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Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2018-0117.R2
Manuscript Type:	Article
Date Submitted by the Author:	02-Aug-2018
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Keyword:	controlled-suction triaxial tests, unsaturated soils, elastoplastic modelling, dilatancy, hardening law
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

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Manuscripts

Submitted to 'Canadian Geotechnical Journal' – Revised version

November 2nd 2018

MECHANICAL BEHAVIOR OF AN UNSATURATED CLAYEY SILT: AN EXPERIMENTAL AND CONSTITUTIVE MODELLING STUDY

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Abstract

This paper reports an experimental study and subsequent constitutive modelling focused on the stress-strain and volumetric response during deviatoric stress application of a partially saturated clayey-silt. The material was statically and isotropically compacted at constant water content towards a pre-defined pre-consolidation stress. A series of strain-controlled triaxial compression tests on a state-of-the-art device and isotropic experiments are presented and discussed. All of the experiments started at the same stress state (i.e. identical matric suction and mean net stress) and were conducted at the same constant suction. Several stress paths under isotropic conditions (i.e. drying/wetting, loading/unloading and wetting/drying) were followed to induce different over-consolidated states before shearing the specimens. The test results are initially interpreted using the elastoplastic Barcelona Basic Model (*BBM*). Independent tests were selected to determine the model parameters associated with the volumetric behavior of the soil. The *BBM* was not able to capture the dilatant behavior observed during shearing in all the samples. An enhancement of the *BBM* is proposed in this work, which consists in including a more general hardening law and sub-loading concepts. Main capabilities and limitations of original *BBM* and enhanced model are discussed and compared. The modified *BBM* was able to handle the dilatancy features observed in the experiments and provided a more realistic description of the experimental stress-strain behavior.

Keywords: controlled-suction triaxial tests, unsaturated soils, elastoplastic modelling, dilatancy, hardening law, sub-loading.

1 INTRODUCTION

A critical step forward to advance the current understanding on the mechanical behavior of unsaturated soils is to conduct high quality experimental campaigns and to analyze the associated results with formal constitutive frameworks. Several efforts have been made in the last years in these two areas. Regarding experimental investigations, important results have been achieved using different cells to explore particular features of unsaturated soil behavior (e.g. Merchán et al. 2011 using a ring shear cell; Cuomo et al. 2016 with a simple shear cell; Hoyos et al. 2015 with a resonant column cell; Alabdullah 2010 with a double-wall biaxial device; Hoyos et al. 2012 using a true triaxial apparatus; Romero et al. 2017 with a hollow cylinder apparatus). In relation to experimental techniques for triaxial testing, emphasis has been placed on the following aspects. To begin with, on the development of systems for controlling suction (e.g. for vapour equilibrium technique: Blatz and Graham 2000; Oldecop and Alonso 2004; Jotisankasa et al. 2007; and Pintado et al. 2009a, b; e.g. for osmotic and axis translation techniques: Delage and Cui, 2008 and Delage et al. 2008). Next, on the improvement of suction and water content measuring techniques (e.g. Lourenço et al. 2011 with tensiometers; Muñoz-Castelblanco et al. 2012 with local monitoring of water content; and Mora Ortiz 2016 with pressure transducers using axis translation technique). And finally, on the improvement of the volume change measurement techniques (e.g. Romero et al. 1997 with local laser-based sensors; Chávez et al. 2009 with local diametrical and axial deformation transducers; Wang et al. 2016 with a double wall system; Li et al. 2016 with photogrammetry-based method; Mora Ortiz 2016 and Ackerley et al. 2017 with local transducers perpendicular to the axis of the sample).

As for the development of mechanical constitutive models for unsaturated soils, significant progresses have been made in recent years. The Barcelona Basic Model

(*BBM*, Alonso et al. 1990) corresponds to a milestone in this field. The *BBM* was developed in the general framework of elasto-plasticity with the aim of extending critical state concepts for saturated soils (the modified Cam-clay model *MCCM* in particular) to the unsaturated condition. Previous modelling efforts related to unsaturated soils were mainly based on linear and nonlinear elastic approaches (e.g. Fredlund and Morgenstern 1976; Lloret and Alonso 1980), or on the upgrade of the Mohr-Coulomb failure criterion to account for suction effects (e.g. Fredlund et al. 1978). In general terms, these types of models were quite limited to reproduce the complex behavior of unsaturated soils. The *BBM* was able to capture the main features of behavior typically observed in unsaturated non-expansive soils, including (amongst others): increase of soil stiffness with suction, increase of soil strength with suction, and volumetric collapse compression behavior under wetting at constant stress. Most of the subsequent mechanical models for unsaturated soils have been aimed at: overcoming some of the basic assumptions associated with the *BBM* (e.g. Wheeler and Sivakumar 1995; Wheeler 1996; Cui and Delage 1996); proposing alternative frameworks for the definition of constitutive stresses (e.g. Loret and Khalili 2002; Gens et al. 2006; Nuth and Laloui 2006; Kodikara 2015); incorporating (explicitly) the water retention-mechanical couplings within the constitutive modelling of unsaturated soils (e.g. Vaunat et al. 2000; Wheeler et al. 2003; Sheng et al. 2008; Lloret et al. 2013; Vecchia et al. 2013, Lloret et al. 2017); extending the *BBM* to deal with the behavior of highly expansive soils (e.g. Gens and Alonso 1992; Alonso et al. 1999; Sanchez et al. 2005), incorporating anisotropic features (e.g. Romero and Jommi 2008; Al-Sharrad et al. 2017), and including chemical effects (e.g. Guimarães et al. 2013). Some few contributions have focused on the modelling of the dilatant behavior of unsaturated soils. For example, Ma et al. (2016) conducted a hydraulic and mechanical study related

to the behavior of an unsaturated silt that exhibits a dilatant response. Ma et al. (2016) adopted the Modified Cam-Clay yield surface, as in the BBM (but without the loading-collapse curve), together with a non-associate flow rule and a hardening law that not only depend on the volumetric plastic strains but also on the deviatoric ones. The volumetric dilation exhibited by this soil were accompanied with a clear post-peak stress-softening behavior. The suggested model was able to properly reproduce the isotropic and shear behaviors of this silt in tests conducted at different constant suction.

Validation exercises and benchmarks aimed at exploring the capability of models and modelers to simulate unsaturated soil behavior have been reported (e.g. Rampino et al. 2000; Geiser et al. 2000; Barrera et al. 2002; Zhou and Sheng 2009; D'Onza et al. 2011; D'Onza et al. 2015).

