### A detailed mathematical programming model for the optimal daily planning of sawmills

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A detailed mathematical programming model for the optimal daily planning of sawmills

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A detailed mathematical programming model for the optimal daily planning of sawmills

Abstract. Daily production planning of sawmills is a critical task when the optimal exploitation of forest resources is pursued. Production planning determines which logs are to be processed taking into account their characteristics with the aim of satisfying the demand for final products. Logs are turned into lumber when they are cut according to a set of available cutting patterns (CP). A key factor to improve sawmills productivity is the development of efficient production planning, and mathematical modeling is a suitable technique to achieve this objective. In this work, a mixed integer linear programming (MILP) model for optimal daily production planning in sawmills is proposed. The model involves a set of CPs for each type of log, which is obtained through an exhaustive algorithm, attaining all possible feasible CPs. The proposed approach determines the optimal number of logs of each type to be cut, the selected CPs to be used, material inventory, demand fulfillment, and other industrial and commercial issues with the objective of maximizing the firm’s benefit, in reasonable computational time taking into account the problem size.

Keywords: production planning, sawmills, cutting patterns, mathematical modeling, forest industry.
1. Introduction

Near 85% of industrial forest plantations are located in the northeastern region of Argentina (NEA), especially in the provinces of Misiones and north of Corrientes, representing approximately 850,000 ha. More than one thousand forest factories (sawmills, plywood mills, pulp mills, and Medium Density Fibreboard factories) are concentrated in this region, where small and medium-sized companies represent 98% of these facilities (Broz et al., 2016).

Forest industries play a very important role in the economic and social development of this region, but inefficient production and high transport costs have a strong impact on competitiveness of this sector. In the particular case of sawmills, efficient production can be achieved through optimal production planning considering raw materials (logs availability, including diameter, length, grade, etc. of each log piece), final products (boards characteristics and demands), industrial and commercial parameters, etc. In the NEA, some sawmills are dedicated to a specific lumber product, for example boards for structural purposes. But due to the unpredictable variations in the market, most sawmills are focused on different types of products. In other words, sawmills are not designed for a specific product, but must often respond to demand requirements.

Different approaches based on mathematical optimization have been developed and successfully implemented in countries with forestry tradition such as Canada, Chile, United States, and Finland, among others. These developments have included specific conditions of each region, characteristics of their production system, planning levels, etc. (Lobos and Vera, 2016; Troncoso et al., 2015; Alvarez and Vera, 2014; Maturana et al., 2010; Gaudreault et al., 2010; Reinders, 1992; Maness and Adams, 1991; etc.).

Several sawmill planning models with different planning horizons can be found in the literature, but daily planning has not been specifically addressed in detail. For example,
Alvarez and Vera (2014) propose an annual planning period, while in Zanjani et al. (2010) and Zanjani et al. (2013) the planning horizon consists of 30 days. Lobos and Vera (2016) propose two levels: a tactical planning horizon with monthly information and an operational level with detailed weekly production. Maness and Adams (1991) solve the problem on a weekly basis and Gaudreault et al. (2010) define a model with a sixty-day planning horizon. Also, several works use a multiperiod approach, where inventory decisions can be incorporated (Maness and Norton, 2002; Maturana et al., 2010, Vanzetti et al., 2018).

Incorporating uncertainty into the model and addressing the problem through robust or stochastic optimization is a suitable approach for long horizons (Lobos and Vera, 2016; Zanjani et al., 2010; Zanjani et al., 2013), although the problem size and its implementation could be prohibitive. Due to model complexity, Gaudreault et al. (2010) resorted to special algorithms (greedy heuristics) because of the poor performance of commercial packages using mathematical programming to solve real industrial-size problems.

Several works presented in the literature have important limitations when applied to real problems, taking into account the different log types and the considered cutting patterns (CPs). For example, Alvarez and Vera (2014) and Zanjani et al. (2010) used 3 and 5 different CPs respectively that are unrealistic applications, because the limited number of CP for the log types considered restricts the optimization space. The cited works do not present the methodology used for CP generation. They are data provided by the firm assuming that each log is processed employing its optimal CP. Nevertheless, it constitutes an important aspect of the decision making process because CP definition is a key input for all models in order to achieve an efficient planning. Other authors only generate CPs that optimize the log value by means of formulations applied to each log class. Todoroki and
Rönnqvist (2002) optimize the cutting pattern for each log using a sawing simulator, until customer demands have been fulfilled in a make-to-order approach. In other work oriented to a make-to-stock processing, Maness and Adams (1991) present a very elaborated proposal to contemplate a detailed set of CP in the weekly planning. Their procedure is very complete although adjusted to the available computational resources at that time. In order to solve the problem they have to resort to a decomposition procedure that, despite limitations in the definition of the included CP, requires very significant computation times. In a later article, Maness and Norton (2002) extend the previous approach to consider a multiperiod time horizon and they focus on to make-to-order production in a specific context in British Columbia. In the same way, Reinders (1992) generates CP that optimizes the performance of each log class, according to the available sawing technology.

As was mentioned in some previous references, the manufacturing type make-to-stock or make-to-order is another aspect to be considered. In “make-to-order” processing, products are produced to satisfy specific customer requirements, while in a “make-to-stock” way, the production of each product seeks to maintain predetermined levels of inventory, that have been determined from sales forecasts, in order to ensure an appropriate response to customer demands. In general, most of the cited works are oriented to make-to-stock approaches, with the exceptions of Maness and Norton (2002) and Todoroki and Rönnqvist (2002), which propose formulations for make-to-order processing. Even, this last article discusses the relationship of these approaches with the type of demand and the value of the products obtained from each log.

Most of the analyzed articles maximize the benefits obtained by the sawmill. However, Vergara et al. (2015) assesses different objective functions to conclude that there are significant variations on production indicators according to the criteria used. Then, this work is continued by Palma and Vergara (2016) who highlight the need to resort to
multiobjective formulations in order to represent the different preferences and criteria involved in this problem. A similar analysis is carried out by Broz et al. (2019) who propose a formulation based on Goal Programming to address this type of problem, emphasizing the role of the decision maker.

Finally, the raw material availability is another aspect that differentiates previous works. In general, it is assumed that the number of available logs with their corresponding characteristics is known. As a special case, Maness and Adams (1991) link the processing in the sawmill with the bucking in the forest in order to supply the logs in appropriate dimensions. Also, Reinders (1992) proposes a decision support system (DSS) to integrate several related activities to a cutting stock system, including the stem processing, from a tactical point of view. Maness and Norton (2002) consider the procurement through rafts with set of logs of different dimensions that must be completely processed. Recently, Vanzetti et al. (2019) integrate bucking and production planning decisions, highlighting the advantages of the simultaneous optimization for the raw material procurement policy.

