Modeling the interactions between loading-induced and creep strains of rockfill materials using a hardening elastoplastic constitutive model

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<th>Journal:</th>
<th>Canadian Geotechnical Journal</th>
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<tr>
<td>Manuscript ID</td>
<td>cgj-2018-0435.R1</td>
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<tr>
<td>Manuscript Type:</td>
<td>Article</td>
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<tr>
<td>Date Submitted by the Author:</td>
<td>26-Aug-2018</td>
</tr>
</tbody>
</table>
| Complete List of Authors: | Fu, Zhongzhi; Nanjing Hydraulic Research Institute, Geotechnical Engineering Department  
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| Keyword: | rockfill, creep, constitutive model, elastoplasticity |
| Is the invited manuscript for consideration in a Special Issue? | Not applicable (regular submission) |

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Modeling interaction between loading-induced and creep strains of rockfill materials
using a hardening elastoplastic constitutive model

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Abstract: An elastoplastic constitutive model taking into account the stress-strain relationship and creep-induced hardening behavior of rockfill materials is proposed in the light of previous experimental observations. It is assumed that the mechanical response during loading and the final amounts of creep strains under a constant stress state are independent of the straining rate. The focus of the proposed model is the coupling effect between loading and creep, including the influence of loading history on subsequent creep strains and the influence of creep history on subsequent loading behavior. An extended yield function, which allows a flexible control over the shape of yield surfaces, is used not only to distinguish loading, unloading and neutral loading but also to manipulate the creep-induced hardening using a plastic strains based hardening parameter. A stress-dependent dilatancy equation is used, instead of a plastic potential function, to define the directions of plastic strains during loading. The hardening law is established based on three different kinds of experimental results. Only routine experiments are required for the calibration of model parameters, and the model can be used in a reduced form according to the available test results.
results. The model is verified using typical experimental data and it is capable of capturing important behavior of rockfill materials, such as the pressure-dependent strength, the shear contraction and dilation, and the creep-induced stiffening.

**Keywords:** rockfill; creep; constitutive model; elastoplasticity

**Introduction**

It has long been recognized that rockfill materials are prone to particle breakage during loading due to the existence of sharp corners and micro-cracks and the resultant highly concentrated inter-particle and intra-particle stresses (Marsal 1967; Lee and Farhoomeand 1967; Hardin 1985; Indraratna et al. 1993; Varadarajan et al. 2003; Frossard et al. 2012; Ovalle et al. 2015; Tapias et al. 2015; Xiao et al. 2016). Particle breakage may also occur under a constant stress state in a delayed manner, leading to the so-called creep strains (Oldecop and Alonso 2007; Zhang et al. 2017; Lade et al. 2009; Kwok and Bolton 2013; Karimpour and Lade 2010; Fu et al. 2018a). For a particular rockfill, the amount of creep strains depends not only on the packing density (Bauer et al. 2012) and the moisture content (Alonso et al. 2016; Bauer 2009), but also on the stress state (Cheng and Ding 2004; Fu et al. 2018a) and the stress history (Zhang et al. 2017; Lade et al. 2009). On the other hand, the deformation behavior of materials during loading may also be altered by previous creep histories, i.e. creep-induced structuration usually results in an evident increase of stiffness in subsequent loading (Mitchell and Soga 2014; Lade et al. 2009; Karimpour and Lade 2013; Zhang et al. 2017).

In recent years, increasing efforts have been made in establishing constitutive models for
granular materials, e.g. sand and rockfill, with special attention paid to their loading-induced particle breakage behavior (Russell and Khalili 2004; Yao et al. 2008b; Salim and Indraratna 2004; Sheng et al. 2008; Desimone and Tamagnini 2005; Liu et al. 2014; Daouadjji and Hicher 2010; Xiao et al. 2014; Kikumoto et al. 2010; Hu et al. 2011). A few constitutive models have also been proposed for rockfill materials taking into account their creep behavior (Oldecop and Alonso 2001; Dolezalova and Hladik 2011; Bauer et al. 2012; Fu et al. 2012). However, these latter ones can only consider the influence of stress history on the following creep strains, while the creep-induced stiffening in subsequent loading seldom captured. In reality, rockfill materials in modern rockfill dams are compacted zone by zone and layer by layer. Each rockfill element experiences vertical loading (weight of overlying materials) and creep repeatedly. A suitable constitutive model is expected to be capable of reflecting the mutual influence between loading-induced strains and creep strains.

This paper aims to establish such a constitutive model for rockfill materials that can capture their deformation behavior during loading and creep as well as their mutual interaction. Previous experimental observations made on granular materials are firstly summarized to provide some keystones for the present study. Then, an elastoplastic constitutive model is proposed by specifying the yield function, the stress-dependent dilatancy equation, the evolution of creep strain and the hardening law. Emphasis is placed on devising a hardening parameter suitable for both loading and creeping. The model is verified using typical experimental results. Throughout the paper, compressive stresses and strains are defined positive and all the normal stresses are effective ones, with the conventional superscript comma omitted for simplicity.
**Previous experimental observations**

Compared with the abundant creep tests performed with various sands (Kuwano and Jardine 2002; Lade et al. 2009; Karimpour and Lade 2010; Karimpour and Lade 2013), experiments on the time-dependent behavior of rockfill materials are considerably less (AnhDan et al. 2006; Zhang et al. 2017; Alonso et al. 2016; Fu et al. 2018b). However, it is reasonable to postulate that the time-dependent behavior of rockfill materials is similar to that of sands. Therefore, laboratory experiments performed with both rockfill and sand are reviewed in this part. Comprehensive reviews of time-dependent behavior of clay and sand and models for characterization of this behavior can be found in Augustesen et al. (2004) and Liingaard et al. (2004).

**Influence of loading rate**

The influence of loading rate on the stress-strain behavior of sands have been studied by many authors (Tatsuoka et al. 2006; Lade and Karimpour 2010; Karimpour and Lade 2013). It was affirmed repeatedly that widely different strain rates produce essentially the same stress-strain relation for sands. For instance, Lade et al. (2009) performed triaxial compression tests on a crushed coral sand under an effective confining pressure of 200 kPa and with five different, constant axial strain rates varying from 0.00665%/min to 1.70%/min. It was found that a 256-fold increase in strain rate resulted in negligible variations of the stress-strain and volume change curves, which were within the normal scatter of such results (Lade and Karimpour 2010). Experiments performed with Virginia Beach sand, consisted of much stronger quartz grains, also shown that a 256-fold increase of strain rate produced negligible effects on the stress-strain and
volume change curves under a low confining stress (250 kPa), while under a higher confining pressure (8 MPa) only small effects could be observed (Karimpour and Lade 2013).

AnhDan et al. (2006) performed a series of large-scale drained triaxial compression tests on a gravelly soil (specimen size: 23 cm × 23 cm × 58 cm), and found that the effects of increasing the axial strain rate from 0.006%/min to 0.06%/min on the overall stress-strain behavior, including the peak strength and the post-peak behavior, were apparently insignificant. However, switching from a constant strain rate to another higher or lower one in a single test resulted in a temporary overshoot or undershoot of the stress-strain curve, which, by further shearing, merged into the reference one obtained from primary loading (Lade et al. 2009; AnhDan et al. 2006; Di Benedetto et al. 2002; Tatsuoka et al. 2002). That is to say, the effect of changing the strain rate is only temporary and the stress-strain and volume change curves obtained under a constant strain rate can be used to calibrate constitutive model parameters, despite of the fact that the loading rate upon rockfill or gravel materials does change in practical engineering.

