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Graphical Abstract

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Evaluation of Equivalent Hydraulic Aperture (EHA) for Rough Rock Fractures

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Abstract

Most existing models for fluid transportation within a single rock fracture tend to use a channel with two smooth parallel plates, whereas real fracture surfaces are usually rough and tortuous, which can produce a flow field significantly different from the smooth plate model. For fluid flow in a rough fracture, there are surface concave areas (SCA), where the fluid velocity is extremely low, contributing little to the fluid transportation. It is of great significance to quantitatively evaluate the impact of rough surfaces on fluid flow. Therefore, we proposed a numerical model for simulating Newtonian fluid through rough fractures, where synthetic surfaces are generated according to statistical analysis of natural rock fractures and can be quantified by several characteristic parameters. Equivalent hydraulic aperture (EHA) is proposed as one quantitative indicator for evaluating the impact of fracture roughness. Systematic studies were conducted for evaluating EHAs of rough fractures, which, combined with characteristic parameters of fractures, are used to build surrogate models for EHA prediction. It is found that the EHA is directly correlated with the fracture roughness, the mean mechanical aperture, and the standard deviation of aperture distribution. The developed surrogate models were verified to have a high accuracy for EHA prediction.

Keywords: Rock fracture; Roughness; Tortuosity; Equivalent hydraulic aperture (EHA); Artificial Neural Network (ANN)

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Introduction

Rock fractures in nature are rough and tortuous, and the fracture surfaces are found to be fractal and self-affine (Hsiung et al. 1993; Mandelbrot 1985; Odling 1994; Power and Tullis 1991; Tse and Cruden 1979b), which makes the flow field complex and analysis of fluid transportation within it more challenging, compared with a channel with flat surfaces (Brown 1987; Brush and Thomson 2003; Noiriél et al. 2007). However, few study has been carried out on how to quantitatively evaluate the impact of fracture roughness on fluid flow. Taking rock grouting as an example, grouting plays an important role in rock engineering nowadays for controlling water seepage and strengthening fractured rock masses. Though grouting has been successfully utilised in many rock engineering projects (Emmelin et al. 2007; Lombardi 2002), grouting design is still mostly based on engineers' experience and the cost is high due to lack of comprehensive geological data and reliable grouting models (Kvartsberg 2013; Xiao et al. 2017). For simplification, most grouting models did not consider the influence of fracture roughness on grout penetration, and rock fractures were modelled either as 1D fracture channel composed of two parallel lines, or as 2D fracture channel with parallel smooth surfaces (El Tani 2012; Eriksson et al. 2000; Hernqvist et al. 2008; Saeidi et al. 2013; Sui et al. 2015; Xiao and Zhao 2017). Compared with grout penetration along smooth parallel plates, the pressure distribution on the surfaces of rock fractures may be greatly different when grout flows through rough rock fractures, since the corresponding velocity distribution can be significantly different under the impact of roughness and tortuosity. As the distribution of fluid pressure is critical to the hydro-mechanical coupling process (Fransson et al. 2010; Gothåll and Stille 2010; Rafi and Stille 2014, 2015; Runslätt and Thörn 2010), the fracture roughness can greatly influence the grouting process under certain conditions.

Over the last two decades, new approaches have been developed to consider the potential impact of roughness and tortuosity of rock fractures on fluid flow. Probably because the Joint Roughness Coefficient (JRC) is defined for 2D profiles, fluid flow, especially water flow, within 2D tortuous and rough fracture channels, has been extensively studied, and the influence of roughness and tortuosity were discussed (Briggs et al. 2014; Briggs 2014; Lipscomb and Denn 1984; Mandelbrot 1985; Power and Tullis 1991; Tsang and Witherspoon 1983; Zhao et al. 2014). Though these studies have limited practical applications due to their idealistic assumptions, they have laid good foundation for future studies in fractal geometry and self-affine structure. Attempts have been made to model 3D rough fracture surfaces, among them were the experiments conducted directly with real rock samples with rough fractures (Crandall et al. 2010; Develi and Babadagli 2015; Lanaro 2000; Rong et al. 2017; Zhang and Nemeik 2013), and simulations conducted with synthetic rock fractures (Amadei and Illangasekare 1994; Brush 2001; Brush and Thomson 2003; Crandall et al. 2010; Ge 1997; Hanssen 2013; Jeong and Song 2005; Ogilvie et al. 2006; Park et al. 1995; Thompson 1991; Wang et al. 2016). However, very few studies have considered the influence of roughness and tortuosity on the distribution of velocity and pressure, especially the variation patterns of effective aperture of rock fractures with different characteristics, under variable flow rates.

Therefore, in the present work, we generated synthetic rock fractures with rough surfaces based on statistics and parameterization of real rock fractures and analysed the characteristics of rough surfaces; then we simulated Newtonian flow through rough fractures utilizing Navier-Stokes equations, and proposed the concept of equivalent hydraulic aperture (EHA) to quantify the roughness impact based on analysis of the characteristics of velocity distribution. In addition, EHAs of rough rock fractures were calculated from theoretical equation utilizing simulation results, then they were compared with that evaluated using the predictive model developed based on Artificial Neural Network (ANN) and Regression analysis. Finally, one

engineering application was presented for grouting design based on the predictive model, utilizing some assumed geological data.

2 Characterization of rough rock fractures

The natural rock fractures have rough surfaces and are often tortuous at the same time. A lot of work have been done to characterize and parameterize these properties by using the fractal and self-affine features (Hsiung et al. 1993; Mandelbrot 1985; Odling 1994; Power and Tullis 1991; Tse and Cruden 1979b). Researchers have proposed different methods for generating synthetic rock fractures (Brush 2001; Brush and Thomson 2003; Chen and Horne 2006; Crandall et al. 2010; Hanssen 2013; Lavrov 2013; Moon and Song 1997; Ogilvie et al. 2003; Ogilvie et al. 2006; Wang et al. 2016; Xiao et al. 2013).

In the present work, Ogilvie's method is adopted for generating synthetic rock fractures (Ogilvie et al. 2006), since their model takes into account all key factors, including fractal dimension, anisotropy property, and the degree of matching between upper and lower surfaces of rock fracture, which makes it easy for quantitative study and sensitivity analysis of different impact factors. Ogilvie's method uses mathematical method to generate two fractal surfaces resulted from a sum of Fourier harmonics, where there are correlations between two surfaces at long wavelengths and no correlation at short wavelengths. Four parameters are used to define the matching properties more elaborately: maximum mismatch fraction (MFMax), minimum mismatch fraction (MFMin), mismatch length (ML), and transition length (TL). ML is defined as the wavelength halfway between the largest wavelength L_1 (where the correlation fraction is MFMin) and the smallest wavelength L_2 (where the correlation fraction is MFMax), and TL equals $(L_2 - L_1)$. The surfaces of real rock fractures are rough and fractal, so two more parameters, fractal dimension (FD) and anisotropy factor (AF), are used for controlling the degree of fracture roughness and ensuring different variation patterns in two orthogonal directions (i.e., directions along fracture width and length) (Ogilvie et al. 2006).

Ogilvie's model is based on statistics from natural rock fractures. The procedure for generating the synthetic rock fractures is as follows: 1) The fracture size and the relevant resolution are determined first, which are set as 100mm×100mm with 512×512 resolution, which is a square model; 2) Then five key parameters are selected, including FD, ML, TL, standard deviation of mechanical aperture of rock fracture, and AF (Ogilvie et al. 2006); 3) A series of rough rock fractures are generated based on orthogonal design table, which is presented in Appendix A1. The ranges of all parameters are determined based on statistics analysis from parameterization of natural rock fractures. In total, 58 different fracture models are generated based on parameter combination listed in Table A1 in Appendix A1, and the original data are further processed with MATLAB programming to generate a rectangular shape fracture channel, with size of 50mm×100mm, as rectangular shape is more general (Crandall et al. 2010; Rong et al. 2017; Wang et al. 2016; Zhang and Nemcik 2013). The direction of water flow is along the length direction, which provides longer flow path for better observation and analysis.

