Full-Scale Monitoring of a Tall, Slender Building with Coupling Viscoelastic Dampers

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science

Department of Civil and Mineral Engineering

University of Toronto

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2019
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2019

Abstract

Tall, slender buildings are sensitive to dynamic vibrations caused by wind, and the design of the building may be governed by occupant motion perception. A method to control dynamic vibrations is to increase the damping of the structure, which may be accomplished by the addition of a supplemental damping system. A novel system, the Viscoelastic Coupling Damper (VCD), was implemented for the first time in a tall, slender building which was under construction during the completion of this thesis. This building was the subject of a year-long plus monitoring program where output-only system identification algorithms were applied to track the development of the dynamic properties through the construction of the building. Additionally, several large amplitude wind events occurred during the program allowing for the tracking of amplitude-dependent phenomenon. The role of the VCD system was evaluated, and numerical finite-element models were constructed with reference to the experimental results.
Acknowledgements

I would like to thank my supervisor, Dr. Constantin Christopoulos, for providing the opportunity to perform such a unique and exciting research project. Thank you for sharing your expertise and inspiring vision for the future of structural engineering with me. It’s been a pleasure working together on this exciting topic, and I look forward to seeing future work on the subject.

I would like to thank my industry advisor, Dr. Michael Montgomery, for your continuous support throughout the completion of this thesis. Your constant dedication and passion for this work always inspired me, and your assistance with both experimental and analytical work, attention to detail, and constant inquisition were essential to the completion of this work and greatly improved the quality of this research.

I would like to thank Dr. Evan Bentz for his thorough review of this thesis. Your review and constructive feedback helped produce a stronger thesis.

I want to thank Xiaoming Sun and Michel Fiss for their work helping me with the instrumentation for the testing program in this work.

I would also like to thank my many colleagues at The University of Toronto. Work is always made better when you thoroughly enjoy working with the people around you, and I couldn’t have asked for a more talented, awesome group to be surrounded with throughout graduate school. Thank you all.

I would like to thank my family for always encouraging me to pursue my passions and to work hard at what I love. Thank you.

Lastly, I would like to thank Jenn for her unconditional support over the last two and half years. I would not be where I am today and could not have done this work without you by my side. Thank you.
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<th>Definition</th>
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<tr>
<td>AMD</td>
<td>Active Mass Damper</td>
</tr>
<tr>
<td>ATMD</td>
<td>Active Tuned Mass Damper</td>
</tr>
<tr>
<td>AVT</td>
<td>Ambient Vibration Testing</td>
</tr>
<tr>
<td>BRB</td>
<td>Buckling Restrained Brace</td>
</tr>
<tr>
<td>CFSMP</td>
<td>Chicago Full-Scale Monitoring Program</td>
</tr>
<tr>
<td>CUHK</td>
<td>City University of Hong Kong</td>
</tr>
<tr>
<td>DAQ</td>
<td>Data Acquisition</td>
</tr>
<tr>
<td>DET</td>
<td>Damping Evaluation Technique</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DVA</td>
<td>Dynamic Vibration Absorber</td>
</tr>
<tr>
<td>ESWL</td>
<td>Equivalent Static Wind Load</td>
</tr>
<tr>
<td>ERA</td>
<td>Eigensystem Realization Algorithm</td>
</tr>
<tr>
<td>EFDD</td>
<td>Enhanced Frequency Domain Decomposition</td>
</tr>
<tr>
<td>FVT</td>
<td>Forced Vibration Testing</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>GMM</td>
<td>Generalized Maxwell Model</td>
</tr>
<tr>
<td>GEF</td>
<td>Gust Effect Factor</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HMD</td>
<td>Hybrid Mass Damper</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>KVM</td>
<td>Kelvin-Voigt Model</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear Time-Invariant</td>
</tr>
<tr>
<td>LS</td>
<td>Least-Squares</td>
</tr>
<tr>
<td>LE</td>
<td>Local Extrema</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transformer</td>
</tr>
<tr>
<td>MAC</td>
<td>Modal Assurance Criteria</td>
</tr>
<tr>
<td>MDOF</td>
<td>Multi Degree of Freedom</td>
</tr>
<tr>
<td>NBCC</td>
<td>National Building Code of Canada</td>
</tr>
<tr>
<td>NExT</td>
<td>Natural Excitation Technique</td>
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<th>Description</th>
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<tbody>
<tr>
<td>OMA</td>
<td>Operational Modal Analysis</td>
</tr>
<tr>
<td>OEM</td>
<td>Original Equipment Manufacturer</td>
</tr>
<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
</tr>
<tr>
<td>PP</td>
<td>Positive Point</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RDT</td>
<td>Random Decrement Technique</td>
</tr>
<tr>
<td>RDS</td>
<td>Random Decrement Signature</td>
</tr>
<tr>
<td>SA</td>
<td>Spectral Analysis</td>
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<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
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<tr>
<td>SR</td>
<td>Slenderness Ratio</td>
</tr>
<tr>
<td>SDS</td>
<td>Supplemental Damping System</td>
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<tr>
<td>SDOF</td>
<td>Single Degree of Freedom</td>
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<tr>
<td>SLS</td>
<td>Serviceability Limit State</td>
</tr>
<tr>
<td>SSI-DATA</td>
<td>Data-Driven Stochastic Subspace Identification</td>
</tr>
<tr>
<td>SID</td>
<td>System Identification</td>
</tr>
<tr>
<td>TMD</td>
<td>Tuned Mass Damper</td>
</tr>
<tr>
<td>TLD</td>
<td>Tuned Liquid Damper</td>
</tr>
<tr>
<td>TSD</td>
<td>Tuned Sloshing Damper</td>
</tr>
<tr>
<td>TLCD</td>
<td>Tuned Liquid Column Damper</td>
</tr>
<tr>
<td>ULS</td>
<td>Ultimate Limit State</td>
</tr>
<tr>
<td>VCD</td>
<td>Viscoelastic Coupling Damper</td>
</tr>
<tr>
<td>VE</td>
<td>Viscoelastic</td>
</tr>
<tr>
<td>VED</td>
<td>Viscoelastic Damper</td>
</tr>
<tr>
<td>VD</td>
<td>Viscous Damper</td>
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Chapter 1: Introduction

1.0 Introduction

1.1 Background and Motivation

1.1.1 Rise of Tall, Slender Buildings in Cities

In recent years, large cities around the world have seen an increasing demand for high-rise structures. In Canada, and specifically in Toronto, Ontario, there are a variety of factors at play that have generated this demand. The Growth Plan introduced by the Ontario government in 2006 mandates that 40% of all development must occur within already existing communities, which results in developers having to “build up and not out” (BILD, 2016). Additionally, due to the increasing average price of new low-rise housing in the Toronto area, many buyers are forced towards the more affordable high-rise housing option (Bishop, 2016).

However, as real estate becomes more expensive and scarce in city cores, buildings are forced to become taller and more slender to create economically viable projects (Hall, 2016). Globally, this is evident as well, as the rate of tall building construction has continued to increase year over year (Figure 1.1).

![Buildings 200 m or Taller Completed Each Year from 1960 to 2018](image)

Figure 1.1 Year by year number of buildings 200 metres or taller completed. Adapted from (CTBUH, 2018).
Chapter 1: Introduction

In addition to the expansive growth of high-rise building construction around the world, the structures themselves are tending to be more slender. Slenderness is defined as the height of the building divided by its smaller base dimension and buildings are typically deemed slender if the Slenderness Ratio (SR) exceeds 10:1 (The Skyscraper Museum, 2016). Building taller and more slender creates the economic benefit of providing more units on smaller plots of land, a highly desirable feature given the price of real estate in large cities like New York City and Toronto. Examples of slender buildings from these cities are shown in Figure 1.2.

![Images of buildings](image)

Figure 1.2 (a) 432 Park Avenue, NYC (SR 15:1) - 425.7 metres, completed 2015 (The Skyscraper Museum, 2016); (b) Steinway Tower, NYC (SR 23:1) - 453.5 metres, completion expected 2019 (The Skyscraper Museum, 2016) (c) YC Condos, Toronto (SR 10.5:1) – 198 metres, completed 2019.

1.1.2 Engineering Challenges of Tall and Slender Buildings

As buildings increase in height and slenderness, they become increasingly sensitive to dynamic vibrations caused by wind, which increases the forces and deformations that the structure must withstand. Additionally, wind-induced vibrations also create the unique problem of occupant motion perception, where occupants may experience varying levels of discomfort, which may become one of the most important parameters for the design of tall and slender buildings.

The response of a structure to wind is governed by the shape of the building, and its mass, stiffness, and damping properties. Through interaction with wind tunnel testing, the aerodynamic shape of the building may be optimized to reduce the dynamic effects of winds. Conventionally,
the stiffness of a structure may be increased to reduce the wind induced response. However, this results in larger structural members and in regions of seismicity, results in higher seismic loads on the structure, which is undesirable. The damping ratio of the structure is a key parameter which can significantly reduce the dynamic loads from wind; however, it is the most uncertain of the structural properties given the significant number of complex mechanisms that contribute to it. Additionally, it has been observed that both the natural frequency and damping vary with amplitude of vibration, increasing the difficulty of accurately assessing the building response to wind at the design stage.

To control the dynamic response of a structure to wind, engineers may implement supplemental damping systems (SDSs) in a building. This has the dual effect of reducing wind-induced motion and mitigating the uncertainty with respect to the inherent damping of a building by engineering the energy dissipation. SDSs may be broadly characterized into two categories; Dynamic Vibration Absorbers (DVAs) and distributed damping systems. DVAs are large masses placed at the top of the structure that have the effect of reducing building motion due to the out-of-phase motion of the added mass. Distributed systems are typically comprised of viscous or viscoelastic dampers spread throughout the structure and located in areas of high relative motion. Both systems have been shown to be effective for reducing wind-induced motion by increasing the overall effective damping of the structure.

To assess a structure’s response to wind and appropriately evaluate the need for and design of SDSs, engineers must be able to construct precise models to simulate the behaviour of the building. This requires understanding of the natural frequency, a function of mass and stiffness, and the damping ratio of the building. However, determining these properties presents a significant challenge to engineers as there is a lack of systematic validation in full scale (Kijewski-Correa, et al., 2013). Full-scale buildings are rarely tested for performance evaluation and to understand how high-rise buildings behave requires in-situ validation, particularly the damping ratio at design level events. Figure 1.3 shows key design ranges and the current status of the database of full-sale damping measurements.
Chapter 1: Introduction

1.1.3 Growth of Full-Scale Building Monitoring

Given these limitations there has been a large thrust over the last several decades to develop methods to monitor the full-scale performance of tall buildings to characterize the in-situ dynamic properties. Some effort has been placed in the field of Forced Vibration Testing (FVT), where the structure is artificially excited, and the response is measured. While this is the most accurate means of testing buildings, there are several logistical challenges with implementing FVT. Organizing and implementing this type of testing requires the collaboration and approval of building owners, which historically has proven to be difficult. Additionally, if the building is occupied there is the concern of occupants to consider in the excitation of the building. These items typically preclude the application of FVT to most tall buildings. Alternatively, the structure may be tested under its operational condition by applying Ambient Vibration Testing (AVT) / Operational Modal Analysis (OMA) methods. This has become the preferred technique for testing of tall buildings since it is non-obtrusive as there is no forced excitation of the building. The testing method relies on the structure being excited in a random nature by wind, building activities, and other random loads, and using state-of-the-art output-only system identification algorithms the dynamic properties may be estimated.
Chapter 1: Introduction

1.1.4 Viscoelastic Coupling Damper (VCD)

Viscoelastic dampers have been used extensively over the last several decades with over 40,000 dampers implemented in over 250 projects in regions of high wind and seismicity around the world (Pant, Montgomery, & Christopoulos, 2019). Given the growth of tall, slender buildings made of reinforced concrete, where the primary lateral force resisting system is typically concrete core walls, demand for a new method of providing supplemental damping in this structural system was desired. This led to the development of the Viscoelastic Coupling Damper (VCD), a novel damper application designed for configuration in reinforced concrete coupled shear wall systems that provides higher efficiency for tall buildings than previous viscoelastic dampers configurations. The damper is comprised of a series of viscoelastic polymers bonded to steel plates, which dissipate energy when deformed in shear (Figure 1.4). [ (Montgomery, 2011), (Christopoulos & Montgomery, 2013), (Christopoulos, Montgomery, & Aiken, 2017)].

![Viscoelastic Coupling Damper (VCD)](Kinetica, 2018)

The first building in the world incorporate this system, YC Condos, was under construction in Toronto, Canada during the completion of this research, and was the subject of a year long plus monitoring program as the test subject for this thesis. The damper configuration in YC Condos as well as the deformation in the system is shown in Figure 1.5.

![Viscoelastic Coupling Damper (VCD)](Kinetica, 2018)
1.2 Objectives of Research

The previous discussion highlights the need for increasing the database of monitored high-rise buildings, as well as full-scale validation of the performance of the VCD in its first application. The objectives of this research are:

- Present a literature review that summarizes the challenges of designing tall and slender buildings and describes the history of tall building dynamics regarding the understanding of the in-situ dynamic properties and their observed amplitude-dependency.
- Explain and evaluate common output-only system identification algorithms, discussing the challenges of applying these methods for the extraction of dynamic properties of tall buildings.
- Determine the in-situ dynamic characteristics (natural period, modal damping, and mode shapes) of the tall, slender test building in Toronto via full-scale monitoring under ambient vibrations and large wind storms.
- Measure the response of the VCDs under various conditions to determine the relationship between the global and local deformation of the structure and to assess the supplemental damping provided by the VCDs.
- Build “As-Built” and calibrated analytical finite element models of the structure on select monitoring days to study the effects of typical engineering modelling assumptions, calculate the estimated supplemental damping provided by the VCD system, and study the
structural behaviour in detail comparing to the observed in-situ experimental properties and behaviours.

1.3 Thesis Organization

The main body of this thesis is comprised of seven chapters:

- Chapter 1 of this thesis provides a brief background regarding the motivation for this research and lists the specific objectives of the investigation.
- Chapter 2 provides a literature review summarizing the current challenges faced by structural engineers when designing tall, slender buildings and describes the historical development of inherent damping models, discussing the implications of amplitude-dependent damping and frequency.
- Chapter 3 reviews the field of system identification and describes the output-only algorithms used in this study, critically assessing the appropriate application of different methods. A numerical study is included to investigate the application of several output-only system identification algorithms.
- Chapter 4 describes the full-scale monitoring experimental program. This chapter provides a description of the test building and the VCD system as well as the experimental procedure, instrumentation and data processing.
- Chapter 5 reviews the results from each phase of the experimental program, including the progression of the dynamic characteristics with construction sequencing, the amplitude-dependency observed during several high-amplitude wind events, and the relationship between the local deformation in the VCDs and the overall building deformation. The implications of these results are discussed in detail.
- Chapter 6 describes the modelling aspect of the research. First, a brief background on structural modelling and modelling the VCDs is provided. Following this, the construction of two “As-Built” models is described. These models were used for a sensitivity study of the effect of temperature and cracking assumptions on the period, damping ratio, and global-local deformation ratio. These results are compared to those obtained experimentally, and a calibrated model is obtained from the best fit from the sensitivity study. Using the calibrated model, the supplemental damping provided by the VCD system
Chapter 1: Introduction

is estimated. Lastly, a discussion regarding the structural behaviours as they relate to the analytical and experimental work is provided.

- Chapter 7 summarizes the work completed for this thesis and provides recommendations for future work to be completed on the topic.
2.0 Background

2.1 Tall, Slender Building Challenges

Tall, slender buildings are sensitive to dynamic vibrations caused by wind, and the design of the building may be governed by occupant motion perception. Understanding this first requires an understanding of the interaction between the wind and the structure. This section highlights the difficulties faced with respect to occupant motion perception and leads into the methods engineers use to mitigate these problems.

2.1.1 The Nature of Wind

Wind is primarily caused by pressure difference in the atmosphere due to differential heating of the earth’s surface by the sun and the rotation of the earth itself, known as the Coriolis force. At the surface of the earth, wind can be simply defined as air movement relative to the earth. As the air passes over the surface of the earth, the terrain causes a horizontal friction force on the air, resulting in turbulence (Figure 2.1) (Holmes, 2007).

![Figure 2.1 Wind interaction with the terrain on the surface of the earth, causing turbulence. Adapted from (Holmes, 2007).](image)

This interaction generates the atmospheric boundary layer; a vertical velocity profile that varies in depth, relative to the surface of the earth, from a few hundred metres to several kilometres (Tamura & Kareem, 2013). Since this is the region in which structures are built, the atmospheric boundary layer is of particular interest for structural engineers.

Within the boundary layer, the wind may be mathematically described using two components; a mean and fluctuating component (Equation 1).
Chapter 2: Background

\[ U(z) = \bar{U}(z) + u(z, t) \]  \hspace{1cm} \text{Equation 1}

Where:
- \( U(z) \): Total wind as a function of height
- \( \bar{U}(z) \): Mean wind component
- \( u(z, t) \): Fluctuating wind component
- \( z \): Height
- \( t \): Time

The mean wind component is generally represented using a power law relationship (Equation 2) (Kijewski, Haan, & Kareem, 2004).

\[ \bar{U}(z) = \bar{U}_{ref} \left( \frac{z}{z_{ref}} \right)^{\alpha} \]  \hspace{1cm} \text{Equation 2}

Where:
- \( \bar{U}(z) \): Mean wind component
- \( \bar{U}_{ref} \): Mean reference velocity
- \( z \): Height
- \( z_{ref} \): Reference height
- \( \alpha \): Constant that varies with the roughness of the terrain

A visualization of this representation is shown in Figure 2.2.

![Figure 2.2 Vertical representation of atmospheric boundary layer. Adapted from (Irwin, Denoon, & Scott., 2013).](image)

The fluctuating wind component is typically quantified by the longitudinal turbulence intensity measure; the ratio of the standard deviation of velocity to the mean wind velocity for a given
direction. While the turbulent component varies on a second-to-second basis, the average and peak components are typically evaluated on a one-hour basis (Irwin, Denoon, & Scott., 2013).

2.1.2 Wind Interaction with Structures

Since wind excitation varies in both space and time, the forces induced by the wind will also vary in space and time. Figure 2.3 shows general definitions for parameters regarding wind loading.

With reference to Figure 2.3, the oncoming wind results in positive pressures on the windward face, or the along-wind direction \((p_w)\). As the wind deflects around the structure, separation of the flow from the building may occur around sharp corners. This creates a region of high negative pressure on the sides of the buildings and the phenomenon known as vortex shedding. Vortex shedding is characterized by turbulent eddies that turn one way and then another behind a bluff body which result in a fluctuating lift, or crosswind, force on the body they are acting on (Figure 2.4). On the leeward face, there is a region of negative pressure, once again acting in the along-wind direction \((p_l)\).
Due to this loading of the structure, there are three structural response components; along-wind, crosswind, and torsional. The along-wind response is primarily due to pressure fluctuation in the approach flow, resulting in swaying in the direction of the wind. The maximum displacement is generally achieved in this direction, and is comprised of a mean, quasi-static displacement, and a varying dynamic resonant component. The crosswind response is primarily due to vortex shedding, resulting in swaying perpendicular to the wind. If the shedding frequency coincides with the natural frequency of the building, large dynamic amplification will occur as the structure experiences resonance. The shedding frequencies are given by Equation 3.

\[ N = S \frac{U}{b} \]  

Equation 3

Where:

\( N \): Frequency at which vortex shedding occurs

\( S \): Strouhal number, dependent on aerodynamic shape. Range in value from 0.1 to 0.3.

\( U \): Wind velocity

\( b \): Building width

Figure 2.5 shows the effect of vortex shedding on increasing the crosswind response in a critical velocity range for a particular structure.
Additionally, once the structure begins to oscillate in the crosswind direction, the vortex shedding frequency may “lock-in” to the natural frequency of the structure. This is due to the interaction between the structure and the wake, resulting in the frequency matching the natural frequency of the building over a larger velocity range than would be predicted using Equation 3. Generally, the maximum acceleration is achieved in this direction, and is a critical consideration for the design of tall, slender towers. Lastly, the torsional response is usually the result of imbalances in the pressure distribution of the wind, and offsets in structural stiffness and mass (Kijewski, Haan, & Kareem, 2004).

The excitation and response spectrum for wind is shown in Figure 2.6, which highlights the distinction between the along-wind and crosswind response of a structure.

The background response under lower wind speeds tends to excite the fundamental modes of the building in each direction (two orthogonal sway modes and one torsional mode, typically).
vortex shedding peak, as previously outlined, is a function of the building shape and wind speed and becomes a key concern for the design of tall, slender buildings. If the shedding frequency is close to the natural frequency of the building, as becomes common for tall, slender buildings, the resonant response in the crosswind direction may become a critical aspect of the design.

An additional consideration for wind loading is the interaction between the moving structure and the aerodynamic forces, known as aeroelasticity. This phenomenon can result in aerodynamic damping, where the movement of the structure coupled with the wind excitation can result in an effective change in the building’s damping. Aerodynamic damping may be positive, as is always the case in the along-wind direction, or negative, which may be the case in the crosswind direction. If the negative aerodynamic damping overcomes the structural damping, the dynamic instability referred to as galloping will occur and the response amplitude will grow significantly. While this is typically negligible for most buildings, it may become a concern for very slender supertall buildings with low natural frequencies (Irwin, Denoon, & Scott., 2013).

2.1.3 Quantifying Wind Interaction with Structures

In Canada, Limit States design approaches are implemented in the National Building Code of Canada (NBCC). In addressing the loads of a particular structure, the structural engineer must ensure that the probability that the structure will collapse is acceptably low, which embodies the concept of ultimate limit state (ULS) design. Additionally, it must be ensured that the probability that the deflections, accelerations, and other habitability limits exceed pre-defined thresholds is acceptably low, which embodies the concept of serviceability limit state (SLS) design.

Using traditional mechanics and a random vibration-based approach, the wind-induced response may be determined. However, building codes generally use a simplified process, known as the Equivalent Static Wind Load (ESWL) procedure that greatly reduces the complexity of the traditional approach. Based on the first mode properties, including the damping, frequency, and mode shape, a statistically derived Gust Effect Factor (GEF) is used to account for the dynamic resonant component of the response. The GEF is the ratio of the maximum expected response and the mean response (Kijewski, Haan, & Kareem, 2004). Generally, codes and standards determine an equivalent static load that is based on the mean wind pressure multiplied by the GEF. Note that there are two main procedures to determine the GEW ($C_g$); the Static and Dynamic procedure.
The Static procedure is applicable to low and medium rise buildings, and the dynamic properties of these structures are not required to conduct the analysis. This approach, while simple and easy to apply, may be overly conservative and inaccurate when applied to tall, slender towers, whereas the Dynamic procedure is intended to capture the amplified resonance response, and is primarily used for these types of structures. As per the NBCC, the Dynamic procedure must be applied when the slenderness ratio exceeds 4, the height of the building is greater than 60 metres, or the lowest natural frequency falls below 1 Hz and above 0.25 Hz. It is applied in the same ways as the static procedure, except that the gust factor ($C_g$) is a function of the height of the tower, the surrounding terrain, and the dynamic properties of the building (natural frequencies and damping). The NBCC specifies that the damping ratio for steel and concrete buildings should be taken as 1% and 2%, respectively.

Alternatively, there is a third procedure, namely the Experimental procedure, which consists of wind tunnel testing that accounts for the dynamic properties of the building. This may be used as an alternative to the Static and Dynamic procedure and is recommended for buildings which may be significantly affected by wind effects such as buffeting and vortex shedding. The NBCC specifies that the Experimental Procedure must be used when the lowest natural frequency of the building is less than 0.25 Hz (National Research Council Canada, 2010). Significant advantage may be gained from employing wind tunnel testing, which can capture the important structural response parameters, such as forces and accelerations, accurately (Gamble, 2003).

Generally, a wind consultant is retained to analyse the wind effects in detail. The structural engineer provides the wind consultant with building properties such as mass, stiffness, damping and modal properties, typically determined using an elastic finite element model with reduced concrete section properties to account for expected levels of cracking (Montgomery, 2011).

Wind tunnel tests simulate the upwind terrain to determine what the wind profile will be at the structure including expected levels of turbulence and buffeting from nearby structures. Depending on the level of testing required, a scaled model is developed by a wind consultant that can adequately capture the desired responses and assess the structural loads. A simple, lightweight model may be constructed of plastic, using 3D printing technology, or in the past of balsa wood or plastic foam. Conversely, detailed fully aeroelastic models, which are generally only used for exceptionally tall or flexible structures, can capture complex responses including damping and
aeroelastic effects. The wind tunnel testing model for the tall, slender test building located in downtown Toronto is shown in Figure 2.7.

Figure 2.7 Wind tunnel study model for tall, slender test building in downtown Toronto.

Peter Irwin, a founding member of notable wind tunnel experts RWDI, stated “Although hundreds of tall buildings are tested annually in commercial wind tunnel laboratories around the world, there is only limited feedback received about the actual performance of these tall buildings” (2008), which highlights the need for further validation of the performance of high-rise buildings in full-scale.

2.1.4 Occupant Motion Perception Criteria

A common challenge faced by tall slender buildings is an exceedance of typically used occupant motion perception criteria for accelerations. Evaluating the acceptance criteria for this is difficult, since human motion perception is highly variable and subjective. This can be dependant on several factors including perception type, which may be kinaesthetic (sensing the motion), visual cues (swinging items), or auditory cues (creaking). The severity of the response can depend on education and experience with building motion, or motion sickness susceptibility (Irwin, Denoon, & Scott., 2013). Additionally, since motion perception is not considered a safety issue, codes and standards do not provide rigidly defined guidelines since this is considered a performance criterion that will be agreed to by the project team. Several studies have investigated human response to motion, with a view to building motion. Burton, Kwok, and Abdelrazaq (2015) conducted a thorough review of the development of motion perception criteria, which is summarized in Table 2.1.
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Table 2.1 Summary of development of occupant motion perception criteria (Burton, Kwok, & Abdelrazaq, 2015).

<table>
<thead>
<tr>
<th>Year</th>
<th>Frequency Dependent</th>
<th>Acceleration Specification Type</th>
<th>Return Period (Years)</th>
<th>Notes</th>
<th>Committee</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>Yes</td>
<td>RMS</td>
<td>6</td>
<td></td>
<td>Academic Research</td>
</tr>
<tr>
<td>1977</td>
<td>No</td>
<td>0.01g - 0.03g</td>
<td>10</td>
<td>Lower for residential, higher for office</td>
<td>NBCC*</td>
</tr>
<tr>
<td>1984</td>
<td>Yes</td>
<td>RMS</td>
<td>5</td>
<td>RMS over 10-minute window</td>
<td>ISO6897-1984</td>
</tr>
<tr>
<td>1989</td>
<td>Yes</td>
<td>Peak</td>
<td>5</td>
<td></td>
<td>AWESC**</td>
</tr>
<tr>
<td>1995</td>
<td>No</td>
<td>5-7 milli-g</td>
<td>1</td>
<td>Residential, Hotels, Commercial</td>
<td>NBCC</td>
</tr>
<tr>
<td>2004</td>
<td>Yes</td>
<td>Peak</td>
<td>1</td>
<td>Different values for usage</td>
<td>AIJ***</td>
</tr>
<tr>
<td>2007</td>
<td>Yes</td>
<td>Peak</td>
<td>1</td>
<td>“</td>
<td>ISO10137-2007</td>
</tr>
</tbody>
</table>

* National Building Code of Canada  
** Australian Wind Engineering Society Commentary  
*** Architectural Institute of Japan

The current standard which is typically referenced is ISO10137-2007 (Figure 2.8 / Equation 4) for one year return periods. Wind tunnel consultants typically also provide their own recommendations acceptable acceleration levels for different building types and return periods, based on their experience.

![Figure 2.8 Evaluation curves for wind-induced vibrations in buildings in a horizontal (x,y) direction for a one-year return period (International Organization for Standardization, 2007).](image)
\[ a_{1\text{-year}} = Rf^{-0.445} \]

Equation 4

Where:

\[ a_{1\text{-year}}: \text{ 1-year peak criterion (milli-g)} \]

\[ R: \text{6.12 for office buildings and 4.08 for residential buildings} \]

\[ f: \text{Natural frequency (Hz)} \]

It is common for tall, slender buildings to exceed these limits, thus requiring some sort of engineering adjustment to meet the objective criteria for the project. Subsequent sections describe how engineers design buildings to meet occupant motion perception criteria.

2.1.5 Designing for Wind

Given that the wind loading varies in both space and time, classical structural dynamic analysis provides the basis for the approach to analysing the wind-induced response of structures. The equation of motion for a lumped mass multi-degree-of-freedom structure is shown in Equation 5.

\[ [M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \]

Equation 5

Where:

\[ [M]: \text{Mass matrix} \]

\[ [C]: \text{Damping matrix} \]

\[ [K]: \text{Stiffness matrix} \]

\[ \{F(t)\}: \text{Vector of time-varying forcing functions} \]

\[ \{x(t)\}: \text{Displacement vector.} \]

\[ \{\dot{x}(t)\}: \text{Velocity vector.} \]

\[ \{\ddot{x}(t)\}: \text{Acceleration vector.} \]

* indicates time derivative.

The dynamic properties of mass, damping and stiffness, together with the aerodynamic shape of the structure, dictate how the structure will dynamically respond to a given set of wind loads. The engineer may have some control over the mass of the structure and the stiffness of the elements through engineering design, however, the effect of mass and stiffness have varying affects on the critical response quantities of load, acceleration, instability, and vortex shedding.
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Damping, on the other hand, is always beneficial to reducing and controlling the dynamic resonant response (Vickery, Isyumov, & Davenport, 1983) as per Equation 6.

\[
Response = f\left(\frac{1}{\sqrt{\zeta}}\right)
\]

Equation 6

Where:

*Response*: Dynamic resonant structural response quantity (acceleration, base moment, etc.)

\(\zeta\): Damping ratio

Figure 2.9 shows the effect of varying the mass, stiffness, and damping for a given structure on accelerations and base moments showing the significant effect of damping on reducing the dynamic response of the structure.

![Figure 2.9 Effect of varying mass, stiffness, and damping on the accelerations and base moment: Adapted from (Irwin, Kilpatrick, Robinson, & Frisque, 2008).](image)

Generally, the building shape is specified by architects and building owners. However, if a wind consultant is retained early enough in the design, significant advantages may be gained from selecting an aerodynamically favourable shape. With reference to Figure 2.5 (Section 2.1.2) the vortex shedding peak in the critical velocity range should be pushed to the right so that it occurs at an extremely high, and likely unattainable wind velocity. To achieve this by increasing the stiffness would be expensive, and potentially cost-prohibitive. However, the height of the peak is sensitive to the aerodynamic shape and may be reduced or eliminated by employing an aerodynamically favourable shape. This has the effect of changing the Strouhal number for the building. There are variety of shaping strategies available (Figure 2.10).
Each of the strategies offers significant improvement to the dynamic behaviour under wind loading, with effects such as 25% reduction in base moment achievable (Irwin, Kilpatrick, Robinson, & Frisque, 2008).

While reductions in wind induced structural response may be achieved by increasing the stiffness of the building or aerodynamic shaping, the most reliable means to achieve this goal is by increasing the damping. Irwin (2008) stated that a change of 0.5% in damping can have a greater effect on the structural response than can be achieved by all reasonable structural measures. While this is the most important parameter for controlling the wind induced response, there is considerable uncertainty with respect to the inherent damping of tall, slender buildings. This is attributed to the fact that there are several mechanisms which contribute to energy dissipation in a structure (Figure 2.11).

The inherent material damping may be attributed to internal straining and friction in the material itself. For structural materials, concrete has generally shown higher values than steel which has been primarily attributed to friction on microcracks. Tall buildings tend to have non-structural components that are designed to accommodate deformations, thus the interaction between structural and non-structural components may be less significant for these types of structures.
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Damping may be generated through friction in structural members, such as slipping along cracks in reinforced concrete and slipping in bolted joints in steel structures, although these mechanisms may require deformation levels in excess of typical service level responses. The expected mechanism of soil damping for tall buildings is primarily due to rocking, which is expected to produce very low levels of damping. Lastly, aerodynamic damping is typically small or negligible for most tall buildings (Smith, Merello, & Willford, 2010), but as previously mentioned, may become a concern for supertall slender buildings with very low natural frequencies in the future (Irwin, Denoon, & Scott., 2013). It is evident that these sources are difficult to deterministically quantify, and the methods used by engineers and that are in development by researchers are discussed thoroughly in Section 2.2.

While the inherent damping is difficult to accurately quantify and is uncertain, the overall damping may be increased and made more reliable through the implementation of a Supplemental Damping System (SDS).

2.1.6 Supplemental Damping Systems

To control the dynamic response of a structure to wind, engineers may implement SDSs in a building. A general classification of select types of SDSs is shown Figure 2.12.

![Classification of Supplemental Damping Systems](image)

**Figure 2.12 Classification of Supplemental Damping Systems.**
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This section provides a brief overview of passive damping systems only, focusing on the most widely used systems including TMDs, TLDs, and VDs, with additional attention paid to VEDs as this is the subject of this thesis. DVAs dominate the SDSs commonly used around the world accounting for 62% of the tallest 50 supplementary damped buildings in the world, whereas VDs account for 20% and VEDs account for only 2% (Figure 2.13).

![Pie chart showing distribution of highest buildings by type](image)

**Figure 2.13** Percentage of 50 tallest buildings with SDSs by type. Adapted from (CTBUH, 2018).

2.1.6.1 Passive Dynamic Vibration Absorbers (DVAs): TMDs and TLDs

There are two main types of DVAs including Tuned Mass Dampers (TMDs) and Tuned Liquid Dampers (TLDs), the latter of which may be subcategorized into Tuned Sloshing Dampers (TSDs) and Tuned Liquid Column Dampers (TLCDs). Mechanical models of each of these systems are shown in Figure 2.14.

![Mechanical models of (a) TMD; (b) TSD; (c) TLCD](image)

**Figure 2.14** Mechanical models of (a) TMD; (b) TSD; (c) TLCD. Adapted from (Montgomery, 2011).

DVAs are tuned to the natural frequency of the structure, and effectively increase the damping by oscillating out-of-phase with the structure. This diverts energy from the structure to the DVA,
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thereby reducing the motion in the primary structure. It can be shown that for a particular tuning of the DVA for a single degree of freedom structure, the motion of the primary structure ($m_s$) in Figure 2.14 may be reduced to zero. The performance of these systems is quantified using the concept of effective damping (Equation 7), which compares the response of the system with and without the DVA (Love, Haskett, & Morava, 2018).

$$\zeta_{eff} = \zeta_{str} \frac{\sigma_{str}^2}{\sigma_{str\text{-damped}}^2}$$  \hspace{2cm} \text{Equation 7}

Where:
- $\zeta_{eff}$: Effective overall damping
- $\zeta_{str}$: Inherent structural damping
- $\sigma_{str}$: RMS response of structure without a DVA
- $\sigma_{str\text{-damped}}$: RMS response of structure with a DVA

For application in tall buildings, the DVA is typically placed near the top of the building where the motion is the largest, and the system is tuned to have a frequency close to the mode requiring supplemental damping. Passive DVAs have been implemented in several notable buildings around the world, including 432 Park Avenue in New York City (Figure 2.15).

Figure 2.15  Tuned mass damper used in 432 Park Avenue, NYC (RWDI, 2018).

Reported supplemental damping values from recent studies monitoring the full-scale response of tall buildings with DVAs are shown in Figure 2.2.
Since DVAs are typically located at the top storey of a building, valuable real estate at the top of high-rise buildings becomes unavailable. Additionally, these systems are heavy, where a TMD may be up to 4.4% of the building’s mass (Love, Haskett, & Morava, 2018), and thus increase the size of gravity members required in the structure and the lateral design loads. Mass dampers are tuned to increase the damping typically only in one mode in each direction, limiting their application to controlling wind-induced dynamic responses only. The damper is tuned relying on the expected natural frequency of the building in the design stage, which is difficult given the uncertainty associated with the modelling of the building, particularly for reinforced concrete buildings (Montgomery, 2011).

2.1.6.2 Distributed Damping Systems

The other general category of SDS are distributed dampers, which are typically viscous or viscoelastic, and are distributed in key areas throughout the structure. The devices are implemented in locations where large relative motion between structural members is expected, such as braces, coupling beams, or outriggers. Depending on the arrangement of the devices and the modes of vibration of the structure, distributed damping systems are typically capable of providing supplemental damping in more than one mode.

When designing a distributed damping system, the stiffness compatibility of the system must be considered. If the damper is designed to be less stiff relative to adjacent members, more proportional force will be driven into the adjacent members, thus reducing the dissipated energy by reducing the force driven across the damper.
2.1.6.2.1 Viscoelastic Dampers (VDs)

Viscous Dampers (VDs) are devices that dissipate energy by pushing a viscous fluid through an orifice which produces a velocity-dependent force from the flow of the viscous fluid across the head of the piston. A schematic showing a typical VD for application in a structure is shown in Figure 2.16.

![Figure 2.16 Schematic of viscous damper (Haskell & Lee, 2007).](image)

A practical application of a VD in an outrigger configuration is shown in Figure 2.17a, while a general force-deformation relationship for the case of a linear viscous damper is shown in Figure 2.17b.

![Figure 2.17 (a) Viscous dampers installed in outrigger configuration (Infanti, Robinson, & Smith, 2008); (b) VD force-displacement relationship.](image)

The energy dissipated by VDs ($E_{VD}$) is shown in Equation 8. 

\[
E_{VD} = \int F_c \delta \, dt
\]
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\[ E_{VD} = C \pi \omega u_0^2 \]  

Equation 8

Where:

\( E_{VD} \): Energy dissipated in VD

\( C \): Damping coefficient

\( \omega \): Angular frequency (rad/s)

\( u_0 \): Displacement

An important note for VDs is that the maximum force occurs when the displacement is zero, meaning the velocity-dependent force does not increase the forces attained at the peak displacement, a desirable feature of this damper type. While VDs have been incorporated in many buildings, a key concern is whether the dampers will maintain their long-term integrity, particularly considering the high variety of loads and long-term deformation patterns. There are several examples of VDs being used in bridges needing to be replaced due to excessive leaking (Matier & Ross, 2013) and concern that the expected force-deformation relationship of the device may not be reliable under such leaking (Konstantinidis & Makris, 2014). For these reasons, generally there are some accessibility and maintenance requirements as well as consideration of degraded viscous damping performance in design (Ooki, Kasai, Takahashi, & Sekiguchi, 2004). Lastly, since fluid damping is utilised in VDs supplemental damping may not be available at low amplitudes (Pant, Montgomery, & Christopoulos, 2019), which may be a concern for designers when implementing these devices to control frequent wind storms.

Reported design damping levels for several buildings incorporating VDs are shown in Table 2.3.

Table 2.3  Reported supplemental damping ratios for VDs in tall buildings.

<table>
<thead>
<tr>
<th>Building</th>
<th>Height</th>
<th>Application</th>
<th>Supplemental Damping</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. France Shangri-La</td>
<td>210 metres</td>
<td>Outrigger</td>
<td>7.5%</td>
<td>(Smith, 2016)</td>
</tr>
<tr>
<td>Grand Hyatt</td>
<td>225 metres</td>
<td>Outrigger</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>West 55th Street</td>
<td>40-storey</td>
<td>Steel Brace</td>
<td>2%</td>
<td></td>
</tr>
</tbody>
</table>
2.1.6.2.2 Viscoelastic Dampers (VEDs)

VEDs are devices that incorporate viscoelastic (VE) material deformation, typically in shear, which transforms the relative motion between two points in a structure into heat, resulting in a reduction of the oscillatory motion of the building. Typically, the VE material is sandwiched between and bonded to a rigid surface which is undergoing motion. An example of this is the VED configuration from the application in the original World Trade Centre towers (Figure 2.18a). The general force-deformation relationship for a VED is shown in Figure 2.18b.

![Figure 2.18](image)

The energy dissipated by a VED ($E_{VED}$) is shown in Equation 9.

$$E_{VED} = C \pi \omega u_0^2$$  

Equation 9

Where:

- $E_{VED}$: Energy dissipated in VED
- $C$: Damping coefficient, function of shear loss modulus ($G_c$) and the size of the damper
- $\omega$: Angular frequency (rad/s)
- $u_0$: Displacement

Note that this is the same equation as that for the VD (Equation 8).

The mechanical properties of VE materials are both frequency and temperature dependent and are weakly dependent on the amplitude of vibration. A key consideration in the design of VEDs is the temperature sensitivity of the VE material. There are two ways in which the temperature of the VED may vary:
1. Environmental condition: This is typically well-controlled in building structures at or near room temperature.