This paper presents an experimental study centered on the mechanical stress-strain behavior and volumetric response of a compacted clayey-silt during shearing, rather than on specific issues related to hydraulic aspects. An advanced controlled-suction triaxial cell with local axial and radial instrumentation (optical laser-based technique) was used to precisely monitor the volume change evolution on shearing. The material was isotropically compacted to avoid inducing any anisotropy on the specimen preparation. Several controlled-suction stress paths under isotropic stress state (drying/wetting, loading/unloading and wetting/drying) were performed to induce different and slightly over-consolidated states before targeting the same initial state (i.e. mean stress and matric suction) for the shear paths. A series of controlled-suction triaxial compression paths were then carried out at the different over-consolidated states previously generated, which also included the normally consolidated state. The aim was to investigate how different ways to induce over-consolidated states in unsaturated soils (i.e. by loading, or drying, or wetting induced volumetric collapse) can affect the soil

response under shearing. This is a valuable aspect to study, because natural or compacted unsaturated soils may undergo different generalized stress paths in the field (i.e. including stress and suction changes), which may affect its over-consolidated condition. Therefore, it results of interest to learn how different generalized-load histories may affect the subsequent shear behavior associated, for example, with the construction of a new structure. The experimental work also includes a comprehensive physical characterization of the soil under study and isotropic tests to determine compressibility parameters.

The *BBM* was initially adopted as the formal approach to analyze the soil response in the different experiments. The model managed to describe satisfactorily some trends of soil behavior observed in these experiments, however was unable to properly capture the volume change with dilatant behavior reported in the shearing tests. To overcome this limitation, an enhanced framework that incorporates a more general hardening law (which is now also function of the deviatoric plastic strains) and sub-loading concepts (e.g. Hashiguchi 1989) is proposed to deal with this complex volume change behavior observed during shearing. The inclusion of sub-loading concepts within the formulation of mechanical models for unsaturated soils has become very popular in the last few years. For example, in Zhang and Ikariya (2011) a *MCCM* was extended to the unsaturated condition by adopting the soil-skeleton-stress and the degree of saturation as independent state variables, and by incorporating sub-loading concepts to account for the over-consolidated unsaturated condition. Yao et al. (2014) proposed the modeling of plastic behavior in over-consolidated unsaturated clays by using a framework built upon the *BBM*, a sub-loading surface and a unified hardening parameter. The Zhou and Sheng (2014) sub-loading hydro-mechanical constitutive model for unsaturated soils is capable of dealing with the effect of soil density and uses the Bishop's effective stress concepts

together with the effective degree of saturation as constitutive variables. Li and Yang (2017) proposed a hydro-mechanical constitutive model for unsaturated soils that is based on Hashiguchi (1989) sub-loading surface, which selects the soil-skeleton-stress and a bonding variable (which depends in turn on degree of saturation) as state variables.

The modification of the *BBM* model is presented in detail in this paper including the main equations associated with the new formulation. The performance of the upgraded model when compared against the experimental data is evaluated and discussed. The paper is organized as follows, first the experimental investigation is introduced including the material, devices and stress paths adopted in the laboratory campaign. Afterwards, the soil behavior is analyzed and discussed using the *BBM*. Finally the enhanced *BBM* is introduced and its comparison against the experimental results is analyzed. The paper finishes with the main conclusions of this research.

2 EXPERIMENTAL INVESTIGATION

This section presents first the soil adopted in the experimental campaign, then a description of the triaxial cell used to conduct the controlled-suction tests, and finally the different stress paths selected for this study. Further details of the experimental investigation are described in Barrera (2002).

2.1 Tested material and compaction procedure

Laboratory tests were conducted on a low plasticity clayey-silt from Barcelona, with a liquid limit $w_L=32\%$, a plastic limit $w_P=16\%$, 15% of particles less than $2\ \mu\text{m}$ with dominant illite clay fraction, and a density of solids $\rho_s = 2.66\ \text{Mg/m}^3$ (Barrera et al. 2000; Barrera 2002). Maximum particle size was limited to ASTM N°16-1.18 mm. Samples at a water content of 11% were prepared following a stress-controlled isotropic

static compaction procedure to reach a given pre-consolidation stress and avoid any induced anisotropy on fabrication (Barrera et al. 2000). In a first stage of the procedure, a low vertical stress of 50 kPa was applied to the soil mass confined in a split mold until reaching a dry density of approximately 1.20 Mg/m^3 that was required to handle the sample. In a second stage, the sample was installed in a conventional triaxial cell and isotropically loaded to a mean net stress of $p=0.6 \text{ MPa}$ (pre-consolidation stress, where p is defined as the difference between the total mean stress and the air pressure, u_a). The initial state of the compacted sample is presented in Table 1. Matric suction (i.e. $s=u_a-u_w$, being u_w the water pressure) was determined with a transistor psychrometer (Woodburn et al. 1993; Cardoso et al. 2007) assuming null osmotic component. Because of end-restraint effect, the sample was subsequently trimmed to achieve uniform sample dimensions of 38 mm in diameter and 76 mm high. The initial state of the sample is also shown in the compaction plot presented in Figure 1. Isotropic static compaction results following the same procedure at different mean net stresses (0.3, 0.6 and 1.2 MPa) are presented. Contours of equal total suction obtained by psychrometer readings are also plotted in this figure, which were obtained from interpolations of data at different water contents and dry densities. Repeated measurements were carried out in the low suction range (lower than 200 kPa), where the psychrometers do not show so good repeatability (Cardoso et al. 2007). Once ensured a well-posed initial state (pre-consolidation stress and matric suction), the samples were subjected to different stress paths to induce different over-consolidated states before the shearing stages.

Include here: Table 1.

Include here: Figure 1.

2.2 Triaxial cell

Figure 2 shows a cross section of the controlled-suction triaxial cell (Romero et al. 1997; Romero 1999; and Barrera et al. 2000) for sample dimensions 38 mm in diameter and 76 mm high. Matric suction was applied simultaneously to both top and bottom ends of the sample ('d' in Fig. 2), maintaining a constant air pressure and controlling water pressure. Top and bottom caps include a combination of two porous stones: a peripheral metallic coarse one (pore sizes $>10\mu\text{m}$) connected to the air pressure line ('k' in Figure 2), and an internal high air-entry value HAEV one (1.5 MPa of air-entry value) connected to the water pressure line ('j' in the same figure). This double drainage system ensured a significantly shorter equalization stage for liquid pressure. However, the system usually traps more occluded air at low suctions during wetting and at mid-height of the sample. Water content changes in the soil were determined by measuring the water volume that crossed both HAEV discs by two automatic burettes with resolution $< 10\text{ mm}^3$. This volume was corrected taking into account the amount of air diffusing through the ceramic discs (Romero 1999; Delage et al. 2008).