Four typical models of rough fracture channels are presented in Fig.1 to illustrate the characteristics of rough rock fracture. As can be seen from Fig.1, Cases 1, 2 and 3 have one thing in common: they all have large aperture and high roughness; and Case 4 is of very small aperture and relatively low roughness. Cases 1 and 2 have similar level of aperture and roughness, but the degree of tortuosity of Case 2 is higher than Case 1.

Another criterion to evaluate the suitability of the fracture model generated is the frequency of its fracture aperture distribution. Fig.2 presents the percentage frequency of the fracture aperture and the relevant probability density distribution. The characteristics of aperture distributions for the four cases are quite different, with different mean aperture and distribution characteristics. Nevertheless, they all satisfy Gaussian distribution, which is taken as reasonable aperture distribution for natural rock fractures (Hakami 1995; Wang et al. 2016).

To better describe the roughness and tortuosity, a quantitative method is needed. The traditional geological definition for roughness is based on a concept within 2D space consisting fracture aperture and length (Tse and Cruden 1979a), and is defined as follows

$$JRC = 32.2 + 32.47 \log(Z_2)$$

$$Z_2 = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_{i+1} - y_i)^2} / \Delta x \quad (1)$$

where JRC is the well-known Joint Roughness Coefficient; Z_2 is a dimensionless parameter corresponding to a fracture channel of length $(N - 1)\Delta x$, and y_i is the aperture height at location $(i - 1)\Delta x$.

It can be seen from Eq. (1) that the value of Z_2 can be used directly to evaluate fracture roughness. However, the natural rock fractures is in 3D domain, thus Z_2 should be extended to 3D form (Belem et al. 2000)

$$Z_{2m} = \left\{ \frac{1}{(m-1)(n-1)} \left[\frac{1}{\Delta x^2} \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \frac{(z_{i+1,j+1} - z_{i,j+1})^2 + (z_{i+1,j} - z_{i,j})^2}{2} + \frac{1}{\Delta y^2} \sum_{j=1}^{n-1} \sum_{i=1}^{m-1} \frac{(z_{i+1,j+1} - z_{i+1,j})^2 + (z_{i,j+1} - z_{i,j})^2}{2} \right] \right\}^{1/2} \quad (2)$$

where $l_x = (m - 1)\Delta x$ and $l_y = (n - 1)\Delta x$ are the fracture length in the x and y directions, respectively; z_{ij} is the fracture aperture at location $(i - 1)\Delta x$ and $(j - 1)\Delta x$.

Belem also proposed a direct method to evaluate the surface roughness and tortuosity coefficient R_m and T_m (Belem et al. 2000), which were modified from the 2D model

$$A_r = (\Delta x \cdot \Delta y) \sum_{i=1}^{m-1} \sum_{j=1}^{n-1} \left[1 + \left(\frac{z_{i+1,j} - z_{i,j}}{\Delta x} \right)^2 + \left(\frac{z_{i,j+1} - z_{i,j}}{\Delta y} \right)^2 \right]^{1/2} \quad (3)$$

$$R_m = \frac{(A_r^t + A_r^b)}{2A_n} = \frac{(R_m^t + R_m^b)}{2} \quad (4)$$

where A_r is the area of fracture rough surface, with A_r^t and A_r^b representing the top and bottom surface areas, respectively; R_m is the overall fracture roughness coefficient, with R_m^t and R_m^b corresponding to the top and bottom surfaces, respectively.

Likewise, the surface tortuosity coefficient T_m is also defined as the ratio of two areas (Belem et al. 2000), where θ will be defined in the following part

$$T_m = \frac{A_r}{A_n} = \frac{A_r}{A_n} \cos \theta = R_m \cos \theta, (0 < \cos \theta \leq 1) \quad (5)$$

where, A_π is the area of the best fitting surface that can pass through the four extreme points of the rough surface, called “ π -surface”, which is defined as

$$\alpha x + \beta y - z + \gamma = 0 \quad (6)$$

The coordinates of the four points can be substituted into Eq. (6), then the value of α and β can be calculated based on the least square regression method. Then, T_m can be derived, as $\cos \theta$ can be expressed as follows,

$$\cos \theta = 1 / \sqrt{\alpha^2 + \beta^2 + 1} \quad (7)$$

If the “ π -surfaces” of the top and bottom surfaces are different, the overall surface tortuosity coefficient of a rough fracture is calculated as,

$$T_m = \frac{A_r^t + A_r^b}{A_\pi^t + A_\pi^b} = \frac{A_r^t + A_r^b}{(A_n / \cos \theta^t) + (A_n / \cos \theta^b)} = 2R_m \frac{\cos \theta^t \cdot \cos \theta^b}{\cos \theta^t + \cos \theta^b} \quad (8)$$

As for the mean mechanical aperture of a 3D fracture, it can be defined as follows

$$h_m = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n (z_{i,j}^t - z_{i,j}^b) \quad (9)$$

where, $z_{i,j}^t$ and $z_{i,j}^b$ are the points located at the top and bottom surfaces with coordinates of $(i-1)\Delta x$ and $(j-1)\Delta y$, respectively.

Therefore, the characteristics of four cases presented in Fig.1 can be quantified, and the results are listed in Table 1. As discussed above, there are differences among different cases in Fig.1, whereas Table 2 describes the differences more directly and accurately. Meanwhile, it can be found that the values of R_m and T_m are extremely close to each other, due to the fact that the tortuosity of rock fractures studied here are very low, which can lead to small θ (close to 0) and large $\cos \theta$ (close to 1), so we have $T_m \approx R_m$ according to Eq. (5) $T_m = R_m \cos \theta$.

Then the characteristic parameters of all rock fractures, generated according to parameters given in Table A1-1 (in Appendix A1), can be described based on Eqs. (2) to (9), and the results are presented in Table A1-2 (in Appendix A1), including the mean mechanical aperture h_m , dimensionless parameter Z_{2m} , the fracture roughness and tortuosity R_m and T_m . As shown in Fig. A1-1 in Appendix A1, the bar diagrams in the diagonal are parameter distributions of Z_{2m} , R_m , and T_m , respectively, and the rest sub-diagrams show correlations between any two specific parameters. It can be easily found that the distributions of three parameters are quite similar; and the relationship between Z_{2m} and R_m (T_m) is approximately linear, whereas R_m and T_m is linearly correlated.

3 Fluid transportation through rough fractures

To study the characteristics of fluid flow through rough rock fractures, including the relationship between inlet flow rate and inlet pressure, the pressure or velocity distribution, and the equivalent fracture aperture, simulations of Newtonian flow through rough rock fracture are conducted based on the Navier-Stokes Equations. The simulation model is shown in Fig.3, where all four models have the same boundary conditions: there is no outflow from lateral boundaries on two sides along the flow direction; the top and bottom rough surfaces are assumed to be impermeable and the coupling effect between fluid flow and rock fracture is not considered here.

A constant inlet flow rate is assumed as the boundary condition at the inlet end, and the mass flow rate applied is 0.1 to 0.7 kg/s, corresponding to 6 to 35 L/min; the pressure is assumed to be zero at the outlet end. Two cuboids are attached to the left and right ends of the fracture model, which are used to reduce the boundary influence, and to make it easier to get convergent solution. The size of the two cuboids is designed in such a way so that the cross section of rock fracture at both ends can be included inside the corresponding cuboids surfaces, as can be seen from Figs. 3 and 4. Normally, their sizes are the same within one fracture model, and they vary among different fracture models.