Loading rate has also some effect on the subsequent creep strains. Specimens sheared at a high strain rate prior to the initiation of creep exhibit high creep deformation rates, whereas specimens loaded under a low strain rate show very small deformation rates at the beginning of creep (Kuwano and Jardine 2002; Karimpour and Lade 2013). Static fatigue-induced particle breakage is thought to be the root of this phenomenon (Lade and Karimpour 2010; Karimpour and Lade 2013). It is argued that particle crushing is a time-dependent process and the loading rate controls the time available for the establishment of a stable structure. The higher the strain rate, the less time will be available and more consequent creep strains will occur. However, it should be noted
that an exact separation of the loading-induced strains and the creep strains from the total ones is
difficult in experiments, and it is very possible that the creep strains obtained after a rapid
straining include certain amounts of loading-induced strains (Fu et al. 2018a). Clear transient
periods have also been observed in humidity-controlled oedometric creep experiments on rockfill
before the creep strain records approach regular linear strain-log time relationships (Oldecop and
Alonso 2007).

**Influence of loading path**

The dependence of mechanical responses of granular materials on the loading path or stress
history has been acknowledged long ago (Lade and Duncan 1976; Nakai 1989; Yao et al. 2008a).
The simplest evidence is the different amounts of plastic strains at a same stress state reached
along different stress paths (Nakai 1989). This important path-dependent property necessitates
that the stress-strain behavior of granular materials modeled in an incremental way. In addition,
the hardening parameter in an elastoplastic constitutive model, usually expressed as a function of
plastic strains, should be carefully devised so that it is independent of the stress path (Nakai 1989;
Yao et al. 2008a).

Loading path also influences the subsequent creep behavior (Lade et al. 2009; Zhang et al.
2017). It was found that the magnitudes of creep strains (both the axial and volumetric strains) of
a sand specimen that experienced a deviatoric stress drop (unloading) are considerably smaller
than those of another specimen creep at the same stress state without unloading (Lade et al. 2009).
This behavior can be explained using Fig. 1, in which the solid curves are primary loading curves
(PLCs) while the dashed ones are termed herein as creep shifted curves (CSCs). The CSCs are obtained by shifting the PLCs by values of creep strains that occur at different stress levels for a given duration (Lade and Karimpour 2010; Karimpour and Lade 2013). If a specimen is loaded to point B as shown in Fig. 1, it will reach point G after creep under a constant stress. On the other hand, if the specimen is first loaded to point C and then unloaded to point D, then the total creep strain that will occur is denoted by the length of DG, less than that of BG.

Creep experiment conducted under an unloaded stress state also shows that if the stress drop prior to creep is large enough, say point F in Fig. 1, then creep will occur in the reverse direction (AnhDan et al. 2006; Lade et al. 2009). It seems that the CSCs plotted in Fig. 1 serve as attractors for the long-term stress-strain states. The directions of creep strains and the strain rates depend on the relative position of the current state point with regard to the CSCs.

**Creep-induced plastic hardening**

Creep-induced plastic hardening, evidenced by the increase of stiffness in post-creep loading, has been observed in experiments on sand, gravel and rockfill materials (Lade et al. 2009; AnhDan et al. 2006; Mitchell and Soga 2014; Zhang et al. 2017). An important field evidence is the increase of penetration resistance after aging (Mesri et al. 1990; Joshi et al. 1995). Currently, hypotheses explaining the causes of this phenomenon fall into two categories (Baxter and Mitchell 2004): mechanical mechanism (Mesri et al. 1990; Schmertmann 1991; Lade et al. 2009; Zhang et al. 2017) and chemical mechanism (Mitchell and Solymar 1984; Joshi et al. 1995; Mitchell and Soga 2014). The chemical mechanism focuses mainly on the formation of cementation bonds at the
grain contact points as a consequence of dissolution and precipitation of silica or other materials such as calcium carbonate. For granular materials that inter-particle bonding is unlikely to play a significant role, it is the continued rearrangement of particles with time, either as a result of grain abrasion or particle breakage (Karimpour and Lade 2010; Karimpour and Lade 2013), that increases the macro-interlocking of particles and the micro-interlocking of surface roughness and results in a more stable structure as reflected by the increase of stiffness after creep (Mesri et al. 1990; Schmertmann 1991; Kuwano and Jardine 2004; Lade et al. 2009; Mitchell and Soga 2014).

From the perspective of constitutive modeling, the creep-induced hardening phenomenon can be explained by the expansion of yield surface due to creep, i.e. the adjustment of particles configuration and interlocking during creep result in an outward movement of yield surface (Lade et al. 2009; Karimpour and Lade 2013; Zhang et al. 2017). For example, a specimen is first loaded to point B (Fig. 1) and the current yield stress is $p_B$ or $q_B$. Afterward, the specimen is allowed to creep to point D and correspondingly the yield stress increases to $p_C$ or $q_C$. As a result, loading after the creep process is initially in the elastic regime (segment DC) until the creep-altered yield stress is reached, and thenceforth the material undergo elastoplastic loading again following approximately the PLCs. Therefore, it is important to establish the relationship between the creep strains and the resultant plastic hardening so as to evaluate the location of the altered yield surface.

Experiments performed with a weakly-weathered granite rockfill by Zhang et al. (2017) also show that the memory of previous creep histories can be eliminated by subsequent loading. A specimen was loaded to a deviatoric stress level and followed by a first-stage creep. Then the
specimen was loaded again to a higher objective stress level and a second-stage creep was allowed and recorded. A second specimen was loaded directly to the same objective deviatoric stress level without pause and the subsequent creep strains were measured. The amounts of axial and volumetric creep strains of the second specimen were found almost the same as those of the first specimen under the same designed deviatoric stress level (Zhang et al. 2017). That is to say, the first-stage creep of the first specimen has no effect on its second-stage creep after reloading. However, this conclusion is valid only when the stress increment is large enough. Zhang et al. (2017) proposed that only elastoplastic loading can eliminate the memory of previous creep histories. This idea is also illustrated in Fig. 1. Considering a specimen first loaded to point B and then creep to point D, it is not until the moment that the stress of the specimen exceeds $p_c$ or $q_c$ during reloading can the memory of accumulated creep strains be eliminated.

**Direction of creep strains**

Since creep occurs under a constant stress state, all the strain components are time-dependent plastic ones. There might be a flow rule governing the direction of creep strains. Lade and co-authors tested Antelope Valley sand (Lade and Liu 1998) and Virginia Beach sand (Karimpour and Lade 2010; Karimpour and Lade 2013), and found that the volume change curves during creep stages do not deviate from the reference curves obtained from loading experiments, indicating that the direction of inelastic creep strain increments is the same as that of the loading-induced plastic strain increments. That is to say, an identical potential function may be used to evaluate the directions of inelastic creep and plastic strains simultaneously. However, this
assumption was invalidated by creep experiments with Carib Sea Geomarine Florida crushed coral sand (Lade et al. 2009), which is more friable compared with previously mentioned ones. The tested coral sand was found much more contractive during creep than during loading.