Include here: Figure 2.

Axial displacements were measured locally using two LVDT transducers ('b' in the figure with resolution $< 1\text{ }\mu\text{m}$). Radial deformations on two diametrically opposite sides were also measured locally by means of an electro-optical laser system ($2\text{ }\mu\text{m}$ resolution) mounted outside the chamber ('c' in Figure 2). This measuring system can be moved throughout the sample height by an electric motor ('l' in the figure). The position of the laser subsection system was determined by an external LVDT ('g' in the same figure, with resolution $3\text{ }\mu\text{m}$). This way, the whole profile of the sample from the

base to the top cap could be measured, giving a better estimation of the global volume change of the sample.

Both stress and displacement controlled systems were used. An axial displacement rate of 1 $\mu\text{m}/\text{min}$ was used during the shearing stages by fluid pushing the axial loading piston at controlled volume rate. Axial load was measured by an internal load cell ('f' in the Figure 2 with resolution $<1\text{ N}$).

Figure 3 presents typical isochrones of the progressive development of local lateral displacements and radial strains at different heights during axial loading at controlled displacement rate 1 $\mu\text{m}/\text{min}$ (the axial displacement rate was applied to the bottom cap, while to top cap was kept in a fixed position). In this test, lateral stress was kept constant at $\sigma_3=0.6\text{ MPa}$, as well as matric suction at $s=0.8\text{ MPa}$. The evolution of the average radial strains is also shown in the same figure to the right by vertical dashed lines. An important aspect to indicate is the development of some membrane wrinkling at the upper part of the sample (laser 2 in Figure 3). The non-contact electro-optical sensor was thus a useful tool that allowed accurately detecting these membrane irregularities, which were more notorious at elevated axial strains during shear. These local membrane effects were corrected when estimating the evolution of the average radial strains. This was particularly important to better assess dilatancy effects at ultimate conditions of shearing.

Include here: Figure 3.

2.3 Stress paths

Two main set of tests are presented in this work, namely: i) tests involving isotropic loading / unloading only at different hydraulic states, and ii) tests that combine different isotropic paths (loading / unloading, wetting / drying and drying / wetting) before

shearing the sample at controlled suction. The first set of tests were selected to determine most of the *BBM* parameters associated with the isotropic behavior. These tests are discussed in detail in Section 3.2. As for the tests ii) involving a shearing stage, different isotropic stress and controlled-suction paths were followed before the deviator stress application, which are identified as Tests A, B, C and D in the $s:p:q$ plane shown in Figure 4, where q is the deviatoric stress (i.e. $q = \sigma_1 - \sigma_3$). Triaxial tests were conducted at a constant matric suction $s = 0.8$ MPa, which is associated with degrees of saturation below 0.548 (Table 1) and thus ensures a good continuity of the air phase and an adequate suction level for axis translation application.

Include here: Figure 4.

A constant air pressure of $u_a = 0.9$ MPa was kept constant throughout the Tests A, B and C, in which the matric suction was always $s \leq 0.8$ MPa. These tests were initiated at $p = 0.03$ MPa and $s = 0.8$ MPa. The as-compacted samples were isotropically compressed at constant $s = 0.8$ MPa in four equalization steps up to the same stress state at A2/B2 ($p = 0.6$ MPa is the same value applied in the static compaction process and shown in Fig. 1).

In Test A, a suction decrease/increase cycle at constant $p = 0.6$ MPa was applied before the shearing stage (A4→A5). First, the sample was subjected to a wetting path A2→A3 along the following matric suction steps: 0.8, 0.1 and 0.01 MPa, followed by a drying A3→A4, up to initial suction 0.8 MPa. In each step, 10 days were considered to reach equalization in terms of deformations (volumetric strain rate $< 0.02\%/day$) and drainage ($< 10 \text{ mm}^3/day$). In this way, a slightly over-consolidated (OC) state was hydraulically imposed to the soil before the beginning of the shearing path.

In Test B, shearing (B2→B3) was applied on a normally consolidated state after the initial isotropic loading B1→B2 at constant $s=0.8$ MPa, which reached the same mean net stress applied on compaction.

In Test C, a previous loading/unloading path was conducted at $s=0.8$ MPa up to a maximum $p=1.6$ MPa (with a subsequent unloading $p=0.6$ MPa) before the shearing stage was performed. The over-consolidation was induced in this case by the mechanical path: C1→C2→C3).

In Test D the over-consolidation condition was induced by imposing a drying from the initial $s=0.8$ MPa up to 87 MPa, with a subsequent wetting that brought the sample back to the initial suction (i.e. D1→D2→D3). Total suction of 87 MPa was applied with vapor equilibrium technique in a desiccator using a saturated solution of magnesium nitrate hexahydrate. Once equilibrated (after 30 days), the sample dimensions were measured and installed in the triaxial cell for the wetting step to $s=0.8$ MPa at a low confining stress of 25 kPa. Then the specimen was isotopically loaded up to $p=0.6$ MPa (i.e. D3→D4). Finally, the sample was sheared (as in the other cases) at a constant suction $s=0.80$ MPa (i.e. D4→D5). In this test the OC state was attained after a strong drying. The initial conditions of the different Tests (A to D) just before the shearing stages are summarized in Table 1 together with a brief description of the stress paths followed. As observed in the table, at the applied matric suction of 0.8 MPa, the water content varied between 8.75 and 11.7% (degree of saturation between 0.397 and 0.548) depending on the hydraulic paths followed and the void ratio attained just before the shearing stages.

During all the shearing stages at a constant rate of strain of $1.32 \times 10^{-5} \text{ min}^{-1}$ (controlled displacement rate of 1 $\mu\text{m/min}$), matric suction and radial stress remained constant

($s=0.8$ MPa and $\sigma_r=0.6$ MPa). The selected axial strain rate ensured constant-suction conditions at both top and bottom boundaries in contact with the ceramic discs. Performing a triaxial test typically involved 11 days. An average drop of the water content of 0.4% was systematically measured along the shearing stages. It could be anticipated that some drying might have locally occurred due to the relative humidity of the applied air phase (that explains the small water content reduction). These small local variations in water content were not greatly affecting the applied matric suction.