Take Case 1 as an example, the velocity distribution inside the rock fracture is shown in Fig.4, where velocity distribution in five longitudinal cross-sections (five slices) are presented, and the red and blue colours refer to high and low velocities, respectively. As it can be seen from Fig.4, the inlet velocity at the cuboid is stable and uniform, indicating its function for stabilising the inlet flow into the rough rock fracture.

One important observation is that bright red colour concentrates mainly at the middle of the fracture, defined as central areas (CA); and the colour close to the fracture wall is dark blue (extremely small velocity), defined as surface concave areas (SCA), especially at the location with large concaves, indicating that these areas do not effectively contribute to the transportation of fluid. The same phenomena can be found for all other cases. Therefore, we can conclude that only part of the rough rock fracture is effective for fluid transportation and propose the “equivalent hydraulic aperture (EHA)” for evaluation of this phenomenon.

To investigate the general situation, we also conducted numerical simulation using Bingham model, the simulation results of which are compared with that under Newtonian model, including the corresponding inlet pressure generated, and the velocity fields. One example is presented in detail in Fig. A2-1 (see Appendix A2). It is observed that the differences of simulation results between the two models are very small, under the same constant inlet flow rates, except that the relevant computational efficiency of Newtonian flow is far higher.

4 Evaluation and prediction of EHA

4.1 Theoretical equation and EHA evaluation

As for water flowing between two rough surfaces, three parameters can be used for describing the fluid field: flow rate Q_m , pressure gradient ΔP , and hydraulic aperture h_e , and their relationship can be expressed as (Thompson 1991)

$$h_e = \left(\frac{12\mu Q_m}{\rho W \nabla P} \right)^{1/3} \quad (10)$$

where, μ and ρ are the viscosity and density of Newtonian flow, respectively; W and L are the width and length of the fracture channel, respectively. Then the rough fracture is equivalent to a fracture channel with two smooth parallel surfaces, as shown in Fig. 5.

As shown in Fig.5, the average pressures at two cross sections in xz plane at $y=0$ and 100 (mm) are taken as P_0 and 0 respectively, then the pressure gradient in Eq. (10) is $\nabla P = P_0/L$.

In generalization, Newtonian flow is a special case of Bingham flow when the corresponding yield stress is taken as zero. Furthermore, for grouting of rock fractures, cement grout with water cement ratio (W/C) around 1 is usually used (Warner 2004), which can be taken as Bingham flow (Rosquoët et al. 2003; Ruan 2005). For Bingham flow, the corresponding EHA can also be deduced from analysis of the corresponding flow rate expression (Xiao et al. 2017)

$$Q_m = \frac{\rho W h_e^3}{12\mu} \left(-\frac{\partial P}{\partial x} \right) \left[1 - 3 \left(\frac{y_0}{h_e} \right) + 4 \left(\frac{y_0}{h_e} \right)^3 \right], y_0 = \tau_0 / (-\partial P / \partial x) \quad (11)$$

where, τ_0 is the yield stress of Bingham flow; $-\partial P / \partial x$ is the pressure gradient, and it is taken as P_0/L , thus the half width of the Bingham plug y_0 is $\tau_0 L / P_0$. Eq. (11) is derived by assuming that grout flows within parallel plates with a distance of h_e .

The relationship between Q_m and h_e is nonlinear, so we cannot express h_e as a function of Q_m directly. Thus, some transformations should be introduced to determine h_e indirectly.

Here, we introduce a dimensionless variable $\phi = y_0 / h_e$. Then, substitute ϕ into Eq. (11)

$$Q_m = \frac{\rho W \tau_0 y_0^2}{12\mu} \left(\frac{1 - 3\phi + 4\phi^3}{\phi^3} \right) \quad (12)$$

Rewrite Eq. (12) as cubic equation of ϕ as follows,

$$(C_0 - 4)\phi^3 + 3\phi - 1 = 0 \quad (13)$$

where, $C_0 = 12\mu Q_m / \rho W \tau_0 y_0^2$ is a dimensionless constant under specific inlet flow rate. We can rewrite C_0 to find the corresponding physical meaning,

$$C_0 = \frac{12\mu Q_m}{\rho W \tau_0 y_0^2} = \left(\frac{12Q_m}{\rho W L^2} \right) \cdot \left(\frac{\mu}{\tau_0} \right) \cdot \left(\frac{P_0}{\tau_0} \right)^2, \left(y_0 = \frac{\tau_0}{P_0/L} \right)$$

For a given rock fracture (W and L are fixed), inlet flow rate (Q_m is fixed), and Bingham flow (μ and τ_0 are fixed), the first two terms inside brackets are constants, so C_0 is positively proportional to square of the ratio of inlet pressure to yield stress, namely $C_0 \propto (P_0 / \tau_0)^2$. Therefore, for the same P_0 , the value of C_0 increase with decreasing τ_0 , indicating that the fluid increasingly resembles Newtonian fluid, which will be validated in the following discussions. Therefore, we define C_0 as the index of Newtonian flow (INF), indicating the degree of similarity between a Bingham flow and a Newtonian flow rises with increasing INF.

For a special situation with $\phi = 1/3$ ($C_0 = 4$), the EHA can be expressed as a simple function of the inlet pressure

$$h_e = 3y_0 = 3\tau_0 L / P_0 \quad (14)$$

Here, Eq. (14) can be used to calculate the EHA when the relationship between the inlet flow rate and pressure is $Q_m = \rho W L^2 \tau_0^2 / 3\mu P_0^2$. It could be very useful for conducting laboratory testing using Bingham fluid, when the test setup satisfies this condition.

Let $\lambda_1 = 3/(C_0 - 4)$, and $\lambda_2 = -1/(C_0 - 4)$ when $C_0 \neq 4$, then Eq. (13) becomes

$$\phi^3 + \lambda_1 \phi + \lambda_2 = 0 \quad (15)$$

To solve the cubic equation, firstly, we should determine the discriminant Δ of Eq. (15),

$$\Delta = \left(-\frac{\lambda_2}{2} \right)^2 + \left(\frac{\lambda_1}{3} \right)^3 = \frac{C_0}{4(C_0 - 4)^3} \begin{cases} > 0, & \text{if } (C_0 > 4) \\ < 0, & \text{if } (C_0 < 4) \end{cases}$$

There is only one real root of Eq. (15) that satisfies $C_0 > 4$, as the other two roots are imaginary numbers, thus the unique solution of Eq. (15) is

$$\phi = \sqrt[3]{-\lambda_2/2 + \sqrt{\Delta}} + \sqrt[3]{-\lambda_2/2 - \sqrt{\Delta}} \quad (16)$$

For typical grouting projects, C_0 is normally a very large number, therefore, Eq. (13) can be simplified as $C_0\phi^3 + 3\phi - 1 = 0$, then $\lambda_1 = 3/C_0$, $\lambda_2 = -1/C_0$, and $\Delta = 1/4C_0^2$. Finally, we have $\phi = C_0^{-1/3}$, so the EHA is $h_e = C_0^{1/3}\tau_0 L/P_0 = (12\mu Q_m L/\rho W P_0)^{1/3}$, which is the same as that given in Eq. (10) for Newtonian flow.

In some laboratory tests, the inlet flow rate or pressure might be very low, so the condition $C_0 < 4$ might occur. In that case, there would be three real roots for Eq. (15)

$$\begin{cases} \phi_1 = 2\sqrt{-\lambda_1/3} \cos\{\alpha/3\} \\ \phi_2 = 2\sqrt{-\lambda_1/3} \cos[(\alpha + 2\pi)/3] \\ \phi_3 = 2\sqrt{-\lambda_1/3} \cos[(\alpha - 2\pi)/3] \end{cases} \quad (17)$$

where, $\alpha = \cos^{-1}[-1/2\sqrt{(4 - C_0)^5}]$, thus $\alpha \in [\pi/2, \pi]$.

