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Probabilistic Seismic Slope Stability Analysis and Design

by Jesse Burgess¹, Gordon A. Fenton², and D. V. Griffiths^{3,4}

Abstract

Deterministic seismic slope stability design charts for cohesive-frictional ($c - \phi$) soils are traditionally used by geotechnical engineers to include the effects of earthquakes on slopes. These charts identify the critical seismic load event which is sufficient to bring the slope to a state of limit equilibrium but they do not specify the probability of this event. In this paper, the probabilistic seismic stability of slopes, modeled using a two-dimensional spatially random $c - \phi$ soil, is examined for the first time using the Random Finite Element Method (RFEM). Slope stability design aids for seismic loading, which consider spatial variability of the soil, are provided to allow informed geotechnical seismic design decisions in the face of geotechnical uncertainties. The paper also provides estimates of the probability of slope failure without requiring computer simulations. How the design aids may be used is demonstrated with an example of slope remediation cost analysis and risk-based design.

Keywords: seismic design, slope stability, probabilistic analysis, random fields, Monte Carlo simulation, design aids, spatial variability, cohesive-frictional soils

¹ MSc, Engineering Mathematics Dept., Dalhousie University, Halifax, Nova Scotia, Canada B3J 2X4.
Jesse.Burgess@dal.ca

² Professor, Engineering Mathematics Dept., Dalhousie University, Halifax, Nova Scotia, Canada B3J 2X4.
Gordon.Fenton@dal.ca

³ Professor, Division of Engineering, Colorado School of Mines, Golden, Colorado 80401-1887, USA.
D.V.Griffiths@mines.edu (F.ASCE)

⁴ Australian Research Council Centre of Excellence for Geotechnical Science and Engineering, University of Newcastle, Callaghan, NSW, Australia

1. Introduction

Earth slopes and embankments are commonly occurring geotechnical structures, be they naturally formed, cut, or constructed. Slopes can often be found near, or as part of, a larger engineered system, such as roadways, bridges, and dams. As such, earth slopes are routinely analyzed to assess their stability against collapse and potential damage to the larger engineered system and/or to life. Traditionally the stability assessment of slopes, such as the one shown in Figure 1, is undertaken by estimating the characteristic shear strength of the soil, through sampling, and then consulting design charts for the slope's particular geometries. For static loading, where only gravitational load is considered, design charts have been developed for a broad range of geometries and soil conditions both deterministically and, to a lesser extent, probabilistically. It is often the case, however, that the serviceability of a slope is in part reliant upon its response to extreme events such as earthquakes, which may cause significant deformation of the slope due to a rapid reduction in the strength of the soil mass.

In general, existing seismic design approaches for slopes are predominately pseudo-dynamic in nature due to the high computational cost and uncertainty of fully dynamic models. See Coduto et al. (2011) for a description of these “pseudo-static” methods. It should be noted that the commonly used term “pseudo-static” is actually a misnomer since the slope analysis involved is still entirely static – there is no “pseudo” about it. In other words, the existing so-called pseudo-static approaches should actually be called “pseudo-dynamic” since it is the dynamic aspects which are approximated. In the remainder of this paper, the approach will be referred to as “pseudo-dynamic.”

For simple homogeneous slope masses, existing seismic slope stability charts may be used, such as those presented by Leshchinsky and San (1994) and Loukidis et al. (2003), provided the

slope in question satisfies a few key assumptions:

- 1) The slope is not expected to experience liquefaction.
- 2) Effects of pore water pressure, u_d , are not important ($u_d = 0$). Pseudo-dynamic slope stability design charts are best used when the ground water table is typically low.
- 3) Seismic coefficients, k , used in pseudo-dynamic design analysis are usually limited to $k \leq 0.3$ (e.g. Hynes-Griffin and Franklin 1984; Baker et al. 2006).
- 4) The static factor of safety, F , of the slope being examined is in the range of $1.0 \leq F \leq 1.7$ (Hynes-Griffin and Franklin 1984).

Despite difficulties in predicting the dynamic response of slopes due to earthquakes, analysis of slope stability subject to seismic loading has been given considerable attention (see, e.g. Mostyn and Small 1987). Geotechnical researchers have developed methodologies to overcome computational limitations and generate rough estimates of slope stability under seismic loading through the application of pseudo-dynamic horizontal loads intended to be roughly equivalent to the true dynamic load. Use of such methods has allowed for the generation of several sets of deterministic seismic design charts (e.g. Loukidis et al. 2003) which estimate the maximum amount of horizontal seismic acceleration a slope can withstand before failing. For simple slopes, use of such charts provide geotechnical engineers a means to quickly estimate the seismic stability of a slope without having to know anything more than the soil strength parameters and the slope geometry. However, the design charts currently used in seismic slope analysis lack the means to account for the spatial variability of soils, which may result in weakened portions of the soil being overlooked. In other words, the risk of slope failure from those weakened sections excited by seismic motion may be missed by such deterministic

evaluations. To provide a procedure that estimates the failure probability of a slope, the analysis presented in this paper seeks to link the traditional factor of safety approaches, currently common in seismic analyses, with the probability of failure, which captures the influence of spatial variability in soil strength parameters. By incorporating the results of this work into seismic design aids, styled similarly to existing charts commonly used in practice, a procedure may be developed to assist geotechnical engineers in the safe design of more realistic slopes when earthquake loading is considered.

The probabilistic assessment of slopes under seismic loading has received some attention over the years. For example, an early study by Grivas and Howland (1980) used a “single random variable” (spatially constant soil properties) limit equilibrium approach with random pseudo-dynamic loading to predict slope failure probability under seismic loading. More recently Xiao et al. (2016) used a random field to model the soil combined with a peak ground acceleration distribution, via a pseudo-dynamic analysis, to estimate the failure probability at a specific site. However, most such studies are interested in estimating the failure probability, and not in the calibration of design factors and aids.

This paper develops and presents a series of design aids which can be used to assist in the probabilistic seismic design of $c-\phi$ slopes having simple geometry (see Figure 1) and composed of a single statistically isotropic and homogeneous (spatially constant mean and standard deviation) ground, ie a single layer, underlain by bedrock. A range of mean cohesion and friction angles are considered, along with pseudo-dynamic seismic coefficients, k , ranging from 0 to 0.3, which is generally sufficient for most locations in Canada. Although the influence of correlation length on failure probability is investigated here, only design aids for an intermediate correlation length ($\theta = 0.2H$) are presented here. The interested reader can find

further such design aids over a wide range of correlation lengths in Burgess (2016). Two examples are provided at the end of the paper to illustrate the use of the design aids.

