Retroreflective Binary Huygens’ Metasurface

by

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Abstract

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This thesis discusses the ability to aggressively discretize a periodic surface to retroreflect an incident wave. The novelty of this work is related to the number of cells that are needed per grating period to obtain perfect retroreflective power efficiency. For an incident angle greater than 19.5° from normal, perfect retroreflection is possible when considering a passive system having only two cells per grating period. This thesis reports the design and demonstration of near-grazing angle retroreflection metasurfaces for both TE and TM polarizations. Such aggressive discretization lessens, as much as possible, the need for small feature sizes, and results in a simple metasurface retroreflector that is extremely power efficient, cost-efficient and scalable to mm-wave and THz frequencies. Because of its simplicity, this approach can be used to realize electrically large retroreflectors that are very challenging for the standard Van Atta approach which requires complex wire routing.
To my loving parents
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Chapter 1

Introduction

"It is difficult to point to another single device that has brought more important experimental information to every field of science than the diffraction grating. The physicist, the astronomer, the chemist, the biologist, the metallurgist, all use it as a routine tool of unsurpassed accuracy and precision, as a detector of atomic species to determine the characteristics of heavenly bodies and the presence of atmospheres in the planets, to study the structures of molecules and atoms, and to obtain a thousand and one items of information without which modern science would be greatly handicapped."[1]

A diffraction grating is a component which has a periodic structure and can diffract light into multiple beams that travel in different directions. A diffraction grating can be used for both reflection and refraction purposes, and thus has the ability to direct an incoming wave into an arbitrary direction. The making of the first diffraction grating can be credited (not without argument) to the American astronomer David Rittenhouse who made rulings out of hair [2]. While this might seem crude, it proved that a grating can diffract light. Thereafter, many others contributed to the making of diffraction gratings such as Henry Rowland, Joseph Fraunhofer, and Sir John Barton [3]. A retroreflector is a device that back-reflects an electromagnetic wave towards the direction of incidence. A diffraction grating can be used as a retroreflector since the output wave can be directed in an arbitrary direction. The only problem with this simple process, is the amount of power that is able to couple into the desired angle.

There are other types of structures which enable the wave to be scattered in a prescribed manner, and one example of that is through artificially structured media. The research of artificially structured media has been going on for a very long time, from the early 1900s, where Robert W. Wood observed the
spectrum of a light source reflecting off an optical metallic diffraction grating. It was noticed that fast variations of the diffracted spectral orders were observed over a narrow frequency band. This was due to the fact that a resonance condition occurred on structure, however during the early 1900s, this was seen as an anomaly that was not well understood [4]. In the past few years, there has been much interest towards metasurfaces since there is the possibility to control the incoming wavefront of electromagnetic waves with a thin surface, such that the reflected or refracted wave can be tailored to suit several designs.


Figure 1.1: Different types of retroreflectors: (a) Corner reflector, (b) Cat’ s-eye retroreflector, (c) Luneberg and (d) Van Atta array.

1.1 Review of Retroreflectors

A retroreflector is a device that can back-reflect an incident electromagnetic wave. There have been a variety of applications for retroreflectors, ranging from satellite communication to autonomous vehicle control. In these and many other applications, the general characteristics which are preferred would be the ability for a wide-angle response, the handling of both TE- and TM-polarized waves, high efficiency, low-profile, and fabrication simplicity. The most simple retroreflector that can be illustrated is a metallic plate, where the electric length is large. In this case, the retroreflected power is very strong when a plane wave impinges perpendicularly to it. However, when the wave diverges from normal incidence, it no longer retroreflects efficiently. The most common retroreflector is known to be the corner cube, as shown in Fig. 1.1a.

This is formed by having two (or three) metallic plates connected perpendicularly to each other. The incoming wave is then reflected two (or three) times, and back-reflects very efficiently. However, this is not the case for all angles, and it can be seen through tests that they work efficiently in the range of $\pm 15^\circ$ from normal, and also when the metallic plates are normal to the incoming wave (i.e. $\pm 90^\circ$) [5]. There are a few disadvantages to using corner cubes outside of the limited angular range. The structure itself can be quite large, as the depth is comparable to the size of the aperture, and in some cases, the retroreflected polarization changed from that of the incident waves polarization. Due to these
limitations, another retroreflector arose; the planar retroreflecting sheet. This uses similar principles to that of a corner cube, however, multiple corner cubes are placed side by side in a closely-packed form. Using this design, it was possible to achieve retroreflection for angles less than $\pm 40^\circ$. However, the problem still arises for retroreflection at stepper angles (i.e. near grazing angles) [6].

There are other types of retroreflectors which do not consist of planar metallic structures, but rather dielectric, plasmonic, and/or disordered metallic materials. The retroreflected intensity of a distribution of latex microspheres was shown to have a peak in the retroreflected direction when the volume density is above 1%. The efficiency is not that strong, and under optimal conditions, 40% efficiency is achievable [7]. This type of retroreflector is commonly known in fields like astronomy, geophysics, and radar applications. The common desire in many of these fields is to understand the physics behind the retroreflection peak and understand the disorder that enabled it. There are generally two mechanisms which are said to be responsible for this backscattering; shadow casting and phase coherence. Shadow casting considers the shadowing effects on a surface, and create backscattering enhancements from the geometric properties due to shadows on a surface. Phase coherent characteristics of a backscattered wave can be explained by understanding the retroreflecting paths that are taken, and the interference that is present in this situation. However, this method also does not allow for efficient coupling, but rather the ability to retroreflect at a wide range of angles [8].

A cat’s eye retroreflector is a reflector that has a focusing system which consists of a primary lens along with a secondary mirror that is located at the focus and is shown in Fig. 1.1b. This type of retroreflector has advantages over other retroreflectors like the corner cube because it has the ability to maintain its polarization. This is very useful in applications involving optical isolation in a laser feedback system [9]. The lateral size and depth of a cat’s eye retroreflector are comparable to that of a corner cube. However, since the incident wave is focused on a particular location, and takes up a small area, it is possible to modulate the wave and perform switching and encoding. A multistage cat’s eye retroreflector takes the basic idea of the single lens and mirror but adds more lenses, and has demonstrated that it can operate effectively up to $\pm 36^\circ$ [10]. The Luneberg Lens retroreflector operates similar to the cat’s eye retroreflector, however in this case the lens and mirror are replaced with a Luneberg lens. The coverage of the metallized cap greatly affects the reflective properties [11]. In Fig. 1.1c, the metallic cap subtends an angle of $2\alpha$, and in the case of $\alpha=45^\circ$, the RCS (radar cross section) drops to zero (i.e. $<-3\text{dB}$) abruptly when the incident angle exceeds $\pm 45^\circ$. Whereas, for $\alpha=90^\circ$, the RCS (radar cross section) drops below $\pm 25^\circ$ but with a less steep slope compared to the case of $\alpha=45^\circ$. While the coverage for the Luneberg lens is quite large, it takes up a large physical area, is quite heavy, and expensive to fabricate. The Eaton lens retroreflector is a lens that is spherically symmetric and thus would be considered a
perfectly omnidirectional retroreflector. This lens is a typical GRIN lens where the index of refraction varies from one to infinity \[12\]. This, therefore, is a retroreflector which can operate at all incident angles, however, the level of precision in the dimensions and permittivity of the material is very high, and therefore impractical in most settings.

The standard planar retroreflector, which is a Van Atta array, is commonly used because of its low profile, and high efficiency. This consists of an array of antennas arranged in a particular manner to retroreflect the incident wave as shown in Fig. 1.1d. For a Van Atta array, the incident power is absorbed by the antennas, and each antenna is then radiating an output wave with a specific phase and amplitude. If retroreflection is desired, the array should be designed such that each antenna is connected to the antenna that is diametrically opposite to it \[11\]. The disadvantages with using the Van Atta array is the complexity that arises when the number of antennas in the array outlay increases. This is because the routing between the antennas will get more complex, and thus add to the overall profile (thickness), and cost of fabrication. Also, as the size of the Van atta array increases, the routing lines will increase, which will in general cause the bandwidth to decrease. The largest angle that can be efficiently retroreflected using a Van Atta array is \(\pm 50^\circ\).

In general, the maximum angle for efficient reflection is around \(\pm 50^\circ\), for a design that is physically implementable. Therefore, designing a retroreflector which can work efficiently for large angles while remaining low-profile and low cost is an area of great interest. Some of the main features that are useful in retroreflectors are power efficiency, cost-effectiveness, simplistic design, angular coverage and the ability for the reflector to scale to different frequencies. These five characteristics collected is something that is not realized in current retroreflectors, as most either suffer from taking up a large physical area, not having the ability to perfectly retroreflect the power, not being able to retroreflect for large incident angles, complex wire routing (e.g. the Van Atta) or being expensive and/or difficult to fabricate due to small feature sizes.

1.2 Motivation for the Retroreflective Metasurface

A metasurface is a layer that consists of two or more materials, that is designed to arbitrarily control the electromagnetic fields. This surface must have a material thickness that is negligible when compared to the wavelength of the incident wave and sub-wavelength unit cells (meta-atoms) such that it can be homogenized with a surface reactance. This is what differentiates it from a metamaterial. These structures allow the control of the transverse impedances enabling a wide range of designs. Previous works have illustrated the possibility to manipulate the reflection of a wave through metasurfaces, and
Chapter 1. Introduction

Figure 1.2: A blazed grating representing the case in where retroreflection occurs (red) and the general case where the incident and reflected angle differ (green).

this is done because of the control of the transverse impedance or reflection phase along the surface which enables efficient power coupling into the desired reflection direction (mode). These works show that in order to get a metasurface to perfectly reflect a plane wave into an arbitrary direction, it requires the material to be both lossy and active. The only situation when that is not the case is for specular and retroreflection [13]. However, the same methodology used in this paper can be applied for designing a reflective metasurface for arbitrary angles without the need of lossy and active material [14].

Designing a metasurface that refracts or reflects perfectly has major differences, and in this thesis, only reflection surfaces will be covered. A major difference in both designs has to do with the fact that when a perfectly refracting metasurface is designed, it is assumed that there is only a single plane wave at every moment in time and space. Designing a perfectly reflective metasurface results in having waves interfering with each other in the same space. Retroreflection of an electromagnetic wave using a simple binary Huygens’ metasurface (BHM) is an area that is of great interest due to it’s possible effectiveness but has not been explored. The typical design methodology that has been used is a phase gradient that mimics a blazed grating. This generally involves multiple cells per grating period with varying reflection phases. In this thesis, it will be shown that it is possible to aggressively discretize the number of cells per grating period to perfectly retroreflect a near grazing incident plane wave for both TE and TM polarizations [15].

