Metamaterial Devices for Wavefront Manipulation from Microwave to Optical Frequencies

by

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Abstract

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This thesis presents highly efficient metamaterial devices for wavefront manipulation from microwave to optical frequencies. In this framework, compact and efficient quarter-wave and half-wave plates for long-wave infrared applications are designed based on elliptical antenna array sheets. Half-wave plate unit cells are used in conjunction with the Pancharatnam-Berry (PB) phase principle for efficient refraction, focusing and polarization discrimination. To avoid high losses introduced by metal in the near-infrared regime, an all-dielectric metalens is proposed. The metalenses comprises an array of tapered nano-holes etched into the indium phosphide (InP) substrate. This tapering of the nano-holes acts as a graded-index matching layer, resulting in metalenses with near unity transmission from 0.9\,\mu m to 1.7\,\mu m. Furthermore, the hole array concept is adapted to designing a grade-index (GRIN) lens at millimeter-wave frequencies. GRIN matching layers are proposed to match the GRIN lens. The optimal permittivity in the GRIN matching layers is calculated through the transfer-matrix method. The resulting matched GRIN lens has very high radiation efficiency. In this thesis, anti-reflection or impedance matching with anisotropic metamaterials is also investigated. With specific material tensors, an anisotropic matching layer can be used to match an arbitrary substrate to free space at an arbitrary incident angle. Realistic metamaterial structures are proposed at microwaves to achieve the required material parameter tensors and perfect matching is demonstrated for either TE or TM polarization at a near grazing angle of 88°. To match TE and TM polarizations simultaneously, a magneto-electric uniaxial matching layer (MEUML) is proposed and matching is demonstrated at 45°. The MEUML is applied to a sandwich radome design at X-band and the resulting radome has an exceptional angular performance. Lastly, a single-layer metamaterial radome is designed at 34.3 GHz. Two coupled metasurfaces are patterned on a regular dielectric substrate to transform it into a homogenized metamaterial slab. By tuning the metasurfaces and their coupling, the metamaterial slab exhibits equal effective permittivity and permeability. The resulting structure is impedance matched to free space. The single layer metamaterial radome is polarization-insensitive, low loss, broadband, and easy to fabricate, making it attractive for many millimeter-wave applications.
Dedication

To my lovely wife:
Guo
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This thesis is not possible without the help from many people, to whom I am forever indebted.

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Chapter 1

Introduction

1.1 Brief history of metamaterials

Metamaterials are artificially structured materials that can be used to control the propagation of electromagnetic waves. The word “meta” means “beyond” in Greek, hinting that metamaterials exhibit extraordinary electromagnetic properties which are not available or not easily obtainable in natural materials. Similar to natural materials with their atoms arranged on 3-dimensional (3D) lattices, metamaterials are composed by arranging man made building blocks arranged or meta-atoms in 2D or 3D lattices (strictly speaking, a 2D metamaterial is a metasurface). For natural materials, their properties are mainly determined by their chemical constituents. In comparison, the properties of metamaterials are derived both from the inherent properties of the materials comprising each building block, as well as from the geometrical arrangement of these building blocks, which can give rise to structural responses. By carefully crafting the building blocks and their structural arrangement, metamaterials can achieve extraordinary functionalities that were not possible before.

An important contribution for the development of metamaterials originated by Victor Veselago in 1968, who summarized that materials with both negative permittivity and negative permeability are theoretically possible [1]. In 1996, John Pendry showed that metallic mesostructures can possess extremely low frequency plasmons in the microwave regime, which can lead to a negative effective permittivity [2]. Later on, he showed the negative effective permittivity can be achieved by arranging thin wires into a periodic array [3]. In 1999, Pendry also showed that negative magnetic permeability could be achieved using an array of split-ring resonators (SRRs) [4]. Building on this, Smith et al. in 2000 proposed that a composite medium with both negative permittivity and permeability can be achieved by combining the SRRs and the wire array [5]. This composite medium can exhibit the interesting property of negative refraction [6], where light rays are refracted at the medium interface with a negative angle (for natural materials, the refraction angle is a positive value). By building such a composite medium with an array of SRRs and copper strips as shown in Fig. 1.1, Shelby et al. demonstrated negative refraction experimentally for the first time at microwave frequencies [7]. The uncommon electromagnetic response of metamaterials generated a lot interest in the scientific community and lead to many remarkable scientific findings over the past twenty years.
Chapter 1. Introduction