The aim behind this experimental campaign was to study how different ways to induce over-consolidated states in unsaturated soils (by controlled isotropic s - p paths) can affect the subsequent shearing behavior.

Two additional triaxial tests at different confining pressures (i.e. $p=0.3$ and 1.2 MPa, Tests E and F, respectively) were also conducted to study the material behavior at different confinements. The initial conditions before the shearing stage of these additional tests are also included in Table 1. The main experimental results associated with all the experiments described above are presented and discussed together with the modelling in Sections 3.2 and 3.3.

3 TEST RESULTS AND INTERPRETATION WITHIN THE BBM

The tests involving different stress paths succinctly presented in the previous Section are discussed hereafter in more detail with the assistance of a formal elastoplastic framework. The intention is to use first the *BBM* (i.e. the most established and perhaps simpler critical state approach for unsaturated soils) to explore the capabilities of this model to explain the behavior observed in the experiments. The main *BBM* equations are presented below, then the procedure adopted for the determination of the main

model parameters is briefly described, afterwards the experimental and modelling results are discussed together.

3.1 Barcelona Basic Model main components

The original *BBM* was developed in the context of hardening elasto-plasticity and extends the modified Cam-Clay model to the unsaturated condition. Two independent stress variables were adopted: the net stress tensor and the suction (a scalar variable). Figure 5 presents schematically the *BBM* yield surface together with possible stress paths that can induce yielding (i.e. by wetting 5a, loading 5b, or drying 5c). The yield function for triaxial stress is given by the following family of ellipses:

$$F = \frac{q^2}{M^2} + (p + p_s)(p - p_0) \quad (1)$$

where M is the slope of the critical state line. The increase in apparent cohesion with suction is given by p_s (which is initially assumed that increases linearly with suction through a constant k_s); and p_0 is the yield stress for isotropic stress conditions that is related to the applied suction through:

$$\left(\frac{p_0}{p_c} \right) = \left(\frac{p_0^*}{p_c} \right)^{\frac{\lambda_{(0)} - \kappa}{\lambda_{(s)} - \kappa}} \quad (2)$$

where p_0^* is the yield net stress for saturated conditions (which acts as a hardening parameter of the model); p_c is a reference stress; κ is the slope related to elastic isotropic unloading–reloading paths; $\lambda_{(0)}$ is the slope of the virgin compression line for saturated conditions; and $\lambda_{(s)}$ is the slope of the virgin compression line for isotropic conditions that depends on suction through:

$$\lambda_{(s)} = \lambda_{(0)} \left[(1-r) \exp(-\beta s) + r \right] \quad (3)$$

where r is a parameter controlling soil compressibility, and β provides the rate of change of $\lambda_{(s)}$ with s . Figure 5 shows a sketch of the yield surface in the (p, q, s) space, in which the trace of the yield locus on the isotropic $p:s$ plane is indicated. This trace is called the *LC* (Loading-Collapse) yield curve because it represents the locus of activation of irreversible deformation ($d\varepsilon_v^p$) due to loading or wetting (i.e. collapse).

The *BBM* considers that wetting/drying processes below the historical maximum suction (s_o) induce volumetric elastic strains changes only, through:

$$d\varepsilon_v^e = \frac{\kappa_s}{(1+e)} \frac{ds}{s + p_{atm}} \quad (4)$$

where κ_s is the compressibility modulus against suction changes, e is the void ratio and p_{atm} is the atmospheric pressure. It is also considered that suction changes beyond s_o (Figure 5) may also induce volumetric plastic strains, as follows:

$$d\varepsilon_v^p = \frac{\lambda_s}{(1+e)} \frac{ds_o}{(s_o + p_{atm})} \quad (5)$$

where λ_s is the slope of the virgin compression line in terms of suction increase.

Include here: Figure 5.

The *BBM* also assumes that the isotropic hardening is controlled by the total plastic volumetric strains $d\varepsilon_v^p$ (i.e. regardless if the plastic deformations are induced by p or s changes beyond the corresponding yield limits), through the following expression:

$$\frac{dp_0^*}{p_0^*} = \frac{1+e}{\lambda_{(0)} - \kappa} d\varepsilon_v^p \quad (6)$$

More details about the model formulation are provided in the Appendix and in Alonso et al. (1990).

3.2 Parameter estimation – Brief description

The behavior of saturated soils corresponds to the boundary condition of the *BBM*. Therefore, the isotropic saturated test was selected first to determine the following parameters: κ , $\lambda_{(0)}$ and p_0^* (i.e. 0.005; 0.085 and 0.71 MPa, respectively). The model outputs based on these constants are presented in the Figure 6. The selected parameters reproduce very satisfactory the soil behavior in general terms, with a slight under-prediction of the strains during unloading.

Include here: Figure 6.

The second test considered in this study corresponds to the isotropic loading-unloading paths contemplated in Test C, which was conducted just before shearing the sample (as illustrated in Figure 4b). Figure 7 presents the variation of the volumetric strains upon applying the loading/unloading cycle at $s=0.80$ MPa. On loading, the sample was subjected to a maximum $p=1.60$ MPa, and then it was unloaded (i.e. path C1→C2→C3, Section 2). The volumetric strain behavior displays clear pre- and post- yield zones. The yield stress is identified around $p_0=0.60$ MPa, which corroborates the maximum fabrication stress attained on isotropic compaction. The normally consolidated path (i.e. C2→C3) was selected to estimate the parameters r and β (i.e. 0.78 and 135 MPa, respectively); which account for the changes in soil compressibility with suction. The model results are also presented in Figure 7. It is worth noting that the elastic parameter κ obtained from the saturated tests discussed above, predicts very satisfactory the initial elastic loading (i.e. C1→ p_0), as well as the unloading path (i.e. C2→C3). As expected, the model predicts a sharp transition between the elastic and plastic states.

Include here: Figure 7.