The problem of CP generation is a special case of the cutting stock problem (CSP) which aims to define appropriate rectangles into a circle to cover its surface. The rectangle has the primary dimension of a board (width and thickness), and the circle has the log dimension (useful diameter). Although two-dimension CSP have been extensively studied (Lodi et al., 2002), the problem of packing a set of rectangles into the circle surface has not been dealt with in depth (Hinostroza et al., 2013). Also, the CP must take into account the kind of technology available in the sawing industry (Todoroki and Rönnqvist, 2002) and, therefore, the problem cannot be reduced to including rectangles in a circle. Exact algorithms only find the optimal solution for small-scale problems (Hinostroza et al., 2013; Birgin and Lobato, 2010) while heuristic (Cassioli and Locatelli, 2011; Burke et al., 2004) and metaheuristic (Martins and Tsuzuki, 2010; Talbi, 2009) techniques can address large-
scale problems. The availability of a complete and exhaustive set of CPs considering the
technology available in the industry is a key aspect for attaining efficient solutions.

Taking into account the previous analysis, this work presents an approach for optimal daily
production planning of sawmills. First, an exhaustive methodology for CP generation
(Module I) is proposed. Then, a formulation for optimal production planning (Module II) is
posed. Module I is an algorithm that searches all possible CP through combination of
boards and log diameters. Usually, the results of Module I do not vary for long periods of
time, unless new dimensions of tables are introduced or other sizes of logs are acquired.

On the other hand, Module II is a mixed integer linear programming model (MILP) that
daily optimizes the sawing process in order to maximize profits. It considers log inventory,
CPs, product demands, and other industrial and commercial issues.

The paper is structured as follows. The next section presents a detailed description of the
real industrial problem. The proposed approach for both modules is presented in sections 3
and 4, respectively. Then, several examples are solved and discussed to assess results and
performance of the model. The last section summarizes the main conclusions derived from
this work.

2. Description of the problem

The considered process includes several assumptions that deserve a detailed explanation.

After analyzing operations management in diverse sawmills of the NEA, some problems
can be noticed in the integration of decisions. Often, low rotating material is listed on the
inventory and there are boards that remain in stock for long periods. Taking into account
that many times logs are cut in order to optimize yield or productivity, but without a link
with the real demand of the firm, primary boards may be accumulated without an
appropriate criterion of their subsequent use. Therefore, in this work, it is assumed that
sawmill planning must be based on final demand fulfillment, with the aim of avoiding unnecessary intermediate stocks of boards that are not going to be used.

The typical sawmill of NEA region processes logs to obtain boards. This cutting process generates intermediate or primary products of medium and large dimension. Then, they are sent to drying chambers where moisture is removed. These dried boards can be directly sent to users but, usually, reprocessing is required to cut these intermediate products into final boards (resized) according to a predefined secondary CP. Thus, a remanufacturing process is required to convert primary products into final dimensional lumber pieces. In this way, the final boards are ready for dispatch to different destinations (Figure 1). In particular, in the sawing process, available logs, considering different lengths and diameters, are cut according to diverse CPs to obtain intermediate boards. An initial cut is made through a primary saw (vertical double cut band, for example) in which a cant and two flitches are obtained. In the secondary saw, the cant is broken down into dimensional lumber pieces and flitches (multi-rip for example); then, flitches undergo a resawing process (vertical resaw and edger, for example). It is important to point out that, the work scheme proposed in this article can be easily adapted to other technologies available in the sawmills.

Insert Figure 1

Although it is known that final products are obtained a few days after the cutting operation, considering, for example, the drying step, the relationship between the demand for final products and the obtained boards in the sawing step must be explicitly considered. Many times, the cutting process is done emphasizing an efficient use of the logs, or a good use of the machines, but the global objective of the firm, profitability, is lost sight of. Assuming a
make-to-stock approach is adopted, production must be planned in order to assure that
appropriate stocks levels are maintained.

Therefore, in order to suitably represent this scenario, products are divided in primary and
final boards. Primary boards, also called intermediate boards, are obtained from the sawing
step. Afterwards, these boards are cut in the remanufacture step to satisfy the requirements
for final boards. These requirements are determined by the firm taking into account diverse
elements: customer demands, forecasts, stock levels, etc. Although the remanufacturing
process is not going to be carried out in the following planning day of the sawmill process,
the relationship between intermediate and final boards must be explicitly considered. By
this approach, sawmill planning can be adjusted to attain an efficient global operation,
without generating excessive useless intermediate stock and fulfilling final demands.

Following this approach, CPs are classified into two types, primary and secondary,
depending on their use at cutting stages in the sawmill facilities. Primary CPs represent
cuts in the sawing step in order to produce intermediate boards, while secondary CPs are
used to obtain final products in the remanufacturing step, representing the relationship
between intermediate and final boards. Primary CPs are obtained from an exhaustive
search algorithm (Module I), while secondary CPs are provided as data by the firm taking
into account the admitted relationships from commercial and industrial criteria between
intermediate and final boards (Figure 2). Each primary CP is applied to a log with a
determined useful diameter or Log Diameter Class (LDC), and is valid for all the different
log lengths with that diameter. The LDC corresponds to the diameter of the largest cylinder
that can be obtained from the log, and it can be determined using scanners. Likewise, each
secondary CP is used on the cross section of an intermediate product and is valid for all
lengths with that cross section. It is important to clarify at this point that the grade of the
logs is not considered in this version of the formulation.
Figure 3 shows how a particular intermediate board can be obtained from different primary cutting patterns applied to logs of different LDC. Also, Figure 3 depicts how an intermediate board can be cut using different secondary cutting patterns to produce final boards. It can be observed how two primary CPs are applied to logs of different LDC. In this case, intermediate products are obtained from each primary CPs: In LDC 1, 4 boards 1\times 4, 2 boards 1/2\times 3, and 2 boards 1\times 3 are obtained; while in LDC 2, 6 boards 1\times 4, 2 boards 1\times 5 and 2 boards 1/2\times 3 are produced. As it can be noticed, the same boards can be obtained from different logs. Besides, in this case, from a 1x4 board, three secondary CPs are defined to obtain the same board (no resized), 2 boards 1\times 2 or 4 batten of 1\times 1.