2. Random Field Model

2.1 Random Fields

When modeling the geotechnical failure mechanisms of slopes, the spatial variability in soil properties can be accounted for by using random fields. A random field is a collection of interdependent random variables, each associated with a spatial location, t . For example, a random field, denoted by $X(t)$, would consist of the random variables $X(t_1) = X_1, X(t_2) = X_2, \dots, X(t_n) = X_n$ at positions $t_1, t_2, \dots, t_n$, where n is the number of random variables used to represent the field. The set of random variables $X_1, X_2, \dots, X_n$ will, in general, have an n -dimensional probability density function, which is usually simplified (as is the case in this paper) by assuming that $X(t)$ is a stationary, isotropic, Gaussian process. Interested readers are referred to Fenton and Griffiths (2008) and Vanmarcke (1984) for more details. The resulting random field is fully described by its mean, standard deviation, and correlation structure, $\rho(\tau)$, where $\tau = |t_a - t_b|$ is the distance between any pair of points $X(t_a)$ and $X(t_b)$. This paper assumes that $\rho(\tau)$, is the Markov correlation function:

$$\rho(\tau) = \exp\left(\frac{-2\tau}{\theta}\right) \quad (1)$$

where θ , the correlation length, is the separation distance within which two random variables are significantly correlated (see Fenton and Griffiths 2008 for the precise mathematical definition). For a two-dimensional, isotropic process, $\rho(\tau)$ is assumed to have the form:

$$\rho(\tau) = \exp \left(- \sqrt{\left(\frac{2\tau_x}{\theta_x} \right)^2 + \left(\frac{2\tau_y}{\theta_y} \right)^2} \right) = \exp \left(\frac{-2\tau}{\theta} \right) \quad (2)$$

where the subscripts ‘ x ’ and ‘ y ’ on τ and θ denote their respective directional components, and the correlation structure was assumed to be isotropic so that $\theta = \theta_x = \theta_y$, and $\tau = \sqrt{\tau_x^2 + \tau_y^2}$ in the rightmost term.

2.2 Random Field Modeling of Soils

The two most important soil strength parameters in slope stability analysis are the cohesion, c , and the internal friction angle, ϕ . Both c and $\tan \phi$ are assumed here to be lognormally distributed with means μ_c and $\mu_{\tan \phi}$, and standard deviations σ_c and $\sigma_{\tan \phi}$, respectively. The lognormal distribution was assumed for these soil properties because it is strictly non-negative, which must be true of these soil properties. As also argued by Fenton and Griffiths (2008), the lognormal distribution is reasonable for soil strength parameters due to the central limit theorem (i.e., low-strength dominated geometric averages tend to a lognormal distribution by the central limit theorem). Lognormally distributed random fields are derived from Gaussian fields using the following simple transformation. If $Y(t)$ is a Gaussian random field, then a lognormally distributed random field $X(t)$ is defined as:

$$X(t) = e^{Y(t)} \quad \Leftrightarrow \quad \ln X(t) = Y(t) \quad (3)$$

For both soil strength parameters, the associated lognormal distribution parameters $\mu_{\ln X}$ and $\sigma_{\ln X}$, where ‘ X ’ is replaced by either c or $\tan \phi$, are derived from μ_X and σ_X through the transformations

$$\sigma_{\ln X}^2 = \ln(1 + \nu_X^2) \quad (5)$$

$$\mu_{\ln X} = \ln(\mu_X) - \frac{\sigma_{\ln X}^2}{2} \quad (6)$$

where $v_X = \sigma_X / \mu_X$ is the coefficient of variation of X (either c or $\tan \phi$).

Phoon and Kulhawy (1999) recommend ranges of the coefficients of variation, v_c and v_ϕ , of the soil strength parameters c and ϕ , to be $0.1 \leq v_c \leq 0.5$ and $0.1 \leq v_\phi \leq 0.2$. The coefficient of variation range considered in this study is 0.1 to 0.3, which the authors consider to be reasonable.

For simplicity, the soil strength parameters c and ϕ , are assumed to be independent, even though they are generally believed to be negatively correlated (as one increases, the other tends to decrease). Fenton and Griffiths (2003, see their Figure 3) examined the effect of cross-correlation between c and ϕ and found that it has a negligible effect on the mean and standard deviation of a foundation's bearing capacity, especially at smaller coefficients of variation ($v \leq 0.3$), with zero correlation (independence) being slightly conservative relative to a negative correlation. Since a coefficient of variation of 0.3 is the maximum considered in this paper, it is not expected that the assumption of independence will make a significant difference to the results presented in this paper.

These soil properties also possess associated correlation lengths, $\theta_{\ln c}$ and $\theta_{\ln(\tan \phi)}$. Low values of θ indicate rapid variability within the soil mass, while large values of θ will result in gradual variation, as illustrated in Figure 2. To further simplify the random field model, it is assumed here that the correlation lengths of the two soil parameters are equal so that $\theta_{\ln c} = \theta_{\ln(\tan \phi)} = \theta_{\ln X}$, which will be referred to as θ henceforth for simplicity. In practice the log-space correlation length, $\theta_{\ln X}$, is not very different from its real-space counterpart θ_X . As such,

the two measures can be used interchangeably for most purposes, especially since the correlation length is generally not well known.

2.3 Single Random Variable Slope Models

Because of their increased complexity, geotechnical engineers have been slow to adopt the use of random fields in their probabilistic studies of slopes. A common alternative makes use of the Single Random Variable (SRV) approach (e.g. Harr 1987; Duncan 2000; Javankhoshdel and Bathurst 2014), which is equivalent to setting θ to infinity in a random field. An infinite correlation length yields a homogeneous field, meaning that the soil strength parameters are constant throughout the soil mass, but random from realization to realization. Probabilistic analysis then typically consists of a combination of limit-equilibrium circular slip surface analyses (e.g. Bishop's simplified method) and Monte Carlo simulations. The soil strength parameters vary randomly from one realization to the next, and the probability of failure is determined by the ratio of the number of failed realizations to the total number of realizations performed. Javankhoshdel and Bathurst (2014) conducted a simulation-based study using the SRV approach to extend deterministic static slope stability design charts into the realm of probabilistic analyses. They considered a wide range of slope angles ($\beta = 10^\circ$ to 90°), various internal friction angles ($\mu_\phi = 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ$, and 45°), and several combinations of coefficients of variation for c and ϕ , based upon recommendations by Phoon and Kulhawy (1999), to produce probabilistic static slope design charts. The traditional factor of safety, F , found through deterministic analyses, was shown to be an imperfect measure of slope failure probability, because some slopes having $F \gg 1$ still exhibited considerable probabilities of failure.

Griffiths and Fenton (2000) have found that the SRV model of slopes is conservative, and often very conservative except at lower values of F (e.g., $F \leq 1.1$) where SRV becomes unconservative. Because homogeneous slope masses are unlikely to occur in real-world slope problems, it makes sense to account realistically for the spatial variability of soils by setting the correlation length to a non-infinite value, as assumed in the Random Finite Element Method (RFEM), discussed next.

