assumption, hence accounts for shear deformation. The first assumption is only valid for high aspect ratio beams, i.e. aspect ratio \(\geq 10\). For low aspect ratio beams, the effect of shear deformation is more pronounced (Timoshenko, 1921).
The effect of shear deformation is important to capture realistic behaviour in cases where the bending moment is non-uniform, beams have low aspect ratios, and/or have less-than-perfect rigid supports. The geometry of tested microcantilevers in (Gong, 2015) is recreated in Figure 4.

The aspect ratio of the beam, $L/h$, is approximately 3.6 for $L = 27.04$ µm and $h = 7.48$ µm. The loading length of 27.04 µm was chosen instead of the total length of the beam for calculating the aspect ratio. The microbeam is loaded in cantilever setup, i.e. non-uniform bending moment. Moreover, as seen in Figure 3, the built-in end connecting the microbeam to the substrate cannot be considered perfectly rigid. Therefore, this brief discussion provides the rationale to use Timoshenko beam theory rather than Euler-Bernoulli beam. Meanwhile, Gong (Gong, 2015) extracted the modulus using Euler-Bernoulli beam theory. A comparison of the extracted modulus from the current study and Gong’s results is shown in Table 1.

It is also critical to understand the effect of the cross-section being right-pentagonal shaped instead of rectangular given that cross-section shape influences the value of $\kappa$, shear correction factor (Timoshenko, 1921), in equation (2). As stated, the cross-section is right-pentagonal shaped, Figure 2, as proposed by Maio and Roberts in (Di Maio & Roberts, 2005). The current authors believe the cross-section was chosen for the simplicity of manufacturing of the beams through the use of FIB cutting.
Assumptions

For the current study, several assumptions are made. Uranium Dioxide is highly anisotropic (Gofryk et al., 2014; Gupta, Pasturel, & Brillant, 2010); however, it is assumed that the material acts as an isotropic material since the microcantilevers are ideally contained within a single crystal-grain. It is reported in (Gong, 2015) that not all microcantilevers are within a single grain; however, without additional information on the number of grains and grain orientation, the assumption will remain. The FE model assumes the beam is solid, homogeneous, and has a constant cross-section free of imperfections. As well, the FE model assumes uniform porosity distribution across the cross-section. Lastly, it is assumed that the effect on Poisson’s ratio for porosities less than 5% is negligible (Asmani, Kermel, Leriche, & Ourak, 2001; Yu, Ji, & Li, 2016).

Reduction in Modulus due to Porosity under Bending

The work of Morrissey and Nakhla (Morrissey & Nakhla, 2018) demonstrated that, when in tension, the effect of pores in low porosity materials, when voids do not interact, can be accurately modelled using relationships that account only for pore volume. However, during bending, the top and bottom portion of the beam are in different states of stress – one is in tension, while the other is in compression. Therefore, for a model to be able to be used for bending, it must also be accurate in both tensile and compressive loading. The work of Handrigan et al. (Handrigan, Morrissey, & Nakhla)\(^1\) extended the work of Morrissey and Nakhla (Morrissey & Nakhla, 2018) to include compression and bending.

\(^1\) Handrigan, S.M., Morrissey, L.S., Nakhla, S. “A finite element model to predict the effect of porosity on elastic modulus in compression and bending of low porosity materials,”
The work of (Handrigan et al.) provided the reduction in modulus due to porosity of a FE model undergoing bending. Initially, they verified the ability of FE analysis to capture the effect of porosity in compression comparing their work to Gibson and Ashby (Gibson & Ashby, 1997). They then modelled a beam with an aspect ratio of 10:1 under bending due to a concentrated tip load. Based on the applied loading, the magnitude of normal stress will change along the beam length, while through the cross-section it will vary from tensile to compressive, hence the location of porosity concentration is also investigated. The reduction in elastic modulus of a cantilever beam undergoing tip loading due to 2.5% and 5% porosity concentrated at various segments along the beam length is reported in Table 2 within from (Handrigan et al.).

