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Study of charged compact stars with class 1 metric under general relativity

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In the present paper we study compact stars under the background of Einstein-Maxwell spacetime, where the 4-dimensional spherically symmetric spacetime of class 1 along with the Karmarkar condition has been adopted. The investigations, via the set of exact solutions, show several important results such as (i) the value of density on the surface is finite, (ii) due to presence of the electric field, the outer surface or the crust region can be considered to be made of electron cloud, (iii) the charge increases rapidly after crossing certain cut off region ($r/R \approx 0.3$), and (iv) the avalanche of charge have a possible interaction between the particles that are away from the center. As the stellar structure supports all the physical tests performed on it, therefore the overall observation is that the model provides a physically viable and stable compact star.

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I. INTRODUCTION

The modelling and description of the compact objects attracts huge attention in the relativistic domain. The theme of four-dimensional spacetime embedded in higher-dimensional flat space is an old one. Schläfli [1] first introduce the embedding problem, just after Riemann introduced the manifold. It was conceived that the Riemannian manifold with positive definition and analytic metric of n dimensions can be isometrically embedded as a submanifold of pseudo-Euclidean space E_n with $N = n(n+1)/2$. The minimum number of dimensions in the pseudo-Euclidean space, i.e. $N - n = n(n-1)/2$ in which the Riemannian space of n dimensions can be immersed is called the embedding class. Much later Nash [2] established the isometric embedding theorem of V_n into E_n . For the non-compact V_n , the manifolds is $N = n(n+1)(3n+11)/2$, whereas, for the compact V_n the manifolds became $N = n(3n+11)/2$. The theorem was improved further by Gunther [3].

Therefore, we may conclude that in some Euclidean space, the Riemannian manifold has a global as well as local, isometrically embedded [4–6]. This embedding class is divided into the following classification scheme: (i) spherically symmetric fields of class 2, (ii) the interior Schwarzschild solution and spherically symmetric field are of class 1 [7], (iii) the exterior solution is of class 2, (iv) the Kerr metric is of class 5 [8], and (v) the cosmological metric Friedmann-Lemaître-Robertson-Walker (FLRW)

metric has been shown to be of class 1. We, along with other scientists, have done a detailed discussion on different aspects of the class 1 metric and their applications in special connection to compact stars [9–22]. Also, neutron stars, like those discussed in the work by Astashenok et al. [23] for magnetic field and by Capozziello et al. [24] for $f(R)$ gravity may be taken into account which would help the reader in enlarging the context of the compact stars with different approach.

The necessary condition of the n -dimensional space time to be embedded in higher dimensional ($> n$) flat space is the Gaussian curvature must be zero. The condition was first introduced by Eiesland [25] in the form of Christoffel curvature tensor. Later on, Karmarkar [26] investigated it in great details, and provide an equation to embed in class one. Though, there was an insufficiency of the equation, which was removed by Pandey [27]. They introduced a critical condition that must be satisfied to hold good the karmarkar condition equally as well as embedding in class 1. The necessary and sufficient condition [28] for a Riemannian space to be of class one is that the second-order symmetric tensor must satisfy the following conditions

$$R_{ijkl} = e(b_{ik}b_{jl} - b_{il}b_{jk}), \quad (1)$$

$$b_{ij;k} - b_{ik;j} = 0, \quad (2)$$

where i, j, k, l are the dummy indices, and $e = \pm 1$, the constant curvature, so that member on the right hand is positive, unless it is zero. The covariant derivatives are with respect to g_{ij} .

In the present work, we confine ourselves in the spherically symmetric charged distribution of the embedding

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class 1. As the system is considered static for simplicity, therefore, magnetic field will not have any role in the model. In connection to these charged models, many investigators [29–39] have been proposed several models which suffer from the negative density or negative pressure of the fluid due to considering of an *ad hoc* equation of state (EOS) in the form $p + \rho = 0$, where ρ is the matter density and p is the fluid pressure. This kind of EOS is known as the ‘pure charge condition’ [31] and also termed in the literature as ‘imperfect-fluid’, ‘false vacuum’, ‘degenerate vacuum’ or ‘ ρ -vacuum’ [40–43]. There are some other works with charge available in the literature [44–46] where special types of metric potentials have been assumed to get the Einstein-Maxwell spacetime solutions. However, in the present work we have employed quite a different technique, by adopting a unique algorithm, to tackle the problem of spherically symmetric charged distribution of the embedding class 1 [9, 15].