1.2 Applications

A great body of scientific papers and books has been published on various designs of 3D metamaterials or 2D metasurfaces [8–17]. To give readers a flavour of these designs and their applications, here we briefly discuss a few examples. One of the earliest application of metamaterials is sub-diffraction imaging. In [6], Pendry suggested that a perfect lens can be constructed with metamaterials having a negative index. In the perfect lens, both propagating and evanescent waves contribute to the resolution of the image, which theoretically should lead to a perfect reconstruction of the object. As a result, the perfect lens can resolve features well below the diffraction limit. The sub-diffraction imaging was experimentally demonstrated with a planar version of the lens [18], which is implemented with transmission lines as shown Fig. 1.2a.

Another application is electromagnetic cloaking [19, 20], in which the concept of metamaterials is combined elegantly with transformation optics. When Maxwell’s equations are transformed to an arbitrary coordinate system, it was found that the material parameter tensors have the same effect as the geometrical properties given by the metric tensor. Hence, the material parameter tensors can be used to mimic the effect of wave propagation under a transformed coordinate system. In the cloaking example shown in Fig. 1.2b, an annular region is undergoing coordinate transformation. A plane wave impinging on the annular region bends internally and circumvent the object placed inside the annular region, and emerges as a plane wave without scattering. Hence, the object is essentially invisible to external observers. One problem encountered by the annular cloak is that after the transformation, the required material parameter tensors may have extreme values and cannot be realized with natural materials. In this case, metamaterials become very handy as these parameter tensors can be realized precisely. In Fig. 1.2b, the cloak is realized with the SRRs, which can achieve the required material parameters tensors along the radial direction and the vertical direction. In addition, the spatial variation of these tensors is also achieved by varying the geometry of the SRRs along the radial direction.

The ability of introducing anisotropy leads to the development of an interesting group of metamaterials called hyperbolic metamaterials (HMM) [21], which have an extreme anisotropy. Such materials have tangential and normal components of the permittivity with opposite signs. For example, if the surface normal of a planar HMM slab is along the z-axis, the $\varepsilon_{zz}$ component can be a positive value whereas $\varepsilon_{xx}$ or $\varepsilon_{yy}$ can be of a negative value, or vice versa. By exploiting the extreme anisotropy,
tapered HMM waveguides were designed for thermal emission engineering [22]. As shown in Fig. 1.2c, a tapered HMM waveguide has been implemented with a stack of alternating metal and dielectric layers. Incident light with different wavelengths is absorbed at different depths along the waveguide. With a total depth around a quarter of the central wavelength of the absorption band, absorption efficiency is higher than 95% from 3 µm to 5.5 µm over a wide angular range.

The metamaterials presented in Fig. 1.2a to Fig. 1.2c can all be considered as volumetric designs since they have appreciable thicknesses. If the thicknesses are sub-wavelength as shown in Fig. 1.2d to Fig. 1.2f, then these designs are considered as 2D metamaterials, or metasurfaces. Fig. 1.2d shows an inhomogeneous metasurface comprising an array of spatially varying V-shaped nano-antennas [23]. Each nano-antenna is designed to stipulate a specific local phase delay. By arranging the nano-antennas in a specific pattern, a linear or a parabolic phase profile can be imposed across the metasurface, which consequently functions as a refracting or focusing surface.