One of the key features of normally consolidated unsaturated soils is the volumetric collapse compression strains generally observed upon wetting at constant net stress. This behavior is explored in the test presented in Figure 8, which consisted of an initial (isotropic) loading stage up to $p \sim 0.6$ MPa, conducted at a constant $s = 0.80$ MPa. A wetting path at constant p (i.e. 0.6 MPa) was performed afterwards, with a suction decrease up to around 0.01 MPa. This suction was hold constant (i.e. $s \sim 0.01$ MPa) for the rest of the experiment, while the sample was loaded-unloaded and reloaded up to a maximum $p = 2.42$ MPa, with a final unloading stage up to $p \sim 0.17$ MPa.

All the model parameters involved in the definition of the LC curve (i.e. Equations 2 and 3), but p_c , were already determined from the previous tests (as discussed above). The volumetric compression deformation observed in the wetting path at $p \sim 0.6$ MPa was used to back-calculate this parameter, obtaining a $p_c = 0.07$ kPa. The model outputs are presented in Figure 8. The volumetric collapse deformation is well reproduced by the model. The elastic loading and unloading paths are also quite well predicted with the parameter κ previously selected. However, some slight differences are observed during the loading at high p and high saturations (i.e. $s \sim 0.01$ MPa). The soil appears to be more compressible for $p > 1$ MPa. Regardless of this slight difference between simulated and experimental results, the global performance of the model in the isotropic tests involving changes in mean net stress and suction can be considered reasonable, including the simulation of the volumetric collapse compression strain upon soaking. Also in this case a sharp transition between plastic and elastic states is predicted by the model.

Include here: Figure 8.

The other two parameters associated with the volumetric behavior (i.e. λ_s and κ_s in Equations (4) and (5), respectively) were directly determined from the wetting-drying cycle conducted in Test D, as explained in the next Section and illustrated in Figure 11.

The determination of the parameters associated with the deviatoric behavior is based on a series of triaxial compression tests conducted at different confining stresses and suctions. Figure 9 compiles shear strength results from triaxial tests carried out on the Barcelona clayey silt specimens at different confining stresses and suctions. These tests have been conducted at UPC and are associated with different research projects and Ph.D. theses (e.g. Barrera et al. 2000, 2002; Barrera 2002; Buenfil 2007; Buenfil et al. 2016). From this experimental data, it was determined that $M=1.155$ and $k_s=0.42$. Finally, the parameter α (see Appendix) is calculated from M , κ , and $\lambda_{(0)}$, as indicated in Alonso et al. (1990).

Include here: Figure 9.

Table 2 lists all the *BBM* parameters adopted in this work. More details about the *BBM* parameters and their determination can be found elsewhere (Alonso et al. 1990; Wheeler et al. 2002; Gallipoli et al. 2010; and D'Onza et al. 2011).

Include here: Table 2.

3.3 Experimental and *BBM* modelling results

This section focuses on the analysis of the experimental data and the corresponding modelling results related to Test A, B, C and D described in Section 2.3. The isotropic stress paths conducted before the shearing stages are analyzed first and the stress-strain curves involving deviatoric loads are discussed afterwards.

In connection with test A, the initial isotropic loading at constant suction (i.e. A1→A2 in Figure 4a) brings the material to the normally-consolidation condition (i.e. $p=p_0$, at $s=0.8$ MPa). Figure 10 also shows the adopted initial LC curve with the main stages of loading for Test A (i.e. A1→A2→A3→A4 as indicated in Figure 4a). In this test, the initial wetting (i.e. from $s=0.8$ MPa up to $s\sim 0.03$ MPa) is associated with an almost vertical branch of the LC curve (Figure 10), anticipating very small plastic compression strains. The subsequent suction decrease (A2→A3) engages two mechanisms, namely: an elastic swelling (controlled by k_s), and a volumetric collapse compression strain that is mainly driven by the suction changes and the shape of the adopted LC curve.

Include here: Figure 10.

Figure 11 shows that in this initial stage (i.e. $0.8 > s > 0.03$ MPa) the elastic swelling prevails, and a net expansion is predicted by the model. However, as suction decreases further, the plastic collapse mechanism is engaged, and a net positive compression strain is predicted. These collapsible plastic strains induce an increment of the hardening parameter p_0^* (i.e. Equation 6), dragging the initial LC locus to its final position after the wetting (Figure 10). The subsequent scanning drying path induces shrinkage (A3→A4 in Figure 10) and produces small elastic deformation only, which has (apparently) no influence on the final yield surface before the shearing stage. In this way, during the initial phase of the shearing stage elastic strains develop only, and plastic strains appear after dragging the yield surface. The selected LC curve was able to satisfactorily capture the main trends observed in these paths. The modelling outputs agree well with the experimental data, however the final volumetric collapse compression strains upon soaking are slightly under-predicted. It is worth noting that all the model parameters involved in these isotropic paths were obtained from independent

tests, so these results can be considered model predictions, and therefore small differences respect to the experimental behavior can be anticipated.

Include here: Figure 11.

Figure 12 presents the volumetric strains associated with the drying-wetting path at constant net stress in Test D (i.e. D1→D2→D3, Figure 4a). Elasto-plastic deformations are anticipated from the beginning of the drying, because suction values above the maximum historical one (i.e. $s_o=0.8$ MPa) were applied. Elastic strains are expected during the subsequent wetting. It is assumed that the plastic strains induced upon drying also lead to an expansion of the *LC* yield curve, because of the hardening law adopted in equation (6). Also in this case, during the initial shearing elastic strains develop only and plastic strains appear after dragging the yield surface.

Include here: Figure 12.

Figure 13a shows the experimental results alongside with the original-*BBM* outputs during shearing for the different tests. The stress-strain curve of Test B resembles the one expected for a normally consolidated soil, with a non-appreciable yield point. The volumetric behavior of this specimen shows a dominant contractive strain at the beginning of the experiment, with a dilatant trend afterwards. As for over-consolidated Tests A, C and D; the soil presents a stiffer slope at the beginning of the shearing (which can be associated with loadings inside the elastic domain), with a faster degradation of the stiffness afterwards. These three (OC) samples also exhibit dilatant volumetric behaviors during shearing, without displaying a strain-softening behavior in the stress-strain curve, as it is usual in saturated and unsaturated soils. For example, the volumetric dilation of the deltaic unsaturated sediment studied by Ma et al. (2016) was always associated with a clear post-peak stress-softening behavior, which represents a

significant difference respect to the Barcelona clay-silt behavior presented here that always tends to harden during yielding. The modelling of the Test B shows that the initial stress point (i.e. $s=0.8$ MPa and $(p_o)_B=0.6$ MPa) is on the yield surface $LC-B$ at the beginning of the shearing stage (B1→B2 in Figure 14). Therefore, plastic straining is occurring from the beginning of the shearing. In terms of stress-strain behavior, the model satisfactorily captures the observed behavior with no apparent pre-yielding in Test B. According to the adopted elastoplastic framework, a volume decrease associated with the shear increase is predicted in this test. As expected, the *BBM* was not able to reproduce the soil dilatancy observed in this normally consolidated sample.