Figure 3 allows to represent the complexity of the problem of production planning of sawmills. As it is shown in the figure, the same final board (1x2 board) can be obtained from different intermediate boards (1x4 and 1x5 boards). In the same way, one of these intermediate boards can be cut from logs of different LDC. Therefore, it is very difficult to evaluate a priori the benefit that a firm obtains from a final board, since it depends on which intermediate board was cut and this, in turn, on which log was obtained. In addition, taking into account that different intermediate boards are obtained from a log and, from each of these, different final boards are cut, it is very difficult to establish the profit that a company obtains by cutting a log in a certain way. For example, as can be noted from Fig. 3, the previous mentioned final board of 1x2 can be obtained from intermediate boards of
1x4 and 1x5 which are made from logs of LDC1 and LDC2, respectively. These logs have different costs, but the final product 1x2 has a determined selling price regardless from which log comes from; therefore, the real profit cannot be calculated since the product traceability is not followed. This also depends on the mix of products included in the demand, whose requirements most likely do not conform to the proportions of boards obtained from a log. As can be noticed, a mathematical programming formulation, that allows representing all the available alternatives and the trade-offs among them, is required.

Several previous works have resorted to decomposition schemes in which a cutting pattern is previously proposed according to the benefit it provides to the firm. As it is derived from Figure 3, this type of formulations does not agree with the idea of optimal solution of the planning of a sawmill, which requires simultaneously considering and integrating all the elements involved in the problem. A complete and exhaustive set of CP allows to balance the availability of raw material and the daily requirements of final products in order to achieve an efficient operation of the sawmill. Finally, this proposal can be easily adapted to consider only the demand for intermediate tables, without taking into account the secondary cut patterns.

In the following two sections, the proposed approaches for CP generation and daily planning are presented, using the following nomenclature.

**Nomenclature**

**Sets**

- \( a \in A \) Width of intermediate products
- \( d \in D \) Diameters of logs
- \( e \in E \) Thickness of intermediate products
Final products
Intermediate products
Length of products and logs
Primary cutting pattern
Secondary cutting pattern
Relationship between width and thickness of intermediate product

\[ f \in F \]
\[ i \in I \]
\[ l \in L \]
\[ p \in P \]
\[ s \in S \]
\[ R \]

**Parameters**

\[ A_d \] Cross sectional area of a log of diameter \( d \)
\[ Cpr_{sl} \] Unit cost for using CP \( s \) with products of length \( l \) ($ / unit)
\[ Cps_{dl} \] Unit cost for processing a log of diameter \( d \) and length \( l \) ($ / unit)
\[ Crm_{dl} \] Cost of a log of diameter \( d \) and length \( l \) ($/unit)
\[ Cs \] Setup cost ($/s)
\[ Cud_{lf} \] Unit cost of unsatisfied demand for product \( f \) of length \( l \) ($/unit)
\[ D_{lf} \] Maximum demand for final products of cross section \( f \) and length \( l \) (units)
\[ f_{p_p} \] Yield of the primary CP
\[ f_{r_{sif}} \] Number of final products of cross section \( f \) obtained from intermediate product of cross section \( i \) when CP \( s \) is used
\[ f_{spdi} \] Number of intermediate products of cross section \( i \) obtained from log of diameter \( d \) applying a CP \( p \) (units)
\[ ll_{ld} \] Availability of logs of diameter \( d \) and length \( l \) (units)
\[ L_l \] Board length
\[ M \] Big M coefficient
\[ S_{lf} \] Unit selling price of final product of cross section \( f \) of length \( l \) ($ / unit)
\[ Tmax \] Maximum operation time for sawmill (s)
\[ tp_{pl} \] Unit processing time for a log of length \( l \), to which the CP \( p \) is applied (s)
$t_s p$  Setup time of CP $p$ (s)

**Binary Variables**

$x_p$  Indicates whether the primary CP $p$ is used

**Continuous variables**

$A_{\text{max}}$  Maximum admissible board width

$Duc$  Total cost of unsatisfied demand ($)

$Du_{lf}$  Unsatisfied demand for final products $f$ of length $l$ (units)

$H$  Height of central block

$I$  Total income ($)

$P$  Total wood loss

$P_{lf}$  Amount of produced final products of cross section $f$ of length $l$ (units)

$Prc$  Total remanufacturing cost ($)

$Psc$  Total cost of the sawmill ($)

$Ps_{li}$  Amount of produced intermediate product of cross section $i$ of length $l$ (units)

$Qf_{lf}$  Amount of sold final products of cross section $f$ of length $l$ (units)

$Qi_{li}$  Amount of intermediate products $i$ of length $l$ for reprocessing (units)

$Ql_{pdl}$  Amount of logs of diameter $d$ and length $l$ to be applied CP $p$ (units)

$Qr_{sli}$  Amount of intermediate products of cross section $i$ and length $l$, to be applied to the CP $s$

$RMc$  Total cost of raw material ($)

$s_L$ and $s_U$  Sagitta for the lateral and upper/lower fitches

$Sc$  Total cost per changeover ($)

$T_{tp_{dl}}$  Total processing time for logs of diameter $d$ and length $l$ using the CP $p$ (s)

$Tis$  Total sawing setup time (seconds).

$Z$  Objective function
3. Primary Cutting Pattern generation (Module I)

As was discussed in the introduction, the availability of primary CP is a key factor for production planning. There are several previous articles that have worked with small sets of CP which are determined in order to optimize the log value or are usually employed by the firm without taking into account the mix of products and the demand of each of them. There are also other articles that incorporate the CP taking into account their individual performance or profitability. On the contrary, in this work, Module I exhaustively determines the primary Cutting Patterns. The objective of the generator is to systematically arrange rectangles (thickness and width of intermediate boards) within circles. The insertion of the rectangles is done by simulating the cutting pattern of logs in sawmills. An exploratory analysis of different sawmills on the Northeastern region of Argentina was carried out and it determined that more than 70% use the generic CP *Cant Sawing* type, according to the classification proposed by Todoroki and Rönnqvist (2002). It divides the log into five sections: a central block, two equal lateral flitches, and an upper and lower one, which are equal to each other (Figure 4). From each of these sections, the different intermediate boards will be obtained. Nevertheless, similar procedures can be developed to different sawmill configurations.

Insert Figure 4

As input data, the generator has a set of diameters \( d \in D \), corresponding to the useful diameter of the log, a set of board widths \( a \in A \) and another set of board thicknesses \( e \in E \). In addition, there is a set of relations \((a, e) \in R\) between widths and thicknesses that establishes the valid and admitted relations between them.
The generator, based on an exhaustive search algorithm, allows obtaining all possible combinations of admitted boards (valid relationships between board width and thickness) which can be cut from a determined log diameter. Next, the steps that explain the algorithm to generate a primary CP are described, taking into account that they are repeated until obtaining all possible valid primary CPs for the available log diameters.