Recently, Fu et al. (2018a) conducted large-scale triaxial compression and creep experiments with a saturated limestone rockfill, and compared the relationships between dilatancy ratio and stress ratio during creep and loading. The positive dilatancy ratio during creep is much higher than that during loading at a same stress ratio, meaning that the tested rockfill is much more contractive in creep (Fu et al. 2018a). Similar phenomenon was also observed in other coarse granular materials (AnhDan et al. 2006; Zhang et al. 2017). It is also interesting to note that the direction of incremental strains does not keep unchanged during creep, as it might be expected. It changes gradually as the time elapses (AnhDan et al. 2006; Bowman and Soga 2003; Zhang et al. 2017; Fu et al. 2018a). This means that the direction of creep strains depends not only on the stress state but also on other factors. However, to avoid an overcomplicated constitutive modeling, it is reasonable to make a simplification that the direction of creep strains keeps constant and the volumetric and deviatoric strains accumulate proportionally. Based on this assumption, the direction of creep strains at a given stress state can be evaluated using the total amounts of creep strains that may occur in creep experiments at this stress state (Fu et al. 2018a).

**Constitutive modeling**

In this section, an elastoplastic constitutive model describing the stress-strain relation and creep
behavior of rockfill materials in a unified way is proposed. In addition to the conventional assumption that the incremental strains can be decomposed into an elastic part and a plastic part, the following three additional assumptions are made based on the experimental observations reviewed above:

(a) The stress-strain relationship is rate-independent, i.e. different straining rates produce an identical stress-strain curve. The amounts and rates of creep strains are neither influenced by the preceding loading rate.

(b) The locations of creep shifted curves (CSCs) are not influenced by the loading path. They can be obtained by shifting the PLCs by values of creep strains that occur at different stress levels for a given duration (Fig. 1).

(c) Creep strains are plastic and time-dependent, the direction of which under a constant stress state does not change during a creep stage. However, it may not coincide with the direction of loading-induced plastic strains at the same stress state.

In the following parts, the elastic behavior, the yield function, the direction of plastic strains, the evolution of creep strains and the hardening law are described sequentially. Stress invariants used hereafter include the mean effective stress, \( p \), and the deviatoric stress, \( q \), i.e.

\[
\begin{align*}
   p &= \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \\
   q &= \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}
\end{align*}
\]  

(1)

in which \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are three principal stresses. Similarly, the two strain invariants used in constitutive modeling are the volumetric strain, \( \varepsilon_v \), and the deviatoric strain, \( \varepsilon_s \), i.e.
\[
\begin{align*}
\varepsilon_v &= \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \\
\varepsilon_s &= \frac{\sqrt{2}}{3} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}
\end{align*}
\] (2)

in which \(\varepsilon_1, \varepsilon_2, \text{ and } \varepsilon_3\) are three principal strains. Hereafter, a variable with a superscript ‘e’, ‘p’ or ‘c’ indicates that the quantity is an elastic, plastic or creep one. A prefix ‘d’ before a variable denotes its increment, and an overhead dot (˙) its time derivative or rate.

**The elastic behavior**

The elastic behavior of rockfill materials can be determined by several different combinations of elasticity quantities, such as the bulk modulus, \(K^e\), and the shear modulus, \(G^e\), i.e.

\[
\begin{pmatrix}
\frac{dp}{dq}
\end{pmatrix} =
\begin{pmatrix}
K^e & 0 \\
0 & 3G^e
\end{pmatrix}
\begin{pmatrix}
\frac{de_v}{dq} \\
\frac{de_s}{dq}
\end{pmatrix}
\] (3)

Unloading stress-strain paths in triaxial experiments show that the elastic modulus depends on the effective confining stress, but is almost independent of the deviatoric stress level from which unloading takes place (Duncan and Chang 1970; Fu et al. 2014). Therefore, the following pressure-dependent elastic moduli are assumed:

\[
\begin{align*}
K^e &= k_v^e \cdot p_a \cdot \left(\frac{p}{p_a}\right)^m \\
G^e &= k_s^e \cdot p_a \cdot \left(\frac{p}{p_a}\right)^n
\end{align*}
\] (4)

in which \(p_a\) denotes the atmospheric pressure (≈100 kPa); \(k_v^e, k_s^e, m, \text{ and } n\) are four parameters. In the case \(m = n\), Eq. (4) gives a constant Poisson’s ratio, as is often assumed in constitutive modeling (Russell and Khalili 2004; Xiao et al. 2014; Daouadji and Hicher 2009; Fu et al. 2014).
The yield function

In elastoplasticity, a yield function is defined with some stress invariants and a certain hardening parameter. It can be used to distinguish loading, unloading and neutral loading conditions. The so-called consistency condition can also be used, with the aid of a flow rule and a hardening law, to determine the magnitude of plastic strains (e.g. Yao et al. 2008a; Russell and Khalili 2004). In this study, the following yield function, \( f \), is used:

\[
f(p, q, H) = \left( \frac{p}{p_a} \right)^{m_v} \left( 1 + \frac{m_v}{2 - m_v} \frac{q^2}{M^2 p^2} \right) - H = 0
\]

(5)

where \( H \) denotes the hardening parameter, and \( M \) the so-called critical stress ratio. \( m_v \) is a power index controlling the behavior under isotropic compression that will be described later. Fig. 2 shows the loci of yield surfaces with a given \( M (1.5) \) and different values of \( m_v \). It can be verified that the line through the origin and the vertical apex on the yield surface always has a slope of \( M : 1 \). Therefore, the critical stress ratio is a basic parameter controlling the shape of the yield surface as in many other constitutive models (e.g. Yao et al. 2008a). Eq. (5) could be seen as an extension of the yield function used in the Modified Cam-Clay (MCC) model, in which \( m_v = 1 \) is imposed.

In the MCC model, the hardening parameter \( H \) is a function of the plastic volumetric strain, which means that the contours of \( \varepsilon_v^p \) in the \( p-q \) space are elliptical. Meanwhile, a larger elliptical yield locus corresponds to a greater value of \( \varepsilon_v^p \). While this feature is appropriate for normally consolidated clay, it is not suitable for granular materials due to the possible dilation during shearing. Fig. 3 shows the stress-strain and volume change data obtained in triaxial compression experiments with a typical rockfill material. The stress ratio increases continuously with the axial
strain in the range applied, and this strain-hardening behavior means that the yield surface expands continuously during shearing. However, all the specimens exhibit considerable dilation after initial contraction as shown in Fig. 3(b). That is to say, if the plastic volumetric strain is used solely in the hardening parameter, it won’t increase monotonically during shearing, which is contradictory with the continuous expanding of the yield surface.

An alternative hardening parameter often used for geomaterials is the plastic work (Lade and Kim 1995; Nakai 1989), $W^p$, i.e.

$$W^p = \int p \cdot \text{d}e^p_v + q \cdot \text{d}e^p_s$$

Nakai (1989) and Yao et al. (2008a) found that the plastic work produced from a stress state to another one is almost independent of the stress path, which makes it an ideal hardening parameter. Fig. 4 compares the contours of the volumetric strain and the input work in the $p-q$ space using the experimental results shown in Fig. 3. The solid and dashed curves marked with two power functions are the envelopes of the peak stress states and the constant volume stress states, respectively. Herein, a constant volume stress state refers to the state where the material starts to dilate. The inclined dashed lines are effective stress paths. It is obvious that the contours of the volumetric strain are far different from ellipses as shown in Fig. 4(a), particularly when the mean effective stress is low. The contours of the input work are relatively more similar to ellipses. However, a tear-drop like yield surface inclined towards the right apex might be an even better choice that coincides with the contours of input work (Lade and Kim 1995). Note that the total strains were used in constructing Fig. 4, as the elastic behavior is unknown. It is believed that small amounts of elastic strains make limited contribution to the total volumetric strain and the
total input work. That is to say, the contours of the plastic volumetric strain and the plastic work do not deviate too much from the ones plotted in Fig. 4. In this study, a plastic strains based hardening parameter is derived as will be presented later.