2.4 Random Finite Element Method

Application of random fields to the slope stability problem has been implemented and extensively investigated by Griffiths and Fenton (2000; 2004). These authors developed a computer program, RSLOPE2D, which combines the eight-nodal finite element model of Griffiths and Lane (1999) and Smith and Griffiths (1988; 1998), with random field simulation (Fenton and Vanmarcke 1990) to realistically account for spatial variability in the soil strength parameters and to allow the failure mechanism to naturally seek out the weakest failure path. In the finite element model slope failure is defined as when the finite element analysis fails to converge within 500 iterations (Griffiths and Fenton 2000, see also Figure 10 in Griffiths and Fenton 2004 which shows that 500 iterations is more than enough to identify slope failure). Two random fields, one each for c and $\tan \phi$, are mapped to a finite-element mesh used to discretize a slope, as illustrated in Figure 2, so that each element within the mesh has associated random variables for c and $\tan \phi$. The size of the elements the random variables are associated with is accounted for by means of the local average subdivision (LAS) algorithm developed by Fenton and Vanmarcke (1990). The random variables are correlated with one another according to the spatial correlation length θ , in accordance with eq. (2). From Figure 2, it may be observed that the homogeneous portions of the soil mass grow as $\theta \rightarrow \infty$, and, consequently, the SRV

approach is a special case of the RFEM. In Griffiths and Fenton (2000), the influence of the correlation length was studied by comparing the RFEM to the SRV approach for cohesive soils. It was found that for $\nu_c < 0.5$ the probability of slope failure increased as the ratio θ/H increased, indicating that the SRV approach is generally conservative, especially at lower failure probabilities. Ignoring the beneficial effects of spatial variability thus generally leads to unnecessarily expensive designs.

3. Validation of Seismic Random Field Model

3.1 General

Considering the pseudo-dynamic analysis method (see, e.g., Coduto et al. 2011), the RSLOPE2D model developed by Griffiths and Fenton (2000; 2004) was modified to include a single constant, destabilizing (acting in the direction of slope failure), horizontal force representative of seismic acceleration. The horizontal acceleration is characterized by a seismic coefficient, k , which is a fraction of gravity, g , as an input parameter to the RSLOPE2D model. The final program, now modified to handle seismic accelerations in a 2-D slope mass, was renamed RSLOPE2A.

Validation of RSLOPE2A was started by conducting a series of deterministic seismic analyses and comparing to existing studies (Leshchinsky and San 1994; Michalowski 2002; Loukidis et al. 2003; Baker et al. 2006). The parameters used in the validation study are shown in Table 1. The word “deterministic” in this paper means non-random, spatially uniform, soil properties held at the mean values, or, in this section, at the values specified in Table 1. It is further noted that only deterministic results are available in the literature with which the RSLOPE2A model can be validated. It is a reasonable assumption that if the finite element model is accurate for spatially constant soil properties, that it will also be reasonably accurate

for spatially variable soil properties. The choice of element size used in this paper (see, e.g., Figure 2), relative to the slope dimension, has been found by the authors over the years to provide reasonably good representation of the slope's response to spatially variable soils.

In Table 1 the value of c corresponding to a prescribed value of λ is obtained by solving the stability factor equation,

$$\lambda = \frac{c}{\gamma H \tan \phi} \quad (7)$$

for $c = \lambda \gamma H \tan \phi$.

For each combination of parameters, RSLOPE2A was run using spatially constant soil properties according to Table 1, starting from the static case ($k = 0$) and increasing the seismic coefficient, k , in steps of 0.01 until the seismic factor of safety was equal to unity ($F_s = 1$) at which point the slope was assumed to have failed. The k value corresponding to the failed $F_s \leq 1$ condition is the critical seismic coefficient, k_c , for that particular combination of stability factor, λ , friction angle, ϕ , and slope angle, β . At k_c , the slope is in a state of limit equilibrium brought on by the combination of vertical and pseudo-dynamic seismic loading. Values of the critical seismic coefficient can then be plotted against the stability factor values to compare to similar plots in other studies.

3.2 Validation of the Seismic Model

Critical seismic coefficients (k_c) can be plotted against the stability factor, λ , to obtain stability charts such as the one displayed in Figure 3. The format of the stability chart in Figure 3 is modeled after Loukidis et al. (2003), where for a particular slope angle, curves for the critical seismic coefficient are plotted for a variety of friction angles, ϕ (hereafter referred to as “ ϕ -curves”). The complete collection of “ ϕ -curve” stability charts from this study may be found in Burgess (2016) for $\beta = 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ, 45^\circ$, and 50° .

To determine the stability of a slope, one first uses eq. (7) to solve for λ using the soil strength characteristics and slope height. If the point (k, λ) is above and/or to the left of the k_c “ ϕ -curve” for that particular slope angle, then the slope is considered to be stable under the seismic load (i.e., $F_s > 1$). However, if (k, λ) is to the right and/or below the k_c curve for that particular slope angle, the slope will have failed with $F_s < 1$. To illustrate this, Figure 4 displays two points, A ($k = 0.1, \lambda = 0.3$) and B ($k = 0.3, \lambda = 0.3$), using the $\phi = 15^\circ, 20^\circ, 25^\circ$ curves of Figure 3. Both points A and B will be unstable when the friction angle is $\phi = 15^\circ$, and stable when the friction angle is $\phi = 25^\circ$. When $\phi = 20^\circ$, point A is stable, since A lies above and to the left of the $\phi = 20^\circ$ curve, while point B is unstable since it is to the right and beneath the curve.

Note that the curves in Figures 3 and 4 end to the right at what appear to be arbitrary points. As discussed by Loukidis et al. (2003) these points correspond to a theoretical limit beyond which the entire slope is expected to move as a single mass sliding over its hard base (the hard layer). This block sliding occurs when k exceeds k_{lim} , where:

$$k_{\text{lim}} = \frac{c}{\gamma DH} + \tan \phi \quad (8)$$

Table 2 illustrates the agreement between deterministic runs of RSLOPE2A and other studies. The data, extracted from Figure 3, were compared to those derived by Leshchinsky and San (1994), Loukidis et al. (2003), and Baker et al. (2006). It can be seen in Table 2 that the deterministic analyses performed here consistently reproduce values in existing studies, and therefore RSLOPE2A is in deterministic agreement with other researchers.

3.3 Alternative Representation of “ ϕ -curves”

Figure 5 presents an alternative representation of the “ ϕ -curves” shown in Figures 3 and 4. The approach taken in Figure 5 was to exchange the roles of β and ϕ , generating “ β -curves” for particular ϕ values. An advantage of this presentation is that k_{lim} (see eq. 8), which is independent of β and dependent upon ϕ , can be plotted as a line alongside the “ β -curves” for each ϕ value (see solid straight line bounding the curves on the right). This representation of the data not only eliminates any confusion caused by the curves ending at seemingly arbitrary points, but also allows for the direct check of slope angles to determine stability when the soil strength is known. As before, points above and to the left of a curve indicate slope survival, $F_s > 1$, while those below and to the right indicate slope failure, $F_s < 1$. A full collection of “ β -curve” stability charts may be found in Burgess (2016) for $\phi = 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ$, and 40° and for the full range of β values considered.