2. Temperature rise of the VE material due to cyclic motion: This may be significant for long-duration wind events and should be considered in the design of the system.

With consideration of the frequency and temperature of the structure in question, the design of a VED system requires the optimization of damper location, stiffness compatibility with other structural members, and force driven across the damper.

VE dampers are able to provide supplemental damping across all amplitude ranges due to the solid damping found in the viscoelastic material, where other mechanical dampers may be ineffective in this low-amplitude range. VCD units were tested down to 0.0025 millimetres (2.5μm) and displayed well-defined hysteresis loops at these small amplitudes (Pant, Montgomery, & Christopoulos, 2019).

The first application of these types of dampers was in the original World Trade Center towers (1969) in New York City, where approximately 10,000 VE dampers were placed between the lower chord of the floor system and the columns (Figure 2.19).

The dampers were tested periodically during their life and showed no signs of deterioration up to the final test in 1996 (Montgomery, 2011). The building performance was monitored during a hurricane in 1978 and it was found that the total damping of the building was approximately 2.5% to 3%, agreeing with design values (Mahmoodi, Robertson, Yontar, Moy, & Feld, 1987).
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Another application of VED was in the Columbia SeaFirst Building in Seattle, a 76-storey 263-metre-tall building, where 260 VEDs were incorporated in a braced configuration (Figure 2.20). While no full-scale test data was provided, the expected increase in damping was from 0.8% inherent to 6.2% for frequent wind events, and to 3.2% for design level wind events.

![Figure 2.20 VED configuration in Columbia SeaFirst Building, Seattle (Samali & Kwok, 1995).](image)

A summary of reported, either experimental or theoretical, supplemental damping ratios for buildings incorporating VEDs is shown in Table 2.4.

<table>
<thead>
<tr>
<th>Building</th>
<th>Height</th>
<th>Application</th>
<th>Supplemental Damping</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original World Trade Centre Towers</td>
<td>415 metres</td>
<td>Between floor and column</td>
<td>2.5% - 3% (total)</td>
<td>(Mahmoodi, Robertson, Yontar, Moy, &amp; Feld, 1987)</td>
</tr>
<tr>
<td>Columbia SeaFirst Building, Seattle</td>
<td>263 metres</td>
<td>Brace</td>
<td>5.6% (frequent wind)</td>
<td>(Samali &amp; Kwok, 1995)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.4% (design wind)</td>
<td></td>
</tr>
</tbody>
</table>

2.1.7 Summary of Tall, Slender Building Challenges

Tall, slender buildings face the challenge of being sensitive to dynamic vibrations caused by wind. This can cause increases in forces in the structure, and the design may commonly be governed by limiting the occupant’s motion perception to acceptable magnitudes. To do so,
engineers may increase the stiffness of the building, aerodynamically shape the building, or add a supplemental damping system. In order to assess the necessity of an SDS, the engineer must have an idea of what the inherent damping of the structure is. While it may be feasible in some circumstances to completely engineer the damping into the structure through SDSs, this may be an overly conservative approach given there is inherent damping the engineer may rely on. This leads into a key background component discussing the development of engineering models to capture the inherent damping of structures, with particular attention paid to amplitude-dependent phenomenon.

2.2 Amplitude-Dependent Damping

Damping plays a significant role in controlling the dynamic response of structures, and is particularly important for tall, slender buildings. The subsequent section reviews the concept of damping from fundamental structural dynamics through to current state-of-the-art models, detailing various complexities and the evolution of the subject matter.

2.2.1 Classical Dynamics and Equivalent Viscous Damping

The dynamic analysis of building structures involves idealizing the structure as a lumped mass multi-degree-of-freedom mass-spring-dashpot system (Figure 2.21).

![Figure 2.21 Three degree-of-freedom mass-spring-dashpot representation of three storey building structure.](image-url)
Each member of the structure contributes to the inertial (mass), elastic (stiffness), and energy dissipation (damping) properties of the structure. The contribution of each of these items is lumped into one of three categories; mass, damping, or stiffness. The general equation of motion for a multi-degree-of-freedom structure was shown in Equation 5 and is shown again in Equation 10 for discussion in this Section.

\[
[M][\ddot{x}(t)] + [C][\dot{x}(t)] + [K][x(t)] = \{F(t)\}
\]

Equation 10

Where:

\[M\]: Mass matrix

\[C\]: Damping matrix

\[K\]: Stiffness matrix

\[\{F(t)\}\]: Vector of time-varying forcing functions

\[\{x(t)\}\]: Displacement vector.

\[\{\dot{x}(t)\}\]: Velocity vector.

\[\{\ddot{x}(t)\}\]: Acceleration vector.

\(*\cdot\) indicates time derivative.

Assuming orthogonality of mode shapes, this equation is rewritten into the uncoupled equations of motions (Equation 11) where the modal superposition method may be utilised.

\[
m_j \ddot{q}_j(t) + c_j \dot{q}_j(t) + k_j q_j(t) = Q_j(t)
\]

Equation 11

Where:

\[m_j\]: Modal mass in mode \(j\)

\[c_j\]: Modal damping coefficient in mode \(j\)

\[k_j\]: Modal stiffness in mode \(j\)

\[Q_j\]: Modal force in mode \(j\)

\[q_j(t)\]: Modal response quantity (acceleration, velocity, displacement)

\(*\cdot\) indicates time derivative.

The damping matrix consists of coefficients which cannot be directly evaluated since there is no clear physical mechanism for damping in structures. A more convenient and common means of performing a dynamic analysis of structures is thus to perform a modal analysis where the damping ratio is used instead of the damping matrix. The damping ratio is a quantification of the energy dissipation per cycle in that mode and is defined in Equation 12.
\[ \zeta = \frac{c}{c_{cr}} = \frac{c}{2m\omega} \]  

Equation 12

Where:
\( \zeta \): Damping ratio
\( c \): Damping coefficient
\( c_{cr} \): Critical damping ratio
\( m \): Mass
\( \omega \): Natural angular frequency

Note that \( c_{cr} \) is the damping coefficient at which the structure would come to rest without oscillating. The uncoupled equations of motion may then be rewritten (Equation 13).

\[ m_j \ddot{q}_j(t) + 2\zeta_j m_j \omega_j \dot{q}_j(t) + m_j \omega_j^2 q_j(t) = Q_j(t) \]  

Equation 13

Thus, to evaluate the dynamic response of the structure the engineer must know the modal mass, modal damping ratio and natural frequency of the mode of interest. While mass and stiffness, and therefore natural frequency, are relatively straightforward to compute based on the structure, the inherent damping is very difficult to evaluate.

For convenience, it was assumed that these sources may be represented in the dynamic analysis by an equivalent linear viscous damper, which produced the linear second order differential equation shown in Equation 10.

The equivalent viscous damping ratio is the ratio of the energy dissipated in one cycle of vibration to the total stored elastic energy of that cycle. Figure 2.22 illustrates this concept and Equation 14 quantifies the equivalent viscous damping ratio. Note that the experiment to determine the equivalent viscous damping should be carried out at resonance and would only be appropriate for application at that frequency (Chopra, 2012).

\[ \zeta_{eq} = \frac{1}{4\pi} \frac{E_D}{E_{S0}} \]  

Equation 14
Figure 2.22 Equivalent viscous damping concept.

Where:

\( \zeta_{eq} \): Equivalent viscous damping ratio

\( E_D \): Energy dissipated in one cycle

\( E_{S0} \): Stored elastic energy in one cycle

Equivalent viscous damping ratios are typically determined by performing vibration tests. Traditionally, this may be done in the form of a free vibration test or by using a mass shaker to artificially excite the structure to generate the frequency response function. More recently, there have been significant developments in the field of full-scale operational vibration testing, which is the subject of this thesis and is discussed in more detail in Section 3.0.

2.2.2 Historical Overview of Damping Models

As previously outlined, the damping ratio plays a significant role in controlling the dynamic response of the structure, and thus it is of particular interest to seek more robust methods for quantifying the inherent equivalent viscous damping ratio of structures. The subsequent section provides a historical overview of the development of damping models for application to tall buildings.
2.2.2.1 Fundamental Development: 1930 - 1975

The notion of equivalent viscous damping (Equation 14) was introduced by Jacobsen (1930), who noted the arbitrary nature of the assumption and emphasized that this approximation was simply a representation of the actual dissipative forces present in the mechanical system. It was later noted that the primary source of damping in structures was slippage in joints and that damping was therefore a function of the amplitude of motion (Jacobsen, 1965). Later, Hart and Vasudevan (1975) showed the increase in damping with amplitude and noted that reinforced concrete structures generally have higher damping than steel structures.

Wyatt (1977) was one of the first researchers to propose a physically robust model for the explanation of amplitude-dependency in damping due to friction mechanisms. Wyatt (1977) proposed a stick-slip mechanism that was comprised of an elastic element in series with a friction element (Figure 2.23).

![Figure 2.23 Wyatt’s (1977) stick-slip model.](image)

Where:

- $F_L$: Force at which friction mechanism slips
- $k_f$: Initial stiffness of stick-slip element
- $k$: Stiffness of the structure

There are two phases in which this system may operate:

1. Force is less than $F_L$, therefore the friction element is locked in the “stuck” phase
2. Force is greater than $F_L$, therefore the friction element has been overcome and the damper is slipping at a constant force.
Chapter 2: Background

If the structure is characterized by several of these types of elements in parallel, each with different slip forces ($F_L$), then the overall building damping may still be calculated using an equivalent viscous damping approach.

2.2.2.2 Developing Phenomenological Models: 1983 - 2006

Up to this date, the proposed models did not possess a significant database of experimental data to refer to. Later, Jeary (1983), (1986), (1996), developed and performed a thorough review of the database of structural damping values in buildings that had been collected up to that time, diligently rejecting those that were unsuitable for detailed analysis. It was noted that damping estimates up to that date were riddled with a myriad of inconsistencies and issues such as:

- Damping values that did not reference amplitude
- Use of spectra from non-stationary data sets in damping estimation
- Modal interference from closely spaced modes

Those estimates that did not adequately account for the above were not considered in the study, and those remaining were used to develop one of the first damping ratio prediction formulae. Based on a database of 10 buildings, Jeary identified three regimes of damping (Figure 2.24).

![Figure 2.24 Generic damping model. Adapted from (Jeary, 1986).](image)

1. Low amplitude plateau
   - Attributed primarily to friction between large structural elements. At low amplitudes, a constant number of structural elements will be engaged, thus representing a constant damping ratio.

2. Non-linear amplitude dependent region
Chapter 2: Background

- As the amplitude increases an increasing number of elements will be engaged, thus increasing the damping with amplitude of vibration.

(3) High amplitude plateau
  - Once all the elements are mobilized, there will be no further increase in the damping prior to yielding of elements which will mobilize further hysteretic damping.

Based on this, and the basic characteristics of the buildings included in the study, Equation 15 was proposed.

\[ \zeta = \zeta_0 + \frac{\zeta_1}{H} \times x_H \]  

Equation 15

Where:
- \( \zeta \): Amplitude dependent damping ratio
- \( \zeta_0 \): Constant low amplitude damping, proportional to natural frequency (Equation 16)
  \[ \zeta_0 = f_0 = \frac{46}{H} \]  

Equation 16
- \( \zeta_1 \): Non-linear damping slope (Equation 17)
  \[ \log_{10} \zeta_1 = \frac{\sqrt{D}}{2} \]  

Equation 17

\( x_H \): Displacement amplitude at the top of the structure
\( H \): Height of the structure
\( D \): Plan dimension in the direction of oscillation

Substituting Equation 16 and Equation 17 into Equation 15 yields Equation 18.

\[ \zeta = \frac{46}{H} + 10 \times \frac{\sqrt{D}}{2} \times \left( \frac{x_H}{H} \right) \]  

Equation 18

In the same year, Davenport and Hill-Carroll (1986) proposed another amplitude-dependent damping ratio prediction model (Equation 19).

\[ \zeta = A \left( \frac{\sigma}{H} \right)^n \]  

Equation 19

Where:
- \( \sigma \): RMS amplitude (millimetres)
- \( H \): Height of the structure (metres)
- \( A, n \): Constants as per Table 2.5.
Chapter 2: Background

Table 2.5 Constants for Davenport and Hill-Carroll's (1986) damping model.

<table>
<thead>
<tr>
<th>Storeys</th>
<th>Material</th>
<th>A</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-20</td>
<td>Steel</td>
<td>0.03</td>
<td>0.075</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>0.03</td>
<td>0.11</td>
</tr>
<tr>
<td>&gt; 20</td>
<td>Steel</td>
<td>0.02</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>0.025</td>
<td>0.11</td>
</tr>
</tbody>
</table>

This was the result of regression analysis on a collection of measurements from several buildings from the same database used by Jeary (1986). These results noted:

- Damping tends to decrease as buildings get taller.
- Concrete buildings tend to have higher damping than steel buildings.

Davenport and Hill-Carroll (1986) considered the primary source of damping to be friction damping and attributed the amplitude-dependent damping to Wyatt’s (1977) stick-slip friction mechanisms. The energy loss due to a single stick-slip mechanism is given in Equation 20.

\[ \Delta E = 4F_L(x - x_0) \]

Equation 20

Where:
- \( \Delta E \): Energy loss
- \( F_L \): Force at which friction mechanism slips
- \( x \): Displacement amplitude
- \( x_0 \): Slip displacement

Elaborating on this concept, Davenport and Hill-Carroll (1986) assumed that there would be many of these stick-slip elements in the structure, and the number of slipping surfaces would increase as a function of amplitude. A theoretical derivation of the equivalent viscous damping assuming a function describing the number of stick-slip mechanisms as an exponential function of amplitude was provided as a proof-of-concept. The paper states that although this is a valid expression for damping based on the exponential growth of the number of stick-slip mechanisms, other expressions may be derived.

Lagomarsino (1993) proposed a damping model that was similar in nature to Rayleigh damping. It was noted that structural damping was comprised of two parts that define the mechanical model for damping:

1. Intrinsic material damping.
2. Friction in structural joints and between structural and non-structural components.
In the first, it was noted that ductile materials contribute very little to the structural damping which was primarily attributed to a truly viscous damping mechanism. Brittle materials, on the other hand, engage friction between microcracks as the primary source of damping and represent a significant contribution to the overall structural damping. This supports the conclusion that other researchers have drawn that the inherent damping of concrete structures is higher than steel structures.

In the second, the stick-slip friction mechanism was used to explain the amplitude-dependency of damping. Using this model, it was shown that once slipping is initiated, damping increases rapidly, remains stable for some amplitude, and then decreases asymptotically to zero, a note shown experimentally later by others. With multiple stick-slip mechanisms in parallel, engaging at different amplitudes, the stable peak is wider than that of a single element. Lagomarsino emphasized that for building structures, a significant portion of the energy dissipated in this manner may be attributed to structural and non-structural elements such as partition walls.

To define the phenomenological aspect of the model, 185 buildings across different heights (most less than 100 metres, some in the 100 metres – 200 metres range, very few above 200 metres) and typology were analysed. No clear discernable trend regarding the amplitude-dependency of damping was established and the focus of the model development was on the baseline damping ratio. This was attributed to limited data from buildings at higher amplitudes. It was found that the only parameter that the damping ratio was correlated with was the period, thus Equation 21 was proposed, which is analogous to Rayleigh’s law.

\[ \zeta_j = \alpha_1 T_j + \frac{\beta_1}{T_j} \]  

Equation 21

Where:
\( \zeta_j \): Damping ratio in mode \( j \)
\( \alpha_1, \beta_1 \): Constants depending on the type of structure (Table 2.6)
\( T_j \): Period of mode \( j \)

**Table 2.6 Constants for Lagomarsino’s damping model (Lagomarsino, 1993).**

<table>
<thead>
<tr>
<th>Constant</th>
<th>Steel</th>
<th>RC</th>
<th>Mixed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 )</td>
<td>0.3192</td>
<td>0.7238</td>
<td>0.2884</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.7813</td>
<td>0.7026</td>
<td>1.2856</td>
</tr>
</tbody>
</table>
Chapter 2: Background

These damping prediction formulae were followed by a robust study by the Architectural Institute of Japan (Satake, Suda, Arawaka, Sasaki, & Tamura, 2003). This study was comprised of the analysis of large array of buildings across typology, height, and testing methods. Phenomenological models were fit to the data with regards to building type, height, plan dimension, and period. While significant scatter was observed in these plots, it was concluded that the damping ratio decreased with building height (Figure 2.25) and was affected by building usage, owing primarily to the density of non-structural components such as partition walls (Figure 2.26).

![Figure 2.25 First mode damping ratio versus height for steel, RC, and mixed buildings (Satake, Suda, Arawaka, Sasaki, & Tamura, 2003)](image)

![Figure 2.26 First mode damping ratio versus height subdivided into building usage categories for steel, RC, and mixed buildings and showing difference between apartments and offices (Satake, Suda, Arawaka, Sasaki, & Tamura, 2003).](image)

A key note regarding the data used in Figure 2.25 and Figure 2.26 is that most of the data is from the low-amplitude regime. These experimental results are indicative that the low-amplitude damping in high-rise buildings is less than the low-amplitude damping in low-rise buildings. This means that while a general decrease in damping is seen with height, confined to the low-amplitude regime, amplitude-dependency is not considered and as such it may not be reasonable to assume that the design level damping, at higher amplitudes, of tall buildings is low. While it was indicated in this study that the damping is correlated with height, a more robust parameter for characterizing...
the low-amplitude damping is the primary deformation mechanisms. As buildings grow taller, the primary mode shapes shift from being primarily a shear-type deformation to primarily a cantilever-type deformation. Since there is less relative motion in cantilever deformations, it follows that less stick-slip friction type mechanisms will be engaged. This was noted and further explored in subsequent research.

Additionally, this study showed that first mode damping ratios in the linear elastic range of the structures increased linearly with vibration amplitude, which was attributed to friction in the joints of the structure and did not share traditional mass and stiffness proportional characteristics of Rayleigh damping. Equation 22 and Equation 23 were proposed first-mode damping ratio prediction models for steel and reinforced concrete buildings, respectively.

\[
\zeta_1 = 0.013f_1 + 400 \left( \frac{x_H}{H} \right) + 0.0029 
\]

Equation 22

Note:

\[
\frac{x_H}{H} \leq 2 \times 10^{-5}
\]

\[30 \text{m} < H < 200 \text{m}\]

\[
\zeta_1 = 0.014f_1 + 470 \left( \frac{x_H}{H} \right) - 0.0018 
\]

Equation 23

Note:

\[
\frac{x_H}{H} \leq 2 \times 10^{-5}
\]

\[10 \text{m} < H < 130 \text{m}\]

Where:

\(\zeta_1\): First mode damping ratio

\(f_1\): First mode natural frequency

\(x_H\): Vibration amplitude at the top of the building

\(H\): Building height

Tamura (2006) elaborated on the amplitude-dependency of tall buildings noting that although many researchers had previously identified an increase in the damping ratio with amplitude, this was restricted to lower end of the amplitude regime, and that there had not yet been
any evidence that damping continues to increase in the high amplitude regime. There was evidence that suggested a plateau in the damping ratio after the initial increase (Jeary, 1986) and results from a study on a 200 metre tall steel building (Figure 2.27) showed that after a certain amplitude was reached, the damping ratio began to decrease with increasing amplitude.

![Amplitude dependency of 200 metre tall steel building (Tamura, 2006).](image)

This amplitude was defined as the critical tip drift ratio, where the damping no longer increases beyond this point, and may in fact decrease. This was primarily attributed to the stick-slip friction mechanisms that have been the primary source to which researchers have attributed the amplitude-dependent nature of damping. Previous researchers (Lagormarsino, 1993), noted that if the primary mechanism of damping was in fact stick-slip mechanisms, then the damping ratio would eventually begin to decrease with amplitude. Tamura evolved this idea by noting that once the friction mechanism is engaged, the damping is characterised by a Coulomb damping relationship since the force becomes constant. Under Coulomb damping, the damping ratio is given by Equation 24.

\[
\zeta = \frac{C}{2m\omega_o} = \frac{2F_L}{\pi x_0 M \bar{\omega} \omega}
\]

Equation 24

Where:

- \(C\): Coulomb damping coefficient
- \(F_L\): Slip force
- \(m\): Mass
- \(\bar{\omega}\): Forcing angular frequency
- \(\omega\): Natural angular frequency
- \(x_0\): Amplitude of excitation
Chapter 2: Background

This shows that under a constant force, the damping ratio will decrease with amplitude. Tamura noted that the critical tip drift ratio may typically be on the order of \(10^{-5}\) to \(10^{-4}\). It was also noted that the equations proposed by several prior researchers, which relied on a frequency-dependent damping term, were baseless, given that there had been no evidence to suggest a frequency effect on damping. It was noted that the primary frequency effect may be attributed to soil-structure interaction. Tamura proposed a small change to the equations proposed by Satake et. al (2003) for steel and RC buildings, respectively (Equation 25 and Equation 26), which replaced the frequency-dependent term with a height-dependent term.

\[
\zeta = \frac{0.65}{H} + 400 \left( \frac{x_H}{H} \right) + 0.0029 \quad \text{Equation 25}
\]

\[
\zeta = \frac{0.93}{H} + 470 \left( \frac{x_H}{H} \right) - 0.0018 \quad \text{Equation 26}
\]

While these may be more reflective of actual structures, no clear attempt to represent the observed increase and subsequent decrease in the damping ratio with respect to amplitude was included.

2.2.2.3 Explaining Phenomenological Models: 2008 - Present

The Chicago Full-Scale Monitoring Program (CFSMP) is a research initiative from the University of Notre Dame that involves monitoring several tall buildings around the world that was initiated in the early 2000s and continues to this day. This group proposed a phenomenological model which suggests that the damping ratio may be closely tied with building typology, specifically the primary deformation mechanism; cantilever or shear action [ (Bentz & Kijewski-Correa, 2008), (Bentz, 2012)]. This is highlighted in the observations of the difference between the amplitude-dependent responses in two modes of a single building, each characterized by a different mode shape (Figure 2.28).
A key note is that the amplitudes investigated were small, characteristic of low amplitude ambient vibrations.

It was proposed that this ratio may be determined by the degree of cantilever action (Bartolini & Kijewski-Correa, 2017) as quantified by the relative contribution of an ideal shear and cantilever mode shape (Equation 27). The concept is illustrated in Figure 2.29.

\[
d_c = 1 - \frac{1}{N} \sum_{i=1}^{N} \frac{\Delta C_i(h_i)}{T_i(h_i)}
\]

Equation 27

Where:
\(d_c\): Degree of cantilever action
\(h_i\): Height
\(\Delta C_i(h_i) = MS(h_i) - C(h_i)\)
\(T_i(h_i) = S(h_i) - C(h_i)\)
\(MS\): Mode shape
\(C(h)\): Ideal cantilever mode shape
\(S(h)\): Ideal shear mode shape
Chapter 2: Background

The degree of cantilever action was used as a regression parameter, fitting to experimental data, resulting in Equation 28 and Equation 29. The equations and plots are shown in Figure 2.30 for damping estimates in the time-domain and those in the frequency-domain. Note that this refers to the domain in which the system identification was performed, a concept discussed further in Section 3.0.

**Frequency Domain**

\[
\zeta = -0.5 \ln(d_c) + 0.75 \quad \text{Equation 28}
\]

**Time Domain**

\[
\zeta = -0.54 \ln(d_c) + 0.26 \quad \text{Equation 29}
\]

While it was shown that the degree of cantilever action was a suitable regression parameter to generate a damping prediction curve, in lieu of simply the building height or period, it did not address what the local physical mechanism behind damping is and did not quantify the observed amplitude-dependency of damping. It follows, based on all previous research, that more relative deformation between structural members and between structural and non-structural members will

---

*Figure 2.29 Illustration of the degree of cantilever action (Bartolini & Kijewski-Correa, 2017)*

*Figure 2.30 Proposed damping models with degree of cantilever action as regression parameter (Bartolini & Kijewski-Correa, 2017).*
result in more stick-slip mechanisms being engaged. This is elaborated on in significant detail in more recent studies, discussed herein.

Recently, Aquino and Tamura (2013) proposed a series of improvements to Wyatt’s (1977) stick-slip model. It was proposed that the total damping ratio may be comprised of three components; (1) Structural, (2) Aerodynamic, (3) Supplemental. It was noted that aerodynamic damping may be neglected for most structures, and that deciding if a supplemental damping system is required is a function of the inherent structural damping. Therefore, special attention must be paid to the formulation of the inherent structural damping.

Noting the drawbacks of Jeary’s (1983, 1986, 1996), Lagomarsino’s (1993), and Satake et al.’s (2003) models in the sense that they do not account for the observed decrease in damping when the amplitude increases beyond the critical tip drift ratio, a more robust model was proposed. It was proposed that the system be comprised of a linear viscous damping component and an amplitude-dependent stick-slip component (Equation 30 and Equation 31).

\[
\zeta(x) = \zeta_{vis} \quad x \leq x_0 \quad \text{Equation 30}
\]

\[
\zeta_s(x) = \zeta_{vis} + \zeta_c(x) \quad x > x_0 \quad \text{Equation 31}
\]

Where:

\(\zeta_s(x)\): Total structural damping

\(\zeta_{vis}\): Viscous damping component

\(\zeta_c(x)\): Stick-slip amplitude-dependent damping component

\(x_0\): Slip amplitude

A detailed formulation of the force-displacement relationship of the stick-slip mechanism was proposed, based on the force-deformation plots shown in Figure 2.31.

![Figure 2.31](image-url)
Chapter 2: Background

Where:

$k_f$: Initial stiffness of stick-slip element

$k$: Stiffness of the structure

$k_{tot}$: Combined stiffness of structure and single stick-slip component

$x_0$: Amplitude at which stick-slip element slips

This results in the reformulated equation of motion, which now presents a non-linear problem (Equation 32 and Equation 33)

$$m\ddot{x}(t) + c\dot{x}(t) + (k + k_f)x(t) = F(t) \quad x \leq x_0 \quad \text{Equation 32}$$

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t) \quad x > x_0 \quad \text{Equation 33}$$

If there are many stick-slip elements, as there would in an actual structure, the equation of motion may be expressed according to Equation 34.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + \sum_{i=1}^{N} k_{fi}x(t) = F(t) \quad \text{Equation 34}$$

The result of this formulation is that the stiffness of the main structure is still accounted for as the stick-slip elements lose their stiffness as their static friction is overcome. The theoretical expression for the damping ratio was then based on the combination of a constant viscous damping component and an amplitude-dependent stick-slip component. The hysteresis and corresponding elastic stored energy is shown in Figure 2.32 for this model.

![Hysteresis and corresponding elastic stored energy](image)

Figure 2.32 Equivalent viscous damping ratio for the structure with a single stick-slip element. Adapted from (Aquino & Tamura, 2013).
By applying the equivalent viscous damping ratio concept, the amplitude-dependent component of the damping ratio may be set according to Equation 35 if all the damping mechanisms are known, thus applying a deterministic approach.

\[
\zeta_c(x) = \sum_{i=1}^{N_{SSC}} 2 \frac{1}{\pi} \left( \frac{1}{1 + \left( \frac{k}{k_{f_i}} \right) \left( \frac{x}{x_{0_i}} \right)} \right) \left( 1 - \frac{x_{0_i}}{x} \right) \quad x > x_{0_i} \tag{Equation 35}
\]

Where:

\( N_{SSC} \): Number of stick slip elements

However, since accurately quantifying all the friction mechanisms in a structure is difficult, a probabilistic approach may be used (Equation 36).

\[
\zeta_c(x) = \sum_{i=1}^{N_{SSC}} 2 \frac{1}{\pi} \left( \frac{1}{1 + \left( \frac{k}{k_{f_i}} \right) \left( \frac{x}{x_{0_i} \rho_{xi}} \right) \rho_{ki}} \right) \left( 1 - \frac{x_{0_i} \rho_{xi}}{x} \right) \quad x > x_{0_i} \rho_{xi} \tag{Equation 36}
\]

Where:

\( \bar{k}_f \): Mean of all \( k_{f_i} \) values

\( x_{0_i} \): Smallest of all \( x_{0_i} \) values

\( \rho_{ki} \): Set of random numbers with mean equal to unity following a probability distribution and covariance such that all \( k_{f_i} \rho_{ki} \) values translate to all \( k_{f_i} \) values

\( \rho_{xi} \): Set of random numbers with mean equal to \( 1/x_{0_i} \) following a probability distribution and covariance such that \( \rho_{xi} x_{0_i} \) translates to \( x_{0_i} \)

It was suggested that a log-normal distribution be used, although several different distribution types were used and were fit to experimental data (Figure 2.33). These fits show that the model is capable of accurately representing the amplitude-dependent damping phenomenon.
Chapter 2: Background

Figure 2.33  Proposed damping model for various probability distributions fit to various experimental datasets (Aquino & Tamura, 2013).

A comparable model was developed by Spence and Kareem (2014) that took a similar approach to quantifying the amplitude-dependent nature of the dynamic characteristics. They developed a more robust model that accounts for the observed change in both the natural frequency and damping ratio in tandem. The equation of motion was reformulated as shown in Equation 37.

\[
M_i \ddot{x}_i(t) + C_i \dot{x}_i(t) + [k_i - \bar{k}(\bar{x}_i)]x_i(t) + \tilde{f}_i \int_0^\infty p_{f_0 x_0}(\eta_1, \eta_2) d\eta_1 d\eta_2 = F_i(t)
\]

Equation 37

Where:

- \(i\): Mode \(i\)
- \(\bar{k}\): Combined stiffness loss resulting from slipping of stick-slip components at \(\bar{x}\)
- \(\bar{x}\): Envelope amplitude of motion
- \(\tilde{f}_i\): Total frictional damping force in mode \(i\)
- \(p_{f_0 x_0}\): Joint PDF between friction force \((f_0)\) and the corresponding slip amplitude \((x_0)\)

The choice to model the frictional damping forces in a probabilistic model stemmed from two primary reasons:
Chapter 2: Background

(1) This allows full saturation of friction mechanisms available as the amplitude goes to infinity, thus enabling the observed growth and subsequent decay of damping with respect to amplitude.

(2) Physical mechanisms that the friction mechanism represents, such as friction in bolted joints, between the structural and non-structural components, etc., are unengineered and difficult to deterministically quantify.

Like the Aquino and Tamura research (2013), the dissipated energy was assumed to be a combination of a baseline viscous damping and an additional friction mechanism damping through stick-slip components as per Equation 30 and Equation 31. The distribution of both the slip displacements and forces were modelled using a log-normal joint probability density function, and by applying the equivalent viscous damping concept the amplitude-dependent damping ratio was expressed as Equation 38.

\[
\zeta(\ddot{x}) = \zeta_{vis} + \frac{4N_{SSC}\ddot{f}_0}{2\pi(k - k(\ddot{x}))} \int_0^{\ddot{x}} \int_0^{\eta_2} p_{x_o}(\eta_1) d\eta_1 d\eta_2
\]

Equation 38

Where:

- \(N_{SSC}\): Total number of stick slip mechanisms in structure
- \(\ddot{f}_0\): Mean value of \(f_0\)
- \(p_{x_o}\): Probability density function of \(x_o\)

It was assumed that the stiffness loss due to stick-slip elements would be small compared to that of the overall structural stiffness (\(K\)), therefore, all constants were collected and Equation 38 was recast as Equation 39.

\[
\zeta(\ddot{x}) = \zeta_{vis} + A \frac{\int_0^{\ddot{x}} \int_0^{\eta_2} p_{x_o}(\eta_1) d\eta_1 d\eta_2}{\ddot{x}^2}
\]

Equation 39

Where:

\[
A = \frac{2N\ddot{f}_0}{\pi K}
\]

Equation 40

The equation was calibrated to high-quality experimental data from three tall buildings that exhibited strong amplitude-dependent damping and frequency and were characterized by different heights, structural systems, and primary construction material. Building 1 was a 200-metre-tall steel frame structure, Building 2 was a 263-metre-tall reinforced concrete core wall with outriggers structure, and Building 3 was a 443-metre-tall steel frame structures. The equation was fit to the
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data by assuming a baseline viscous damping ($\zeta_{\text{vis}}$) and performing a non-linear least squares minimization of the parameters influencing the probability density function ($P_{X0}$). The results of this calibration to the amplitude-dependent damping data from the three tall buildings are shown in Figure 2.34a. This highlights the model’s high fidelity at capturing the varying amplitude-dependent trends observed from three unique buildings, exhibiting similar success to the model developed by Aquino and Tamura (2013). Additionally, the model was fit to a much larger database to be used as a predictor tool for the inherent damping ratio of a structure (Figure 2.34b). This figure highlights the damping ratios as computed from buildings with known primary deformation mechanisms (shear or cantilever dominated), showing that at a particular amplitude the buildings dominated by cantilever deformation exhibit lower damping ratios, supporting the notion claimed by earlier researchers.

![Figure 2.34](a) Calibration of Equation 39 to experimental data from three buildings; (b) Amplitude dependency of buildings by primary deformation type and model fit of Equation 39 (Spence & Kareem, 2014).

2.2.3 Amplitude-Dependent Damping Model Comparison

A comparison of the proposed amplitude-dependent damping models is shown in Figure 2.35. Several settings were selected for the models, which are summarized in Table 2.7.
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Damping Model Comparison

![Graph showing damping model comparison]

Figure 2.35 Comparison of amplitude-dependent damping predictor models. Experimental data adapted from Spence and Kareem (2014) database.

Table 2.7 Assumed building properties for amplitude-dependent damping predictor models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Height (m)</th>
<th>Plan Dimension (m)</th>
<th>Material</th>
<th>Period (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>198</td>
<td>18.7</td>
<td>RC</td>
<td>5</td>
</tr>
</tbody>
</table>

All of these models were based on the assumption of the undefined stick-slip mechanisms being the primary source of amplitude-dependent damping and are a combination of a baseline viscous damping component with an amplitude-dependent stick-slip component (with the exception of the 1986 Davenport and Hill-Carroll model). Each model was calibrated to a subset of experimental data from full-scale testing of a wide array of buildings, which grew as more data was available from additional full-scale building tests as time progressed. The database used by Spence and Kareem (2014) was the most robust, considering data from:

- 76 buildings between 100 metres and 282 metres from Satake (2003) database.
- 19 buildings from Chicago Full Scale Monitoring Program from 100 metres to 443 metres in height.
- 3 tall buildings from CSMIS database, a subset of tall buildings monitored on the west coast under high amplitude vibrations during earthquakes.

Note that the experimental data points in Figure 2.35 are comprised of the Spence and Kareem (2014) database.
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What has been historically evident is that as more data is obtained from a wider array of buildings, the predictive models for damping become better at capturing the amplitude-dependent phenomenon. This concept calls for the continued monitoring of buildings to further expand the database.

2.3 Amplitude-Dependent Frequency

Varying in tandem with the previously outlined amplitude-dependency of damping, the natural frequency of structures has been shown to decrease with amplitude (Figure 2.36)

Figure 2.36 Amplitude-dependent frequency; (a) From CFSMP (Kijewski-Correa, Kareem, Guo, Bashor, & Weigand, 2013); (b) From City University of Hong Kong (Li & Yi, 2016).

This follows the previously developed stick-slip framework, which has been shown to explain the amplitude-dependency of damping. As the amplitude grows and more elements slip, the stiffness in the undefined, but generically used, stick-slip element is lost resulting in a drop in the overall stiffness and subsequently a drop in the frequency.

Historically, less emphasis has been placed on developing phenomenological models to predict the frequency since physical modelling has consistently produced relatively accurate estimations of frequency. Unlike damping, the mass and stiffness matrices of Equation 10 may be directly formulated, thus placing less emphasis on experimental observations. However, some recent effort has been placed on building expressions for amplitude-dependent frequency. Spence and Kareem (2014), developed, in a unified theory, expressions for amplitude-dependent damping (Equation 39) and frequency (Equation 41). Results from the calibration of Equation 41 are shown in Figure 2.37.
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\[ f(\ddot{x}) = \frac{1}{2\pi} \sqrt{\frac{B_1 - B_2 \int_0^\infty \frac{p_{x_0}(\eta_2)}{\eta_2} d\eta_2}{m}} \]

Equation 41

Where:

- \( p_{x_0} \): Calibrated PDF from Equation 39.
- \( B_1 \): Zero amplitude stiffness
- \( B_2 \): Constant

The model effectively captures the observed amplitude-dependency, thus further validating the concept of stick-slip mechanisms as a primary driver for amplitude-dependent damping and frequency.

2.4 Summary of Key Points in Amplitude-Dependent Model Development

It is evident from this review that considerable effort has been placed on developing better models to understand and predict the inherent damping of structures. Early in the development of dynamic analysis of buildings, the critical assumption of equivalent viscous damping was suggested, which has long been held as a convenient and effective way of representing the variety of complex mechanisms which contribute the inherent damping of a structure. This concept is still used today, however, several refinements have been made based on observations of full-scale damping data.

As experimental testing of tall buildings began to mount, attention began to be paid to the observed amplitude-dependent properties of damping and frequency. The models available at that time were insufficient to capture this behaviour; however, it was explained by the presence of many stick-slip elements which can capture the observed rise and fall of damping with increasing amplitude of motion. It has also been observed that the overall deformation mechanisms of the structure are related to the observed inherent damping. This follows with the stick-slip element
concept as deformation mechanisms that are dominated by shear deformations will likely engage more stick-slip elements, resulting in a higher damping ratio when compared to cantilever type deformations which result in less relative deformation in the structure.

Recent models have sought to reformulate the equations of motion and develop explicit equations describing the variation of damping with amplitude as a function of many stick-slip elements. These models effectively capture observed amplitude-dependent trends; however, they have still generically cast stick-slip elements as being a combination of friction in structural members (bolted joints), between structural and non-structural members, and in microscopic material imperfections, among other possible sources.

Overall, the research on the development of damping models has produced several key observations:

- The low-amplitude inherent damping of a structure is a function of friction between large structural elements (Jeary, 1986) and the inherent material damping (Smith, Merello, & Willford, 2010).

- Inherent damping is a function of the amplitude of vibration and the amplitude-dependent component may be modelled by applying the equivalent viscous damping concept to the energy dissipated due to many randomly distributed stick-slip friction mechanisms. Generally, the damping ratio tends to increase with amplitude until a critical tip deflection is reached, after which the damping ratio plateaus or decreases with amplitude. Varying in tandem with this, the natural frequency varies with amplitude as a result of the stiffness loss associated with stick-slip mechanisms.

- The damping ratio tends to decrease with building height, which may be attributed to an increase in the cantilever action of the mode shapes, which is quantified by calculating the degree of cantilever action which describes the similitude of the mode shape to an ideal cantilever. Buildings with primarily shear deformations have been shown to exhibit higher inherent damping ratio than buildings dominated by cantilever action. While this trend has been consistently observed by several researchers, these results are primarily confined to the low-amplitude regime, and thus are not indicative of possible increases in damping ratio at higher amplitudes.

- Buildings constructed of reinforced concrete have been shown to exhibit higher damping ratios than steel or mixed material buildings, although this may not be true for all tall
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buildings, as Spence and Kareem (2014) showed two steel buildings exhibiting higher damping ratios than the reinforced concrete building. This logic has primarily been attributed to friction on microcracks in the concrete, which may represent a significant portion of the energy dissipated.

The objectives of the research to date has been to better quantify the inherent damping of structures through observations of full-scale building behaviour. While many buildings have been monitored by several researchers over the last several decades, there is ongoing need to further increase this database. Additionally, while an understanding of the inherent damping ratio is key, novel and unique devices such as the VCD must be monitored in full-scale to assess their effect on the structural performance. This leads to the next phase of the research which was concerned with the testing methods and system identification algorithms for full-scale building monitoring.
Chapter 3: Key Aspects of System Identification for Tall Buildings

3.0 Key Aspects of System Identification for Tall Buildings

This chapter introduces the field of vibration testing and system identification for tall buildings. Discussion on various testing methods is provided, as well as a summary of several typically used system identification algorithms. A numerical study is included to study the effects of each techniques’ parameters on the identified dynamic properties. A discussion is provided regarding the application range for each technique.

3.1 Vibration Testing Methods for Buildings

Given the growth of tall building construction around the world there has been a concerted effort over the last several decades to develop methods to validate the full-scale behaviour of these buildings. With the scale of construction, it is not possible to build and robustly test trial buildings considering the price tag in the hundreds of millions of dollars for typical high-rise projects. Comparing to other fields, such as the automotive industry, full-scale testing plays a key role in assessing the performance of the product. Unfortunately, structural engineers are not typically able to robustly test tall buildings to failure simply to better understand the behaviour, which has led to the development of vibration testing of buildings to assess the dynamic properties.