Once a diameter \( d \) is selected, the first step is to define sections of the CP within the log. To do this, a width \( a \) of set \( A \) is selected and placed centrally to the vertical axis of the circle (Figure 5a). This defines the width of the central block. The height \( (H) \) of the block is calculated as follows (Figure 5b):

\[
H = 2 \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{a}{2}\right)^2}
\]  

(1)

Once the dimensions of the central block are defined, the five sections of the primary cutting pattern are determined, as shown in Figure 5c. The next step is to define the boards that will be obtained from each section.

**Insert Figure 5**

**Flitches**

The procedure on the flitches is here described. The algorithm is similar for the four generated sections, although the same cut is assumed to be applied to both upper and lower flitches, on the one hand, and lateral flitches, on the other hand. First, sagitta \( s_L \) and \( s_U \) for the lateral and upper/lower flitches, respectively, is calculated (Eq. 2) (Figure 6a):

\[
s_L = \frac{d}{2} - \frac{a}{2} \quad s_U = \frac{d}{2} - \frac{H}{2}
\]  

(2)
Then, thickness $e$ is taken from the set of thicknesses $E$, so that $e < s$ and $e \in E$.

Thicknesses are selected in ascending order. The maximum admissible board width within the section (Figure 6b) is then calculated as follows (Eq. 3):

$$A^{max} = 2 \sqrt{\left(\frac{d}{2}\right)^2 - \left(\frac{d}{2} - s\right)^2}$$

Then, this width $A^{max}$ is compared with those widths $a \in A$ allowed for thickness $e$ previously selected, i.e. $(a,e) \in R$. The largest allowed width $a^*$ that can be inscribed inside the piece is selected (Figure 6c). This ensures the greatest yield of wood. The following expression is used:

$$a^* = \max\{a / a \in A, (a,e) \in R, a \leq A^{max}\}$$

Then, a board of width $a^*$ and thickness $e$ is added to the pattern. If no value for $a^*$ is found, a new thickness $e$ is proposed.

Then, the new sagitta is recalculated with the rest of the flitch; and the procedure is repeated. These steps continue until the sagitta size is smaller than all thicknesses $e$ or until a certain number of established cuts are reached (Figure 6d).

Insert Figure 6

Central block

For the central block, boards will be obtained with the same width $a$, so that only those related thicknesses $e$, i.e. such that $(a,e) \in R$, can be used.

The algorithm for this section begins by adopting a thickness: the smallest $e$, with $(a,e) \in R$. Thicknesses are selected in ascending order. Then, the remaining height of the block is calculated: $H_r = H - e$ (Figure 7a). These actions will be repeated until the remaining height is lower than all admitted thicknesses for width $a$, or until a certain number of cuts
is reached (Figure 7b). The number of admitted boards is limited taking into account the characteristics of the available equipment.

**Cutting Pattern generation**

For a certain diameter, once all the alternatives for the central block and related flitches have been determined, all feasible primary CPs can be identified by combining the five sections (Figure 8).

Finally, the yield of the primary CP ($f_{p_p}$), which represents the percentage of wood converted into boards, must be calculated. It is determined as the quotient between the sum of the the area of the cross sections of the intermediate products that integrate the CP, and the area of the cross section of the log of useful diameter $d$. Usually, this is compared with a tolerance proposed by the firm: only primary CPs with a reasonable yield are stored in a database; otherwise they are discarded. The database contains the types and amounts of cross sections that form each CP, and their yield.

Usually the database of primary CP is maintained for long periods, unless new boards are incorporated or new log sizes are acquired, alternatives that are not so usual in a sawmill.
4. Production planning optimization (Module II)

In this section, the MILP formulation for the optimal daily planning of the sawing process is proposed. Both the different constraints and the considered objective function are presented.

4.1. Raw Material

A known amount of logs $I_{dl}$ is daily available at the plant. Logs are classified according to its useful diameter $d$ and length $l$.

The CP $p$ will be used to cut an amount $Q_{dl}p$ of logs of diameter $d$ and length $l$. Eq. (4) states that the total number of logs cut at the sawing stage cannot exceed the number of available logs for each type. The inventory at the end of the day can be calculated by subtracting the logs used from the available logs.

$$\sum_{p} Q_{dl}p \leq I_{dl} \quad \forall d, l \quad (4)$$

4.2. Sawmill process

The binary variable $x_p$ indicates if CP $p$ is used in the daily sawmill planning:

$$x_p = \begin{cases} 1 & \text{if CP } p \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

If CP $p$ is not used, no logs are cut by this CP, as states Eq. (5) through a Big-M type constraint:

$$\sum_{d} \sum_{l} Q_{dl} \leq M x_p \quad \forall p \quad (5)$$

where $M$ is a sufficiently large number that can be calculated taking into account the amount of available logs.
As a result of the sawmill planning, the number of units for each type of intermediate board must be determined. Each intermediate board is characterized by a cross section and a length. Cross section $i$ corresponds to a valid relationship between width $a$ and thickness $e$, i.e., each $i$ corresponds to an element $(a,e) \in R$. For each primary CP $p$, the parameter $fs_{pdi}$ indicates the number of boards with cross section $i$ obtained when $p$ is applied. Eq. (6) determines the amounts of intermediate products $Ps_{li}$ with cross section $i$ and length $l$ that are obtained when all selected primary CPs $p$ are used.

$$\sum_p \sum_d f_{s_{pdi}} Q_{l_{pdi}} = P_{s_{li}} \quad \forall \ l_i$$ (6)

Production time and the time required to change CP cannot exceed the maximum available time $T_{max}$. In order to calculate operation time, parameter $tp_{pl}$ is the time needed to process a log of length $l$ with primary CP $p$. Besides, $ts_p$ is the required time to adapt or install primary CP $p$ in the saw. Then, the following expression limits the consumed time:

$$\sum_p \sum_d \sum_l t_{p_{sl}} Q_{l_{pdt}} + \sum_p t_{s_p} x_p \leq T_{max}$$ (7)

This expression establishes that the available time cannot be exceeded, where the first term corresponds to operation time that depends on the number of cut logs, and the second one is setup time, which depends on the number of employed primary CPs.

Eq. (8) calculates total wood loss $P$, where $fp_p$ is the yield of each CP $p$ obtained in the generator, $A_d$ is log transversal area of diameter $d$, and $L_l$ is its length. $P$ is not employed in the model and only is incorporated to be used in the results discussion.

$$\sum_p \sum_d \sum_l (1 - f_{p_{pl}}) A_d L_l Q_{l_{pdt}} = P$$ (8)
4.3. Remanufacturing process

Once the $P_{si}$ intermediate boards with cross section $i$ and length $l$ are obtained, they are converted into final boards in the remanufacturing plant applying new cuts. The relationship between intermediate and final boards used in this work allows for an adequate use of production boards, optimizing sawmill performance and inventory management.