As $\alpha \in [\pi/2, \pi]$, the range of angle ϕ in Eq. (17) can be estimated as,

$$\begin{cases} \alpha/3 \in [\pi/6, \pi/3] \\ \alpha + 2\pi/3 \in [5\pi/3, \pi] \\ \alpha - 2\pi/3 \in [-\pi/2, -\pi/3] \end{cases} \Rightarrow \begin{cases} \phi_2 < 0 \\ \phi_1 > \phi_3 > 0 \end{cases}$$

As for the negative solution, ϕ_2 should not be accepted as a useful solution, as ϕ should be positive to ensure that the relevant pressure drop per unit length $P_0/L = \tau_0/(\phi h_e) > 0$. In the meanwhile, consider the expression of pressure drop $P_0/L = \tau_0/(\phi h_e)$, the conditions $\phi_1 > \phi_3 > 0$ indicate that the relevant inlet pressure corresponding to ϕ_3 is higher than ϕ_1 for the same fracture. If a lower pressure can make the cement grout move with a flow rate of Q_m , a higher pressure can lead to higher flow rate. Thus, ϕ_1 is taken as the root of Eq. (15)

$$\begin{cases} \phi_1 = 2\sqrt{1/4 - C_0} \cos(\alpha/3) \\ \alpha = \arccos[-1/2\sqrt{(4 - C_0)^5}] \end{cases} \Leftarrow \begin{cases} C_0 = 12\mu Q_m / \rho W \tau_0 y_0^2 \\ y_0 = \tau_0 L / P_0 \end{cases} \quad (18)$$

Then, the EHA can be calculated as $h_e = y_0/\phi_1$, according to Eq. (18).

A summary is presented in Table 2 for calculation of EHA under different situations. In grouting practice, the dimensionless parameter $C_0 = 12\mu Q_m / \rho W \tau_0 y_0^2$ is normally far larger than 4, then Eq. (16) degenerate into Eq. (10) when used for calculating the EHA for Bingham fluid, which is beneficial to engineering practice.

All 58 different rough fracture models generated in Section 2 are used for numerical simulation, similar to those conducted in Section 3, where the fluid type used is Newtonian flow, and the corresponding inlet flow rate ranges from 0.1 to 0.7 kg/s within an interval of 0.1 kg/s. The inlet pressure P_0 from all numerical simulations are recorded, then the corresponding h_e is calculated according to Eq. (10). The simulation results and the EHAs calculated are presented in Table A1-3 (see Appendix A1). It can be found that the EHA

decreases with increasing mass flow rate, directly indicating that the effective aperture for fluid transportation is shrinking with an increasing flow rate. In fact, it can be seen from Eq. (10) that $h_e = (12\mu Q_m L / \rho W P_0)^{1/3}$ would decrease with increasing Q_m only when the corresponding inlet pressure P_0 is of higher rate of rise than that of Q_m , indicating that the energy lost increase faster than its supply from some energy sources, like injection pump. This phenomenon exists for fluid flow within both rough fractures and smooth fractures, which can be seen from Fig. 6, where the ratios of Q_m/P_0 (the unit of this ratio is ignored) for 58 cases are presented, and the mean mechanical apertures of rough fractures listed Table A1-2 (see Appendix A1) are taken as the apertures of all smooth fractures correspondingly.

As can be seen from Fig. 6, for both smooth and rough fractures, the ratio Q_m/P_0 is declining with increasing inlet flow rate Q_m , whereas the rate of declining for rough fractures is faster than that for smooth fractures, indicating that the existence of surface roughness and tortuosity can lead to higher and faster energy lost.

4.2 Development of surrogate model for EHA prediction

It would be of great significance if we can build a surrogate model for predicting the EHA of rough rock fractures just based on some readily available geological data, without knowing details of fracture surfaces, as the concept of EHA is important not only to the field of hydraulics but also to other areas. Again, field grouting, for instance, it would be very useful if we can know in advance the injection pressure corresponding to a specific injection flow rate, as the injection pressure can be used to evaluate the potential fracture (or rock mass) deformation, which is critical to the stability and safety of tunnel or rock cavern under excavation. It is known that the characteristic parameters of rock fractures can be evaluated from geological data, and the potential injection flow rate can be read and determined from the grouting pump, which are all data that can be used to predict the injection pressure. Through data analysis, we found that a predictive model of high accuracy, for predicting the EHA of a fracture, can be built utilizing only geological data and the correlated injection rate based on Artificial Neural Network (ANN), and more details will be introduced in the following sections.

Therefore, we can indirectly calculate the injection pressure of a specific rough rock fracture via transformed Eq. (10),

$$P_0 = \frac{12\mu Q_m L}{\rho W h_e^3} \quad (19)$$

The process for building an ANN based predictive model and evaluating injection pressure is presented in a schematic diagram in Fig.7, where, σ_m , FD, and AF are standard deviation, fractal dimension, and anisotropy factor of rock fractures.

The ANN based predicative model is developed using data from Tables A1-1, A1-2, and A1-3 (see Appendix A1), which can be adopted to calculate the injection pressure under given injection rate and geological conditions (characteristics of rough rock fractures).

4.2.1 ANN based model

Data mining is usually based on intelligent algorithms, among which ANN is very popular, whose original goal was to solve problems in the same way that a human brain would (Hagan et al. 1996). So far, ANNs have been used in many areas, such as computer vision, speech recognition, machine translation, social network filtering, playing board and video games and medical diagnosis (Acharya et al. 2003; Brause et al. 1999; Evangelou et al. 2001; Kim and

Han 2000; Patil and Kumaraswamy 2009). In the field of civil engineering, there are also many applications, geological data was used for developing predictive model for dynamically evaluating engineering parameters, such as ground settlement, grout penetration range, where the relationship between geological data and parameters to be evaluated are highly nonlinear but the results obtained in literature were good (Li et al. 2017; Suwansawat and Einstein 2006; Zhong et al. 2015). The schematic diagram of ANN used in the present work is shown in Fig.7, where there are 8 nodes in the input layer, 6 hidden nodes in the hidden layer, and 1 output node in the output layer. The detailed algorithm and principles behind neural network will not be presented here, as it has been introduced in many papers (Hagan et al. 1996; Mahdevari and Torabi 2012; Suwansawat and Einstein 2006; Zurada 1992).

One sample is defined as a rough rock fracture under one inlet flow rate. In total, there are 58 rough rock fractures and 7 types of inlet flow rate, so the total sample size equals $58 \times 7 = 406$. All samples are used for developing the ANN based predictive model, and they were divided randomly into three parts: training set (70%, 284 samples), validation set (15%, 64 samples), and test set (15%, 64 samples), where the training and validation sets work together to build a predictive model, whereas the test set is for estimating the performance of the model trained. The performance of the predictive model developed is shown in Fig.8, where the coefficient of correlation (R) for three data sets and all data are evaluated, and its definition is as follows

$$R = \frac{Cov(x_1, x_2)}{\sqrt{var(x_1)}\sqrt{var(x_2)}} = \frac{\sum_{k=1}^n (x_{k1} - \bar{x}_1)(x_{k2} - \bar{x}_2)}{\sqrt{\sum_{k=1}^n (x_{k1} - \bar{x}_1)^2} \sqrt{\sum_{k=1}^n (x_{k2} - \bar{x}_2)^2}} \quad (20)$$

where x_1 and x_2 are two variables, and, in the present work, they represent target and output, respectively; $Cov(x_1, x_2)$ is the covariance of two variables; $\sqrt{var(x_1)}$ and $\sqrt{var(x_2)}$ represent standard deviation of two variables, respectively; $\bar{x}_1$ and $\bar{x}_2$ are mean values of two variables; n is the number of components in two variables; x_{k1} and x_{k2} are the k th components of two variables, respectively.