The vibration testing of buildings can be broadly split into 3 categories; Forced Vibration Testing (FVT), Free Vibration Testing, and Ambient Vibration Testing (AVT). This classification and certain methods for each type are shown in Figure 3.1.

![Figure 3.1 Vibration testing methods for buildings classification.](image-url)
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For each of these testing methods, response quantities such as acceleration, displacement, strain, among others are recorded and are used to identify the dynamic properties of the building.

FVT involves mechanically exciting the structure using mass shakers or synchronized human excitation where the excitation frequency is varied to determine the frequency response function for the structure. Alternatively, free vibration testing is performed by applying an initial displacement or velocity to a system and measuring the ensuing free vibration response. A recent example of the application of this technique where AMDs were used to excite the structure at its natural frequency, and then were turned off to allow the structure to freely vibrate is shown in Figure 3.2.

![Figure 3.2 Measured acceleration response in two orthogonal directions during a FVT (Shi, Shan, & Lu, 2012).](image)

While this is the most accurate means of testing buildings, there are several logistical challenges with implementing FVT. Organizing and implementing this type of testing requires the collaboration and approval of building owners, which historically has proven to be difficult. Additionally, if the building is occupied there is the concern of occupants to consider in the excitation of the building. These items typically preclude the application of FVT to most tall buildings.

Conversely, AVT, also sometimes referred to as Operational Modal Analysis (OMA), involves monitoring the building during operating conditions. OMA is the process of characterizing the dynamic properties of an elastic structure by identifying its natural modes of vibration from the operating response (Brincker & Ventura, 2015).

This method eliminates the concerns associated with FVT, however, the testing and analysis procedures become more complicated due to the low levels of vibrations. Sensitive
instrumentation is required, and sophisticated output-only system identification techniques must be used. However, the challenges of FVT are eliminated. Generally, the steps involved in performing AVT are as follows:

1. Instrumenting the structure with several accelerometers (low noise, resolution on the order of micro-g’s) that capture the ambient acceleration response of the structure.
2. Depending on the sensors used and the measured response, various signal processing tools may be used to process the data.
3. Following the general signal processing, system identification techniques are applied to determine the dynamic characteristic (natural frequencies, damping, mode shapes).

### 3.2 Output-Only System Identification

Systems analysis involves the description of the behaviour of a system by defining an input, system, and output. Usually, two of these three parameters are known, which gives rise to the three general problems of systems:

1. Input and system known, output determined (classic structural dynamics problem).
2. System and output known, input determined (classic controls problem).
3. Input and output are known, system determined (classic laboratory experiment). This is system identification (SID).

For the monitoring of structures, the output is known through the measured data, usually in the form of accelerations, while the input is not directly measured. The interest is in determining what the system properties, in this case the natural frequencies ($f$), damping ratios ($\zeta$), and mode shapes ($\Phi$), are. As previously discussed, for the system identification of large structures, output-only techniques must be implemented. These parameters are illustrated in Figure 3.3 for the AVT problem.

![Generalized system identification parameters.](Figure 3.3)
Several important assumptions are typically made when applying output-only SID. First the input statistics must be assumed, which are usually split into two categories; stationary and non-stationary. For stationary analyses, it is assumed that the input is a stationary, broadband, random process. This means that the statistics of the input signal do not change in time. For non-stationary analyses, this assumption is violated. The second key assumption is that the system is linear and time-invariant (LTI). This means that structure’s properties (mass, stiffness, and damping) are linear, and those properties do not change in time.

3.2.1 **Stationary Algorithms**

To apply most output-only system identification methods, assumptions about the input must be made. Traditionally, it is assumed that the input is a stationary, broadband, random process. Since the wind is exciting the structure, the excitation will be stationary if the mean amplitude and direction of the wind are invariant with time. Practically, this is difficult to establish accurately. Since it was assumed that the structure is a LTI system, if the response is a stationary process then this is sufficient to establish the stationarity for the signal. Note that the amplitude-dependent properties complicate this since a high-amplitude stationary input will result in a non-linear system.

Stationarity may be verified by several tests where each of these checks involves dividing the time-history into several blocks and applying the stationarity checks to each block. Three common tests applied to tall building wind response data are:

1. **Runs Test**: RMS of each block compared to RMS of overall record. If the block RMS is above the RMS of the overall record, the check takes on a value of 1, otherwise the check takes on a value of 0. Runs are defined as one or more consecutive occurrences, and so the number of runs for the signal is computed and compared to the number of runs expected for truly random data.

2. **Reverse Arrangement Test** (Bendat & Piersol, 2010): A test to detect any underlying trend in data by detecting the number of times that a sub-segment is above or below the previous. This is tested against the number of times this will occur random data at select significance.

3. **Montpellier Check** (Montpellier, 1996): Each block is divided into 3 sub-segments and the RMS for each sub-segment is computed. If the RMS of each sub-segment is within 15% of the RMS of the block, the record may be deemed stationary.
Different researchers have applied different combinations of these tests and passing criteria to establish the stationarity of a signal (Table 3.1). It was noted that very few records pass the Runs test (Kijewski-Correa, et al., 2006), thus explaining why few groups rely on this check.

Table 3.1 Summary of stationarity checks performed by various researchers.

<table>
<thead>
<tr>
<th>Group</th>
<th>Checks Used</th>
<th>Passing Criteria</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFSMP (General)</td>
<td>Runs</td>
<td>Passing at least two of these tests</td>
<td>(Kijewski-Correa, et al., 2006)</td>
</tr>
<tr>
<td></td>
<td>Reverse Arrangement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Montpellier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFSMP (Erwin Thesis)</td>
<td>Reverse Arrangement</td>
<td>85% of blocks must pass</td>
<td>(Erwin, 2009)</td>
</tr>
<tr>
<td></td>
<td>Montpellier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CFSMP (Guo Work)</td>
<td>Runs</td>
<td>80% of blocks pass at least one test</td>
<td>(Guo, Kareem, Ni, &amp; Liao, 2012)</td>
</tr>
<tr>
<td></td>
<td>Reverse Arrangement</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Montpellier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>City University of</td>
<td>Reverse Arrangement</td>
<td>Failing blocks removed</td>
<td>(Li &amp; Li, 2018)</td>
</tr>
<tr>
<td>Hong Kong</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Generally, the verification of stationarity is somewhat subjective, as each of the references applies different techniques and criteria to their data. Ultimately, it is important that a baseline consideration be given to the stationarity of the signal when applying output-only SID methods and significantly non-stationary data should not be analysed using these methods. A further caveat to the verification of stationarity is the application range of the method being applied. This is further discussed in subsequent sections.

The following sections discuss a variety of techniques used by researchers to determine the dynamic properties of structures from output-only data. These include:

- Spectral Analysis (SA)
- Random Decrement Technique (RDT)*
- Natural Excitation Technique (NExT)
- Eigensystem Realization Algorithm (ERA)
- Enhanced Frequency Domain Decomposition (EFDD)
- Data-Driven Stochastic Subspace Identification (SSI-DATA)

* Capable of dealing with minor non-linearities (mild non-stationarity).
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3.2.1.1 Spectral Analysis (SA)

One of the more traditional approaches to output-only system identification is performing a spectral analysis. This is performed by computing the power spectral density (PSD) function for a signal (Equation 42).

\[
\hat{S}_{xx}(f) = \frac{1}{N_s T_{\text{win}}} \sum_{j=1}^{N_s} |\hat{X}_j(f)|^2
\]

Equation 42

Where:
\(\hat{S}_{xx}(f)\): Power Spectral Density (PSD)
\(N_s\): Number of Fourier Spectra being averaged
\(T_{\text{win}}\): Length of window used for computing Fourier Spectra
\(\hat{X}_j(f)\): Fourier Spectra of segment \(j\) as a function of discrete Fourier Frequencies (f).

A key note in applying this transform is the inherent bias and variance errors, which affect the damping estimation in opposing ways. Bentz (2012) reviewed this in detail and suggested minimizing bias error as it always leads to an increase in damping by way of controlling the frequency resolution (Equation 43).

\[
\Delta f_r = \frac{2\zeta f_n}{4}
\]

Equation 43

Where:
\(\Delta f_r\): Frequency resolution (Hz)
\(\zeta\): Damping ratio
\(f_n\): Natural frequency (Hz)

By setting the frequency resolution, the number of points in the FFT (NFFT) may be set according to Equation 44.

\[
\text{NFFT} = \frac{1}{\Delta f_r} f_s
\]

Equation 44

Where:
\(f_s\): Sampling Frequency (Hz)

The NFFT is rounded up to the nearest power of two for application in the FFT algorithm, which gives the required length of the segments (Equation 45).
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\[ T = \frac{NFFT}{f_s} \] \hspace{1cm} \text{Equation 45}

Once the PSD is obtained for a signal, the frequency may be determined by finding the peak in the spectrum, referred to as the peak-picking technique, and the damping ratio may be evaluated using the well-known Half-Power Bandwidth approach (Figure 3.4 and Equation 46).

\[ \zeta_n = \frac{f_b - f_a}{2 f_n} \] \hspace{1cm} \text{Equation 46}

Where:
\[ \zeta_n \]: Damping ratio of mode \( n \)
\[ f_b, f_a \]: Frequencies at \( 1/\sqrt{2} \) peak amplitude
\[ f_n \]: Natural frequency of mode \( n \)

If the response of multiple degrees of freedom are measured, the mode shapes may be determined by the spectral ratio method where the ratios of the peaks of the PSD from two measurements at positions \( m \) and \( n \) are equivalent to the mode shapes (Brownjohn & Ang, 1998). The relationship is summarized in Equation 47.

\[ \frac{\hat{X}_m(f)}{\hat{X}_n(f)} \approx \frac{\Phi_{rm}}{\Phi_{rn}} \] \hspace{1cm} \text{Equation 47}

Where:
\[ \Phi_{rk} \]: Mode shape for mode \( r \) at DOF \( k \).
The sign of the mode shape may be determined by investigating the angle of the cross power spectral density between the two responses. If the angle is 0, the DOFs move in-phase, and if the angle is 180, the DOFs move out-of-phase (Areemit & Christopoulos, 2009).

Despite the management of the inherent errors of the SA approach for output-only system identification, the presence of mild non-linearities by way of amplitude-dependent damping and frequency introduce another challenge in applying this technique. As shown by several researchers, the frequency tends to reduce with amplitude. If a PSD is computed from a signal in which this frequency softening is present, the bandwidth of the spectral peak will grow, leading to larger damping estimates (Bentz, 2012). Given these challenges, the SA approach is most applicable to highly stationary, low amplitude data only, when applied to tall buildings. Additionally, this technique has been thoroughly researched for application to tall buildings and is generally found to be useful as a first check on the frequencies and damping, but more refined techniques are usually applied for characteristic extraction. Beyond this brief background, this technique was not explored further in this thesis.

### 3.2.1.2 Damping Evaluation Techniques

Several of the subsequently discussed SID techniques (RDT, NExT, EFDD) may be viewed as data-conditioning techniques which produce proportional free vibration responses for a system that is excited by white noise input. Therefore, these are not exactly SID techniques themselves, and must be combined with a Damping Evaluation Technique (DET) to extract the dynamic properties from the signal.

If the system is a single degree of freedom (SDOF) the period, and therefore frequency, may be extracted from the time between peaks. The damping ratio may be extracted using the logarithmic decrement (Equation 48) or by a least squares (LS) fitting of a decaying sin function (Equation 49)

\[
\delta = \frac{1}{p} \ln \left( \frac{x_n}{x_{n+p}} \right) = 2\pi\zeta
\]

Where:
\(\delta\): Logarithmic decrement
\(x_n\): Amplitude at \(n\)
\(x_{n+p}\): Amplitude at \(n+p\)
Chapter 3: Key Aspects of System Identification for Tall Buildings

$p$: Number of full cycles between peaks

$\zeta$: Damping ratio (if small damping is assumed)

$$y(t) = Ae^{Bt}(sinCt + D)$$  \hspace{1cm} \text{Equation 49}$$

Where:

$A$: Amplitude

$B = -\zeta \omega$

$C = \omega$

$\zeta$: Damping ratio (if small damping is assumed)

$\omega$: Natural frequency (rad/s) (if small damping is assumed; $\omega \approx \omega_d$)

$t$: Time (s)

$D = \Phi$: Phase shift

The least-squares fit was selected as the DET for this research.

3.2.1.3 Eigensystem Realization Algorithm (ERA)

If the system is an MDOF, the Eigensystem Realization Algorithm (ERA) may be used to extract the damping ratio, frequency, and mode shapes from the free-vibration response. The ERA is a system identification technique developed for modal parameter identification (Juang & Pappa, 1985). The second order MDOF differential equation of motion is rewritten into a first order matrix differential equation, known as the state-space model (Equation 50 and Equation 51) (Juang & Phan, 2001).

$$x(k + 1) = Ax(k) + Bu(k)$$  \hspace{1cm} \text{Equation 50}$$

Where:

$x(k)$: State vector

$A$: Dynamic state matrix

$B$: Input matrix

$u(k)$: Input vector

$$y(k) = Cx(k) + Du(k)$$  \hspace{1cm} \text{Equation 51}$$

Where:

$y(k)$: Output vector

$C$: Influence (output) matrix
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D: Direct-transmission term

The ERA uses the principle of minimum realization to obtain the smallest order state-space model that gives an equivalent response to the real system. There are an infinite number of matrices A, B and C of different dimensions (order) that can describe the system. However, the smallest order will minimize the number of states. The fundamental principle of the ERA is to find the state matrix A from the free vibration response of the dynamic system, thus the ERA requires the input to be in the form of a free vibration. The two most common means of achieving this are the RDT and NExT (outlined subsequently), if an authentic free vibration is not available. The natural frequencies, damping ratios, and mode shapes of the system can be identified by performing an eigenvalue analysis on A. The theory is well developed by Juang and Pappa (1985), thus only a summary of the implementation algorithm, adapted from Caicedo (2011), is summarized herein.

The first step in performing the ERA is to assemble the response measurements (M channels) into the block Hankel Matrix (Equation 52):

\[ H(k) = \begin{bmatrix} y(k+1) & y(k+2) & \cdots & y(k+m) \\ y(k+2) & y(k+3) & \cdots & y(k+m+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(k+n) & y(k+n+1) & \cdots & y(k+m+n) \end{bmatrix} \]

Equation 52

Where:

y(k): Measurement vector (M x 1) from all channels at time k.
m: Number of block columns
n: Number of block rows

Next, the singular value decomposition of Equation 52 is performed (Equation 53).

\[ H = USV^T \]

Equation 53

Where:

U: m x m orthonormal matrix
V: n x n orthonormal matrix
S: m x n diagonal matrix

Under ideal conditions, the matrix S will be in the form of Equation 54.

\[ S = \begin{bmatrix} S_g & 0 \\ 0 & 0 \end{bmatrix} \]

Equation 54

Where:

S_g: g x g diagonal matrix
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$g$: System order

Since noise is present in the system the diagonal terms in $D$ will all be non-zero. By eliminating smaller values, a minimum realization of model order $g$ is obtained. The system matrices can then be calculated according to Equation 55 and Equation 56.

$$A = S_g^{-\frac{1}{2}} U^T H(1) V S^{-\frac{1}{2}}$$  \hspace{1cm} \text{Equation 55}

$$C = E^T U S^{-\frac{1}{2}}$$  \hspace{1cm} \text{Equation 56}

Where:

$H(1)$: Shifted Hankel matrix

$E = [I_{MxM} \ 0]$

Next, an eigenvalue analysis is performed on $A$. Note that the eigenvectors ($V_e$) and eigenvalues ($z_r$) are complex conjugates, thus contain information about the frequency and damping, and must be converted to real numbers. First, the discrete time values must be converted to continuous time (Equation 57).

$$s_r = \frac{\ln(z_r)}{\Delta \tau}$$  \hspace{1cm} \text{Equation 57}

Where:

$z_r$: Discrete complex eigenvalues

$s_r$: Continuous complex eigenvalues

$\Delta \tau$: Data sampling interval

Note that the continuous time complex values take on the form of Equation 58.

$$s_i = a_r + b_r i$$  \hspace{1cm} \text{Equation 58}

Where:

$a_r$: Real component

$b_r$: Imaginary component

Then, the corresponding natural frequencies (Equation 59) and damping ratios (Equation 60) may be calculated.

$$\omega_r = \sqrt{a_r^2 + b_r^2}$$  \hspace{1cm} \text{Equation 59}

$$\zeta_r = -\frac{a_r}{\sqrt{a_r^2 + b_r^2}}$$  \hspace{1cm} \text{Equation 60}

The complex mode shapes were obtained using Equation 61.
\[ \Phi = CV_e \]  

Equation 61

The main parameters involved in implementing the ERA are the number of block rows and columns in the Hankel matrix (Equation 52), which are generally chosen based on the expected number of modes in the system. A classic rule of thumb is to use four times the number of expected modes for the number of columns, although at a minimum at least two times the number of expected modes must be used. Typically, the number of rows is selected based on the number of points in the free vibration responses. The objective is to use as much of the clearly decaying data as possible without including the noisy data typically found at the end of these functions (if built from the RDT or NExT). The relationship between the block columns, rows, sampling frequency, and time is given in Equation 62.

\[
\text{Time} = \frac{\text{Block Rows} + \text{Block Columns} - 1}{F_s}
\]

Equation 62

In order to validate the model order, a stabilization diagram may be developed. The maximum model order is selected, and the Hankel matrix is formed for this order. Then, the model order considered is varied, which affects the amount of the decomposed Hankel matrix that is considered in the ERA and the identified dynamic properties are plotted against the model order. Stable modes will be identified in most of the model orders.

Additional filters may be applied to remove spurious modes. These include damping filters and the Modal Assurance Criteria (MAC). Modes with unrealistic damping ratios, such as negative or very large values, may be eliminated. The MAC provides a measure of the correlation between mode shapes \( a \) and \( b \) identified between different model orders at a certain frequency (Equation 63).

\[
\text{MAC}_{a,b} = \frac{\left\{ \sum_{j=1}^{n} \Phi_{a,j} \Phi_{b,j} \right\}^2}{\left\{ \sum_{j=1}^{n} (\Phi_{a,j})^2 \right\} \left\{ \sum_{j=1}^{n} (\Phi_{b,j})^2 \right\}}
\]

Equation 63

Where:
\( \Phi_{a,j} \): \( j^{th} \) coordinate of mode shape \( a \)

Modes with a MAC of less than 0.95 are usually discarded. Employing these filters can greatly increase the accuracy of the ERA.

The ERA is commonly applied in conjunction with the RDT (RDT-ERA) or the NExT (NExT-ERA) or may be applied directly to free vibration data.
3.2.1.4 Random Decrement Technique (RDT)

The Random Decrement Technique (RDT) is an output-only time-domain system identification technique that is widely used for operational testing of tall buildings. Developed in 1973 (Cole Jr, 1973) for damage identification of aerospace structures, the technique has seen widespread use in the structural engineering community over the past several decades for identifying the dynamic properties of high-rise buildings.

Fundamentally, the RDT is a data conditioning technique which takes the output response of a structure and produces a Random Decrement Signature (RDS) which has been shown to be proportional to the auto-correlation function (Vandiver, Dunwoody, Campbell, & Cook, 1982) for the specific case of a linear, time-invariant system excited by a zero-mean, stationary, random process (Equation 64).

\[
D_{X_0}(\tau) = \frac{R_X(\tau)}{R_X(0)} X_0
\]

where:

- \(D_{X_0}\): Random decrement signature
- \(X_0\): Trigger condition
- \(R_X(\tau)\): Auto-correlation function of random process X(t)
- \(\tau\): Delay \((t_2 - t_1)\)
- \(R_X(0)\): Auto-correlation function at \(\tau = 0\)

The RDS is computed by selecting a triggering condition, which specifies amplitude and may specify slope, recording a time-history of pre-selected length each time the trigger condition is satisfied, and subsequently performing an ensemble average of these time histories. Inherent in the application of the ensemble average is the assumption that the process is ergodic, meaning that the statistics of a sample of the signal are representative of the entire signal. Since it is assumed that the input is a zero-mean, stationary random process it follows that the random components of the response will tend to cancel out with significant averaging and what remains is the response of the system subject to the initial condition specified by the triggering condition. For a single degree of freedom system, under these assumptions, the RDS is exactly proportional to the free vibration response of the system under the specified triggering condition. The RDS may be expressed mathematically according to Equation 65, as the conditional mean of the random signal on the triggering condition.
\[ D_{X_0}(\tau) = E[X(t_2)|X(t_1) = X_0] \]  

Equation 65

Where:

\[ X(t) \]: Time history

Numerically, for application to limited duration discrete data, this may be expressed according to Equation 66.

\[ D_{X_0}(\tau) = \frac{1}{N} \sum_{n=1}^{N} (X_n(\tau)|X_n(0) = X_0) \]  

Equation 66

Where:

\[ N \]: Number of samples

\[ n \]: Trigger index

There are four main types of triggering conditions (Table 3.2):

**Table 3.2** RDT triggering conditions (Kijewski T., 2003)

<table>
<thead>
<tr>
<th>Trigger Name</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level Crossing</td>
<td>( X(0) = X_0 )</td>
</tr>
<tr>
<td>Zero Up-Crossing</td>
<td>( X(0) = 0 ) and ( \dot{X}(0) &gt; 0 )</td>
</tr>
<tr>
<td>Positive Point</td>
<td>( X_{low} &lt; X(0) &lt; X_{high} )</td>
</tr>
<tr>
<td>Local Extrema</td>
<td>( X_{low} &lt; X(0) &lt; X_{high} ) and ( \dot{X}(0) = 0 )</td>
</tr>
</tbody>
</table>

Positive point and local extrema are the relevant triggering methods for this study, since the primary application of this technique is the investigation of amplitude-dependent properties. Practical limitations with discrete sampling of the response preclude the exact use of the level crossing method, since it is unlikely that the response will take on the exact value of the trigger in the time-history. The zero up-crossing method does not reference amplitude, and therefore is generally not used, since the primary advantage of the RDT is the reference to a certain amplitude. The positive point triggering method is a variation of the level crossing technique, specifying an amplitude and a certain tolerance on that amplitude to find several viable candidates within a few percent of the desired amplitude, making this method appropriate for application to discrete data. The initial condition of amplitude with no velocity is enforced by significant averaging. Since no slope criteria is specified, it is expected that a similar number of positive and negative slopes will be captured. When averaged, this produces a zero-slope condition at the specified amplitude. Lastly, the local extrema method may be applied, which a stricter version of the positive point triggering, adding the specification of a zero-slope condition.
The variance of the RDS was derived by Vandiver et al. (1982) and is shown in Equation 67.

\[
\text{Var}[D_X(\tau)] = \frac{1}{N} R_X(0) \left[ 1 - \frac{D_X^2(\tau)}{X_0^2(0)} \right] 
\]

Equation 67

This was derived based on the assumption of a zero mean, random and stationary process, so the results should be taken with reservations when applied to non-ideal data. This indicates that as the number of segments, \( N \), grows, the variance decreases. As the lag (number of cycles) increases, the variance also increases. These are key parameters when applying the RDT to full scale building response data. A key note is that it was assumed that each segment was uncorrelated, meaning if an RDS of 100 seconds was selected, no other segment may be triggered for 100 seconds after this trigger. It was noted that several other researchers had noted that there was an observed benefit of allowing correlation by increasing the number of segments. Vandiver et al. (1982) argued that less segments would be required if the correlation condition was enforced.

While the application of the RDT had been established as a viable means to determine the dynamic characteristic of a structure by measuring the output of the system, a key benefit of the technique is the investigation of amplitude-dependent properties (Jeary, 1986) by referencing the estimate of damping and frequency to the trigger amplitude. This is highly relevant for tall buildings since damping plays a key role in controlling the response of the building at design level events, thus an understanding of the properties at higher amplitudes is critical for structural engineers. Jeary (1992) noted several key obstacles in practical application that may cause RDS degradation when applied to non-stationary responses of non-linear (amplitude-dependent) structures:

- Drops and spikes in the amplitude in the time history
- Sequences in which sudden changes in the RMS of the signal occur
- Beating between modes

These items must be overcome for proper application of the RDT. Careful selection of the records and regions of analysis may overcome the first two points, and appropriate bandpass filtering about the mode of interest may overcome the last.

It was shown that by varying the triggering level, the amplitude-dependent damping may be computed using the RDT on relatively short duration records (Figure 3.5) (Jeary, 1996).
A refinement of the RDT was developed by Tamura and Suganuma (1996) which added zero-slope criteria to the triggering condition, defining the local extrema triggering method.

$$D_{X_0}(X_0; \tau) = E\{\text{sgn}[X(t)]X(t + \tau)|\dot{X}(t) = 0, X(t) = X_0\}$$  \hspace{1cm} \text{Equation 68}

This technique was established to overcome the possible amplitude variations in actual response signals, which was identified as a problem by Jeary (1992), by specifying a certain range of local peaks in the time-history for triggering, thus generating a sort of self-stationary set of data. This process is shown schematically in Figure 3.6.

The RDT is traditionally applied to SDOF response signals. Since real structures are MDOF systems, digital bandpass filters are typically used to isolate each modal response for subsequent processing using the RDT. However, if the modes are closely spaced than bandpass filters may not able to sufficiently resolve the modal responses. In this case, the RDT may be
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generated using the unfiltered responses. The first step in doing so is to select a reference channel, which should contain response contributions from all modes of interest. The RDS for this channel is computed based on the selected triggering method, and each time this triggering threshold is met the RDS from each subsequent channel is captured. By matching the time index between signals, the phase and amplitude relationship between degrees of freedom are preserved, allowing for the mode shapes to be estimated. When processing in this fashion for an MDOF system, the Eigensystem Realization Algorithm (ERA) must be applied, which was discussed in Section 3.2.1.3.

The CFSMP researchers have instrumented several tall buildings around the world and several key Theses have been published from this work [ (Kijewski T. , 2003), (Erwin, 2009), (Pirnia, 2009), (Bentz, 2012), (Guo Y. , 2015)], detailing the development and application of the monitoring system, stationary and non-stationary system identification, and structural behaviours and how they relate to the dynamic properties. Some of the key items from this research pertaining to the RDT are summarized herein.

Kijewski (2003) formally investigated the effect of correlation on the damping estimation using the RDT and found that the benefit of the additional segments generated by allowing correlation of segments outweighed the cons of correlation. This work also introduced the concept of local averaging to increase the repeatability of the results by selecting several triggers within a few percent of the target trigger, and averaging the results from each RDS. Typically, a local averaging tolerance of 3% had been implemented in the group’s research. It was also found that the best amplitude range for application of the RDT was between the RMS and 3 times the RMS of the signal, citing excessively high correlation in the low amplitude range and insufficient data at higher amplitudes. Enforcing some restrictions on correlation in the low amplitude range improved the performance of the RDT but detracted from the performance of the RDT at high amplitude (by reducing the number of segments). This may be a relevant concern for generating a robust amplitude-dependent curve, as some correlation criteria may need to be enforced in the low amplitude range to achieve better results without compromising the quality of the high amplitude estimates.

Kijewski (2003) also investigated the effect of non-stationarities and non-linearities. Previously, Jeary (1992) highlighted the applicability of the RDT to mildly non-stationary responses by implementing a pre-treatment of the data to remove drops or spikes, regions of highly
varying amplitude, and the presence of more than one mode. Kijewski (2003) confirmed that this pre-treatment of the data allows the RDT to be appropriately applied, in conjunction with the damping being estimated only over the first few cycles of the RDS.

Erwin (2009) applied the local extrema triggering condition to short duration records of low to mid-rise buildings. These records ranged in duration from 24.5 minutes to 76.4 minutes (inferred from information provided in thesis Appendix). Estimates from these records were made using a local averaging scheme around the standard deviation of the signal. Only a single amplitude was evaluated in this study for each of the 22 RC, 13 steel, and 10 steel/RC buildings. There was no explicit information given on the number of segments used for these analyses, and 3-5 cycles were included in the RDS computation.

Pirnia (2009) applied the local extrema triggering condition to long duration records of tall buildings. Interestingly, Pirnia used 10 cycles in the computation of the RDS, even though amplitude-dependent characteristics were being calculated, in spite of the previous research indicating that 3-5 cycles were more appropriate for this. For this study, the data was pre-screened to verify that it was stationary, and the amount of data retrieved is summarized in Table 3.3.

Table 3.3 Summary of record length used in Pirnia (2009) thesis.

<table>
<thead>
<tr>
<th>Building</th>
<th>Data</th>
<th>Amplitude Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Korean Tower</td>
<td>128 hrs</td>
<td>1.5 milli-g</td>
</tr>
<tr>
<td>Chicago 1</td>
<td>24 hrs</td>
<td>0.75 milli-g</td>
</tr>
<tr>
<td>Chicago 2</td>
<td>12 hrs</td>
<td>0.25 milli-g</td>
</tr>
<tr>
<td>Chicago 3</td>
<td>56 hrs</td>
<td>0.65 milli-g</td>
</tr>
</tbody>
</table>

Bentz (2012) developed and implemented a method to identify the dynamic properties using a non-stationary wavelet-based system identification framework. This was the focus of the thesis, but a small portion was dedicated to the RDT. For application of the RDT several similar events were used, circumventing the challenge of recording a continuous response, since transient wind events are typically short duration in nature. 26 hours of data was generated for this analysis; however, it was stated in the thesis that an insufficient number of segments were obtained to establish a reliable RDS.

The City University of Hong Kong (CUHK) has produced a significant number of research papers related to the application of full-scale testing to tall and super tall buildings in Asia over the past several decades. Several buildings have been monitored through significant wind events
(typhoons), and the primary system identification technique used by the group for the identification of dynamic properties in these events was the RDT. Typical implementation involved bandpass filtering the mode of interest, and applying the local extrema triggering method. A recent paper (Li & Li, 2018), utilised the time-histories from 14 tropical cyclones, providing a total time-history of high amplitude data of 360 hours. The local extrema triggering method was effectively used to determine the amplitude dependency of the dynamic properties for the building using a minimum of 500 segments.

The various parameters that affect the RDT were explored in more detail in Sections 3.3.1.1, 3.3.2.1, and 3.3.3.1.

3.2.1.5 Natural Excitation Technique (NExT)

The Natural Excitation Technique (NExT) was developed for the modal testing of wind turbines under wind action (James, Carne, & Lauffer, 1993). It was shown that by assuming that the input is white noise, the cross-correlation function between two channels of acceleration has the same characteristics as a free vibration response of an MDOF system (Equation 69) (Caicedo, 2011).

\[ M\ddot{R}_{y,y_i} + C\dot{R}_{y,y_i} + K\ddot{R}_{y,y_i} = 0 \]  

Equation 69

Where:
\( \dddot{R}_{y,y_i} \): Cross-correlation function between acceleration vector and reference acceleration signal

Therefore, the cross-correlation function provides all the dynamic characteristics of the system. The cross-correlation function may be determined by calculating the cross power spectral density, traditionally using Welch’s method, between all the channels and a selected reference channel and subsequently performing an inverse Fourier transform to take the signal back to the time domain. Note that in the case of an SDOF system, there is only one channel and the cross power spectral density may be taken with itself (auto-spectral density function). In the case of an MDOF system, the reference channel should be selected such that is not near any nodes of the modes of interest and has a high signal-to-noise ratio.

The key parameters in calculating the cross-correlation function functions are:

- **Record length**: Longer records are required for noisy data and for low period structures, such as tall buildings, since more data will be available in each average when computing the cross power spectral density, thus improving SID results.
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- **Number of points in the FFT**: The number of points used in the FFT is equal to the number of points in the cross-correlation function. Therefore, too few points considered in the FFT may produce too short of a cross-correlation function.

When implemented in Matlab, Welch’s method defaults to dividing the record of length \(N\) (number of points) into eight segments and using a Hamming window with 50% overlap. This method was used in its default settings in this thesis.

If the system is an SDOF and once the cross-correlation function is obtained, the frequency and damping may be estimated from the zero-crossing points and the logarithmic decrement method (Equation 48) or by a least squares fit (Equation 49).

Note that the NExT is similar in nature to the RDT in the sense that it may be used to produce a proportional free vibration signal from ambient response data. In the case of modes that are not closely spaced, these techniques may be applied to a bandpass filtered signal, or if mode shapes are of interest multi-channel signals may be processed using the ERA.

3.2.1.6 Enhanced Frequency Domain Decomposition (EFDD)

Enhanced Frequency Domain Decomposition (EFDD) is an extension of the basic peak-picking technique, from SA. The drawback of the classical technique is that it does not work when closely spaced modes are present, and the estimation of damping is highly uncertain. Developments in the EFDD technique allow for the identification of closely spaced modes as well as accurate estimations of the damping. The theoretical components of the EFDD are well summarized in (Brincker, Zhang, & Andersen, 2000) and (Brincker, Ventura, & Anderson, 2001), and so the implementation algorithm from these references is summarized herein.

First, the power spectral density matrix is formed by determining the cross power spectral density matrix between each of the response channels. Note that this creates an \(m \times m \times n\) matrix, where \(m\) is the number of channels, and \(n\) is the number of points in the series. For the case of a 3-channel matrix, the PSD matrix indices are shown in Equation 70.

\[
PSD = CPSD \begin{bmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \\ 31 & 32 & 33 \end{bmatrix}_N
\]  
Equation 70

The singular value decomposition of the matrix at each frequency is performed (Equation 71), where the first singular vector and value (from \(U\) and \(S\), respectively), represents the principal component at that frequency. If only a single mode is present at a frequency, there will only be
one singular value, whereas if there are multiple modes present at a single frequency, there will be more than one singular value, of appreciable magnitude. It is through this decomposition that closely spaced modes may be identified.

\[
\hat{G}_{yy}(j\omega_i) = U_i S_i U_i^T
\]

Where:
- \(\hat{G}_{yy}(j\omega_i)\): Output PSD
- \(U_i\): Unitary matrix holding singular vectors \(u_{ij}\)
- \(S_i\): Diagonal matrix holding scalar singular values \(s_{ij}\)

The first singular values from \(S\) are then plotted at all frequencies, producing the SVD of the PSD spectrum. Peaks in the SVD spectrum indicate modes, and in the case of modes that are not closely spaced, the first singular value will be dominant, and the first singular vector is an estimation of the mode shape (Equation 72).

\[
\Phi_i = u_{i1}
\]

Where:
- \(\Phi_i\): Estimate of mode shape
- \(u_{i1}\): First singular vector

The corresponding singular value is the auto spectral density function of a corresponding SDOF system. By identifying singular vectors that have a high MAC value (Equation 63) with the mode of interest, an SDOF auto spectral density function may be recognised around the peak. If all values that are not greater than the specified MAC (usually 0.8) are set to zero, the inverse Fourier Transform of the auto spectral density function provides the auto correlation function of the SDOF system. Thus, as noted previously, the damping and frequencies may be obtained from this function using a time-domain DET, while the mode shape was obtained from the first singular vector from the SVD.

Note the similarities to the NExT, given that the cross-power spectral density function is computed, and the inverse Fourier transform is used to return the signal to the time domain. The primary difference between techniques is that the EFDD applies what is effectively a bandpass filter to the signal in the frequency domain through the MAC and SVD filtering process to isolate each mode.
3.2.1.7 **Data-Driven Stochastic Subspace Identification (SSI-DATA)**

Data-Driven Stochastic Subspace Identification (SSI-DATA) is a time-domain system identification technique that utilises the measured output data directly. The algorithm was thoroughly developed in (Van Overschee & De Moor, 1996). The implementation of SSI-DATA was well presented in (Weng, et al., 2008), (Brincker & Anderson, 2006) and (Brincker & Ventura, 2015) and is briefly summarized herein.

To apply the SSI-DATA method, the system is modelled as a stochastic state-space model (Equation 73). Note that the input matrix (B) is replaced by the stochastic process noise component, and the direct transmission term (D) is replaced by the measurement noise component.

\[
\begin{align*}
x^s(k + 1) &= Ax^s(k) + w(k) \\
y^s(k) &= Cx^s(k) + v(k)
\end{align*}
\]

Equation 73

Where:

- \( s \): Denotes stochastic
- \( w(k) \): Process noise
- \( v(k) \): Measurement noise
- \( A / C \): System matrices (Dynamic / Output)
- \( y^s(k) \): Output measurement

First, the output block Hankel matrix is formed directly from the measurements (Equation 74).

\[
H = \begin{bmatrix}
y_0^s & y_1^s & \cdots & y_{j-1}^s \\
y_1^s & y_2^s & \cdots & y_j^s \\
\vdots & \ddots & \vdots \\
y_{i-1}^s & y_i^s & \cdots & y_{i+j-2}^s \\
y_i^s & y_{i+1}^s & \cdots & y_{i+j-1}^s \\
y_{i+1}^s & y_{i+2}^s & \cdots & y_{i+j}^s \\
\vdots & \ddots & \vdots \\
y_{2i-1}^s & y_{2i}^s & \cdots & y_{2i+j-2}^s
\end{bmatrix} = \begin{bmatrix} Y_p^s \\ Y_f^s \end{bmatrix}
\]

Equation 74

Where:

- \( H \): Output Hankel matrix
- \( y^s \): Measurement vector of accelerations
- \( Y_p^s \): Past matrix
- \( Y_f^s \): Future matrix
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The Hankel matrix is partitioned into the past and future components, which simply splits the matrix in half. The user must select the number of block rows \(i\), block columns \(j\), and data points \(r\) - Equation 75 which will be included in the Hankel matrix.

\[ r = 2i + j - 1 \]  
\[ \text{Equation 75} \]

Next, the orthogonal projection of the future \((Y_f^s)\) onto the past \((Y_p^s)\) is performed (Equation 76), which is the conditional mean of the observed random signals. Functionally, this projection may be computed according to Equation 77. Brincker and Ventura (2015) noted the similarities between the projection and the RDS, stating that since the projection is a conditional mean of the observed random signals, the projection is basically the same as the RDS.

\[ O_i^s = E[Y_f^s | Y_p^s] \]  
\[ \text{Equation 76} \]

\[ Y_f^s / Y_p^s = Y_f^s Y_p^s T \left(Y_p^s Y_p^s T\right)^H Y_p^s = O_i^s \]  
\[ \text{Equation 77} \]

Where:

\(T\): Transpose operator
\(H\): Pseudo-inverse operator
\(O_i^s\): Orthogonal projection matrix

Next, the SVD of the projection matrix is performed (Equation 78).

\[ O = USV^T \]  
\[ \text{Equation 78} \]

The smaller singular values in \(S\) are truncated, leaving a matrix of dimension \(g\), which indicates the model order in a manner like the ERA. The reduced SVD matrices capture the principle components of the projection matrix, which functionally describes the dynamics of the system being analysed and reduces the effect of noise. Using the reduced matrices from the SVD, the extended observability matrix is computed.

\[ I_i^1 = U_g S_g^2 \]  
\[ \text{Equation 79} \]

Finally, the system matrix \((A)\) may be calculated according to Equation 80, and the influence matrix may be calculated as the first \(l\) rows of \(I_i\) (Equation 81)

\[ A = I_i^H I_i \]  
\[ \text{Equation 80} \]

\[ C = I_i^1 (1:l,: ) \]  
\[ \text{Equation 81} \]

Where:
$I_i^H$: $I_i$ without the last $l$ rows

$\overline{I_i}$: $I_i$ without the first $l$ rows

$l$: Number of measurement channels

Performing an eigenvalue decomposition of the system matrix yields the natural frequencies and damping ratios of the system, and the mode shapes may be determined according to Equation 61. Like the ERA, a stabilization diagram may be constructed to identify stable modes.

### 3.2.2 Non-Stationary Algorithms

Obtaining records that are suitable for the previously discussed stationary SID algorithms requires long duration stationary signals. Practically, this limits the amplitude range for investigation to the low amplitude range where signals of this nature may be obtained. However, higher amplitude events, like those from large wind storms, which are of primary interest to structural engineers, are typically characterized by transient or non-stationary signals. Unfortunately, all the techniques discussed previously require records of a certain length of stationary data (with the exclusion of the RDT), and thus are not applicable for analysing non-stationary signals, which tend to be shorter in duration.

Non-stationarity may be manifested in a signal in two ways:

1. **Time-varying amplitude:** Large variance in the excitation amplitude will result in varying dynamic properties due to amplitude-dependency, resulting in time-varying system properties.

2. **Time-varying frequency:** If the input excitation is not truly white noise and in fact has some harmonic component, the broadband excitation frequency assumption will not be met, which violates the inherent assumptions for most output-only SID.