An important comparison that should be made when designing a retroreflective metasurface, is how it compares with a traditional blazed grating. A blazed grating is a type of diffraction grating which consists of a grating period \( \Lambda_g \) as in the general case derived earlier, however, per grating period, there are physical steps (periods) which consist of sawtooth-shaped cross sections. These steps are tilted in a particular manner in order to achieve the desired reflection characteristics. The case when retroreflection is achieved is called the Littrow configuration, as shown by the red rays in Fig. 1.2. If the incident angle is not not the prescribed angle for retroreflection, it will reflect at another angle, as shown by the green rays.
The blazed grating is designed to obtain the maximum grating efficiency possible in the desired diffraction mode. This essentially suppresses all other modes including specular and thus is a very powerful diffraction grating. The difficulty with a blazed grating has to do with the fabrication process. When comparing the process of fabricating a blazed grating to that of the TE- or TM-metasurface, it is quite obvious that the process for the latter is much more simple. The metasurface design only requires that a thin layer is etched as shown in Fig. 1.3b, whereas a blazed grating generally requires a linear ramp to be etched as shown in Fig. 1.3a. Another way in which a blazed grating is fabricated is through depositing a material on top of a surface. Generally, this involves multiple steps but can be done with several techniques including photolithography, electron-beam writing, and laser beam writing [16].

There has been recent work done on designing an equivalent planar metasurface that models the blazed gratings phase profile [17]. This is done by modelling the phase accumulating by the different sections of the blazed grating and producing a metasurface that is discretized to follow the same phase profile, as shown in Fig. 1.4a&b. This phase progression is modelled in Section 2.4.1 for the case of retroreflection. The difficulty in this situation is the minimum number of cells needs in order to perfectly reflect all the power into the retroreflection mode.
1.3 Thesis Outline

The outline for the remainder of this work will be summarized here. Chapter 2 will carry out the theoretical framework for the proposed binary retroreflective Huygens’ metasurface. An in-depth Fourier analysis is performed in order to determine the length of a grating period, and the minimum number of cells needed per grating period in order to simultaneously suppress specular reflection, and efficiently retroreflect that power. Chapter 3 covers the design and simulation of the retroreflective metasurface. First an infinite Floquet simulation was done to determine the appropriate unit cell designs in order to achieve the correct phasing, and then a finite simulation followed to determine the farfield radiation pattern. This was done for both TE- and TM-polarized incident wave. The chapter then goes on to cover important details regarding the simulated results and explores methods to further analyze the data. Chapter 4 goes through the monostatic and bistatic RCS (radiation cross section) measurements that were obtained for both the TE and TM retroreflective metasurfaces that were fabricated, and analysis was done on these measurements to see how well it fits with the simulated results obtained in chapter 3. Chapter 5 goes on to discuss further areas of study that were done in relation to the binary cell design procedure which includes the design methodology to obtain wide-angle and/or wide-band response, and the ability to absorb all the incident wave power such that no wave propagates. Chapter 6 then ends with concluding remarks, and future directions that can be taken to expand on the methods and ideas mentioned in this thesis.
Chapter 2

Theoretical Background

2.1 Theoretical Background of Huygens’ Metasurfaces

The retroreflective metasurface that is designed in this thesis is called a binary Huygens’ metasurface (BHM) because of the fact that it consists of two cells per grating period (binary), while maintaining Huygens’ source characteristics, where each cell behaves as an electric and a magnetic current source. Metamaterials differ from metasurfaces because the former are usually described with effective permittivity and permeability, which strictly speaking is not possible with surfaces. Metasurfaces are modelled by effective surface parameters, which can be modelled in electromagnetic problems using effective boundary conditions. These effective boundary conditions can be formulated using surface impedances (or admittances), surface polarizabilities, or surface susceptibilities. The physical intuition behind these boundary conditions has to do with the average tangential field applied on a thin layer of polarizable particles which induces an electric ($\vec{J}_s$) and magnetic ($\vec{M}_s$) surface current [18]. This can be seen in Fig. 2.1.

When an applied field interacts with a boundary, it refracts and reflects, and this occurs because the tangential components of the electric and magnetic fields are conserved. An intuitive way to understand this behaviour is that given the continuity of the tangential fields across a surface boundary, it can manipulate and control the reflected and refracted fields. Therefore, it is possible to artificially engineer a boundary with electric and magnetic surface currents at an interface to completely control the scattered wave [19].
Chapter 2. Theoretical Background

2.1.1 Discontinuity in Electromagnetic Fields

Since passive reflective metasurface are being considered, the focus will be on impedance and admittance surfaces rather than using active dipole arrays or slot arrays. Using this method, the metasurface will be a surface which scatters the field in a prescribed fashion. These passive surfaces can be described by an impedance and/or admittance surface with boundary conditions found by understanding the discontinuity shown in Fig. 2.2.

The case for impedance surfaces gives (2.1-2.3),

\[
\hat{n} \times [E_2 - E_1] = 0, \quad (2.1)
\]

\[
\hat{n} \times [H_2 - H_1] = J_s, \quad (2.2)
\]

\[
-\hat{n} \times (\hat{n} \times E_1) = Z_s J_s. \quad (2.3)
\]

The case for an admittance surfaces gives (2.4-2.6),

\[
\hat{n} \times [E_2 - E_1] = M_s, \quad (2.4)
\]

\[
\hat{n} \times [H_2 - H_1] = 0, \quad (2.5)
\]

\[
-\hat{n} \times (\hat{n} \times H_1) = Y_s M_s. \quad (2.6)
\]

Both \(Z_s\) and \(Y_s\) are complex values. There are differences in both types of surfaces and how they are designed. To design a surface that obtains both magnetic and electric currents, specific geometries are generally desired. In the case of an incident TE-polarized wave, a planar metallic sheet would allow for
Figure 2.2: Both electric and magnetic surface currents produce a discontinuity on a boundary

Figure 2.3: Patch generating both electric and magnetic currents (i.e. Huygens’ source).

a surface current $\vec{J}_s$. This would then be mirrored by the ground plane but the current on the image reverses direction. These two electric currents together generate an effect similar to a current loop which imitates a magnetic current. The ground plane cancels out the electric current, and this cancellation is performed in a way to cancel the transmitted field. It is seen from this analysis that the electric and magnetic fields generate a one-sided response suggestive of a Huygens’ source. This is shown in Fig. 2.3 and is considered a Huygens’ source because orthogonal electric and magnetic currents are obtained. Both the electric and magnetic currents are required for a Huygens’ source to alter the scattered wave and so two degrees of freedom are needed to do that. If a TM-polarized incident wave is considered, it will have a slot design (Bookers Principle, discussed in Section 3.2.1), which will also be considered a Huygens’ source [19]. The top view designs for the TE- and TM-polarized are shown in Fig. 2.4.

2.2 Principle of Diffraction

Diffraction is a phenomenon that occurs when a wave is presented with an obstacle. This can be described in classical physics as the Huygens-Fresnel principle, where at every point in which the wave is disturbed, it will behave as a secondary point source propagating as a spherical wave, and the interference of these
secondary waves determines the phase front of the wave after the disturbance. A component that is vastly used with diffraction is the diffraction gratings. It is used to direct a diffracted wave into different directions which can be reflective or transmissive in nature. An example of a diffraction grating is shown in Fig. 2.5 and its operation can be summed up with (2.7) below

$$m\lambda = \Lambda_g (\sin \theta_i - \sin \theta_r), \quad (2.7)$$

where $m$ is the diffraction order, $\lambda$ is the wavelength, $\Lambda_g$ is the grating period, and $\theta_i$ and $\theta_r$ represent the incoming wave and the reflected wave respectively. It is important to note that only reflected waves were considered, but the same equation can be used for refracted waves where material properties of the grating would have to be taken into consideration.

### 2.2.1 Fourier Analysis of a Diffraction Grating

The reflected wave from the diffraction grating can be analyzed by first calculating the electromagnetic field in the near-field and using this field distribution to predict the farfield in the forward half-space ($z \geq 0$). If the common rectangular aperture is considered and illuminated by a horn as shown in Fig. 2.6, the tangential electric field on the aperture can be described as

$$E(x, y) = E_0 f(x, y)e^{j\omega t}, \quad (2.8)$$
where \( f(x,y) \) describes the geometry of the aperture. When Huygens’ principle is taken into consideration, the aperture can be thought of as a generation source of secondary waves that form a diffraction pattern. To formulate this, the electric field can be expressed at the aperture as a linear combination of infinite plane waves and transform it to \( \theta \)-space (\( k \)-space). To simplify the results, the one-dimensional case will be considered below

\[
E(\theta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} E(x)e^{-jk_y y}dy, \quad (2.9)
\]

where \( k_y = k \sin \theta \) and it is important to note that this is the Fourier transform of the spatial electric field at the aperture. The inverse transform will be

\[
E(x) = \int_{-\infty}^{\infty} E(\sin \theta)e^{-jk_y \sin \theta}d(\sin \theta), \quad (2.10)
\]

where \( k = 2\pi/\lambda \). The electric field distribution \( E(\sin \theta) \) (the angular spectrum) represents the distribution of plane waves. A finite aperture (length \( a \)) angular spectrum is the same as the far-field pattern, \( E(\theta) \), therefore, (2.9) can be revised to be:

\[
E(\theta) = \frac{1}{2\pi} \int_{-a/2}^{a/2} E(x)e^{-jk_y \sin \theta}dy. \quad (2.11)
\]

A plane wave incident normal to a 1D aperture of length \( a \) will have \( E(x) = 0 \) outside of \(-a/2 \leq y \leq a/2\), and \( E(x) = E_0 \) within that region, resulting in:

\[
E(\theta) = \frac{aE_0 \sin \beta}{2\pi} \beta d\theta; \quad \beta = \frac{\pi a \sin \theta}{\lambda}. \quad (2.12)
\]
Figure 2.7: The convolution of a single aperture with two Dirac delta functions

The irradiance in the far-field is the time-averaged electric field which gives:

$$I(\theta) = I_0 [\text{sinc}(\beta)]^2$$

(2.13)

Diffraction due to multiple apertures can also be calculated using the fact that the Fourier transform of the convolution of two functions is simply the product of the individual Fourier transforms. As an example, the irradiance of two apertures having slot widths of $a$ and a separation length of $\Lambda_g$ can be found through the convolution of a single aperture with two Dirac delta functions, as shown in Fig. 2.7.