R is usually used for measuring how strong one variable is correlated to another variable, and $R=1$ indicates a strong positive relationship. As can be seen from Fig. 8, all the R involved are close to 1, indicating that the target (practical h_e) can be well described by the output (predicted h_e). To examine the degree of accuracy of the predictive model, the distribution of relative error between real and predicted h_e is shown in Fig.9 using box plot, where the relative error between 12.6976% and -11.0681% accounts for 96.06% of all sample points, among which only 16 are outliers (shown as “+”). Here, define two variables: UA (upper adjacent) and LA (lower adjacent), with $UA = q_3 + w(q_3 - q_1) = 12.6976$ and $LA = q_1 - w(q_3 - q_1) = -11.0681$, where w is the maximum whisker length of default value 1.5; $q_3 = 3.5046$ and $q_1 = -2.6624$ are the 75th and 25th percentiles, respectively. A percentile is the value less than which a given percentage of sample points in a set of sample points fall. The outliers are those points with relative error larger than UA and less than LA.

It can be found from the above discussions that the EHA can be well described by the ANN based predictive model utilizing only geological data and the corresponding inlet rate. In other words, the EHA is a function of geological data and inlet rate, whereas the function behind the ANN algorithm is too complex, it is hard to derive an expression for the function with obvious physical meaning. Therefore, in the following discussion, we will manage to derive a simple expression for the EHA.

4.2.2 Regression based model

As discussed in Section 4.2.1, the EHA h_e is a function of both the inlet flow rate and the characteristics of rough rock fracture, including its roughness R_m , tortuosity T_m , mean mechanical aperture h_m , the fractal dimension, standard deviation σ_m , and so on. To derive a simple form expression, all the data should be analysed and pre-processed. Firstly, there are seven inlet rates, and every time all fractures (generated according to Table A1-1 in Appendix A1) is under only one inlet rate, so Q_m is taken as a special factor. As for the remaining seven parameters, it is important to evaluate the importance of each factor and reduce the number of input parameters required. After step-by-step analysis, it is found that the relationship between h_e and σ_m , R_m , and h_m can be expressed by a linear polynomial, the coefficients of which are linear function of Q_m .

From stepwise regression analysis based on MATLAB programming, it is found that fracture roughness R_m , standard deviation of fracture aperture σ_m , and the mean mechanical aperture h_m are the three most important impact factors to the EHA under one inlet rate. The fractal dimension and anisotropy factor are not considered separately, since there is causal relationship between them and the remaining factors. Meanwhile, among Z_{2m} , R_m and T_m , only R_m is taken as the most critical factors, since the magnitude and variation trends of these three parameters are more or less the same, as discussed in Section 2, so it is reasonable to assume that R_m alone can represent their overall impact. The mean mechanical aperture h_m and its standard deviation σ_m describe the magnitude and the degree of concentration of fracture aperture, respectively, so they are bound to be among the most critical factors.

In building a prediction model correlating EHA h_e to the fracture roughness R_m , the standard deviation of fracture aperture σ_m , and the mean mechanical aperture h_m , an explicit expression in a polynomial form is used, i.e. $h_e = \sum_{n=0}^N (b_n R_m^i h_m^j \sigma_m^k)$, where $i \in [0, m_1]$, $j \in [0, m_2]$, and $k \in [0, m_3]$, N is the number of polynomial terms; m_1 , m_2 , and m_3 are the maximum exponents corresponding to R_m , σ_m , and h_m , respectively; and a constant term exists when $i = j = k = 0$. The EHA h_e obtained thus far is under a specific inlet flow rate, and it may vary under different inlet flow rates, so we need to establish the relationship between inlet flow rates Q_m and the coefficients b_n , expressed as $b_n = f_n(Q_m)$, ($n \in [0, N]$). Then, EHA can be expressed as $h_e = \sum_{n=0}^N [b_n(Q_m) R_m^i h_m^j \sigma_m^k]$. Finally, the EHA expression should be validated with extra cases. The detailed process for deriving the prediction model is described in the following sections.

In the present work, linear regression is tried first

$$h_e = b_0 + \sum_i^N (b_i x_i) \quad (21)$$

where, $N = 3$ is number of impact factors, x_i and b_i represent the impact factor and the regression parameters, x_1 represents the standard deviation of fracture aperture σ_m , x_2 represents the fracture roughness R_m , and x_3 represents the mean mechanical aperture h_m .

The parameter array $B = [b_0, b_1, \dots, b_N]$ is obtained when the minimum value of the sum of squared residual error (SSE or RSS) between the predicted $\hat{h}_e$ and real h_e is reached,

$$B = \arg \min_B \left(\sum_{i=1}^N (h_e - \hat{h}_e)^2 \right) \quad (22)$$

Then, the predicted EHA can be obtained based on Eq. (21). Statistical distribution of the relative error between real h_e and that predicted by the regression based analytical model is present in Fig.10, where the relative error in more than 90% of all cases is within the interval of $[-20, 20]$, and that in more than 65% of all cases is within the interval of $[-10, 10]$, indicating that the regression based analytical model derived is of high accuracy.

The function derived so far is still under some specific inlet rate, so we should integrate the special parameter Q_m into the expression in the next step. Hence, plot the parameter array of B in Fig.11, to see the relevant variation patterns. As can be seen, the value of b_1 and b_2 is negative, indicating that the existence of aperture deviation and fracture roughness can reduce the relevant EHA, which is reasonable and acceptable.

Moreover, with increasing Q_m , b_2 and b_3 are increasing, and b_0 and b_1 are decreasing. Therefore, it is reasonable to express the relevant relationship with linear equation, which is obtained through finding the best fitting lines passing through all the points corresponding to four coefficients, respectively. The linear regression curves are also drawn in Fig.11, where there is very good linear fitting.

The final prediction model, which includes the standard deviation of fracture aperture σ_m , fracture roughness R_m , mean mechanical aperture h_m , and flow rate Q_m can be expressed as

$$h_e = b_0(Q_m) + b_1(Q_m) \cdot \sigma_m + b_2(Q_m) \cdot R_m + b_3(Q_m) \cdot h_m \quad \begin{cases} b_0(Q_m) = -0.327Q_m + 0.5 \\ b_1(Q_m) = 0.311Q_m - 0.5 \\ b_2(Q_m) = 0.144Q_m - 0.232 \\ b_3(Q_m) = -0.546Q_m + 0.761 \end{cases} \quad (23)$$

Eq. (23) are used to calculate the EHAs of all cases under grout inlet rate of 0.1, 0.6, and 0.7 kg/s, respectively. The prediction results are compared with that based on numerical simulations and Eq. (10), as presented in Fig.12. The input parameters for the prediction model are only the characteristics of rock fractures (R_m , h_m , and σ_m), and the specific grout inlet flow rate Q_m . It can be found from Fig. 12 that there are small differences between the EHA predicted by the regression based analytical model and that from numerical simulations for most cases, which verifies that the derived predictive model has a high degree of accuracy, and it can predict the EHA of rock fracture under a specific inlet flow rate.

The accuracy of the prediction model for the cases under inlet flow rate $Q_m = 0.7 \text{ kg/s}$ is slightly lower than that under $Q_m = 0.1 \text{ kg/s}$ and $Q_m = 0.6 \text{ kg/s}$, since the latter two inlet flow rates are closer to the flow rate interval $[0.2, 0.5] \text{ (kg/s)}$ used for model training. This indicates that a wider range of data used for training can lead to a better prediction model, i.e., with higher accuracy.