This calls for a new branch of system identification algorithms which can handle these transient features. This a growing field and several novel techniques have been proposed in recent years, primarily out of the Notre Dame research groups [(Kijewski T., 2003), (Bentz, 2012), (Guo Y., 2015)]. The Wavelet based methods discussed in this work are described in the subsequent section.
3.2.2.1 Wavelet-Based Methods

Wavelet-based methods were briefly explored in this thesis, as expanding the study to the full range of non-stationary SID techniques was outside of the scope for the work. Wavelet transforms are a useful technique for preserving the time-varying frequency content of signals. Functionally, the wavelet basis functions, or mother wavelet, are localized in both time and frequency. The mother wavelet is scaled to produce daughter wavelets, which vary the frequency content of the wavelet. Each of the daughter wavelets is then convoluted with the time-domain signals to determine the frequency content in time. The general expression for the wavelet transform is shown in Equation 82.

$$W(a, t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\tau) \psi \left( \frac{\tau - t}{a} \right) d\tau$$  \hspace{1cm} \text{Equation 82}

Where:

- $W(a, t)$: Wavelet coefficients
- $a$: Dilation parameter
- $\psi(t)$: Mother wavelet

This may be computed in the time domain by calculating a sliding dot product between the scaled daughter wavelet and the signal at each time instant. Conversely, this may be computed in the frequency domain by pointwise multiplication of the Fourier transform of the signal and the daughter wavelet, followed by an inverse Fourier transform to return the signal to the time domain. The second method is significantly less computationally expensive and was adopted for the application of the wavelet transform.

There are many options for the selection of the mother wavelet, although the most common for structural engineering applications is the Morlet wavelet (Equation 83).

$$\psi(t) = e^{i2\pi f_0 t} e^{-\frac{t^2}{2}}$$  \hspace{1cm} \text{Equation 83}

Where:

- $f_0$: Central frequency of the mother wavelet

This mother wavelet is commonly used in structural engineering applications since it is ideal for analyzing time varying harmonics in a signal. The complex Morlet wavelet is shown in Figure 3.7 for a centre frequency of 1 Hz.
For the complex Morlet wavelet, there is a unique relationship between the scale and the frequency of the wavelet, allowing for simple application in the wavelet transform framework (Equation 84).

\[ a = \frac{f_0}{f} \]  

Equation 84

The centre frequency and support range for the mother wavelet must be specified before applying the transform. These are both key parameters for the transform to ensure that the proper frequency range and resolution is found. The transform produces the complex valued wavelet coefficients, which when squared produces the wavelet scalogram.

While the time-varying frequency may be readily identified by the ridges in the wavelet scalogram, identifying the damping ratio requires the presence of free vibrations in the signal, which may be present following peak amplitudes in the non-stationary structural response. Two methods for finding these free vibration-like signals in a non-stationary structural response have been proposed recently including:

1. **Bentz (2012): Impulse Response Function detector wavelet.** A new wavelet that has the direct characteristics of a free vibration response of an SDOF system was proposed. This is applied to the signal which produces a scalogram that highlights regions where the signal appears to be a free vibration. User intervention is then required to review the identified region to ensure that the identified signal segment is in fact a viable free vibration, and not a mirror-image increase in amplitude. The Morlet wavelet is then applied to the region of
interest, and various damping evaluation techniques may then be applied to extract the damping ratio and frequency.

2. **Guo (2015): Laplace wavelet filtering.** Guo (2015) proposed using the Laplace wavelet to filter the wavelet coefficients along a frequency ridge to extract free vibration-like responses from the non-stationary signal. This technique has the benefit of directly computing the dynamic properties (damping ratio, frequency) through the extraction of the free-vibration-like responses.

Both methods have been shown to be effective for locating free-vibration-like responses in non-stationary structural response signals. Note that the frequency ridge produced by the Morlet wavelet transform is essentially a tight bandpass filter around the frequency of interest.

### 3.2.2.2 Non-Stationary Algorithms Discussion

While the field of non-stationary SID is growing and shows great promise, a key challenge in this framework is that the signal must have an authentic free-vibration response in the non-stationary time-history. It is not a guarantee that all non-stationary signals will be characterized in this way, meaning that a large data set of non-stationary data may be required to find free-vibration-like responses that achieve sufficient quality as to perform SID on these traces. Note again, that the investigation into non-stationary SID was limited in this work and should be explored in greater detail in future research applications.

### 3.2.3 Output-Only System Identification Summary

This chapter has summarized a wide array of techniques that may be used for SID of structures. The distinction between stationarity and non-stationarity was discussed, and the implications regarding non-stationary amplitude’s effect on the non-linear (amplitude-dependent) response of structures was highlighted. Additionally, several output-only SID methods were described. A breakdown of system identification, with emphasis placed on the topics discussed in this thesis, is shown in Figure 3.8.
All techniques may be applied effectively to tall building responses, provided the appropriate assumptions are not excessively violated. With respect to tall buildings, one of the primary interests is the amplitude-dependent damping and frequency, therefore an ideal technique is one that can track this phenomenon. This is the main reason that the RDT with bandpass filtering has seen such widespread use in the tall building community. The other techniques discussed, including RDT-ERA, NExT-ERA, EFDD, and SSI-DATA are useful for analysing MDOF responses if there is interest in obtaining mode shapes. These techniques are effective at processing response data from multi-component sensor arrays, meaning that if several floors are instrumented with several channels on each, these techniques may be ideal for processing all the data simultaneously.

The next section reviews the practical application of several of the previously outlined SID techniques to numerically simulated data to develop and validate the output-only SID algorithms.
3.3 Stationary Excitation Numerical Simulation

To develop and validate the described system identification algorithms, and to study the effectiveness when applied to linear and non-linear data, a numerical study was performed on four system types:

- Linear SDOF
- Linear MDOF
- Non-Linear SDOF
- Non-Linear MDOF

For each, the dynamic properties were known and numerical simulations using white noise inputs were performed. The response quantities from each system were then used in several SID algorithms to study the effectiveness and the required parameters for each method.

3.3.1 Linear SDOF

The system properties for the linear SDOF case are listed in Table 3.4. The system was excited by a zero-mean, white noise, Gaussian forcing function and the response of the system was solved numerically using Newmark’s average acceleration time-stepping algorithm (Appendix I).

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>10000</td>
</tr>
<tr>
<td>Damping Ratio (%)</td>
<td>1.5</td>
</tr>
<tr>
<td>Period (s)</td>
<td>5</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>0.2</td>
</tr>
<tr>
<td>Damped Natural Frequency (Hz)</td>
<td>0.19998</td>
</tr>
</tbody>
</table>

Records of 1, 10, 100, and 250 hours were generated for processing using the output-only SID methods. An example of the excitation and response for the 10-hour record is shown in Figure 3.9 with samples obtained at 10 Hz.
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3.3.1.1 RDT

The RDT is the most commonly used technique for tall buildings, given the ability to investigate amplitude-dependency by varying the selected triggering level as well as the technique’s physical motivation. Various combinations of parameters have been used by different researchers and this section investigates the key parameters for the RDT including the triggering type, number of segments, number of cycles, and duration of the record. The objective was to establish baseline criteria for applying the RDT to tall building test data. For all analyses unless otherwise noted the RDT was applied using a 250-hour signal, 10 cycles, and a tolerance of 10% on amplitude.

3.3.1.1.1 Triggering Type

Two different triggering types were investigated including positive point (PP) and local extrema (LE). Note that since the system under analysis was a linear system, thus no amplitude-dependence should be observed. Despite this, the effect of varying the trigger level was

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Figure 3.9 Random force input and displacement time history for linear SDOF system.

---

Full-Scale Monitoring of a Tall, Slender Building with Coupling Viscoelastic Dampers

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investigated to understand the applicable amplitude range for the technique. The results using each of these triggering methods are shown in Figure 3.10.

![Graph](image)

**Figure 3.10** Effect of triggering type on RDT results for linear SDOF system.

These results indicate that at low amplitudes, below 1σ (RMS) of the signal (2.74 millimetres), the technique does not adequately capture the frequency or the damping ratio. As the amplitude grows beyond the RMS, both triggering methods converge closely to the correct property estimates, with the frequency being within 0.35% of the exact frequency and the damping being within 15%. At higher amplitudes, an increase in variability is evident in both the frequency and damping estimates. This indicates the triggering amplitude influences the estimated properties and that there may be an optimal amplitude range to apply the RDT. This is investigated further in subsequent sections. Note that the frequency converges to a value less than the specified, which may be attributed to the effect of damping on reducing the natural frequency to the damped natural frequency and errors in the damping estimation manifesting as errors in frequency. Several research groups have used different triggering methods (Table 3.5).
Table 3.5 Previous applications of triggering method for RDT.

<table>
<thead>
<tr>
<th>Method</th>
<th>Primary Application</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level crossing</td>
<td>Offshore structure, mathematical validation</td>
<td>(Vandiver, Dunwoody, Campbell, &amp; Cook, 1982)</td>
</tr>
<tr>
<td>Level crossing</td>
<td>Tall building</td>
<td>(Jeary A., 1992)</td>
</tr>
<tr>
<td>Local Extrema</td>
<td>Air traffic towers (in this paper). Tall buildings in subsequent research.</td>
<td>(Tamura &amp; Suganuma, 1996)</td>
</tr>
<tr>
<td>Local Extrema/Positive Point</td>
<td>Numerical study. LE for long duration, PP for short duration.</td>
<td>(Kijewski T., 2003)</td>
</tr>
<tr>
<td>Local Extrema</td>
<td>Tall buildings, long records</td>
<td>(Pirnia, 2009)</td>
</tr>
<tr>
<td>Local Extrema</td>
<td>Short duration records, short buildings</td>
<td>(Erwin, 2009)</td>
</tr>
<tr>
<td>Local Extrema</td>
<td>Tall/Supertall Buildings</td>
<td>(Li &amp; Li, 2018)</td>
</tr>
</tbody>
</table>

3.3.1.1.2 Number of Segments

Since the RDT relies on successive averaging to remove the random component of the structural response, it follows that increasing the number of segments will result in a more accurate RDS. This is reflected mathematically in the work by Vandiver et al. (1982), which shows that as the number of segments increase, the variance of the RDS decreases. The number of segments from each triggering method is shown in Figure 3.11.

![Figure 3.11 Variation of number of segments as a function of amplitude.](image)

Comparing these techniques, the main advantage is that the PP technique produces significantly more segments than the LE technique. In an idealized system this is not an obvious advantage, but when looking at real structural responses on limited duration records this is advantageous, as more segments may be generated in a shorter duration record, which is beneficial in reducing the variance of the RDS. A caveat to this conclusion is that the PP triggering method
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inherently requires more segments to enforce the zero-slope initial condition, while the LE triggered segments are closer to the specified condition inherently, therefore requiring fewer segments.

Figure 3.11 shows that the number of segments grows until the RMS of the signal is reached for the PP method and just after for the LE method, after which the number of segments decreases. This follows since the system simply does not reach the higher amplitudes as frequently as the lower ones. Below the RMS, the growth may be attributed to the discrete sampling nature of the technique. Since the rate of change of displacement is at its maximum as the system oscillates through zero, and the response is sampled at a constant frequency, there is a larger difference in amplitude in this region. This means that if the trigger threshold is low, some segments may simply be missed. Closer to the RMS of the signal there will be the highest density of data points, which corresponds to the highest number of segments. This provides a useful insight into practical application, as with shorter duration records data amplitudes below $1\sigma$ may need to be discarded, providing a lower bound on the application range for the RDT.

Kijewski (2003) found that the optimal range for the RDT application was approximately $1\sigma$-$3\sigma$. In the low amplitude range ($<1\sigma$), the segments are likely to be highly correlated, which can increase the variance of the RDS (Vandiver, Dunwoody, Campbell, & Cook, 1982). This, and the lower density of points in the low amplitude range serve to reduce the quality of the RDS in this range. As the amplitude grows, there will simply be less points, leading to Kijewksi’s (2003) recommendation for truncation at $3\sigma$.

Several researchers have recommended different minimum segment numbers required (Table 3.6).

<table>
<thead>
<tr>
<th>Number of Segments</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Best results at 1000 segments, 100 segments acceptable (PP)</td>
<td>(Jeary, 1996)</td>
</tr>
<tr>
<td>Several hundred peaks can yield sufficient results (LE)</td>
<td>(Tamura &amp; Suganuma, 1996)</td>
</tr>
<tr>
<td>100 segments insufficient</td>
<td>(Kareem &amp; Gurley, 1996)</td>
</tr>
<tr>
<td>Minimum of 100 segments (LE)</td>
<td>(Pirnia, 2009)</td>
</tr>
<tr>
<td>Minimum of 500 segments (LE)</td>
<td>(Li &amp; Li, 2018)</td>
</tr>
</tbody>
</table>

The effect of the diminishing number of segments at higher amplitudes was investigated by plotting the percent difference of the frequency and damping estimates against amplitude (Figure 3.12).
First it is important to note the difference in the magnitude of the percent difference between the damping and frequency estimates. The damping ratio estimates appear to be generally within approximately 5\% of the true value (when greater than the RMS), until the 1000 segment limit is reached where the percent difference grows, whereas the frequency estimates are generally within approximately 0.30\% of the true value. The implications of this are significant; the estimation of the frequency using the RDT may be regarded as highly reliable, whereas the estimation of damping in a perfect idealized system is characterized by error an order of magnitude greater than the frequency. Extending this method to true vibration response signals of actual structures it is evident that the damping estimation will be highly variable. Literature suggests that 1000 segments may be considered a conservative cut-off of segment number, which is supported by increased scatter after the 1000 segment limit in the numerical results shown in Figure 3.2.

Moving forward, the effect of the algorithm parameters will be investigated for the PP triggering only. If a long-term monitoring system was implemented, the LE method with a narrow tolerance would be preferred to achieve a better resolution of the trigger amplitude, but the additional segments generated using PP by removing the slope criteria is beneficial for short duration records. This is supported by Kijewski (2003), who also concluded that the benefit of the additional segments generated by the PP technique makes it more suitable for shorter duration records. Since most of the data obtained from the test structure comprised of short duration records,
and the frequency was constantly changing as the structure was built, the PP triggering was the most suitable for this study.

### 3.3.1.1.3 Tolerance

A key parameter in the practical application of the PP triggering technique is the specified tolerance. In the previous investigation, a tolerance of 10% was used. The CFSMP researchers typically use a tolerance of 3%. This group introduced the concept of local averaging (Kijewski T., 2003), where the RDS results within the specified tolerance are analysed as a single point statistically (average, variance) to improve the reliability of a specific trigger amplitude. One of the primary advantageous of this concept is that additionally segments may be generated at each trigger amplitude.

If the system were to exhibit amplitude-dependent characteristics the larger tolerance would blend the results over the amplitude range being considered. Too large of a tolerance, and there would be no evidence of amplitude-dependence. Too small of a tolerance and the number of segments will be reduced leading a degrading RDS quality (for short duration records). Functionally, the selection of the tolerance should be made based on the length of available records and the number of segments that may be generated for that length (discussed further in Section 3.3.1.1.5).

### 3.3.1.1.4 Number of Cycles

The primary concern when selecting the number of cycles to include in the RDS is the expected amplitude-dependency. If too many cycles are included, effectively a larger amplitude range is being considered which may obscure the amplitude-dependent phenomenon. Given this there is little benefit to investigating the effect of the number of cycles considered in the RDS on a linear system, since there is no amplitude-dependency in the system. However, this concept is further investigated in Section 3.3.3.1.2 for a non-linear system. Several researchers have recommended different numbers of cycles to include in the analysis. (Table 3.6).
Table 3.7 Previous applications of number of cycles for RDT.

<table>
<thead>
<tr>
<th>Number of Cycles</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>First few cycles 3-5</td>
<td>Tamura and Suganuma (1996)</td>
</tr>
<tr>
<td>Accurate within the first few cycles of the RDS decay</td>
<td>Kareem &amp; Gurley, 1996</td>
</tr>
<tr>
<td>10</td>
<td>Pirnia, 2009</td>
</tr>
<tr>
<td>8</td>
<td>Li &amp; Li, 2018</td>
</tr>
</tbody>
</table>

3.3.1.1.5 Duration of Record

As previously outlined one of the primary challenges of applying the RDT is establishing enough segments. While this can be influenced by the triggering criteria selected and the tolerance, the easiest way to increase the number of segments is to simply increase the duration of the record. The effect of increasing the duration of the record analysed is shown in Figure 3.13 and Figure 3.14.

![Figure 3.13](image1)

**Figure 3.13** Effect of signal duration on the frequency estimates using the RDT.

![Figure 3.14](image2)

**Figure 3.14** Effect of signal duration on the damping estimates using the RDT.
Evidently, the estimation of frequency and damping stabilizes over all amplitudes as the duration increases, including the region below $1\sigma$. This may be attributed to the significant increase in number of segments as the length of the record increases (Figure 3.15).

![Figure 3.15 Effect of signal duration on number of segments generated for the RDT.](image)

For the 100 and 250-hour duration records, higher amplitudes were able to be investigated since the number of segments at those amplitudes is larger for the longer duration records. While higher amplitudes were investigated, the quality of the estimation begins to diminish once the number of segments drops below 1000 segments for all duration records, giving further support to 1000 segments as an upper bound for the amplitude.

Most notably the estimation of damping ratio stabilizes to within 2% of the exact value for the 250-hour duration record over the amplitude range from $1\sigma$ to 1000 segments, showing the importance of obtaining long duration records. Therefore, it is highly desirable to achieve long duration stationary responses to establish amplitude-dependent damping and frequency estimates using the RDT. A caveat to this analysis is that the simulated signal is perfectly stationary, which is practically difficult to achieve in the field. Recent studies (Li & Li, 2018) have shown success in establishing longer duration high-amplitude signals by collecting high-amplitude data over many years and collating these signals together into a single signal for a single RDT analysis to investigate the dynamic properties at higher amplitudes.

An example RDS computed at the RMS of the 250-hour signal is shown in Figure 3.16. Note the high-quality fit of the decaying sin curve ($R^2 = 1.0000$). This RDS provides a damping estimation of 1.5219% and a frequency estimate of 0.1997 Hz.
3.3.1.1.6 *Selected RDT Parameters*

Based on the literature review and the above investigation, the selected parameters for application of the RDT to field data are shown in Table 3.8.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triggering</td>
<td>Positive Point</td>
</tr>
<tr>
<td>Number of Segments</td>
<td>Minimum 1000 segments</td>
</tr>
<tr>
<td>Number of Cycles*</td>
<td>10</td>
</tr>
<tr>
<td>Tolerance**</td>
<td>10%</td>
</tr>
<tr>
<td>Amplitude Range</td>
<td>1σ – 1000 segments</td>
</tr>
</tbody>
</table>

* To be investigated in Section 3.3.3.1.2.

** As longer duration records become available in subsequent studies, the tolerance may be reduced as more eligible segments will be available.

3.3.1.2 NExT

Unlike the RDT, the NExT does not rely on several different parameters, since the cross / auto correlation functions are calculated across the entire duration of the signal. Given this, the key parameter for estimations using the NExT is the duration of the signal. By increasing the duration of the signal, the number of points in each window in the cross power spectral density computation increases, leading to a smoother spectrum, which translates to a smooth cross-correlation (auto-correlation in the case of an SDOF) function after completing the inverse Fourier transform. Four signal durations were investigated including 1, 10, 100, and 250-hour signals and the results are shown in Figure 3.17. Note that ten cycles of the auto-correlation function were considered in the analysis.
Therefore, the estimation of dynamic properties using the NExT captures the dynamic properties of the system. The estimation of damping improves with an increase in the duration of the signal, as expected since the number of points used in the FFT computation for the cross power spectral density computation increases.

3.3.1.3 Linear SDOF Summary

While the RDT is capable of investigating amplitude-dependency directly, the NExT does not have this capability and is only suitable for stationary records. Obtaining long duration signals significantly improves the quality of the SID for both the RDT and the NExT.

3.3.2 Linear MDOF

The linear MDOF system and its properties are shown in Figure 3.18 and the dynamic properties are summarized in Table 3.9.
Like the SDOF example, the system was excited by a zero-mean, white noise, forcing function and the response of the system was solved numerically using Newmark’s average acceleration time-stepping algorithm (Appendix I). The response of each degree of freedom is shown in Figure 3.19 for a 1-hour simulation sampled at 10 Hz.
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Figure 3.19  MDOF linear response to white noise excitation for 10-hour record.

PSDs for several durations are shown in Figure 3.20. It can be observed that as the record length increases, the spectrum becomes less jagged.

Figure 3.20  PSDs for MDOF system for 1 and 100-hour record lengths.
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Based on the PSDs of the acceleration responses it can be seen that all three modes are well-present in DOF 3, thus it was selected as the reference channel for all analyses requiring one. Records were generated for 1, 10, 100 and 250-hours to study the effect of signal duration on the SID results.

3.3.2.1 RDT-ERA

For application to the MDOF system, amplitude-dependency was not investigated. Instead, the RDT–ERA was used as pointwise estimator for the dynamic properties at the RMS of the acceleration signal, where the number of segments was maximized. This is a useful framework for analysing low amplitude signals for the purpose of cross-validation with other MDOF SID techniques. A MAC passing criteria of 0.95 was used and a damping filter between 0 and 2%, since the prescribed damping ratios were between 0.5% and 1%. The RDS for each DOF and the stabilization diagram for the 250-hour record are shown in Figure 3.21 and Figure 3.22, respectively.

![RDS from each Channel](image)

**Figure 3.21** RDS from each channel for 250-hour record. DOF 3 used as reference channel. RMS 0.8313 milli-g.
Figure 3.22  Stabilization diagram for RDT-ERA for linear MDOF system; 250-hour record.

The stabilization diagram indicates three stable modes, and a model order of 7 was selected since the singular value had reduced to less than 1% of its maximum value at this order. 120 block columns were used with a 50 second signal at 10 Hz, calling for 380 block rows. This over defines the necessary model but was found to be successful at identifying the properties. The results found from varying the duration of the record are shown in Table 3.10 and the identified mode shapes are shown in Table 3.11.

Table 3.10 RDT-ERA frequency and damping results for linear MDOF system; 250-hour record.

<table>
<thead>
<tr>
<th>Duration (hours)</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
<th>Number of Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>0.178</td>
<td>0.382</td>
<td>0.518</td>
</tr>
<tr>
<td>1</td>
<td>0.178</td>
<td>0.379</td>
<td>0.513</td>
</tr>
<tr>
<td>10</td>
<td>0.178</td>
<td>0.380</td>
<td>0.514</td>
</tr>
<tr>
<td>100</td>
<td>0.178</td>
<td>0.380</td>
<td>0.513</td>
</tr>
<tr>
<td>250</td>
<td>0.178</td>
<td>0.380</td>
<td>0.513</td>
</tr>
</tbody>
</table>
Table 3.11 RDT-ERA mode shapes results for linear MDOF system; 250-hour record.

<table>
<thead>
<tr>
<th>Duration (hours)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>u₁</td>
<td>u₂</td>
<td>u₃</td>
</tr>
<tr>
<td>Exact</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.234</td>
<td>0.681</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.233</td>
<td>0.689</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>0.231</td>
<td>0.688</td>
<td>1</td>
</tr>
<tr>
<td>250</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
</tr>
</tbody>
</table>

As expected, the estimation of the dynamic properties improves as the signal considered becomes longer, which may be directly attributed to the increasing number of segments at the RMS of the signal. The frequency estimates are accurate for all durations, whereas the damping estimates become much more accurate as the signal duration increases.

### 3.3.2.2 NExT-ERA

The cross-correlation functions and the stabilization diagram for the 250-hour record processed using the NExT-ERA are shown in Figure 3.23 and Figure 3.24, respectively. The same parameters used for the RDT-ERA were used for the NExT ERA (block rows, duration, etc.). Note that the cross-correlation functions are proportional to the RDSs generated using the RDT, showing that these two techniques are effectively producing the same results.
The results found from varying the duration of the record are shown in Table 3.12 and the identified mode shapes are shown in Table 3.13.

Table 3.12 NExT-ERA frequency and damping results for linear MDOF system; 250-hour record.

<table>
<thead>
<tr>
<th>Duration (Hours)</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exact</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.178</td>
<td>0.382</td>
</tr>
<tr>
<td>10</td>
<td>0.178</td>
<td>0.380</td>
</tr>
<tr>
<td>100</td>
<td>0.178</td>
<td>0.380</td>
</tr>
<tr>
<td>250</td>
<td>0.178</td>
<td>0.380</td>
</tr>
</tbody>
</table>
Table 3.13 NExT-ERA mode shapes results for linear MDOF system; 250-hour record.

<table>
<thead>
<tr>
<th>Duration (Hours)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_1$</td>
<td>$u_2$</td>
<td>$u_3$</td>
</tr>
<tr>
<td>Exact</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.237</td>
<td>0.683</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
</tr>
<tr>
<td>250</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
</tr>
</tbody>
</table>

Since the duration of the signal results in a smoother PSD and therefore cross-correlation function, the results improve as the signal duration increases. Like the RDT, all estimates, except the first mode damping ratio in this case, underestimate the exact properties (frequency and damping).

### 3.3.2.3 EFDD

For application of the EFDD method, a critical MAC of 0.8 was selected for modal peak auto spectral density function identification. After the IFFT was complete, ten cycles of decay were considered, and the LS fit method was used to extract the damping ratio and frequency. The plot of the SVD of the PSD matrix is shown in Figure 3.25, and the corresponding free decay plots identified are shown in Figure 3.26. Note that the first singular values dominate, indicating clear modal separation in this case.
The peak identification by MAC is effectively applying a bandpass filter to the signal based on the mode shape as opposed to a specified frequency band. By performing the inverse Fourier transform
on these regions, which were originally computed from the SVD of the CPSD matrix which decomposes the spectral matrix into a set of auto spectral density functions, the computed time-domain signal represents the auto-correlation function for the SDOF system defined by the mode shape. The results from the EFDD method are shown in Table 3.14 and Table 3.15.

Table 3.14 EFDD frequency and damping results for linear MDOF system; 250-hour record.

<table>
<thead>
<tr>
<th>Duration (Hours)</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>0.178</td>
<td>0.382</td>
</tr>
<tr>
<td>1</td>
<td>0.178</td>
<td>0.381</td>
</tr>
<tr>
<td>10</td>
<td>0.178</td>
<td>0.380</td>
</tr>
<tr>
<td>100</td>
<td>0.178</td>
<td>0.380</td>
</tr>
<tr>
<td>250</td>
<td>0.178</td>
<td>0.380</td>
</tr>
</tbody>
</table>

Table 3.15 EFDD mode shapes results for linear MDOF system; 250-hour record.

<table>
<thead>
<tr>
<th>Mode Shapes</th>
<th>Duration (Hours)</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>u_1</td>
<td>u_2</td>
<td>u_3</td>
<td>u_1</td>
</tr>
<tr>
<td>Exact</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.235</td>
<td>0.688</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0.231</td>
<td>0.688</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>250</td>
<td>0.232</td>
<td>0.688</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Once again, extending the length of time considered improves the quality of the SID.

### 3.3.2.4 SSI-DATA

SSI-DATA is computationally expensive since it relies on forming the Hankel matrix directly from the output data as opposed to after the processing to a free vibration response. Processing of long records, specifically computing the projection matrix for long records, exceeded the computation capacity of the local computer used for these analyses. Therefore, only two record lengths were analysed; 1 hour and 5 hours. First, the first 5 hours of the 250-hour signal were analysed independently as 1-hour signals. For this analysis, 100 block rows were used and 35900 block columns. Second, the entire 5 hours was analysed as one signal, with 100 block rows and 179000 block columns. The stabilization diagram for the 5-hour record is shown in Figure 3.27.
Figure 3.27 Stabilization diagram for SSI-DATA for linear MDOF system; 5-hour record.

A model order of 7 was selected based on the reduction in the magnitude of the singular values. The identified dynamic properties and mode shapes are shown in Table 3.16 and Table 3.17, respectively.

Table 3.16 SSI-DATA frequency and damping results for linear MDOF system.

<table>
<thead>
<tr>
<th>Duration</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact</td>
<td>0.178 0.382 0.518 0.500 0.750</td>
<td>1</td>
</tr>
<tr>
<td>Hour 1</td>
<td>0.178 0.381 0.513 0.448 0.645</td>
<td>1.036</td>
</tr>
<tr>
<td>Hour 2</td>
<td>0.178 0.380 0.514 0.578 0.734</td>
<td>0.962</td>
</tr>
<tr>
<td>Hour 3</td>
<td>0.178 0.380 0.513 0.459 0.672</td>
<td>0.944</td>
</tr>
<tr>
<td>Hour 4</td>
<td>0.178 0.381 0.514 0.653 0.779</td>
<td>1.140</td>
</tr>
<tr>
<td>Hour 5</td>
<td>0.177 0.380 0.513 0.548 0.688</td>
<td>1.011</td>
</tr>
<tr>
<td>Average</td>
<td>0.178 0.380 0.513 0.537 0.704</td>
<td>1.018</td>
</tr>
<tr>
<td>Hour 1-5</td>
<td>0.178 0.380 0.513 0.528 0.700</td>
<td>1.017</td>
</tr>
</tbody>
</table>
The first five individual hours analysed showed some variability in all estimates. However, the average of these results approached reasonable estimates for all properties. Additionally, the averaged results from the first five individual records agreed well with the results from SSI-DATA applied to the entire first five hours. Therefore, as the record length increases the results improve, like the other SID techniques investigated. Additionally, a useful note for application to field data is that shorter individual records may be analysed and averaged to produce estimates when long-duration records are unavailable.

### 3.3.2.5 Linear MDOF Summary

Each SID technique applied to the linear MDOF system was successful at identifying the dynamic properties including the natural frequencies, damping ratios, and mode shapes. It was found that the accuracy always improved with longer signal durations, and cross-validation between multiple techniques may be useful for reducing variability.

### 3.3.3 Non-Linear SDOF

A non-linear SDOF system characterized by directly specified amplitude-dependent damping and frequency was analysed to investigate the fidelity of applying the previously discussed SID techniques to non-linear data. The method proposed by Pirnia (2009), involving modifications to Newmark’s method and applying Newton-Raphson iterations to prevent errors, was used to analyse the system (Appendix I). However, displacement-dependent damping was
specified rather than acceleration-dependent damping. A baseline damping and frequency of 1% and 0.2 Hz was used, respectively, and the displacement-dependent expressions are shown in Equation 85 and Equation 86.

\[ \zeta(u) = 0.001u(t) + 0.01 \]  \hspace{1cm} \text{Equation 85}

\[ f(u) = -0.002u(t) + 0.2 \]  \hspace{1cm} \text{Equation 86}

Pirnia (2009) updated the dynamic properties based on the peak accelerations attained, such that the properties were constant over the entire cycle. While this was a reasonable setting selected for that study, it was not representative of the expected displacement-dependent nonlinearities due to stick-slip mechanisms. To represent this, the damping and frequency were updated at every time step based on the calculated displacements. Since this means that the damping and frequency will be changing at every step in a cycle, the effective damping was computed by running a free vibration analysis and computing the damping on each cycle using the logarithmic decrement (Equation 48), and the frequency was computed by the time between peaks. For the purpose of comparison and validating this approach, the peak displacement method was used as well. The results from the peak displacement method are shown in Figure 3.28.

![Figure 3.28](image)

As expected, the effective frequency and damping match the specified almost perfectly (note that this was Pirnia’s suggested method). This indicates that the effective damping and frequency may
be estimated from the free vibration response as proposed. The results from the suggested method (updating properties on every time step) are shown in Figure 3.29.

![Damping and Frequency Graphs](image)

Figure 3.29 Non-Linear SDOF amplitude-dependent frequency and damping by displacement updating; specified vs. effective.

Using the proposed displacement updating method, the effective damping is less than the specified, and the effective frequency is higher than the specified. This follows since over the course of one cycle at, for example, 4 millimetres, the system must sway through all amplitudes between 4 millimetres and 0. Therefore, the nominal damping on a 4 millimetres cycle would be the specified, and the effective must therefore be less. The effective damping and frequency parameters for a linear function are shown in Table 3.18.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Intercept</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping</td>
<td>0.999</td>
<td>0.0509</td>
</tr>
<tr>
<td>Frequency</td>
<td>0.2</td>
<td>-0.0016</td>
</tr>
</tbody>
</table>

3.3.3.1 RDT

For the purpose of checking the performance of the positive point triggering method under the new simulated conditions, the four cases investigated by Pirnia (2009) were simulated to understand the effect of each component on the SID results. These four cases include:

- **Case 1**: Constant frequency and damping
- **Case 2**: Amplitude-dependent frequency and constant damping
- **Case 3**: Amplitude-dependent damping and constant frequency
Case 4: Amplitude-dependent frequency and damping

For this analysis 250-hour time-history simulations were performed for each case, each with the same random forcing function.

3.3.3.1.1 First-Run Applying Parameters Selected for Linear System

The RDT-PP was applied with the same parameters as specified in Section 3.3.1.1.6 and the results are shown in Figure 3.31.

**Figure 3.30** Non-Linear SDOF amplitude-dependent frequency using RDT.

**Figure 3.31** Non-Linear SDOF amplitude-dependent damping using RDT.
Generally, the damping estimates appear to diminish in quality after the 3σ amplitude and were therefore truncated after this point for the linear trend line fitting. Therefore, care should be taken when evaluating the results from the RDT-PP above the 3σ limit with amplitude-dependent data. The truncated plots are shown in Figure 3.32 and the linear best-fit line properties are shown in Table 3.19.

![Figure 3.32 Non-linear SDOF truncated numerical results using RDT.](image)

<table>
<thead>
<tr>
<th>Case</th>
<th>Damping</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Effective (Reference)</strong></td>
<td>Intercept</td>
<td>Slope</td>
</tr>
<tr>
<td>Case 1: Constant Frequency and Damping</td>
<td>1.017</td>
<td>-0.0023</td>
</tr>
<tr>
<td>Case 2: Amplitude-Dependent Frequency and Constant Damping</td>
<td>1.47</td>
<td>0.0386</td>
</tr>
<tr>
<td>Case 3: Constant Frequency and Amplitude-Dependent Damping</td>
<td>1.170</td>
<td>0.0148</td>
</tr>
<tr>
<td>Case 4: Amplitude-Dependent Damping and Frequency</td>
<td>1.575</td>
<td>0.0402</td>
</tr>
</tbody>
</table>

The results indicate:

- **Case 1: Constant Frequency and Damping**: The RDT-PP is effective for predicting the constant properties. The frequency and damping were well predicted over the entire range.
Case 2: Amplitude-Dependent Frequency and Constant Damping: Amplitude-dependent frequency manifests itself as both an offset as well as an induced amplitude-dependency in the damping estimation. This may be attributed to the low-level stiffness non-linearities introduced by the amplitude-dependent frequency, resulting in a slight widening of the force-deformation response (hysteresis) with respect to amplitude. Since the amount of energy dissipated is proportional to the area in the loop, and the envelope of the loop is constantly changing by a prescribed amount as a function of the amplitude, the amplitude-dependency follows. The slope of the damping was close to the effective, however, this may be viewed as a coincidence due to the parameters selected for the simulation itself. While there should be some slope, this slope has no relationship with the specified amplitude-dependent damping, which was not simulated in this case. The slope of the frequency amplitude-dependency is lower than specified and the intercept is lower. Although a linear variation with amplitude was captured, the correct variation was not. This was further investigated by studying the number of cycles used in the RDS damping estimation. It follows that since the system is non-linear, the RDS will also be non-linear, meaning that it may be important to consider fewer cycles as suggested by Tamura and Suganuma (1996). This was investigated in Section 3.3.3.1.2.

Case 3: Constant Frequency and Amplitude-Dependent Damping: The amplitude-dependent damping was captured by the RDT-PP; however, the slope was lower than the effective and the intercept was higher. Again, this may be due to the number of cycles being considered resulting in an averaged value over the amplitude range considered. The frequency estimation is close to the specified value of 0.2 Hz over the entire range considered, indicating the amplitude-dependent damping does not have as significant an effect on the frequency as the amplitude-dependent frequency has on the damping, as expected.

Case 4: Amplitude-Dependent Damping and Frequency: The amplitude-dependent damping was captured; however, the slope was still less than the effective and the intercept was higher. This follows since the addition of the specified amplitude-dependent damping should increase the slope of the estimated damping due to the combined effect of the amplitude-dependent damping effect from the amplitude-dependent frequency and the specified amplitude-dependent damping. For a system with stick-slip elements, the
simulated amplitude-dependent damping ratio would not be present, and all the amplitude-dependent damping may be attributed to the low-level displacement-dependent non-linearities introduced by the slipping friction mechanisms, as was simulated in Case 2.

The frequency estimates are like that of Case 2, again indicating that amplitude-dependent damping does not have a significant effect on the amplitude-dependent frequency estimates.

To investigate the slope softening observed, the effect of the number of cycles was investigated.

### 3.3.3.1.2 Effect of Number of Cycles on RDT Application to Non-Linear SDOF

The number of cycles was noted as an important consideration for the application of the RDT to amplitude-dependent data (Tamura & Suganuma, 1996). To further investigate this, the number of cycles considered in the RDT-PP was varied to study the effect of cycle numbers on the estimation of amplitude-dependent damping and frequency. For this, only Case 2 and Case 3 were investigated, and within each case only the amplitude-dependent component was checked. The results are plotted in Figure 3.33 and the linear fit parameters are shown in Table 3.20.

![Figure 3.33 Effect of number of cycles on frequency and damping estimations for non-linear SDOF.](image)}
Table 3.20 SDOF effective amplitude-dependent damping linear parameters for various cycle numbers considered.

<table>
<thead>
<tr>
<th>Case</th>
<th>Number of Cycles</th>
<th>Damping</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Intercept</td>
<td>Slope</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 2: Amplitude-Dependent</td>
<td>10</td>
<td>-</td>
<td>0.999</td>
</tr>
<tr>
<td>Frequency and Constant Damping</td>
<td>6</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Case 3: Constant Frequency and Amplitude-Dependent Damping</td>
<td>10</td>
<td>1.17</td>
<td>0.0148</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.14</td>
<td>0.0201</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.04</td>
<td>0.0386</td>
</tr>
</tbody>
</table>

Therefore, a reduction in the number of cycles tends to generally improve the estimations of both the amplitude-dependent damping and frequencies. Including more cycle numbers softens the slope of both, indicating that the RDT is establishing an average frequency and damping ratio over the amplitude range considered in the RDS. Referencing Table 3.7, the typical recommended number of cycles for evaluating amplitude-dependent properties was 3-5. With consideration to the literature and the results of this numerical study, 5 cycles were considered in the practical application of the RDT moving forward.

3.3.3.1.3 Updated Selected RDT Parameters

The non-linear analysis provided further clarification on the number of cycles to include in the analysis. For the practical application of the RDT in this study the parameters shown in Table 3.21 were used.

Table 3.21 Updated selected RDT parameters for field data analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triggering</td>
<td>Positive Point</td>
</tr>
<tr>
<td>Number of Segments</td>
<td>Minimum 1000 segments</td>
</tr>
<tr>
<td>Number of Cycles</td>
<td>5</td>
</tr>
<tr>
<td>Tolerance</td>
<td>10%</td>
</tr>
<tr>
<td>Amplitude Range</td>
<td>$1\sigma - 1000$ segments</td>
</tr>
</tbody>
</table>

While the non-linear SDOF investigation indicated some diminishing quality in RDT estimates above $3\sigma$ of the signal, most research applies segment limits on the amplitude. This may be attributed to the fact that in real signals the responses following the highest amplitude tend to be
like the free vibrations that the technique is attempting to construct. Therefore, less segments are
required at the higher amplitudes to ensure adequate results using the RDT, a conclusion supported
by (Bentz, 2012). Therefore, the segment limit of 1000 was used in the practical application of the
RDT in this study.

\[ \text{3.3.3.2} \quad \text{NExT} \]

The NExT was applied to Case 2 and Case 3 from the previous analysis. The results are shown in Table 3.22.

Table 3.22 Non-linear SDOF NExT estimates and effective amplitudes.