Include here: Figure 13.

Include here: Figure 14.

In Tests A, C and D the *BBM* predicts a stiff transition between elastic and plastic states. This response is related to the initial elastic stress-path lying inside the yield surface, which was dragged by the previous p and s paths involved in those tests, becoming the samples in an over-consolidated state. However, yielding is very sudden on the predicted curves and further refinements seem necessary to provide a smoother transition between elastic and plastic states and to be closer to the actual observed behavior. Furthermore, an (apparent) over-prediction of the elastic range (as it happened in Test A and D) is not recommendable when dealing with practical engineering problems.

In terms of the influence of the different paths selected to induce the over-consolidated states, on one hand the loading/unloading path associated with Test C is the one that induced the larger hardening in the material with a $p_o \sim 1.77$ MPa (i.e. previous to the

shearing stage). On the other hand, the drying/wetting path related to Test D is the one that affected less the apparent pre-consolidation pressure, $p_o \sim 0.90$ MPa. The intermediate behavior corresponded to the wetting/drying path induced in Test A, $p_o \sim 1.05$ MPa. The observed and predicted shear strengths in the different experiments was practically not affected by the p - s pre-shearing paths. However, the volumetric strains were influenced, Tests A and C were the ones that exhibit the lower contractive strains and larger dilation, while Test D contracted more and dilated less than them. Sample B presented the larger compressive volumetric strains, with some dilation at the final stages of the shearing. The *BBM* managed to qualitatively predict this pattern, i.e. with larger strains for Test B and lower for Tests A and C, with specimen D showing and intermediate behavior. However, it was not able to predict the dilatant behavior observed in all the slightly OC samples. An upgraded model is proposed in the next Section to overcome some of the problems observed in these experiments with the aim of achieving a closer representation of the experimental results.

4 ENHANCED BBM

The limitation of the *BBM* to reproduce the dilatant behavior of the soil (as discussed above) is in fact inherited from the modified Cam-Clay model, which is the reference framework adopted in the *BBM*. The *MCCM* (as many others strain-hardening elastoplastic models) predicts a dilatant response in soils when the stress path is above the M line (i.e. $p/q > M$, this implies that the stresses are on the left or ‘dry’ side of the yield surface). Furthermore, this dilatant response is always associated with a softening behavior in the stress-strain curve (feature that was not observed in these tests). These combinations of soil-hardening with contractive-strains, or soil-softening with dilatant-response, are quite typical behavior of soils and this is why the *MCCM* and *BBM* are often used with success to simulate the response of saturated and unsaturated soils,

respectively. However, these models are not able to predict the behavior of other type of soil that exhibits untypical behavior, as the one studied in this paper.

The main aim is to maintain the mechanical model as simple as possible and introduce some few changes, which are based on well-known concepts by the geotechnical community that will assist to a better description and understanding of the experimental data. In this Section the main components of an upgraded *BBM* are presented first, followed by the application of the proposed approach using the experimental results discussed in the previous sections, plus the analysis of some additional tests.

4.1 Model formulation

The suggested changes are related to two main aspects: a more general hardening law, and the incorporation of subloading concepts. The prediction of a sharp transition between elastic and plastic states is a common characteristic of typical elastoplastic models. Sub-loading concepts allow overcoming this shortcoming. With this technique is also possible to model irreversible strains that (sometimes) are observed inside the yield surface. Detailed information about the sub-loading theory can be found elsewhere (e.g. Hashiguchi 1989). Only those equations of the proposed model that are different from the classical *BBM* (introduced in Section 3.1) are discussed hereafter.

The elastic behavior is modelled as in the *BBM* (i.e. through equations A1 to A3 in Appendix). As reported in previous contributions, some (few) limitations associated with the elliptical yield surface can be overcome by adopting a more advanced formulation (e.g. Muhunthan et al. 1996; and Whittle and Kavadas 1994). However, for the sake of the simplicity, the original *BBM* yield surface proposed by Alonso et al. (1990) was maintained in this work (i.e. equation 1), and a sub-loading yield surface

located inside the *BBM* one is added. This new yield surface has the same shape of the *BBM* one and is expressed as follows:

$$F = \frac{q^2}{M^2} + (p + p_s)(p - Rp_0) \quad (7)$$

where R is the sub-loading ratio (with $0 < R < 1$), which evolution is expressed as:

$$dR = -\mu \ln R |d\varepsilon^p| \quad (8)$$

where $d\varepsilon^p$ is the increment of total plastic strain; and μ is the parameter that controls the rate of the evolution of the sub-loading yield surface, which is related to the smoothness of the elastic to plastic transition. This parameter can be estimated from isotropic or triaxial tests. The initial R value (R_0) is determined as the ratio between the initial mean net stress and the mean yield stress, as follows:

$$R_0 = \frac{p_{mi}}{p_0} \quad (9)$$

Table 3 lists the R_0 values corresponding to the different tests. More information about sub-loading concepts and parameters can be found elsewhere (e.g. Hashiguchi 1989; Gai and Sanchez 2017). Equations (7), (8) and (9) correspond to the three main components associated with the incorporation of sub-loading concepts.

Include here: Table 3.

