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Ultimate Seismic Bearing Capacities and Failure Mechanisms for Strip Footings Placed Adjacent to Slopes

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ABSTRACT

The assessment of the ultimate bearing capacity of foundations adjacent to slopes is complex as it is highly dependent on the slope geometry and soil properties. Seismic loading may impact both the critical failure mechanism and its associated bearing capacity. The existing approaches for analyzing the seismic bearing capacity of footings near slopes typically employ coefficients developed to fit the conventions of Terzaghi's bearing capacity equation, herein referred to as the "superposition method". In this study, a rigorous assessment of the seismic bearing capacity is performed using an upper-bound limit state plasticity framework known as discontinuity layout optimization (DLO), which makes few prior assumptions concerning the failure geometry. The results show that soil properties, slope configuration, and pseudostatic seismic loading all influence the realized failure mechanism and associated bearing capacity. The use of bearing capacity coefficients that fit within the conventional superposition method may underestimate limit loads when the underlying soil provides a relative increase in resistance but may greatly overestimate bearing capacity when the self-weight of the soil is destabilizing in nature. A set of design charts using direct computational methods for a variety of geometric, geotechnical and seismic conditions is provided.

Keywords: Bearing capacity; Earthquakes; Slopes; Foundations; Failure mechanism.

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INTRODUCTION

The original theoretical research on bearing capacity was performed by Prandtl (1921) to assess the bearing pressure on a weightless material. This solution was modified by Terzaghi (1943) to estimate the bearing capacity while considering the combined influences of cohesion, footing embedment, and soil weight. Throughout the rest of this study, the conventional bearing capacity equation proposed by Terzaghi (1943) is referred to as the “superposition method.”

In practice, structures built near a slope are common (e.g., Clark et al. 1988; Bathurst et al. 2003; Blatz and Bathurst 2003; Lee and Manjunath 2000; Georgiadis 2009, 2010; Turker et al. 2014). Meyerhof (1957) adopted the conventions of the superposition method and assumed failure mechanism geometries a priori in an investigation of the static bearing capacity for footings placed adjacent to slopes. Later, several studies (Kumar and Kumar 2003; Chakraborty and Rao 2006; Askari and Farzaneh 2003; Kumar and Ghosh 2006; Kumar and Chakraborty 2013; Chakraborty and Mahesh 2016; Chakraborty and Kumar 2015; Kumar and Rao 2003) expanded the assessment of bearing capacities near slopes to account for seismic loading. Among these studies, investigations (e.g., Kumar and Rao 2003; Kumar and Ghosh 2006; Chakraborty and Kumar 2015; Chakraborty and Mahesh 2016) have attempted to provide bearing capacity factors that fit within the superposition framework (i.e., the values of N_c and N_γ were determined separately by assuming that $\gamma = 0$ and $c \neq 0$ or that $\gamma \neq 0$ and $c = 0$, respectively). They generally assumed that failure occurred directly through the slope face (i.e., assuming a stable slope of

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infinite height), and ignored the complex relationships among slope geometries, material properties, and seismic loads that can also influence the realized failure mechanisms. Recent studies have demonstrated that static bearing capacity near slopes is a function of the soil properties and geometric parameters (Leshchinsky 2015; Leshchinsky and Xie 2017; Zhou et al. 2018), which often results in complex failure mechanisms. The presence of earthquake loading adds more complexity to the problem.

The main objective of this technical note is to explore the influence of horizontal pseudostatic seismic coefficients on the failure mechanism and the ultimate load for footings located near slopes. The suitability of bearing capacity factors compatible with the superposition method for analyzing the capacity of footings near slopes subject to seismic conditions is discussed. Finally, a set of design charts is presented.

METHODOLOGY

The discontinuity layout optimization (DLO) procedure (Smith and Gilbert, 2007) implemented in the LimitState: GEO v3.4 software (LimitState, 2013) was adopted for this study. DLO is an efficient tool for directly obtaining upper-bound collapse loads and critical failure mechanisms. An advantage of this method is that it works without assuming the failure mode a priori. The bearing capacity of shallow foundations on slopes under static conditions has successfully been assessed by Leshchinsky (2015), Leshchinsky and Xie (2017) and Zhou et al. (2018) using the DLO procedure. Fig. 1 shows a schematic of the model used in this study. A weightless, rigid strip footing of width B was placed adjacent to the slope crest with a

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slope angle β and a slope height H . The seismic stability of these systems was conventionally analyzed using pseudostatic conditions, and seismicity was incorporated through a horizontal pseudostatic acceleration coefficient k_h applied to both the soil and the footing. The interface between the rigid footing and the soil was assumed to be rough.

Comparison to Prior Research

A brief comparison of the DLO procedure was made against seismic bearing capacity factors available from the prior literature, considering a footing placed on a slope with $\beta = 15^\circ$ and $\varphi = 40^\circ$, as shown in Fig. 2. Kumar and Rao (2003) applied the stress characteristic method, finding a slightly lower solution than that provided by DLO. In contrast, Kumar and Kumar (2003) applied a limit equilibrium (LE) approach which delivered higher ultimate bearing capacities than those from DLO. However, under high k_h values, both the DLO and LE methods show consistent results. The differences between the DLO results and those from the lower-bound limit analysis of Kumar and Chakraborty (2013) are typically within 4.0%, demonstrating that the DLO procedure provides a reasonable evaluation of bearing capacity factors.