5 Potential application

Under the condition that Eq. (23) is obtained through a large number of training cases, the coefficients a_{i0} and a_{i1} should be reliable to be used in $b_i(Q_m) = a_{i1}Q_m + a_{i0}$, ($i = 0, 1, 2, 3$), which can be used to predict the EHAs using onsite data, then other characteristics of rock fractures can be calculated based on the EHA obtained, finally we can utilize all information derived to estimate the volume of grout in need. Moreover, the ANN based predictive model can also be used here to calculate the corresponding h_e .

Consider an isolated fracture in rock mass, of which both the width and length are unknown, but some geological data can be obtained via site investigation. A simplified schematic

diagram of a designed experiment is shown in Fig.13, where the rock fracture is assumed to be a rectangular with finite width and length. The pressure measured from holes *H1* and *H2* (when the flow is stable) can be used for calculating the pressure difference between two holes, which is taken as the nominal inlet pressure; meanwhile, the steady flow rate of outflow water from hole *H2* is recorded. The top and bottom surfaces are rough but are not illustrated here.

The basic information concerning the characteristics of rock fractures, including the mean mechanical aperture h_m , the fracture roughness R_m , and its standard deviation σ_m , can be estimated according to data analysis of the rock samples from early site investigation. The distance between the two holes is $L = 5\text{m}$, which can be taken as the nominal fracture length. The fracture width W is the unknown parameter, but it is a critical parameter for grouting design, as the void volume of the rock fracture V_{frac} can be calculated as $V_{frac} = WLh_m$, which is the volume of grout required for filling the fracture. However, the width W can be calculated through the following equation transformed from Eq. (10)

$$W = \frac{12\mu Q_m L}{\rho P_0 h_e^3} \quad (24)$$

In Eq. (24), the EHA can be calculated based on Eq. (23) utilizing the site data. As shown in Table 3, three sets of data from the designed experiment are used to derive three pairs of P_0^i and Q_m^i ($i=1,2,3$) for better estimation of fracture width W .

Substitute the water flow rate Q_m^i ($i=1,2,3$), and the characteristic information of rock fractures into Eq. (23), we have three EHAs h_e^i ($i=1,2,3$)

$$\begin{cases} b_0(Q_m^1) + b_1(Q_m^1) \cdot \sigma_m + b_2(Q_m^1) \cdot R_m + b_3(Q_m^1) \cdot h_m = h_e^1 \\ b_0(Q_m^2) + b_1(Q_m^2) \cdot \sigma_m + b_2(Q_m^2) \cdot R_m + b_3(Q_m^2) \cdot h_m = h_e^2 \\ b_0(Q_m^3) + b_1(Q_m^3) \cdot \sigma_m + b_2(Q_m^3) \cdot R_m + b_3(Q_m^3) \cdot h_m = h_e^3 \end{cases} \quad (25)$$

where, the coefficients a_{i0} and a_{i1} for expression $b_j(Q_m^i) = a_{j1}Q_m^i + a_{j0}$, ($i=1,2,3; j=0,1,2,3$) is the same as that presented in Eq. (23).

By using the EHA h_e obtained from Eq. (25), the fracture width W via Eq. (24) can be calculated, which are presented in Table 4, where three fracture widths calculated are very close to each other, indicating good prediction accuracy and the feasibility of the proposed potential application.

Thereafter, the volume of the cement grout required for sealing this fracture is taken as

$$V_{frac} = W \cdot L \cdot h_m = 2.145e^{-3} (m^3)$$

6 Conclusions

In the present work, a model was developed to generate synthetic rough surfaces of rock fractures, and the fracture characteristics were analysed based on a MATLAB program. A numerical model for fluid transportation simulation based on the proposed rock fractures was also developed, and the simulation results were used for demonstrating and calculating the concept and value of equivalent hydraulic aperture (EHA) of rock fractures, respectively. Two aperture prediction models were built based on the simulation data using different methods. A designed experiment was introduced to demonstrate the potential application of the proposed predictive model for grouting design. The fluid type used for numerical simulation is

Newtonian flow, which is a special case of Bingham flow, so three equations were derived for calculating EHA corresponding to Bingham flow under different conditions.

The following conclusions can be obtained:

1. Larger mean mechanical aperture of a rough rock fracture is correlated with higher fracture roughness and tortuosity.
2. Compared with ideal fracture channel with the same mean mechanical aperture, effective spaces for fluid transportation in rough rock fracture are considerably reduced due to the existence of surface concave areas, which can be quantitatively evaluated by EHA.
3. With the increase of inlet flow rate, the inlet pressure is increasing, but the corresponding EHA is decreasing, probably because the corresponding energy lost increases faster than the corresponding increase rate of energy supply.
4. The EHA can be calculated based on a predictive model consisting of grout inlet flow rate, standard deviation and mean mechanical aperture of rock fracture, and fracture roughness, and it is negatively related to the standard deviation and roughness of a rock fracture.
5. The concept of EHA has feasibility and potential in practical application.

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Figure captions

Figure in the main text

Fig. 1 Four typical synthetic models of rough rock fractures

Fig. 2 Percentage frequency of the aperture of synthetic rock fractures

Fig. 3 Fracture models for simulating Newtonian flow within rough rock fractures

Fig. 4 Velocity distributions (Unit: m/s) presented in five different slices of Newtonian flow within rough rock fracture (Case 1 in Fig. 3)

Fig. 5 Schematic diagram for equivalent channel for rough fracture in Fig. 3 (Thompson 1991)

Fig. 6 Comparison between smooth and rough fractures for varying Q_m/P_0

Fig. 7 Schematic diagram for an ANN based predictive model and its application for field grouting (for grout flow showing Newtonian characteristics only)

Fig. 8 Correlation coefficient (R) of the predictive model developed based on ANN: Target (T) is the real h_e ; Output (Y) is the predicted h_e

Fig. 9 Box plot of the relative error between real and predicted h_e : Relative error = $100 * (\text{Output} - \text{Target})/\text{Target}$

Fig. 10 Box plot for the distribution of relative error between real h_e and that calculated based on Eq. (21): Relative error = $100 * (\text{Output} - \text{Target})/\text{Target}$

Fig. 11 Linear relationship between parameter b_i and inlet flow rate Q_m via fitting

Fig. 12 Validation of the prediction model for calculating EHA based on the characteristics of rock fracture and the corresponding inlet flow rate. “Prediction model” is based on Eq. (23), and “Simulation” represents calculation utilizing numerical simulation results and Eq. (10)

Fig. 13 Simplified schematic diagram of a rock fracture with length of 5m and width to be determined using the predictive model following Eq. (23): a designed experiment

Figure in Appendix A1

Fig. A1-1 Correlation between Z_{2m} , R_m , and T_m

Figure in Appendix A2

Fig. A2-1 Comparison of grout flow within two rough surfaces based on Bingham and Newtonian model

Table 1 Characteristic parameters of four rock fractures shown in Fig. 1

Case NO.	h_m (mm)	Z_{2m}	R_m	T_m
1	2.14	2.3274	2.3353	2.3351
2	2.46	2.4110	2.4002	2.4001
3	3.66	2.2863	2.3035	2.3031
4	0.63	1.1805	1.4931	1.4931

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Table 2 Equations for calculating EHA

Fluid type of cement grout	Equation Number
Bingham flow	Eq. (14) ($C_0 = 4$)
Bingham & Newtonian flow	Eq. (16) / (10) ($C_0 > 4$)
Bingham flow	Eq. (18) ($C_0 < 4$)