The *BBM*, as most of the mechanical models for soils (e.g. *MCCM*), assumes that the hardening/softening behavior depends on the plastic volumetric strains only. However, as discussed in Nova (1988), a more general description of soil behavior can be achieved if the history variable of the model is a function of both, volumetric and shear

plastic strain increments ($d\varepsilon_q^p$). Following this idea, the hardening law associated with the pre-consolidation pressure is expressed as:

$$\frac{dp_0^*}{p_0^*} = \frac{1+e}{\lambda-\kappa} d\varepsilon_v^p + D_s \frac{1+e}{\lambda-\kappa} d\varepsilon_q^p \quad (10)$$

where D_s is an experimental parameter associated with the soil dilation at failure (Nova, 1988). As in the *BBM*, the total plastic volumetric strains (i.e. the ones induced by p and s changes) are considered in the first term of the right-hand side. If D_s is set equal to zero, the original *BBM* hardening law is recovered. Gai and Sanchez (2018) used a similar hardening law to model bio-cemented soils dilatancy. Previous contributions (e.g. Collins and Hilder 2002; Collins 2005; and Houlsby and Puzrin 2006) have dealt with the formal aspects related to the inclusion of the plastic shear strains in the hardening law. Collins and Hilder (2002) states that such an approach requires the rotation of the yield surface in the stress space, becoming the material response anisotropic. However, a yield surface rotation was not implemented in the current modeling. A similar approach has been followed in other contributions (e.g. Wilde 1977; Nova 1988; Boulon and Nova 1990; Anandarajah 1994; Ma et al. 2016). A consistent thermomechanical framework for developing elasto-plastic models based on thermodynamics principle (including the corresponding methodology for handling the yield function rotation) is discussed in detail in Collins and Hilder (2002).

The upgraded *BBM* model includes two extra parameters compared with the original *BBM*. Almost the same parameters adopted in Section 3.2 for the *BBM* are selected for the new version of the model, as shown in Table 4.

Include here: Table 4.

4.2 Experimental and enhanced *BBM* results

The tests presented in Sections 3.2. and 3.3 were simulated using the new model, some of these results are presented in this Section. For example, Figure 15a shows the modelling of the isotropic saturated test, and Figure 15b the results of the isotropic loading on an unsaturated sample followed by a soaking at constant p , with the subsequent unloading-reloading and unloading stages. A very satisfactory description of the experiments is achieved in both cases. After comparing these outputs against the *BBM* ones (i.e. Figure 7 and 9), it is observed that a smooth (and more realistic) transition between elastic and plastic states in the isotropic plane is obtained with the new model.

Include here: Figure 15.

Figure 13b presents the enhanced-*BBM* results for the four tests involving a shearing stage. The new model resembles very satisfactorily the observed experimental behavior, predicting a very smooth transition between elastic and plastic states during shearing, which are very similar to the actual material behavior. The extent of the elastic domain is not over-predicted in these cases, as it happened in some cases analyzed with the *BBM* (e.g. Test A). As for the volumetric response, the new model is not only able to improve the results at earlier stages of the tests, but also captures very satisfactorily the dilatant behavior observed later on. In general terms, it can be said that the stress-strain behavior is well captured by the enhanced-*BBM*.

Two additional triaxial compression tests conducted at different confining pressures were selected to further study the original and the enhanced *BBM*. The new experiments are related to the Test B and involved cell pressures that are lower (i.e. $\sigma_3=0.3$ MPa) and higher (i.e. $\sigma_3=1.2$ MPa) than the reference one (i.e. $\sigma_3=0.6$ MPa). The test at the lower

pressure is over-consolidated (i.e. OCR 2), while the other two are normally consolidated. The suction of all three tests was maintained at 0.8 MPa. Figures 16 compares the performance of the two models (i.e. *BBM* and enhanced-*BBM*, Figure 16a and b, respectively) and the experimental results are again included in both Figures to facilitate the discussion. As expected and observed in the previous shear tests, the *BBM* was not able to predict the dilation observed in these experiments, however the new model manages to capture qualitatively well the dilatant behavior observed in these tests, but with a slight under-prediction of the volumetric dilation. The new model was also capable of reproducing very satisfactorily the stress-strain behavior observed in the experiment. Looking at Figures 13, 15 (7 and 8), and 16, it can be concluded that the new model managed to capture very well in a qualitative manner the observed material behavior, and in most of the cases the results were also satisfactory in quantitative terms. After comparing the original-*BBM* and enhanced-*BBM* performances, it is evident that the new model represents an improvement respect to the original one, particularly when simulating more complex unsaturated soils, like the one studied in this paper.

Include here: Figure 16.

5 SUMMARY AND CONCLUDING REMARKS

Controlled-suction isotropic and triaxial experiments were conducted on an isotropically compacted silt to provide insights into the stress-strain behavior and volumetric response of this kind of soil. Experiments involving normally consolidated and lightly over-consolidated (OC) states were conducted. The preparation of the OC samples was done following different test protocols. For example, in one specimen the OC state was achieved mechanically by loading/unloading maintaining a constant suction. For the other specimens two different hydraulic paths were conducted, namely: a wetting path

(at a constant mean net stress) followed by a drying; and a drying (at a constant mean stress) and then a soaking. After these isotropic paths, a deviatoric load was applied until soil failure. In addition to these experiments, tests involving isotropic loads only were also conducted on saturated and unsaturated specimens. The reported experiments were conducted in a state-of-the-art triaxial cell that allows conducting tests at constant suction with a precise measurement of the volumetric deformations by means of a laser scanning.

The tests were first analyzed using the Barcelona Basic Model (*BBM*), which is formulated in the framework of hardening elasto-plasticity. The procedure followed for determining the model parameter was explained in detail. Comparisons with wetting/drying and loading/unloading test results, showed the ability of the model to capture in a satisfactory manner most of the qualitative trends of the experimental observations and in many cases the quantitative tendencies were well reproduced as well. The model also allowed simulating the shearing response including the gradual yielding from the beginning of the test of the normally consolidated sample and the pre- and post- yield responses of the lightly over-consolidated specimens. However, it failed in predicting the post-yield transition between contraction and dilatancy observed at advanced stages of shearing in all the samples, including the normally consolidated specimen. The *BBM* also predicts unrealistic sharp transitions between the elastic and plastic behavior under isotropic and shearing conditions.

To overcome the shortcomings discussed above, enhancements were introducing into the original *BBM* by incorporating sub-loading concepts and extending the hardening law to include the effects of the shear plastic deformations as well. Two additional model parameters are associated with these changes. The performance of the new model was compared against the tests discussed above and the original *BBM*. The enhanced

BBM was able to reproduce the dilatant behavior observed in all the samples. The maximum shear strength observed in the different tests was also well captured by the model. Smooth transitions between pre- and post- yielding behaviors were predicted by the model, which reproduce more closely the actual response observed in the experiments.

Based on the models results and observed soil behavior, it was concluded that the pre-shearing loading/unloading stage (i.e. Test C) was the one that induced a more significant effect in the material behavior, with a larger increase in the apparent pre-consolidation pressure. While Test D, in which the over-consolidated state was induced by a drying weeding path was the one that affected less this variable, and intermediate behavior correspond to Test A (i.e. over-consolidation indices by a wetting-drying path). These previous isotropic paths did not affect the maximum shear strength of the soil, but it did has a significant impact on the volumetric behavior.