4. Probabilistic Seismic Slope Design Aids

4.1 General

Having determined that RSLOPE2A is capable of capturing seismic effects upon slopes having deterministic properties, a parametric study using random field models of the soil over a series of correlation lengths, θ , was performed to examine the probability of failure, p_f , of cohesive-frictional ($c - \phi$) slopes subjected to various degrees of seismic loading. The design aids presented in the next subsection were generated using Monte Carlo simulations based upon the parameters shown in Table 3. Simulations proceeded by producing 2000 realizations of the spatially variable slope and observing the proportion of these realizations which fail ($F_s < 1$) for each combination of the parameters given in Table 3. The soil finite elements were of size $0.1H$ by $0.1H$. Both the cohesion, c , and the tangent of the friction angle, $\tan \phi$, are assumed to be lognormally distributed random fields with the same correlation lengths and approximately the same coefficients of variation. The probability of failure is then estimated by dividing the number of realizations which failed by the total number of realizations. Using 2000 realizations the standard deviation of the estimated probability of failure is $0.022\sqrt{p_f(1-p_f)}$; $0.022\sqrt{p_f}$. For example, if $p_f = 0.001$, then $\sigma_{p_f} = 0.022\sqrt{0.001} = 0.0007$, so that $p_f = 0.001$ is really the lower limit of resolution of 2000 realizations.

The parameters given in Table 3 are similar to those used for the deterministic analysis with the addition of the statistical parameters ν and θ . The slope angles, constant soil parameters, height, and depth factor remain the same as those in the previous analysis. Furthermore, the pore water pressure, u_d , remains ignored. The parameter values that have been changed, or added, are discussed as follows:

- 1) The range in stability factor, λ , values was selected based upon the seismic coefficient, k , values. The k values were chosen to represent the static case ($k = 0$), as well as three design seismic loads within the typical restrictions (see, e.g., Melo and Sharma 2004) set upon pseudo-dynamic analyses ($k \leq 0.3$). The λ values were then selected to include slopes which have some likelihood of failing.
- 2) The mean cohesion, μ_c , values were obtained by solving eq. (7) for c given the particular λ values chosen in Table 3, where c and ϕ are replaced by μ_c and μ_ϕ in eq. (7),
- 3) Selection of the coefficients of variation, v , was roughly based upon the work of Phoon and Kulhawy (1999) who suggest v_c values between 0.1 and 0.5 and v_ϕ values between 0.1 and 0.2; we assume that $v_{\tan \phi}$ has the same range as v_ϕ . In this study, the two separate values of coefficients of variation were taken to be equal with $v_c = v_{\tan \phi} = v$ between 0.1 and 0.3. It should be noted that the v associated with the friction angle in this study is for the distribution of $\tan \phi$. The unit weight, γ , is assumed deterministic and is held constant at the same value selected in the deterministic analysis.
- 4) The normalized correlation length, θ/H , which defines the spatial correlation structure of the soil, is varied over a wide range to investigate whether a “worst-case” correlation length exists. The “worst-case” correlation length would possess the highest slope failure probability.

Note that the maximum seismic coefficient, k , experienced during the target lifetime of a slope is unknown, and is thus a random variable, but assumed to be known in this study. Consequently, the failure probability results presented below are actually conditional

probabilities of failure, given that the selected k value is the maximum experienced during the target lifetime. The actual lifetime failure probability would have to be computed using the total probability theorem over a range of possible k values, weighted by their probability of occurring. The results given below can still be used to determine the total failure probability calculation using the probabilities of occurrence of the k values over the target lifetime. This is a topic of ongoing research by the authors.

4.2 Results

4.2.1 Effects of λ , k , and v

For each correlation length, θ , four probabilistic design aids were generated (one for each value of k considered). In each chart are a collection of twenty curves, five associated with the deterministic factor of safety (one per λ value), and fifteen probability curves (three per λ corresponding to the three v values). The deterministic factor of safety is obtained using a non-random soil field having properties μ_c and $\mu_{\tan \phi}$ everywhere. Because of the finite sample size, the probability estimates show some sample error which was smoothed by fitting the following logistic growth curve to the sample estimates:

$$p_f = 1 - \frac{1}{1 + \exp((\beta - \beta_0) / \kappa)} \quad (9)$$

where β_0 is the slope angle for which the probability of slope failure, p_f , is 50%, and κ governs the steepness of the fitted curve. For a given β_0 , low κ indicates a steep curve, high κ indicates shallow curves. Comparison between the fitted curve (eq. 9) and one set simulation-based estimates is illustrated in Figure 6. The fitted logistic curve can be seen to closely approximate the sample data and has the advantage of smoothing out sampling error, particularly at the small probabilities of interest. Even at small failure probabilities, the fit

appears very reasonable, erroring slightly on the conservative side.

Figures 7 to 10 are probabilistic design aids for $\theta = 0.2H$. For given values of β and $\lambda_i = i/10$, where $i = 1, 2, 3, 4, 5$, these charts provide the deterministic factor of safety evaluated at μ_c and $\mu_{\tan\phi}$ (solid lines) and the failure probabilities, p_f , for $v = 0.1, 0.2$, and 0.3 (dashed lines). The fitted β_0 and κ values, for the $v = 0.3$ case and for each λ value in Figures 7 to 10, are provided in Tables 4 to 7. These values can be used to solve for the slope β value which results in a specified probability of failure by rearranging eq. (9) as follows:

$$\beta = \beta_0 + \kappa \ln \left(\frac{p_f}{1 - p_f} \right) \quad (10)$$

The collection of design aids for alternative correlation lengths can be found in Burgess (2016).

Figures 7 to 10 also illustrate the effects of the stability number, λ , and the seismic coefficient, k , on the probability of slope failure. These parameters have opposing roles in the slope stability problem – as λ increases, the probability of failure, p_f , for a particular slope angle decreases, while as k increases, p_f also increases. In Figures 7 to 10, from left to right, increases in λ are observed to shift the probability of failure curve to higher β values, implying an increase in slope stability and a decreased risk of failure. This is as expected, since for a fixed friction angle and slope height an increase in λ implies an increase in mean cohesive strength. Alternatively, when k is increased from one figure to the next, all of the λ curves are observed to shift to lower β values, which is consistent with the expected loss of stability, and thus increased risk of failure brought on by increased seismic loading. As would be also expected, the deterministic factor of safety is observed to increase for increasing λ and

reduce for increasing k .

The effects of the coefficient of variation, v , can also be seen in Figures 7 to 10. Increasing the value of v both decreases the β_0 value, shifting the p_f curves to the left, and increases the range of β values over which there is a significant risk of failure (shallower curves correspond to an increase in the steepness parameter κ). For a fixed slope angle β , increasing v typically increases the probability of failure, except when p_f is already high (e.g. above about 0.8 for the $\theta = 0.2H$ case).

4.2.2 Effects of θ

Figures 7 to 10 present a set of probabilistic seismic design aids for the $\theta = 0.2H$ case. To determine if there is a “worst-case” θ value, associated with the highest failure probability, Figures 11-13 plot p_f versus θ for the three seismic coefficients ($k = 0.1, 0.2, 0.3$) considered here. The $\theta/H \rightarrow \infty$ case, which yields a SRV Monte Carlo simulation, was found by Burgess (2016) to be closely approximated by the $\theta/H = 10$ case, and so only θ/H values ranging from 0.2 to 10 are shown in these figures. The β_0 and κ values for the curves depicted in Figures 11-13 are given in Table 8.