In order to obtain the far-field pattern for these two apertures, the inverse Fourier transform needs to be taken of the convolution of the single aperture and two Dirac delta functions

$$\mathcal{F}^{-1}\left\{\left[ E_{slit} \right] * \left[ \delta \left( y + \frac{\Lambda_g}{2} \right) + \delta \left( y - \frac{\Lambda_g}{2} \right) \right] \right\} = \mathcal{F}^{-1}\left\{ E_{slit} \right\} \mathcal{F}^{-1}\left\{ \delta \left( y + \frac{\Lambda_g}{2} \right) + \delta \left( y - \frac{\Lambda_g}{2} \right) \right\}$$

(2.14)

where,

$$\mathcal{F}^{-1}\left\{ \delta \left( y + \frac{\Lambda_g}{2} \right) \right\} = \frac{1}{2\pi} (e^{-jk_y \Lambda_g / 2}).$$

(2.15)

Using (2.13) it can be concluded that the irradiance of two apertures will follow the equation shown below:

$$I(\theta) \propto \text{sinc}^2 \left( \frac{k_y a}{2} \right) \cos^2 \left( \frac{k_y \Lambda_g}{2} \right).$$

(2.16)

The main difference between (2.13) and (2.16) is the modulation that is brought forth from the $\cos$ factor. The distance between the maxima will be determined by this finer $\cos$ pattern. Since the period of a $\cos^2$ function is $\pi$, the periodic nature of the intensity is shown below

$$\frac{k_y \Lambda_g}{2} = m\pi \implies \Lambda_g \sin(\theta) = m\lambda,$$

(2.17)

where $m$ is an integer which indicates the order of the reflected wave.

The specific case where the incident plane wave is at normal incidence ($90^\circ$) with the two apertures
is shown in (2.17). If the incident wave was at an angle that is not at normal incidence, the equation would change to account for this initial wave vector in the $y$-direction ($k_{iy}$),

\[
\frac{\Lambda_g(k_{ry} - k_{iy})}{2} = m\pi \implies \Lambda_g\left(\sin(\theta_r) - \sin(\theta_i)\right) = m\lambda. \tag{2.18}
\]

It is important to note that $\theta$ can be either positive or negative. If the incident $\theta_i$ is chosen to be positive, then the reflected $\theta_r$ will be positive if it lies on the opposite side of the normal, and negative if it lies on the same side (Fig. 2.8). If the number of apertures increases, the maxima positions remain in the same location, where the only difference will be how sharp the modes are. In the case where the number of apertures increases to infinity, the modes will look more like delta functions. This is the convolution between a Dirac comb function and a single slit.

### 2.2.2 Principle of Retroreflection

The principle of diffraction is directly correlated to retroreflection. If the grating period of the meta-surface is such that the reflected mode power is in the direction of the incident wave, it will achieve retroreflection. Using (2.18) it is seen that if the incident and reflected angle have the same magnitude but opposite signs, the equation will be:

\[
\Lambda_g = -\frac{m\lambda}{2\sin(\theta_i)} \quad \text{or} \quad \Lambda_g = \frac{\lambda}{2\sin(\theta_i)}. \tag{2.19}
\]

The integer value of $m$ was chosen to be -1 because the grating length $\Lambda_g$ should be as small as possible.
2.3 Floquet Theory

A diffraction grating is a periodic structure. In order to understand the way to analyze these types of
structures, Floquet theory can be quite valuable. Periodic structures are common in nature and through
man-made innovations. This can be in one, two or three dimensions. A simple example in the field of
Electromagnetics is a dielectric waveguide with grooves (perturbations) along the y-direction (Fig. 2.5).
The electromagnetic field components of the incoming wave can be represented by

\[ E(x, y + d, z) = e^{j k_y d} E(x, y, z). \] (2.20)

The exponential term represents the complex phase shift \( k_y = -j \alpha + \beta \) between adjacent unit cells.
Given that this is a one-dimensional structure, it is clear that the structure changes along the y-direction
and this expression is called the Floquet theorem.

The electric field components that describe the structure can be written as follows

\[ E(x, y, z) = e^{j k_y y} G(x, y, z), \] (2.21)

where \( G \) is the periodic function which is a result of the repeated grooves per unit cell of length \( d \). Since \( G \) is periodic, it can be written for a general substrate as

\[ G(x, y, z) = \sum_{-\infty}^{\infty} a_m(x, z) e^{j \frac{2\pi m}{d} y}, \] (2.22)

where \( a_m \) is a function of \( x \) and \( z \). Using both (2.21) and (2.22), the electric field can be expressed
as

\[ E(x, y, z) = \sum_{-\infty}^{\infty} a_m(x, z) e^{j k_{ym} y}, \] (2.23)

where

\[ k_{ym} = k_y + \frac{2\pi}{d} m, \quad m = 0, \pm 1, \pm 2, \ldots \]

(2.23) is called the Floquet spatial harmonic expansion. This is a simple demonstration of Floquet
analysis, as in the full-wave simulation software (HFSS) in order to determine the general grating cell
design, the 3D equivalent model needs to be taken into consideration. This is referred to as the Floquet-
Figure 2.9: Diagrams showing an incident (a) TM polarized and (b) TE polarized plane-wave reflecting off a general metasurface at $z = 0$.

Bloch theorem, and thus in general the electric field components will be:

$$E(r) = E_0 e^{jkr} G(r).$$  \hspace{1cm} (2.24)

## 2.4 Metasurface Design Methodology

The method that was used to determine the properties of the BHM was done through surface impedance analysis. It was noted in [19] that discontinuities can be implemented in an electromagnetic wave through passive admittance or impedance surfaces. In this section, the required impedance will be determined in order to achieve retroreflection [19].

### 2.4.1 Surface Impedance Analysis

An illustration of a plane wave diffracting off a general surface is shown in Fig. 2.9. Both TE- and TM-polarized incident waves are shown in the figure because the design of the metasurface largely depends on the polarization, and such, the impedance analysis is generally done for both polarizations. Once a general surface impedance analysis is done, the specific case can be analyzed where $\theta_r = -\theta_i$. Analysis on the TM-polarized incident wave will be done here. Other works have done the TE analysis, but both follow a similar derivation [13].
TM Surface Impedance Analysis

The incident and reflected fields for the TM case is described below,

\[
E_i = E_{i0} \exp \left( -j k_0 (\sin \theta_i y - \cos \theta_i z) \right) \cdot (\cos \theta_i \hat{y} + \sin \theta_i \hat{z}),
\]

\[
H_i = \frac{E_{i0}}{\eta} \exp \left( -j k_0 (\sin \theta_i y - \cos \theta_i z) \right) \hat{x};
\]

and

\[
E_r = E_{r0} \exp \left( -j k_0 (\sin \theta_r y + \cos \theta_r z + \phi) \right) \cdot (-\cos \theta_r \hat{y} + \sin \theta_r \hat{z}),
\]

\[
H_r = \frac{E_{r0}}{\eta} \exp \left( -j k_0 (\sin \theta_r y + \cos \theta_r z + \phi) \right) \hat{x},
\]

where \( \lambda_0 \) is the free-space wavelength, \( k_0 = 2\pi/\lambda_0 \) is the spatial frequency, and \( \phi \) is a phase constant which describes the offset phase between the incident and reflected wave at \( y = 0 \). The tangential components of the incident and reflected wave on the surface \( (z = 0^+) \) is given by:

\[
E_{i,t} = E_{i0} \cos \theta_i \exp \left( -j k_0 \sin \theta_i y \right) \hat{y}
\]

\[
H_{i,t} = \frac{E_{i0}}{\eta} \exp \left( -j k_0 \sin \theta_i y \right) \hat{x}
\]

\[
E_{r,t} = E_{r0} \cos \theta_r \exp \left( -j k_0 \sin \theta_r y + \phi \right) \hat{y}
\]

\[
H_{r,t} = \frac{E_{r0}}{\eta} \exp \left( -j k_0 \sin \theta_r y + \phi \right) \hat{x}
\]

Two relationships will be introduced in order to simplify the derivation which will follow. The first is the phase of the reflection coefficient

\[
\Delta \Phi(y) = \angle(E_{r,t}/E_{i,t}) = -k_0 \sin \theta_r y + \phi + k_0 \sin \theta_i y.
\]

This represents the phase difference between the incident and reflected waves. Taking the derivative of (2.28) gives us the gradient between the reflected and incident angles. This gives us

\[
k_0 (\sin \theta_i - \sin \theta_r) = \frac{d\Phi(z)}{dz}.
\]
The second relationship is

\[ E_{r0} = \sqrt{\cos \theta_i \cos \theta_r} E_{i0}, \]  

(2.30)

which relates the plane-wave amplitudes of the incident and reflected waves in the case of a perfectly designed reflection metasurface.

With (2.28) and (2.30) the amplitude of the reflected wave \( E_r \) can be found, which will provide full power reflection from the metasurface, and generally there is a phase reflection that varies linearly with the surface along the \( y \)-direction. This approach will be greatly simplified with the aggressively discretized design that has been formulated, and this will be covered in section 2.4.2.

Using (2.28) and (2.30) the surface impedance as a function of the location on the metasurface (along the \( y \)-direction) can be formulated. This will allow us to generate the desired reflection given a prescribed incidence. The surface impedance for the case of a TM-polarized wave is shown below

\[ Z_{s,TM} = \eta \cos \theta \left( \frac{1 - e^{-j \Delta \Phi(y)}}{1 + e^{-j \Delta \Phi(y)}} \right), \]

(2.31)

where \( \eta \cos \theta \) represents the wave impedance for the incident and reflected TM-polarized waves.

Since retroreflection is of interest, \( \theta_r = -\theta_i \Rightarrow \cos \theta_r = \cos \theta_i \). Redefining \( \theta = |\theta_i| = |\theta_r| \), the following is obtained,

\[ Z_{s,TM} = j Z_{0,TM} \tan \left( \frac{\Delta \Phi(y)}{2} \right), \]  

(2.32)

It is generally more intuitive to work with reflection coefficients as opposed to surface impedances. The single plane wave retroreflection scenario has been used, and the reflection coefficient can be found as follows:

\[ \Gamma_{TM} = \frac{Z_{s,TM} - Z_{0,TM}}{Z_{s,TM} + Z_{0,TM}} = -e^{-j \Delta \Phi(y)}. \]  

(2.33)

Therefore, if the retroreflective metasurface satisfies the impedance surface characterized by (2.32) or the reflection coefficient (2.33), the surface should efficiently reflect all the power in the retroreflected direction. This, however, is not a surface that is easily implementable since it is a continuous distribution. Discretizing the board is essential, however, there come complications with discretization as will be seen
when designing a metasurface using this method.

Using (2.32) the periodicity of the metasurface can be determined by expanding the \( \tan \) function and knowing that the periodicity of a \( \tan \) function is \( \pi \):

\[
\pi = \frac{\Delta \Phi(y)}{2} = 2k_0 y \sin \theta_i \quad \text{solving for } y = \frac{\lambda_0}{2 \sin \theta_i}. \tag{2.34}
\]

Therefore, the metasurface can be thought of as a surface that repeats with a period equal to the equation given in (2.34). It is important to note that this is the same period as that obtained from (2.19). Therefore, there are similarities between a grating and this particular metasurface as the grating period is equal to the period the metasurface. However, when designing the metasurface cells, the surface boundary conditions are considered and the grating equation is not considered. It is also generally the case that gratings require device thickness (in terms of electrical length), whereas the metasurface is theoretically free from this limitation.