<table>
<thead>
<tr>
<th>Case</th>
<th>Signal RMS (mm)</th>
<th>Damping (%)</th>
<th>Frequency (Hz)</th>
<th>Effective Amplitude (mm)</th>
<th>Amplitude Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2: Amplitude-Dependent Frequency and Constant Damping</td>
<td>3.33</td>
<td>1.93</td>
<td>0.190</td>
<td>6.44</td>
<td>1.93</td>
</tr>
<tr>
<td>Case 3: Constant Frequency and Amplitude-Dependent Damping</td>
<td>3.02</td>
<td>1.25</td>
<td>0.200</td>
<td>4.98</td>
<td>1.65</td>
</tr>
</tbody>
</table>

Back-calculating the expected amplitude for each analysis based on the specified effective equations (Table 3.18) yields effective amplitude estimates of 6.44 mm for Case 2 and 4.98 mm for Case 3. This yields an amplitude ratio, the effective divided by the RMS, of 1.93 and 1.65 for each case. While it may be intuitive to suggest that the reference amplitude for the NExT should be the RMS of the signal, this does not appear to be the case. It follows that the non-linearity introduced leads to errors in the computation of the PSD (as described in Section 3.2.1.1) leading to inflated damping ratios. Therefore, the NExT is not appropriate for evaluating non-linear response data and should be constrained to the low amplitude response regime only.
3.3.3.3 Non-Linear SDOF Summary

To summarize the non-linear SDOF study, it was found the RDT adequately captures both the amplitude-dependent damping and frequency of a non-linear SDOF system, while the NExT does not. Care must be taken when applying the RDT to ensure that the amplitude-dependency is not obscured by considering too many cycles or too large of a tolerance. The application of the NExT should be constrained to the low-amplitude response regime only.

3.3.4 Non-Linear MDOF

Expanding upon the test performed on the non-linear SDOF system, a 3-DOF non-linear MDOF system was simulated. To simplify the analysis, only amplitude-dependent damping was simulated, given the observed complexities with the amplitude-dependent frequency for the SDOF system. Amplitude-dependent damping allows for the application of the RDT-ERA, NExT-ERA, EFDD and SSI-DATA to be evaluated for a non-linear system.

The same 3-DOF system analysed in Section 3.3.2 was analysed here, with the addition of amplitude-dependent damping expressions for each mode. To simplify, the damping was updated in each mode based on the third DOF displacement \( u_3(t) \) (Equation 87, Equation 88, and Equation 89).

\[
\begin{align*}
\zeta_1(u_3) &= 0.005 + |u_3(t)| \\
\zeta_2(u_3) &= 0.0075 + |u_3(t)| \\
\zeta_3(u_3) &= 0.01 + |u_3(t)|
\end{align*}
\]

Equation 87

Equation 88

Equation 89

Where:

\( \zeta_i \): Modal damping ratio

\( u_3(t) \): DOF 3 displacement in millimetres

Since only the damping ratio was updated, the mode shapes and frequencies remain constant in time, therefore the modal superposition method in conjunction with Newmark’s time-stepping procedure was used. Note that geometric displacements were computed on each time step to allow for the damping ratio updating to reference the desired amplitude. Details of the implementation algorithm are provided in Appendix I. The effective damping in each mode was determined by applying initial displacements such that the system was deformed in the exact mode...
shape of interest, releasing the system, and computing the damping using the log-decrement technique as described in Section 3.3.3. An example of the free vibration response for the first mode is shown in Figure 3.34.

![Non-Linear MDOF System Response](image)

Figure 3.34 First mode free-vibration response for effective damping computation.

The linear trend line parameters for the effective amplitude-dependent damping ratios for each mode are shown in Table 3.23.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Intercept</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.500</td>
<td>0.0413</td>
</tr>
<tr>
<td>2</td>
<td>0.750</td>
<td>0.0417</td>
</tr>
<tr>
<td>3</td>
<td>1.00</td>
<td>0.0409</td>
</tr>
</tbody>
</table>

Additionally, it was found that the specified frequencies remained constant over all cycles, as expected since variable damping has no direct effective on mass or stiffness.

Having established the expected dynamic properties, a 250-hour response to white noise was generated. Since it was already validated that the SDOF SID using the RDT was effective, there was no need to verify that bandpass filtering the MDOF signal would be successful. Therefore, the other 4 MDOF SID techniques discussed were checked against the non-linear data including RDT-ERA, NExT-ERA, EFDD, and SSI-DATA.

### 3.3.4.1 RDT-ERA

The RDT-ERA was applied at several different triggering threshold from 1σ-3σ, using DOF 3 as the reference channel. This ensures that the triggering amplitude references the amplitude at which the dynamic properties were updated in the non-linear simulation. A 250-hour long record was considered, an amplitude tolerance of 10% was used, and 30 second RDSs were
computed. A typical model order of 7 was selected, which ensures a minimum of three modes were present. The results from the analysis are shown in Figure 3.35 and Figure 3.36.

**Non-Linear MDOF Damping RDT Results**

![Graph](image1)

**Figure 3.35** Non-Linear MDOF damping RDT-ERA Results. Dashed lines are linear trendlines.

**Non-Linear MDOF Frequency RDT Results**

![Graph](image2)

**Figure 3.36** Non-Linear MDOF frequency RDT-ERA Results.

The results indicate that the RDT-ERA can resolve the amplitude-dependent damping by varying the triggering threshold. The slope was under-estimated in the first and second mode, but over-estimated in the third mode while the intercept was over-estimated in the first and second mode and underestimated in the third mode. Constant frequency was estimated at all amplitudes, as prescribed. Approximate estimates of the damping ratio were still obtained, and accurate predictions of the frequencies were obtained, indicating that the RDT-ERA is applicable for extracting amplitude-dependent data from a non-linear MDOF system.
3.3.4.2 NExT-ERA, EFDD, SSI-DATA

The results from the remaining SID techniques are discussed collectively, since none can reference a particular amplitude. The results from applying each technique to the 250-hour non-linear MDOF random response are shown in Table 3.24 and Table 3.25.

Table 3.24 NExT-ERA, EFDD, SSI-DATA: Frequency and damping estimates for non-linear MDOF system.

<table>
<thead>
<tr>
<th>Method</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
<td>M2</td>
</tr>
<tr>
<td>Exact</td>
<td>0.178</td>
<td>0.382</td>
</tr>
<tr>
<td>NExT-ERA</td>
<td>0.178</td>
<td>0.380</td>
</tr>
<tr>
<td>EFDD</td>
<td>0.178</td>
<td>0.380</td>
</tr>
<tr>
<td>SSI-DATA*</td>
<td>0.177</td>
<td>0.380</td>
</tr>
</tbody>
</table>

* 5 one-hour segments analysed and averaged.

Table 3.25 NExT-ERA, EFDD, SSI-DATA: Mode shape estimates for non-linear MDOF system.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>u1</td>
</tr>
<tr>
<td>Exact</td>
<td>0.232</td>
</tr>
<tr>
<td>NExT-ERA</td>
<td>0.232</td>
</tr>
<tr>
<td>EFDD</td>
<td>0.233</td>
</tr>
<tr>
<td>SSI-DATA*</td>
<td>0.232</td>
</tr>
</tbody>
</table>

* 5 one-hour segments analysed and averaged.

All three techniques provide reasonable estimates of the frequencies and mode shapes, which follows since they are constant in the analysis. However, here it becomes evident why the RDT gained significant popularity for tall buildings; the estimates of damping have no reference to amplitude. All three SID techniques provide similar estimates of the frequencies, damping ratios, and mode shapes. It could be hypothesized that since these techniques rely on computing correlation functions over the entire signal, it may be appropriate to assume that the property estimates may be referenced to the RMS of the signal. However, working backwards from these estimates, the NExT-ERA, EFDD, and SSI-DATA are providing estimates at approximately 6.54 mm, or about 1.77σ, which is not a statistically significant amplitude value.

This indicates that these techniques are not applicable for identifying the natural frequencies, damping ratios, and mode shapes, of amplitude-dependent systems. However, these
techniques have practical applicability when applied to ambient vibration levels where little-to-no amplitude-dependency is expected. In the literature this is the primary application of these techniques, where low-level vibrations from bridges or buildings are obtained using a large sensor arrays to inform the baseline dynamic properties as well as to estimate the mode shapes, when needed. Cross-validating these estimates with the RDT-ERA results can inform the applicable amplitude range in which these methods may produce reasonable results.

### 3.3.4.3 Non-Linear MDOF Summary

To summarize the non-linear MDOF study, it was found that the RDT-ERA was effective for evaluating the amplitude-dependent dynamic properties of a non-linear MDOF system. NExT-ERA, EFDD, and SSI-DATA were found to be ill-suited for application to amplitude-dependent systems. However, these methods are suitable for low-amplitude ambient level responses where little-to-no amplitude dependency is expected.

### 3.4 Non-Stationary Excitation Numerical Simulation

The last numerical simulation investigated was that of a non-stationary input signal to an SDOF system. The key implications of non-stationary excitation are the violation of two fundamental assumptions in output-only SID; the input is stationary, and the system is linear. Since the amplitude will be variable over the duration of the signal, the properties will be varying by different amounts throughout the signal as well, thus violating both the stationary and linear assumptions. While this complicates the SID, this is a more realistic structural response under a large amplitude windstorm. An example of the structural response from a single structure subjected to 14 typhoons collated together to produce 360 hours of data from a recent study (Li & Li, 2018) is shown in Figure 3.37.
Chapter 3: Key Aspects of System Identification for Tall Buildings

Figure 3.37  Structural response for 14 tropical cyclones for supertall building (Li & Li, 2018).

To simulate this, both the linear and non-linear SDOF system were excited by a Gaussian windowed white noise input, which produces the rise and subsequent fall in response amplitude shown in the typhoon responses in Figure 3.37. An example of the simulated force time-history is shown in Figure 3.38.

Note that 250 hours were simulated here, which is exceptionally long for a storm of this nature. However, the cumulative effect of collating the signals as in Figure 3.37 is that a similar signal to that shown in Figure 3.38 is effectively obtained. The responses for the linear and non-linear SDOF are shown in Figure 3.39. Note that only amplitude-dependent damping was simulated.
Figure 3.39 shows that the non-linear response generally does not obtain the same magnitude of amplitude, since the increased damping at higher amplitudes tends to attenuate the motion. For the sake of establishing some repeatability, three realizations of the random process were simulated for both the linear and non-linear systems.

### 3.4.1 Linear SDOF

Both the RDT and NExT were applied to the linear non-stationary response data.

#### 3.4.1.1 RDT

The results from the application of the RDT to the non-stationary linear system are shown in Figure 3.40. Note that based on the results obtained from the non-linear SDOF analysis, 5 cycles were considered in the RDT.
The key limits are shown including the $\sigma$, $3\sigma$, and 1000 segment amplitudes. The RDT was able to accurately resolve both the damping and frequency and showed little variation over the amplitude range considered. Therefore, the RDT is applicable to non-stationary linear data.

### 3.4.1.2 NExT

The results from the application of the NExT to the non-stationary linear system are shown in Table 3.26.

Table 3.26 Non-stationary linear SDOF NExT results.

<table>
<thead>
<tr>
<th>Realization</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>1.012</td>
</tr>
<tr>
<td>2</td>
<td>0.2</td>
<td>0.999</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
<td>1.001</td>
</tr>
</tbody>
</table>

The results from three realizations indicate that the NExT can resolve the damping and frequency for a non-stationary excitation of the nature simulated (i.e. one with a gradual change in amplitude).

### 3.4.2 Non-Linear SDOF

Both the RDT and NExT were applied to the non-linear non-stationary response data.
3.4.2.1 RDT

The RDT-PP was applied to the non-linear non-stationary SDOF system response. For this, only non-linear Case 2 and Case 3 were investigated, and within each case only the amplitude-dependent component was checked. The results are shown in Figure 3.41 and the linear trend line parameters are shown in Table 3.27.

![Frequency vs Trigger Displacement](image1)

![Damping Ratio vs Trigger Displacement](image2)

**Figure 3.41 Non-stationary non-linear SDOF RDT results.**

**Table 3.27 SDOF effective amplitude-dependent frequency and damping linear parameters for non-stationary excitation.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 2: Amplitude-Dependent Frequency and Constant Damping Frequency Line Parameters</th>
<th>Case 3: Constant Frequency and Amplitude-Dependent Damping Damping Line Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intercept</td>
<td>Slope</td>
</tr>
<tr>
<td><strong>Effective (Reference)</strong></td>
<td>0.2</td>
<td>-0.0016</td>
</tr>
<tr>
<td>Realization 1</td>
<td>0.196</td>
<td>-0.0012</td>
</tr>
<tr>
<td>Realization 2</td>
<td>0.196</td>
<td>-0.0012</td>
</tr>
<tr>
<td>Realization 3</td>
<td>0.196</td>
<td>-0.0012</td>
</tr>
<tr>
<td>Segment Weighted Average</td>
<td>0.196</td>
<td>-0.0012</td>
</tr>
</tbody>
</table>

The frequency estimates were consistent between each realization, slightly underestimating the intercept and the slope of the effective amplitude-dependent frequency, which may be attributed to some averaging over the 5 cycles considered in the RDS. While the frequency estimates were meaningful on each realization, there was significant variability in the damping estimation from
each realization. While all three realizations displayed amplitude-dependent trends, none successfully captured the effective amplitude-dependent trend precisely. However, it was observed that in the three realizations tested that the average value of the three realizations at each amplitude appeared to better capture the amplitude-dependent trend. A segment weighted average was computed at each triggering threshold and is shown in both frequency and damping plots, which most accurately captures the amplitude-dependent damping and frequency. While the application of a segment weighted average may be replaced by collating the signals, this approach may provide cross-validation and may be useful for comparing across many records, or for buildings under construction.

3.4.2.2 NExT

The results from the application of the NExT to the non-stationary non-linear system are shown in Table 3.28. Note that the inflated damping estimations in Case 2 are due to the hysteretic damping from the low-level non-linearities introduced by softening the stiffness with amplitude.

Table 3.28 Non-stationary non-linear SDOF NExT results.

<table>
<thead>
<tr>
<th>Realization</th>
<th>Case 2: Amplitude-Dependent Frequency and Constant Damping</th>
<th>Case 3: Constant Frequency and Amplitude-Dependent Damping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMS (mm)</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>2.00</td>
<td>0.192</td>
</tr>
<tr>
<td>2</td>
<td>2.11</td>
<td>0.191</td>
</tr>
<tr>
<td>3</td>
<td>2.13</td>
<td>0.191</td>
</tr>
<tr>
<td>Average</td>
<td>2.08</td>
<td>0.191</td>
</tr>
<tr>
<td>Effective Amplitude (mm)</td>
<td>-</td>
<td>5.48</td>
</tr>
<tr>
<td>Amplitude Ratio</td>
<td>-</td>
<td>2.63</td>
</tr>
</tbody>
</table>

Each realization produced similar RMS and estimates of frequency and damping. The expected effective amplitudes and amplitude ratios were computed based on the average result of the three realizations. The effective amplitude bears no significance, indicating that the NExT is not appropriate for evaluating non-stationary non-linear responses, with inconsistencies exceeding that of the stationary non-linear case. Additionally, the amplitude ratio was larger than the stationary...
non-linear case, indicating further errors due to the PSD estimates due to the variation of the dynamic properties with amplitudes leading to inflated damping ratios.

3.5 Key Aspects of System Identification for Tall Buildings Summary

In summary, several state-of-the-art SID techniques were introduced and discussed. Numerical simulations were performed to evaluate the fidelity of these techniques under various conditions including stationary and non-stationary inputs, and linear and non-linear systems. The key takeaways from this chapter include:

- All SID techniques discussed are effective for evaluating the response of SDOF and MDOF linear systems excited by a stationary random process.
- All SID techniques benefit from longer duration records, which aids in reducing the noise and randomness in the response from the random excitation.
- The RDT is effective for evaluating the amplitude-dependent frequency and damping for non-linear systems, both SDOF and MDOF, excited by both stationary and non-stationary excitations. This makes the RDT the prime SID technique for application to tall buildings.
- SID techniques that do not explicitly reference amplitude, including NExT, EFDD, SSI-DATA, are not suitable for application to non-linear systems. However, these techniques may be applicable to the low-amplitude response regime for tall buildings, which is useful for establishing baseline dynamic characteristics.

Moving forward, these techniques were applied to the data obtained from the full-scale test structure in their respective applicable amplitude ranges.

Note that no numerical studies were performed for the wavelet-based non-stationary SID algorithms. Preliminary analysis of the data obtained in the experimental program yielded no adequate impulse-response-function-like segments, and further exploration of non-stationary SID was outside the scope of this work. Interested readers are referred to (Bentz, 2012) and (Guo Y., 2015) for more information on this subject.
4.0 Experimental Program

As highlighted in the previous sections, monitoring the vibration response of tall buildings is critical to improving engineers’ understanding of the dynamic behaviour of structures. This chapter reviews the experimental program for this work, introducing the test building and describing the testing procedure.

4.1 Building Introduction

YC Condos is a 62-storey, 198-metre-tall building located in downtown Toronto, Canada. The lateral force resisting system consists of a reinforced concrete coupled shear wall system with a combination of steel and reinforced concrete coupling beams. Photos of the completed building are shown in Figure 4.1.

![Figure 4.1 (a) Photo looking south along Yonge Street; (b) Photo looking north along Yonge Street](image)

Due to the building’s slenderness (10.5:1), initial evaluation of the structure noted acceleration and drift problems. Two supplemental damping systems were evaluated, including a TSD and Viscoelastic Coupling Damper (VCD) system. It was found that both systems offered the desired supplemental damping, however, the VCD system was selected for several key reasons:

- The developer was able to reclaim valuable real-estate at the top of the structure that otherwise would have been occupied by the TSD tank.
Chapter 4: Experimental Program

- No maintenance or monitoring required for the system.
- VCD system effective despite concrete property assumptions at the design stage.
- VCD system offered improved seismic performance.

The objective was to add supplemental damping in the slender direction (first mode) of 0.9% in order to meet the ISO 1-year occupant motion perception criteria (International Organization for Standardization, 2007) and wind tunnel 10 year return period criteria to supplement the assumed 1.5% inherent damping for a total of 2.4% damping for SLS (Montgomery, MacLean, Christopoulos, & Kokai, 2014)

The deformed shape and VCD configuration/deformation are shown in Figure 4.2a. The VCDs were placed from levels 20 – 40, along the strongwall lines (Figure 4.2b). A conservative design approach was adopted, utilising the VCDs’ stiffness and damping for SLS loading (building motion and drift) only. A temperature bounded analysis was performed (between 20 °C and 30 °C) which found that the expected supplemental damping at SLS in the first and third mode was 0.95% to 1.29%, and 1.16% to 1.34%, respectively, thereby meeting the supplemental damping requirement (Montgomery & MacLean, 2014) utilizing the engineer of record’s ETABS model. Although the building only required damping in the first mode, the VCDs provided supplemental damping in the torsional directions as well, related to the locations of the dampers and the kinematics of the system under lateral loads.

![Figure 4.2](image)

**Figure 4.2** (a) VCD layout in elevation. Adapted from (Christopoulos, Montgomery, & Aiken, 2017); (b) VCD layout in plan. Adapted from (Montgomery & MacLean, 2014).
4.2 Viscoelastic Coupling Dampers (VCDs)

A brief background on the system for the application in the test building is provided in the subsequent sections.

4.2.1 VCD Background

While VE dampers have been used in several different configurations historically (Section 2.1.6.2.2), these layouts are less efficient for the newer generation of tall buildings constructed from reinforced concrete and utilising coupled shear walls with or without outriggers as the lateral force resisting system (Pant, Montgomery, & Christopoulos, 2019). Since these structures’ deformation is governed by cantilever type deformation, there is limited interstorey (shear) drift over the height of the structure, thus reducing the efficiency of dampers that are activated by shear type of deformation, such as dampers in wall or brace configurations. Conversely, the coupling beams and outriggers in current tall buildings undergo more significant relative deformations. To take advantage of these deformations, the Viscoelastic Coupling Damper (VCD) was developed at the University of Toronto to add supplemental damping to these tall RC structures [(Montgomery, 2011), (Christopoulos & Montgomery, 2013), (Christopoulos, Montgomery, & Aiken, 2017), (Pant, Montgomery, & Christopoulos, 2019)]. The VCDs replace coupling beams or are added between outriggers and columns in outrigger configurations (Figure 4.3).

Figure 4.3 (a) VCD configuration in coupling beams; (b) VCD configuration in coupling beams and outriggers; (c) VCD unit; (d) VCD configuration in outriggers. Adapted from (Christopoulos, Montgomery, & Aiken, 2017).
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The VCD unit itself consists of multiple layers of high damping viscoelastic material sandwiched between and bonded to steel plates, which are anchored to built-up steel sections which in turn are anchored to the structure (Figure 4.4).

![VCD unit diagram](image)

**Figure 4.4 VCD unit (Montgomery, 2011)**

The VE material used is produced by 3M and is referred to as ISD-111H. The properties of the VE material are strain, frequency, and temperature-dependent, but are stable under design-level amplitudes. The amplitude and frequency-dependent properties are captured well through established VE models; however, the effect of temperature is more difficult to quantify. Material tests are performed at various temperatures to characterize the stiffness and damping coefficient properties for the VCD. For design applications, bounded analyses are appropriate for establishing building performance under a range of temperatures. The modelling of VCDs is further discussed in Chapter 6.

When the structure undergoes lateral deformation due to wind, the VCD undergoes shear deformation as the walls or adjacent elements rotate (Figure 4.5).
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The VCD transfers the forces between the two walls, primarily in shear, resulting in a velocity-dependent viscous and displacement-dependent elastic restoring force. While the focus of this work is on the response of structures to wind, the VCD provides supplemental damping under all levels of excitation from occupant motion perception level wind to Maximum Considered Earthquake (MCE) level seismic activity. For seismic applications, the built-up steel component of the VCD may be detailed as a capacity designed yielding element. The range of hysteretic responses of the system is shown in Figure 4.6.

Figure 4.5 Exaggerated deformed shape. Adapted from (Montgomery, 2011).

Figure 4.6 VCD design concept. Adapted from (Montgomery, 2011).
4.2.2 VCD Details in Test Building

On the levels that the VCDs were installed (20-40), the slab thickness above the VCD was reduced from 200 millimetres to 150 millimetres. Additionally, a 1.5” notch was introduced in the slab above the VE damper panels at the locations where the VE panel starts on both sides. This detail creates a mechanism above the dampers such that the stiffness contribution of the adjacent elements is reduced to maximize the proportional force being driven into the damper.

During the construction of the building, temporary steel channels were used in place of the VCDs as placeholders until all stories containing VCD units were built. This allowed a dedicated crew to install the VCDs on a full-time basis. Photos of the VCD installation process are shown in Figure 4.7 and the VCD installation progress is summarized in Appendix II.
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4.3 Instrumentation

Three types of sensors were used in this study including accelerometers, Linear Variable Differential Transformers (LVDTs), and GPS receivers.
4.3.1 Accelerometers

Kinematics Episensor ES-U2 force-balance accelerometers were used to capture the vibration response of the structure. These sensors have a variety of user-selectable dynamic ranges from +/- 0.25 g to +/- 4 g, and for this study a full-scale range of +/- 1/2 g was selected with a differential +/-5 V output. The sensors have a frequency bandwidth of DC to 200 Hz and possess very low self noise (1.43x10^-5 g RMS) making them highly appropriate for the expected levels of acceleration in tall buildings. The accelerometers were mounted on custom made plates that ensured that the accelerometers axes were orthogonal. The accelerometers and the mounting plate are shown in Figure 4.8.

![Figure 4.8](a) Kinematics Episensor ES-U2 accelerometer; (b) Sensors setup on site using orthogonal mounting plate.

4.3.2 Linear Variable Differential Transformers

LVDTs were used to measure the local deformation in the VCDs. The LVDTs used were Measurement Specialities +/- 0.25-inch sensors. Magnetic clips were used to attach the LVDTs to the reference frame in the test. The LVDTs and magnetic clips are shown in Figure 4.9.

![Figure 4.9](a) LVDTs; (b) Magnetic clips.
The kinematic relationship showing an exaggerated deformation of the VCD assembly is shown in Figure 4.10, showing a single LVDT attached to the steel embed. The steel embeds provide a rigid reference frame upon which the LVDTs were attached. Most of the deformation in the system was expected to occur in the VE material, although Montgomery (2011) noted that there may be some losses in the built-up steel sections. In this study, since the LVDTs were placed at the endpoint of the VE material, the measured signal was directly the shear deformation in the VE component (Equation 90). However, there still may be some flexibility of the cantilevered steel plates. This means that the VE deformation averaged over the entire VE damper panel would be slightly larger than those measured in this study.

\[ u_{VE} = u_{LVDT} \]  

*Equation 90*

Where:

- \( u_{VE} \): Shear deformation in the VE material
- \( u_{LVDT} \): Measured displacement in the LVDT

Photos showing the LVDTs mounted on the steel embed measuring the shear deformation in the VCD are shown in Figure 4.11. After mounting and levelling the LVDTs, the reading was zeroed just prior to initiating the data acquisition. Since the system was already in motion, there was potential for the zeroed displacement to be offset from the true zero-displacement location of the VCD. The average of all four LVDTs used was computed to represent the average shear deformation in the VCD.
4.3.3 GPS Receivers and Time Synchronization

A key aspect of the sensor selection and data acquisition system design was to develop two independent, versatile systems such that two different structures could be monitored simultaneously, or multiple stories in a single structure could be monitored simultaneously. Monitoring multiple stories simultaneously required accurate time synchronization to ensure that the phase relationship between responses at different stories was maintained. In the interest of versatility, the objective was to develop a wireless synchronization strategy. To achieve this, two options were explored; GPS synchronization and computer clock synchronization.

Utilizing Garmin GPS 18x OEM receivers allowed for GPS time to be sampled along with the response channels on each story. Since both GPS receivers reference the same clock, the time stamps from the GPS channel may be used to synchronize the data. Unfortunately, the GPS receivers available for the study provided a 1 Hz sampling rate. This means that the best time
synchronization possible was one second. However, since the structure had a relatively long period, GPS synchronization could be applied as a first run of synchronization to two independently initiated data acquisition units. After aligning the signals to within 1 second of each other, it was assumed that the first mode shape had constant phase over the height of the structure. This means that peaks in the first mode response would be entirely synchronized. The offset within the one second synchronization windows was observed, and the signals were shifted accordingly to line up these peaks, thus achieving synchronization between multiple floors. The Garmin 18x OEM receiver is shown in Figure 4.12.

![Garmin GPS 18x receiver](image)

Figure 4.12  Garmin GPS 18x receiver.

While the GPS technology used in this thesis was insufficient to establish higher resolution on the time synchronization, other units are available with a higher sampling rate. For example, the Garmin 18x 5 Hz model provides a faster sampling rate, allowing for synchronization up to 0.2 seconds.

Alternatively, the clocks on the laptops used for the data acquisition may be used to for synchronization. Since the clocks update at 1 Hz, manual synchronization of the onboard clock may be performed prior to initiating data acquisition. Upon starting the data acquisition, the initial time stamp may be used to synchronize the data. Over time these clocks may drift at different rates, thus this application may only be suitable for short duration tests.

The GPS synchronization method was used for the multi-floor tests performed on May 4, 2018. More information on this testing procedure is provided in Section 4.4.3.

4.3.4 Data Acquisition

Two different data acquisition systems were used to digitize the data using the CATMAN software, depending on the stage of monitoring. Early in the testing program, the HBM MGC Split

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(Figure 4.13a) data acquisition system was used and later, HBM QuantumX (Figure 4.13b) 24-bit modules were used for the testing. Either 4 or 8 channel QuantumX modules were used, depending on the channel requirements for each test.

(a) Test setup with HBM MGC Split DAQ       (b) Test setup with HBM QuantumX

All tests were sampled at 200 Hz and the built-in Bessel low pass filter at 20 Hz was applied. Full time-histories were recorded for all tests and saved locally on the on-site laptop hard drive. Typically, files were saved each hour to avoid potential data loss due to power outages.

4.4 Testing Procedure

This section describes the three tiers of testing types applied in the study; baseline vibration monitoring, local element monitoring, and multi-floor monitoring with local element monitoring. Several testing photos for each test type are shown in Appendix III.

4.4.1 Baseline Vibration Monitoring

This test type comprised solely of acceleration measurements. Early in the testing program these were short duration tests where the primary objective was characterizing low amplitude frequency, damping and mode shapes. Later, this evolved into longer monitoring sessions, sometimes overnight and over several days. The longer monitoring sessions typically targeted higher amplitude wind events in the interest of obtaining amplitude-dependent data.

As the test program evolved, many lessons were learned on practical application of field vibration monitoring. Early in the testing program, a roving test procedure was applied. This involved placing a single accelerometer at a location where all the modes of interest would be excited, in this case the south-west corner of the structure, facing east, and roving the other
accelerometers to different locations in the structure. This allowed for more detailed mode shapes to be determined since all SID could be normalized to the reference channel. For each configuration, tests of 30 minutes to 1 hour were performed, in line with the tests performed by Zhang et.al (2016). For these tests, three accelerometers were used and typically three configurations were tested (Figure 4.14).

![Figure 4.14 Baseline vibration monitoring accelerometer layout for roving tests.](image)

Over time, it became clear that the benefit gained from more detailed mode shapes from the roving configuration was unnecessary and was outweighed by the benefit of having longer duration signals in a single test configuration. Since only a single floor was monitored and only the first three modes were of interest, placing the accelerometers at key locations offset from the centre of the structure, such as configuration 1 and 3 (Figure 4.14), always provided enough information to determine approximate planar mode shapes. As shown in the numerical study section (Section 3.3), longer data recordings were required to establish higher confidence in the dynamic property estimations, particularly the damping estimation. Later in the testing procedure
Chapter 4: Experimental Program

this testing method was adjusted to monitor in a single setup for several hours. A summary of baseline monitoring tests is shown Table 4.1

Table 4.1 Baseline vibration monitoring test summary.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Levels Complete</th>
<th>Structure Height (m)</th>
<th>Floor Monitored</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 28, 2017</td>
<td>10:00am – 3:00pm</td>
<td>36</td>
<td>111.41</td>
<td>30</td>
</tr>
<tr>
<td>October 4, 2017</td>
<td>9:15am – 12:00pm</td>
<td>37</td>
<td>114.36</td>
<td>30</td>
</tr>
<tr>
<td>October 19, 2017</td>
<td>9:15am – 12:30pm</td>
<td>39</td>
<td>120.26</td>
<td>31</td>
</tr>
<tr>
<td>November 8, 2017</td>
<td>9:30am – 6:00pm</td>
<td>41</td>
<td>126.16</td>
<td>34/20*</td>
</tr>
<tr>
<td>November 21, 2018</td>
<td>9:15am – 2:30pm</td>
<td>43</td>
<td>132.06</td>
<td>38/41</td>
</tr>
<tr>
<td>December 6, 2017</td>
<td>9:15am – 12:30pm</td>
<td>46</td>
<td>140.91</td>
<td>41</td>
</tr>
<tr>
<td>December 19, 2017</td>
<td>9:15am – 12:30pm</td>
<td>47</td>
<td>143.86</td>
<td>43</td>
</tr>
<tr>
<td>January 19, 2018</td>
<td>9:15am – 12:30pm</td>
<td>49</td>
<td>149.76</td>
<td>46</td>
</tr>
<tr>
<td>February 8, 2018</td>
<td>9:00am – 1:00pm</td>
<td>52</td>
<td>158.61</td>
<td>48</td>
</tr>
<tr>
<td>February 22, 2018</td>
<td>9:15am – 12:30pm</td>
<td>54</td>
<td>164.51</td>
<td>50</td>
</tr>
<tr>
<td>February 27, 2018</td>
<td>9:00am – 12:15pm</td>
<td>55</td>
<td>167.76</td>
<td>50</td>
</tr>
<tr>
<td>March 8, 2018</td>
<td>8:00am – 11:00am</td>
<td>57</td>
<td>174.26</td>
<td>52</td>
</tr>
<tr>
<td>March 16, 2018</td>
<td>7:45am – 11:00am</td>
<td>58</td>
<td>177.51</td>
<td>52</td>
</tr>
<tr>
<td>March 20, 2018</td>
<td>7:45am – 11:00am</td>
<td>58</td>
<td>177.51</td>
<td>52</td>
</tr>
<tr>
<td>March 22, 2018</td>
<td>7:45am – 11:00am</td>
<td>59</td>
<td>179.56</td>
<td>54</td>
</tr>
<tr>
<td>March 26, 2018</td>
<td>9:00am – 12:30pm</td>
<td>59</td>
<td>179.56</td>
<td>54</td>
</tr>
<tr>
<td>March 28, 2018</td>
<td>9:00am – 12:30pm</td>
<td>59</td>
<td>179.56</td>
<td>54</td>
</tr>
<tr>
<td>April 19, 2018</td>
<td>9:00am – 12:00pm</td>
<td>61</td>
<td>188.33</td>
<td>57</td>
</tr>
<tr>
<td>June 11, 2018**</td>
<td>8:00am – 12:00pm</td>
<td>62</td>
<td>198</td>
<td>56</td>
</tr>
<tr>
<td>July 18, 2018</td>
<td>8:00am – 12:00pm</td>
<td>62</td>
<td>198</td>
<td>56</td>
</tr>
<tr>
<td>September 20-22, 2018</td>
<td>5:00pm – 9:30am</td>
<td>62</td>
<td>198</td>
<td>58</td>
</tr>
<tr>
<td>November 6-7, 2018</td>
<td>9:00am – 9:00am</td>
<td>62</td>
<td>198</td>
<td>49</td>
</tr>
</tbody>
</table>

* Started on L34., shifted to L20 for the afternoon, returned to L34 for after hours monitoring
** From June 11, 2018 onwards, all tests were in a single configuration

4.4.2 Local Element Monitoring

The second type of test involved monitoring both the accelerations of the overall building as well as the local deformation in the VCD. After installing the VCDs there was a short window of time before the drywall was installed around them, effectively blocking access. This window provided a rare opportunity to access the VCDs and measure the deformation in the unit and compare it to the overall building motion. Typical tests involved placing four accelerometers on the level being monitored, and four LVDTs on the VCD as outlined in Section 4.3.2. A summary
of the local monitoring tests performed are shown in Table 4.2. Note that for all the tests shown in Table 4.2, the north VCD was tested.

Table 4.2  Local element monitoring test summary.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Levels Complete</th>
<th>Building Height (m)</th>
<th>Floor Monitored</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 30 – April 2, 2018</td>
<td>3:40pm – 6:40am</td>
<td>59</td>
<td>179.56</td>
<td>29</td>
</tr>
<tr>
<td>April 4-5, 2018</td>
<td>11:00am – 7:00am</td>
<td>60</td>
<td>184.61</td>
<td>29</td>
</tr>
<tr>
<td>April 6-7, 2018</td>
<td>1:00pm – 9:00am</td>
<td>60</td>
<td>184.61</td>
<td>30</td>
</tr>
<tr>
<td>April 13-16, 2018</td>
<td>3:00pm – 10:00am</td>
<td>60</td>
<td>184.61</td>
<td>35</td>
</tr>
<tr>
<td>April 27-30, 2018</td>
<td>4:00pm – 7:00am</td>
<td>61</td>
<td>188.33</td>
<td>37</td>
</tr>
<tr>
<td>June 13-14, 2018</td>
<td>3:30pm – 7:00am</td>
<td>62</td>
<td>198</td>
<td>25</td>
</tr>
<tr>
<td>August 27-29, 2018</td>
<td>4:30pm – 6:30am</td>
<td>62</td>
<td>198</td>
<td>25</td>
</tr>
</tbody>
</table>

4.4.3  Multi-Floor Monitoring with Local Element Monitoring

On one test day (May 4, 2018) the most advanced testing type was applied; multi-floor monitoring with local element monitoring. Two independent data acquisition systems were setup on level 29 and level 47, each with three accelerometers and on level 29, the south VCD was tested. The Garmin GPS receivers were used to synchronize the signals accurately to within 1 second of each other, and through post-processing the signals were synchronized based on the assumption that the first mode was a sway mode only. The details of the test are summarized in Table 4.3.

Table 4.3  Multi-floor monitoring with local element monitoring test summary.

<table>
<thead>
<tr>
<th>Date</th>
<th>Time</th>
<th>Levels Complete</th>
<th>Building Height (m)</th>
<th>Floor Monitored</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 4, 2018</td>
<td>9:30am – 8:00pm</td>
<td>62</td>
<td>192.05</td>
<td>29 &amp; 47</td>
</tr>
</tbody>
</table>

One of the accelerometer channels on level 47 failed due to a cable malfunction, and as such only two channels were available on this level. The accelerometers were placed in the same location on each level.

4.5  Data Processing

An example of the data processing for the May 4, 2018 event is shown in this section. The overall process applied to each record was:

1. Down-sampling and high pass filtering to remove low frequency offset/drift.
2. Numerically integrating to produce velocity and displacement signals.
4. Bandpass filtering to isolate each of the first three modes.

All data processing was completed in Matlab. The signals shown in this example are the three accelerometers placed on the level 29 and the four LVDTs used to measure the shear deformation in the south VCD on level 29. Note that this VCD was effectively in the level 30 floor slab but was in the ceiling area on level 29.

4.5.1 Down-Sampling and High Pass Filtering

First, the raw acceleration signals (Figure 4.15) and LVDT signals (Figure 4.16) were down-sampled to 10 Hz. Since the frequencies of interest were below 1 Hz, this still provided a margin of safety with respect to the Nyquist frequency. The built-in Matlab function ‘decimate’ was used to down-sample the signal from 200 Hz to 10 Hz, which utilizes a lowpass Chebyshev Type I IRF filter of order 8.

![Raw Accelerometer Measurements](image)

**Figure 4.15** Raw accelerometer measurements from May 4, 2018.

The accelerometer offset may be attributed to the DC (0 Hz) frequency content of the signal. The low frequency drift in the signal may be attributed to temperature variations over the course of testing, resulting in a slight change in the accelerometer properties. Both effects are removed by applying a high-pass filter to the signal.
The LVDTs were zeroed just prior to initiating the data acquisition, when the amplitude of motion was low prior to the wind speed increasing later in the day. The gradual offset of the LVDT measurement may be attributed to the static deformation of the structure. Unlike the accelerometers, the LVDTs measure the absolute displacement of the element, therefore the static deformation is effectively captured. The higher frequency content superimposed on the static displacements represents the dynamic displacements (Figure 4.17).

The PSDs for the accelerometer signals are shown (Figure 4.18a) and for the LVDT signals are shown in (Figure 4.18b).
A key observation may be made here regarding the spectral estimates for non-stationary data. Evidently the time-history for May 4, 2018 is non-stationary as the amplitude changes significantly over time, resulting in a time-varying RMS. This is manifested in a wider peak around the first mode frequency (0.215 Hz) where several peaks are clearly discernable. This may be attributed to the expected decrease in frequency as the amplitude grows, resulting in the wider peak.

Additionally, three modes are clearly present in the accelerometer PSDs at 0.22 Hz, 0.29 Hz, and 0.36 Hz, which represent the first three modes of the structure. Conversely, only two modes are present in the LVDT PSD at 0.22 Hz and 0.36 Hz. This indicates that there is little to no deformation in the VCD at the second mode frequency. This was expected since the VCDs were designed to only add supplemental damping in the first and third modes.

A sixth order Butterworth high-pass filter was used to remove the offset and drift components from the signals. When applied to the accelerometer time-histories, this produces a pure acceleration signal (Figure 4.19) and when applied to LVDT signal, produces a pure dynamic shear deformation signal (Figure 4.20).
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4.5.2 Numerical Integration

Next, the high-pass filtered accelerometer signals were numerically integrated using the trapezoid rule to produce velocity (Figure 4.21) and displacement (Figure 4.22) signals. Note that a high-pass filter was applied after each integration to prevent any low-frequency offset errors from the initial filtering to accumulate.
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4.5.3 Kinematic Projections

The three (or sometimes four, depending on the day) accelerometers were used to generate kinematic projections onto the primary axes of the structure. Assuming rigid body motion of the floor plate (rigid diaphragm), the overall X-direction displacement, Y-direction displacement, and rotation were computed. An illustrative example is shown in Figure 4.23 for May 4, 2018.

\[
\Delta_1 = \Delta X + dy_1 \theta \quad [1]
\]

\[
\Delta_2 = -\Delta Y + dx_1 \theta \quad [2]
\]

\[
\Delta_3 = \Delta X + dy_2 \theta \quad [3]
\]


\[
\theta = \frac{\Delta_3 - \Delta_1}{dy_2 - dy_1} \quad [4]
\]

Rearranging [1] and [3] and substituting [4]:

\[
\Delta X_1 = \Delta_1 - dy_1 \theta \quad [5]
\]

\[
\Delta X_2 = \Delta_3 - dy_2 \theta \quad [7]
\]

\[
\Delta X = \frac{\Delta X_1 + \Delta X_2}{2} \quad [8]
\]

Rearranging [2] and substituting [4]:

\[
\Delta Y = -\Delta_2 + dx_1 \theta \quad [9]
\]
For days when there were four accelerometers, a second displacement average was computed for the Y-direction response. These projections were applied to the filtered acceleration, velocity, and displacement signals. The overall displacement and rotation responses are shown in Figure 4.24.