The p_f curves in Figures 11-13 are observed to flatten as θ/H increases, corresponding to a rise in κ values in Table 8. When slope angles are below β_0 , approximately, all three figures show that longer correlation lengths lead to higher failure probabilities – in which case the worst-case correlation length is infinity (the SRV model). Conversely, when slope angles exceed β_0 , approximately, the opposite is true and shorter correlation lengths lead to higher failure probabilities so that the worst-case correlation length becomes zero. In other

words, longer correlation lengths are conservative for shallower slopes, while shorter correlation lengths are conservative for steeper slopes having high failure probability. For slope angles in the vicinity of β_0 , there will be an intermediate worst-case correlation length. For static loading, this worst-case correlation length is examined more carefully in Zhu et al. (2018). Its nature for pseudo-dynamic loading is a subject for future study.

4.2.3 Effects of $\mu_{\tan\phi}$

Figure 14 illustrates how the probability of failure, p_f , varies with slope angle, β , for mean friction angle values ranging from $\tan 10^\circ$ to $\tan 25^\circ$ in the static case. The corresponding β_0 and κ values are given in Table 9.

Unsurprisingly, changes in $\mu_{\tan\phi}$ have a significant effect upon the probabilistic slope stability curves. Figure 14 illustrates that increasing the mean friction angle $\mu_{\tan\phi}$ results in failure occurring at higher slope angles, as expected. Changes in the mean friction angle have a strong influence on the β_0 value, as observed in Table 9, where the shift in β_0 for each 5° step in $\mu_{\tan\phi}$ is roughly 20° for the combination of parameters examined. There is an observable jump in the κ value, which controls curve steepness, from $\mu_{\tan\phi} = \tan 10^\circ$ to $\mu_{\tan\phi} = \tan 15^\circ$ after which the steepness factor only very slowly rises, with slightly less steep curves, at higher friction angles.

To further illustrate the influence of $\mu_{\tan\phi}$, Figure 9 was repeated except at mean friction angle values of $\tan 15^\circ$, $\tan 25^\circ$, and $\tan 30^\circ$ in Figures 15, 16, and 17, respectively. Due to space constraints, only the $k = 0.2$ seismic loading case is shown here. The results shown in Figure 15 indicate that the lower mean friction angle of $\tan 15^\circ$ results in a significant decrease in slope

stability for the λ values examined. Comparatively, an increase of the mean friction angle to $\tan 25^\circ$ or $\tan 30^\circ$, as shown in Figures 16 and 17, results in a significant increase in slope stability – the larger λ valued slopes are stable for almost all slope angles considered. Corresponding β_0 and κ values for $\nu = 0.3$ in Figures 15, 16, and 17 are shown in Tables 10, 11, and 12, respectively.

5. Example 1: Deterministic Analysis

Geographical hazard maps can be used to determine the strongest seismic coefficient, k , that a slope within a particular region is expected to experience over its design lifetime. Soil samples taken from the site may be analyzed to determine estimates of the soil strength parameters μ_c , μ_ϕ , and γ . Measurements can be taken to determine the approximate slope angle (β) and height (H), and borings taken to estimate the depth to the hard layer (DH). Based upon the soil strength parameters and the height of the slope, the stability factor, λ , can be calculated. The seismic coefficients obtained from the hazard maps are compared to the critical seismic coefficient, k_c , obtained from the pseudo-dynamic stability charts for the particular λ , β , and $\mu_{\tan \phi}$ values. The slope remains stable under the expected seismic loading if $k < k_c$ (indicating $F_s \geq 1.0$).

Consider a slope, having the parameters outlined in Table 13, and assume that it is expected to experience a maximum seismic event with $k = 0.2$ sometime during its design lifetime. Evaluation of the slope's stability, using the pseudo-dynamic slope stability design aids shown above, proceeds as follows:

- 1) The depth factor, D , is simply taken as the depth to the hard layer divided by the height of the slope, $D = 10 \text{ m} / 5 \text{ m} = 2$. The stability factor, λ , is determined by eq. (7), using

mean values, as follows:

$$\lambda = \frac{\mu_c}{\gamma H \mu_{\tan \phi}} = \frac{13 \text{ kN/m}^2}{(18 \text{ kN/m}^2)(5 \text{ m})(\tan 20^\circ)} \approx 0.4$$

- 2) The deterministic factor of safety curve in Figure 7 ($k = 0$, or static loading) for $\lambda = 0.4$ and $\beta = 55^\circ$ was used to determine a static factor of safety of $F = 1.37$ for the example slope described in Table 13. This factor of safety indicates a statically stable slope, but one which also falls within the range that can be investigated using a pseudo-dynamic analysis, according to Hynes-Griffin and Franklin (1984).
- 3) Since $\mu_{\tan \phi} = \tan 20^\circ$, the deterministic pseudo-dynamic slope stability chart in Figure 5 is consulted to determine if the slope remains stable under seismic loading. For $\beta = 55^\circ$, a slope with $\lambda = 0.4$ has $k_c \approx 0.25$. Since $k < k_c$ the slope is considered to remain stable by the pseudo-dynamic analysis.

6. Example 2: Probabilistic Analysis

Now let c and $\tan \phi$ be random fields having lognormal distributions with means μ_c and $\mu_{\tan \phi}$, standard deviations σ_c and $\sigma_{\tan \phi}$, and correlation length θ . If the correlation length is $\theta \approx \theta_{\ln c} \approx \theta_{\ln(\tan \phi)} \approx 0.2H$ then Figure 18 can be constructed as a modified version of Figure 9, pertaining to slopes having correlation $\theta / H \approx 0.2$, mean $\mu_{\tan \phi} = \tan 20^\circ$, $k = 0.2$, and displaying only the curves where $\lambda = 0.4$.

For the example slope discussed in the previous section having $\beta = 55^\circ$ (see Table 13), Figure 18 shows a seismic factor of safety $F_s = 1.06$. The pseudo-dynamic factor of safety is

lower than the static $F = 1.37$ found in the previous section, as expected, but is greater than 1.0, which makes sense since $k < k_c$ (where k_c is approximately 0.25 for this example, as noted in the previous section). Despite the fact that $F_s > 1.0$, the probability of slope failure ranges from about 2% to 48% for ν ranging from 0.1 to 0.3. Probabilities of failure of such magnitude are typically unacceptable, except perhaps where the failure of the slope would have no significant consequences. In many cases, this example slope would be considered too unsafe and must be remediated by excavating to a shallower slope angle, if this is an existing slope, or by using a stronger fill material, if this is a constructed slope. How the probability of failure is reduced by reducing the slope angle is illustrated in Figure 19. For example, slope angles of 50° , 45° , and 40° (with static factors of safety $F = 1.46$, 1.57 , and 1.69 , respectively) have failure probabilities of 16.8%, 4.3%, and 1.0%, respectively. If a target failure probability of at most $p_f \leq 1\%$ is desired, given that a maximum earthquake event having $k = 0.2$ occurs during the target lifetime, remediation would require that the slope angle be reduced to $\beta \leq 40^\circ$.

6.1 Comparison to the SRV approach

Suppose that, instead of using the estimate of the correlation length in the previous subsection, the described slope was investigated using the SRV approach ($\theta/H \rightarrow \infty$). Here, the condition that $\theta/H \rightarrow \infty$ is roughly approximated by the $\theta/H = 10.0$ case, as shown to be a reasonable approximation by Burgess (2016). Figure 20 gives the deterministic factors of safety and the probabilities of failure over the same slope angles considered in Figure 19.