The nano-antennas shown in Fig. 1.2d can be considered as small dipoles arranged in a V shape, which primarily respond to electric fields. Fig. 1.2e shows a Huygens’ metasurface [24] that responds to both electric and magnetic fields with its building blocks comprised of co-located small dipoles and loops, which induce orthogonal electric and magnetic currents. Hence, each building block can be considered as a Huygens’ source. By exciting Huygens’ sources with a certain phase profile, an arbitrary wave front can be constructed. In addition, each Huygen’s source is intrinsically matched to free space. The metasurface in [24] refracts a normally incident wave to 45° without reflection in the transmission mode. This is not possible with the nano-antenna metasurface, which is only efficient in the reflection mode.

Fig. 1.2f shows a tensor metasurface [25] which can achieve sophisticated polarization control. The design has two cascaded Huygens’ metasurfaces with one of them rotated around the normal to the surface with respect to the other one. This design behaves as chiral structure which enables linear polarization rotation, i.e, a TE polarization is converted to TM polarization (at normal incidence). Attributed to each Huygen’s metasurface, the design is reflection-less. Furthermore, the tensor metasurfaces can be used for circular-polarization (CP) selection as in [26]. Here, three cascaded metasurfaces enable full transmission of a right-handed CP wave, and full reflection of a left-handed CP wave. From all the examples described above, one can see that metamaterials and metasurfaces are powerful tools for manipulation electromagnetic waves.

1.3 Design Strategies

Metamaterials usually consist of periodically arranged building blocks that have a size and spacing much smaller than the wavelengths of incoming electromagnetic radiation. Consequently, the microscopic details of these individual building blocks cannot be resolved by the wave. On a macroscopic level, the assemblage of these discrete building blocks is approximated as a continuous substance, which can be characterized with macroscopic material properties such as permittivity and permeability. Hence, the effort is mainly focused on designing individual building blocks.

The standard design approach of metamaterial building blocks is the unit cell analysis as illustrated in Fig. 1.3. An individual building block is contained inside an air box with sizes equal to the lattice constants. By applying periodic boundary conditions (PBC), the unit cell analysis simulates an infinite-array of identical building blocks. Consequently, the interaction between the building blocks can be taken into account. To obtain the effective permittivity and permeability of the metamaterial, a parameter
retrieval method can be used [27]. This is done by assigning Floquet ports to the top and bottom surfaces of the air box. Exciting the Floquet ports emulates a plane wave incidence, and with a full-wave simulator [28], scattering parameters (S-parameters) can be used for extracting the effective material parameters. For metasurfaces, instead of characterizing the surfaces with effective permittivity and permeability, it is more appropriate to use more pertinent parameters such as effective polarizabilities [29,30] or surface impedances [31,32], which can also be extracted from the S-parameters. For inhomogeneous designs, such as the metasurface with spatially varying nano-antennas in Fig. 1.2d, the unit cell analysis, which assumes the building blocks to be identical, is still an effective tool. Since the size and spacing are usually sub-wavelength, the variation between adjacent building blocks is usually small, and quasi-periodic boundary conditions can be assumed. Hence, for each building block in an inhomogeneous
Figure 1.3: By applying periodic boundary conditions, an infinite array of building blocks can be analyzed with a unit cell that contains a single building block. Floquet ports are used to emulate a plane wave incidence and S-parameter can be extracted for calculating relevant material parameters.

design, it should have similar behaviors as if it is in an infinite array; hence, the unit cell analysis can be applied. The assumption of quasi-periodic boundary conditions has been successfully applied to many metamaterial and metasurface designs.