At the remanufacturing stage, secondary CPs $s$ are used. Each secondary CP $s$ is applied to an intermediate board in order to obtain a set of final boards. It is assumed that secondary CPs are provided by the firm taking into account that operational and commercial factors and it is not just a question of physical limits of the cut. Final boards are characterized by cross section $f$ and length $l$.

Variable $Q_{rsli}$ corresponds to the number of intermediate boards of cross section $i$ and length $l$ cut by secondary CP $s$. The number of intermediate boards consumed at the remanufacturing stage cannot exceed the available amount:

$$\sum_s Q_{rsli} \leq P_{si} \quad \forall l, i$$

Intermediate boards that are not used for producing final products in the manufacturing process, can be left in storage to be utilized for next periods. The amount of stored intermediate boards of cross section $i$ and length $l$, is obtained from the subtraction between the produced ($P_{si}$) and the used intermediate boards ($\sum_s Q_{rsli}$).

Eq. (10) defines $P_{fj}$, the number of final boards of cross section $f$ and length $l$ obtained by applying a secondary CP $s$ to intermediate product $i$. Parameter $fr_{si}$ determines the quantity of final boards of cross section $f$ obtained by applying secondary CP $s$ to a given intermediate board with cross section $i$. 
\[
\sum_{s} \sum_{i} f_{r_{sif}} Q_{r_{sli}} = P_{f_{lf}} \quad \forall \ l, f
\]  

(10)

\(Q_{lf}\), the amount of final boards of cross section \(f\) and length \(l\) devoted to fulfill the demand, cannot surpass the generated boards:

\[Q_{lf} \leq P_{lf} \quad \forall \ l, f\]  

(11)

At the same time, \(Q_{lf}\) cannot exceed the maximum demand for this board \(D_{lf}\) established by the firm:

\[Q_{lf} \leq D_{lf} \quad \forall \ l, f\]  

(12)

Again, the stored final boards of cross section \(f\) and length \(l\) can be obtained from the subtraction between the number of produced \((P_{lf})\) and selling \((Q_{lf})\) final boards of that size. In the examples presented below, these inventories are calculated in order to compare the production performance in each studied scenario.

4.4. Objective Function

The proposed objective function \(Z\) maximizes profits given by the difference between income from product sales and raw material, sawing production, changeover, and remanufacturing costs:

\[\text{max } Z = I - (RMc + Psc + Sc + Prc)\]  

(13)

Income from sales \(I\) is represented by Eq. (14), where \(S_{lf}\) corresponds to the unit price of the final board of cross section \(f\) and length \(l\):

\[I = \sum_{l} \sum_{f} S_{lf} Q_{lf}\]  

(14)
Raw material cost $RMc$ depends on unit cost $Crm_{dl}$ of logs of diameter $d$ and length $l$ and the number of used logs $Ql_{ndl}$:

$$RMc = \sum_p \sum_d \sum_l Crm_{dl} Ql_{ndl}$$ (15)

Production cost at the sawing stage $Psc$ is proportional to the number of logs cut by each primary CP. $Cps_{pl}$ represents the unit cost of processing a log of length $l$ with primary CP $p$:

$$Psc = \sum_p \sum_d \sum_l Cps_{pl} Ql_{ndl}$$ (16)

Setup cost $Sc$ considers the proper preparation of sawmill facilities (saw configuration) to use CP $p$, and it is proportional to the required time to prepare it according to cost $Cs$:

$$Sc = Cs \sum_p ts_p x_p$$ (17)

Remanufacturing cost $Prc$ takes into account cost $Cpr_{si}$ of applying secondary CP $s$ to an intermediate board of length $l$ and depends on the number of intermediate boards $i$ that are cut:

$$Prc = \sum_s \sum_l \sum_i Cpr_{si} Qr_{si}$$ (18)

In short, the MILP model for the daily operation planning in a sawmill is given by the maximization of Eq. (13) subject to constraints (4)-(12), and (14)-(18).

5. Results
For the case study, 5 diameters, 6 log lengths, 24 cross sections of intermediate boards, and 35 cross sections of final products are proposed. Also, 8-hour work shifts are considered. It is important to note that the CPs generator and the optimizer are solved separately. The primary CPs obtained by the generator are used as input data by the daily planning optimization model. In the proposed example, the generator produces a total of 5117 primary CPs. In addition, 361 secondary PCs are considered, assuming that they are proposed by the firm as appropriate cut alternatives. The example was implemented and solved in GAMS (Rosenthal, 2013) using CPLEX solver in an Intel(R) Core(TM) i7-3770, 3.40 GHz. The model involves 274362 equations, 666347 continuous variables, and 5117 discrete variables.

In order to perform a more detailed analysis of the different model elements and their performance, 7 case studies are presented. The first one constitutes the base case, and then some variants are introduced in the following cases.

### 5.1. Case Study

For the base case, a demand for 7119 final boards distributed in 10 cross sections and 6 lengths is considered (Table 1). To cover this demand, there are 796 available logs that are classified according to 5 diameters and 6 lengths (Table 2). Optimal daily production planning gives a total benefit of $107013.6. Income and costs are listed in Table 3.

From the total available logs, 69% are used in the sawing process, spending 7.2 hours, and leaving an unsatisfied demand of 4.1%. Even if time and logs remain available to continue production and decrease the percentage of unsatisfied demand, this does not occur because the remaining logs (Table 4) are not profitable to produce any of the missing boards.
### Table 1. Maximum demand [units]

<table>
<thead>
<tr>
<th>Lengths</th>
<th>f1</th>
<th>f3</th>
<th>f6</th>
<th>f10</th>
<th>f11</th>
<th>f18</th>
<th>f21</th>
<th>f26</th>
<th>f30</th>
<th>f35</th>
</tr>
</thead>
<tbody>
<tr>
<td>l1</td>
<td>98</td>
<td>159</td>
<td>171</td>
<td>82</td>
<td>59</td>
<td>58</td>
<td>9</td>
<td>111</td>
<td>125</td>
<td>98</td>
</tr>
<tr>
<td>l2</td>
<td>148</td>
<td>148</td>
<td>198</td>
<td>176</td>
<td>18</td>
<td>45</td>
<td>19</td>
<td>276</td>
<td>178</td>
<td>101</td>
</tr>
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<td>137</td>
<td>19</td>
<td>74</td>
<td>29</td>
<td>152</td>
<td>166</td>
<td>130</td>
</tr>
<tr>
<td>l4</td>
<td>168</td>
<td>102</td>
<td>190</td>
<td>150</td>
<td>33</td>
<td>115</td>
<td>0</td>
<td>144</td>
<td>156</td>
<td>139</td>
</tr>
<tr>
<td>l5</td>
<td>135</td>
<td>238</td>
<td>155</td>
<td>132</td>
<td>24</td>
<td>100</td>
<td>0</td>
<td>140</td>
<td>185</td>
<td>102</td>
</tr>
<tr>
<td>l6</td>
<td>195</td>
<td>156</td>
<td>120</td>
<td>152</td>
<td>12</td>
<td>45</td>
<td>0</td>
<td>206</td>
<td>180</td>
<td>74</td>
</tr>
</tbody>
</table>