The direction of loading-induced plastic strains

The direction of loading-induced plastic strains is determined conventionally by the so-called plastic potential function, \( g \), i.e.

\[
\left( \frac{d\varepsilon_v^p}{d\varepsilon_s^p} \right) = \lambda \left( \frac{\partial g}{\partial \varepsilon} \right)
\]

(7)

in which \( \lambda \) is a proportionality that can be determined using previously mentioned consistency condition. Eq. (7) indicates that an explicit definition of the plastic potential function is actually not necessary. A simple dilatancy ratio can be used instead as follows:

\[
\left( \frac{d\varepsilon_v^p}{d\varepsilon_s^p} \right) = \lambda \left( \frac{d}{1} \right)
\]

(8)

in which \( d \) is the dilatancy ratio, i.e. the ratio between the plastic volumetric strain increment and the plastic deviatoric strain increment \( (d = d\varepsilon_v^p/d\varepsilon_s^p) \). For granular materials, the dilatancy ratio depends not only on the stress state but also on the density (Li and Dafalias 1999; Xiao et al. 2014). However, a simple stress-dependent dilatancy equation is used in this study.

Fig. 5(a) shows the relationship between the dilatancy ratio and the stress ratio using the test results plotted in Fig. 3. Data scatterness is very evident when the stress ratio is lower than 1.0, particularly at the beginning of shearing. This phenomenon may be attributed to several factors, such as the effect of initial fabric (Been and Jefferies 2004) and the bedding error at specimen
ends (Jardine et al. 1984; Clayton 2011). Scatterness can also be seen in the dilative zone. The stress ratio at zero dilatancy \((d = 0)\) decreases when the confining pressure is increased. This has already been illustrated in Fig. 3 by the curved envelope of constant volume stress states in the \(p-q\) space. Herein, a power function is used to represent this particular curve, i.e.

\[
\frac{q}{p} = r_c \left( \frac{p}{p_a} \right)^{n_c}
\]

in which the two parameters are shown in Fig. 4, i.e. \(r_c = 2.31; n_c = 0.92\). Eq. (9) gives the stress ratio at zero dilatancy, \(M_c\), directly as follows:

\[
M_c = r_c \left( \frac{p}{p_a} \right)^{n_c - 1}
\]

To reduce the data scatterness in the dilative zone, the stress ratio is normalized by the one evaluated in Eq. (10), and the results are plotted in Fig. 5(b). Much more condensed data points are obtained and they can be approximated by the following nonlinear equation, i.e.

\[
d = d_0 \left[ 1 - \left( \frac{\eta}{M_c} \right)^4 \right]
\]

in which \(d_0\) is the initial dilatancy ratio when \(\eta = 0\). For the current rockfill material, \(d_0 = 0.75\). It is interesting to note that the authors performed large-scale triaxial experiments with several different rockfill materials, and found that the power function given in Eq. (11) with a power index of 4 well fits most of the stress-dependent dilatancy data (Fu et al. 2018a, 2018b).

**The evolution of creep strains**

Post-construction observations made on rockfill dams show that the settlement of rockfill dams accumulates relatively quickly at the beginning of operation. The rate of settlement, however,
decreases rapidly as time elapses (Oldecop and Alonso 2007; Fu et al. 2015). In many cases, the post-construction settlement rate decreases exponentially with time (Fu et al. 2015), which means the relationship between creep strains and time also follows an exponential function. Take the volumetric creep strain as an example, the evolution equation reads:

\[ \varepsilon_v^c = \varepsilon_v^f \left[ 1 - \exp \left( -\frac{t}{t_0} \right) \right] \]  

(12)

in which \( \varepsilon_v^c \) denotes the volumetric creep strain, and \( t \) the creep time. \( t_0 \) is a reference time, and \( \varepsilon_v^f \) the final amount of volumetric creep strain when \( t \to \infty \). Eq. (12) gives the volumetric creep strain rate as follows:

\[ \dot{\varepsilon}_v^c = \frac{\varepsilon_v^f - \varepsilon_v^c}{t_0} \]  

(13)

which indicates that the current creep strain rate is proportional to the unfinished creep strain.

In order to take the influence of loading path on the creep behavior into account, an additional term, the accumulated plastic strain, \( \varepsilon_v^p \), is added to the second term of the numerator in Eq. (13), i.e.

\[ \dot{\varepsilon}_v^c = \frac{\varepsilon_v^f - (\varepsilon_v^p + \varepsilon_v^c)}{t_0} \]  

(14)

where the plastic volumetric strain should not be interpreted as the total amount of plastic strain accumulated. Instead, it is the plastic volumetric strain that accumulates when the material is loaded from the current stress to the maximum one it has ever experienced, or where it has been unloaded (see Fig. 1). If no unloading has ever occur, or the unloaded material is reloaded to a stress level beyond the current yield surface, \( \varepsilon_v^p \) in Eq. (14) should be set to zero and Eq. (13) is
recovered.

Similar to Eq. (14), the deviatoric creep strain rate could also be determined according to the following equation:

\[
\dot{\varepsilon}_s^c = \frac{\varepsilon_s^f - (\varepsilon_s^p + \varepsilon_s^c)}{t_0}
\]

(15)

in which \(\varepsilon_s^f\) denotes the final amount of deviatoric creep strain when \(t \to \infty\), and \(\varepsilon_s^p\) the plastic deviatoric strain which should be interpreted similarly as the \(\varepsilon_v^p\) in Eq. (14).

By incorporating the loading-induced plastic strains in Eqs. (14) and (15), the influence of loading history upon subsequent creep behavior and the memory eliminating effect of reloading can be effectively modeled. In addition, if the amounts of loading-induced plastic strains exceed the final amounts of relevant creep strains, negative rates will be predicted and correspondingly reversed creep strains can be captured. Since the locations of CSCs are assumed independent of the previous loading history, the maximum potential amounts of creep strains depend only on the stress state. The authors conducted large-scale triaxial creep experiments on a limestone rockfill, and found that the maximum potential (final) amounts of creep strains can be modeled by the following equations (Fu et al. 2018a):

\[
\varepsilon_v^f = c_1 \left( \frac{p}{p_a} \right)^{n_1} + c_2 \left( \frac{q}{p_a} \right)^{n_2}
\]

(16)

and

\[
\varepsilon_s^f = c_3 \left[ \frac{p}{p_a} \right]^{n_3} \left( \frac{\eta}{M_p - \eta} \right)^{n_4}
\]

(17)

in which \(c_1 \sim c_3\) and \(n_1 \sim n_4\) are parameters. \(M_p\) is the stress ratio at the peak states. A power function
similar to Eq. (9) is used to represent the curved envelope of the peak stress state points shown in
Fig. 2, i.e.

\[
\frac{q}{P_a} = r_p \left( \frac{p}{P_a} \right)^{n_p}
\]  

(18)

which gives the peak stress ratio, \(M_p\), as follows:

\[
M_p = r_p \left( \frac{p}{P_a} \right)^{n_p-1}
\]  

(19)

The values of the two parameters are given in Fig. 3, i.e. \(r_p = 2.77; n_p = 0.88\). Eq. (17) indicates
that the well-known creep rupture will occur when \(\eta\) approaches \(M_p\). Note also that the long-term
peak stress ratio, which might be lower than \(M_p\), is not used in Eq. (17) because of the difficulty
in its determination by experiments.