[insert Table 2]

*Finite Element Model – Three-Dimensional Beam*

To build the FE model, a three-dimensional, deformable solid part was created in Abaqus. The substrate was sketched and extruded to create a cube. From the front face, the geometry was sketched and extruded to create the beam. The beam from (Gong, 2015) and the currently developed FE model are shown in Figure 5. The beam and substrate were then partitioned to allow for separate modification of material properties and mesh development. The beam was further partitioned into three segments of equal length, Figure 6, allowing for different material properties to be applied to each segment.

[insert Figure 5]

[insert Figure 6]
Mesh convergence studies were completed for the beam and substrate and it was determined that the beam required 11,088 20-noded 3D stress hex quadratic reduced integration elements (C3D20R), while the substrate required 4,464 of the same elements.

All boundary conditions were applied to the substrate instead of the beam. All substrate sides, except the front and top face are fixed boundary condition, as shown in Figure 7. In doing so, no boundary conditions were imposed on the beam and there was no influence on the behaviour of the beam.

Prior to continuing with the model, another 3D FE model is developed to study two beams – one with a rectangular cross-section and the other with a right-pentagonal cross-section, as shown in Figure 8. Both beams possess identical aspect ratios, L/h, of 2, 5, 10, and 20. The tip displacement of both beams from the FE model and Timoshenko beam theory, using equation (2), are compared in Table 3.

The consistent accuracy of Timoshenko beam theory in predicting deflection when compared to the FE analysis is shown in Table 3. It is worth noting that the same shear correction factor as that of a rectangular cross-section is used for the right-pentagonal cross-section as well. Therefore, it can be concluded from this example that the right-pentagonal cross-section is similar to the rectangular cross-section in shear. Therefore, the shear correction factor of a rectangular cross-section is used henceforth. The equation for the
shear correction factor for a rectangular cross-section developed in (Cowper, 1966) is in seen in equation (3).

\[
\kappa = \frac{10(1 + \nu)}{12 + 11\nu}
\]

As shown in Figure 3, the substrate is too large when compared to the beam cross-section. Therefore, an optimized substrate size is determined for the FE model to prevent inadvertent effects on the beam's boundary conditions while also maintaining computational efficiency of the model. Four substrate sizes were considered with their dimensions identified as multiples of the beam’s largest cross-sectional dimension, i.e., height. A schematic of the 3D FE model with labels is shown in Figure 9. Substrate sizes were labelled A through D, with A being the smallest and D the largest. Substrate size A was approximately the same size as the height of the beam – an 8 µm x 8 µm x 8 µm cube. Substrate sizes B, C, and D were approximately two-times (15 µm x 15 µm x 15 µm), three-times (20 µm x 20 µm x 20 µm), and six-times (40 µm x 40 µm x 40 µm) larger than the beam height, respectively. With each substrate size, the model was tested under constant tip load and the load-deflection data was used to calculate elastic modulus. In addition, the stress distribution in the substrate was also observed in order to determine its size.

[insert Figure 9]

In this study, a nominal modulus of 219 GPa is assigned to the material for a perfect, non-porous beam. After applying the load, inverse calculations based on Timoshenko beam theory is used to obtain the modulus. A comparison of the modulus obtained for each of the
four cases is shown in Table 4. In addition, Figure 10 provides a comparison of the normal stress throughout the model for each substrate size with red indicating positive stress (tension) and blue indicating negative stress (compression). As it can be seen, the size of the substrate does affect the response of the beam. With a small substrate, the boundary conditions influence the beam behaviour since it approximates a perfectly rigid connection, thus the deformation energy is contained mostly within the beam. However, as the substrate size increases, the deformation energy is divided amongst the beam and the substrate since the effect of the boundary condition to stiffen the beam becomes less significant. Because of this, there will be an apparent reduction in modulus as not all deformation occurs in the beam. Therefore, the size of the substrate was increased until the calculated apparent modulus converged to a constant value. It was determined that as long as the substrate is at least three-times the height of the beam, the influence of boundary conditions is negligible. As such, a balance between removing the unwanted effect of the boundary conditions to stiffen the beam and computational efficiency was completed. For the current study, Substrate size D was chosen for the final 3D FE model.

[insert Figure 10]

[insert Table 4]

Observing the calculated modulus in Table 4, it is noted to be 18% less than the nominal modulus for a perfect, non-porous beam. This apparent reduction in modulus will become the base case to which the models with porosity will be compared. This is to isolate the effect of porosity and its location on reducing the modulus such that the effect on elastic modulus can be clearly described. If comparisons are made to the nominal modulus, the
apparent reduction in modulus due to the beam’s non-ideal boundary conditions (-18%) is included; thus, the reduction in modulus due to porosity and its location alone is not clearly presented.