The plan of study of the present work can be outlined as follows: in Sect. 2, using Karmarkar condition in the static spherically symmetric metric and the Einstein-Maxwell field equations, we obtained spacetime of the matter distribution of embedding class 1. We construct a set of new general solutions, based on an algorithm for class 1 metric in Sect. 3. In Sect. 4, we introduce the boundary conditions and inter relationship of the system and further discussed to find out the constants. Different physical properties of the system are verified in Sect. 5 to find the validity of the stellar structure. Finally some remarks are done in Sect. 6.

II. BASIC MATHEMATICAL BACKGROUND

Let us consider the static spherical symmetric metric to be

$$ds^2 = -e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + e^\nu dt^2. \quad (3)$$

If the above metric satisfies the Karmarkar condition [26] then, it may represent the spacetime of embedding class 1.

A. The Karmarkar condition

The Karmarkar condition given for the metric (3) to be of class 1 has the form in Riemannian components as follows

$$R_{1414}R_{2323} = R_{1212}R_{3434} + R_{2142}R_{3143}, \quad (4)$$

where the components of Riemann curvature tensor R_{hijk} for metric (3) are given as

$$R_{2323} = -\sin^2\theta(e^\lambda - 1)r^2e^{-\lambda}, \quad R_{1212} = \frac{\lambda'r}{2},$$

$$R_{3143} = 0, \quad R_{1414} = -e^\nu\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4}\right],$$

$$R_{2142} = 0, \quad R_{3434} = -\sin^2\theta\frac{\nu're^{\nu-\lambda}}{2},$$

where $R_{2323} \neq 0$.

By inserting the components of R_{hijk} in Eq. (4), we get the following differential equation

$$e^\nu\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4}\right] = \frac{1}{4}e^\nu(e^\lambda - 1)^{-1}\lambda'\nu'. \quad (5)$$

Karmarkar considered the metric potentials ν and λ are the function of (r, t) , though for the simplicity we consider our metric potentials are only function of radial coordinate (r) .

The solution of the above differential equation provides the gravitational potential ν as

$$\nu = 2 \ln \left[A_1 + B_1 \int \sqrt{(e^{\lambda(r)} - 1)} dr \right], \quad (6)$$

where A_1 and B_1 are non-zero arbitrary constants of integration.

B. The Einstein-Maxwell field equations (EMFE)

Eq. (3) describes the charge perfect fluid distribution. Hence, the functions $\lambda(r)$ and $\nu(r)$ must satisfy the Einstein-Maxwell field equations

$$G^i_j = R^i_j - \frac{1}{2}Rg^i_j = 8\pi(T^i_j + E^i_j), \quad (7)$$

where $G = c = 1$ in the relativistic geometrized units.

The matter within the star is assumed to be a locally perfect fluid and consequently T^i_j and E^i_j , the energy-momentum tensors for the fluid distribution and the electromagnetic field tensors, are respectively defined by

$$T^i_j = [(\rho + p)v^i v_j - p\delta^i_j], \quad (8)$$

$$E^i_j = \frac{1}{4\pi}(-F^{im}F_{jm} + \frac{1}{4}\delta^i_j F^{mn}F_{mn}), \quad (9)$$

where v^i is the four-velocity as $e^{-\lambda(r)/2}v^i = \delta^i_4$.

The anti-symmetric electromagnetic field tensor F_{ij} in Eq. (9), denotes the field strength tensor and can be defined as

$$F_{ij} = \frac{\partial A_j}{\partial A_i} - \frac{\partial A_i}{\partial A_j}. \quad (10)$$

This should satisfy the Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0, \quad (11)$$

and

$$\frac{\partial}{\partial x^k}(\sqrt{-g}F^{ik}) = -4\pi\sqrt{-g}J^i, \quad (12)$$

where g is the determinant of quantities g_{ij} in Eq. (12) and is given by

$$g = \begin{pmatrix} e^\nu & 0 & 0 & 0 \\ 0 & -e^\lambda & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{pmatrix}.$$

For the spherically symmetric metric (3), the Einstein-Maxwell field equations can be expressed in the following form

$$-8\pi T^1_1 = \frac{\nu'}{r} e^{-\lambda} - \frac{(1 - e^{-\lambda})}{r^2} = 8\pi p - \frac{q^2}{r^4}, \quad (13)$$

$$\begin{aligned} -8\pi T^2_2 = -8\pi T^3_3 &= \left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r} \right] e^{-\lambda} \\ &= 8\pi p + \frac{q^2}{r^4}, \quad (14) \end{aligned}$$

$$-8\pi T^4_4 = \frac{\lambda'}{r} e^{-\lambda} + \frac{(1 - e^{-\lambda})}{r^2} = 8\pi \rho + \frac{q^2}{r^4}, \quad (15)$$

where the prime denotes differentiation with respect to the radial coordinate r .