1.4 Thesis Objectives and Organization

In this thesis, we explore the possibility of applying metamaterial concepts in designing various metamaterial devices for wavefront control in the microwave and optical regimes. In particular, we want to maintain high transmission efficiency while achieving these functionalities. We propose three methods to improve efficiency. The first one is to use a multilayer structure. By having more layers, we have more degrees of freedom in controlling the transmission magnitude and phase. This idea is explored in Chapter 2. The second method is to use only dielectric materials in the design. By eliminating metal, which is quite lossy at optical or millimetre-wave frequencies, high efficiency can be obtained. This idea is explored in Chapter 3 and Chapter 4. The third method is to design impedance matching or anti-reflection layers. Specifically, we want to utilize anisotropy as an additional degree of freedom for designing the matching layer. This idea is explored in Chapter 5, Chapter 6, and Chapter 7. Due to the variety of metamaterial devices presented in this thesis, it is not possible to cover all the background information in this introduction. Hence, a more detailed background is given in the introduction of each chapter. The thesis is organized as follows.

In Chapter 2, metamaterial unit cells based on multilayer elliptical antenna array sheets (AAS) are introduced for designing far infrared quarter-wave and half-wave plates. These wave plates can achieve
polarization control by introducing different phase retardations for different incident polarizations. At the same time, high efficiency can be achieved. Based on the half-wave unit cell, a refraction lens, a focusing lens, and a polarization discriminator for circular polarization are presented.

In Chapter 3, we investigate methods to improve the light collection efficiency for a near-infrared focal plane array, which is an array of light-sensitive pixels placed at the focal plane of the optical imaging lens. For each pixel, since the light-sensitive or the absorption region, which converts incident photons to electrons, does not occupy the entire pixel area, a portion of the incident photons is lost during the collection process. To improve the collection efficiency, a microlens array is proposed to focus the incident photons into the absorption region. The microlenses are further transformed into nano-hole based dielectric metamaterial lenses (metalens) with equivalent focal lengths. Simulations and measurements are used to fully characterize the performance of these metalenses.

Based on the concept of the metalens, a hole-array based grade-index (GRIN) lens for millimeter-wave (mm-wave) applications is presented in Chapter 4. The lens is designed on a high-index substrate, which can result in strong reflections at the lens surfaces due to the impedance mismatch. Furthermore, the GRIN lens cannot be matched with conventional matching layers due to the inhomogeneity in the lens. We propose to match the GRIN lens with GRIN matching layers. The required index distribution in the GRIN matching layers is solved with the transfer-matrix method. The matched GRIN lens is verified with both simulations and measurements.

In Chapter 5, we explore the possibility of designing anti-reflection or impedance matching layers with anisotropic metamaterials. We derive the required permittivity and permeability for matching an arbitrary substrate at an arbitrary incident angle for either transverse-electric (TE) or transverse magnetic (TM) polarization. Realistic layer structures are synthesized to demonstrate perfect matching at extreme angles of 88° in simulations and 60° in measurements.

In Chapter 6, we present an extension to the anisotropic matching layers, which are designed to match either TE or TM polarizations. To achieve polarization-insensitive matching, a magneto-electric uniaxial matching layer (MEUML) with special permittivity and permeability tensors is presented. Perfect matching is demonstrated at 45° for both polarizations. Based on the MEUML layer, a super wide-angle multi-layer radome is designed for maximizing transmission from 0° to 85°.

In Chapter 7, we present a simpler single-layer metamaterial radome (meta-radome) for millimeter-wave applications. This single-layer meta-radome is impedance matched to free space at normal incidence when its tangential permittivity is equal to the tangential permeability, irrespective of its thickness. This meta-radome is realized by transforming an ordinary dielectric substrate into an impedance matched metamaterial slab by patterning two metasurfaces on both sides of the substrate. By utilizing the electric and magnetic coupling between the metasurfaces, the effective permittivity and permeability can become equal. This concept is verified through simulations and measurements.