Figure 20 displays a flatter probability of failure curve than the one seen in Figure 19, while the corresponding deterministic pseudo-dynamic factor of safety curve remains

unchanged. Since F_s is a deterministic quantity evaluated at the mean values of the soil strength parameters, neglecting spatial variability, Figures 19 and 20 are expected to provide the same F_s values. A comparison of the probabilities of failure of the $\theta/H = 0.2$ case with the $\theta/H \rightarrow \infty$ case is shown in Table 14 for $\nu = 0.3$. When $\beta \geq 55^\circ$, p_f is lower under the SRV approach ($\theta/H = 10$) than when $\theta = 0.2H$, which is unconservative. However, p_f becomes significantly higher under the SRV case than under the $\theta = 0.2H$ case as β is reduced. The SRV appears to yield a very conservative estimate of the probability of slope failure when failure probability is less than about 0.4.

6.2 Cost analysis

To further expand upon the application of the probabilistic seismic slope design aids, suppose the objective of the analysis in the previous example is to remediate a slope, such as the one displayed in Figure 21, which is assumed to extend 100 m into the page. Remediation of the slope requires excavation of the slope from its initial slope angle, $\beta_1 = 55^\circ$ to a reduced slope angle, β_2 , so as to achieve a target maximum probability of failure, p_m . A comparison is made between the RFEM and SRV approaches. It will be assumed that the soil strength characteristics ($\mu_c, \mu_{\tan\phi}, \sigma_c, \sigma_{\tan\phi}$) can be estimated from two CPT borings of depth 10 m, and that this is sufficient information to conduct the SRV analysis. For the RFEM analysis the additional parameter θ the correlation length, needs also to be estimated. In this example it was assumed that taking 10 CPT borings would be sufficient to estimate θ , at least in the vertical direction. Ten CPT borings are probably not sufficient to accurately estimate horizontal correlation length – at most sites the horizontal correlation length would be estimated to be some

multiple of the vertical correlation length. In this simple example, the correlation length is assumed to be isotropic. A cost of \$50 per meter depth of CPT borings was assumed in this example. The cost of sampling is thus $C_s = \$50 / \text{m} * (\text{number of borings}) * (10\text{m depth/boring})$, so that $C_s = \$1000$ for two borings (SRV model) and $C_s = \$5000$ for ten borings (RFEM model).

Required remediation slope angles, β_2 , are obtained from the probabilistic seismic slope design aids (see Figures 11-13) for $\theta / H = 0.2$, $\theta / H = 2.0$ and $\theta / H = 10$. It should be noted that the probabilistic seismic slope design aids are not extended to slopes having angles less than $\beta = 10^\circ$; for slope angles that are less than 10° , eq. (9) was used to estimate the probability of failure, using the β_0 and κ values from Table 8. The required β_2 values are displayed in Table 15 for various target probabilities of failure, P_m .

The average cost of excavating soil in 2016, according to the California Department of Transportation Highway Price Index (CalTrans 2016), was stated to be \$27.60/m³. Using this value, the excavation cost for 100m length of slope is then:

$$C_R = \text{area}(\beta_1, \beta_2, H) * 100\text{m} * \$27.60 / \text{m}^3 \quad (11)$$

where $\text{area}(\beta_1, \beta_2, H)$ represents the area of the shaded region in Figure 21. Table 16 displays the total calculated costs for the SRV and RFEM approaches.

The results of the cost analysis in Table 16 show that considering spatial variation can lead to significant cost savings when the correlation is found to be smaller than the $\theta / H \rightarrow \infty$ assumption made by the SRV case. Furthermore, the savings of the RFEM case would still be considerable even if the number of CPT borings was increased to, say, 40 in order to more

accurately estimate the horizontal correlation length. In this case, C_s only increases to (40 borings) * (10 m depth/boring) * (\$50/m) = \$20,000.

6.3 Risk-based Design

To investigate the use of probabilistic seismic design aids for risk-based design, consider again the $\beta = 55^\circ$ slope described by Table 13. Suppose that the slope has a cost of failure of \$1,000,000 if it were to collapse and has a design target lifetime of 50 years. If records of previous seismic events indicated that $k = 0.1$ earthquake events occur roughly once every ten years, $k = 0.2$ events occur once every fifty years, and $k = 0.3$ events occur once every two-hundred and fifty years, then the expected number of seismic events, $E[N_{e,k}]$, where $N_{e,k}$ is the number of earthquakes of intensity k , during the design life is 5, 1, and 0.2 for $k = 0.1, 0.2$ and 0.3 respectively. The expected number of failures, $N_{f,k}$, at each intensity k is thus:

$$E[N_{f,k}] = E[N_{e,k}] p_{f,k} \quad (12)$$

where $p_{f,k}$ is the probability of slope failure under seismic loading k . The total expected cost of failure, $E[C_{f,k}]$, for the “do-nothing” case (no remediation of the slope, $C_R = 0$) is presented in Table 17 for $k = 0.1, 0.2$, and 0.3 according to:

$$E[C_{f,k}] = E[N_{f,k}] * \$1,000,000 + C_R + C_s = \$1,000,000 * E[N_{e,k}] p_{f,k} + C_R + C_s \quad (13)$$

where $C_R = 0$, and C_s is the cost of site investigation (assumed to be \$1,000 for the $\theta/H \rightarrow \infty$ case, and \$5,000 for other θ/H values).

Table 17 shows that the $\theta/H = 10$ case predicts a lower expected total cost of failure

when $k > 0.1$ and $p_{f,k} > 0.4$. This is because when the failure probability is high, the SRV case is unconservative. Since it is the goal of the geotechnical engineer to reduce the probability of slope failure to a more reasonable target value, such as $p_m = 0.01$, then Table 17 suggests that excavation of the slope should be considered. Table 18 gives the total expected cost of failure after excavation of the slope to an angle β_2 , where β_2 is the slope angle required to reduce the maximum acceptable probability of slope failure to $p_m = 0.01$. The total expected cost, $E[C_{f,k}]$, after remediation, is found from eq. (13) with C_R evaluated from eq. (11).

Clearly indicated in Table 18 is the significantly lower total expected costs for the RFEM approach, where spatial variability is accounted for, compared to the SRV approach, where spatial variability is not accounted for. Table 18 shows that both non-infinite correlation lengths exhibited cost savings over the SRV case for the lower seismic events. The $k = 0.3$ case indicated that the target probability of failure, p_m could not be achieved for both the $\theta/H = 2$ and SRV scenarios, and the difference in their $E[C_{f,k}]$ was simply the difference in their C_s value. When $\theta/H = 0.2$ the additional sampling cost compared to the SRV approach is well worth spending, thanks to the reduced excavation costs required to remediate the slope.

7. Conclusions

The seismic slope stability charts developed in this paper are applicable to cohesive-frictional slopes subjected to seismic loading. These design aids were developed using the random finite element method (RFEM) and built upon the RSLOPE2D program developed by Griffiths and Fenton (2000; 2004). Two examples were provided to illustrate the use of the design aids. The probabilistic seismic slope stability design aids presented provide geotechnical engineers with an effective alternative to computer-based analyses.