III. NEW SOLUTION OF CHARGED EMBEDDING CLASS 1 SPACETIME

Let us take the metric function e^λ to solve the Einstein field equations as

$$e^\lambda = 1 + ar^2 \sinh^2(br^2 + c), \quad (16)$$

where in general the constants $a \neq 0$, $b \neq 0$ and $c \neq 0$, however the unit of a and b is length^{-2} .

Then from Eq. (6), we get

$$\nu = 2 \ln [A + B \cosh(br^2 + c)], \quad (17)$$

where $A = A_1$ and $B = \frac{B_1 \sqrt{a}}{2b}$ are constants.

A. Modified form of the EMFE

We have used the following notations for our ongoing study

$$e^\lambda = 1 + ar^2 \sinh^2 \theta = \phi, \quad (18)$$

$$e^{\frac{\nu}{2}} = [A + B \cosh \theta] = \psi, \quad (19)$$

where $\theta = c + br^2$.

From Eqs. (13) - (17), we obtain

$$8\pi p - \frac{q^2}{r^4} = -\frac{(-8bB + 2aA \sinh \theta + aB \sinh 2\theta) \sinh \theta}{2\psi\phi}, \quad (20)$$

$$\begin{aligned} 8\pi p + \frac{q^2}{r^4} &= -\frac{1}{2\phi^2\psi} [a \sinh 2\theta (2Abr^2 + B \sinh \theta) + 2Aa \sinh^2 \theta \\ &\quad + 4Bb(ar^2 - 2) \sinh \theta - 8Bb^2 r^2 \cosh \theta], \quad (21) \end{aligned}$$

$$8\pi \rho + \frac{q^2}{r^4} = \frac{a(3 \sinh^2 \theta + ar^2 \sinh^4 \theta + 2br^2 \sinh 2\theta)}{\phi^2}. \quad (22)$$

FIG. 1: Variation of the gravitational metric potentials e^λ (top) and e^ν (bottom) with respect to the fractional radius (r/R). For the purpose of plotting these graphs, we have employed the values of parameters and arbitrary constants as follows: (i) $a = 0.12 \text{ Km}^{-2}$, $b = 0.00025 \text{ Km}^{-2}$, $c = 0.212$, $A = -21.78722$, $B = 22.04938$ for *Her X-1*, (ii) $a = 0.1743 \text{ Km}^{-2}$, $b = 0.00037 \text{ Km}^{-2}$, $c = 0.2$, $A = -18.63205$, $B = 18.8712$ for *Cen X-3*, (iii) $a = 0.1262 \text{ Km}^{-2}$, $b = 0.00028 \text{ Km}^{-2}$, $c = 0.22$, $A = -20.9777$, $B = 21.2086$ for *4U 1538-52*, and (iv) $a = 0.1913 \text{ Km}^{-2}$, $b = 0.0004 \text{ Km}^{-2}$, $c = 0.195$, $A = -18.1098$, $B = 18.34105$ for *PSR J1903+327*. Henceforth this data set will also be followed for the other plots where the compact stars are presented as follows: *Her X-1* with dot, *Cen X-3* with dashed dot, *PSR J1903+327* with long dash and *4U 1538-52* with solid line

On the other hand, from the pressure isotropy condition we obtain

$$\begin{aligned} \frac{q^2}{r^4} &= -\frac{r^2}{4\psi\phi^2} (2b \cosh \theta - a \sinh^3 \theta) \times \\ &\quad (-4bB + 2aA \sinh \theta + aB \sinh 2\theta). \quad (23) \end{aligned}$$