### Table 2. Raw material availability [units]

<table>
<thead>
<tr>
<th>Lengths</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>d4</th>
<th>d5</th>
</tr>
</thead>
<tbody>
<tr>
<td>l1</td>
<td>45</td>
<td>41</td>
<td>24</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>l2</td>
<td>32</td>
<td>46</td>
<td>19</td>
<td>26</td>
<td>5</td>
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<tr>
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<td>45</td>
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<td>4</td>
</tr>
<tr>
<td>l4</td>
<td>48</td>
<td>51</td>
<td>37</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>l5</td>
<td>53</td>
<td>29</td>
<td>36</td>
<td>34</td>
<td>3</td>
</tr>
<tr>
<td>l6</td>
<td>40</td>
<td>43</td>
<td>13</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

### Table 3. Economic report [thousands of $]

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Incomes</td>
<td>480.71</td>
</tr>
<tr>
<td>Raw material costs</td>
<td>190.14</td>
</tr>
<tr>
<td>Sawing process cost</td>
<td>109.97</td>
</tr>
<tr>
<td>Setup cost</td>
<td>6.00</td>
</tr>
<tr>
<td>Remanufacture cost</td>
<td>67.60</td>
</tr>
<tr>
<td><strong>Net benefit</strong></td>
<td><strong>107.01</strong></td>
</tr>
</tbody>
</table>
Table 4. Remaining logs [units]

<table>
<thead>
<tr>
<th>Lengths</th>
<th>d1</th>
<th>d2</th>
<th>d3</th>
<th>d4</th>
<th>d5</th>
</tr>
</thead>
<tbody>
<tr>
<td>l1</td>
<td>28</td>
<td>17</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l2</td>
<td>9</td>
<td></td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l3</td>
<td>22</td>
<td>6</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l4</td>
<td>38</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>l5</td>
<td>15</td>
<td>11</td>
<td>29</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l6</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5 shows the number of logs classified according to diameter and length, and used primary CP. It is noticed that only 5 primary CPs are used, and that they are employed for different log lengths, thus reducing set up cost. Table 6 displays the type and quantity of intermediate products obtained by each CP when used once. Without taking into account the length \( l \) of the board and considering only its cross section \( i \), it is noted that several boards are obtained from different CP. For example \( i_{24} \) is produced using \( p\text{l105}, p\text{2525} \) and \( p\text{4853} \). A total of 41 intermediate boards are left in stock, which are not going to be processed in the next remanufacturing stage (Table 7).

Table 5. Logs processed according to primary CP, diameter and length [units]

<table>
<thead>
<tr>
<th>Diameter</th>
<th>Primary CP</th>
<th>l1</th>
<th>l2</th>
<th>l3</th>
<th>l4</th>
<th>l5</th>
<th>l6</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
<td>p\text{91}</td>
<td>16</td>
<td>22</td>
<td>21</td>
<td>9</td>
<td>37</td>
<td>15</td>
</tr>
<tr>
<td>d2</td>
<td>p\text{324}</td>
<td>23</td>
<td>46</td>
<td>38</td>
<td>44</td>
<td>29</td>
<td>43</td>
</tr>
<tr>
<td>d3</td>
<td>p\text{l105}</td>
<td>13</td>
<td>19</td>
<td>35</td>
<td>37</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>d4</td>
<td>p\text{2525}</td>
<td>13</td>
<td>8</td>
<td>3</td>
<td>5</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>d5</td>
<td>p\text{4853}</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 6. Intermediate products generated by each CP

<table>
<thead>
<tr>
<th>Primary CP</th>
<th>i1</th>
<th>i2</th>
<th>i3</th>
<th>i5</th>
<th>i6</th>
<th>i8</th>
<th>i10</th>
<th>i13</th>
<th>i18</th>
<th>i20</th>
<th>i21</th>
<th>i23</th>
<th>i24</th>
</tr>
</thead>
<tbody>
<tr>
<td>p91</td>
<td>2</td>
<td></td>
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<td></td>
<td></td>
<td>2</td>
<td>1</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>p324</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>p1105</td>
<td>2</td>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>p2525</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>p4853</td>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

Table 7. Intermediate boards in stock

<table>
<thead>
<tr>
<th>Lengths</th>
<th>Intermediate boards</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>i1</td>
</tr>
<tr>
<td>l3</td>
<td></td>
</tr>
<tr>
<td>l4</td>
<td></td>
</tr>
<tr>
<td>l6</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 8 shows the number of intermediate products processed by each secondary CP. As it can be noticed, more than one secondary CP can be used for the same intermediate board. From the sum of the boards in Tables 7 and 8, the total production of intermediate products is obtained. Only 26 secondary CPs are used. From the production of final boards, 6830 are used to meet the demand and 8 are stored. These boards correspond to final product f5, which has no demand in this case. Table 9 displays final boards production. Comparing this table with Table 1, the unsatisfied demands can be obtained. Besides, considering Tables 8 and 9, it can be easily concluded that the final boards are obtained from different secondary CP applied to different intermediate boards.
Table 8. Intermediate products processed according to secondary CP, diameter, and length [units]