A similar approach can be taken for the case of an incident TE-polarized wave (Fig. 2.9b). This is shown in more detail in [13]. The resulting surface impedance will be

\[
Z_{s,TE} = \frac{\eta}{\sqrt{\cos \theta_i \cos \theta_r}} \frac{\sqrt{\cos \theta_i} e^{-j\Delta \Phi(y)}}{\sqrt{\cos \theta_r} - \sqrt{\cos \theta_i} e^{-j\Delta \Phi(y)}}. \tag{2.35}
\]

In the case of retroreflection, (2.35) reduces to

\[
Z_{s,TE} = -jZ_{0,TE} \cot \left( \frac{\Delta \Phi(y)}{2} \right), \tag{2.36}
\]

where \( Z_{0,TE} = \eta/\cos \theta \) represents the wave impedance for the incident and reflected TE-polarized waves. The resulting reflection coefficient is

\[
\Gamma_{TE} = \frac{Z_{s,TE} - Z_{0,TE}}{Z_{s,TE} + Z_{0,TE}} = e^{-j\Delta \Phi(y)} = -\Gamma_{TM}. \tag{2.37}
\]

2.4.2 Discretization Effects of Periodic Metasurface

In general, diffraction gratings have implemented a continuous pattern that is discretized into several elements per grating period, to produce a blazed grating (retroreflective grating). In this section, a unique way that makes it possible to aggressively discretize a number of cells per grating period to produce efficient retroreflection will be shown. It was shown in (2.18) and (2.19) how to obtain the length of the grating period, and this will also provide conditions for only allowing lower-order modes
to propagate. Therefore using (2.19) into (2.18) gives us

\[
\frac{\lambda}{2\sin \theta_i} \left( \sin \theta_r - \sin \theta_i \right) = m\lambda
\]

\[
\sin \theta_r = \sin \theta_i (2m + 1).
\]

(2.38)

Setting the appropriate limits to the above equation gives us

\[
|\sin \theta_r| = |\sin \theta_i (2m + 1)| \leq 1
\]

\[
\frac{-1}{2m + 1} \leq \sin \theta_i \leq \frac{1}{2m + 1}.
\]

(2.39)

With this simple equality, it can be easily shown that the minimum angle required for only the specular mode \((m = 0)\) and retroreflective mode \((m = -1)\) to propagate is \(\theta_i = 19.47^\circ\). The metasurface that will be designed has an incident angle of \(-82.87^\circ\), and thus it will only allow the propagation of these two modes, while every other mode will be evanescent in nature.

While the above analysis gives us the reasoning behind only having two modes that propagate, it does not give any information about the amplitudes of the propagating modes or the process of determining the minimum number of cells per grating period. Therefore, a diffraction-based approach will be employed to determine sufficient conditions for discretization of a periodic metasurface. This is shown in Fig. 2.10 where again \(k_g = 2\pi/\Lambda_g\), with \(\Lambda_g\) being the grating period. When a wave impinges on the metasurface, multiple diffraction orders reflect depending on whether they are in the propagating regime. The range for which \(k_y\) is in the propagating regime is \(\in [-k_0, k_0]\), where \(k_0\) represents the wave vector of the incident plane wave. These \(k\)-vectors on the right-most figure represent the plane waves that scatter off the metasurface and radiate into the farfield, with unique propagation angles

\[
\sin \theta = \frac{k_y}{k_0} \text{ for } k_y \leq k_0.
\]

(2.40)

This is essentially the mapping from the spatial frequency domain \(k_y\) to the angular domain \(\theta\). The \(k_y\)-vectors which lie outside of this region remain within the near-field regime and do not propagate to the farfield.

It was discussed earlier in (2.39) what the bounds on the minimum angle \(\theta\) were in order to get retroreflection while only allowing the specular and retroreflection modes to be in the propagating regime. Now, the bounds on the number of diffraction modes that propagate in a general system will be
Figure 2.10: Diagram representing how a periodic metasurface produces diffraction modes. The arrows represent the existence of a diffraction mode alone and do not describe the amplitude or phase. The purple box on the leftmost plot represents the propagation regime.

Figure 2.11: Diagram showing three cases of the propagation regime. (a) represents a general case (b,c) represents the extreme case where there is a mode $k_y = 0$.

considered, and then focus on retroreflection. This can be represented by

$$ N = \begin{cases} 2 \times \lceil \frac{k_0}{k_g} \rceil + 1 & \text{if } \frac{k_0}{k_g} \text{ is } \in \mathbb{Z} \\ 2 \times \lceil \frac{k_0}{k_g} \rceil & \text{otherwise,} \end{cases} $$

(2.41)

where $\lceil \cdot \rceil$ is the ceiling (round up) operator. The upper bound on the number of diffraction modes that propagate is shown above in (2.41). This equation can be extended to the design of the metasurface itself. Given that there are $N$ diffraction orders, it is required for the metasurface to have $N$ independent degrees of freedom to allow for characterizing the diffraction into the different modes. Therefore, if there are $N$ Huygens’ sources within a grating period $\Lambda_g$, it is possible to have complete control over the propagating diffraction modes. (2.42) is represented visually in Fig. 2.11 where three cases are provided to describe three different types of metasurfaces and the diffraction modes that may arise.

In the case of retroreflection, the discretization requirements can be relaxed to be

$$ N = 2 \times \lfloor \frac{k_0}{k_g} \rfloor, $$

(2.42)

where $\lfloor \cdot \rfloor$ is the rounding operator. If (2.19) is combined, (2.40) and (2.42) the same relationship will
arise form that of (2.39).

\[ \theta_i \geq 19.5^\circ \Rightarrow k_g > \frac{2}{3}k_0 \Rightarrow N = 2. \]  

(2.43)

The inequality represented in (2.43) shows that retroreflection is possible with only \( N=2 \) cells per grating period if the incident angle is larger than \( \theta = 19.5^\circ \). Therefore, if the incident angle of \( \theta_i = 82.87^\circ \) was used, the determination of the phasing along a metasurface could be found by using (2.33) and (2.37), because this is the phase profile that enables an incident wave to perfectly retroreflect. This in turn means that the specular reflection will be completely suppressed. The equation below shows the phasing that is needed using two cells that are equally spaced:

\[ \Gamma_{TM} = -e^{-j\Delta \Phi(y)}, \]

\[ \{ \Delta \Phi(y) = 2k_0y \sin(\theta_i) \Rightarrow y = 0mm, 3.15mm \}, \]

\[ \Gamma_{TM}(y = 0mm) = -e^{-0}, \]

\[ \Gamma_{TM}(y = 3.15mm) = -e^{-\pi}. \]

If these two cells are placed an equal distance from each other, it is clear from the derivation above that perfect retroreflection is obtainable, if the two cells have unity reflection and have a relative phase of \( 180^\circ \) between them. It is possible however to have these two cells placed at different locations within the period, and in that case, a different relative phase must be present between the two cells. To illustrate this correct phasing, Fig. 2.12 shows the phasing along a period for two different cases. The more obvious design choice would be Fig. 2.12a.

Maximal Discretization using Fourier Analysis

In order to understand the reason why the minimum number of cells per grating period can be characterized by (2.41) and (2.42), a Fourier analysis will be done on the scattered field. The scattered wave for a periodic metasurface can be described by

\[ E_{\text{scat}} = \sum_mE_m \exp(jk_my)\hat{y}, \]  

(2.44)

where \( k_m \) \((k_m = k_{iy} + mk_g)\) represents the propagating and evanescent orders, and \( E_m \) represents the complex phasor for the \( m^{th} \) mode. The respective magnetic field for each mode can be represented
Figure 2.12: Diagram showing two cases which incorporate two cells per period. (a) Represents the case where the two cells are evenly distributed, (b) represents the case where the cells are not evenly distributed per period.

by

\[
\mathbf{H}_m = \frac{E_m}{\eta_0} \left( \sqrt{k_0^2 - k_m^2} \mathbf{\hat{y}} + k_m \mathbf{\hat{z}} \right),
\]

(2.45)

where \( \eta_0 \) represents the intrinsic impedance of free-space. The addition of (2.45) does not provide any extra information, and thus this equation will not be considered since the \( E_m \) field gives all the information that is needed.

The index will shift from \( m \) to \( n \) to only focus on the propagating modes. This means that the original equation will be changed to be represented in (2.44) from \( n = 1 \) to \( N \) (where \( N \) is the total number of propagating modes), and obtain a result that represents the propagating farfield. The assumption that will be made is that if complete control is desired of the set \( \{E_n\} \) for \( n = 1 \) to \( N \), it is sufficient to control \( N \) points of the metasurface in the near-field electric field (scattered electric field) on the surface of the metasurface. The obvious choice of these \( N \) points within the metasurface is to choose equidistant points within one period. These specific points of interest will be represented by the locations \( y_p \) and the respective complex scattered field \( a_p \), for \( p = 1 \) to \( N \). Therefore, given \( \{a_p, y_p, k_n\} \), \( E_n \) can be determined through the Fourier transform

\[
\mathbf{E}_n = C \sum_{p=1}^{N} a_p \exp(jk_n y_p),
\]

(2.46)

where \( C \) represents the power normalization constant.
If (2.46) is written in matrix form, the following is obtained

\[
\vec{E} = M \vec{A}
\]

where

\[
\vec{E} = \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_N
\end{bmatrix},
M = C \begin{bmatrix}
e^{-jk_1 y_1} & e^{-jk_1 y_2} & \ldots & e^{-jk_1 y_N} \\
e^{-jk_2 y_1} & e^{-jk_2 y_2} & \ldots & e^{-jk_2 y_N} \\
\vdots & \vdots & \ddots & \vdots \\
e^{-jk_N y_1} & e^{-jk_N y_2} & \ldots & e^{-jk_N y_N}
\end{bmatrix},
\text{ and } \vec{A} = \begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_N
\end{bmatrix}. \tag{2.47}
\]

This shows that it is possible to transform between \(a_n\) and \(E_n\) if there are \(N\) independent points within a period. Therefore, \(N\) equidistant points within a metasurface period are enough to characterize the \(N\) diffraction modes in the farfield. The \(N \times N\) matrix \(M\) is also invertible because \(M^{-1}\) is the inverse Fourier transform of the matrix, and so \(N\) independent parameters are generated to uniquely define all the modes. Therefore, the desired scattered waves can be completely determined by inverting \(M\).