Also, the gradient form of the pressure, can be obtained from Eqs. (13) and (20) - (21) as follows

$$\frac{dp}{dr} = -\frac{M_G(r)(p + \rho)}{r^2} e^{(\lambda - \nu)/2} + \frac{q}{4\pi r^4} \frac{dq}{dr}, \quad (24)$$

where M_G is the gravitational mass within the sphere of radius

FIG. 2: Variation of the effective pressure p (top) and the effective density ρ (bottom) with respect to the fractional radius (r/R). The compact star corresponding to the curves are presented as follows: *Her X-1* with dot, *Cen X-3* with dashed dot, *PSR J1903+327* with long dash and *4U 1538-52* with solid line for same parameter values as used in Fig.1.

r and is given by

$$M_G(r) = \frac{1}{2} r^2 \nu' e^{(\nu-\lambda)/2}. \quad (25)$$

The Eq. (24) represents the stellar charged generalized Tolman-Oppenheimer-Volkoff (TOV) equation for the perfect fluid system [48, 49].

The explicit mathematical equations for the pressure and density gradients are as follows:

$$\frac{dp}{dr} = \frac{r}{4\pi} \left[\frac{f_{q1} + f_{q2} + f_{q3} + (2br^2 \cosh \theta + \sinh \theta) f_{q4} + f_{q5}}{4\psi^2 \phi^3} - \frac{f_{p1} + f_{p2} + f_{p3}}{\psi^2 \phi^2} \right] \quad (26)$$

$$\frac{d\rho}{dr} = \frac{r}{4\pi} \left[\frac{a(f_{p1} - f_{p2})}{\phi^3} + \frac{f_{q1} + f_{q2} + f_{q3} + (2br^2 \cosh \theta + \sinh \theta) f_{q4} + f_{q5}}{4\psi^2 \phi^3} \right] \quad (27)$$

where the values of the f_{p1} , f_{p2} , f_{p3} , f_{p1} , f_{p2} , f_{q1} , f_{q2} , f_{q3} , f_{q4} and f_{q5} are provided in the Appendix 1.

IV. BOUNDARY CONDITIONS

The above mentioned physical system is solved under the condition that the radial pressure $p = 0$ at the surface. The

interior metric (3) also satisfy the Reissner-Nordström metric [50] at the surface of sphere, so that we get

$$ds^2 = -\frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)} - r^2(d\theta^2 + \sin^2 \theta d\phi^2) + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2, \quad (28)$$

$$e^{-\lambda(R)} = 1 - \frac{2M}{R} + \frac{Q^2}{R^2}, \quad (29)$$

$$e^{\nu(R)} = 1 - \frac{2M}{R} + \frac{Q^2}{R^2}, \quad (30)$$

$$q(r=R) = Q, \quad (31)$$

$$p(r=R) = 0. \quad (32)$$

The pressure is zero on the boundary $r = R$ and hence we obtain

$$\frac{A}{B} = -\frac{[-8b + 2abR^2 \cosh^2 \theta - 4b^2 R^2 \coth \theta - 6abR^2 \sinh^2 \theta + \Phi_1(R)]}{a[2bR^2 \cosh \theta + 2 \sinh \theta + aR^2 \sinh^3 \theta]} = \chi, \quad (33)$$

where $\Phi_1(R) = a^2 R^2 \cosh \theta \sinh^3 \theta + a \sinh 2\theta$.

Again, at the boundary $e^{-\lambda(R)} = e^{\nu(R)}$, which gives

$$B = \frac{1}{\sqrt{1 + aR^2 \sinh \theta \sinh \theta (\chi + \cosh \theta)}}. \quad (34)$$

V. PHYSICAL ACCEPTABILITY CONDITIONS FOR THE ISOTROPIC STELLAR MODELS

Now we need to perform more physical tests for the isotropic stellar configuration of embedding class one. For this purpose we take data set for different compact stars given in Table 1.

TABLE I: Numerical values of physical parameters a , b , c , A , B , R (km) [51], M ($M_\odot$), M/R

Compact stars	$a(km^{-2})$	$b(km^{-2})$	c	A	B	R (km)	M ($M_\odot$)	M/R
Her X-1	0.12	0.00025	0.212	-21.7872	22.0494	8.10	0.85	0.1548
Cen X-3	0.1743	0.00037	0.2	-18.6321	18.8712	9.178	1.49	0.2395
PSR J1903+327	0.1913	0.0004	0.195	-18.1098	18.3411	9.438	1.667	0.2605
4U 1538-52	0.1262	0.00028	0.22	-20.9777	21.2086	7.866	0.87	0.1631

A. Regularity and Reality Conditions

From Eqs. (18) and (19), we get that the central potentials of the system are $e^{\lambda(0)} = 1$ and $e^{\nu(0)} = (A + B \cosh c)^2$ respectively. This clearly states that the metric potentials are positive and finite at the centre, i.e., the system is singularity free. Also, Fig. 1 claims that the system has finitely increasing functions.