RESULTS AND DISCUSSION

Applicability of the Superposition Method for Assessing Bearing Capacity

The bearing capacity of footings is commonly determined using bearing capacity factors compatible with the superposition method, which is defined as follows:

$$q_{u,s} = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma \quad (1)$$

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For a footing placed on horizontal ground, the conventional bearing capacity approach is considered conservative, and the error induced by this superposition assumption is less than 30% (Zhu et al. 2003). Griffiths (1982) reported that the error stems from the non-linearity of N_γ . Table 1 shows a comparison of the bearing capacity $q_{u,s}$ obtained using the superposition method with $q_{u,D}$ obtained from direct computation using numerical tools (DLO and finite element analyses from Griffiths, 1982). The unit weight of soil γ is 20 kN/m³. The superposition error obtained with the DLO method is -7.4%, consistent with that of Griffiths (1982), which was calculated to be -9.5%. Nonetheless, the superposition method is commonly used in engineering practice for footings on a horizontal ground.

Two cases of a footing on slopes are presented that highlight limitations in the use of the superposition method. Note that the effect of the footing embedment is not considered. Fig. 3 (a) shows a comparison between the direct computation of seismic bearing capacity $q_{u,D}$ using DLO and the bearing capacity $q_{u,s}$ using the superposition method (DLO and prior studies from Kumar and Rao 2003, and Kumar and Chakraborty 2013) for $\beta = 30^\circ$, $\varphi = 40^\circ$, and $c/\gamma B = 1.0$. The superposition method used in prior studies isolated N_γ for the superposition method by assuming that when $c = 0$, the slope is stable when $k_h < \tan(\varphi - \beta)$; hence, only solutions for $k_h < 0.176$ were presented. The $q_{u,D}$ determined from the direct computational method was compared to that the previous studies assuming a stable slope of infinite height, but two finite slope heights are also shown for illustrative purposes (i.e., $H/B = 1$ and 4). For $H/B =$

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4, the superposition method slightly underestimates the bearing capacity, which is similar to that of a footing on a horizontal ground. For $H/B = 1$, the $q_{u,D}$ is much larger than $q_{u,S}$ values, but it achieves a consistent result as k_h increases beyond 0.4 because the stability of the slope governs. That is, the stability of simple slopes can be expressed as a function of c , γ , and H , suggesting that the convergence with large k_h stems from a decrease in the influence of the footing load relative to the large body forces acting on the unstable slope. Fig. 3 (b) shows another comparison between the direct computational method and the superposition method both using DLO for $\beta = 40^\circ$, $\phi = 20^\circ$, and $c/\gamma B = 0.75$. Slope heights of $H/B = 2, 4$, and 8 are considered for $q_{u,D}$ values. An overestimate of the bearing capacity is observed for the superposition method as the N_c value is significantly overestimated from the process of isolating its value. That is, the slope is fully stable when using the assumption of $\gamma = 0$ when evaluating the N_c value in the superposition method. Consistent results are observed for $H/B = 1$, whereas major discrepancies are observed for $H/B = 4$ and 8 for k_h larger than 0.35 and 0.1, respectively. Again, the increasing discrepancies demonstrate the geometric transition in the mechanism where the influence of footing load is small with respect to the failing slope.

Influence of the Observed Failure Mechanism

Leshchinsky (2015), and Leshchinsky and Xie (2017) reported that the static bearing capacity of a footing on slopes is governed by its critical failure mechanism. Failures that pass through the toe of the slope yield the lowest relative bearing capacity as it is associated with a minimum in soil passive resistance and a maximum in the driving

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forces associated with soil self-weight. Zhou et al. (2018) defined the threshold between bearing capacity and slope stability mechanisms based on whether the soil weight contributes to the passive resistance or acts as a destabilizing force, and six failure modes were observed, as shown in Fig. 4.

Fig. 5 illustrates observed seismic failure mechanisms using the direct computational method. For slopes with a small height (i.e., $H/B = 1$), the use of the superposition method may provide results of seismic bearing capacity that is significantly underestimated for the given geometry, particularly when k_h is small (Fig 3(a)). This result is because the realized failure goes underneath the toe of the slope (e.g., $k_h = 0.1$ in Fig. 5(a)). The passive resistance occurring from passing beneath the toe is omitted as a face failure is assumed in superposition approaches, resulting in an underestimate of bearing capacity. As k_h increases (e.g., $k_h = 0.3$ in Fig. 5(a)), the maximum depth of the rigid wedge underlying the footing is not significantly affected by the seismic acceleration, whereas the critical slip surface becomes slightly shallower. Finally, a face failure occurs for large k_h values (e.g., $k_h = 0.5$ in Fig. 5(a)), leading to the consistency of bearing capacity between the $q_{u,S}$ and $q_{u,D}$ values. When $\beta > \phi$, a face failure occurs for small k_h values (e.g., $k_h = 0.2$ in Fig. 5(b)). As the value of k_h increases (e.g., $k_h = 0.35$ in Fig. 5(b)), the active wedge shrinks, and the slip surface is greatly extended. In this case, a toe failure occurs, and the corresponding bearing capacity is reduced significantly (Fig. 3 (b)) because the soil weight acts as a destabilizing force, thus resulting in the slope becoming unstable. Higher k_h values (e.g., $k_h = 0.45$ in Fig. 5(b)) result in a deepened slope failure where the shear surface

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extends beyond the outer edge of the footing, and the observed bearing capacity decreases greatly.