### 2.5 Power Flow Analysis

The amount of power that can couple from the incident electromagnetic field to the metasurface is highly dependent on the projected area of the metasurface. The projected area of the metasurface is its projection onto a plane normal to the incident plane wave. As shown in Fig. 2.13, the angle \(\theta\) is defined as the angle that the antenna makes with respect to the normal of the metasurface. The projected area can be defined as,

\[
A_p = A_{MS} \cos \theta,
\]

where \(A_{MS}\) is the area of the metasurface.

The method to determine the strength or efficiency of the retroreflected power as a function of the incident antenna emitting power can be done using the Friis transmission equation. This is an equation which relates the power transmitted and received if the separation distance \(R\) satisfies

\[
R > \frac{2D^2}{\lambda}, \tag{2.49}
\]

where \(D\) is the largest effective dimension of the metasurface.

Fig. 2.14 represents the general case of a transmitting antenna emitting a wave which reflects off a metasurface and is received by another antenna. In our case \(R_1\) and \(R_2\) are equal and the transmit and receive antennas are the same, and thus \(\theta_1\) and \(\theta_2\) are also equal.
Figure 2.13: The effective area of the metasurface (black rectangle) is found by the projection of the horn onto the metasurface (shown by the blue shaded area). Calculating the effective area is done by multiplying the area of the metasurface with the angle ($\theta$) that the horn makes with the normal of the metasurface.

Figure 2.14: A transmit antenna emitting a plane wave at a distance of $R_1$ from a metasurface. This wave reflects off a metasurface of area $A_{MS}$ and then power is received from a receiving antenna a distance of $R_2$ away.
Chapter 2. Theoretical Background

There are two parts to understanding the power efficiency of the retroreflective metasurface. The first part consists of the total power that is received from the metasurface, and the second is the total power that is transmitted from the metasurface. The first part is denoted by

\[ P_r = P_{tar} \left( \frac{\lambda}{4\pi R} \right)^2 G_{tar} G_r, \]  

(2.50)

where \( P_r \) is the power received at the receiving antenna, \( P_{tar} \) is the power reflected from the target, \( G_{tar} \) is the gain of the target, and \( G_r \) is the gain of the receiving antenna. \( P_{tar} \) must be expanded to understand what happens at the transmitting antenna location. This will give us:

\[ P_{tar} = \left( \frac{P_t G_t}{4\pi R^2} \right) \cos \theta \left( \frac{\lambda}{4\pi R} \right)^2 G_{tar} G_r, \]  

(2.51)

where \( P_t \) is the power transmitted from the transmitting antenna, and \( G_t \) is the gain of the transmitting antenna. It is assumed that the metasurface itself is the rectangular aperture. To solve for \( G_{tar} \) the directivity of a rectangular aperture needs to be known, which is given by [20]:

\[ D_{tar} = \left( \frac{4\pi}{\lambda^2} \right) A \cos \theta. \]  

(2.52)

Also, if the radiation efficiency is assumed to be equal to unity, the gain will equal the directivity. Therefore, the final equation for the power efficiency of a retroreflective surface is:

\[ \frac{P_r}{P_t} = \left( \frac{G_t A \cos \theta}{4\pi R^2} \right)^2. \]  

(2.53)

Therefore, the ratio of the power received compared to the power that is transmitted can be found using the gain of the antenna, the distance from the antenna to the retroreflector and the effective aperture of the retroreflector. These are the only constraints determining the power efficiency of a retroreflector. However, in general this efficiency will be quite low because the loss due to the propagation of the wave will be very significant. The important component of this efficiency is how well it compares with a metallic plate with an equivalent aperture size. Thus, when finding the power received from the receiving horn, the comparison that needs to be made is that with the ratio defined by (2.53) with that from the retroreflector with an equivalent aperture size.
Chapter 3

Design Procedure and Simulations

The design of the retroreflective metasurface can be described by Fig. 3.1. When a typical smooth surface is considered, it generally reflects in the specular direction, whereas the design that is being aimed for is to suppress the specular reflection mode and couple all the energy into the retroreflection mode. There are two designs in mind because both TE and TM polarizations are considered. A TE polarized wave is where the $E$-field is directed in the $x$-direction, whereas a TM polarized wave is such that the $H$-field is directed in the $x$-direction. The two modes that will be in the propagating regime will be the specular ($m = 0$), and the retroreflection ($m = -1$) mode, however as was explained earlier, the power coupled into the specular reflection must be greatly suppressed such that efficient retroreflection can be obtained. In this design, a 24GHz wave will be incident on the retroreflective metasurface at a near-grazing incidence angle of $\theta_i = -82.87^\circ$ with respect to the normal. Using (2.19), a grating period will be

![Diagram showing the basic geometry of an incident wave which specularly reflects and retroreflects.](image)

Figure 3.1: Diagram showing the basic geometry of an incident wave which specularly reflects and retroreflects.
Figure 3.2: Metasurface design for an incident TE-polarized 24GHz wave. (a) The geometry of an individual cell, with dimensions: $U_x = U_y = 3.149\text{mm}$, $S_z = 1.575\text{mm}$, $P_y = 0.5\text{mm}$. (b) Top view of the two-cell (single grating period) TE metasurface. These dimensions are $P_y = 1.5\text{mm}$, $P_{x1} = 2.16\text{mm}$, $P_{x2} = 2.35\text{mm}$.

$\Lambda_g = 6.30\text{mm}$.

Using (2.42) the grating period can be discretized into two cells, such that each unit cell will have a size equal to

$$U_y = \frac{\Lambda_g}{2} = 3.15\text{mm}.$$ 

### 3.1 Design Procedure for TE Polarized Metasurfaces

#### 3.1.1 TE Metasurface Element Design

The TE-polarized metasurface was designed using a ground-backed dipole array. This was used because it was shown in previous works [21, 22] that this simple structure contains Huygens’ source characteristics when considering reflection mode operation, and it has the ability for $\Gamma_{TE}$ to have a phase range of 360° with minimal loss by tuning the length of the dipole. The individual cell is shown in Fig. 3.2a.

The simulation for the scattering properties of this design was done using a full-wave electromagnetic simulation software, Ansys HFSS. Periodic boundary conditions were applied to the $x$- and $y$-directed walls, with the appropriate phase shifts related to the incident plane wave at $\theta_i = -82.87°$. The excitation source was a Floquet port which was applied from the $+z$ boundary. The understanding of the Floquet ports is done in Section 2.3. This 2D infinitely periodic simulation uses a Floquet port as an incident plane wave at $-82.87°$ and then the reflection coefficients ($\Gamma_{TE}$) are observed. The dipole length from $P_x = 1.5\text{mm}$ to $3\text{mm}$ is swept and Fig. 3.3 shows the variation in amplitude and phase of the reflection.
Figure 3.3: Single grating period metasurface design for an incident TE-polarized 24GHz wave. Plot of the reflection coefficient of the specular reflection $\Gamma_{TE}$, as $P_x$ varies from 1.5mm to 3mm.

coefficient as the dipole length is increased. The results are as expected with a phase range approaching $360^\circ$, with minimal loss in the reflection amplitude. It was already determined that a $180^\circ$ phase difference was required between the two single-cells which are contained inside a grating period, and thus from the polar plot, it is clear that at $P_{x1} = 2.16$mm and $P_{x2} = 2.35$mm, the phase difference is approximately $180^\circ$ and the amplitude of the reflected coefficient is nearly unity. This metasurface was designed on a RT/Duroid 5880 Laminate board from Rogers Corp., with a substrate thickness $S_z = 1.575$mm and 1/2 oz. copper cladding for the ground plane and the patch. The two single-cells have the same dipole width $P_y = 0.5$mm, and the cell size is $U_x = U_y = 3.15$mm.

3.1.2 Periodic Metasurface Simulation

Once $P_{x1}$ and $P_{x2}$ were found using the single-cell design, they were placed beside each other as shown in Fig. 3.3b and two simulations were done to observe the scattering properties of the grating-cell (two-cell). The first simulation was an extension to the simulation done above for the single-cell, however in this case a 2D infinitely periodic Floquet simulation was done for two cells (grating-cell) which is essentially a period of the metasurface. With this simulation, it was found that the scattered power coupled to the retroreflection and specular modes were 94% and 6% respectively. This shows that it is possible to obtain efficient retroreflection while simultaneously suppressing specular reflection. In the second simulation, a 1D finite simulation was performed where the metasurface was truncated to 136 cells in the $y$-direction and infinite in the $x$-direction. The choice of 136 cells was to have consistency with the fabrication board.
Figure 3.4: The simulated results for the TE retroreflection metasurface operating at 24GHz. (a) The geometry of the finite element (1D Finite) simulation. The $x$-directed (blue) faces are infinitely periodic boundaries; the $y$- and top $z$-directed faces are radiation/absorbing boundaries; the bottom $z$-directed face is a copper layer. (b) A simulated scattering pattern for $\phi = 90^\circ$ (the $yz$-plane) was plotted where the maximum reflection power was used to normalize the plot.

Figure 3.5: The simulated results for the TE retroreflection metasurface operating at 24GHz. The simulated monostatic RCS for the metasurface (blue, solid) plotted alongside the theoretical RCS of a metallic plate with the same effective aperture, placed normal to the incident radiation (red, dotted) (see Fig. 2.13 for further explanation).
size that was possible. The $x$-direction is infinitely periodic to conserve computational resources. This is described in Fig. 3.4a. An air gap of approximately $\lambda_0/2$ is also left in the $\pm x$- and $\pm z$-directions, and there are radiation boundary conditions on the $y$- and top $z$-directed faces, which are perfectly matched layers (PMLs). Fig. 3.4b shows the bistatic radiation cross section (RCS) scattering pattern for an incident plane wave impinging on the metasurface at $\theta_i = -82.87^\circ$. This is the scattering pattern with a cross section at $\phi=90^\circ$ ($yz$-plane). It is clearly visible that there is a strong retro-reflected component, while simultaneously having a weak specular reflection. The monostatic RCS is shown in Fig. 3.5, also with a cross section at $\phi=90^\circ$ ($yz$-plane). The blue line represents the simulated results and shows the symmetric properties of the metasurface, given the fact that it consists of two cells that periodically repeat and thus are independent of whether the incoming angle is $\theta_i = \pm 82.87^\circ$. The dotted red line represents the power that would be retro-reflected if an equivalent metallic plate was put in the place of the metasurface that covered the same effective aperture as shown in Fig. 2.13. The size of the effective aperture scales with $\cos \theta$ and the power shown with the dotted red line scales with $\cos^2 \theta$ [23]. The plot was normalized such that at broadside the equivalent metallic plate would have an aperture efficiency of 100%. When comparing the effective aperture to the normalized measurements, the difference can be used to determine the efficiency of metasurface when compared to a perfect reflector. An angle of interest is $\pm 82^\circ$, and the BHM achieves a monostatic RCS level that is only 0.3dB lower than the effective aperture, which is equivalent to an aperture efficiency that is 93%. Therefore, this BHM design is able to produce very efficient retroreflection at near grazing angles. Further optimization could be done however to increase the retroreflection by incorporating the effects of mutual coupling between the two cells, and this is something that will be covered in the BHM for TM-polarized waves.