Integration of Eq. (14) and (15) yields the following relations:

\[
\varepsilon_v^c = \left( \varepsilon_v^f - \varepsilon_v^p \right) \left[ 1 - \exp \left( -\frac{t}{t_0} \right) \right]
\]  

(20)

and

\[
\varepsilon_s^c = \left( \varepsilon_s^f - \varepsilon_s^p \right) \left[ 1 - \exp \left( -\frac{t}{t_0} \right) \right]
\]  

(21)

from which the dilatancy ratio during creep can be derived, i.e.

\[
d = \frac{\dot{\varepsilon}_v^c}{\dot{\varepsilon}_s^c} = \frac{\varepsilon_v^f - \varepsilon_v^p}{\varepsilon_s^f - \varepsilon_s^p}
\]  

(22)

It is evident that the dilatancy ratio under a given stress state keeps constant during creep, and
this assumption is the result of using a same reference time \(t_0\) in Eqs. (14) and (15).

The hardening law
As shown in the previous part, a combined use of the plastic volumetric and deviatoric strains in the hardening parameter is a better choice than the plastic volumetric strain for the used yield function. Herein, the consistency condition during loading is used in an incremental form to derive such a hardening parameter, i.e.

\[
\frac{\partial H}{\partial \varepsilon^p_v} \cdot \frac{\partial H}{\partial \varepsilon^p_s} \left( \begin{array}{c} \varepsilon^p_v \\ \varepsilon^p_s \end{array} \right) = \left( \begin{array}{c} \frac{\partial f}{\partial p} \\ \frac{\partial f}{\partial q} \end{array} \right) \left( \begin{array}{c} dp \\ dq \end{array} \right)
\]

The relationship between the incremental plastic strains and the incremental stresses can be formally written as follows:

\[
\left( \begin{array}{c} d\varepsilon^p_v \\ d\varepsilon^p_s \end{array} \right) = \left[ C^p \right] \left( \begin{array}{c} dp \\ dq \end{array} \right) = \left[ \begin{array}{cc} \frac{1}{K^p_{vv}} & \frac{1}{K^p_{vs}} \\ \frac{K^p_{vv}}{K^p_{ss}} & \frac{K^p_{ss}}{K^p_{vs}} \end{array} \right] \left( \begin{array}{c} dp \\ dq \end{array} \right)
\]

in which \([C^p]\) denotes the second-order plastic flexibility matrix. The stiffness quantities will be given later. Eq. (24) can also be expressed in an inversed strain-driven form, i.e.

\[
\left( \begin{array}{c} dp \\ dq \end{array} \right) = \left[ D^p \right] \left( \begin{array}{c} d\varepsilon^p_v \\ d\varepsilon^p_s \end{array} \right) = \left[ \begin{array}{ccc} K^p_{vv} \cdot K^p_{ss} \cdot K^p_{sv} & K^p_{vv} \cdot K^p_{ss} \cdot K^p_{sv} & K^p_{vv} \cdot K^p_{ss} \cdot K^p_{sv} \\ K^p_{vv} \cdot K^p_{sv} \cdot K^p_{ss} & K^p_{vv} \cdot K^p_{sv} \cdot K^p_{ss} & K^p_{vv} \cdot K^p_{sv} \cdot K^p_{ss} \\ K^p_{vv} \cdot K^p_{ss} \cdot K^p_{sv} & K^p_{vv} \cdot K^p_{ss} \cdot K^p_{sv} & K^p_{vv} \cdot K^p_{ss} \cdot K^p_{sv} \end{array} \right] \left( \begin{array}{c} \varepsilon^p_v \\ \varepsilon^p_s \end{array} \right)
\]

where \([D^p]\) is the plastic stiffness matrix, i.e. \([D^p] = [C^p]^{-1}\). Herein, the plastic stiffness matrix is assumed invertible, which excludes the strain-softening behavior from being modeled.

Inserting Eq. (25) into Eq. (23) yields

\[
dH = \left( \frac{\partial f}{\partial p} \frac{\partial f}{\partial q} \right) \left[ D^p \right] \left( \begin{array}{c} d\varepsilon^p_v \\ d\varepsilon^p_s \end{array} \right)
\]

in which
Eq. (26) is the differential hardening parameter, which can be integrated step by step to model the outward expanding of the yield surface, i.e.

\[
H = \int \left( \frac{\partial f}{\partial p}, \frac{\partial f}{\partial q} \right) \begin{bmatrix} D^p \gamma \end{bmatrix} \begin{bmatrix} d\epsilon_v^p \\ d\epsilon_v^p \end{bmatrix}
\]

(28)

In elastoplastic loading, the plastic strains can be evaluated using Eq. (24) and the hardening parameter be calculated by the yield function, Eq. (5), directly. During creep, on the other hand, the plastic creep strains can be evaluated by Eqs. (14) and (15) while the creep-induced plastic hardening be estimated by Eq. (28). In the latter case, the yield surface parameters, \(M\) and \(m_v\), control the contributions of the plastic volumetric and deviatoric strains to the hardening. The plastic flexibility and stiffness matrices play a central role in estimating the loading-induced plastic strains and the creep-induced hardening. In this study, the following experiments are used to establish the representations of the moduli in Eq. (24).

(a) Isotropic compression

During isotropic compression, both the deviatoric stress and the deviatoric strain vanish and Eq. (24) degrades to a single one, i.e.

\[
d\epsilon_v^p = \frac{1}{K_{\nu v}^p} dp
\]

(29)

which can be added to the elastic volumetric strain increment to yield the following equation:
\[
\frac{d\varepsilon_v}{dp} = \frac{d\varepsilon_v^e}{dp} + \frac{d\varepsilon_v^p}{dp} = \frac{1}{K} dp = \left( \frac{1}{K^e} + \frac{1}{K_{vv}^p} \right) dp \tag{30}
\]

Fig. 6 plots the measured volumetric strains of the previously referred rockfill at different mean effective stresses during isotropic compression and the regression curve. The volumetric strain behavior can be modeled by the following power function:

\[
\varepsilon_v = \varepsilon_v^e \left( \frac{p}{p_a} \right)^{m_v} \tag{31}
\]

which, by differentiation, yields the representation of the bulk modulus, \( K \), i.e.

\[
K = \frac{dp}{d\varepsilon_v} = k_v \cdot p_a \cdot \left( \frac{p}{p_a} \right)^m \tag{32}
\]

in which \( k_v = \frac{1}{(m_v / c_v)} \); \( m = 1 - m_v \). In the current case, \( k_v = 306.4 \); \( m = 0.32 \). It deserves to note that a same power index, \( m \), is used in Eqs. (32) and (4) for the bulk moduli so as to reduce the number of parameters. Once both the bulk moduli are obtained, the plastic bulk modulus can be calculated as follows:

\[
K_{vv}^p = \left( \frac{1}{K} - \frac{1}{K^e} \right)^{-1} \tag{33}
\]