Chapter 8 briefly concludes the thesis, and outlines the contributions made during the course of this Ph.D. work.
Chapter 2

Infrared Antenna Array Sheets as Wavefront Manipulation Devices

2.1 Introduction

A wave-plate is an essential part of modern optics and photonics applications. A conventional wave-plate is constructed out of a birefringent crystal, for which the index of refraction is different for different orientations of light passing through it [33]. For example, when light polarized in the $x$-axis enters the wave plate, it will exit the wave plate with a different phase retardation to the light polarized in the $y$-axis. By adjusting the thickness of the birefringent crystal, the difference in the phase retardations between the two polarizations can be controlled precisely. The most common types of wave-plates are the quarter-wave plates (QWP) and the half-wave plates (HWP). For the QWP, the phase difference is $90^\circ$ between the two orthogonal polarizations. Thus, the QWP can transform a linearly polarized light into a circularly or elliptically polarized light and vice versa. For the HWP, the phase difference is $180^\circ$ between the two orthogonal polarizations. It rotates the polarization of a linearly polarized light by twice the angle between its optic axis and the initial direction of polarization.

In the long wavelength infrared (LWIR) regime, a quarter-wave plate (QWP) and a half-wave plate (HWP) can find applications in industrial CO$_2$-laser machining to control the polarization state for offering more uniform and efficient cutting [34]. A HWP can also be found in beam splitters for polarization-sensitive imaging applications [35, 36]. A conventional LWIR wave-plate is usually made by polished anisotropic dielectric materials such as cadmium thiogallate with thicknesses in the millimeter range including the mount [37]. The bulky profiles of the conventional wave-plates make them difficult to be integrated into miniaturized photonic devices and sensors. Thus, this has generated an interest in developing thin wave-plates. In this regard, recent research in nano-metallic metamaterial structures has provided a low cost method to fabricate thin wave-plates. Reflective QWPs and HWPs have been introduced in [38–41] and transmissive wave-plates based on single layer nano-metallic structures utilizing meander-lines, strips, slits and corrugated gratings are demonstrated in [42–48]. However, the transmission efficiencies of those designs are low. For example, a classical transmitting single-layer meander-line QWP [42] operating in the range of 8 µm–12 µm has a power transmission efficiency around 20%.

To improve the transmission efficiency of the metamaterial based wave-plates, we propose a multi-layer structure. By having more than one layer, we have more degrees of freedom of controlling the
transmission phase and amplitude. In [49], a multi-layer meander-line QWP was introduced in to improve efficiency from a single layer design, but the total efficiency is found be to less than 70%. To further improve efficiency, we propose to use stacked antenna array sheets (AAS) as the unit cell in our multilayer structure. Each AAS consists of elliptical metallic patches, which in theory should incur less Ohmic loss than the thin traces in the meander-line QWP. The geometry of the AAS is designed based on microwave phase-shifter theory [50]. The AAS based QWP and HWP operate in the LWIR region in the transmission mode with a design frequency of 30 THz (or a design wavelength of 10 µm).

With the designed HWP unit cell, we show that by locally rotating the unit cells, the local transmission phase of the incident circularly polarize wave can be controlled precisely while maintaining a high transmission efficiency. This enables a variety of functionalities such refraction, focusing, and polarization discrimination, which can be very attractive for LWIR imaging applications.