After the base case was developed, the percent reduction values from Table 2 within, as reported in (Handrigan et al.), were applied to the nominal elastic modulus for Uranium Dioxide and a new modulus was calculated for when porosity is concentrated within different sections of the beam. A similar process was completed for uniform porosity distribution over the entire length of the beam.

The reduced modulus for each case was applied to specific sections of the beam where the porosity was to be concentrated while the remaining sections of the FE model, including the substrate, were considered to be equal to the nominal modulus of 219 GPa for Uranium Dioxide (Olander, 1976).

Results And Discussion

*Finite Element Model – Three-Dimensional Beam*

In Figure 11, the Abaqus FE models completed for the current study are compared with experimental load versus deflection data reported in (Gong, 2015). The limits of the results to which all other models should be contained within are the experimental results from (Gong, 2015) and the pore free Perfect beam FE model from the current study. These limits are clear since a higher modulus cannot be predicted for a beam with pores than a beam without pores, and a lower modulus should not be predicted than the experimental data. From this, it is evident that the FE models in the current study are within these limits and capture the trend of the experimental results from (Gong, 2015) with an average error of
+14.8%, as compared with the calculated approximate error of +38% reported in (Gong, 2015). It can be observed from Figure 11, the FE models for total porosities of 2.5% concentrated in various segments along the beam show that there is a minimal effect to reduce the modulus of the beam. A similar trend is found in the 5% porosity FE models, with the exception of the FE model where the 5% porosity is concentrated at the root. As such, it is evident that unless porosities are concentrated towards the root, the effect on reducing the load-deflection curve, thus reducing the modulus, is minimal.

[insert Figure 11]

Some of the error in the FE models shown above can be contributed to the assumptions made in the current study. The FE models completed in the current study assumed a uniform, constant cross-section, free of imperfections, which is not the case when observing SEM images of the beam in (Gong, 2015) and Figure 3 within. As well, the FE models were assumed isotropic due to the single-grain assumption, but as reported in (Gong, 2015) this was not true. Lastly, the porosity concentration in the FE models completed in the current study do not include the effect of pores away from the neutral plane – it is assumed the porosity is concentrated uniformly across the cross-section with no bias away from the neutral plane. Without more information provided in (Gong, 2015) on the exact location and size of pores it is difficult to model various pore sizes in various locations and obtain accurate results which replicate the experiment. As well, this last assumption may have a large effect on the accuracy of the FE model, since for a cantilever beam undergoing bending due to tip loading, the maximum normal stresses are located at the root, but away from the neutral plane and towards the upper and bottom surfaces. Therefore, if the porosity, especially if it is
a singular large pore, is located at the point where normal stress is approaching the maximum, the effect on reducing elastic modulus will be the greatest. The presence of a large pore would also violate the assumption that the cross-section remained constant as a large pore would cause the cross-section to vary along the beam length throughout the pore. As well, if the large pore is located close to the surface of the beam it is probable it would be present as a surface defect or hole acting as a stress concentrator. In addition, this surface defect would also change the cross-section along the length. Such defects can be seen in SEM images reported in (Gong, 2015). Due to the above assumptions and the inability to enhance the FE model any further, the authors are satisfied with the accuracy of the model.

Next, a comparison between all FE models by the current study and by Gong (Gong, 2015) is made with the experimental data in Figure 12. It is first noted that several of Gong's FE models predict a higher modulus than the Perfect beam FE model, which is not possible as previously stated. From Gong's results, it is evident that the Single Pore Root FE model has the largest effect, comparable to the current study's FE model for 5% porosity concentrated at the root. This single pore is located approximately in the location of maximum normal stress – the root; however, the exact placement from the neutral plane is unknown and as such it is difficult to compare to the FE models completed in the current study. From observing Figures 11 and 12, it is evident that porosity location does have an effect on the elastic modulus of a cantilever.

[insert Figure 12]
A final comparison of porosity uniformly distributed throughout the entire length of the beam to porosity concentrated at various locations is made for 2.5% and 5% porosities in Figures 13 and 14, respectively.