(b) Triaxial compression

Triaxial compression under a constant cell pressure is the most widely used testing condition. Two equations can be established from this type of experiments. First, the deviatoric stress and strain relation reads:

\[
d\varepsilon_s = d\varepsilon_s^e + d\varepsilon_s^p = \frac{1}{3G} dq + \frac{1}{K_{sv}^p} dp + \frac{1}{K_{ss}^p} dq \tag{34}
\]

since \( dp = dq/3 \), Eq. (34) can be rearranged to give the shear modulus, \( G \), i.e.
To establish the representation of the shear modulus, the values of initial shear modulus, $G_0$, are evaluated based on experiments and they are plotted against the mean effective stress in Fig. 7(a). The data trend can be well reproduced by a power function, i.e.

$$G_0 = k_s \cdot p_a \left( \frac{p}{p_s} \right)^n$$  \hspace{1cm} (36)$$

in which $k_s$ and $n$ are two parameters. For the tested rockfill, $k_s = 450.5$; $n = 0.34$. Note a same power index, $n$, is also used in Eqs. (36) and (5) for the shear moduli. The shear modulus during loading are then divided by the right-hand side of Eq. (36), and plotted against the normalized stress ratio, $(\eta/M_p)$, in Fig. 7(b). The data points concentrated within a narrow band and the trend can be modeled satisfactorily by the following function:

$$G = \left[ 1 - \left( \frac{\eta}{M_p} \right)^\alpha \right] \cdot k_s \cdot p_a \cdot \left( \frac{p}{p_s} \right)^n$$ \hspace{1cm} (37)$$

in which $\alpha$ is a dimensionless parameter. Herein, $\alpha = 1.30$.

The second condition that can be used is the stress dilatancy equation, i.e.

$$\frac{d \varepsilon_p}{d \varepsilon_s} = \frac{1}{K_{sv}^p} + \frac{3}{K_{ss}^p}$$ \hspace{1cm} (38)$$

Three plastic stiffness quantities exist in Eqs. (35) and (38), and for solvability an additional equation is required. An oedometer test meets the end.

(c) Oedometric compression

\[
\frac{1}{3G} \frac{d \varepsilon_s}{dq} = \frac{1}{3G^e} + \frac{1}{3K_{sv}^p} + \frac{1}{K_{ss}^p}
\]

(35)
Two conditions can also be used in oedometric compression tests. First, the ratio between the volumetric strain increment and the deviatoric strain increment is always a constant, i.e.

\[
\frac{d\varepsilon_v}{d\varepsilon_s} = \frac{\frac{1}{K^e} dp + \frac{1}{K^p_{vv}} dp + \frac{1}{K^p_{vs}} dq}{\frac{1}{3G^e} dq + \frac{1}{K^p_{sv}} dp + \frac{1}{K^p_{ss}} dq} = \frac{3}{2}
\]

which can be rewritten as follows:

\[
\frac{dq}{dp} = \frac{\frac{2}{K^e} + \frac{2}{K^p_{vv}} - \frac{3}{K^p} + \frac{1}{G^e} + \frac{3}{K^p_{ss}} - \frac{2}{K^p_{sv}}}{\frac{3}{2}}
\]

Eq. (40) gives the stress increments ratio in an oedometer.

The relationship between the mean effective stress and the volumetric strain can be used to derive another condition, i.e.

\[
\frac{d\varepsilon_v}{dp} = \frac{1}{K^e} dp + \frac{1}{K^p_{vv}} dp + \frac{1}{K^p_{vs}} dq
\]

Inserting Eq. (40) into Eq. (41) yields the bulk modulus in an oedometer, \(K'\), i.e.

\[
\frac{1}{K'} = \frac{d\varepsilon_v}{dp} = \frac{1}{K^e} + \frac{1}{K^p_{vv}} + \frac{1}{G^e} + \frac{3}{3K^p_{ss}} - \frac{2}{2K^p_{sv}}
\]

Fig. 8 plots the oedometric compression results of the tested rockfill and the best-fit curve. The mean effective stress is evaluated herein with an earth pressure coefficient at rest being set to 0.2. This simplification will be verified later. As can be seen in Fig. 8, the bulk modulus can be expressed similarly as the one for isotropic compression, i.e.
\[ K' = k'_{v} \cdot p_{a} \cdot \left( \frac{p}{p_{a}} \right)^{m'} \]  

(43)

in which the two modulus parameters are evaluated as \( k'_{v} = 726.7 \); \( m' = 0.14 \). Comparison of Fig. 8 with Fig. 6 indicates that the compressibility in an oedometer is less than that in isotropic compression. Shear stress-induced dilation in an oedometer is responsible for this particular observation, as it can compensate the contraction caused by the mean effective stress.

Solving Eqs. (35), (38) and (42) gives the representations of the involved moduli, i.e.

\[
\begin{align*}
K_{ss}^{p} &= 3 \left( \frac{1}{G} - \frac{1}{G^{e}} \right) \left( \frac{1}{K} \right)^{2} \\
K_{sv}^{p} &= \left( \frac{1}{K'} - \frac{1}{K} \right) K_{sv}^{p} - 3 \left( \frac{1}{K'} - \frac{1}{K} \right) K_{sv}^{p} - K_{sv}^{p} - 2 K' \\
K_{ss}^{p} &= 3 \left( \frac{1}{G} - \frac{1}{G^{e}} - \frac{1}{K_{sv}^{p}} \right)^{-1}
\end{align*}
\]

(44)

Once the shear and bulk moduli are obtained via Eqs. (4), (32), (37) and (43), all the entries in the flexibility and stiffness matrices can be determined by Eqs. (33) and (44).

It is clear that in the current study the plastic strains are evaluated by Eq. (24) directly based on experiments, and then the hardening parameter can be updated correspondingly. This logic is opposite to the conventional elastoplastic constitutive modeling, in which a hardening law is defined based on experiments and the plastic strains are estimated with the aid of a flow rule and the consistency condition (Lade and Kim 1995; Yao et al. 2008a, 2008b).

**Parameters and calibration**

There are five groups of parameters in the proposed model: (a) the shear moduli parameters, \( k_{s} \),
$k_s^c$, $n$, $\alpha$; (b) the bulk moduli parameters, $k_v, k_v^c, m, k_v', m'$; (c) the characteristic stress states parameters, $M, r_c, n_c, r_p, n_p$; (d) the dilatancy parameter, $d_0$; and (e) the creep parameters, $t_0, c_1, c_2, c_3$ and $n_1, n_2, n_3, n_4$. Although the number of parameters seems large, their determination are simple. Routine tests including isotropic, oedometric and triaxial compression experiments are enough for the determination of the first four groups of parameters, as illustrated in the previous parts. The creep parameters can be determined using triaxial creep experiments conducted under different confining stresses and stress levels, as described in Fu et al. (2018a).

Model verification

Triaxial compression and creep experiments on typical rockfill materials are used to verify the capability of the proposed model in capturing their loading-induced deformation and creep-induced hardening behavior.

Oedometric and triaxial compression experiments on rockfill

Large-scale oedometric and triaxial compression experiments on the rockfill referred previously (Fig. 2 and Fig. 7) are firstly modeled with the calibrated parameters listed in Table 1. The experiments used to calibrate the parameters are also included as a summary. Note the critical state stress ratio, $M$, is not calibrated herein as it is not a necessary parameter in this particular case, i.e. no loading and unloading need to be judged and the plastic strains are calculated explicitly by Eq. (24). In addition, $k_s^c = 5k_s$ and $k_v^c = 5k_v$ are set.