The pressure at the centre of the system is given by

$$p(0) = \frac{(4Bb - aB \sinh c \cosh c - aA \sinh c) \sinh c}{8\pi (A + B \cosh c)}. \quad (35)$$

The pressure vanishes on the boundary of the system. The variation of pressure is shown in the Fig. 2 (left panel).

The central density of the system is

$$\rho(0) = \frac{3a \sinh^2 c}{8\pi}. \quad (36)$$

The variation of density is shown in the Fig. 2 (right panel). The density is a decreasing function, which is minimum on the surface of the system.

B. Stability condition

The stability condition of the matter distribution can be given by the condition of causality: $0 \leq v^2 \leq 1$ [52, 53]. Fig. 3 shows that our system satisfy this condition, i.e., the sound velocity lies between $0 \rightarrow 1$ through out the region. Thus, our model provides a stable system.

FIG. 3: Variation of speed of sound $v = \sqrt{dp/d\rho}$ with respect to fractional radius (r/R). The compact star corresponding to the curves are presented as follows: *Her X-1* with dot, *Cen X-3* with dashed dot, *PSR J1903+327* with long dash and *4U 1538-52* with solid line for same parameter values of Fig.1

C. TOV equation

At equilibrium the sum of all forces must be equal to zero [48, 49], i.e.

$$F_g + F_h + F_e = 0. \quad (37)$$

The first, second and third term of the above equation are the gravitational, hydrostatic and electrostatic forces respectively.

In our present model following TOV equation the different forces are given by

$$F_g = - \left[\frac{2bBr[4bB+4aAbr^2 \cosh \theta + 4abBr^2 \cosh 2\theta + F_{g1}] \sinh^2 \theta}{8\pi\psi^2\phi^2} \right], \quad (38)$$

$$F_h = -\frac{dp}{dr}, \quad (39)$$

$$F_e = \frac{r}{4\pi} \left[\frac{F_{e1}+F_{e2}+F_{e3}+(2br^2 \cosh \theta + \sinh \theta)F_{e4}+F_{e5}}{4\psi^2\phi^3} \right], \quad (40)$$

where the expressions of F_{g1} , F_{e1} , F_{e2} , F_{e3} , F_{e4} and F_{e5} are provided in the Appendix 2.

FIG. 4: Variation of different forces F_g , F_h and F_e with respect to the fractional radius (r/R) are shown in the plots where the legends are presented as follows: gravitational force (F_g) with dashed dot, electrostatic force (F_e) with long dash and hydrostatic force (F_h) with solid line for same parameter values as used in Fig.1.

D. Energy Conditions

The system must satisfy the different energy conditions, e.g., the null energy condition (NEC), weak energy condition (WEC) and strong energy condition (SEC). For an isotropic

TABLE II: The numerical values of electric charge for different compact stars ($\times 10^{20} C$)

r/R	<i>Her X-1</i>	<i>Cen X-3</i>	<i>PSR J1903+327</i>	<i>4U 1538-52</i>
0.0	0.000	0.000	0.000	0.000
0.2	0.022	0.033	0.033	0.027
0.4	0.089	0.141	0.146	0.110
0.6	0.199	0.338	0.362	0.245
0.8	0.348	0.622	0.685	0.424
1.0	0.525	0.961	1.073	0.632

system the following inequalities have to hold:

$$\begin{aligned}
 (i) \text{ NEC} : \rho + \frac{E^2}{4\pi} &\geq 0, \\
 (ii) \text{ WEC} : \rho + p &\geq 0, \\
 (iii) \text{ SEC} : \rho + 3p - \frac{E^2}{4\pi} &\geq 0.
 \end{aligned}$$

The above energy conditions are drawn in Fig. 5, shows that our proposed model satisfies all the energy conditions.