[insert Figure 13]

[insert Figure 14]

As can be seen from Figures 13 and 14, the effect of the location of porosity concentration diminishes if the total porosity is uniformly distributed along the beam or is concentrated beyond half the beam length. This is intuitive since, if the porosity is concentrated in a region of lesser stress – away from the root – its effect to reduce modulus is less pronounced. Similarly, if the same porosity concentration is distributed over the entire length there is less “total” porosity in each segment. In other words, 2.5% total porosity distributed evenly results in approximately 0.8% total porosity in each segment (root, middle, and tip); thus, resulting in less porosity in a region of higher stresses such as the root. Comparatively, 5% total porosity distributed over the entire length results in approximately 1.7% porosity in each beam segment. Since this value is less than 2.5%, which showed to have minimal effect on reducing the modulus even when concentrated at the root, the 5% total porosity distributed over the beam length will also have minimal effect on reducing the modulus.

The FE models completed in the current study, which assume uniform porosity distribution across the cross-section throughout various segments of the beam, have shown with certainty that porosities of 5% have a large effect on the behavior of the beam when concentrated close to the root. However, if these large porosities are uniformly distributed
over the length, or concentrated in a location away from the root – at or beyond half the beam length – the effect decreases drastically. As for porosities of 2.5% and lower, it can be concluded that there is minimal effect on the beam’s modulus regardless of distribution and concentration throughout the length.

**Conclusions**

The current study supports the results from Morrissey and Nakhla – a macroscale FE model is able to capture the effect of a microscale property such as microporosity on elastic modulus. The current study has also proven that to perform the analysis of microcantilevers effectively and accurately, one should utilize Timoshenko beam theory paired with a 3D FE model. This is because Timoshenko beam theory works with various aspect ratios, non-uniform bending, and less-than-ideal rigid connections. In addition, 3D FE can capture Poisson’s effect and allows for the modelling of non-ideal boundary conditions; thus, 3D FE allows for realistic modelling of the experiment. Lastly, the current study has proven that the amount of porosity and its location have a clear effect on the elastic modulus. Several cases were analyzed and it was determined that porosities concentrated at the root have the largest effect on the elastic modulus of a cantilever, while porosities uniformly distributed over the length, or concentrated away from the root, have minimal effect on elastic modulus.

**Acknowledgements**

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**References**


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List of Tables

Table 1 - Comparison of Calculated Modulus
Table 2 – Reduction in modulus reported in (Handrigan et al.)
Table 3 – Tip Deflection of Cantilever Beams
Table 4 - Comparison of substrate size
List of Figures

Figure 1 – Experimental Load-Deflection data from (Gong, 2015)

Figure 2 - Schematic of beam cross-section

Figure 3 - SEM image of microcantilever and nanoindenter from (Gong, 2015)

Figure 4 – Cantilever beam geometry recreated from (Gong, 2015) (a) detailed cross-section, (b) beam geometry

Figure 5 - SEM Picture of Experimental Setup (Gong, 2015) (left), Abaqus FE model the current study (right)

Figure 6 - Beam Sections (From left to right: Tip, Middle, Root, Substrate).

Figure 7 - Boundary Conditions (Top, Front, and Beam are free surfaces; all other sides fixed).

Figure 8 – Beam geometry studied for cross-section comparison, (a) Rectangular, (b) Right-Pentagonal

Figure 9 - Geometry of 3D FE Model

Figure 10 – Stress in Substrates: A) Substrate Size A, B) Substrate Size B, C) Substrate Size C, D) Substrate Size D

Figure 11 - Load-Deflection Comparison of the Current Study with Experimental (Gong, 2015)

Figure 12 - Load-Deflection Comparison of All Data.

Figure 9 - Load-Deflection Comparison for 2.5% Porosity

Figure 14 - Load-Deflection Comparison for 5% Porosity
Table 1 - Comparison of Calculated Modulus

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Table 2 – Reduction in modulus reported in (Handrigan et al.)

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Table 3 – Tip Deflection of Cantilever Beams

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Table 4 - Comparison of substrate size

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<tr>
<td>D</td>
<td>180</td>
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Figure 14 - Load-Deflection Comparison for 5% Porosity