![Overall Displacement Responses](image1)
![Overall Rotation Responses](image2)

**Figure 4.24** Overall displacement and rotation responses for May 4, 2018.

### 4.5.4 Bandpass Filtering

To analyse the modal responses of the structure, bandpass filters were used to isolate the first three modes. A fourth order Butterworth bandpass filter was used to isolate each mode in both the accelerometer and LVDT signals (Figure 4.25).
To ensure that the PSDs were not compromised by the filtering, the pre and post filtered PSDs were compared (Figure 4.26). The results show that the filters effectively isolate each spectral peak. Note that the additional frequency content at approximately 0.4 Hz that was not captured by the mode 3 filter may be attributed to a spurious on-site mode. This is further discussed in Section 5.1.
The overall modal responses are shown in Figure 4.27.
4.5.5 Data Processing Summary

This concludes the data processing for each testing day. For MDOF system identification the high-pass filtered acceleration/displacement results were used, whereas for SDOF system identification for investigating amplitude dependency using the RDT, the bandpass filtered modal responses were used.
5.0 Experimental Results and Discussion

This chapter reviews the results from the experimental program including the progression of the dynamic characteristics throughout the construction sequencing of the building, the amplitude-dependent properties observed under high-amplitude wind events, and the relationship between the VCD deformation and overall building motion.

The overall objectives of the experimental program were to build a database of experimental results that could be referenced to the finite element models analysed in Section 6.0, establish the amplitude-dependent trends, and determine the overall damping such that the effect of the VCDs could be investigated.

5.1 Construction Sequencing

The first phase of testing involved tracking the progression of the dynamic properties as the building was constructed. This provided an opportunity to benchmark the finite element models and the inherent-supplemental damping against several different stages of construction.

5.1.1 Record Filtering for Low Amplitude Regime

For the construction sequencing analysis, the estimates were constrained to low-amplitude vibrations, most of which were typical ambient vibrations with some random pulses due to construction activities. An example of a typical construction sequencing signal is shown in Figure 5.1. Note the low-amplitude acceleration; with a maximum in this 30-minute signal of approximately 0.55 milli-g (tested on level 38, constructed building at level 43). No high-amplitude wind events were considered in this section to ensure comparable amplitude levels.

![Figure 5.1 Construction sequencing response time-history on November 21, 2017.](image-url)
Only records where the maximum tip drift ratio, computed as the maximum displacement obtained from all channels divided by the building height at its current construction stage, was less than 1.2x10^{-5} were considered in this analysis to avoid the influence of amplitude on the dynamic characteristic estimates. For each test day the files were recorded in several short duration records. The length of these records and the number that were below the amplitude cut-off are shown in Table 5.1.

Table 5.1 Summary of typical sub-record lengths considered in analysis.

<table>
<thead>
<tr>
<th>Date</th>
<th>Sub-Record Length</th>
<th>Number of Sub-Records Considered</th>
</tr>
</thead>
<tbody>
<tr>
<td>September 28, 2017</td>
<td>20 minutes</td>
<td></td>
</tr>
<tr>
<td>October 4, 2017</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>October 19, 2017</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>November 8, 2017</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>November 21, 2018</td>
<td>30 minutes</td>
<td>8</td>
</tr>
<tr>
<td>December 6, 2017</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>December 19, 2017</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>January 19, 2018</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>February 8, 2018</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>February 22, 2018</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>February 27, 2018</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>March 8, 2018</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>March 16, 2018</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>March 20, 2018</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>March 22, 2018</td>
<td>1 hour</td>
<td>2</td>
</tr>
<tr>
<td>March 26, 2018</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>March 28, 2018</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>April 19, 2018</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>June 11, 2018</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>July 18, 2018</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>September 20-22, 2018</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>November 6-7, 2018</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.1.2 Spurious Mode Influence

Early in the testing program it was observed that four modes in the range of expected structural modes were present. Initially, all four were considered as structural modes, since there is no distinction between structural modes and harmonics in OMA. However, as the construction progressed, three of the four modes’ natural frequencies progressed as expected, while one, at
approximately 0.4 Hz, remained constant throughout all construction days. PSDs from select testing days are shown in Figure 5.2 showing the spurious mode near 0.4 Hz.

![Figure 5.2 PSDs for select testing days showing the spurious mode at approximately 0.4 Hz.](image)

A challenge arose with respect to SID when the natural frequencies passed through the spurious mode as the building was constructed. The effect was that the structural modes became indiscernible from the spurious mode, since the frequencies were either closely spaced or overlapping. In either case, SID became increasingly difficult and the SID results were discarded when they were in the range of the spurious mode. The spurious mode generally resulted in inflated damping ratio estimates for the influenced mode, whereas the frequencies were still able to be identified.

It was hypothesized that this spurious mode may be associated with induced vibrations from the crane. Since the crane used on site was a climber, meaning it rose with the structure and was always anchored a few stories below the top storey under construction (Figure 5.3), it follows that the crane would be responding in its natural mode as well during all tests. The crane’s natural frequency would remain constant throughout the construction sequencing, since the structural properties of the crane remained constant. Since the crane was anchored to the structure near the
top, the result of this would be a forcing harmonic at the natural frequency of the crane applied at the anchor point (Equation 91).

\[ F_{Crane} = F_0 \sin(\omega_{crane} t) \]  

Equation 91

Where:
- \( F_{Crane} \): Induced force from crane
- \( F_0 \): Force amplitude
- \( \omega_{crane} \): Natural angular frequency of the crane
- \( t \): Time

Figure 5.3 (a) Base of crane; (b) Crane anchorage to slab; (c) General drawing of crane anchoring to structure.

Literature suggests that the first mode frequency of typical construction cranes braced against the structure may be approximately 0.3 – 0.5 Hz (Ju, Choo, & Cui, 2006), (Ju & Choo, 2002). Therefore, it follows that there may be a forcing function from the crane at its natural frequency, which explains the detection of a mode at 0.4 Hz. This harmonic remained in all signals while the crane was onsite, however, once the crane was removed after the completion of the building, the harmonic was no longer present (Figure 5.4), thus confirming that the spurious mode was from the crane vibrations.
5.1.3 Construction Sequencing Results: Low Amplitude Regime

Four output-only SID techniques were applied to each sub-record considered including RDT-ERA (at the RMS of reference channel), EFDD, NExT-ERA, and SSI-DATA. The average from all techniques and all records was computed along with the standard deviation and a single value of frequency and damping was reported for each monitoring session. The mode shapes were investigated for a few select days only.

5.1.3.1 Mode Shapes

The mode shapes from three select days were computed; one from early in the testing program (October 4, 2017), one from the middle (January 19, 2018), and one from late (September 20, 2018) to investigate any changes in the mode shapes as the building was constructed. Since only a single floor was monitored on these days, the planar mode shapes were determined.

The mode shapes were determined based on considering each accelerometer as a degree-of-freedom and calculating the corresponding deformed shape assuming a rigid diaphragm based
on the kinematic projection concept described in Section 4.5.3. The mode shapes on the selected days are shown in Table 5.2.

Table 5.2  Mode shapes throughout construction sequencing.

<table>
<thead>
<tr>
<th>Date</th>
<th>Mode 1 Primarily X</th>
<th>Mode 2 Primarily Y+Tor</th>
<th>Mode 3 Primarily Tor+Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
<td>θ rad/m</td>
</tr>
<tr>
<td>Oct.4/17</td>
<td>1.000</td>
<td>0.033</td>
<td>0.003</td>
</tr>
<tr>
<td>Jan.19/18</td>
<td>1.000</td>
<td>0.034</td>
<td>0.001</td>
</tr>
<tr>
<td>Sept.20/18</td>
<td>1.000</td>
<td>-0.016</td>
<td>0.001</td>
</tr>
</tbody>
</table>

The results indicate that the planar mode shapes remain relatively constant throughout the construction of the building. This follows since the program was initiated when the structure was already taller than 100 metres and well into the regular floor plan section of the tower.

5.1.3.2  Frequency

The variation of the frequencies of the first three modes with building height and time are shown in Figure 5.5.
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The frequencies for all three modes followed a decreasing trend with height. As the structure grew in height the mass was increasing and the stiffness was decreasing, both of which resulted in a decrease in the frequency. The structural member mass increased in a linear fashion with respect to height, as each storey was comprised of approximately the same volume of materials and finishes, which trailed the floor construction by several stories. The overall lateral stiffness decreased inversely proportional to the height cubed. These two elements varying in tandem resulted in a decreasing frequency versus height plot.

Viewing the frequency variation in time plots, the frequency remained constant after the structure topped out. While the structure itself was complete, only a fraction of the non-structural components had been installed. The progression of non-structural component installation on each monitoring day is shown in Figure 5.6. Note that the difference between these categories was subjective and was based on the researcher’s observations on site at the time of inspection.
Notably, the structure topped out between the May 4, 2018 and June 11, 2018 monitoring sessions at which point complete non-structural finishes were installed up to storey 34, a little over half complete, with variable finishes complete up to level 53 where the cladding was installed. Over the time between the completion of the structure (June 11, 2018) and the final low-amplitude monitoring session (September 20-22, 2018), at which point most of the non-structural components were fully installed, the natural frequencies of the first three modes exhibited little change. The first and third mode showed slight decreases while the second mode remained virtually constant. In viewing the possible effects on the structural properties (mass, damping, stiffness) and the corresponding effect on frequency, it follows that the more notable effect in the amplitude range considered was the increase in total mass of the non-structural components resulting in a decrease in the frequency. This does not indicate that the non-structural components do not contribute to the stiffness, it simply shows that the additional mass gained from their installation governs the frequency in the low-amplitude regime.

To develop more conclusive evidence for the role of non-structural components on the frequency, particularly low-amplitude frequency, more studies must be conducted on buildings throughout their construction, in particular starting the monitoring earlier in the construction sequencing. A crucial aspect of furthering structural engineers’ understanding of tall building behaviour is continued effort in the field of structural monitoring.
5.1.3.3 Damping

To highlight the difference between SID techniques when applied to full-scale data, first the damping results are shown by method (Figure 5.7, Figure 5.8, and Figure 5.9) as the structure was built. The results presented are the overall effective equivalent viscous damping ratios in each mode. Inherent in the application of the RDT is the assumption that the only source of energy dissipation is viscous in nature. This means that the results shown represent all the sources of energy dissipation in the structure, including the effect of amplitude-dependent stick-slip mechanisms, the VCDs, and the baseline inherent damping of the structure, and the cumulative effect of these sources was represented as an equivalent viscous damping ratio. Note that estimates when the spurious mode overlapped the natural frequency of the mode in question are not shown.

![1st Mode Damping](image)

Figure 5.7 First mode damping (construction sequencing) by SID method.
The results displayed scatter across methods, which may be attributed to:

- Variability in the loading due to construction activities.
- Short duration signals.

Since the VCDs replace elastic elements with viscoelastic, any force in the coupling beam would now contain a viscous component as well. The behaviour of the VCDs at extremely low amplitudes (2.5 μm) displacements was shown by Pant, Montgomery and Christopoulos (2019) to maintain a stable hysteresis, indicating that supplemental damping is provided by the VCDs at all
amplitude ranges. Given the variability associated with the damping estimates in the low-amplitude regime due to the short duration records and the variability in loading due to construction activities, conclusive statements cannot be made regarding the amount of supplemental damping provided in the low amplitude range. However, general increases were evident in the first and third mode during the window in which the VCDs were installed, indicating some additional damping in the low amplitude regime due to the dampers.

To overcome this challenge in future applications, long duration signals should be obtained overnight or during weekends to avoid the influence of construction activities and increase the duration of records available.

### 5.1.4 May 4, 2018 Low-Amplitude Regime: Multi-Floor Monitoring

On May 4, 2018, two floors were monitored including level 47 and level 29. Two accelerometers were used on level 47 and three were used on level 29. Level 47 was monitored from 10:30am until 5:30pm, just after the peak of the storm while level 29 was monitored from 9:30am until 8:30pm, well after the peak of the storm. The signals were synchronized according to methods described in Section 4.3.3. The synchronized signals are shown in Figure 5.10.

![May 4, 2018: Synchronized Signals](image)

**Figure 5.10** May 4, 2018 synchronized acceleration signals.

The primary advantage from instrumenting multiple floors was information regarding the vertical mode shapes. Analysing the low-amplitude data from early in the signal (first three hours) using the MDOF SID techniques, including RDT-ERA at the RMS of the reference channel, NExT-ERA, EFDD, and SSI-DATA, the vertical mode shapes were computed by considering the five accelerometers as degrees-of-freedom. The mode shapes, as well as frequency and damping
estimates from the low-amplitude regime early in the signal are tabulated in Table 5.3. Note that the mode shapes reference the degrees-of-freedom associated with the accelerometer locations shown in Figure 5.11.

Table 5.3 SID results from multi-floor monitoring on May 4, 2018: Low amplitude regime.

<table>
<thead>
<tr>
<th>Mode</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (Hz)</td>
<td>0.22</td>
<td>0.30</td>
<td>0.37</td>
</tr>
<tr>
<td>Damping (%)</td>
<td>1.31</td>
<td>0.98</td>
<td>1.51</td>
</tr>
<tr>
<td>L47 – Ch.1 [u₁]</td>
<td>0.011</td>
<td>0.461</td>
<td>-1</td>
</tr>
<tr>
<td>L47 – Ch.2 [u₂]</td>
<td>1</td>
<td>-1</td>
<td>-0.94</td>
</tr>
<tr>
<td>L29 – Ch.1 [u₃]</td>
<td>0.44</td>
<td>0.042</td>
<td>0.043</td>
</tr>
<tr>
<td>L29 – Ch.2 [u₄]</td>
<td>0.008</td>
<td>0.25</td>
<td>-0.55</td>
</tr>
<tr>
<td>L29 – Ch.3 [u₅]</td>
<td>0.46</td>
<td>-0.47</td>
<td>-0.48</td>
</tr>
</tbody>
</table>

These results were used to compare to finite element models in Section 6.7.

5.1.5 Free Vibration Extraction from Non-Stationary Events

Throughout the testing program there were a few instances in which the structure was subjected to loading which produced a free-vibration-like response (Table 5.4).
Table 5.4 Free vibration response extraction from non-stationary events.

<table>
<thead>
<tr>
<th>Date</th>
<th>Excitation Type</th>
<th>Equivalent Testing Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 8, 2017</td>
<td>Concrete pumping</td>
<td>Resonance</td>
</tr>
<tr>
<td>February 22, 2018</td>
<td>Crane activity</td>
<td>Pull and release</td>
</tr>
<tr>
<td>February 27, 2018</td>
<td></td>
<td></td>
</tr>
<tr>
<td>June 13, 2018</td>
<td>Short duration high intensity wind event</td>
<td>Pull and release</td>
</tr>
</tbody>
</table>

These responses provided an opportunity to extract the damping and frequency in a more direct manner than the output-only SID techniques applied to the data previously and provided a strong point of comparison for the results. Additionally, by referencing the initiation amplitude the properties may be referenced to amplitude, which is a useful comparison for the results discussed in Section 5.2. Note that this is a similar approach as taken by applying non-stationary SID techniques such as the Wavelet-Transform with Laplace Wavelet Filtering method discussed in Section 3.2.2. However, instead of automating the search in a noisier signal, the free vibrations were visually extracted based on response form and qualitative observations by the author regarding on-site experience.

5.1.5.1 Free Vibration due to Concrete Pumping

The first case of free vibration observed when testing was the effect of concrete pumping on the structure. On November 8, 2017 monitoring was performed after construction had ceased for the day, ensuring that there was no other excitation other than the ambient vibrations. A late concrete pour was occurring early in this window, which was completed by pumping concrete from the ground to the top storey using a typical built-in concrete pump. It was observed that the pulsing motion of the pumping served to excite the structure in a resonance like fashion (Figure 5.12).
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Figure 5.12 Free vibration response from concrete pumping effect.

The concrete pumping was initiated at 100 seconds into the signal and terminated at 375 seconds. Over this window the amplitude of excitation gradually increased. After the pumping was completed, the structure returned to rest in a free vibration-like fashion. In this case, only the second mode was adequately excited, providing an estimation of the damping ratio in this mode only (Figure 5.13). Note that the free vibration response was not perfect, however this response represents an approximate extraction based on the observed signal.

Figure 5.13 November 8, 2017; Second mode free vibration response from concrete pumping effect.

This provided an estimate of a second mode damping ratio of approximately 0.5% at 0.2 mm. This agrees well with the output from Section 5.1.3.3, adding further support to the robustness of the SID techniques applied in the low-amplitude range.

5.1.5.2 Free Vibration due to Crane Activity

The second form of induced free vibrations that was observed during the testing program was crane-induced motion. It was observed that when the crane made certain large picks or drops, the structure experienced an impulse-like load, initiating a free vibration response. An example from February 22, 2018 is shown in Figure 5.14.
These responses were measured during active construction days, and as such after the initial impulse subsequent activities inhibited a pure free vibration response, rendering the signals unsuitable for direct damping evaluation. However, there may be potential to use crane motion to induce free vibrations as a method of field testing of tall buildings when a climbing crane is used.

Controlled crane motions were tested in July of 2018; however, it was found that without a large payload, there was insufficient clarity on the induced free vibrations, and the small excitations that were applied resulted in low amplitude spikes, thus rendering the information gained from the test redundant when compared to low amplitude AVT tests. Tests of this nature have been performed in the past (Glanville, Kwok, & Denoon, 1996) and could be used to induce these types of motions in future tests. However, this involves significant coordination and permissions from contractors and site representatives and creates a noteworthy obstacle. As outline at the onset of this thesis, the simplicity of AVT makes it desirable over these types of FVT.

5.1.5.3 Free Vibration due to Transient Wind Storm

The last type of induced free vibration observed during the testing program was a short duration wind storm on June 13-14, 2018. The response of the structure spiked as a severe storm passed through Toronto (time 2900 seconds to 3120 seconds) and after the abrupt end to the event, the structure exhibited a free-vibration like response (from time 3120 seconds until 3260 seconds). The response time-history is show in Figure 5.15, and the short duration transient wind event is highlighted in Figure 5.16.
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5.2 Amplitude-Dependency of Frequency and Damping

In the spring of 2018, Toronto was subjected to several large amplitude wind events, resulting in several opportunities to retrieve amplitude-dependent response data from the test structure. This phase of the results discusses the amplitude-dependent dynamic properties computed using the RDT for these high-amplitude wind events which have the most relevance to the structural engineering community.

5.2.1 Wind Events

The wind data retrieved from the Toronto Islands weather station (Government of Canada, 2018) over the course of the monitoring program is shown in Figure 5.17.
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During the months of April and May of 2018 there were several large amplitude wind events (Figure 5.18).

The events that were monitored over this period are summarized on the wind rose showing the directionality and maximum gust (km/hr) for each event (Figure 5.19).
The full response time-histories for each event are shown in Appendix IV. Note that a key challenge in the interpretation of results is the variable temperature between test days. As noted previously, temperature plays an important role in the VE properties of the VCD, which translates to notable effects on the supplemental damping and natural periods. On the wind event days, the temperature distribution over the record was difficult to accurately quantify. Since the structure was incomplete, there was inconsistent temperature distribution throughout the building. During cold days, localized heaters were present throughout the structure and operated by on-site workers to maintain workable environments in their current area of work. These were intermittently dispersed throughout the structure, and inconsistently used. This challenge is further discussed in subsequent sections.

### 5.2.2 Signal Selection and Reference Amplitude

The amplitude-dependent properties for each of the high-amplitude wind events were investigated using the RDT-PP based on the parameters obtained from the numerical simulation study (Section 3). The RDT-PP was applied to the bandpass filtered overall modal time-histories. The reference amplitude for each mode is shown in Table 5.5.
Table 5.5 Reference amplitudes for RDT-PP.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Reference Amplitude</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Overall X</td>
<td>Mode is primarily X sway</td>
</tr>
<tr>
<td>2</td>
<td>Overall Y</td>
<td>Mode is primarily Y sway</td>
</tr>
<tr>
<td>3</td>
<td>Arc length projection in X</td>
<td>Computed by taking the lever arm to the extreme North or South end of</td>
</tr>
<tr>
<td></td>
<td>direction at edge of building</td>
<td>the structure (20.35m) and multiplying by the modal rotation.</td>
</tr>
</tbody>
</table>

5.2.3 Vertical Mode Shape Projection

Since each monitoring session was performed on a different level, and the structure itself was still under construction during several of the sessions, approximate vertical mode shapes were used to project the modal responses to the top storey such that the response amplitudes from each event could be compared. These mode shapes were obtained using the finite element model of the building where a bounded analysis on stiffness and mass was performed, with the model that provided the closest match to the experimental frequency from the bounded analysis being used for the vertical mode shape projection. These models were constructed to reflect the as-built condition on the day of monitoring, ensuring that the proper structural height and VCD installation stage were captured. More detail on the modelling bounded analysis is provided in Section 6.3. The vertical mode shapes projections for each day and mode are shown in Table 5.6.

Table 5.6 Vertical mode shape projection for each test day.

<table>
<thead>
<tr>
<th>Date</th>
<th>Floor Tested</th>
<th>Top Storey</th>
<th>Mode 1 Factor</th>
<th>Mode 2 Factor</th>
<th>Mode 3 Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>March 30-April 2, 2018</td>
<td>29</td>
<td>59</td>
<td>2.9967</td>
<td>2.3646</td>
<td>2.2806</td>
</tr>
<tr>
<td>April 4-5, 2018</td>
<td>29</td>
<td>60</td>
<td>3.0182</td>
<td>2.3397</td>
<td>2.2150</td>
</tr>
<tr>
<td>April 6-7, 2018</td>
<td>30</td>
<td>60</td>
<td>3.0135</td>
<td>2.3591</td>
<td>2.2944</td>
</tr>
<tr>
<td>April 13-16, 2018</td>
<td>35</td>
<td>60</td>
<td>2.2672</td>
<td>1.9463</td>
<td>1.7871</td>
</tr>
<tr>
<td>April 27-30, 2018</td>
<td>37</td>
<td>61</td>
<td>2.1282</td>
<td>1.8671</td>
<td>1.7157</td>
</tr>
<tr>
<td>May 4, 2018</td>
<td>29</td>
<td>62</td>
<td>3.4446</td>
<td>2.7020</td>
<td>2.5419</td>
</tr>
<tr>
<td>June 13-14, 2018</td>
<td>25</td>
<td>63</td>
<td>4.8154</td>
<td>3.4444</td>
<td>3.3942</td>
</tr>
<tr>
<td>August 27-29, 2018</td>
<td>25</td>
<td>63</td>
<td>4.8154</td>
<td>3.4444</td>
<td>3.3942</td>
</tr>
<tr>
<td>September 20-22, 2018</td>
<td>58</td>
<td>63</td>
<td>1.1507</td>
<td>1.5000</td>
<td>1.0820</td>
</tr>
<tr>
<td>November 6-7, 2018</td>
<td>47</td>
<td>63</td>
<td>1.5601</td>
<td>1.4247</td>
<td>1.4845</td>
</tr>
</tbody>
</table>

Note that all amplitudes referenced for the amplitude-dependent curves reference the projected top storey displacement in the primary direction of the mode in question.
5.2.4 Amplitude-Dependent Frequency

For the purpose of brevity only the results from three select high-amplitude events, including April 4-5, 2018, May 4, 2018, and November 6-7, 2018 are shown for the amplitude-dependent frequency. Full results are shown in Appendix V. The results for the amplitude-dependent frequencies for each mode on the select days are shown in Figure 5.20, Figure 5.21, and Figure 5.22. Note that the results for the first two modes are shown for April 4-5, 2018 and May 4, 2018 but are only shown for November 6-7, 2018 for the third mode. The spurious mode interference resulted in unreliable results for the third mode during the two key high-amplitude events.

**Figure 5.20** First mode frequency amplitude-dependency from two events; April 4-5, 2018 and May 4, 2018.

**Figure 5.21** Second mode frequency amplitude-dependency from two events; April 4-5, 2018 and May 4, 2018.
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In all cases the frequency of each mode exhibited a decrease with an increase in amplitude. The slope of linear fits to the data were computed to compare the degree of amplitude-dependency observed and are tabulated in Table 5.7.

**Table 5.7 Degree of amplitude-dependency of frequency for select days.**

<table>
<thead>
<tr>
<th>Day</th>
<th>Degree of Frequency Amplitude-Dependency (Hz/mm)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>April 4-5, 2018</td>
<td>May 4, 2018</td>
<td>November 6-7, 2018</td>
</tr>
<tr>
<td>Mode 1</td>
<td>-1.51x10^{-4}</td>
<td>-1.65x10^{-4}</td>
<td>-</td>
</tr>
<tr>
<td>Mode 2</td>
<td>-1.33x10^{-3}</td>
<td>-1.88x10^{-3}</td>
<td>-</td>
</tr>
<tr>
<td>Mode 3</td>
<td>-</td>
<td>-</td>
<td>-2.30x10^{-3}</td>
</tr>
</tbody>
</table>

These results indicate that the degree of amplitude-dependency increased with each mode. While there was a difference between the first and second mode on the order of 10, the third mode was only incrementally larger than the second, which showed that the second and third mode exhibit similar degrees of amplitude-dependency. This means that the mechanisms responsible for the amplitude-dependency may be similar in the second and third mode.

The variability in the plots may be attributed to the short duration records used in the study. The plots indicate that the slope of the frequency amplitude-dependency was generally linear but may begin to soften with amplitude. This supports the theory that the amplitude-dependency may be attributed to stick-slip mechanisms. In a structure there would be a finite amount of these mechanisms, and as the amplitude grows there will be some point at which all the mechanisms have slipped (Tamura’s critical tip drift ratio). Based on the distributions used by Spence and Kareem (2014), it follows that there would be a decreasing number of these mechanisms available.
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Therefore, the results of this study indicate that the amplitude-dependent phenomenon fits with the currently proposed state-of-the-art models, attributing the variation with amplitude to stick-slip mechanisms.

5.2.5 Amplitude-Dependent Damping

The amplitude-dependent damping was investigated in two phases; first, the effect of amplitude was investigated looking at signals obtained during the installation of the VCDs and second, was investigated after the installation of all the VCDs.

5.2.5.1 During VCD Installation

The results for the two key high-amplitude events that occurred during and just after the completion of the installation of the VCDs are shown in Figure 5.23 and Figure 5.24. Note that due to spurious mode influence on these testing days, reliable data was not obtained for the third mode.

![1st Mode Damping](image1)

**Figure 5.23** First mode damping amplitude-dependency from two events; April 4-5, 2018 and May 4, 2018. Dashed lines are linear trendlines.

![2nd Mode Damping](image2)

**Figure 5.24** Second mode damping amplitude-dependency from two events; April 4-5, 2018 and May 4, 2018.
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The first mode exhibited an amplitude-dependent response, highlighted specifically by an offset in the damping ratio computed for the April 4-5, 2018 and May 4, 2018 events. Both curves exhibited a linear trend, and the trend line parameters are shown in Table 5.8.

Table 5.8 Degree of amplitude-dependency of damping for select days.

<table>
<thead>
<tr>
<th>Date</th>
<th>Slope (%/mm)</th>
<th>Intercept (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 4-5, 2018</td>
<td>0.0279</td>
<td>1.3576</td>
</tr>
<tr>
<td>May 4, 2018</td>
<td>0.0356</td>
<td>1.7111</td>
</tr>
</tbody>
</table>

In the range where the results overlap, the average difference was approximately 0.43% damping. On the April 4-5, 2018 monitoring session, exactly half of the VCDs were installed, while on the May 4, 2018 session all the VCDs were installed. A key note in the amount of damping added by the VCDs is the effective stiffness, often attributed to the degree of cracking, of adjacent structural members. As these members become more cracked at higher return period events, the amount of supplemental damping provided grows (this is discussed further in Section 6.5) due to the relationship between stored elastic energy and dissipated energy. If the stiffness is decreased due to increased cracking, or stiffness losses due to stick-slip elements, the stored elastic energy decreases. Since the energy dissipated by the VCDs remains constant (a function of the damper properties), the overall effective equivalent viscous damping grows. Note that the VCD system itself is not significantly amplitude-dependent, but the additional damping generated due to the cracking of the structure of stiffness losses in stick-slip elements may be viewed as a second-order effect of these stiffness losses.

While the concept of stick-slip components is commonly used to describe non-structural components, the extension may also be made to friction mechanisms in structural members themselves. This indicates that if structural elements in parallel with the VCDs exhibit stick-slip behaviour through the mechanism of slipping or opening/closing of microcracks in the reinforced concrete, then the supplemental damping will also grow with amplitude as the adjacent members slip and lose stiffness. Therefore, an increase in overall damping and the slope of the damping may be expected due to the VCDs. This was reflected in the amplitude-dependent results, as both an offset and an increase in the slope were observed. Note that while theoretically true, the temperature was different between the two days, which may also contribute to a portion of the difference. The effect of temperature on the supplemental damping estimations is discussed further in Section 6.5.
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The second mode did not exhibit a dependency on the amplitude. While this was true over the amplitude range considered, the low-amplitude damping estimates performed in Section 5.1.3.3 indicated a damping ratio of approximately 0.9% in the second mode. Based on the reliable range for RDT application (greater than the RMS of the signal), there may in fact be some variation in the amplitude range lower than that considered for the two high amplitude events. This will be explored further in Section 5.2.5.2. The estimations of the damping ratio between each day were approximately equal, indicating no effect in the second mode due to the installation of additional VCDs between the test days. This follows since the VCDs were not designed to have any effect in the second mode.

5.2.5.2 After VCD Installation

The results for the segment weighted amplitude-dependent damping are shown in Figure 5.25, Figure 5.26, and Figure 5.27 and were computed according to the method described in Section 3.4.2.1. In addition to the average, the segment weighted standard deviation was computed according to (Equation 92) and was plotted as error bars on the results.

$$\zeta = \sqrt{\frac{\sum_{i=1}^{N} w_i (\zeta_i - \zeta_w)^2}{\frac{N'}{N'} \sum_{i=1}^{N'} w_i}}$$

Equation 92

Where:

\(w_i\): Number of segments
\(\zeta_i\): Damping estimate
\(\zeta_w\): Segment weighted average damping ratio
\(N'\): Number of non-zero weights
\(N\): Number of averages

These results were computed considering monitoring sessions after all VCDs were installed (Table 5.9).
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Table 5.9 Monitoring sessions considered in segment weighted average.

<table>
<thead>
<tr>
<th>Date</th>
<th>Storeys</th>
<th>Modes Considered</th>
<th>VCD Completion</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 27-30, 2018</td>
<td>61</td>
<td>1,2</td>
<td></td>
</tr>
<tr>
<td>May 4, 2018</td>
<td>62</td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>June 13-14, 2018</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>August 27-29, 2018</td>
<td></td>
<td></td>
<td>100%</td>
</tr>
<tr>
<td>September 20-22, 2018</td>
<td>63</td>
<td>1,2,3</td>
<td></td>
</tr>
<tr>
<td>November 6-7, 2018</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

While the structure was changing over these monitoring dates, the results from Section 5.1.3.3 before the VCDs were installed indicated that the inherent damping ratio may not have been changing significantly in the background over this time. Therefore, no significant effects on the overall damping characteristics were expected by the construction completions that occurred over these dates, allowing the damping ratio to be compared equivalently across all dates. Note that violations of this assumption would be reflected by increased deviation in the segment weighted averages.

A key note regarding the computation of the segment weighted average was the variability in the temperature of the structure across the monitoring sessions. Since the VCDs are temperature sensitive, the supplemental damping provided varies depending on the temperature. Over the test days the temperature distribution over the record was difficult to accurately quantify due to inconsistent temperature maintenance in the building. The ambient temperature varied significantly from sub zero degrees Celsius in March and early April up to high 20 degrees Celsius in August. On cold days, localized heaters were present throughout the structure and operated by on-site workers to maintain workable environments in their current area of work. These were intermittently dispersed throughout the structure, and inconsistently used. Additionally, as the structure neared completion the regular building temperature control was installed, properly regulating the temperature on completed stories. All of this adds to the deviation of the damping estimations. In future studies of this nature, temperature should be monitored along with accelerations and local element monitoring to develop a more accurate representation of the on-site conditions during the test, particular when VE dampers are used and during the construction phase of the building when the temperature is less controlled.
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Figure 5.25  First mode damping amplitude-dependency presented as segment weighted average after all VCDs installed.

Figure 5.26  Second mode damping amplitude-dependency presented as segment weighted average after all VCDs installed.

Figure 5.27  Third mode damping amplitude-dependency presented as segment weighted average after all VCDs installed.
The low-amplitude damping was computed by removing the requirement for the low-amplitude cut-off at the RMS of the signal. Since several long-duration records were considered, the number of segments was significantly increased in the low-amplitude range, with the first, second, and third mode initiating with 153285, 166609, and 99940 segments, respectively.

All three modes displayed amplitude-dependent trends with similar magnitudes of standard deviation. On average the standard deviation was 0.24% in the first mode, 0.12% in the second mode, and 0.13% in the third mode. The three modes converged to low-amplitude damping estimations of 1.19% in the first mode, 0.878% in the second mode, and 1.46% in the third mode. Note that these were comparable to the estimates obtained in Section 5.1.3.3 after the installation of the VCDs was complete.

The first mode showed a steady increase in a linear fashion over the entire amplitude range considered, peaking at 2.61% at 22.25 millimetres. The free vibration response extracted from the transient wind event in Section 5.1.5.3 agrees with the segment weighted average results. The second mode showed an initial increase from 0.88% at 0.05 millimetres to 1.36% at 0.6 millimetres, after which the damping ratio remained relatively constant. The second mode showed some variability in this plateau range, indicating a maximum of 1.50% at 3.05 millimetres. Lastly, the third mode showed a consistent linear increase over the entire amplitude range considered (0 - 1.3 millimetres), peaking at 1.93%. Note that the third mode was constrained to lower amplitudes due to the spurious mode influence on the high-amplitude testing days. The degree of amplitude-dependency was quantified by the slope of the linear portion of the plots and is summarized for each in (Table 5.10).

<table>
<thead>
<tr>
<th>Mode</th>
<th>Degree of Damping Amplitude Dependency (% / mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0576</td>
</tr>
<tr>
<td>2</td>
<td>0.7206</td>
</tr>
<tr>
<td>3</td>
<td>0.2897</td>
</tr>
</tbody>
</table>

These results indicate that the second mode exhibited the highest degree of amplitude-dependency in the low-amplitude range, but levels off quickly, followed by the third mode and then the first mode. Further research is required to continue to understand the degree of amplitude-dependency of damping and how it relates to overall and local deformation mechanisms in structures.
Lastly, the overall results for the first three modes were compared to Spence and Kareem’s (2014) predictor model in Figure 5.28, which represents the current state-of-the-art in damping predictor models.

![Amplitude-Dependent Damping](image)

Figure 5.28  Amplitude-dependent damping compared to Spence and Kareem (2014) predictor model.

This indicated that the test building for this research generally had higher damping than the structures used in the calibration of the predictor model, which were namely buildings without supplemental dampers. While this follows for the first and third mode, the second mode did not show any evidence of supplemental damping from the VCDs. This may be attributed to the deformation mechanism, since Spence and Kareem (Spence & Kareem, 2014) showed that the structures dominated by shear deformations in their experimental database tended to produce higher damping ratios (Figure 2.34b). This is discussed further in Section 6.8.

5.2.6  Amplitude-Dependent Results Summary

In summary, the high amplitude wind events experienced in Toronto in the Spring of 2018 allowed for the tracking of amplitude-dependent phenomenon with respect to both frequency and damping. The RDT was applied to extract the amplitude-dependent trends and the key conclusions include:

- The frequency of the first three modes displayed a decrease with respect to amplitude. The slope showed mild indications of gradually softening with amplitude, indicating that the
stiffness losses are reducing as the amplitude grows. This agrees with the hypothesis that stick-slip components are responsible for the observed amplitude-dependent trends.

- The damping ratio of the first three modes displayed an increase with respect to amplitude. The first and third mode displayed consistent linear trends, whereas the second mode showed an initial linear increase followed by a plateauing of the damping with respect to amplitude. The damping ratio for the first mode, second mode, and third mode varied with amplitude from 1.19% to 2.61%, 0.878% to 1.50%, and 1.46% to 1.93%, respectively, over the reliable amplitude ranges analysed using the RDT.

- The VCDs resulted in an absolute increase in damping as well as an increase in the degree of amplitude-dependency due to the increasing amount of energy dissipation as adjacent member stiffness’ are reduced.

5.3 Global – Local Deformation Relationships

The last thrust of the experimental program was the local monitoring of the VCD unit on available floors in conjunction with the overall building monitoring. This allowed the relationship between the global building motion and the local element deformation to be established.

5.3.1 VCD Deformation Time-Histories

On most of the test days listed in Table 4.2 four LVDTs were used, however, on some days cables failed resulting in only three LVDTs. The average deformation in the VCD was computed as the average of all reliable LVDT signals. On some testing days, certain sensors were disrupted during the tests due to construction activities and were discarded from the analysis due to unreliable data. The averaged signal for the May 4, 2018 event is shown in Figure 5.29.

![Average shear deformation in VCD for May 4, 2018 event.](image-url)  

Figure 5.29 Average shear deformation in VCD for May 4, 2018 event.
Additional time-histories for the VCD deformations are shown in Appendix IV.

5.3.2 Global-Local Deformation

Measuring the response of the VCD allowed for the overall global deformation to be compared to the corresponding local deformation. The results as a function of time and amplitude are discussed.

5.3.2.1 Global-Local Deformation in Time

A sample plot showing an excerpt of the normalized first mode overall X-direction motion and the normalized first mode average VCD response is shown in Figure 5.30.

![Figure 5.30](image.png)

**Figure 5.30** Excerpt from normalized global and local time-histories for May 4, 2018.

Figure 5.30 shows a lag of approximately 0.2 - 0.3 seconds between the peak of the overall structural displacement and the VCD displacement. The lag was consistent throughout the signal and was present in a similar magnitude for all days except the April 4-5, 2018 event (Figure 5.31), in which the trend was reversed (VCD displacement leads structural displacement).

![Figure 5.31](image.png)

**Figure 5.31** Excerpt from normalized global and local time-histories for April 4-5, 2018.
The cause of the reversed trend on April 4-5, 2018 was unclear, as the same testing and data analysis procedure was used for all testing days. This indicates an error with regards to the testing synchronization on this day. Despite this, the lag was present in all other tests. Figure 5.32 shows plots of the overall global deformation and the local VCD deformation. The lag may be attributed to the relative magnitude of the damping coefficient and stiffness coefficient or may be due the complex modal damping associated with discrete dampers.

Figure 5.32 First and third mode global-local deformation plots for May 4, 2018.

5.3.2.2 Amplitude-Dependency of the Global-Local Deformation Ratio

To investigate the amplitude-dependency of the global-local deformation relationship, the deformation ratio was computed at an array of amplitudes. This was achieved by finding peaks within a small amplitude range (5%) in the overall structural response and matching that peak to the corresponding peak in the VCD response. In a single time-history, there would be several peaks in each amplitude bin, and so the average of all the deformation ratios found was computed and reported, referencing the selected amplitude bin. The deformation ratio plots for all days with local element monitoring for the first mode are shown in Figure 5.33. Note that since different levels were monitored on each day, the magnitude of the ratio between the tip deformation and VCD deformation will inherently be different in magnitude between each test day, although the responses were expected to follow the vertical mode shape for the first mode. Additionally, since the LVDTs were mounted between the steel embeds and the steel plate of the VCD, approximately 1.2% additional average deformation was expected in the VE material based on an ABAQUS model developed by the damper designer (Montgomery, personal communication, January 25, 2019).
Figure 5.33 Amplitude-dependent deformation ratio for all local monitoring tests.
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With the exception of the April 13-16, 2018 event, all deformation ratios displayed a decreasing trend with respect to amplitude. This was true for the April 13-16, 2018 event as well after the initial increase measured. Note that the computed deformation ratio is of prime interest for the comparison of experimental results to numerical models, which is further discussed in Section 6.0.