E. Electric charge

The Fig. 6 (left panel) shows variation of the charge from its minimum value $q(0) = 0$ at the centre to its maximum value on the surface. Therefore, the expression for the total charge can be obtained from Eq. (23) as

$$Q = \sqrt{\frac{R^6 (2b \cosh \theta_R - a \sinh^3 \theta_R) (4bB - 2aA \sinh \theta_R - aB \sinh 2\theta_R)}{4\psi_R \phi_R^2}} \quad (41)$$

The variation of amount of charges at different points from the centre to the boundary is shown in Table II.

F. Effective mass-radius relation and surface redshift

The maximum allowable mass-radius ratio for an isotropic fluid sphere is given by Buchdahl [54] in the form $2M/R \leq 8/9$. Böhmer [55] have also shown that for a charged compact object, $Q(< M)$, the lower bound for the mass-radius ratio can be constrained as

$$\frac{3Q^2}{4R^2} \left(\frac{1 + \frac{Q^2}{18R^2}}{1 + \frac{Q^2}{12R^2}} \right) \leq \frac{M}{R}, \quad (42)$$

for the constraint $Q < M$.

This upper bound of the mass-radius ratio for charged fluid sphere was generalized by [53] who proved that

$$\frac{M}{R} \leq \frac{1}{R} \left(\frac{\sqrt{R}}{3} + \sqrt{\frac{R}{9} + \frac{Q^2}{3R}} \right)^2. \quad (43)$$

We have obtained lower bound and upper bound of the mass-radius ratio for different charged compact star which are mentioned in Table III. From this Table III, it can be

FIG. 5: Variation of different energy conditions with respect to fractional radius (r/R) are shown in the plots where the legends are presented as follows: SEC with dot, WEC with dash and NEC with solid line for same parameter values as used in Fig. 1.

observed that the mass-radius ratio is satisfying its lower and upper bounds nicely.

In the present model, we find the effective mass as

$$M_{eff} = 4\pi \int_0^R \left(\rho + \frac{E^2}{8\pi} \right) r^2 dr = \frac{1}{2} R (1 - e^{-\lambda(R)}), \quad (44)$$

which may be expressed as

$$M_{eff} = \frac{aR^3 \sinh^2(bR^2 + c)}{2[1 + aR^2 \sinh^2(bR^2 + c)]}. \quad (45)$$

In terms of the compactification factor $u(r) = m(r)/r$ we now define the redshift Z as

$$Z = (1 - 2u)^{-\frac{1}{2}} - 1. \quad (46)$$

The red-shift can be expressed as

$$Z = \sqrt{1 + ar^2 \sinh^2(br^2 + c)} - 1. \quad (47)$$

The variation is shown in Fig. 6 (right panel).

FIG. 6: Variation of the electric charge q (top) and redshift Z (bottom) with respect to the radial coordinate (r/R) . The compact star corresponding to the curves are presented as follows: *Her X-1* with dot, *Cen X-3* with dashed dot, *PSR J1903+327* with long dash and *4U 1538-52* with solid line for same parameter values as used in Fig.1.

G. Pressure and density relationship of the compact star

In the present paper, we have studied the physical properties of the charged compact stars, e.g. *Her X-1*, *Cen X-3*,

TABLE III: The lower bound and upper bound of mass-radius ratio for different charged compact stars

Compact stars	Lower Bound $\frac{3Q^2 \left(1 + \frac{Q^2}{18R^2}\right)}{4R^2 \left(1 + \frac{Q^2}{12R^2}\right)}$	Mass-Radius Ratio $\frac{M}{R}$	Upper Bound $\frac{1}{R} \left(\frac{\sqrt{R}}{3} + \sqrt{\frac{R}{9} + \frac{Q^2}{3R}} \right)^2$
Her X-1	0.00171361	0.154784	0.445966
Cen X-3	0.0031756	0.239458	0.447263
SAX J1808.4-3658	0.00357377	0.260524	0.447616
4U 1538-52	0.0020501	0.163139	0.446265

TABLE IV: The central density, surface density and central pressure for different compact stars

Compact stars	Central Density (gm/cm^3)	Surface Density (gm/cm^3)	Central Pressure $(dyne/cm^2)$	Surface Redshift
Her X-1	8.778×10^{14}	6.162×10^{14}	3.489×10^{34}	0.190
Cen X-3	1.138×10^{15}	6.619×10^{14}	9.882×10^{34}	0.344
PSR J1903+327	1.181×10^{15}	6.608×10^{14}	1.221×10^{35}	0.387
4U 1538-52	9.953×10^{14}	6.778×10^{14}	4.122×10^{34}	0.204