2.2 Design of AAS based QWP and HWP

2.2.1 Theory

Many wave plates based on single layer nano-metallic structures [42, 43, 43–48] work using a similar principle: a QWP is usually constructed with a single AAS, and the unit cell of the AAS comprises two perpendicular detuned dipoles. The longer dipole acts as an inductor and introduces a phase delay in the transmitted field, whereas the shorter dipole acts as a capacitor and introduces a phase advance. Thus, by adjusting the lengths of the two perpendicular dipoles, a 90° phase difference is achieved between two orthogonal polarizations. The AAS is essentially a frequency selective surface (FSS) [51], which is equivalent to a filter and its transmission through it can be described by a corresponding transfer function \( H(s) \). Such an AAS of detuned dipoles has a transfer function equivalent to a FSS modeled as a series resistor-inductor-capacitor (RLC) band-reject filter. An AAS with two perpendicular detuned dipoles arranged in a cross can be modeled with two separate transfer functions, one for each polarization. To introduce a 90° phase difference between two polarizations, transfer function \( H_x(s) \) for the X-polarization has to introduce a +45° degree phase shift and transfer function \( H_y(s) \) for the Y-polarization has to introduce a −45° phase shift, as shown in Fig. 2.1a. \( H_x(s) \) and \( H_y(s) \) are band-reject filters with different rejection frequencies. The magnitudes of \( H_x(s) \) and \( H_y(s) \) are exactly \( 1/\sqrt{2} \) when the corresponding phases are ±45° [52], indicating a total 50% of transmitted power which constitutes the upper bound for this type of single-layer AAS based QWP. Similarly, for a HWP, a ±90° phase shift is required for the two transfer functions, as shown in Fig. 2.1b. However, as the phase difference approaches 180°, the magnitudes of both transfer functions approach zero. In fact, it is not possible to achieve exactly 180° phase difference with a thin single layer AAS made of crossed dipoles. A similar statement also applies to single layer cross-slot designs which can be modeled as band-pass filters.

The transmission limitation of a single layer AAS based QWP or HWP can be improved by using multilayer designs [51, 53]. However, it is desirable to achieve unity transmission while satisfying the required phase shifts in the QWP and HWP. Since the transmission through an AAS or a FSS can be modeled with transmission-line theory [51], we realize that the QWP with unity transmission can be designed based on a microwave phase shifter [50]. The circuit model of such phase shifter based QWP is shown in Fig. 2.2. It consists of two layers of identical AAS, which are modeled as shunt reactances. The AAS are separated by a quarter-wave thick dielectric, which is modeled as a transmission line
Figure 2.1: Magnitude and phase of the transmitted fields of a QWP and a HWP made from single layer AAS of cross dipoles. The intersection of the magnitude curves determines the ideal operating frequency since the same power is transmitted for both polarizations. (a) For a QWP, to achieve a 90° phase difference at the operating frequency, at most 50% power is transmitted for both polarizations. (b) For a HWP, as the phase difference approaches 180°, the transmission magnitudes approach zero.

2.2.2 Design and Simulation of a QWP

For a QWP design, we assume the reactance $B_x$ leads to a +45° phase shift and reactance $B_y$ leads to a −45° phase shift, the normalized values of $b_x$ and $b_y$ are calculated to be +4 and −4. In free space, $B_x$ having an impedance of $Z_0$. Due to the orthogonality between the X-polarization and Y-polarization, we assume each polarization propagates through separate transmission networks without coupling to each other. In other words, each polarization is passed through an individual phase shifter and phase shifts are controlled separately by the shunt reactances associated with each polarization, that are, $jB_x$ and $jB_y$. With appropriate values for $jB_x$ and $jB_y$, we can control the phase difference between the output X-polarized and Y-polarized waves. It should be pointed out that in the proposed transmission-line model, the corresponding transmission-line impedances should be adjusted according to the angle of incidence. For simplicity, we only consider here the case of normal incidence. For unity transmission, the phase difference $\Delta \phi$ between the transmitted $E_x$ and $E_y$ fields can be computed by (2.1) with $b_x$ and $b_y$ normalized with respect to $Z_0$.