The findings of the cost analysis and risk-based design indicate the potential for significant savings if spatial variability is properly considered. The paper shows that the assumption $\theta / H \rightarrow \infty$ adopted by the SRV approach can lead to very conservative estimates of slope failure probability for flatter slopes with lower p_f . For steeper slopes with higher p_f however, the SRV approach can be unconservative. The paper also shows that a relatively low cost of additional sampling can lead to significant remediation cost savings when the spatial correlation length of the soil, θ , is estimated more accurately and not just assumed to be infinite.

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List of Symbols

CPT	Cone penetration test for in-situ soil testing
LAS	Local average subdivisions
RFEM	Random finite element method
SRV	Single random variable approach
c	cohesion
C_R	cost of remediation through excavation
C_S	cost of sampling
D	depth factor
DH	depth to hard layer
$E[C_{f,k}]$	total expected cost of failure for a seismic event of intensity k
$E[N_{f,k}]$	expected number of failures for a seismic event of intensity k
$E[N_{e,k}]$	expected number of earthquake events of intensity level k
F	static factor of safety
F_S	seismic factor of safety
g	gravity
H	height of the slope
k	seismic coefficient
k_c	critical seismic coefficient
k_{lim}	theoretical limiting seismic coefficient
n	size of a random field
p_f	probability of failure
$p_{f,k}$	probability of failure for a seismic event of intensity k
p_m	target probability of failure
t_i	spatial positions in a random field. $i = 1, 2, \dots, a, b, n$
u_d	dynamic pore-water pressure
v	coefficient of variation
v_c	coefficient of variation for cohesion, c
v_x	coefficient of variation for a variable/parameter ' x '
v_γ	coefficient of variation for the unit weight, γ
v_ϕ	coefficient of variation for the internal friction angle, ϕ

$v_{\tan \phi}$	coefficient of variation for the tangent of ϕ
$X(t)$	a lognormally distributed random field
X_i	random variable at position t_i
$Y(t)$	a Gaussian random field
β	slope angle
β_0	slope angle at which $p_f = 50\%$
β_1	initial slope angle
β_2	remediated slope angle
γ	unit weight
θ	correlation length
θ_i	correlation length in direction i
$\theta_{\ln c}$	log-space correlation length for cohesion
$\theta_{\ln(\tan \phi)}$	log-space correlation length for $\tan \phi$
$\theta_{\ln x}$	log-space correlation length for a variable/parameter 'x'
κ	steepness factor
λ	stability factor
μ_c	mean cohesion
$\mu_{\ln x}$	mean value of a lognormal variable/parameter 'x'
$\mu_{\tan \phi}$	mean of $\tan \phi$
$\rho(\tau)$	correlation structure
σ_c	standard deviation of cohesion
$\sigma_{\tan \phi}$	standard deviation of $\tan \phi$
$\sigma_{\ln x}$	standard deviation of a lognormal variable/parameter 'x'
τ	spacing between positions
τ_i	spacing between positions in direction i
ϕ	friction angle

Figure Captions

- Figure 1** Sample geometry used in slope analysis
- Figure 2** Sample random field realizations for slopes with a) short correlation length, and b) longer correlation length
- Figure 3** Critical seismic coefficients, k_c , lie along the “ ϕ -curves” above for a $\beta = 45^\circ$ slope
- Figure 4** Slope stability of slopes A and B according to the “ ϕ -curves” from Figure 3
- Figure 5** Critical seismic coefficients, k_c , for slopes with $\phi = 20^\circ$
- Figure 6** Comparison of the simulation-based failure probability estimates to the fitted curves (eq. 9) for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.2$, $\nu = 0.2$, $\theta = 0.2H$, and $\lambda_i = i/10$.
- Figure 7** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_S .
- Figure 8** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.1$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_S .
- Figure 9** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.2$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_S .
- Figure 10** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.3$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_S .
- Figure 11** Influence of correlation length on the probability of slope failure for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.1$, $\lambda = 0.4$, $\nu = 0.3$
- Figure 12** Influence of correlation length on the probability of slope failure for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.2$, $\lambda = 0.4$, $\nu = 0.3$
- Figure 13** Influence of correlation length on the probability of slope failure for $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.3$, $\lambda = 0.4$, $\nu = 0.3$

- Figure 14** Influence of mean friction angle on the probability of slope failure for $\theta = 0.2H$, $k = 0$ (static case), and $\lambda = 0.3$
- Figure 15** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 15^\circ$, $k = 0.2$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_s .
- Figure 16** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 25^\circ$, $k = 0.2$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_s .
- Figure 17** Probabilistic pseudo-dynamic slope stability design aid for $\mu_{\tan\phi} = \tan 30^\circ$, $k = 0.2$, $\theta = 0.2H$, and $\lambda_i = i/10$. Solid lines plot F_s .
- Figure 18** Probabilistic pseudo-dynamic slope stability design aid for a slope having $\theta/H = 0.2$, $\mu_{\tan\phi} = \tan 20^\circ$, $k = 0.2$, and $\lambda = 0.4$. The sloped solid line and curved dashed lines are from Figure 9. Horizontal lines show the failure probabilities for various coefficients of variation and the deterministic F_s value corresponding to $\beta = 55^\circ$.
- Figure 19** Deterministic factors of safety, F_s , and failure probabilities, p_f , versus slope angle for the example problem with $\nu = 0.3$, $k = 0.2$, and $\theta = 0.2H$.
- Figure 20** Deterministic factors of safety, F_s , and failure probabilities, p_f , versus slope angle for the example problem with $\nu = 0.3$, $k = 0.2$, and $\theta = 10H$ case (approximately SRV).
- Figure 21** Reduction of a slope from slope angle β_1 to β_2 through excavation of the shaded region

Table 1 Input parameters used in the validation of RSLOPE2A

Parameter	Values Considered
Slope angle, β	10° to 85°
Friction angle, ϕ	10°, 15°, 20°, 25°, 30°, 35°, 40°
Stability factor, λ	0 to 1
Cohesion, c	determined by λ
Seismic coefficient, k	0 to 1
Poisson's ratio	0.3
Young's Modulus	10 ⁵ kN/m ²
Unit weight, γ	18 kN/m ³
Height, H	5 m
Depth factor, D	2

Table 2 Comparison of k_c from Figure 3 with k_c from other studies

λ	ϕ	k_c			
		Figure 3	Leshchinsky & San	Loukidis et al.	Baker et al.
0.10	25°	0.00	0.02	0.01	0.00
0.10	30°	0.14	0.15	0.15	0.14
0.10	40°	0.37	>0.25	0.39	0.40
0.20	25°	0.21	0.25	0.22	0.25
0.20	30°	0.36	>0.25	0.37	0.36
0.30	20°	0.19	0.20	0.20	0.20
0.35	15°	0.05	0.05	0.05	0.05
0.35	20°	0.26	>0.25	0.27	0.27
0.40	20°	0.32	>0.25	>0.30	0.03
0.50	15°	0.20	0.20	0.20	0.20
0.75	15°	0.37	>0.25	0.37	0.39
1.00	10°	0.24	0.25	0.24	0.25

Draft

Table 3 Parameters used in the parametric study

Parameter	Values Considered
Slope angle, β	10° to 85°
Mean friction angle, $\mu_{\tan \phi}$	$\tan 10^\circ$ to $\tan 30^\circ$
Stability factor, λ	0.1, 0.2, 0.3, 0.4, 0.5
Mean cohesion, μ_c	determined by λ
Coefficient of variation, ν	0.1, 0.2, 0.3
Seismic coefficient, k	0, 0.1, 0.2, 0.3
Poisson's ratio	0.3
Young's Modulus	10^5 kN/m ²
Unit weight, γ	18 kN/m ³
Height, H	5 m
Depth factor, D	2
Correlation, θ / H	0.1, 0.2, 0.3, 0.5, 0.7, 1.0, 2.0, 3.0, 5.0, 7.0, 10.