PSR J1903+327 and *4U 1538-52* without taking any particular equation of state (EOS). From Eqs. (20-21), it is difficult to predict a exact form of EOS for the proposed charged compact stellar model. However, we have analyzed from Eq.(20-21) that the pressure is zero at the boundary of the star ($r = R$) while the matter density is not vanishing at the boundary. Therefore EOS of this solution will not be in the usual form $p = \omega\rho$ or a polytropic EOS of the form $p = \omega\rho^\gamma$. So, we have plotted the graph of the pressure (p) versus density (ρ) which is shown by Fig. (7)

FIG. 7: Variation of the pressure p with respect to the density (ρ). The compact star corresponding to the curves are presented as follows: *Her X-1* with dot, *Cen X-3* with dashed dot, *PSR J1903+327* with long dash and *4U 1538-52* with solid line

From Fig. 7, it is clear that the pressure is zero at the initial phase and is increasing with increase of the density ρ . From this observation we can say that the pressure is propositional to the quantity $(\rho - \rho_s)$, where ρ_s is the surface density. From this relation one can strongly suggest that EOS will be of the following form [15, 56, 57]

$$p_r = \alpha (\rho - \rho_s), \quad (48)$$

where α is a constant. However, following Maurya et al. [58], we would like to mention that in the present study our focus

mainly on the macroscopic properties of the compact stellar model under embedding class 1 technique without any detailed analysis based on the nuclear and particle physics to determine the exact form of EOS of the matter distribution.

VI. DISCUSSION AND CONCLUSION

In the present work, we have studied compact stars under the framework of Einstein's general relativity. In addition we have also considered the Maxwell field equations in the presence of electric charge and applied the Karmarkar condition along with the algorithm as provided by Maurya et al. [9, 15]. Thus, in the investigation basically we formulate the equations of equilibrium of compact stars taking into account of properties of the electric charge distribution of compact star having spherically symmetric distribution of the embedding class 1. Through an analytical scheme we have found that due to electric charge distribution the limiting star configuration can have more mass and a smaller radius relatively to the limiting star with zero charge, i.e. neutral stellar system. This is expected since the electric charge distribution has a repulsive effect, adding to the pressure as a force that withstands the star. Through the boundary conditions we have arrived at expressions for the various physical attributes, like the central density, surface density, central pressure as well as mass and radius of different compact stars.

Here, we have determined the values of constants A and B from the boundary conditions whereas the free parameters a , b and c have been chosen by following a trial and error method so that we can obtain specific values of radius and mass of a specific star which are comparable to the observed values as suggested in the literature and physically acceptable models. For the sake of clarity we would like to explain that here the free parameters are only a , b and c as other constants/parameters depend upon a , b and c . First, we fixed range of the parameter a by using the central density of the star (Eq. 36). Next, we chose the predicated radius from the paper of Gangopadhyay et al. [51] and we select the values of parameters a , b and c for which we get the values of observed mass and physical quantities, like the pressures, density, velocity of sound etc for different compact stars. The set of parameters we chose in such way that all the physical quantities must be well behaved. However, other set of values are also possible for the same mass and radius where some of the physical quantities may not be well behaved.

Since, the system is physically acceptable (consistence with the cracking condition, TOV equation etc.) and values of the free parameters have been obtained for the specific mass and radius of a star, therefore we can consider Table I is the best suitable set for the specific star.

The solution set for the Einstein-Maxwell spacetime shows several usual as well as interesting results. Some salient features of the model are therefore can be discussed as follows:

- (i) The metric function $e^{\lambda(r)}$ cannot be equal to unity except at the centre. Though for $e^{\lambda(r)} = 1$ (which represents $R_{2323} = 0$), the system satisfies the Karmarkar relation. However, it ceased to be of class 1 [27], i.e. the four dimensional system cannot be embedded in five dimensional flat space-time geometry.
- (ii) The variation of density in Fig. 2 (right panel) shows that the value of density on the surface is finite.
- (iii) Due to the presence of electric field, the outer surface or the crust region can be considered to be made of electron cloud.
- (iv) From Fig. 6 (left panel) we observe that the charge increases rapidly after crossing a certain cut in region ($r/R \approx 0.3$). One may interpret this effect as the avalanche of charge interactions between the particles that are away from the center.
- (v) The stellar structure supports all the physical tests done on it, viz. the regularity and reality conditions, stability condition, TOV equation, electric charge content, effective mass-radius relation and surface redshift.