3.2 Design Procedure for TM Polarized Metasurfaces

3.2.1 Babinet's and Booker's Principle

The design of the TM metasurface is done in a similar fashion to that of its TE counterpart, however, the metasurface element will no longer be made up of two dipoles. At angles near-grazing the electric field component of a TM-polarized incident wave is directed predominately in the $z$-direction (as shown in Fig. 2.9a), which would not couple adequately with a metallic dipole element. A design methodology to get around this is to use elements that consist of slots, such that the magnetic field couples to the elements rather than the electric field. This is very similar to the impedance and admittance surfaces which induce electric and magnetic currents covered in Section 2.1. A slightly different approach
Chapter 3. Design Procedure and Simulations

3.2.2 TM Metasurface Element Design

Fig. 3.7a shows the general schematic for a single-cell TM metasurface element. This element consists of a rectangular slot which is etched on the substrate, with a length and width of $P_x$ and $P_y$ respectively. The periodicity is the same as that of the TE metasurface, since it only depends on the incident angle ($U_x = U_y = 3.149$mm), however, in this case, the substrate is thicker ($S_z = 3.175$mm). This metasurface is again designed using a Rogers RT/Duroid 5880 laminate board with 1/2 oz. copper cladding on both sides.
A parametric sweep of the length of the slot \( P_x \) is done from 0mm to 3.149mm to accurately tune the coupling dynamics of the incoming/outgoing waves with the ground-backed slot array. This in turn tunes the reflection coefficient \( \Gamma_{TM} \). Fig. 3.8 plots \( \Gamma_{TM} \) as the slot length \( P_x \) is swept from 0mm to 3.149mm. The simulation conditions followed the same format as that for the TE metasurface as described in Section 3.1.1. It is observed when analyzing this plot that \( \Gamma_{TM} \) achieves a magnitude of approximately 1 (100% reflection), however, the phase variation as \( P_x \) varies is only around 190°, which is quite different than what is achievable with the TE-polarized design of nearly 360°. The reason for the limited phase variation is due to the transformation of the metasurface from the TE-polarized to TM-polarized design. Babinet’s and Booker’s equivalent models were not perfectly applied in this case, as the dielectric and ground plane remained unchanged (in terms of material). If a design that perfectly modeled the equivalent TM metasurface was desired, the Rogers dielectric would need to be replaced with a material containing a specific magnetic permeability. The ground plane would also have to be replaced with a magnetic conductor, which is not physically realizable. Despite this practical limitation, the reflection response is sufficient for our particular design, as only 180° is needed between the two slots per grating period. By analyzing Fig. 3.8, the initial operation points were chosen to be \( P_{x1} = 0.8 \text{mm} \) and \( P_{x2} = 3.149 \text{mm} \). This is displayed in Fig. 3.7b.

### 3.2.3 Periodic Metasurface Simulation

Upon choosing the slot lengths \( P_{x1} \) and \( P_{x2} \), a Floquet analysis can again be performed which enables the understanding of the scattering parameters for a 2D infinite extension of this BHM. The scattered
power into the retroreflected mode and specular modes were 84.3% and 15.5% respectively. This level is not comparable to the TE metasurface where the retroreflected power was 94%, and so a parametric sweep of $P_{x1}$ was done in order to determine if a greater retroreflection efficiency was obtainable. It was found that when $P_{x1} = 1.6$mm, the power into the retroreflected mode and specular mode was 99.1% and 0% respectively. This shows that if the two-cell design is optimized, it is possible to achieve even greater retroreflection efficiency, while simultaneously suppressing specular reflection greatly. The reason for the discrepancy between the one-cell and two-cell results could be due to the mutual coupling between the two slots. In the case of the dipole design (for the TE metasurface), there was significant isolation between adjacent cells. In the slot design, however, there is direct metallic contact between two adjacent cells and could cause the effects of mutual coupling to be much more significant.

A 1D finite simulation was then performed where the metasurface was again truncated to 136 cells, to determine the scattering properties of a finite metasurface. Boundary conditions were applied in a fashion that was similar to the TE metasurface case. The simulated bistatic RCS pattern is shown in Fig. 3.9a in the $\phi = 90^\circ$ plane ($yz$-plane). This shows that again retroreflection is strong, while the specular reflection is very weak. Fig. 3.10 shows the monostatic RCS and achieves nearly 100% retroreflection efficiency for $\pm 82^\circ$. 

Figure 3.9: The simulated results for the TM retroreflection metasurface operating at 24GHz. (a) A simulated scattering pattern for $\phi = 90^\circ$ (the $yz$-plane) was plotted. (b) A simulated scattering pattern with lossy material at the ends of the substrate for $\phi = 90^\circ$ (the $yz$-plane) was plotted.
3.2.4 Spurious Reflection

While observing Fig. 3.9a it is evident that it achieves desirable results, however, there is spurious reflection at $-37^\circ$. According to grating theory, there should not exist another mode within the propagating regime besides the specular and retroreflection mode. Therefore this spurious reflection is quite peculiar. It was concluded that since it was not due to grating theory, it was due to the coupling of the incident wave to spurious radiation (surface waves), which then re-radiates.

The first approach that was taken to solve this problem was to place a PEC on the leftmost wall of the substrate. The reason for doing this was because if surface waves did travel within the substrate, these PECs may allow us to understand whether placing a perfect reflector on one of the walls changes any of the results. Two simulations were run, one with an incident plane wave at $-82.87^\circ$ and then at $+82.87^\circ$. The bistatic RCS is shown in Fig. 3.11, and it is clear that when the incident field is impinging on the metasurface from $-82.87^\circ$ there is a suppression of the spurious mode, whereas when the incident field impinges from $+82.87^\circ$ the spurious mode is strengthened. This showed that the reflection from the walls of the substrate may be the cause for the increase in power of this spurious mode.

The second approach was to create a material that was lossy on both ends of the metasurface. This was necessary because if there existed surface waves within the substrate, the lossy material on both ends would absorb the energy, resulting in the suppression of this spurious reflection. This lossy material
Figure 3.11: The simulated results for the TM retroreflection metasurface operating at 24GHz. There is also a PEC on the leftmost wall of the substrate (a) A simulated scattering pattern with an incident plane wave at $-82.87^\circ$ for $\phi = 90^\circ$ (the $yz$-plane) was plotted. (b) A simulated scattering pattern with an incident plane wave at $+82.87^\circ$ for $\phi = 90^\circ$ (the $yz$-plane) was plotted.

Figure 3.12: The metasurface with lossy walls on both ends. The purple in this diagram represents the metasurface. There are then lossy electric material placed on both sides of the metasurface with increasing loss as denoted by the gradient. In this picture only the leftmost wall is shown, however the rightmost wall follows the same process.
was composed of 6 layers that each increased in loss in order to reduce reflections. This method was used because it approximates the way in which perfectly matched layers (PMLs) operate, and is shown in Fig. 3.12. Fig. 3.9b shows the resulting bistatic RCS, and there indeed is a reduction in the spurious mode, and thus proves that it originates from coupling into surface waves and reradiation at the edge of the metasurface. The problem with this additional lossy material is the loss of power in the retroreflected direction. The retroreflected power was reduced by 0.8dB while the specular reflection was increased by 2.2dB, which shows a major trade-off between the two designs.

3.2.5 Optimization

It was noticed that in the finite simulations, the reflected modes are not exactly the angle that is desired. If Fig. 3.9a is observed, it shows that the direction of the reflected mode is maximal at around -80°, whereas the desired direction is -82.87°. The reason for this is due to the finite size of the metasurface. Therefore, a way to counteract this is by designing a metasurface with an incident angle of -82.87° and a reflected angle of something that is closer to -90°, such that when the metasurface is reduced to a finite size, it will operate as a retroreflector. The more obvious way to counteract this is by having a metasurface that is larger since that will enable the reflected mode to be closer to the reflection direction. This is shown in Fig. 3.13, where as the size of the metasurface increases from 100 cells to 200 cells, the reflected angles changes from -79° to -81°. The TE metasurface was considered as the visual interpretation of the plots are more clear. However, while this method is the obvious solution, it may not always be practical when there is a restriction in space or material cost to consider.
3.3 Near-field to Far-field Transformation

In order to verify the results given by the simulation in HFSS, it was desired to use the near-field components of the electric and magnetic fields in order to determine the farfield pattern. This was initially done because of some inconsistencies with the results in HFSS in the early stages of the simulation. There are different methods that can be used to find the farfield pattern from the near-field, but the method used here involved electric and magnetic surface currents.

3.3.1 Using E & H Fields

Generally, the near-field equations can be quite complex, and so when the farfield is considered ($\beta r >> 1$), the equations generally simplify. In the farfield $E$-field and $H$-field components become orthogonal to each other and form TEM modes to $r$ [25]. In the case of determining the farfield components from a rectangular aperture, it can be found through

\[
E_r \simeq 0, \\
E_\theta \simeq -\frac{j\beta e^{-j\beta r}}{4\pi r} (L_\phi + \eta N_\theta), \\
E_\phi \simeq +\frac{j\beta e^{-j\beta r}}{4\pi r} (L_\theta - \eta N_\phi),
\]

\[
H_r \simeq 0, \\
H_\theta \simeq +\frac{j\beta e^{-j\beta r}}{4\pi r} (N_\phi + \frac{L_\theta}{\eta}), \\
H_\phi \simeq -\frac{j\beta e^{-j\beta r}}{4\pi r} (N_\theta + \frac{L_\phi}{\eta}),
\]

where $N_\theta$, $N_\phi$, $L_\theta$ and $L_\phi$ are represented by the equations below,

\[
N_\theta = \iint_S (J_x \cos \theta \cos \phi + J_y \cos \theta \sin \phi - J_z \sin \theta) e^{+j\beta r' \cos \Psi} \, dx' \, dy',
\]

\[
N_\phi = \iint_S (-J_x \sin \phi + J_y \cos \phi) e^{+j\beta r' \cos \Psi} \, dx' \, dy',
\]

\[
L_\theta = \iint_S (M_x \cos \theta \cos \phi + M_y \cos \theta \sin \phi - M_z \sin \theta) e^{+j\beta r' \cos \Psi} \, dx' \, dy',
\]

\[
L_\phi = \iint_S (-M_x \sin \phi + M_y \cos \phi) e^{+j\beta r' \cos \Psi} \, dx' \, dy',
\]
where $S$ represents the aperture. The general geometry of the problem is shown in Fig. 3.14, and $J_x$, $J_y$, $M_x$ and $M_y$ represent the surface currents along the aperture and can be defined as

$$ J_s = \hat{n} \times H_a, \quad (3.11) $$

$$ M_s = -\hat{n} \times E_a, \quad (3.12) $$

where $E_a$ and $H_a$ represent the total electric and magnetic fields over the surface $S$ respectively, and $\hat{n}$ is the normal unit vector to the surface $S$.