Draft

Table 4 β_0 and κ values for the $\nu = 0.3$ curves in Figure 7 ($k = 0$)

λ	β_0	κ
0.1	32.0030°	1.39905
0.2	45.9141°	2.19347
0.3	61.2197°	2.50769
0.4	75.2870°	2.81716
0.5	N/A	N/A

Draft

Table 5 β_0 and κ values for the $\nu = 0.3$ curves in Figure 8 ($k = 0.1$)

λ	β_0	κ
0.1	23.2063°	1.14525
0.2	35.1545°	2.12784
0.3	51.6128°	2.83533
0.4	66.6297°	2.96735
0.5	79.4626°	3.23835

Draft

Table 6 β_0 and κ values for the $\nu = 0.3$ curves in Figure 9 ($k = 0.2$)

λ	β_0	κ
0.1	13.6653°	0.83178
0.2	24.0265°	1.73406
0.3	38.6696°	2.50769
0.4	55.3294°	3.33257
0.5	69.9127°	3.32908

Draft

Table 7 β_0 and κ values for the $\nu = 0.3$ curves in Figure 10 ($k = 0.3$)

λ	β_0	κ
0.1	N/A	N/A
0.2	N/A	N/A
0.3	20.8694°	2.46922
0.4	39.7755°	3.45482
0.5	57.3369°	3.90297

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Table 8 β_0 and κ values for the curves in Figures 11-13

θ / H	Figure 11		Figure 12		Figure 13	
	β_0	κ	β_0	κ	β_0	κ
0.2	66.6297	2.96735	55.3294	3.33257	39.7755	3.45482
1.0	66.8617	6.69605	55.9982	7.65354	40.2951	9.01331
2.0	67.2043	8.39566	56.1372	9.51748	40.7369	11.0862
5.0	68.0272	9.63529	57.0866	10.8158	41.1308	13.5800
∞	67.0964	10.1903	56.8543	10.9420	42.2103	14.3812

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Table 9 β_0 and κ values for the curves in Figure 14 ($\nu = 0.3$)

$\mu_{\tan \phi}$	β_0	κ
tan 10°	20.4977°	0.77708
tan 15°	40.1144°	2.24544
tan 20°	61.5054°	2.50536
tan 25°	78.7003°	2.83115
tan 30°	N/A	N/A

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Table 10 β_0 and κ values for the $\nu = 0.3$ curves in Figure 15

λ	β_0	κ
0.1	N/A	N/A
0.2	N/A	N/A
0.3	13.4172°	1.13611
0.4	22.8955°	1.97247
0.5	36.5580°	3.09388

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Table 11 β_0 and κ values for the $\nu = 0.3$ curves in Figure 16

λ	β_0	κ
0.1	24.5847°	1.52871
0.2	41.1284°	2.83948
0.3	61.2667°	3.07380
0.4	77.0748°	3.06750
0.5	N/A	N/A

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Table 12 β_0 and κ values for the $\nu = 0.3$ curves in Figure 17

λ	β_0	κ
0.1	36.1687°	2.15009
0.2	58.5937°	2.90434
0.3	77.8568°	2.96345
0.4	N/A	N/A
0.5	N/A	N/A

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Table 13 Input parameters for an example slope subjected to seismic loading

Parameters	Values Considered
Slope angle, β	55°
Mean friction angle, $\mu_{\tan \phi}$	$\tan 20^\circ$
Mean cohesion, μ_c	13 kN/m ²
Poisson's ratio	0.3
Young's Modulus	10 ⁵ kN/m ²
Unit weight, γ	18 kN/m ³
Height, H	5 m
Depth to hard layer	10 m

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Table 14 Probabilities of slope failure for the example slope with $\nu = 0.3$ when $\theta / H = 0.2$ and $\theta / H = 10$ (approximately SRV)

β	P_f	
	$\theta / H = 0.2$	$\theta / H = 10$
65°	0.9479	0.6780
60°	0.8024	0.5714
55°	0.4753	0.4577
50°	0.1681	0.3480
45°	0.0431	0.2525
40°	0.0100	0.1760

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Table 15 Excavation slope angles (β_2) required to achieve specified P_m values for $\theta/H = 0.2$ and $\theta/H = 10$ (approximately SRV)

P_m	β_2	
	$\theta/H = 0.2$	$\theta/H = 10$
0.1000	48°	32°
0.0100	40°	6°
0.0010	32°	<1°
0.0001	25°	<1°

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Table 16 Cost of excavation and sampling for the $\theta / H = 0.2$ (RFEM) and $\theta / H = 10$ (approximately SRV) approaches at specific p_m values

p_m	$C_R + C_s$	
	$\theta / H = 0.2$	$\theta / H = 10$
0.1000	\$11,906	\$32,054
0.0100	\$21,958	\$304,088
0.0010	\$36,054	>\$1,953,346
0.0001	\$54,828	>\$1,953,346

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Table 17 Expected cost of failure for $\theta / H = 0.2$, $\theta / H = 2.0$ and $\theta / H = 10$ (approximately SRV) at specific k values for the “do-nothing” scenario

k	$\theta / H = 0.2$		$\theta / H = 2.0$		$\theta / H = 10$	
	$p_{f,k}$	$E[C_{f,k}]$	$p_{f,k}$	$E[C_{f,k}]$	$p_{f,k}$	$E[C_{f,k}]$
0.1	0.020	\$105,000	0.189	\$951,700	0.234	\$1,170,000
0.2	0.475	\$480,300	0.470	\$474,900	0.458	\$458,700
0.3	0.988	\$202,600	0.958	\$196,360	0.709	\$142,740

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Table 18 Total expected costs for $\theta / H = 0.2$, $\theta / H = 2.0$, and $\theta / H \rightarrow \infty$ with excavation to angle β_2 for $p_m = 0.01$

k	$\theta / H = 0.2$		$\theta / H = 2.0$		$\theta / H \rightarrow \infty$	
	β_2	$E[C_{f,k}]$	β_2	$E[C_{f,k}]$	β_2	$E[C_{f,k}]$
0.1	53°	\$56,840	28°	\$95,727	25°	\$100,828
0.2	40°	\$31,958	12°	\$153,152	6°	\$315,088
0.3	24°	\$60,331	<1°	>\$1,959,346	<1°	>\$1,955,346

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p_f