Fig. 9 shows the comparison between the experimental data and the model predictions. The
stress-strain and volume change curves match the experiments rather well. The pressure-dependent strength and stiffness, the initial shear contraction and the subsequent shear dilation can be reproduced well by the model. Only slight deviation can be observed in the volume change curves, i.e. the maximum compressive strains under low confining pressures (0.4 and 0.8 MPa) are overestimated while those under higher confining pressures (1.2, 2.0 and 3.0 MPa) underestimated. This deviation may be due to the scatter of the data points, particularly those at the beginning of shearing, used to calibrate the dilatancy parameter as shown in Fig. 4.

Isotropic and oedometric compression experiments are also modeled, and the obtained results coincide exactly with the approximating curves plotted in Fig. 6 and Fig. 8. Therefore, they are not shown again. Fig. 10(a) plots the stress path in an oedometric compression test. The horizontal stress increases almost proportionally with the vertical pressure, with a ratio of 0.21. This value is very close to the earth pressure coefficient at rest (0.20) used in plotting Fig. 8. Fig. 10(b) shows the shear-induced volumetric strain during vertical loading. Negative labels along the ordinate signify the dilation caused by shearing in the oedometric compression test with the specific rockfill.

**Triaxial compression and creep experiments on rockfill**

Zhang et al. (2017) performed drained triaxial compression and creep experiments with a crushed weakly-weathered granite to study the coupling effect between loading and creep. Table 2 lists the constitutive parameters determined using the published experimental data. Note that the parameters \( k_v' \) and \( m' \) are not calibrated as oedometric compression experiment was not
performed. As a compensation, a high value ($10^8$) is prescribed to $K_{sv}^p$ in all the simulations carried out for this rockfill. A consequence of this simplification is the definite invertibility of the plastic flexibility matrix, $C^p$ in Eq. (24), as the determinant of which is always positive. Fig. 11 compares the results obtained from triaxial compression experiments under different confining pressures and the corresponding predictive curves. The agreement between the model predictions and the corresponding experimental data are once again acceptable, indicating the reliability of the stress-strain model and the calibrated parameters. However, slight deviation can also be seen in the volume change curves under low confining pressures (0.2 and 0.4 MPa).

Fig. 12 shows the stress-strain and volume change curves obtained from two coupling tests conducted under $\sigma_3 = 0.4$ and 1.0 MPa, respectively. The model predictions are also included for comparison. Both tested specimens exhibit evident increase of stiffness after creep and this creep-induced hardening behavior is successfully captured by the current model. It deserves to point out that the creep strains under each deviatoric stress level are not calculated using Eqs. (16) and (17). Herein, the measured creep strains given in Zhang et al. (2017) are substituted into Eq. (28) to update the hardening parameter directly. The agreement of the experimental and predictive results seems to prove the effectiveness of the hardening law devised in the way presented in this study.

As pointed out previously, the parameters $M$ and $m$ ($m_v=1-m$) control the creep-induced hardening in the proposed model. Fig. 13 shows the sensitivity of the creep-induced hardening to the two yield surface parameters. A higher value of $M$ corresponds to a higher degree of creep-induced hardening. However, the sensitivity is low as an increase of $M$ from 1.20 to 1.80
only results in a slight shift of the stress-strain curve. The creep-induced hardening is also more evident with a lower value of \( m \) as can be seen in Fig. 13(b). It can also be seen from Fig. 13 that the volumetric strain curve almost does not change with the yield surface parameters.

To demonstrate the capability of the model in capturing the dependence of creep behavior upon previous loading history, an one-stage creep experiment conducted under a confining pressure of 0.6 MPa (Zhang et al. 2017) is used as a reference as shown in Fig. 14. The specimen is loaded to a deviatoric stress of 2.4 MPa (A), and then it is allow to creep. The evolution of creep strains as well as the creep-induced hardening can be seen in Fig. 14. Note that the exponential evolution rules given by Eqs. (20) and (21) could not reproduce the creep strains at the initial stage well. However, this is not the focus of the current study. Improvement may be achieved by using other evolution rules (e.g. Fu et al. 2018). Now assume that a second specimen is loaded to a deviatoric stress of 2.7 MPa (B) and then unloaded to 2.4 MPa (C), both the final amounts of creep strains and their rates are considerably smaller than those of the first specimen that has not experienced unloading, as is shown in Fig. 14 (b). No experimental results are available for a quantitative comparison herein. However, the model predictions agree qualitatively with previous experimental observations (Lade et al. 2009).

Verification performed with the experimental results of two rockfill materials in this section demonstrates that the salient mechanical behavior of rockfill materials can be effectively captured and satisfactorily reproduced by the proposed model, and the calibration of relevant parameters is straightforward. In addition, the constitutive model can be used in a simplified way according to the available experimental data.
Summary and conclusions

An elastoplastic constitutive model considering the stress-strain and creep behavior of rockfill materials is proposed in the light of experimental observations. The stress-strain behavior is assumed independent of the loading rate as in most other constitutive models. The final amounts of creep strains are also assumed independent of the previous loading rate. However, previous loading path has an evident effect on subsequent creep strains, and the creep strains accumulated under a constant stress state also changes the subsequent loading behavior. This coupling effect is modeled in two steps. First, the loading-induced plastic strains, which accumulates from the current stress state to the one where the material is unloaded, is evaluated and taken into account in predicting the rates of subsequent creep strains. Second, the creep strains happen in a given stage are used to update the hardening parameter and therefore the location of the creep-altered yield surface. The model is capable of capturing many important behavior of rockfill materials, including the pressure-dependent strength, the shear contraction and dilation, and the creep-induced stiffening. In addition, only simple routine laboratory tests, including oedometric compression, isotropic compression, triaxial compression and creep experiments are required for the calibration of the model parameters.

The limitations of the proposed model also deserves to be mentioned. First, the model is formulated in the $p-q$ space, in which only two stress and strain invariants are used. Therefore, it is in principle only suitable for describing the material behavior under triaxial compression stress states. Extension to true triaxial stress states can be fulfilled by different ways. However, it is not
attempted in the current study due to the lack of true triaxial experiments. Second, post-peak strain softening behavior is often observed in triaxial compression experiments on granular materials, during which both the deviatoric stress and the volume changes tend to level off. These particular behavior cannot be reproduced by the current model as can be seen in the model predictions. Incorporation of some additional state variable(s) and use of the critical state concept can strengthen the model in this aspect. Third, the model can neither reproduce the stress relaxation behavior of materials although the mechanism of which is thought to be the same as that for creep.

Despite of the above limitations, the model is simple in concept and effective in capturing salient features of rockfill materials. The most important conclusion obtained from this study is the feasibility of using the hardening plasticity to model the creep-induced hardening behavior, which is of practical value in predicting deformation distribution of rockfill dams constructed over a long period. A more relevant engineering problem that the model may be used for is the evaluation of stress and deformation behavior of a heightened rockfill dam, in which the original dam has already experienced a long-term operation under nearly constant stresses and new loads are expected to be applied.

Acknowledgements

This work is supported by the National Key Research and Development Program of China (No. 2017YFC0404806) and the National Natural Science Foundation of China (No. 51779152). The
authors are also grateful to Prof. Zhang Binying in Tsinghua University for providing the experimental results used in this study.