Draft

Table 3 Data from thought experiment and the fracture characteristics

Data sets	Testing pressure (Pa)	Water volume (L)	Time taken (min)	Fracture characteristics	
1	212370	120	10	σ_m	0.75
2	480353	126	7	R_m	1.64
3	864671	216	9	h_m	1.13

Draft

Table 4 Determination of fracture width W

Data NO.	h_e (mm)	W (m)	Final $W(m)$
1	0.53	0.3795	0.3797
2	0.46	0.3850	
3	0.42	0.3746	

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Fig.1 Four typical synthetic models of rough rock fractures

224x159mm (300 x 300 DPI)

Fig.2 Percentage frequency of the aperture of synthetic rock fractures
293x205mm (150 x 150 DPI)

Fig.3 Fracture models for simulating Newtonian flow within rough rock fractures

224x159mm (300 x 300 DPI)

Fig.4 Velocity distributions (Unit: m/s) presented in five different slices of Newtonian flow within rough rock fracture (Case 1 in Fig. 3)

219x94mm (300 x 300 DPI)

Fig. 5 Schematic diagram for equivalent channel for rough fracture in Fig. 3 (Thompson 1991)

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Fig.6 Comparison between smooth and rough fractures under different Q_m/P_0
148x111mm (300 x 300 DPI)

Fig. 7 Schematic diagram for an ANN based predictive model and its application for field grouting (for grout flow showing Newtonian characteristics only)

254x190mm (300 x 300 DPI)

Fig. 8 Correlation coefficient (R) of the predictive model developed based on ANN: Target (T) is the real h_e ; Output (Y) is the predicted h_e

185x185mm (300 x 300 DPI)

Fig. 9 Box plot of the relative error between real and predicted h_e : Relative error = $100 \times (\text{Output} - \text{Target}) / \text{Target}$

162x114mm (300 x 300 DPI)

Fig. 10 Box plot for the distribution of relative error between real h_e and that calculated based on Eq. (21):
Relative error=100*(Output-Target)/Target

162x108mm (300 x 300 DPI)

Fig.11 Linear relationship between parameter b_i and inlet flow rate Q_m via fitting

209x164mm (300 x 300 DPI)

Fig. 12 Validation of the prediction model for calculating EHA based on the characteristics of rock fracture and the corresponding inlet flow rate. "Prediction model" is based on Eq. (23), and "Simulation" represents calculation utilizing numerical simulation results and Eq. (10)

203x152mm (300 x 300 DPI)

Fig. 13 Simplified schematic diagram of a rock fracture with length of 5m and width to be determined using the predictive model following Eq. (23): a designed experiment

Appendix A1

Table A1-1 Orthogonal design table on key parameters for generating fracture model

Case NO.	TL	ML	σ_m	FD	AF
1	20	7	0.89	2.6	0.96
2	30	7	0.96	2.65	0.92
3	80	8	0.75	2.75	0.96
4	50	4	0.89	2.85	0.98
5	70	5	0.47	2.8	0.96
6	50	7	0.47	2.75	1.02
7	50	3	0.61	2.95	0.94
8	50	2	0.75	2.65	1.06
9	30	6	0.68	2.75	1.06
10	40	7	0.4	2.85	1
11	50	9	0.4	2.9	0.96
12	80	4	0.47	3	1.04
13	70	3	0.96	2.6	0.98
14	80	6	0.4	2.7	0.98
15	80	9	0.61	2.6	1.02
16	40	5	0.89	2.65	1.08
17	70	6	0.54	2.65	1.02
18	30	5	0.61	2.9	1
19	80	5	0.54	2.85	0.92
20	40	2	0.68	2.6	1.04
21	20	2	0.54	2.95	1
22	50	1	0.68	2.8	1
23	60	5	0.82	2.6	1.06
24	20	3	0.4	2.8	1.06
25	10	7	0.82	2.7	0.94
26	30	2	0.4	3	1.02
27	50	5	0.96	2.7	1.04
28	40	4	0.82	2.8	1.02
29	20	6	0.61	2.85	1.04
30	10	2	0.47	2.9	0.98
31	50	8	0.54	2.6	1.08
32	90	8	0.61	2.8	0.92
33	20	9	0.82	2.75	1.08
34	60	7	0.54	2.8	0.98
35	70	4	0.4	2.95	1.08
36	60	9	0.47	2.95	0.92
37	20	1	0.47	2.65	0.94
38	10	4	0.61	2.65	0.96
39	70	9	0.75	2.7	1
40	90	4	0.54	2.9	1.06
41	10	3	0.54	2.75	1.04
42	70	8	0.68	2.85	0.94
43	10	1	0.4	2.6	0.92
44	20	4	0.68	2.7	0.92
45	90	6	0.47	2.6	1
46	90	5	0.4	2.75	0.94
47	30	4	0.75	2.6	0.94
48	30	1	0.54	2.7	0.96
49	90	9	0.68	2.65	0.98
50	60	2	0.61	2.7	1.08

51	40	8	0.47	2.7	1.06
52	90	3	0.89	2.7	1.02
53	40	9	0.54	3	0.94
54	80	3	0.82	2.65	1
55	30	3	0.47	2.85	1.08
56	70	1	0.4	2.9	0.96
57	20	6	0.96	2.7	0.98
58	10	7	0.47	2.6	0.94

Notes*: TL - transition length; ML - mismatch wavelength; σ_m - standard deviation; FD - fractal dimension; AF - anisotropy factor.

A fractal dimension (FD) is an index for evaluating fractal patterns or sets, utilizing the ratio between the variation in detail to the variation in scale to measure their complexity. Therefore, FD is 0 for points (0D sets); 1 for lines (1D sets having length only); 2 for surfaces (2D sets with length and width); and 3 for volumes (3D sets with length, width, and height). For common geometries, the theoretical FD is their Euclidean or topological dimension (TD). If the theoretical FD of a set exceeds its TD, then its geometry is fractal.

Ideally, rock fractures are usually represented by two smooth parallel plates, the fractal dimension of which is 2. The surfaces of real rock fractures are rough, but they are still surface not volume, so the corresponding fractal dimension is capped at 3. In general, the fractal dimension of synthetic rough rock fractures should range between 2 and 3. It has been verified that the surface fractal dimension of natural rock fracture ranges from 2 to 2.5 (Ogilvie et al. 2006; Persson 2014; Thompson 1991). However, referring to Ogilvie's research work, FD ranging from 2.6 to 3 should be used as program input to generate real rock fractures, with the corresponding anisotropy factor varying around 1. Determination of other three parameter ranges also follows the same procedure, ensuring that the rough fractures created are practical and reasonable.

Table A1-2 Characteristics of fracture model generated (based on data from Table A1)