For each of these events a decreasing trend with respect to amplitude was observed, and on some there was some softening of the slope over the initial 3 millimetres, after which the slope remained relatively constant. In the sub 3 millimetres range, it appeared that there was a higher stiffness near the VCD, such that it required more deformation of the structure to generate the same deformation in the VCD. This is indicative of stick-slip mechanisms, since at higher amplitudes the slope of the deformation ratio plot softens, indicating that stiffness losses are occurring in the structure. In this case, the slipping mechanisms result directly in an increased proportion of deformation being driven into the VCDs, indicating the the stick-slip mechanism may be related to structural elements in parallel with the VCDs. It follows that friction on microcracks in the structural elements may be one of many stick-slip mechanisms in a structure. This is discussed further in Section 6.8.

The April 13-16, 2018 event appeared as an outlier compared to the other seven events recorded. In the low amplitude range the deformation ratio experienced an increase with amplitude, opposite to the trend observed on all other days. It is possible that the weather pattern in Toronto for this event may be responsible for the outlier result. The wind directionality was opposite to all the other storms recorded, thus the actual applied loading may have had some effect on the response quantities. Additionally, it may be that the localized heaters used were intermittently used over certain time periods when the amplitude was constant, resulting in variable VCD properties over the course of the time-history. The test was performed from Friday, April 13, 2018 from 4:00pm until Monday, April 16, 2018 at 10:00am. It was common for workers to be on-site on Saturday, so it may be assumed that the heaters were on for the majority of the low-amplitude portion of the response early in the signal, and were off for the high-amplitude portion of the response later in the signal (Figure 5.34).
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5.3.3 Local Behaviour and Amplitude-Dependent Damping and Frequency

At lower temperatures, it was expected that the deformation ratio would be higher due to the increased stiffness of the VCDs at low temperatures. Therefore, the deformation ratio computed in the sub 3-millimetre amplitude range may have been calculated for a softer VCD, where the deformation ratio computed in the higher amplitude range may have been calculated for a stiffer VCD. This highlights the importance of measuring the temperature of VE materials when monitoring over varying conditions and is recommended to be included in future work on the subject.

Figure 5.34 Tip response time-histories for April 13-16, 2018.

The observed trend of a softening slope for the deformation ratio plot with amplitude were like those observed for the amplitude-dependent frequency and damping. This indicated that they may be driven by the same underlying phenomenon, namely stick-slip mechanisms that themselves are a function of amplitude. If the global-local deformation represents a quantification of a portion of these mechanisms, it follows that frequency and damping are related to the deformation ratio. The plots for the first mode frequency and damping as a function of the first mode deformation ratio are shown in Figure 5.35.
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Figure 5.35  First mode frequency and damping as a function of deformation ratio for April 4-5, 2018 and May 4, 2018.

While there is significant scatter in the data, the damping is shown to decrease with an increase in the deformation ratio and the frequency is shown to increase with an increase in the deformation ratio for the both events.

It has been well established experimentally that the global behaviour with respect to the amplitude-dependency of frequency and damping may be explained theoretically by many stick-slip mechanisms randomly distributed throughout the structure. To further this modelling approach, the local elements that are directly responsible for the stick-slip phenomenon must be defined. These local elements are generically cast as being a combination of a variety of friction mechanisms in the structure such as friction in bolted joints, friction between structural and non-
structural, and in microscopic cracks in the materials, among others. There was some evidence relating the deformation ratio to the frequency and damping ratio, it follows that a portion of these stick-slip friction type mechanisms may in fact be attributed to microcracking in the reinforced concrete elements in parallel with the damper. This hypothesis is explored further in Section 6.0.

### 5.3.4 Global-Local Deformation Relationships Summary

In summary, the local element monitoring provided insights into the local structural behaviours and their possible effects on the computed amplitude-dependent frequency and damping. It was hypothesized that microcracks in the slab may represent a portion of the observed amplitude-dependency of frequency and damping.

### 5.4 Experimental Results and Discussion Summary

The frequency and damping ratio were found to be dependent on the amplitude of motion, with the frequency decreasing with respect to amplitude and the damping increasing. This behaviour supports the established theory that the mechanisms of damping may be attributed to stick-slip elements randomly distributed throughout the structure. The VCDs resulted in an absolute increase in the damping ratio as well as an increase in the degree of amplitude-dependency. As the structure reaches higher amplitudes and the frequency softens due to stiffness losses in stick-slip elements, the proportion of dissipated energy in the damper to elastic stored energy grows, resulting in an increase in the degree of damping amplitude-dependency.

Lastly, it was observed that the ratio of the global-local deformation exhibited an amplitude-dependent trend, indicating that there was more local deformation at higher amplitude per unit of building motion than at low amplitudes. This was a notable finding, showing that the physical deformation of the VCDs and the adjacent members may be represented by a series of stick-slip type mechanisms. It was hypothesized that a portion of these stick-slip elements may be manifested in microcracks in reinforced concrete elements.
6.0 Modelling

While the retrieval of full-scale monitoring data is valuable, key aspects of the data is using the information to inform the numerical modelling of similar structures and to aid structural engineers in the design and analysis process for tall buildings. This chapter reviews:

- Typical approaches taken by structural engineers in the modelling of high-rise buildings.
- VE material models and how the VCD is modelled in commercial software (ETABS).
- Bounded analysis throughout construction sequencing.
- Development of as-built models for two key high amplitude events.
- Sensitivity study regarding the effect of temperature and assumed gross section cracking parameters on the fundamental periods, modal supplemental damping, and global-local deformation ratio.
- Development of calibrated models for two key high amplitude events and estimation of the supplemental damping ratio.

Note that the modelling components of this work discuss the period in lieu of the frequency, as period is more typically used by structural engineers.

6.1 Modelling Background

Before performing the sensitivity studies and building the refined models, a brief background on the finite element monitoring of tall buildings and the modelling of the VCDs is provided.

6.1.1 Typical Modelling Assumptions

Engineers build analytical models of structures in order to calculate loads and deformations in structural elements under a variety of external loading. Design models typically make conservative assumptions in order to ensure that a design is adequately safe under a certain limit state condition. Therefore, it is assumed that the structure has experienced the level of forces and deformations associated with that limit state. Given this approach, models typically deviate from in-situ conditions (Isyumov, et al., 2010), particularly early in the structure’s life when it has likely not been subjected to any significant wind loading.
Chapter 6: Modelling

Structural engineers use often three-dimensional finite element models to analyse and design structures. The model is drawn to accurately represent the geometry of the structure, and various element types are used to model the physical behaviour of the members. Frame elements are used to model columns and beams while shell elements can be used to model floor slabs and walls. The floor system is assumed to be much stiffer in-plane than out-of-plane, and as such is usually modelled as a rigid diaphragm, except for levels containing outriggers. While the shell element formulation is capably of dealing with in-plane deformations, the rigid diaphragm assumptions simplifies the computational effort. Generally, linear elastic modelling is used but more advanced software is also utilized when necessary (i.e. for non-linear time history analyses, detailed local element non-linear modelling, etc.).

The mass is usually lumped at each storey level and includes the self-weight of the structure and some portion of the Superimposed Dead Load (SDL) and the Live Load (LL) based on the state of the structure being investigated in the model. The stiffness of reinforced concrete members is calculated based on the effective stiffness of the gross section properties, which greatly simplifies most analyses. This means that for a reinforced concrete section, the moment of inertia in the numerical model is calculated based on the overall cross section. This leads to a notable challenge in the modelling of reinforced concrete structures; properly assessing the effective stiffness, which is generally related to the degree of cracking of each member at a given deformation state. This is considered in linear elastic models by applying gross section reductions to cross-sectional properties such as the moment of inertia or area to simulate the expected stiffness loss due to cracking. Note that in the remainder of this chapter effective stiffness and cracking are used interchangeably.

Engineers may assess the level of cracking in an iterative nature by applying a set of loads, determining the amount of deformation and relying on experimental evidence, experience, or sophisticated non-linear finite element analyses, determine gross section reductions. ACI 375 provides recommended levels of cracking for wind analysis for SLS and ULS (Table 6.1)

<table>
<thead>
<tr>
<th>State</th>
<th>Columns</th>
<th>T-Beams (Coupling)</th>
<th>Slabs</th>
<th>Shear Walls Uncracked / Cracked</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLS</td>
<td>1</td>
<td>0.5</td>
<td>0.36</td>
<td>1 / 0.5</td>
</tr>
<tr>
<td>ULS</td>
<td>0.7</td>
<td>0.35</td>
<td>0.25</td>
<td>0.7 / 0.35</td>
</tr>
</tbody>
</table>
While these recommendations provide some assistance in assessing cracking, the exact level of cracking is highly uncertain and is further complicated due to variable cracking over the height of the structure. Typically, structural engineering consultants develop their own internal guidelines for cracking levels in reinforced concrete structures for different loading conditions.

There are several other uncertainties associated with structural modelling that are generally neglected in the model:

- **Non-Structural Components:** The effect of non-structural components on the lateral stiffness depends on the density (Satake, Suda, Arawaka, Sasaki, & Tamura, 2003) and nature of partition walls. Additionally, the primary structural deformation mechanisms as well as the allowances provided between structural and non-structural components may be important considerations in the modelling of non-structural components. For example, masonry partitions in a low-rise building may be more important than drywall partitions in a high-rise building.

- **Dynamic Modulus:** Under dynamic loading materials have been shown to have different stiffness’s than when loaded statically (Paulay & Priestley, 2002). For tall, slender buildings this becomes a concern since the dynamic response to wind is a key design condition.

- **Composite Material:** The composite effect of the reinforcement steel in the concrete is generally neglected in typical models (Isyumov, et al., 2010).

- **Soil-Structure Interaction:** The soil that supports the building as well as the foundation type may have some flexibility associated with it (Isyumov, et al., 2010). Additionally, there may be lateral support provided to below grade levels through the backstay effect.

### 6.1.2 Modelling VCDs

The hysteretic behaviour of the viscoelastic material used in the VCDs is strain, frequency, and temperature dependent. There are two primary modelling approaches used to model viscoelastic behaviour including the Kelvin-Voigt Model (KVM) and the Generalized Maxwell Model (GMM). There is a trade off between simplicity and robustness between the two, with the KVM being appropriate only under a certain strain, frequency, and temperature whereas the GMM is frequency independent. Details of each model are provided in subsequent sections, followed by a description of the modelling of VCDs in commercial software.
6.1.2.1 **Kelvin-Voigt Model (KVM)**

The viscoelastic component of the VCD may be modelled using the KVM, which consists of a spring and viscous dashpot in parallel (Figure 6.1).

![Figure 6.1 Kelvin-Voigt Model (KVM).](image)

Since the VE material primarily deforms in shear, the shear stress may be expressed according to Equation 93.

\[ \tau(t) = G_E \gamma(t) + G_C \dot{\gamma}(t) \]  \hspace{1cm} \text{Equation 93}

Where:
- \( G_E \): Shear storage modulus
- \( G_C \): Shear loss modulus
- \( \gamma \): Shear strain
- \( \dot{\gamma} \): Shear strain per second

Typically, VE manufacturers supply the shear storage modulus \( G_E \) and in lieu of the shear loss modulus the loss factor is provided (Equation 94).

\[ \eta = \frac{G_C \omega}{G_E} \]  \hspace{1cm} \text{Equation 94}

Where:
- \( \eta \): Loss factor
- \( \omega \): Natural angular frequency

Given these properties, the elastic stiffness and viscous damping coefficient may be determined for a given frequency, temperature, and strain amplitude according to Equation 95 and Equation 96, respectively.

\[ k = \frac{G_E A}{h} \]  \hspace{1cm} \text{Equation 95}

Where:
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$k$: Elastic stiffness
$A$: Shear area of VE material
$h$: Thickness of shear area

\[ c = \frac{G_c A}{h} \]  

\text{Equation 96}

Where:
$c$: Damping coefficient

This yields the equation of motion associated with the VCD (Equation 97)

\[ F(t) = c\dot{u}(t) + ku(t) \]  

\text{Equation 97}

Where:
$\dot{u}(t)$: Velocity
$u(t)$: Displacement
$F(t)$: Force

The KVM is simple and easy to implement, however, it is only valid if the strain, frequency, and temperature are known and are not expected to vary over the analysis. Given these shortcomings an alternative is to apply the GMM.

6.1.2.2 Generalized Maxwell Model (GMM)

The viscoelastic component of the VCD may also be modelled using the Generalized Maxwell Model (GMM). The GMM contains several ($n$) Maxwell Models (MM), each of which consist of a spring and dashpot in series (Figure 6.2), in parallel with a KVM (Figure 6.3a).

![Figure 6.2 Maxwell Model (MM)](image)

The GMM, consisting of a KVM model in parallel with $n$ Maxwell elements is shown in Figure 6.3a. The GMM requires VE material characterization tests at different temperatures and frequencies which are used to calibrate the modelling parameters (Figure 6.3b). The various parameters of the GMM are determined through a least-squares regression analysis of the
manufacturer specified properties at the specified temperature and frequency according to Equation 98 and Equation 99.

\[
G_E(\omega, T) = G_0 + \sum_{m=1}^{n} \left\{ \frac{\left( \alpha(T,T_{ref}) \omega \psi_{m,ref} \right)^2}{1 + \left( \alpha(T,T_{ref}) \omega \psi_{m,ref} \right)^2} \right\} G_m \]
\[
G_C(\omega, T) = \alpha(T,T_{ref}) \psi_{0,ref} G_0 + \sum_{m=1}^{n} \left\{ \frac{\left( \alpha(T,T_{ref}) \omega \psi_{m,ref} \right)^2}{1 + \left( \alpha(T,T_{ref}) \omega \psi_{m,ref} \right)^2} \right\} G_m \]

Where:
\( \omega \): Angular frequency of excitation
\( T \): VE temperature
\( G_0/G_i \): Calibrated spring stiffness coefficients
\( \beta_0/\psi_i \): Modelling parameters calibrated at the reference temperature
\( T_{ref} \): Reference temperature
\( \alpha_T \) is a shifting function used to adjust the parameters from a reference temperature \( T_{ref} \) to a temperature \( T \) according to Equation 100.

\[
\alpha_T(T,T_{ref}) = \left( \frac{T}{T_{ref}} \right)^p
\]

Where:
\( p \): Calibrated shifting parameter
Note that typical finite element analysis programs are not able to implement the temperature shifting function, which accounts for self-heating of the VE material due to friction of the VE polymers under long-term loading. In lieu of this, bounded analyses at a range of temperatures may be performed to properly assess the effect of temperature on the structure.

While the GMM is more complex than the KVM, it can account for variations in frequency over the duration of the loading. For practical applications in this thesis, since the frequency and temperature were varied in subsequent chapters for the modelling investigation, the GMM was selected as the VE model to simplify the changes required on each iteration of the models.

### 6.1.2.3 Modelling VCD Unit in Commercial Software

The GMM was used to model the VCD assembly in the commercial software ETABS (Computers and Structures Inc., 2018). The VCD model is comprised of two Maxwell Models in parallel with a spring, which provides a balance between simplicity and accuracy. In this case, the spring accounts for the static stiffness of the assembly, and the MMs account for the dynamic response component. An additional spring is connected in series with the GMM, which accounts for the connecting steel element’s stiffness. The kinematics of the deformed VCD and the GMM model is shown in Figure 6.4. The GMM is frequency independent, and by considering VE material properties at appropriate temperatures and small strains the model captures the behaviour well (Montgomery, 2011).

In ETABS, the exponential Maxwell Damper element was used to model the MMs in the VCD (Equation 101).
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\[ f = k d_k = c \dot{d}_c^{c_{exp}} \quad \text{Equation 101} \]

Where:
\( k \): Spring constant
\( d_k \): Deformation across the spring
\( c \): Damping coefficient
\( \dot{d}_c \): Deformation rate across damper
\( c_{exp} \): Damping exponent set to 1 for VCDs.

A linear spring link element was used to model the static stiffness of the VCDs, and frame elements corresponding to the steel connecting components were used to connect the VCD links to the main structure.

### 6.1.3 SLS Design Models

Two SLS models of the test structure were provided by the industry collaborator; one with the temporary steel channels in the VCD locations and one with the VCDs modelled. These models were characterized by typical assumptions for SLS models including a reduced service level mass, 0.8SDL and 0.15LL, and gross section cracking for RC members. These models predicted the periods of the first three modes to be 6.5 seconds, 4.64 seconds, and 3.82 seconds for the first to third mode, respectively.

These service level models were used to:

- Perform a bounded analysis throughout construction sequencing.
- Construct two as-built models for key high-amplitude events on April 4-5, 2018 and May 4, 2018.
- Perform a sensitivity study regarding the effect of temperature and cracking on the dynamic properties and deformation ratio.
- Build calibrated models for April 4, 2018 and May 4, 2018.
- Analyse and understand the structural behaviour and relate it to the experimental results.

### 6.2 Extracting Periods, Supplemental Damping, and Deformation Ratios from ETABS

There were three primary parameters of interest to retrieve from the models to compare to the experimental work:
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1. Periods of the first three modes.
2. Supplemental damping in mode 1 and 3.

Since the VCD elements are modelled using non-linear damping elements and their discrete nature results in a non-classically damped structure, typical linear solvers are not capable of adequately capturing the effect of the VCD in the model. A common means of assessing the effect of discrete dampers is by performing a free vibration analysis (non-linear time history) on an analytical model which accurately represents the structure. While this is effective, isolating coupled modes is difficult [ (Smith R. , 2016), (Christopoulos, Montgomery, & Aiken, 2017)], since initiating a modal response relies on displacing the structure into a deformed shape identical to the mode of interest. While this is simple for pure sway modes, it is difficult to apply effectively to coupled modes in three-dimensional finite element models.

To overcome the challenge of analysing coupled modes the ERA was used to process the MDOF free vibration response of the structure. To ensure that the three fundamental modes of interest were excited, the structure was displaced in both the X and Y direction and rotated. A ramp force was slowly applied to push the structure into the displaced position where the structure was held to allow any transient vibrations to subside, after which the load was suddenly removed to initiate the free-vibration response (Figure 6.5).

![Free Vibration Loading Protocol](image)

**Figure 6.5** Free vibration loading protocol for numerical assessment.

To ensure that the only source of energy dissipation in the model was the VCDs, all possible sources of damping, including modal and material, were set to zero. Using the ERA, the periods and supplemental damping ratios were evaluated. The global-local deformation ratio was evaluated in the first mode only. Since the first mode was primarily sway in the X-direction (and
as such is uncoupled) the same free vibration loading protocol (Figure 6.5) as the ERA was used, however, the force was applied laterally (X-direction) only, to initiate a free vibration response in the first mode. The tip deflection and the shear deformation across the VCD link elements were plotted and used to compute the deformation ratio. An example is provided in Appendix VI.

### 6.3 Construction Sequencing Bounded Analysis

Models were constructed reflecting approximate as-built conditions throughout the construction sequencing monitoring. Since it was difficult and complex to construct best-estimate as-built models, primarily due to the uncertainty of the effective stiffness of structural members and to a lesser extent mass, as was performed in Section 6.4, a bounded analysis was performed. Baseline increases of 20% were applied to the modulus of elasticity for all concrete elements, (in line with expectations from as-built models and used by Isyumov et al. (2010)), and four models were built for several days throughout the construction sequencing monitoring (Table 6.2). The models varied between considering no mass and no cracking to SLS level mass and cracking as per ACI 375 recommendations (ACI 375, 2004). Note that effective stiffness modifications were applied uniformly over the height of the building. For this analysis, no distinction was made between the reduced slabs above the VCDs and the remainder of the slabs in the structure (i.e. Slab refers to all of the slabs in the structure).

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass</th>
<th>Cracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETABS 1</td>
<td>No LL or SDL</td>
<td>No cracking</td>
</tr>
<tr>
<td>ETABS 2</td>
<td>No LL or SDL</td>
<td>0.36 Slab, 0.5 CB</td>
</tr>
<tr>
<td>ETABS 3</td>
<td>0.15 LL + 0.8 SDL</td>
<td>No cracking</td>
</tr>
<tr>
<td>ETABS 4</td>
<td>0.15 LL + 0.8 SDL</td>
<td>0.36 Slab, 0.5 CB</td>
</tr>
</tbody>
</table>

Before the VCDs were installed, the Modal Ritz method was used to solve for the periods whereas after, the previously discussed ERA method was used. The results from the bounded analysis are shown in Figure 6.6 and Figure 6.7, while Figure 6.8 shows the different finite element models constructed to represent different stages of construction.
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Figure 6.6 Bounded modelling analysis through construction sequencing.

Figure 6.7 Difference between bounded analysis models and experimental frequencies.
The results shown in Figure 6.6 and Figure 6.7 show that model one and two are the closest estimates for the first mode frequency, and model two and three are the closest for the second and third mode frequencies. This follows since the expected mass while the structure was under construction was less than the service level, and the member effective stiffness was expected to be between nominal and service levels. This was further explored in Section 6.6.

This exercise adds confidence to the quality of current state-of-the-art finite element modelling methods used by structural engineers. With minor changes to more accurately represent the in-situ condition, the four models considered bound the experimental results well. This indicates that the fundamental modelling assumptions discussed in Section 6.1.1 adequately capture the physical behaviours of the structure, and that further refinements should focus on more difficult parameters to quantify such as effective stiffness. Note that additional models of this nature were constructed to determine the vertical mode shape projections used in Section 5.2.3.
6.4 As-Built Models

The provided SLS models were adjusted to the as-built condition for the two key high-amplitude events on April 4, 2018 and May 4, 2018. The flow of the development of these models is shown in Figure 6.9.

![Flow chart for as built model development.](image)

The objective of the construction of the as-built model was to make reasonable assumptions to model the as-built condition of the structure, avoiding the typical conservative assumptions made for design models. The following subsections detail the implementation of the construction of the May 4, 2018 as-built condition. For brevity, the April 4-5, 2018 development is not included, only the results are discussed.

6.4.1 Stage#1: Geometry/Baseline Stiffness Adjustments

Since the structure was not complete on these days, the first step in building the as-built model was to model the current construction progress on the day in question (Figure 6.10). On May 4, 2018, the slab on level 62 had been poured, and several of the walls/columns had been formed/poured.
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In addition to removing the incomplete structure, all stiffness modifiers were set to 1 to create a baseline model which could be varied in later stages. Additionally, the below-grade levels were laterally braced to simulate the backstay effect.

The status of the VCD installation is shown in Table 6.3. The VCDs were partially installed on April 4-5, 2018 and were full installed on May 4, 2018.

Table 6.3  VCD installation levels on key monitoring days.

<table>
<thead>
<tr>
<th>Level</th>
<th>April 4-5, 2018</th>
<th>May 4, 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>35 – 39</td>
<td>Channels in</td>
<td>VCDs Installed</td>
</tr>
<tr>
<td>34</td>
<td>Channels out</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>Channels in</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Channels out</td>
<td></td>
</tr>
<tr>
<td>30-31</td>
<td>VCDs In, Locked</td>
<td></td>
</tr>
<tr>
<td>19-29</td>
<td>VCDs Installed</td>
<td></td>
</tr>
</tbody>
</table>

For the levels where the channels were still installed, frame elements were used to model the channels. Where they were removed, these frame elements were deleted from the model. On levels where the VCDs were installed but the temporary lock-up plates (used for shipping) were still
installed, they were simply modelled as typical VCD elements. It was assumed that the additional stiffness from the thin lock-up plates would be small in comparison to the stiffness of the overall VCD system. Lastly, where VCDs were observed to have been installed VCD elements were used in the model.

6.4.2 Stage#2: Mass

An audit of the structure was complete on the monitoring days to assess the level of completion of non-structural components (partitions, cladding, etc.). The progression of finishes installation is shown in Figure 6.11. Note that the difference between these categories was subjective and was based on the researcher’s observations on site at the time of inspection. Photos indicating the levels of finishes associated with each tier are shown in Figure 6.12.

**Progression of Finishes Installation**

![Progression of Finishes Installation](image)

*Figure 6.11 Progression of finishes installation.*
The selection of the peak assumed SDL of 0.1 kPa was based on rough calculations approximating the amount of finishes on a typical floor, as well as recommendations from the HBSC (CISC, 2016). The calculations of these loads are shown in Appendix VII. The level of non-structural components was split into four tiers based on observations (Table 6.4).

Table 6.4 Load assumptions for non-structural finishes.

<table>
<thead>
<tr>
<th>Tier</th>
<th>1: Skeleton</th>
<th>2: Some Walls</th>
<th>3: Most Walls</th>
<th>4: All Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (kPa)</td>
<td>0.025</td>
<td>0.05</td>
<td>0.075</td>
<td>0.1</td>
</tr>
</tbody>
</table>

These loads were applied on each level range as indicated in Figure 6.11 as uniformly distributed loads across the interior floor surface. Note that no load was applied on the balconies. In addition to the approximated SDL, the cladding design load was applied on levels where the cladding was observed to have been installed (as per Figure 6.11). Finally, an additional mass of 1130 kN (as per on-site crane drawings) was added to the structure at the location of the crane. No other additional gravity loads were considered in the model.
6.4.3 Stage#3: Modulus of Elasticity

The last parameter for building the as-built model was to update the modulus of elasticity of the concrete. This involved considering the in-situ concrete strength, aging effects, strain rate effects, and composite action from the steel reinforcement.

6.4.3.1 In-Situ Concrete Strength

Specified concrete strengths and their associated moduli were used in the design model, however, as is common the in-situ concrete strength exceeded the specified, in some cases significantly. The in-situ cast concrete strengths were obtained from the industry collaborator and were averaged to produce representative concrete strengths by element and storey bin. Each storey is reported as an average of several pours for each element. Since there were four different concrete strengths specified for vertical elements, decreasing in strength at higher stories, the average for each element (slabs and verticals) was computed over that storey range. This produced nine different concrete strengths that were used in the as-built model (Table 6.5).

Table 6.5 Average in-situ concrete strengths.

<table>
<thead>
<tr>
<th>Strength (MPa)</th>
<th>Slabs</th>
<th>Verticals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P4-15</td>
<td>16-29</td>
</tr>
<tr>
<td>57.1</td>
<td>50.7</td>
<td>49.9</td>
</tr>
</tbody>
</table>

6.4.3.2 Time-Dependency of Concrete Strength

The in-situ concrete strengths were used to calculate approximate elastic moduli. First, the average time since casting for each regime was calculated (Table 6.6).
Table 6.6  Average number of days since casting concrete for each regime.

<table>
<thead>
<tr>
<th>Element</th>
<th>April 4, 2018</th>
<th>May 4, 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slabs (P4-15)</td>
<td>514</td>
<td>544</td>
</tr>
<tr>
<td>Slabs (16-29)</td>
<td>278</td>
<td>308</td>
</tr>
<tr>
<td>Slabs (30-45)</td>
<td>185</td>
<td>215</td>
</tr>
<tr>
<td>Slabs (46-63)</td>
<td>58</td>
<td>88</td>
</tr>
<tr>
<td>45 MPa Verticals</td>
<td>58</td>
<td>88</td>
</tr>
<tr>
<td>55 MPa Verticals</td>
<td>185</td>
<td>215</td>
</tr>
<tr>
<td>65 MPa Verticals</td>
<td>278</td>
<td>308</td>
</tr>
<tr>
<td>75 MPa Verticals</td>
<td>514</td>
<td>544</td>
</tr>
</tbody>
</table>

Using this time and the time-dependent strength of concrete from ACI (ACI 209, 2008) the compressive strength on the day in question was computed (Equation 102).

\[
(f'_c) t = \frac{t}{\alpha + \beta t} (f'_c)_{28}
\]  
Equation 102

Where:

- \((f'_c) t\): Compressive strength of concrete at any time \(t\)
- \(\alpha\): Constant – 4.0 for Moist Cured Type I cement (assumed)
- \(\beta\): Constant – 0.85 for Moist Cured Type I cement (assumed)
- \(t\): Time (days)
- \((f'_c)_{28}\): Compressive strength of concrete at 28 days (MPa)

It was found that the difference in computed compressive strength between each of the key dates was minimal, so the average compressive strength was calculated and used on both days.

### 6.4.3.3 Modulus of Elasticity of High-Strength Concrete

The recommended relationship for modulus of elasticity for high strength concrete (HSC) from ACI 363 (ACI 363, 2010) based on compressive strength was used to calculate the updated modulus of elasticity for each regime (Equation 103).

\[
E_c = 3320(f'_c)^{0.5} + 6900 \text{ (MPa)} \text{ for } 21 \text{ MPa} < f'_c < 83 \text{ MPa}
\]  
Equation 103
6.4.3.4 Strain Rate Effects

Since the structure is loaded dynamically, the dynamic amplification factor for the modulus of elasticity from Paulay and Priestly (2002) was used (Figure 6.13) to approximate dynamic strain rate effects. An increase of 2.5% (Dynamic Magnification Factor of 1.025) to approximate the strain rate effects in the concrete was used. Note that the concrete strengths under consideration significantly exceed those tested by Paulay and Priestly (2002) and the amplitude range was much less. This was used as approximate indication of the dynamic strain rate effects.

![Figure 6.13 Dynamic Magnification Factor for strain rate effects in concrete (Paulay & Priestley, 2002)](image)

6.4.3.5 Effect of Reinforcement (Composite Action)

Lastly, the effect of composite action in reinforced concrete was considered according to Equation 104.

\[
E_{RC} = E_c (1 - \rho) + E_s \rho
\]

Equation 104

Where:

- \(E_{RC}\): Modulus of elasticity for reinforced concrete
- \(\rho\): Reinforcement Ratio (assumed to be 1% in slabs and 2% in verticals)
- \(E_s\): Steel modulus of elasticity, assumed to be 200 GPa

6.4.3.6 Overall In-Situ Modulus of Elasticity

Each of these effects resulted in an increase in the modulus of elasticity for the in-situ condition compared to the design values (Table 6.7).
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Table 6.7  Comparison between design and as-built model modulus of elasticity for each element.

<table>
<thead>
<tr>
<th>Element</th>
<th>Design (MPa)</th>
<th>Final As-Built (MPa)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slabs (P4-15)</td>
<td>30266</td>
<td>36574</td>
<td>20.8</td>
</tr>
<tr>
<td>Slabs (16-29)</td>
<td>30266</td>
<td>34962</td>
<td>15.5</td>
</tr>
<tr>
<td>Slabs (30-45)</td>
<td>30266</td>
<td>34756</td>
<td>14.8</td>
</tr>
<tr>
<td>Slabs (46-63)</td>
<td>30266</td>
<td>34615</td>
<td>14.4</td>
</tr>
<tr>
<td>Slabs</td>
<td>30266</td>
<td>36574</td>
<td>15.3</td>
</tr>
<tr>
<td>45 MPa Verticals</td>
<td>31070</td>
<td>36994</td>
<td>19.1</td>
</tr>
<tr>
<td>55 MPa Verticals</td>
<td>33570</td>
<td>40189</td>
<td>19.7</td>
</tr>
<tr>
<td>65 MPa Verticals</td>
<td>35850</td>
<td>41453</td>
<td>15.6</td>
</tr>
<tr>
<td>75 MPa Verticals</td>
<td>37960</td>
<td>42471</td>
<td>11.9</td>
</tr>
</tbody>
</table>

The percent difference between the design and as-built modulus of elasticity is in line with that of the estimate of 1.2 used by Isyumov et al. (2010), and adds support to the increased moduli used in Section 6.3 for the construction sequencing bounded analysis.

6.4.4  As-Built Model Results

The results from each stage of the construction of the as-built model (Figure 6.9) for both April 4-5, 2018 and May 4, 2018 are shown in Table 6.8. Note that an initial assumed temperature of 22 °C was used for both events. Note that model number refers to the changes indicated in Figure 6.9.

Table 6.8  As-built model results.

<table>
<thead>
<tr>
<th>Date</th>
<th>April 4-5, 2018</th>
<th>May 4, 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model Number</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1st Mode Period (s)</td>
<td>5.04</td>
<td>4.79</td>
</tr>
<tr>
<td>2nd Mode Period (s)</td>
<td>3.32</td>
<td>3.16</td>
</tr>
<tr>
<td>3rd Mode Period (s)</td>
<td>2.86</td>
<td>2.72</td>
</tr>
</tbody>
</table>

The final as-built models (Model#3) are a best representation of the structure on the day of monitoring and were used in a sensitivity study to understand the effect of temperature and cracking and to build a calibrated model for the purpose of supplemental damping evaluation.
6.5 Sensitivity Study

To investigate the effects of temperature and effective stiffness, a sensitivity study was conducted. First, the temperature was varied to establish a best fit to the experimental data. After selecting an expected temperature, the effective stiffness of each element was varied in isolation between the nominal stiffness and the SLS level cracking recommended in Table 6.1.

6.5.1 Temperature

The as-built models were studied to understand the effect of temperature on the estimation of supplemental damping, period, and deformation ratio. As noted previously, temperature plays a key role in the VE properties of the VCD, which translates to notable effects on the supplemental damping and natural periods. On the key event days, the temperature distribution over the record was difficult to accurately quantify for several reasons:

- **Ambient temperature variation.** The ambient temperature measured at the Toronto Island Airport (Government of Canada, 2018) for each of the key event days is shown in Figure 6.14 and Figure 6.15.

- **Inconsistent heating in the building due to localized heaters.** Localized heaters were present throughout the structure and operated by on-site workers to maintain workable environments in their current area of work. These were intermittently dispersed throughout the structure, and inconsistently used. Note that there was no observation of localized heaters being used during the May 4, 2018 event.

![April 4-5, 2018: Ambient Temperature](image)

*Figure 6.14  April 4-5, 2018 ambient temperature.*
Given this uncertainty, VCD properties at temperatures between 0 °C and 28 °C were investigated. The effect of temperature on the first and third mode periods, damping ratios, and the global-local deformation ratio for both April 4-5, 2018 and May 4, 2018 are shown in Figure 6.16 through Figure 6.19.

Figure 6.15  April 4-5, 2018 and May 3-4, 2018 ambient temperatures.

Figure 6.16  Period temperature sensitivity for April 4-5, 2018.
Figure 6.17 Supplemental damping ratio and deformation ratio temperature sensitivity for April 4-5, 2018.

Figure 6.18 Period temperature sensitivity for May 4, 2018.

Figure 6.19 Supplemental damping ratio and deformation ratio temperature sensitivity for May 4, 2018
As the temperature increases, the period of mode one and three elongated as the dampers became softer. As they became softer, the amount of overall structural deformation per unit deformation in the VCD decreased, indicating that there was more motion in the VCD. Over the temperature range considered, both the first and third mode damping ratios showed an initial increase, followed by a subsequent decrease. This is reflective of the design temperature where the dampers (VE material) were optimized to provide the peak damping at room temperature, at approximately 22 °C. Since the application of the VCD is primarily in buildings, the temperature is expected to remain well-controlled. For the cracking sensitivity study, a temperature of 10 °C was selected for the VCD properties for May 4, 2018.

6.5.2 Effective Member Stiffness

A key uncertainty with respect to structural modelling is the gross section reductions applied to simulate cracking in various structural elements. To understand the effect of these assumptions, several elements were cracked between nominal (1) and the ACI SLS effective stiffness level (Table 6.1) in isolation (while all other remained at nominal effective stiffness) and the effect on period, supplemental damping ratio, and deformation ratio was studied. These elements are shown in Figure 6.20. Note that X-CB refers to coupling beams in the X-direction, Y-CB refers to coupling beams in the Y-direction, CB refers to all coupling beams, and slabs (general) refers to all slabs except the highlighted regions above the VCDs. For brevity, only the results for May 4, 2018 are presented (Figure 6.21, Figure 6.22, Figure 6.23).
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**1st Mode Period**

![Graph showing effective stiffness sensitivity study period results for the 1st mode.](image1)

**2nd Mode Period**

![Graph showing effective stiffness sensitivity study period results for the 2nd mode.](image2)

**3rd Mode Period**

![Graph showing effective stiffness sensitivity study period results for the 3rd mode.](image3)

Figure 6.21 Effective stiffness sensitivity study period results. Each element’s effective stiffness was varied while all others held constant at 1.
The results indicated:

- Slabs above the VCDs had a minimal effect on the period in the first three modes. Since the slabs above the VCD represented a small portion of the total slab area, this small effect on the period was expected.
- The first mode period was primarily dependent on the effective stiffness of the general slab.
- The second and third period modes had similar dependencies to the different effective stiffness parameters. First, the coupling beams overall had the most significant effect on the period, with the Y-CB dominating this dependence over the X-CB. Following this the slab had a significant effect on the period in modes one and three. This is an indication that the deformation mechanisms between the two modes were similar in nature.

Figure 6.22 shows the effect of effective stiffness variations on the supplemental damping ratio in the first and third mode.

![Graph showing the effect of effective stiffness variations on the supplemental damping ratio in the first and third mode.](image)

Figure 6.22 Effective stiffness sensitivity study damping results. Each element’s effective stiffness was varied while all others held constant at 1.

The results indicate:

- Damping in the first and third mode was primarily governed by the effective stiffness of X-CB. As the X-CB effective stiffness was reduced, an increasing proportion of the coupling force in the X-direction lateral system was taken by the VCDs, therefore
increasing the energy dissipated while simultaneously reducing the elastic stored energy, resulting in an increased supplemental damping ratio.

- The slabs above the VCDs had the next critical effect on the supplemental damping, followed by the general slabs throughout the structure. Reducing the effective stiffness in the slabs above the VCD resulted in a drop in the stiffness of the elements adjacent to the VCD (in parallel), therefore resulting in an increased proportion of the force in the coupled area (VCD and adjacent slab) between the walls being transmitted through the VCD, thus increasing the damping. This was similar to the effect observed for the X-CB.

- The decrease in the damping ratio in mode 3 as the Y-CBs effective stiffness is reduced was unexpected. Typically, it may be assumed that any decrease in effective stiffness will produce an increase in the supplemental damping when VE dampers are used. However, in this case reducing the effective stiffness of the Y-CB softens the structure in the Y-direction, shifting the third mode shape to be more like the second mode, where the dampers have a negligible effect. This is a product of varying the cracking of the elements in isolation, as it is unlikely that in an actual structure only the Y-CB would be cracked.

Figure 6.23 shows the effect of effective stiffness variations on the deformation ratio.

Figure 6.23  Effective stiffness sensitivity study deformation ratio results. Each element’s effective stiffness was varied while all others held constant at 1.
These results indicated:

- The X-CB had the most significant effect on the deformation ratio, followed by the slabs above the VCDs. This dependency is similar in nature to that of the first mode supplemental damping ratio.

Overall, these results indicate that increased cracking in elements in parallel with the VCDs resulted in an increased efficiency of the damper. Functionally, this follows as a reduction in the elastic stiffness coupled with an increase in the energy dissipated (due to more proportional force being driven across the damper) both are proportional to increases in overall equivalent viscous damping. This may be explained by inspecting the equivalent viscous damping equation considering only energy dissipated by the VCDs (Equation 105).

\[
\zeta_{eq} = \frac{1}{4\pi} \frac{E_{VCD}}{E_s^0}
\]

Where:

- \( \zeta_{eq} \): Equivalent viscous damping
- \( E_{VCD} \): Energy dissipated by VCDs
- \( E_s^0 \): Stored elastic energy

If only the damping due to the VCDs are considered as energy dissipation sources, then when elements in parallel lose stiffness, either due to stick-slip mechanisms or due to irrecoverable cracking and effective stiffness reductions, the stored elastic component in the denominator decreases. All else equal, the decrease in stored elastic energy with constant dissipated energy results in an increase in the equivalent viscous damping.

This concept was identified and considered by the designers through the reduction of the slab thickness above the VCDs and the introduction of the notch detail to drive a higher proportion of the force into the VCD. A side-effect of the cracking of coupling beams in parallel with the VCDs is an even more significant increase in the supplemental damping. This is due to the increased proportion of the force being driven through the VCDs, resulting in an increase the energy dissipation. However, the reduction in coupling beam stiffness corresponds to significant softening of the lateral system and period elongation, which may increase overall and inter-storey drift and may be undesirable.
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6.6 Model Calibration

Using the as-built models and the insight gained from the sensitivity study, calibrated models for the two key high-amplitude events were constructed. Calibrated models were built by selecting temperature and cracking values such that the first three modes and the deformation ratio were all within +/- 5% of the experimental averages from the peak 5 millimetres from the RDT results (Table 6.9).

Table 6.9 Targeted average properties on key high amplitude event days for as-built models.

<table>
<thead>
<tr>
<th>Date</th>
<th>1st Mode Period (s)</th>
<th>2nd Mode Period (s)</th>
<th>3rd Mode Period (s)</th>
<th>Deformation Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 4-5, 2018*</td>
<td>4.31</td>
<td>3.20</td>
<td>2.62</td>
<td>110.14</td>
</tr>
<tr>
<td>May 4, 2018**</td>
<td>4.72</td>
<td>3.46</td>
<td>2.82</td>
<td>189.66</td>
</tr>
</tbody>
</table>

* Range: 10.5 mm – 15.5 mm for April 4-5, 2018.  
** Range: 20.5 mm – 25.5 mm for May 4, 2018

Note that the objective of the calibration was to produce a model that accurately represented the experimental results. The calibrated model produced was used to study the estimated supplemental damping and to understand the overall structural behaviours as they relate to the experimental observations.