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Notation

\( \sigma_1, \sigma_2, \sigma_3 \) maximum, intermediate, and minimum principal stresses

\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) maximum, intermediate, and minimum principal strains

\( p \) mean effective stress

\( q \) deviatoric stress

\( \eta \) stress ratio

\( dp, dq \) mean effective stress increment, and deviatoric stress increment

\( \varepsilon_v, \varepsilon_s \) volumetric strain, and deviatoric strain

\( \varepsilon_v^p, \varepsilon_s^p \) plastic volumetric strain, and plastic deviatoric strain

\( \varepsilon_v^c, \varepsilon_s^c \) volumetric creep strain, and deviatoric creep strain

\( \varepsilon_v^f, \varepsilon_s^f \) final amounts of creep strains

\( d \varepsilon_v^p, d \varepsilon_s^p \) plastic volumetric strain increment, and plastic deviatoric strain increment

\( d \) dilatancy ratio

\( K, G \) bulk and shear moduli

\( K^e, G^e \) elastic bulk and shear moduli

\( K' \) bulk modulus in oedometric compression

\( G_0 \) initial shear modulus in triaxial compression

\( f \) yield function

\( g \) plastic potential function

\( H \) hardening parameter
$W^p$ plastic work

$M$ critical state stress ratio

$M_c$ constant volume stress ratio

$M_p$ peak state stress ratio

$t$ time

$t_0$ reference time

$[C^p]$ plastic flexibility matrix

$[D^p]$ plastic stiffness matrix

$p_a$ atmospheric pressure

$K_{vv}^p, K_{vs}^p$ volumetric strain related plastic moduli

$K_{sv}^p, K_{ss}^p$ deviatoric strain related plastic moduli

$k_v, k_v^e, n, \alpha$ shear moduli parameters for triaxial compression

$k_v, k_v^e, m$ bulk moduli parameters for isotropic compression

$k_v', m'$ bulk moduli parameters for oedometric compression

$r_c, n_c$ constant volume stress states parameters

$r_p, n_p$ peak stress states parameters

$d_0$ initial dilatancy parameter

$c_1, c_2, n_1, n_2$ volumetric creep strain parameters

$c_3, n_3, n_4$ deviatoric creep strain parameters
Figure Captions

Fig. 1 Schematic illustration of the coupling effects between loading and creep (a) void ratio $e$ v.s. mean effective stress $p$; (b) deviatoric stress $q$ v.s. deviatoric strain $\varepsilon_d$

Fig. 2 Loci of the yield surfaces on the $p$-$q$ plane with different parameters

Fig. 3 Triaxial compression experiments with a typical rockfill (a) stress ratio $\eta$ v.s. axial strain $\varepsilon_1$; (b) volumetric strain $\varepsilon_v$ v.s. axial strain $\varepsilon_1$

Fig. 4 Contours of the volumetric strain and the input work in triaxial compression (a) Contours of $\varepsilon_v$; (b) Contours of $W/p_a$

Fig. 5 Stress dilatancy relationships in triaxial compression (a) dilatancy ratio $d$ v.s. stress ratio $\eta$; (b) dilatancy ratio $d$ v.s. normalized stress ratio $\eta/M_c$

Fig. 6 Isotropic compression data of the tested rockfill and the regression curve

Fig. 7 Relationships between the shear modulus and the mean stress and the stress ratio (a) $G_0/p_a$ v.s. $p/p_a$; (b) $G/[k_e p_a (p/p_a)^n]$ v.s. $\eta/M_p$

Fig. 8 Oedometric compression data of the tested rockfill and the regression curve

Fig. 9 Comparison of the model predictions and the experimental results (a) deviatoric stress $q/p_a$ v.s. axial strain $\varepsilon_1$; (b) volumetric strain $\varepsilon_v$ v.s. axial strain $\varepsilon_1$

Fig. 10 Model predictions of the oedometric compression experiment (a) effective stress path; (b) shear-induced volumetric strain

Fig. 11 Comparison of the model predictions and the experimental results by Zhang et al. (a) deviatoric stress $q/p_a$ v.s. axial strain $\varepsilon_1$; (b) volumetric strain $\varepsilon_v$ v.s. axial strain $\varepsilon_1$

Fig. 12 Comparison of the model predictions and the creep experiments by Zhang et al. (a) $\sigma_3 = 0.4$ MPa; (b) $\sigma_3 = 1.0$ MPa

Fig. 13 Sensitivity of creep-induced hardening to yield surface parameters ($\sigma_3 = 0.6$ MPa) (a) Influence of parameter $M$; (b) Influence of parameter $m$.

Fig. 14 Influence of loading history on subsequent creep strains ($\sigma_3 = 0.6$ MPa). (a) stress-strain and volume change curves; (b) evolution of creep strains
Fig. 1 Schematic illustration of the coupling effects between loading and creep
(a) void ratio $e$ v.s. mean effective stress $p$; (b) deviatoric stress $q$ v.s. deviatoric strain $\varepsilon_s$

Fig. 2 Loci of the yield surfaces on the $p$-$q$ plane with different parameters
Fig. 3 Triaxial compression experiments with a typical rockfill
(a) stress ratio $\eta$ v.s. axial strain $\varepsilon_1$; (b) volumetric strain $\varepsilon_v$ v.s. axial strain $\varepsilon_1$

Fig. 4 Contours of the volumetric strain and the input work in triaxial compression
(a) Contours of $\varepsilon_v$; (b) Contours of $W/p_a$
Fig. 5 Stress dilatancy relationships in triaxial compression
(a) dilatancy ratio $d$ v.s. stress ratio $\eta$; (b) dilatancy ratio $d$ v.s. normalized stress ratio $\eta/M_c$

Fig. 6 Isotropic compression data of the tested rockfill and the regression curve
**Fig. 7** Relationships between the shear modulus and the mean stress and the stress ratio

(a) $G_0/p_a$ v.s. $p/p_a$; (b) $G/[k_s·p_a·(p/p_a)^n]$ v.s. $\eta/M_p$

**Fig. 8** Oedometric compression data of the tested rockfill and the regression curve
Fig. 9 Comparison of the model predictions and the experimental results

Fig. 10 Model predictions of the oedometric compression experiment
   (a) effective stress path; (b) shear-induced volumetric strain
Fig. 11 Comparison of the model predictions and the experimental results by Zhang et al. 
(a) deviatoric stress \(q/p_a\) v.s. axial strain \(\varepsilon_1\); (b) volumetric strain \(\varepsilon_v\) v.s. axial strain \(\varepsilon_1\)

Fig. 12 Comparison of the model predictions and the creep experiments by Zhang et al. 
(a) \(\sigma_3 = 0.4\) MPa; (b) \(\sigma_3 = 1.0\) MPa
Fig. 13 Sensitivity of creep-induced hardening to yield surface parameters ($\sigma_3 = 0.6$ MPa)
(a) Influence of parameter $M$; (b) Influence of parameter $m$.

Fig. 14 Influence of loading history on subsequent creep strains ($\sigma_3 = 0.6$ MPa).
(a) stress-strain and volume change curves; (b) evolution of creep strains.
# Tables

## Table 1 Constitutive parameters of the tested rockfill

<table>
<thead>
<tr>
<th>Description of parameters</th>
<th>Experiments in use</th>
<th>Notations</th>
<th>Values</th>
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## Table 2 Constitutive parameters of the tested rockfill by Zhang et al.

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