Case number	Mean aperture h_m	Roughness factor Z_{2m}	Roughness R_m	Tortuosity T_m
1	2.03	1.419	1.656	1.656
2	2.68	1.798	1.930	1.930
3	3.33	1.907	2.013	2.013
4	3.41	2.995	2.872	2.872
5	2.18	1.382	1.628	1.628
6	2.13	1.195	1.502	1.502
7	2.14	2.613	2.561	2.561
8	1.13	1.403	1.644	1.644
9	2.53	1.730	1.879	1.879
10	1.88	1.346	1.602	1.602
11	2.29	1.528	1.730	1.730
12	2.24	2.239	2.265	2.265
13	1.92	1.529	1.732	1.732
14	1.6	0.876	1.303	1.303
15	2.71	0.973	1.360	1.360
16	2.19	1.669	1.834	1.834
17	2.13	1.011	1.384	1.384
18	2.41	2.327	2.335	2.335
19	2.79	1.818	1.942	1.942
20	0.91	1.081	1.428	1.428
21	1.58	2.312	2.323	2.323
22	1.04	1.997	2.082	2.082
23	2.36	1.311	1.580	1.580
24	1.01	1.175	1.488	1.488
25	2.24	1.794	1.927	1.927
26	1.28	1.905	2.008	2.008
27	2.87	2.101	2.161	2.161
28	2.46	2.411	2.400	2.400
29	2.19	2.052	2.121	2.121
30	1.25	1.793	1.925	1.925
31	2.49	0.863	1.296	1.296
32	3	1.792	1.925	1.925
33	2.72	2.087	2.150	2.150
34	2.6	1.586	1.773	1.773
35	1.71	1.715	1.865	1.865
36	2.64	2.015	2.092	2.092
37	0.48	0.880	1.305	1.305
38	1.21	1.143	1.467	1.467
39	3.41	1.641	1.813	1.813
40	2.72	2.060	2.127	2.127
41	1.15	1.374	1.623	1.623
42	3.66	2.286	2.303	2.303
43	0.36	0.638	1.176	1.176
44	1.61	1.488	1.703	1.703
45	1.96	0.751	1.233	1.233
46	1.66	1.019	1.387	1.387
47	1.31	1.196	1.501	1.501
48	0.63	1.180	1.493	1.493
49	3.05	1.271	1.553	1.552
50	1.07	1.333	1.596	1.596
51	2.06	1.028	1.394	1.394

52	2.28	1.950	2.043	2.043
53	3.18	2.574	2.529	2.529
54	1.78	1.534	1.736	1.736
55	1.29	1.582	1.769	1.769
56	0.86	1.525	1.730	1.730
57	2.56	2.101	2.162	2.162
58	1.03	0.750	1.232	1.232

Fig. A1-1 Correlation between Z_{2m} , R_m , and T_m

Table A1-3 Water inlet pressure and the corresponding equivalent hydraulic aperture (EHA)

Case NO.	$Q_m = 0.2\text{kg/s}$		$Q_m = 0.3\text{kg/s}$		$Q_m = 0.4\text{kg/s}$		$Q_m = 0.5\text{kg/s}$	
	P_0	h_e	P_0	h_e	P_0	h_e	P_0	h_e
1	28963	1.034	64879	0.904	116440	0.819	184180	0.757
2	13254	1.342	29518	1.176	52817	1.066	83341	0.986
3	5795	1.768	12704	1.558	22588	1.415	35534	1.311
4	6644	1.689	14444	1.492	25250	1.363	39074	1.270
5	14857	1.291	32139	1.143	56874	1.040	89354	0.964
6	15616	1.270	33679	1.125	59462	1.025	93246	0.950
7	23661	1.106	51201	0.979	89268	0.895	137910	0.834
8	212370	0.532	480353	0.464	864672	0.420	1369121	0.388
9	13443	1.335	29998	1.170	53763	1.060	84993	0.980
10	24545	1.092	54155	0.961	96686	0.872	152699	0.806
11	13776	1.324	30058	1.169	53314	1.063	83816	0.985
12	19707	1.175	44015	1.029	78897	0.933	124666	0.863
13	37317	0.950	84024	0.830	151083	0.751	239187	0.694
14	38937	0.937	84289	0.829	148954	0.755	233838	0.699
15	6807	1.675	14513	1.490	25448	1.360	39750	1.263
16	25777	1.075	58033	0.939	104315	0.850	165102	0.785
17	17836	1.215	38744	1.074	68507	0.978	107484	0.906
18	16785	1.240	37374	1.087	66898	0.985	105585	0.912
19	8561	1.552	18739	1.368	33281	1.244	52344	1.152
20	301014	0.474	671593	0.415	1200117	0.376	1892054	0.348
21	84300	0.724	188300	0.634	335380	0.576	526400	0.534
22	154737	0.591	343362	0.519	608515	0.472	951223	0.438
23	16433	1.249	36538	1.095	65364	0.993	103222	0.919
24	179132	0.563	399964	0.493	716630	0.447	1132287	0.413
25	22405	1.126	50264	0.985	90230	0.892	142700	0.825
26	57338	0.823	129477	0.718	232720	0.650	367815	0.601
27	12203	1.379	27317	1.207	48910	1.094	77148	1.012
28	21145	1.148	47542	1.003	85275	0.909	134639	0.841
29	11538	1.405	25796	1.230	46264	1.114	73111	1.030
30	137010	0.616	309806	0.537	557552	0.486	882623	0.449
31	4153	1.975	8765	1.763	15279	1.612	23770	1.499
32	6871	1.670	15077	1.471	26834	1.336	42264	1.237
33	6703	1.684	15007	1.473	26906	1.335	42493	1.235
34	5305	1.820	11632	1.604	20691	1.457	32583	1.349
35	19630	1.177	44115	1.029	79449	0.930	126023	0.859
36	4997	1.857	10966	1.636	19522	1.486	30754	1.375
37	834949	0.337	1855639	0.296	3307897	0.268	5206459	0.249
38	54772	0.836	122504	0.732	220125	0.662	348565	0.612
39	2319	2.399	5060	2.117	8989	1.924	14150	1.781
40	10113	1.468	22345	1.290	39854	1.171	62820	1.084
41	157749	0.588	354237	0.514	635138	0.465	1002985	0.430
42	4227	1.964	9283	1.729	16504	1.571	25960	1.455
43	2502868	0.234	5442952	0.207	9604089	0.188	15015033	0.175
44	55325	0.833	124456	0.728	223906	0.659	354696	0.609
45	19618	1.177	41671	1.048	72856	0.958	113666	0.890
46	36856	0.954	80339	0.842	142417	0.766	223883	0.710
47	94483	0.697	211312	0.610	378730	0.553	598547	0.511
48	1026360	0.315	2275008	0.276	4038238	0.251	6328947	0.233
49	5652	1.782	12249	1.577	21665	1.435	34004	1.330

50	208066	0.536	466318	0.469	834726	0.425	1316857	0.393
51	16597	1.245	35942	1.101	63522	1.003	99695	0.929
52	26067	1.071	58711	0.935	105489	0.847	166832	0.783
53	6277	1.721	13714	1.518	24321	1.381	38191	1.280
54	48040	0.873	108103	0.763	194155	0.691	307113	0.639
55	111194	0.660	250221	0.577	449330	0.522	710375	0.483
56	417068	0.425	931502	0.372	1661962	0.338	2614934	0.313
57	18196	1.207	40903	1.055	73399	0.955	115978	0.884
58	114158	0.654	248014	0.578	440022	0.526	692705	0.487

Notes: The unit of Q_m , P_0 , and h_e are kg/s, Pa, and mm, respectively.

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Appendix A2

Fig. A2-1 Comparison of grout flow within two rough surfaces based on Bingham and Newtonian model

For Bingham flow, the most important parameters are its dynamic viscosity μ and yield stress τ_0 . It is known that the corresponding theoretical model is a stepwise function, thus it is hard to reach convergent solution. Therefore, Bingham-Papanastasiou model is adopted (Mitsoulis 2007). The apparent viscosity η is expressed as $\eta = \mu + \left(\tau_0 / |\gamma| \right) \left[1 - e^{(-m|\gamma|)} \right]$, where γ is shear rate, and m is a constant number controlling fitting accuracy, and it is normally taken as 3000. The corresponding dynamic viscosity μ and yield stress τ_0 are taken as 0.01 Pa·s and 0.1 Pa. The inlet flow rate is 0.1 kg/s. The simulation results based on two fluid models are shown in Fig. A2-1. The relevant velocity distributions look the same. The relevant inlet pressure corresponding to Bingham model and Newtonian model are 214.52 Pa and 194.62 Pa. The relevant EHA calculated based on Eq. (16) is 1.55 mm, which is close to that presented in Fig. 8, for Case 23 which is 1.50 mm.