6.6.1 Temperature

The expected temperature for the construction of the calibrated model was selected based on consideration of the environmental condition and the deformation ratio. Since cracking the structure will decrease the deformation ratio, a temperature which produced a deformation ratio higher than the average was selected. The selected temperature for each day compared with the average ambient temperature and the corresponding deformation ratios are shown in Table 6.10 with reference to the temperature plots shown in Figure 6.14 and Figure 6.15.

Table 6.10 Temperature selection for VCD properties for calibrated models.

<table>
<thead>
<tr>
<th>Date</th>
<th>Ambient Temperature*</th>
<th>Ambient Temperature**</th>
<th>Experimental Deformation Ratio</th>
<th>Selected Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>April 4-5, 2018</td>
<td>3.8 °C</td>
<td>-1.6 °C</td>
<td>110.14</td>
<td>20 °C</td>
</tr>
<tr>
<td>May 4, 2018</td>
<td>12.6 °C</td>
<td>12.5 °C</td>
<td>189.66</td>
<td>12 °C</td>
</tr>
</tbody>
</table>

* 12 hours before monitoring commenced  
** During monitoring
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For the April 4-5, 2018 event the temperature was difficult to assess due to the intermittent usage of localized heaters. However, based on qualitative observations on site the working conditions were comfortable, indicating a temperature much greater than the ambient temperature of -1.6 °C during monitoring. With consideration given to the deformation ratio, 20 °C was selected as the calibrated temperature for the April 4-5, 2018 event. For the May 4, 2018 the ambient temperature was more likely to be representative of the onsite conditions and the temperature of the VCDs, as there were not observations of localized heaters being used during monitoring. Considering the 12 hours before monitoring and over the course of the 10-hour monitoring session the average temperature remained constant at approximately 12 °C, and with attention to the deformation ratio, 12 °C was selected as the calibrated temperature for the May 4, 2018 event.

6.6.2 Cracking Calibration

The calibration process is shown in Figure 6.24 and the reference for the cracking parameters in each model is shown in Table 6.11. Note that VCD refers to the 150-millimetre-thick slabs immediately above the VCDs, SLAB refers to all other slabs in the structure, and CB refers to reinforced concrete coupling beams.

![Figure 6.24 Model calibration progression.](image)
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Table 6.11 Model calibration cracking parameters.

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Parameter</th>
<th>April 4-5, 2018</th>
<th>May 4, 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical</td>
<td>Experimental</td>
</tr>
<tr>
<td>April 4-5, 2018</td>
<td>Baseline</td>
<td>VCD -0.4</td>
<td>VCD - 0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SLAB -1</td>
<td>SLAB - 0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CB - 1</td>
<td>CB - 1</td>
</tr>
<tr>
<td>May 4, 2018</td>
<td>Baseline</td>
<td>VCD -0.4</td>
<td>VCD - 0.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SLAB -1</td>
<td>SLAB - 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>CB - 1</td>
<td>CB - 1</td>
</tr>
</tbody>
</table>

Note that there are other combinations of cracking that could produce similar results, and no consideration was given to variable cracking over the height of the structure. The final calibrated model for April 4-5, 2018 was model 5 and for May 4, 2018 was model 4.

The model calibration was successful as each of the four calibration parameters was within +/- 5% of the experimental results. Each of the effective stiffness parameters was found to be greater than the SLS levels. Given that the building was newly constructed and had not yet experienced an SLS design level event, it follows that the effective stiffness would not yet be at SLS levels, which is supported by the calibration results.

6.6.3 Calibration Results

The results for the final calibrated model are shown in Table 6.12.

Table 6.12 Calibrated model results

<table>
<thead>
<tr>
<th>Mode</th>
<th>Parameter</th>
<th>April 4-5, 2018</th>
<th>May 4, 2018</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Analytical</td>
<td>Experimental</td>
</tr>
<tr>
<td>1</td>
<td>Period (s)</td>
<td>4.49</td>
<td>4.31</td>
</tr>
<tr>
<td></td>
<td>Supplemental Damping (%)</td>
<td>0.64</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Deformation Ratio</td>
<td>109.53</td>
<td>110.14</td>
</tr>
<tr>
<td>2</td>
<td>Period (s)</td>
<td>3.06</td>
<td>3.20</td>
</tr>
<tr>
<td></td>
<td>Supplemental Damping (%)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Period (s)</td>
<td>2.43</td>
<td>2.62</td>
</tr>
<tr>
<td></td>
<td>Supplemental Damping</td>
<td>0.85</td>
<td>-</td>
</tr>
</tbody>
</table>

The results from the model calibration indicate a supplemental damping in the first mode and third mode of approximately 0.88% and 0.85%, respectively, on the May 4, 2018 event when all the VCD units had been installed at the average amplitude-dependent properties in the peak amplitude range (frequency, deformation ratio).
6.6.4 Discussion for Design and Performance

The targeted supplemental damping from the system was 0.9% assuming an inherent damping ratio of 1.5% for a total first mode damping of 2.4%.

While the damping ratio predicted from the calibrated numerical model for May 4, 2018 for the first mode fell just below the target, there are three important considerations to this fact. First, the structure was not complete on the monitoring days and was not subjected to service level mass. Second, the baseline stiffness was higher than the design values and the effective stiffness had not yet reached SLS levels. Referring to the effective stiffness sensitivity study (Section 6.5.2), the results indicated that at the expected levels of cracking at higher amplitude events the supplemental damping ratio would be higher than that found for the May 4, 2018 event. Lastly, the operational temperature of the VCD (12 °C) was below the optimal temperature of 22 °C, where the expected supplemental damping was maximized. Adjusting the VCD properties to reflect an operating temperature of 22 °C in the calibrated May 4, 2018 model (Model 4) produced supplemental damping ratios of 1.00% and 1.16% in the first and third mode, respectively.

To further investigate how the measured performance compared to the design intent, the results from the calibrated study were extended to estimate the final supplemental damping ratio for the entire structure at SLS cracking. A model of the full building was modified to more accurately represent the conditions for the final as-built structure.

- **Mass**: Typical SLS mass of 0.85 SDL and 0.15 LL was used to model the serviceability limit.
- **Concrete Modulus**: Updated moduli and vertical distribution from Table 6.7 were used.
- **Effective Stiffness**: SLS effective stiffness (Table 6.1).
- **Temperature**: A typical room temperature of 22 °C was used.

The results from this model are shown in Table 6.13 and are compared to the results from the design predictions.
### Table 6.13 Final period and damping estimations for as-built SLS model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Parameter</th>
<th>Full Building</th>
<th>SLS Design (Montgomery &amp; MacLean, 2014)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>As-Built SLS (22 C°)</td>
<td>20 C°</td>
</tr>
<tr>
<td>1</td>
<td>Period (s)</td>
<td>5.85</td>
<td>6.51</td>
</tr>
<tr>
<td></td>
<td>Supplemental Damping (%)</td>
<td>1.29</td>
<td>1.29</td>
</tr>
<tr>
<td>2</td>
<td>Period (s)</td>
<td>4.00</td>
<td>4.64</td>
</tr>
<tr>
<td></td>
<td>Supplemental Damping (%)</td>
<td>0.0237</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>Period (s)</td>
<td>3.33</td>
<td>3.80</td>
</tr>
<tr>
<td></td>
<td>Supplemental Damping (%)</td>
<td>1.35</td>
<td>1.34</td>
</tr>
</tbody>
</table>

The periods from the final as-built model for the full structure were less than those predicted at the design stage, which may primarily be attributed to the modulus of elasticity of the reinforced concrete and to a lesser degree the effective stiffness reductions. For analysis models at the design stages, the in-situ modulus should be used, which can be approximated as 1.2 E_c (design modulus). The damping ratios from the final as-built model for the full structure matched those predicted at the design stage well and exceeded the target of 0.9%. This follows since cracking is the primary mechanism which controls the supplemental damping, and SLS level cracking was used in the final model. With reference to the experimental results, a total damping of 2.61% was achieved at a displacement of 22.25 millimetres, thereby meeting and exceeding the target for total damping.

### 6.7 Mode Shapes

Since the VCD elements include non-linear elements, the Modal Ritz method was not suitable to determine the mode shapes. In lieu of this, the ERA was applied to a non-linear time history free vibration response in a similar manner to earlier. Four time-histories were output on every fifth floor to develop vertical mode shapes. To represent the torsional component in the same way as the lateral, the arc length (rθ) was computed (Figure 6.25). The vertical mode shapes computed using the ERA for the May 4, 2018 calibrated model are shown in Figure 6.26.
The mode shapes were found to be primarily X, primarily Y + Torsion, and primarily Torsion + Y for the first, second, and third mode, respectively. The second and third mode were found to be coupled and to have similar mode shapes, indicating that both modes contain similar deformation mechanisms.

This model was further validated by comparing the vertical mode shapes to those obtained from the multi-floor monitoring on May 4, 2018 (Section 5.1.4). A comparison of the mode shape
ordinates between level 29 and 47 are shown in Table 6.14. Note that the model results reference the exact location where the accelerometers were placed during the test to ensure a direct comparison.

Table 6.14 Comparison of mode shapes between calibrated model and multi-floor monitoring for May 4, 2018

<table>
<thead>
<tr>
<th>DOF</th>
<th>Analytical</th>
<th></th>
<th></th>
<th>Experimental</th>
<th></th>
<th></th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode 1</td>
<td>Mode 2</td>
<td>Mode 3</td>
<td>Mode 1</td>
<td>Mode 2</td>
<td>Mode 3</td>
<td></td>
</tr>
<tr>
<td>L47 [u1]</td>
<td>0.01</td>
<td>0.50</td>
<td>-1</td>
<td>0.01</td>
<td>0.46</td>
<td>-1.00</td>
<td>0.84</td>
</tr>
<tr>
<td>L47 [u2]</td>
<td>1</td>
<td>-1</td>
<td>-0.98</td>
<td>1.00</td>
<td>-1.00</td>
<td>-0.94</td>
<td>1.00</td>
</tr>
<tr>
<td>L29 [u3]</td>
<td>0.42</td>
<td>0.09</td>
<td>0</td>
<td>0.44</td>
<td>0.04</td>
<td>0.04</td>
<td>0.96</td>
</tr>
<tr>
<td>L29 [u4]</td>
<td>0</td>
<td>0.30</td>
<td>-0.57</td>
<td>0.01</td>
<td>0.25</td>
<td>-0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>L29 [u5]</td>
<td>0.45</td>
<td>-0.43</td>
<td>-0.59</td>
<td>0.46</td>
<td>-0.47</td>
<td>-0.48</td>
<td>0.99</td>
</tr>
</tbody>
</table>

These results indicate that the vertical mode shapes show good agreement between the experimental results and the calibrated analytical model, further validating the calibration and adding confidence to the supplemental damping estimation.

6.7.1.1 Degree of Cantilever Action

Based on the calculated mode shapes and the CFSMP degree of cantilever action model (Section 2.2.2.3), the estimated damping ratio was calculated. This estimation represents the expected level of inherent damping for the structure based on the degree of cantilever action reflected in the mode shape for the first two modes. The third mode was not investigated since the primary action was torsion, and the model was developed based on primarily sway modes only. The primary component of the vertical mode shapes are plotted again in Figure 6.27 against ideal cantilever and shear mode shapes.
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The calculations using the CFSMP model are shown in Table 6.15 (Bartolini & Kijewski-Correa, 2017).

Table 6.15 Degree of cantilever action calculations for first and second mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Degree of Cantilever Action</th>
<th>Time Domain Estimation</th>
<th>Frequency Domain Estimation</th>
<th>Average Damping Ratio Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.67</td>
<td>0.48%</td>
<td>0.95%</td>
<td>0.72%</td>
</tr>
<tr>
<td>2</td>
<td>0.29</td>
<td>0.92%</td>
<td>1.36%</td>
<td>1.14%</td>
</tr>
</tbody>
</table>

Qualitatively, the first mode is closer to an ideal cantilever than an ideal shear deformation while the second mode is a more shear type deformation. This is supported by the degree of cantilever calculations, indicating the first mode has a higher degree of cantilever than the second mode.

The average damping ratio estimates from the time and frequency domain models were 0.72% in the first mode and 1.14% in the second mode, which were in the approximate range of the estimated values from the before the installation of the VCDs from Section 5.2.5.
6.7.1.2 Local Forces and Stresses in Each Mode

The last element explored in the numerical work was the local forces and stresses in the structural elements as a function of the mode shape. The body of this work up to this point has indicated that the primary deformation mechanisms are indeed related to the inherent damping. To push beyond the use of the vertical mode shapes only, the implications in terms of local element forces and stresses should be explored in further detail. It follows that the microscopic material imperfections and cracks in a reinforced concrete structure would be more activated in regions of high stresses and deformation. This work provides a brief overview of the local stresses in the floor slab and forces in coupling beam obtained from the calibrated model from the May 4, 2018 event in the first three modes. These elements were selected since they showed the most significant effect on the parameters investigated in the sensitivity study.

A key limitation in this section was practical limitations imposed by the software used (ETABs). Full time-histories of slab stresses or frame forces were not available, instead only the maximums obtained for each element were reported, regardless of whether these occur at the same time. To work around this, the structure was statically pushed into the deformed shape of each mode, referencing the vertical mode shapes from Section 6.7. This method was used simply to qualitatively investigate the force and stress distribution in key structural elements in each mode. The distribution of the slab stresses and bending moment in the coupling beams on level 29 from the May 4, 2018 calibrated model are show in Figure 6.28.
This result adds further support to several conclusions that have been drawn earlier in this thesis:

- First mode frequency was governed by the slab behaviours. Since the slabs showed a high stress distributed throughout the floor plate area, it follows that reductions in the effective stiffness in this key element resulted in changes in the frequency.

- The dependency of the second and third mode frequency on the coupling beams, particularly the X-CB was highlighted by the larger relative moments obtained in these coupling beams compared to the Y-CB.

- Given the observed dependency of the first mode frequency on the slab stresses, it follows that the subsequently observed amplitude-dependency of the damping in the first mode also depends on the friction mechanisms engaged in the high-stressed regions of the slab. This
dependency could be attributed to engaging more friction type mechanisms in microscopic material imperfections/cracks in the reinforced concrete.

- The second and third mode displayed almost identical stress distributions in the slab, with the exception of additional stresses caused by the additional torsion in the third mode in the region of the VCDs. Additionally, the bending moment distribution in the coupling beams was similar between modes two and three. As shown in several other instances, the similar behaviour of the amplitude-dependent damping and frequency may be attributed to similar deformation mechanisms. Beyond the overall mode shape being similar, the local element deformations were also similar. It is possible that in these types of local deformations stick-slip friction due to microcracks sliding or opening and closing is highly engaged, triggering a portion of the amplitude-dependency.

The qualitative observations of the force and stress distribution in the coupling beams and slab supports the observations drawn regarding the amplitude-dependent frequency and damping.

### 6.8 Overall Damping Ratio Discussion

The overall damping of the test building may be expressed as a summation of all the mechanisms that dissipate energy according to Equation 106.

\[
ζ_{total}(u) = ζ_{vis} + ζ_{ssc}(u) + [ζ_{VCD}(u) + ζ_{VCD-Low-Amplitude}] 
\]

Equation 106

Where:
- \(u\): Displacement
- \(ζ_{total}(u)\): Overall damping ratio of structure
- \(ζ_{vis}\): Inherent viscous damping (low-amplitude damping)
- \(ζ_{ssc}(u)\): Amplitude-dependent damping due to stick-slip components
- \(ζ_{VCD}(u)\): Supplemental damping from the VCDs
- \(ζ_{VCD-Low-Amplitude}\): Low amplitude damping provided by VCDs

The baseline inherent viscous term represents the low amplitude damping which may be attributed to ever-present damping sources such as inherent material damping and friction between large elements.

The amplitude-dependent term captures all the effects of stick-slip mechanisms in the structure. The overall deformation mechanisms trigger certain local element deformations, which are the source of the amplitude-dependent damping, whether these mechanisms are friction
between structural and non-structural members or cracks in reinforced concrete. The overall mode shape is simply an indication of the mechanisms being engaged but does not specifically account for the mechanisms themselves. These stick-slip elements may be a combination of friction in structural joints, friction between structural and non-structural components, and in microscopic material imperfections and are likely to be triggered in regions of high force/stress and regions of high relative displacement. Further research in this area is needed to more accurately quantify the amplitude-dependent damping sources. The fitting of phenomenological models from full-scale data is useful to understand global behaviours and develop databases, but a view to formulation of mechanical models that may be used by structural engineers is a key aspect of future work on this topic. The VCD term is amplitude-dependent due to the effect of adjacent member cracking reducing the stiffness and driving more proportional force into the VCD. The additional cracking gained at higher amplitudes itself is likely responsible for inherent amplitude-dependent damping, but a side effect in a building incorporating VCDs is the amplitude-dependency introduced in the supplemental damping term.

Paying attention to the May 4, 2018 event, the measured or analytically predicted components of Equation 106 are shown in Table 6.16. Note that the viscous component, the VCD component, and the total were determined, leaving only the stick-slip component remaining.

<table>
<thead>
<tr>
<th>Damping Component</th>
<th>Mode</th>
<th>Damping Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta_{viss} + \zeta_{SSC}(u) )</td>
<td>1</td>
<td>1.73**</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.50**</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-*</td>
</tr>
<tr>
<td>( \zeta_{VCD}(u) + \zeta_{VCD-Low-Amplitude} )</td>
<td></td>
<td>0.85</td>
</tr>
<tr>
<td>( \zeta_{total}: \text{Peak on May 4, 2018} )</td>
<td></td>
<td>2.61</td>
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* No high-amplitude high quality damping data available
** Computed from Equation 106

Given the established benchmarks for the damping, including the baseline inherent, the supplemental, and the measured total damping, it is evident that the amplitude-dependent inherent damping represents a large portion of the total amount of damping. Given the difficulty of accurately quantifying the various mechanisms that are responsible for this at the design stage, significantly more research is required in this area. Of particular note is the projection to higher amplitude events, which structural engineers are primarily concerned with. Understanding the
inherent damping mechanisms at a fundamental level is a critical next step in supporting tall building dynamics research and full-scale building monitoring.

6.9 Modelling Summary and Conclusions

In summary, this chapter reviewed the basics of modelling of tall buildings and the VCDs, and presented several studies aimed at understanding the structural behaviour as it related to the experimental results. The key conclusions from this section include:

- Typical assumptions made by structural engineers when building analytical models of structures adequately capture the behaviour, and with minor adjustments to reflect in-situ mass, elastic modulus, and cracking levels the models can accurately capture in-situ frequencies and global-local deformation ratios.

- The key parameter that affects the accuracy of the model was the stiffness of the elements, which is dependent on the elastic modulus and the specified cracking. To more accurately represent the modulus, an increase of 20% may be applied. Continued effort is required to refine the effective stiffness estimations with further experimental and analytical studies.

- Under the key high amplitude wind event (May 4, 2018), calibrated models with best estimates of mass and stiffness and a VE material of 12 °C (on-site temperature on May 4, 2018) the VCDs added an estimated 0.88% and 0.85% supplemental damping in the first and third mode, respectively. While this fell below the design level, the expected levels of cracking were not that of a SLS event and the operating temperature was below optimal. By projecting the final as-built model to the SLS state, the target supplemental damping was reached.

- At the expected temperature of 22 °C and service conditions, including larger deformations resulting in lower reinforced concrete effective stiffness, the expected supplemental damping was 1.29% and 1.35% in the first and third mode, respectively.

- The overall measured damping on May 4, 2018 was 2.61% at 22.25 millimetres, therefore the overall target supplemental damping was reached.

- It was hypothesized that a portion of the structural amplitude-dependent damping may be due to microcrack activation in highly stressed regions of the structure, particularly the slab and the coupling beams. Further research is required on this topic.
Chapter 7: Summary and Conclusions

7.0 Conclusions and Recommendations for Future Work

7.1 Conclusions

The first objective of this thesis was to present a state-of-the-art literature review regarding the understanding of the in-situ damping and the development of amplitude-dependent damping models. This discussion ranged from the introduction of the concept of equivalent viscous damping up to the current robust amplitude-dependent models based on probabilistically distributed stick-slip mechanisms. While the overall behaviour has become well-defined and modelled using stick-slip mechanisms and overall deformation mechanisms, the innate nature of these mechanisms is no better understood now than in the early days of the model development.

The second objective was to evaluate and study several state-of-the-art output-only system identification algorithms. A thorough literature review was provided, and a robust analytical study performed to understand the implications of violating the inherent assumptions of the methods, as is typically required in application to tall buildings. It was found that the RDT was capable of accurately resolving the inherent damping and frequency, with proper parameterization of the technique, even when excessively violating the two fundamental assumptions of output-only system ID; stationary input and linear systems. It is due to the robustness of this technique that it has seen such widespread usage in the tall building community. Alternative methods such as NExT, EFDD, and SSI-DATA are useful for processing the data from large sensor arrays and for defining the mode shapes of the structure. However, these techniques are only applicable in the low-amplitude range to ensure that the input signal remains stationary and the inherent non-linearity of the frequency and damping is minimized.

The third objective involved performing a robust monitoring program for a tall, slender building through its construction sequencing and under high-amplitude wind events. The non-structural components were found to have little effect on the natural frequency near the end of construction when the primary structure had been completed. Little change in the natural frequency was observed over several months of exclusive changes to non-structural components. The high-amplitude response data to large wind storms was used to inform that amplitude-dependency of the dynamic properties. It was found that the frequency decreased with amplitude, and the damping
increased with amplitude, supporting the theory of stick-slip components being the primary driver of amplitude-dependent phenomenon.

The fourth objective of this thesis was to measure the local response of the VCDs and use this information to inform the relationship between overall motion and local deformation. It was found that the ratio of the global-local deformation exhibited an amplitude-dependent trend, indicating that there was more local deformation at higher amplitude per unit of building motion than at low amplitudes. This was a notable finding, showing directly that the physical deformation of the VCDs and the adjacent members may be represented by a stick-slip type mechanism.

The fifth and final objective of this thesis was to build calibrated models of the structure to understand the key effects of typical engineering modelling assumptions, to estimate the supplemental damping provided by the VCDs, and to explore the local structural behaviour. It was found that the typical modelling strategies used by structural engineers are effective at estimating the natural frequencies. The fundamental parameter which controls the accuracy of the model is the stiffness of the elements, specifically informed by the modulus of elasticity and the assumed cracking levels. The modulus may be reasonably estimated from in-situ concrete strengths, and this research showed an approximate 20% increase over the design values. The assumed cracking levels are more difficult to accurately quantify, requiring expertise, test data, or advanced finite element models to accurately represent. The overall finding of the modelling aspect of the research was that with more accurate representations of the mass and stiffness, the current state-of-the-art finite element models accurately capture the natural frequencies and overall deformation mechanisms. The calibrated models were used to estimate the supplemental damping provided by the VCDs. It was found that the expected levels of cracking were below that of the design level, as expected given the load history of the building to date, resulting in supplemental damping estimates of 0.88% and 0.85% in the first and third mode, respectively, during the May 4, 2018 event. Lastly, the local stresses and forces in the slab and coupling beams were investigated, and it was hypothesized that microcracks in the reinforced concrete in these regions may be responsible for a portion of the observed amplitude-dependency.

Overall, this thesis was focused on the dynamic properties of tall, slender buildings and informing the links between in-situ properties, particularly the natural frequencies and damping ratios, and the modelling of structures at the design stage. It was found that full-scale monitoring of structures is of paramount importance in forging this link. While traditional acceleration
monitoring is fundamental to continuing to further structural engineers’ understanding of tall building dynamics, supplemental local element monitoring is critical to developing links between the overall building motion and the local element deformations. It is in these local element deformations that more precise insights into the mechanisms of damping may be obtained, and the relationship between the global deformation mechanisms, local deformation mechanisms, and the inherent dynamic properties may be improved. Continued effort in establishing long-term robust monitoring studies of tall buildings is a key baseline aspect to eventually developing precise predictive models for damping that may be used by structural engineers in the design stage.

7.2 Recommendations for Future Work

There are three branches where recommendations for future work were made including monitoring, system identification, and the mechanisms of damping.

**Monitoring:**

- Continue to expand database of tested buildings to increase the size of the library of dynamic properties and observed amplitude-dependency.
- Strive for long-term monitoring when possible to ensure all high-amplitude events are captured in their entirety and the challenges of short-duration records may be negated.
- Supplement overall acceleration monitoring with local element monitoring. Expand this to several element types and investigate improved sensing technologies for this, such as fibre-optic sensors.
- Target buildings under construction to understand the progression of dynamic characteristics with construction. This will aid in understanding the role of non-structural components in the overall dynamics of the building. Additionally, begin the construction sequencing monitoring earlier in the construction, to inform the development of the dynamic properties over the entire height of the building.
- Include temperature measurements when monitoring structures with viscoelastic dampers.
- For the test building in this thesis additional long-term monitoring should be performed to further validate the conclusions of this thesis and overcome the challenges of short-duration records and temperature variability.

To do this, collaborative relationships with designers and builders must continue to be built, and the community must continue to educate and encourage smarter structures by way of monitoring.
Chapter 7: Summary and Conclusions

System Identification:
- Explore further non-stationary SID techniques to increase the amplitude range where damping and frequency may be evaluated to inform the highest-amplitude responses. While the low – mid amplitude responses investigated in this thesis are important for overall understanding of tall building dynamics, the fact is that the most relevant information is that close to the design states.

Mechanisms of Damping:
- Continue to build understanding of the effect of non-structural components on overall buildings. Lab scale testing and full-scale local element testing should be performed to better understand this interaction, particularly for tall building applications.
- Investigate the proposed structural mechanism, microcracking in slabs and coupling beams, as potential sources of amplitude-dependent damping and frequency. Full scale local element monitoring and lab scale testing should be performed to better understand this behaviour. The objective should be to forge a link between construction material, mode shapes (primary deformation mechanism), structural layout, and local element deformation mechanisms that can be developed to better understand the inherent damping of structures.
- Work towards the construction of a mechanically driven, data informed, predictive model for damping.
References


ACI 375. (2004). *Performance Based Design of Concrete Buildings for Wind Loads*. ACI.


References


References


References


References


References


Appendix I: Numerical Simulation Algorithms

The following algorithms were adapted from the work of (Pirnia, 2009), which was based on the Modified Nonlinear Newmark’s Average Acceleration method with Newton Raphson iterations described by Chopra (2001). Note that to perform the linear case of either the SDOF of MDOF case, the amplitude-dependent expressions were set to constants.

**Modified Newton-Raphson Iteration Procedure**

**Step 1:** Given
\[ u_i, (f_s)_i, a, \Delta t \]

**Step 2:** Initialize Data
\[ \Delta R^{(1)} = \Delta \hat{p}_i \]
\[ \hat{k}_T = \hat{k}_i \]
\[ k = k_i \]

**Step 3:** Initial Calculations
3.1: \[ \Delta u^{(1)} = \frac{\Delta R^{(1)}}{k_T} \]
3.2: \[ u_i^{(1)} = u_i + \Delta u^{(1)} \]
3.3: \[ (f_s)_{i+1}^{(1)} = (f_s)_i + k \Delta u^{(1)} \]
3.4: \[ (\Delta f_s)^{(1)} = (f_s)_{i+1}^{(1)} - (f_s)_i + \frac{a}{\Delta t} \Delta u^{(1)} \]
3.5: \[ \Delta R^{(2)} = \Delta R^{(1)} - (\Delta f_s)^{(1)} \]

**Step 4:** Calculations for each iteration, \( j=2,3,4,\ldots \)
4.1: \[ \Delta u^{(j)} = \frac{\Delta p^{(j)}}{k_T} \]
4.2: \[ u_i^{(j+1)} = u_{i+1}^{(j-1)} + \Delta u^{(j)} \]
4.3: \[ (f_s)_{i+1}^{(j)} = (f_s)_{i+1}^{(j-1)} + k \Delta u^{(j)} \]
4.4: \[ (\Delta f_s)^{(j)} = (f_s)_{i+1}^{(j)} - (f_s)_{i+1}^{(j-1)} + \frac{a}{\Delta t} \Delta u^{(j)} \]
4.5: \[ \Delta R^{(j+1)} = \Delta R^{(j)} - (\Delta f_s)^{(j)} \]

**Step 5:** Repetition for next iteration. Replace \( j \) with \( j+1 \) and repeat calculation steps 3.1-3.5.

**Note:** Since the imposed non-linearities are small and the sampling period is high relative to the period, very few iterations are required to minimize the error.
Appendix I: Numerical Simulation Algorithms

**SDOF Algorithm:**

**Step 1: Given**

\[ f_{n_0}, \Delta f_{n_0}, \zeta_0, \Delta \zeta_0, m, \Delta t, p, u_0, \dot{u}_0 \]

\[ \beta = \frac{1}{4}, \gamma = \frac{1}{2} \]

**Step 2: Initial Calculations**

2.1: \[ c_0 = \zeta_0 2m (2\pi f_{n_0}) \]

2.2: \[ a_0 = \frac{4}{\Delta t} m + 2c_0 \]

2.3: \[ k_0 = \left(2\pi f_{n_0}\right)^2 m \]

2.4: \[ b = 2m \]

2.5: \[ (f_s)_0 = k_0 u_0 \]

2.6: \[ \ddot{u}_0 = \frac{p_0 - c_0 \dot{u}_0 - (f_s)_0}{m} \]

2.7: \[ \Delta p_0 = p_1 - p_0 \]

2.8: \[ \Delta \dot{p}_0 = \Delta p_0 + a_0 \ddot{u}_0 + b \dot{u}_0 \]

2.9: \[ \hat{k}_0 = k_0 + \frac{2}{\Delta t} c_0 + \frac{4}{(\Delta t)^2} m \]

2.10: Solve for \( u_1, \Delta u_0, \) and \((f_s)_1\) using the Modified Newton-Raphson Iteration.

2.11: \[ \Delta u_0 = \sum_{j=1}^{i} \Delta u_j \]

2.12: \[ u_1 = u_0 + \Delta u_0 \]

2.13: \[ \Delta \dot{u}_0 = \frac{2}{\Delta t} \Delta u_0 - 2\dot{u}_0 \]

2.14: \[ \dot{u}_1 = \dot{u}_0 + \Delta \dot{u}_0 \]

2.15: \[ \Delta \ddot{u}_0 = \frac{4}{(\Delta t)^2} \Delta u_0 - \frac{4}{\Delta t} \dot{u}_0 - 2\ddot{u}_0 \]

2.16: \[ \ddot{u}_1 = \ddot{u}_0 + \Delta \ddot{u}_0 \]

**Step 3: Calculations for each time step, \(i=1,2,3,4,...\)**

Equations 3.1 and 3.2 generically represents the specified amplitude-dependent frequency and damping.

3.1: \[ f_{n_i} = f_{n_{i-1}} + \Delta f_{n_{i-1}} \]

3.2: \[ \zeta_i = \zeta_{i-1} + \Delta \zeta_{i-1} \]

3.3: Solve for \( c_i, a_i, \) and \( k_i \) using steps 2.1-2.3 \((0 \rightarrow i, 1 \rightarrow i+1)\)

3.4: Solve for \( \dot{u}_{i+1} \) and \( \ddot{u}_{i+1} \) using steps 2.7 – 2.16
Appendix I: Numerical Simulation Algorithms

3.5: Determine $\Delta f_{n_{i-1}}$ and $\Delta \zeta_{i-1}$ based on computed conditions and prescribed amplitude-dependent relationship.

**Step 4: Time-Stepping:** Replace $i$ with $i+1$ and implement steps 3.1-3.5 for each time step.

**MDOF Algorithm:**

To simplify the MDOF simulations only the damping was varied with amplitude thus the modal superposition method was used. To allow the modal damping to be updated based on the specified physical displacements, the response was simulated in a nested for-loop. The outer loop stepped through time and the inner loop stepped through the modes. On each time step, the mode shapes were used to determine the physical displacements from the computed modal displacements and the corresponding damping ratios were updated. Modal damping was used, and the modal damping ratios were updated based on the third DOFs physical displacement.

**Step 1: Given**

$$\begin{bmatrix} M, C \end{bmatrix} = \begin{bmatrix} \zeta_1 2m_1 (2\pi f_1) & 0 & 0 \\ 0 & \zeta_2 2m_2 (2\pi f_2) & 0 \\ 0 & 0 & \zeta_3 2m_3 (2\pi f_3) \end{bmatrix}, [K]$$

1.2: Compute mode shapes and frequencies.
1.3: Compute modal mass and stiffness:
$$M_i = \Phi_i^T [M] \Phi_i$$
$$K_i = \Phi_i^T [K] \Phi_i$$
1.4: Compute modal initial conditions:
$$q_i = \frac{\Phi_i^T [M] u}{M_i}$$

**Step 2: Time-Stepping**

2.1: Perform SDOF algorithm for each mode at time step $i$, replacing $u$ with $q$.
2.2: Combine modal responses at time step $i$.
$$[u] = [\Phi][q]^T$$

**Step 3: Update damping ratios.** For each mode compute:
$$\zeta_i = \zeta_{i-1} + \Delta \zeta_{i-1}$$
## Appendix II: VCD Installation Progress

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## Appendix II: VCD Installation Progress

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Appendix III: Testing Photos

September 28, 2017

October 19, 2017

March 20, 2018

March 26, 2018

April 4, 2018

June 13, 2018
Appendix III: Testing Photos

April 13, 2018

April 16, 2018

June 13, 2018

June 13, 2018
Appendix IV: Time-Histories

March 30-April 2, 2018

April 4-5, 2018

Full-Scale Monitoring of a Tall, Slender Building with Coupling Viscoelastic Dampers 244
Appendix IV: Time-Histories

April 6-7, 2018

April 13-16, 2018
April 27-30, 2018

April 27-30, 2018: VCD Shear Deformation

May 4, 2018

May 4, 2018: VCD Shear Deformation

May 4, 2018: Acceleration Measurements
Appendix IV: Time-Histories

**June 13-14, 2018**

![June 13-14, 2018: VCD Shear Deformation](image)

![June 13-14, 2018: Acceleration Measurements](image)

**August 27-29, 2018**

![VCD Shear Deformation](image)

![August 27-29, 2018: Dynamic Displacement](image)

* Displacement shown to some spikes in acceleration due to nearby construction workers.
Appendix IV: Time-Histories

September 20-22, 2018

[Graph showing acceleration measurements for September 20-22, 2018]

November 6-7, 2018

[Graph showing acceleration measurements for November 6-7, 2018]
Appendix V: Amplitude-Dependent Frequency and Damping

The following shows the results from all independent analyses utilising the RDT as applied to the long-term data records.

March 30 – April 2, 2018

March 30 - April 2, 2018: 1st Mode Frequency

March 30 - April 2, 2018: 1st Mode Damping

March 30 - April 2, 2018: 2nd Mode Frequency

March 30 - April 2, 2018: 2nd Mode Damping

- Third mode not considered due to spurious mode
Third mode not considered due to spurious mode
Appendix V: Amplitude-Dependent Frequency and Damping

April 6-7, 2018

April 6-7, 2018: 1st Mode
Frequency

April 6-7, 2018: 1st Mode
Damping

April 6-7, 2018: 2nd Mode
Frequency

April 6-7, 2018: 2nd Mode
Damping

- Third mode not considered due to spurious mode
Appendix V: Amplitude-Dependent Frequency and Damping

April 13-16, 2018

- Third mode not considered due to spurious mode
Appendix V: Amplitude-Dependent Frequency and Damping

April 27-30, 2018

April 27-30, 2018: 1st Mode
Frequency

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April 27-30, 2018: 1st Mode
Damping

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<th>0.2232</th>
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<th>0.2244</th>
<th>0.2248</th>
</tr>
</thead>
<tbody>
<tr>
<td>ζ (%)</td>
<td>0.8</td>
<td>1</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
</tr>
</tbody>
</table>

April 27-30, 2018: 2nd Mode
Frequency

<table>
<thead>
<tr>
<th>Displacement (mm)</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency (Hz)</td>
<td>0.223</td>
<td>0.2234</td>
<td>0.2236</td>
<td>0.224</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
</tr>
</tbody>
</table>

April 27-30, 2018: 2nd Mode
Damping

<table>
<thead>
<tr>
<th>Displacement (mm)</th>
<th>0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Frequency (Hz)</td>
<td>0.223</td>
<td>0.2234</td>
<td>0.2236</td>
<td>0.224</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
</tr>
</tbody>
</table>

- Third mode not considered due to spurious mode
May 4, 2018: 1st Mode Frequency

May 4, 2018: 2nd Mode Frequency

May 4, 2018: 2nd Mode Damping

- Third mode not considered due to spurious mode
Appendix V: Amplitude-Dependent Frequency and Damping

June 13-14, 2018

June 13-14, 2018: 1st Mode
Frequency

June 13-14, 2018: Damping

June 13-14, 2018: 2nd Mode
Frequency

June 13-14, 2018: 2nd Mode
Damping

June 13-14, 2018: 3rd Mode
Frequency

June 13-14, 2018: 3rd Mode
Damping

Full-Scale Monitoring of a Tall, Slender Building with Coupling Viscoelastic Dampers
Appendix V: Amplitude-Dependent Frequency and Damping

August 27-29, 2018

August 27-29, 2018: Frequency

August 27-29, 2018: Damping

August 27-29, 2018: 2nd Mode Frequency

August 27-29, 2018: 2nd Mode Damping

August 27-29, 2018: 3rd Mode Frequency

August 27-29, 2018: 3rd Mode Damping

Full-Scale Monitoring of a Tall, Slender Building with Coupling Viscoelastic Dampers 256
Appendix V: Amplitude-Dependent Frequency and Damping

September 20-22, 2018

September 20-22, 2018: Frequency

September 20-22, 2018: Damping

September 20-22, 2018: 2nd Mode Frequency

September 20-22, 2018: 2nd Mode Damping

September 20-22, 2018: 3rd Mode Frequency

September 20-22, 2018: 3rd Mode Damping
Appendix V: Amplitude-Dependent Frequency and Damping

November 6-7, 2018

November 6-7, 2018: 1st Mode
Frequency

November 6-7, 2018: 1st Mode
Damping

November 6-7, 2018: 2nd Mode
Frequency

November 6-7, 2018: 2nd Mode
Damping

November 6-7, 2018: 3rd Mode
Frequency

November 6-7, 2018: 3rd Mode
Damping
Appendix VI: Dynamic Property Extraction from ETABS

The following describes the approach used to extract the dynamic properties from the analytical ETABS model using the ERA and the global-local deformation ratio.

**Damping Ratio and Frequency:**

**Step 1:** Apply loads such that all target modes are excited.

**Step 2:** Apply free-vibration time-history.
Appendix VI: Dynamic Property Extraction

**Step #3:** Output time-history at select locations where all modes of interest are excited.

![Displacement Records](image)

**Step #4:** Process response using ERA.

![Singular Value Distribution](image) ![Stabilization Diagram](image)

**Step #5: Identify properties:**

Frequencies (Hz):

- 0.2052
- 0.3034
- 0.3634

Periods (s):

- 4.8735
- 3.2958
- 2.7518

Damping Ratios (%):

- 0.8294
- 0.0128
- 0.7787
Appendix VI: Dynamic Property Extraction

Mode Shapes:
-1.0000 0.7563 0.9973
-0.0497 1.0000 -0.0434
-0.9095 -0.7732 -1.0000
-0.0105 0.3092 -0.9452

Part 2: Global-Local Deformation Ratio:

**Step#1**: Apply loads such that first mode is excited only.
**Step#2**: Output time-history of displacement and link deformation of interest.
**Step#3**: Find matching peaks and compute average.
Appendix VII: NSC Load Calculations

1/2” Gypsum : 10.2 kg/m² x 2.75m (height) = 28.05 kg/m

Insulation: 25 kg/m³ x 0.05m x2.75 m = 3.44 kg/m

Approximately 210m of drywall on each floor: 210m x [28.05 kg/m + 3.44 kg/m] = 6600 kg

Convert to load and smear over surface area: 6600 kg x 9.81 N/kg x 1kN/1000N / (40.7x18.9m²) = 0.086 kPa

HBSC: Gypsum Wallboard per 10mm = 0.08kPa

Assume total load is 0.1 kPa