It is to note that in Introduction we have mentioned about the modified Einstein-Maxwell theories and compared them a bit with the model. However, the compact star solutions, e.g. in the models given by Sert [59, 60] may be compared with our results in a more specific way. A detailed observation on the work by Sert [60] where a model of compact stars in the non-minimal coupled electromagnetic fields to gravity has been studied, shows that the solution related to pressure as well as density suffers from singularity. Moreover, in the study on radiation fluid stars in the non-minimally coupled $Y(R)F^2$ gravity Sert [59] has considered the radiative equation of state where the pressure and energy density is not singularity free. However, the solutions are physically acceptable, since total mass and charge are finite.

We also note that in their article Mak and Harko [61] studied the strange star under the comformal symmetry. They used MIT bag model equation of of state where the pressure and energy density are not singularity free. However, the solutions are physically acceptable, since the total mass and charge are finite. In addition, they found the maximum mass and corresponding radius for a defined Bag value.

In contrary, we are concerned with finding out a physically acceptable and singularity free solution of a compact stellar. No defined relation is used between the energy density and pressure. However, we have used the Karmarkar condition to describe the metric of the system in the embedding class system.

As a final comment, the overall observation regarding our investigation is that the model provides a singularity free and hence physically viable and stable configuration of compact stars.

Appendix 1

In this Appendix, we are providing the expressions of the constants which arise in the explicit calculation of Eqs. (26) and (27) as follows:

$$f_{p1} = b\phi\psi[2a(A \cosh \theta + B \cosh 2\theta) \sinh \theta + (-8bB + 2aA \sinh \theta + aB \sinh 2\theta) \cosh \theta],$$

$$f_{p2} = a\psi(2br^2 \cosh \theta + \sinh \theta)(-8bB + 2aA \sinh \theta + aB \sinh 2\theta) \sinh^2 \theta,$$

$$f_{p3} = \phi bB[-8bB + 2aA \sinh \theta + aB \sinh 2\theta] \sinh^2 \theta,$$

$$f_{\rho1} = \phi[2b \sinh 2\theta(5 + 2ar^2 \sinh^2 \theta) + 8b^2 r^2 \cosh 2\theta + 2a \sinh^4 \theta],$$

$$f_{\rho_2} = 4[br^2 \sinh 2\theta + \sinh^2 2\theta],$$

$$f_{q_1} = 2abr^2\psi\phi(A \cosh \theta + B \cosh 2\theta)(a \sinh^3 \theta - 2b \cosh^2 \theta),$$

$$f_{q_2} = \psi\phi(2b \cosh^2 \theta - a \sinh^3 \theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta),$$

$$f_{q_3} = bBr^2\phi(2b \cosh \theta - a \sinh^3 \theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta) \sinh \theta,$$

$$f_{q_4} = 2ar^2\psi(2b \cosh^2 \theta - a \sinh^3 \theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta) \sinh \theta,$$

$$f_{q_5} = \frac{1}{2}[br^2\psi\phi(4b - 3a \sinh 2\theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta) \sinh \theta].$$

$$F_{e2} = 3\psi\phi(2b \cosh^2 \theta - a \sinh^3 \theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta),$$

$$F_{e3} = bBr^2\phi(2b \cosh \theta - a \sinh^3 \theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta) \sinh \theta,$$

$$F_{e4} = 2ar^2\psi(2b \cosh^2 \theta - a \sinh^3 \theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta) \sinh \theta,$$

$$F_{e5} = \frac{1}{2}[br^2\psi\phi(4b - 3a \sinh 2\theta)(4bB - 2aA \sinh \theta - aB \sinh 2\theta) \sinh \theta].$$

Appendix 2

In this Appendix, we are providing the expressions of the constants which arise in the explicit calculation of Eq. (40) as follows:

$$F_{g1} = 2aA \sinh \theta + aB \sinh 2\theta,$$

$$F_{e1} = 2abr^2\psi\phi(A \cosh \theta + B \cosh 2\theta)(a \sinh^3 \theta - 2b \cosh^2 \theta),$$

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