For simplicity, only the E and H fields on top of the aperture were taken. A more complete analysis could have been done to generate the near-field components from all sides of the metasurface and then summed in the farfield. Fig. 3.15 shows the comparison between the HFSS farfield results and that obtained using (3.1-3.10) in Matlab. There are similarities in terms of the retroreflected component, however, the spurious reflection in the HFSS simulation is not present in Fig. 3.15b, and the magnitude of the retroreflected components are different. This may result in believing that either one of the simulations is incorrect, however, it is important to note that this comparison is between the farfield results generated by HFSS using all sides of the metasurface, whereas the one generated through Matlab only used the top surface. It is quite interesting to note that the spurious mode is not present when only considering the top aperture, and shows that it could be caused by leakage on the sides of the aperture.

### 3.3.2 Understanding the Magnetic Currents, and the Voltages

In order to understand the effects of the incident TM-polarized wave on the slots, an analysis of the slots was done by extracting the voltages across the slots and then analyzed within Matlab. When observing the magnetic current in Fig. 3.17, there is a shift in the strength from both ends of the same slot,
Figure 3.15: Bistatic RCS for the TM metasurface using two different methods. (a) Using the near fields, HFSS was used to generate the farfield projection. (b) Using Matlab to generate the farfield projection using the near-fields on top of the metasurface.

Figure 3.16: The potential difference across the slots were extracted from HFSS where the the two cells per grating period are called "Slot A" and "Slot B". However “Slot A” and “Slot B” is quite consistent in magnitude when comparing both slots A and B. The phase between adjacent slots are also interesting to observe as shown in Fig. 3.18, because the phase difference between “Slot A” and “Slot B” is approximately 180°, which follows the theoretical results. Fig. 3.18b also shows that the phase change from the side walls of the slots are very minimal.
Figure 3.17: This represents the magnetic current magnitude.

Figure 3.18: This represents the magnetic current phase distribution along the slots. (a) Shows the entire array for a 130 cell simulation. (b) Shows a zoom-in where it compares Slot A with Slot B.
Chapter 4

Retroreflective Metasurface Measurements

4.1 TE Metasurface

The TE metasurface was fabricated with 87 cells in the $x$-direction and 136 cells in the $y$-direction (the same number of cells as the 1D finite simulation). The total area of this metasurface is $428\text{mm} \times 275\text{mm}$. Two types of measurements were done on the metasurface: a monostatic and bistatic RCS. The setup for both the monostatic and a bistatic RCS is shown in Fig. 4.1.

4.1.1 Monostatic RCS

The monostatic RCS measurement was done in an Anechoic chamber, where a vertically polarized K-band horn was placed at one end of the chamber, and the metasurface was placed at the other end of the chamber, $5.3\text{m}$ away. This distance corresponds to the farfield regime of the wave. A VNA was used to determine the retroreflected component at every angle. The $S_{11}$ signal was measured using the VNA, however, reflections produced by the horn caused the reflections from the metasurface to become drowned out. Therefore, the time-gate function was applied. This allowed us to only measure the received signal from the horn that is represented by the time it takes the wave to reach the metasurface and reflect back towards the horn. This is shown in Fig. 4.3. Along with the time gating function, a background subtraction was done such that any background noise from the platform or the chamber would be cancelled out.

The monostatic RCS was measured and then compared with a copper plate with the same effective
Figure 4.1: (a) The monostatic RCS setup with the metasurface (green) on a rotating stage. The effective area is shown as the projection of the horn onto the aperture. (b) The bistatic RCS setup: there are limitations on the variable angle in this setup because the movable receiver horn can only measure angles $+3^\circ$ from the fixed horn.

Figure 4.2: Bistatic RCS measurement setup.
Figure 4.3: (a) Diagram showing the time it takes to reach the end of the horn \( t_h \) and the metasurface \( t_s \). (b) An example of \( S_{11} \) as a function of time, with region that is time gated.

aperture. Fig. 4.4 shows the measured results. At \( \pm 81^\circ \) the retroreflected power is 0.1dB smaller than that of the effective copper plate. This is equivalent to an aperture efficiency of 90%. Therefore, when the effective aperture is taken into account, the retroreflected power is very strong near the desired angle. The reason for the shift from \( \pm 82.87^\circ \) to \( \pm 81^\circ \) is again due to the finite size of the board.

4.1.2 Bistatic RCS

The bistatic RCS measurement setup is shown in Fig. 4.1b. This was built in-house with the purpose of making the measurements for this metasurface. The final setup outlay is shown in Fig. 4.2, where the metasurface is placed on a platform located between two arms. The \( S_{21} \) signal is measured from the Rx horn using a VNA with the same functions applied as the monostatic measurement (i.e. time gating and background noise subtraction). The measured results are shown in Fig. 4.5. The measurement includes the metasurface and an equivalent copper plate in order to make a comparison of the power efficiency. It can be seen that the retroreflected power at -82.87° is equivalent to 93% of the power that specularly reflects off an equivalent copper plate at 82.87°. Specular reflection around 82.87° is reduced to only 10% when compared to the same copper plate. The exact reflection at 82.87° is much less than 10%, however, the finite size of the metasurface will cause the reflected angle to be slightly different. At 79° there is appreciable reflection, and therefore greater efficiency is obtained if the metasurface size increased.
Figure 4.4: Monostatic RCS measurement for the TE metasurface (blue, solid) at 24GHz, and an effective aperture (red, dotted). The angle is with respect to broadside incidence.

Figure 4.5: Bistatic RCS measurement comparing the TE metasurface (blue, solid) with an equivalent copper plate (red, dotted) at 24GHz. The figure is normalized to the specular reflection power of the equivalent copper plate.
Chapter 4. Retroreflective Metasurface Measurements

Figure 4.6: Monostatic RCS measurement for the TM metasurface (blue, solid) at 24GHz, and the theoretical effective aperture (red, dotted).

4.2 TM Metasurface

The TM metasurface was also fabricated with 87 cells in the $x$-direction and 136 cells in the $y$-direction (the same number of cells as the 1D finite simulation). The total area of this metasurface is 428mm × 275mm. The two types of measurements (monostatic and bistatic RCS) done for the TE metasurface were also done for the TM metasurface.

4.2.1 Monostatic RCS

Fig. 4.6 shows the measured results for the monostatic RCS of the TM metasurface, and when compared with the effective copper plate at ±82.87°, there is only a difference of 0.2dB, which signifies an aperture efficiency of 95%. This metasurface again shows that most of the power is coupled into the retroreflected mode, which simultaneously means that the specular reflection is suppressed. Fig. 4.6 also corroborates with the simulated results where the retroreflected power at ±82.87° and ±37° lies in the range of -18dB to -15dB. If the retroreflection of the metasurface is compared to that of a 4-inch corner cube with approximately the same effective aperture, it is found that the magnitude of the retro-reflected power is within 1 dB of each other, and thus gives us reason to trust that the BHM retroreflector can compare well to traditional retroreflectors, while maintaining a low profile, and simple design.
4.2.2 Bistatic RCS

The bistatic RCS experiment was done slightly differently than the test done for the TE metasurface. The reason for this is due to the fact that it was observed from the simulated and measured results that there was optimal retroreflection at $-81^\circ$ rather than at $-82.87^\circ$. This is largely due to the specific size of the metasurface, as was described in Section 3.2.5. Fig. 4.7 shows the bistatic RCS for the TM metasurface, where it can be seen that it follows the simulated results. The retroreflected power at $-81^\circ$ is approximately 93% of what would specularly reflect off the equivalent copper plate. Once more, this proves that there is a strong coupling of the incident wave to the retroreflected mode.
Chapter 5

Advanced Metasurfaces: Angular Range Extension and Absorption

5.1 Chirped Design

When looking at the simulated (Fig. 3.5 & Fig. 3.10) and measured (Fig. 4.4 & Fig. 4.6) results for the TE and TM metasurfaces, it is quite clear that the angular spread in the range in which retroreflection can occur is very small. The retroreflected power does not work efficiently when it operates at angles that differ from the designed angle. In this section, alternative designs are found which could possibly allow for broad-angle operation. It will also be shown that using this same design, it is possible to achieve a broader bandwidth retroreflective response.

5.1.1 Wide-Angle Design

The first design was to create three sections of a metasurface which each retroreflects at a specific angle. The angles that were chosen were -87.6°, -82.9°, and -76° as shown in Fig. 5.2a. The choice of these three angles was arbitrary, but in general, any angle choice is possible. The design assumed a TM-polarized incident plane wave and the analysis follows the exact same procedure as that found in Section 3.2, where the only difference would be the angle of incidence (and retroreflection consequently). However, there are three angles to consider because there has to be analysis done such that the 180° phase shift is present within the three sections. The design parameters are given in Fig. 5.1.

The second design used the same three angles, however, there was a gradual shift from -87.6° to -76°, rather than three distinct metasurfaces put together. This was referred to as the chirp retroreflective
Chapter 5. Advanced Metasurfaces: Angular Range Extension and Absorption

Figure 5.1: The parameters of the three different angles. Only $P_{x1}$ and $U_y$ are shown because those are the two parameters that changed from the three angle designs.

<table>
<thead>
<tr>
<th></th>
<th>Cell A: 87.6°</th>
<th>Cell B: 82.9°</th>
<th>Cell C: 76°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{x1}$</td>
<td>1 mm</td>
<td>0.8 mm</td>
<td>2.42 mm</td>
</tr>
<tr>
<td>$U_y$</td>
<td>3.13 mm</td>
<td>3.15 mm</td>
<td>3.22 mm</td>
</tr>
<tr>
<td>$\theta$</td>
<td>87.6°</td>
<td>82.9°</td>
<td>76°</td>
</tr>
</tbody>
</table>

Figure 5.2: (a) Chirp design that contain three different designed angles. (b) Chirp design that has a gradual change from $-87.6°$ to $-76°$.

metasurface and is shown in Fig. 5.2b. When considering Fig. 5.1, the gradual design will start with the “Cell A” design and then shift linearly to “Cell B”, and then shift linearly to “Cell C”.