Design Charts

The aforementioned discussion demonstrates that the superposition method cannot capture all the geometric interdependencies that stem from transitions in critical failure mechanism when assessing the seismic bearing capacity of footings on slopes. Therefore, a comprehensive set of design charts was created using DLO to capture the relationship of cohesion (represented as $N_s = \gamma H/c$), B/H , ϕ , β , and k_h , as shown in Figs. 6 - 8. The results, presented in terms of the normalized bearing capacity $q_u/\gamma B$, were established for slope angles (β) of 15° , 30° , 45° , and 60° and B/H ratios of 0.1, 0.4, and 1. N_s values of 2, 10, and ∞ (indicating cohesionless soil) were considered for the bearing capacity of the footings adjacent to the slope face. For large N_s values (i.e., 10 and ∞), limited results are presented because the slopes are inherently unstable under many conditions. A significant reduction ratio is observed for larger B/H values (i.e., $B/H = 0.1$) because slope stability conditions often govern (i.e., the soil weight acts as the primary destabilizing force), and the bearing capacity is relatively sensitive to the increasing k_h . Similar observations can be made for larger N_s values, as the results approach zero for increasing k_h . Additionally, the ultimate load decreases as the slope angle β increases, and it increases with the soil strength (particularly for large friction angles). Conversely, slopes with large friction angles are also the most sensitive to changes in k_h .

CONCLUSIONS

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In this technical note, the DLO procedure was applied to demonstrate that previous studies have provided accurate solutions for seismic bearing capacity factors when the observed critical failure mechanism is through the slope face. However, the bearing capacity factors compatible with the superposition method do not consider the realistic interdependencies of geometric parameters and soil properties on seismic bearing capacity. Specifically, the available bearing capacity factors for seismic conditions may underestimate or overestimate the ultimate load when the soil weight contributes to the bearing capacity or acts as a destabilizing force, respectively. This method is only theoretically valid when a face failure occurs. The direct computational method leads to improved estimates of ultimate bearing capacity without errors caused by superposition or prescribing the collapse mechanism geometry a priori. A set of design charts expressed by $q/\gamma B$ was presented for ease of application.

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Figure captions

Fig. 1. Schematic of the model.

Fig. 2. Comparison between the ultimate seismic bearing capacity obtained using the DLO method and previous studies ($\varphi = 40^\circ$).

Fig. 3. Comparison between the direct computation and superposition methods.

Fig. 4. Illustration of the potential failure mechanism of footings on slopes (Zhou et al. 2018).

Fig. 5. Transition of failure mechanisms for various seismic accelerations.

Fig. 6. Design charts for normalized seismic bearing capacity $q/\gamma B$ with $k_h (N_s = 2)$.

Fig. 7. Design charts for normalized seismic bearing capacity $q/\gamma B$ with $k_h (N_s = 10)$.

Fig. 8. Design charts for normalized seismic bearing capacity $q/\gamma B$ with $k_h (N_s = \infty)$.

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Table captions

Table 1. Assessment of the superposition approach to bearing capacity on level ground.

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Figures

Fig. 1. Schematic of the model.

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Fig. 2. Comparison between the ultimate seismic bearing capacity obtained using the DLO method and previous studies ($\phi = 40^\circ$).

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(a) $\beta = 30^\circ, \varphi = 40^\circ$

(b) $\beta = 40^\circ, \varphi = 20^\circ$

Fig. 3. Comparison between the direct computation and superposition methods.

(a) Bearing capacity failure type

(b) slope stability failure type

Fig. 4. Illustration of the potential failure mechanism of footings on slopes (modified from Zhou et al. 2018).

Fig. 5. Transition of failure mechanisms for various seismic accelerations.

Fig. 6. Design charts for normalized seismic bearing capacity $q/\gamma B$ with k_h ($N_s = 2$).

Fig. 7. Design charts for normalized seismic bearing capacity $q/\gamma B$ with k_h ($N_s = 10$).

Fig. 8. Design charts for normalized seismic bearing capacity $q/\gamma B$ with k_h ($N_s = \infty$).

Tables

Table 1. Assessment of the superposition approach to bearing capacity on level ground.

φ (°)	c (kPa)	$q_{u,D}$ (kPa)	$q_{u,S}$ (kPa)	Superposition error (%)	
				Griffiths (1982)	This study
20	20	409	370	-9.5	
20	20	370	342		-7.4

$q_{u,T}$, bearing capacity from Terzaghi’s bearing capacity definition
 $q_{u,D}$, bearing capacity from the direct computational method