<table>
<thead>
<tr>
<th>Intermediate Products</th>
<th>Secondary CP</th>
<th>L1</th>
<th>L2</th>
<th>L3</th>
<th>L4</th>
<th>L5</th>
<th>L6</th>
</tr>
</thead>
<tbody>
<tr>
<td>i1</td>
<td>s2</td>
<td>26</td>
<td>16</td>
<td>6</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i2</td>
<td>s7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>s9</td>
<td>26</td>
<td>38</td>
<td>15</td>
<td>51</td>
<td>43</td>
<td>26</td>
</tr>
<tr>
<td>i3</td>
<td>s13</td>
<td>19</td>
<td>31</td>
<td>39</td>
<td>18</td>
<td>38</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>s16</td>
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<td>13</td>
<td>4</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>i5</td>
<td>s32</td>
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<td>11</td>
<td>14</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>s42</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>i6</td>
<td>s48</td>
<td>5</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>s65</td>
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<td>4</td>
<td>2</td>
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</tr>
<tr>
<td>i8</td>
<td>s101</td>
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</tr>
<tr>
<td></td>
<td>s121</td>
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<td>26</td>
<td>14</td>
<td>14</td>
<td>16</td>
<td>22</td>
</tr>
<tr>
<td>i10</td>
<td>s168</td>
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<tr>
<td></td>
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<tr>
<td>i24</td>
<td>s350</td>
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<td>158</td>
<td>137</td>
<td>137</td>
<td>120</td>
<td>144</td>
</tr>
</tbody>
</table>
A great advantage of working with an exhaustive CP generator is that a great number of alternatives can be selected to produce a board, either intermediate or final. Figure 9 represents the different paths to obtain particular products. For example, intermediate product \( i_{24} \) may be obtained from logs of diameter \( d_3 \) using primary CP \( p_{1105} \) or from logs of diameter \( d_5 \) using CP \( p_{4853} \). In addition to this product, each of these primary CPs generates a set of additional intermediate products. Similarly, product \( f_3 \) may be obtained from intermediate boards \( i_2 \) and \( i_6 \) which were generated with the abovementioned primary CPs. In turn, starting from intermediate product \( i_6 \), final product \( f_3 \) can be obtained using two secondary CPs, \( s_{48} \) and \( s_{65} \); meanwhile, it may be also produced using \( s_7 \) from \( i_2 \). Taking into account that not all available alternatives are represented in Figure 9, the number of contemplated options is very significant. Besides, when a log or an intermediate board is cut, the produced boards do not exactly adjust to the proportions of the product mix in the demand. Therefore, from a more global perspective, it is necessary to balance the log dimensions to be cut, taking into account the available raw material, with the mix of products of the demand. The key factor to find efficient solutions is the availability of a broad set of CP from which choose an appropriate production planning.
Taking into account the results here presented, the formulation proposed in this work allows reaching highly efficient solutions.

Insert Figure 9

Following, several cases are solved by varying the base case data so as to assess formulation capabilities. Table 10 presents general results, mainly the objective function; and Table 11 shows different performance indicators to assess how the proposed model can efficiently plan sawmill production subject to different scenarios.

5.2. Case study 2 and 3

In these cases the demand varies: in case 2, demand decreases; and in the case 3, it increases. In both cases, they present a 50% variation with respect to the base case.

The second and third columns of Table 10 show the main economical results while in Table 11 the daily plan is depicted for these cases respectively. In case study 2, the net benefit has a 50% decrease when compared to the base case, similarly to demand. Demand is fully covered in 3.7 hours, using only 30% of available logs. Since more alternatives are available, CPs are chosen producing a smaller amount of boards to be stored: only 10 intermediate units. Even though demand significantly grows in case study 3, net profit only increases 22% since operation time is exhausted. The unsatisfied demand rises to 23.4%. On the other hand, raw material is more efficiently used, reducing losses but generating a greater quantity of intermediate and final products in stock. Comparing the base case and case 3, although there is a reduced idle time in the former (only 0.8 hr, i.e. 10 %),
production is significantly increased in the latter (almost 32%). Although the total number of used CPs is equal for both cases, the selected CPs change in order to take further advantage of the available time and increase production.

Table 10. Economic report [thousands of $]

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incomes</td>
<td>480.71</td>
<td>243.34</td>
<td>560.03</td>
<td>325.94</td>
<td>487.98</td>
<td>488.23</td>
<td>551.92</td>
</tr>
<tr>
<td>Raw Material cost</td>
<td>190.14</td>
<td>94.76</td>
<td>215.09</td>
<td>125.61</td>
<td>190.84</td>
<td>194.05</td>
<td>207.12</td>
</tr>
<tr>
<td>Process cost at sawing stage</td>
<td>109.97</td>
<td>54.00</td>
<td>123.65</td>
<td>74.49</td>
<td>108.72</td>
<td>112.83</td>
<td>116.06</td>
</tr>
<tr>
<td>Setup cost</td>
<td>6.00</td>
<td>4.80</td>
<td>6.00</td>
<td>6.00</td>
<td>7.20</td>
<td>6.00</td>
<td>9.60</td>
</tr>
<tr>
<td>Remanufacturing Process cost</td>
<td>67.60</td>
<td>34.93</td>
<td>84.72</td>
<td>50.89</td>
<td>70.39</td>
<td>69.12</td>
<td>79.16</td>
</tr>
<tr>
<td>Net benefit</td>
<td>107.01</td>
<td>54.85</td>
<td>130.57</td>
<td>68.95</td>
<td>110.83</td>
<td>106.24</td>
<td>139.98</td>
</tr>
</tbody>
</table>

Table 11. General results of the different case studies

<table>
<thead>
<tr>
<th></th>
<th>Base Case</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production time [h]</td>
<td>7.2</td>
<td>3.7</td>
<td>8.0</td>
<td>5.2</td>
<td>7.2</td>
<td>7.4</td>
<td>8.0</td>
</tr>
<tr>
<td>Unsatisfied demand [%]</td>
<td>4.1</td>
<td>0.0</td>
<td>23.4</td>
<td>31.8</td>
<td>0.2</td>
<td>0.0</td>
<td>13.4</td>
</tr>
<tr>
<td>Intermediate products in stock [units]</td>
<td>41</td>
<td>10</td>
<td>76</td>
<td>0</td>
<td>25</td>
<td>79</td>
<td>154</td>
</tr>
<tr>
<td>Final products in stock [units]</td>
<td>8</td>
<td>0</td>
<td>26</td>
<td>10</td>
<td>0</td>
<td>10</td>
<td>147</td>
</tr>
<tr>
<td>Wood loss [%]</td>
<td>24.1</td>
<td>23.1</td>
<td>21.1</td>
<td>22.1</td>
<td>23.1</td>
<td>24.2</td>
<td>19.8</td>
</tr>
<tr>
<td>Primary CP</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Secondary CP</td>
<td>26</td>
<td>18</td>
<td>21</td>
<td>21</td>
<td>22</td>
<td>26</td>
<td>68</td>
</tr>
<tr>
<td>Logs used [units]</td>
<td>549</td>
<td>248</td>
<td>588</td>
<td>382</td>
<td>509</td>
<td>568</td>
<td>501</td>
</tr>
<tr>
<td>Logs used [%]</td>
<td>69</td>
<td>32</td>
<td>74</td>
<td>100</td>
<td>43</td>
<td>72</td>
<td>63</td>
</tr>
</tbody>
</table>
5.3. Case study 4 and 5

For case study 4 and 5, raw material availability is decreased and increased by 50%, respectively, and results are shown in the fourth and fifth columns of Table 10 and 11.