The monostatic RCS of the different metasurface designs is shown in Fig. 5.3. In the figure, $RCS_{83}$ refers to the simulation that is described in Section 3.2. $RCS_{(3)}$ refers to the case where the metasurface is split into three sections where each section is assigned a certain retroreflection angle design as described by Fig. 5.2. $RCS_{\text{chirp}}$ refers to the case where the metasurface is designed to gradually change its retroreflective angle from $-87.6°$ to $-76°$. It is important to note that the effective aperture of all three simulations are the same, and thus their relative power can be compared to determine their aperture efficiency.

Fig. 5.4 shows a close up on the area of interest, namely around the retroreflected angles. It is quite obvious that $RCS_{(3)}$ is not a relevant design, as $RCS_{\text{chirp}}$ performs better than $RCS_{(3)}$ at almost every point. Therefore, when comparing $RCS_{83}$ to $RCS_{\text{chirp}}$, there is a larger angular coverage with the chirp design, however, there is a shift in the peak retroreflection angle. The reason for this can be explained by the same reasoning found in Section 3.2.5 and Fig. 3.13. Since the chirped metasurface is no longer designed solely for $-82.87°$ but rather for a larger range of angles, it is equivalent to a metasurface being designed for a single angle, with a reduction in the number of cells. This is because only a fraction of the cells operate to retroreflect at specific angles.
Figure 5.3: Plot comparing the monostatic RCS of different metasurface designs.

Figure 5.4: Plot comparing the monostatic RCS of different metasurface designs where there is a zoom-in in the area of interest.
5.1.2 Wide-band Design

With this same metasurface that can retroreflect a larger range of angles, it is possible to also use it to retroreflect a larger bandwidth of angles at $82.87^\circ$. The reason for this is because the relationship between the grating period $\Lambda_g$ and the frequency of the incident wave is represented by (2.19) where,

$$\Lambda_g = \frac{\lambda}{2 \sin(\theta_i)} = \frac{c/\nu}{2 \sin(\theta_i)}$$

If it is rearranged for $\nu$ the following is obtained,

$$\nu = \frac{c}{2 \Lambda_g \sin(\theta_i)}.$$  \hspace{1cm} (5.1)

Therefore, the corresponding frequency range that this chirped metasurface was designed to operate will range from 23.5-24.1GHz, since the grating period $\Lambda_g$ has a range from 6.26-6.44mm. Therefore, when the frequency response for the retroreflective metasurface of a single-angle design is compared to the chirped design, a very interesting result is obtained as shown in Fig. 5.5. The frequency response for an incident plane wave at $78^\circ$ was also done because of the wide-band response as shown in Fig. 5.6. It is evident that when there is a comparison between the retroreflection properties of the chirped metasurface and the single frequency (24GHz) design, there is a wider frequency retroreflection response. Therefore, this proves that the same metasurface that is designed to retroreflect at a wider range of angles can also be used to enable the bandwidth of the metasurface to increase.

5.2 Absorbing Metasurface

The simulations that were done in this thesis predominately covered the case for efficient retroreflection. In this section, however, the focus will be on how to create a metasurface using the binary Huygens’ methodology to efficiently absorb all the incident energy and not allow any mode to propagate efficiently.

5.2.1 Design, Simulation and Results

This design no longer uses an RT/Duroid 5880 Laminate because there is no desire for the board to reduce loss. Therefore, FR4 will be used with a thickness of 3.175mm. The period that was chosen was 5.7mm because this would only allow the specular mode to be in the propagating regime, while leaving every other mode in the evanescent regime. The two cells that make up the grating period are half the entire period (2.85mm) and the incident wave of interest is the TM-polarized plane wave. The
Figure 5.5: The retroreflective power for an 82.87° incident plane wave comparing the frequency response of the TM design (red line) compared to the chirped design (blue line).

Figure 5.6: The retroreflective power for a 78° incident plane wave comparing the frequency response of the TM design (red line) compared to the chirped design (blue line).
Figure 5.7: A plot showing the bistatic RCS signal in dB scale for a 24GHz plane wave incident at 82.87° for an (a) absorbing metasurface with $S_z=3.175\text{mm}$ and an (b) absorbing metasurface with $S_z=2.86\text{mm}$.

The same could have been done for the TE-polarized case, however, this work focused on expanding on the TM-polarized metasurface.

There were three parametric sweeps that were done in conjunction with each other to determine which design would absorb the incident wave the most efficiently. These three sweeps involved the thickness of the substrate $S_z$ and the slot widths $P_{x1}$ and $P_{x2}$. It was found that the optimal design involved the thickness $S_z=2.86\text{mm}$ and the two cells having slot widths of $P_{x1}=1.3\text{mm}$ and $P_{x2}=1.4\text{mm}$, and the slot length $P_y=2\text{mm}$. This design did not have a particular phase distribution to follow since there is no particular reflected wave direction of interest. In order to better understand the design layout, Fig. 3.7 describes it with the same variable definitions.

It is important to note that the thickness of the metasurface is generally determined by the manufacturer and obtaining a thickness of $S_z=2.86\text{mm}$ is not going to be as straightforward when compared to a metasurface of thickness $S_z=3.175\text{mm}$. Therefore, a metasurface with the standard thickness of $S_z=3.175\text{mm}$ was also designed with the dimensions of the slot being optimized for this case. In this case, the optimal dimensions were $P_{x1}=2.12\text{mm}$, $P_{x2}=2.48\text{mm}$ and $P_y=1\text{mm}$. The comparison between these two designs is shown in Fig. 5.7.

When the bistatic RCS of Fig. 5.7a & Fig. 5.7b is compared with each other, the scattered power with the substrate $S_z=2.86\text{mm}$ is 5dB lower than the metasurface with the substrate $S_z=3.175\text{mm}$.
Figure 5.8: A plot showing the bistatic RCS signal in dB scale for a 24GHz plane wave incident at 82.87° of a metallic plate. This power level is arbitrary, but is used as a comparison with Fig. 5.7a&b.

(b) The general reflection from a PEC on top of a dielectric substrate ($\theta_i = 82.87°$). (c) An incident wave that is coupled into evanescent modes ($\theta_i = 82.87°$).

At the specular angle (82.87°). At the retroreflected angle, the scattered power of the substrate with $S_z=2.86\text{mm}$ is about 4dB lower than the metasurface of the substrate with $S_z=3.175\text{mm}$. When comparing Fig. 5.7b with Fig. 5.8a, it also shows a major difference in the power that is reflected in the specular direction between the absorptive metasurface and a metallic plate, respectively. Fig. 5.7b shows a decrease in 18.6dB in the amount of power that is specularly reflected when compared to the PEC (Fig. 5.8a). It is important to note that while this absorber works very efficiently for the design with $S_z=2.86\text{mm}$ at the specular direction, there are no other angles at which the scattered power exceeds -56dB. This shows that most of the power is absorbed by the metasurface. The understanding of this absorption can be due to the similar process that has been considered in the design of the retroreflective metasurface. Since the two cells within a grating period had a $\pi$ phase shift between each other for the retroreflective metasurface, this enabled the suppression of the specular mode by coupling all the energy into the retroreflective mode. In the case of the absorbing metasurface, there is only the specular mode propagating. If the phasing between the two cells per grating period were such that the energy is coupled to an evanescent mode, this would suppress the specular reflection. Therefore, if this condition is satisfied it will result in no propagating modes, and thus no reflections from the metasurface.
Chapter 6

Conclusion

6.1 Summary

In this thesis, it was shown that a binary Huygens’ metasurface has the ability to achieve strong retroreflection at near-grazing angles (~82.87°). There were two separate designs for a TE- and TM-polarized incident plane wave, where both required two cells per grating period to efficiently couple all the energy into the retroreflected mode. The TE-polarized design used a ground-backed dipole as the individual cells, whereas the TM-polarized design used a ground-backed slot. The design procedure was shown and the simulated and measured results both demonstrate a very efficiency retroreflector. The simulated results show an aperture efficiency of 93% and approximately 100% for TE- and TM-polarized waves respectively. The measured results show an aperture efficiency of 93% for both TE- and TM-polarized waves. When comparing the design of retroreflectors, the binary Huygens’ metasurface enables a design that is a single-layer structure, with aggressively discretized cells which enable the use of larger unit cells, and simple slot and dipole elements. This ultimately means that the fabrication cost will be greatly reduced when compared to other efficient retroreflectors at near-grazing incident angles. The binary Huygens’ metasurface, therefore becomes an attractive option for the design of efficient retroreflectors for use in mm-wave and THz frequency applications.

6.2 Future Directions

The current progress that was taken with regards to generating retroreflective metasurfaces using the binary Huygens’ methodology has been analyzed thoroughly. The three designs that need further work would be the chirped metasurface that contains two designs in one (the wide-angle retroreflector and the
larger bandwidth retroreflector), and the absorbing metasurface. Thus far there are preliminary results through simulations that show it is possible to create these different metasurfaces with similar design methodologies to the binary Huygens’ metasurface. Another design that could be implemented but has not be investigated is a dual-polarized retroreflective metasurface.

### 6.2.1 Chirped Retroreflective Metasurface

**Optimized Wide-angle Retroreflective Metasurface**

The wide-angle retroreflector design in Section 5.1 has a very simple design procedure where the cell design is varied linearly from period to period. If other distributions were tried and compared, an optimal distribution would likely be determined. Fig. 5.1 shows the three different cell designs for the three different angles of interest, and the gradual design implemented a linear variation. However, the optimal transition from “Cell A” to “Cell B” to “Cell C” may not be linear, and thus analysis in that area could help increase the efficiency over the angular range.

**Wide-Bandwidth Retroreflective Metasurface**

The retroreflector that works for a large range of frequencies was designed using the exact same metasurface used for the wide-angle retroreflector. If further analysis was done on this specific design, greater efficiency in the amount of retroreflected power could be achievable over a larger bandwidth.

### 6.2.2 Absorptive Metasurface

The absorptive metasurface that was designed was quite efficient, however, the understanding of how to determine the design parameters to couple the incident wave into evanescent modes was not well understood. Therefore, if analysis was done on the specific wave vectors that these evanescent modes belong to, there may be a systematic way of determining the parameters that will absorb all the incident energy.

### 6.2.3 Dual-Polarized Retroreflective Metasurface

A dual-polarized retroreflective metasurface is something that could be implemented with our current design methodology because both the TE- and TM-polarized incident waves do not couple strongly to dipoles and slots respectively. Therefore, if a single cell contained both a dipole and slot design, it could be possible to have the TE- and TM-polarized waves interact only with the respective component that will couple with the plane wave. It is likely the case that the individual cells will contain a shape
that is different than a slot and a dipole (i.e. a cross) that can work to achieve this dual-polarization requirement.

6.3 Contributions

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Bibliography