$$\Delta \phi = \tan^{-1} \left[ -\frac{b_x + \left(1 - \frac{1}{2}b_x^2\right)}{1 - b_x} \right] - \tan^{-1} \left[ -\frac{b_y + \left(1 - \frac{1}{2}b_y^2\right)}{1 - b_y} \right]$$ (2.1)
is 1500Ω and \( B_y \) is −1500Ω. We can relate \( B_x \) and \( B_y \) to the self-reactance of a dipole and approximate the corresponding dipole lengths [54]. The dipole lengths needed are approximated to be 0.75\( \lambda \) and 0.25\( \lambda \) respectively. Therefore, by using an array of crossed dipoles of lengths 0.75\( \lambda \) and 0.25\( \lambda \) for each layer, a QWP with transmission close to unity can be implemented. Fig. 2.3 shows a double layer AAS design with elliptical patches. The working principle of the patch is the same as for the cross dipoles: Current flow along the two axes \( L_x \) and \( L_y \) resembles current flowing along a long dipole and a short dipole. The elliptical patch design is inspired from the hemispherical dipole as in [54] for increasing the bandwidth. The patch is made of gold of 50 nm thickness. The behavior of the gold is characterized as a Drude-Lorentz model with corresponding parameters [55]. The major axis \( L_x \) is 3.3\( \mu \)m and the minor axis \( L_y \) is 1.3\( \mu \)m. The bottom gold layer sits on a common dielectric substrate intended for LWIR applications such as ZnSe with \( \epsilon_1 = 5.8 \) [56]. Between the bottom and top layer gold, a 1.1\( \mu \)m ZnSe thin film which acts as a quarter-wavelength transmission line at the design frequency of 30 THz can be deposited with the CVD method [57]. The lengths of \( L_x \) and \( L_y \) correspond to 0.77\( \lambda \) and 0.3\( \lambda \) in the dielectric respectively, which match well to the approximated cross dipole lengths of 0.75\( \lambda \) and 0.25\( \lambda \) computed in the previous section. A 1.6\( \mu \)m quarter-wave transformer made of YbF\(_3\) with \( \epsilon_2 = 2.3 \) [58] is deposited on top of another 0.2\( \mu \)m ZnSe to match to free space. The total thickness of the design is 2.9\( \mu \)m.

![Figure 2.2](image1.png)

**Figure 2.2:** The circuit model for a normally incident field \( E_0 \) containing both X and Y polarizations with equal magnitude passing through a 2-layer AAS. The dielectric is modeled as a transmission line with impedance \( Z_0 \) and the AAS layers are modeled as shunt reactances. The AAS layer has an admittance \( jB_x \) for the X-polarized field and \( jB_y \) for the Y-polarized field. Depending on the values of \( jB_x \) and \( jB_y \), a 90° differential phase shift can be achieved with a combined transmission magnitude of unity.

![Figure 2.3](image2.png)

**Figure 2.3:** A unit cell design of the proposed QWP using double layer elliptical patches made of gold with dielectrics of ZnSe with \( \epsilon_1 = 5.8 \) and YbF\(_3\) with \( \epsilon_2 = 2.3 \). The major axis \( L_x \) is 3.3\( \mu \)m, corresponding to 0.77\( \lambda \) in the dielectric; the minor axis \( L_y \) is 1.3\( \mu \)m, corresponding to 0.3\( \lambda \). The values of \( L_x \) and \( L_y \) match the theoretically approximated lengths of 0.75\( \lambda \) and 0.3\( \lambda \).
The simulated result of the transmitted field for each polarization is shown in Fig. 2.4. The transmission magnitudes of both the X and Y components are about 0.95, leading to 90% transmitted power. The axial ratio (AR) and phase difference between the two polarizations are shown in Fig. 2.5. The bandwidth of the QWP can be defined by satisfying both the AR limit and the phase difference limit which are 0.8 – 1.2 and 80° – 100° respectively. This bandwidth of the QWP is 8.6 µm–10.4 µm, or 19% of the design frequency. In Table. 2.1, the AAS based QWP is compared to meander-line based QWPs. The AAS has 20% higher efficiency and smaller AR than the best meander-line QWP; however, the meander-line designs has better bandwidth. Hence, AAS QWP can be a good alternative for applications that require high efficiency and limited bandwidth.