The proposed framework appears as an attractive alternative to the *BBM* for those cases in which the soil exhibits dilatant behavior when yielding takes place with stresses on the ‘wet side’ of the yield surface (i.e. $p/q < M$), soil behavior feature that the *BBM* cannot handle. The simulation of a realistic smooth transition between elastic and plastic states is another advantage of the proposed model.

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Figure captions

Figure 1. Static compaction curves for three isotropic stresses (0.3, 0.6 and 1.2 MPa). Solid lines indicate contours of equal total suction and dashed lines contours of equal degree of saturation.

Figure 2. Scheme of the triaxial cell and picture. a) Specimen; b) local LVDT (axial strain); c) laser displacement sensor (radial strain); d) high air-entry ceramic disc surrounded by coarse metallic porous stone; e) Perspex wall; f) internal load cell; g) LVDT (vertical displacement of laser sliding subjection); h) confining air pressure; i) axial stress pressure chamber; j) water pressure (to volume change measuring system); k) air pressure; l) vertical displacement electric motor.

Figure 3. Isochrones of local lateral strains (Test A: OC-collapse).

Figure 4. Stress paths followed in $q:p:s$ space: a) Tests A and B; b) Tests C and D.

Figure 5. Yield surfaces of the *BBM* model and their expansion during: a) wetting, b) loading, and c) and drying.

Figure 6. Isotropic loading on saturated sample.

Figure 7. Isotropic loading and unloading at constant suction (i.e. cycle path $C1 \rightarrow C2 \rightarrow C3$).

Figure 8. Isotropic loading on unsaturated sample with the subsequent wetting and unloading-reloading-unloading paths.

Figure 9. Experimental results related to Barcelona clayey silt shear strength compiled from triaxial tests.

Figure 10. Evolution of LC and yield surface (A2→A3→A4→A5).

Figure 11. Variation of volumetric strain with suction (A2→A3→A4).

Figure 12. Variation of volumetric strain with suction (D1→D2→ D3).

Figure 13. Shearing paths at constant suction: a) original-*BBM* and experimental results, and b) enhanced-*BBM* and experimental results.

Figure 14. Evolution of LC and yield surface (B1→B2).

Figure 15. Experimental observations and modeling results obtained with the enhanced *BBM*: a) saturated isotropic test; and b) isotropic loading on an unsaturated sample with the subsequent wetting and unloading-reloading-unloading paths.

Figure 16. Shearing at different confining pressure (i.e. 0.3, 0.6 and 1.2 MPa): a) original-*BBM* and experimental results; and b) enhanced-*BBM* and experimental results.

APPENDIX A1

Alonso et al. (1990) proposed a non-linear mechanical model for estimating the elastic strains induced by net stress inside the yield locus, as follows:

$$d\varepsilon_v^e = \frac{\kappa}{1+e} \frac{dp}{p} \quad (A1)$$

$$d\varepsilon_q^e = \frac{dq}{3G} \quad (A2)$$

where $d\varepsilon_v^e$ and $d\varepsilon_q^e$ are the increments in volumetric and shear elastic strains, respectively; and G is the shear modulus that is calculated in terms of bulk modulus (K) through:

$$G = \frac{3(1-2\nu)}{2(1+\nu)} K \quad (A3)$$

where ν is the Poisson's ratio.

A non-associated flow rule is assumed through:

$$\frac{d\varepsilon_q^p}{d\varepsilon_v^p} = \frac{2 q \alpha}{M^2 (2p + p_s - p_0)} \quad (A4)$$

where α is a constant related to non-associative flow rule. More details can be found in Alonso et al. (1990).

Table 1. Initial states and stress paths

Initial state	Dry density* ρ_d (Mg/m ³)	Void ratio* e	Water content* w (%)	Degree of saturation* S_r (%)	Matric suction* s (MPa)	Stress paths
As-compacted	1.63	0.632	11.0	46.3	0.8	isotropic static compaction
Test A	1.70	0.567	11.7	54.8	0.8	A1-A2: isotropic loading A2-A3-A4: wetting/drying paths A4-A5: shearing
Test B	1.63	0.632	11.0	46.3	0.8	B1-B2: isotropic loading B2-B3: shearing
Test C	1.72	0.550	10.4	50.3	0.8	C1-C2-C3: isotropic loading/unloading paths C3-C4: shearing
Test D	1.68	0.587	8.75	39.7	0.8	D1-D2-D3: drying/wetting paths D3-D4: isotropic loading path D4-D5: shearing
Test E	1.64	0.624	10.8	46.0	0.8	isotropic loading to 0.30 MPa shearing
Test F	1.69	0.575	9.90	45.8	0.8	isotropic loading to 1.2 MPa shearing
Isotropic loading (saturated)	1.61	0.650	24.4	1.0	0.0	isotropic loading/unloading (saturated)
Isotropic loading with subsequent wetting/loading paths	1.63	0.632	11.0	46.3	0.8	isotropic loading with subsequent wetting and loading/unloading paths (saturated)

*Initial state before shearing stage (Tests A to F)

Table 2. Adopted parameters for the original-*BBM*.

Parameter	Value	Parameter	Value
κ	0.005	κ_s	0.0015
$\lambda_{(0)}$	0.085	λ_s	0.0078
p^*_0	0.071 MPa	p^c	0.07 kPa
r	0.78	β	135 MPa
ν	0.33	α	0.6
M	1.15	k_s	0.42

Table 3. Initial R_θ values –Enhanced-BBM

Initial state	Confining pressure (MPa)	p_θ (MPa)	R_θ
Test A	0.60	1.052	0.57
Test B	0.60	0.600	1.00
Test C	0.60	1.771	0.33
Test D	0.60	0.897	0.66
Test E	0.30	0.600	0.50
Test F	1.20	1.200	1.00

Table 4. Adopted parameters for the upgraded-*BBM*.

Parameter	Value	Parameter	Value
κ	0.005	κ_{σ}	0.0015
$\lambda_{(0)}$	0.085	λ_s	0.0078
p^*_0	0.071 MPa	p^c	0.07 kPa
r	0.78	β	135 MPa
ν	0.33	α	0.6
M	1.00	k_s	0.42
D_s	0.3	μ	100

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