In the optimal solution of case 4, total available raw material is consumed, fulfilling 68.2% of the maximum demand. CPs have been selected not only to cover the demand, but also to generate a low quantity of products in stock and wood losses. As well as Case 2, when there is available production time, CPs selection allows reducing products in stock.

In case 5, the number of available logs is greater than in the base case; and therefore, in the optimal solution the maximum demand is better fulfilled in the same production time, leaving only 0.2% of unsatisfied demand. It is worth highlighting that the total number of consumed logs is lower than the base case, since the greater availability of raw material makes log selection more flexible. Also, CPs are chosen in order to reduce products in stock and increase productivity.

5.4. Case study 6

In this case, a cost or penalty for unsatisfied demand \( (D_{uc}) \) is considered. The following objective function, which replaces Eq. (13), is proposed:

\[
\max Z = I - (RM_c + P_{sc} + S_c + P_{rc} + D_{uc})
\]  

(19)

The unsatisfied demand is determined as the difference between the maximum demand and the amount of final product used to cover that demand (Eq. 20).

\[ D_{uf} = D_{lf} - Q_{lf} \]  

(20)
The total unsatisfied demand cost, $D_{uc}$, is calculated using the following expression, where $C_{udf}$ represents unit cost of unsatisfied demand for final product $f$ of length $l$ ($$/u$):

$$D_{uc} = \sum_{lf} C_{udf} D_{uf}$$  \hspace{1cm} (21)

The optimal solution considering objective function in Eq. (19) reaches a net benefit equal to $106237.5$, slightly lower than the base case due to unprofitable products must be produced. The maximum demand is completely satisfied, attaining a zero cost for the unsatisfied demand. Obviously this result significantly depends on the value assigned to parameter $C_{udf}$, which in this case has been proposed as $25\%$ of the unit price of the final board $S_{lf}$. Although the percentage of wood loss is similar to the base case, a larger amount of remaining products is generated. Even though inventory increases, these products are produced to avoid the penalty for unsatisfied demand.

<table>
<thead>
<tr>
<th>Table 12. Primary CPs used in each case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>$p91$</td>
</tr>
<tr>
<td>$p216$</td>
</tr>
<tr>
<td>$p251$</td>
</tr>
<tr>
<td>$p252$</td>
</tr>
<tr>
<td>$p324$</td>
</tr>
<tr>
<td>$p332$</td>
</tr>
<tr>
<td>$p784$</td>
</tr>
<tr>
<td>$p844$</td>
</tr>
<tr>
<td>$p1103$</td>
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<tr>
<td>$p1105$</td>
</tr>
<tr>
<td>$p1627$</td>
</tr>
<tr>
<td>$p2522$</td>
</tr>
<tr>
<td>$p2525$</td>
</tr>
<tr>
<td>$p4853$</td>
</tr>
<tr>
<td>$p4866$</td>
</tr>
</tbody>
</table>
5.5. Case study 7

In this case, the total number of demanded units is maintained, but their variety increases from 10 cross sections to 35.

In the optimal solution, the net benefit grows 31% from the base case value, leaving an unsatisfied demand of 13.4% and using the entire production time. Both the number of consumed logs and wood losses are reduced. The number of used CPs increases, taking into account that a greater variety of boards must be produced. Also, CP selection is more complicated and results in a greater number of boards in stock.

Finally, Table 12 shows the different primary CPs used in the different analyzed cases. It can be concluded that the solutions take advantage of the wide availability of options to choose those CPs that better fit the problem data and the proposed objectives. The optimal solution simultaneously selects the amounts and dimensions of logs to be cut among the available raw material as well as the CP to be employed. As was previously discussed, CP selection allows satisfying different problem requirements: raw material availability, demands, production requirements, etc. Table 12, determined using the same products and raw materials and varying only their quantities, is key because it shows that there are not CPs that are appropriate a priori. The first 6 columns (the last one introduces a more significant variation) present the wide variation that exists between the selected CPs. These CPs satisfy the aforementioned requirements for each particular case. Previous articles used to argue that a CP is very efficient because it has a very low loss and, thus, it was usually adopted for the sawmill for a determined diameter and length of the log. However, when a CP is applied, a set of boards is cut from a log, some of which may not be included in the demand or the relationships among boards do not correspond to the demanded product mix. Therefore, this approach overcomes previous proposals that worked with a
limited set of efficient CPs and those that calculate and incorporate efficient CPs for each
available log class.

6. Conclusions

This work presents a detailed model for optimal daily planning in sawmills. The proposed
approach is structured in two modules. First, an exhaustive generator allows providing a
significant number of feasible cutting patterns for properly planning sawmills production.
Second, a MILP model is formulated to address the problem of daily operations planning
in sawmills, considering log processing for obtaining final products.
Demand for final products has been specially considered in order to attain an efficient
production, avoiding useless inventories. Beyond achieving an optimal use of logs,
production is efficiently performed to satisfy the demand.
Daily production planning of sawmills is a problem with several trade-offs that must be
appropriately considered and addressed. The detailed proposed model simultaneously
considers raw material availability, production capacity, current technology (processing
and setup times), commercial aspects (demands), etc. The exhaustive procedure to generate
primary CPs allows selecting the suitable CPs to achieve an efficient operation plan. These
elements are properly balanced by all considered costs. Thus, planners can adjust model
parameters to attain well organized and profitable production plans.
Analyzing the results of the different solved examples, the MILP model achieves a great
performance: solutions are adjusted to problem conditions; demand is fulfilled in the best
possible way; and raw material is efficiently used so as to obtain the greatest economic
benefit. From the point of view of management and operation, several interesting
indicators are obtained: the number of produced and stored intermediate and final boards,
percentage of wood loss, operation time, etc. Taking into account that results include the

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selection of CP, production traceability is possible, so that the production plan can inform
the origin of the different cut boards.

Although the defined objective function aims at profitability, taking into account its
different terms and the posed constraints, an efficient employment of available logs and a
reduced inventory are also achieved. New elements can be also added as demonstrated in
Case 6. The proposed formulation is a good basis on which other objective functions such
as yield or production, can be evaluated in future works, since these objectives are often
antagonistic and lead to different solutions. Also, using this formulation for an extended
time horizon considering several days is an interesting alternative that deserves research.
Therefore, the proposed approach in this work is a useful tool for daily production
planning of sawmills, and its implementation allows improving productivity in these
plants.

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SIUTIFE0005246TC, respectively.

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