![Figure 2.4: Magnitude and phase of the transmitted fields in the X and Y polarizations for QWP.](image1)

![Figure 2.5: The axial ratio and phase difference between the X and Y polarizations. The bandwidth of the QWP is 8.6 µm–10.4 µm, which is the frequency range where both AR and phase difference curves stay within the gray region.](image2)
Table 2.1: Comparison of AAS QWP to meander-line based QWP. Simulated power transmission, phase, and AR are compared at the design wavelength. Bandwidth is defined for phase between $90^\circ \pm 10^\circ$.

<table>
<thead>
<tr>
<th>Design</th>
<th>Design wavelength</th>
<th>Power Transmission</th>
<th>Phase</th>
<th>AR</th>
<th>Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAS QWP</td>
<td>10 µm</td>
<td>90%</td>
<td>90º</td>
<td>1.03</td>
<td>8.6 µm–10.4 µm</td>
</tr>
<tr>
<td>[42] 3-layer QWP</td>
<td>9.2 µm</td>
<td>23%</td>
<td>80º</td>
<td>1.5</td>
<td>9.5 µm–12 µm</td>
</tr>
<tr>
<td>2-layer QWP</td>
<td>10.6 µm</td>
<td>67%</td>
<td>89.4º</td>
<td>1.23</td>
<td>8 µm–13 µm</td>
</tr>
<tr>
<td>3-layer QWP</td>
<td>10.6 µm</td>
<td>59.4%</td>
<td>82º</td>
<td>1.26</td>
<td>8 µm–13 µm</td>
</tr>
</tbody>
</table>
2.2.3 Design and Simulation of a HWP

For a HWP design, the required phase difference $\Delta \phi$ is $\pi$ in Eq. (2.1). However, the arctan function has a periodicity of $\pi$, which leads to the same value for $jB_x$ and $jB_y$. As a result, it is not possible to design a HWP of unity transmission using 2 layers. This problem can be solved by cascading one more layer [53, 59]. By adding one more quarter-wavelength dielectric layer and gold elliptical patches, the HWP can be designed with good transmission. Fig. 2.6 shows a triple layer AAS design with elliptical patches. The major axis $L_x$ of the elliptical patch remains 3.3 $\mu$m but the minor axis $L_y$ is changed slightly to 1.45 $\mu$m for the required 180° phase difference. The total thickness of the HWP is 3.8 $\mu$m. The transmission of each polarization is shown in Fig. 2.7. The transmission magnitudes at the operating frequency are about 0.9 leading to 80% of transmitted power. The phase difference is 180° as desired. We can define the bandwidth of this HWP by satisfying both the AR and the phase difference limits which are 0.8 - 1.2 and 160° - 200° respectively. As shown in Fig. 2.8, this bandwidth of the HWP is from 9.2 $\mu$m–11 $\mu$m.

Both the QWP and the HWP designs presented herein utilize gold layers with the same patterning (homogeneous loading) and fixed interlayer spacing of $\lambda/4$. The specific choice of $\lambda/4$ is to achieve the widest bandwidth possible, since in this case the incident and reflected waves are out of phase and cancel each other. This is consistent with [50] which shows that indeed the bandwidth is widest when
the interspacing is $\lambda/4$ between the loading elements. As the QWP or the HWP is sandwiched between a glass substrate and air with different impedances, a quarter-wave transformer is required to achieve good matching. It may be possible that with inhomogeneous loading (different metallization patterning) and different dielectric interspace materials, the quarter-wave matching could be eliminated. This is even more likely for more broadband designs that contain more layers, as it is well known from microwave filter theory [60]. However, adding more layers lead to increased Ohmic loss in the gold AAS and decreases total transmission efficiency. Due to the additional layer, the HWP has a total efficiency of 80%, which is 10% lower than the QWP. To obtain even higher transmission, a pure dielectric design that utilizes diamond sub-wavelength grating can be used [61]. The grating based HWP has a comparable bandwidth to the AAS HWP, but with a transmittance more than 90%. However, the material cost of such HWP can be prohibitive. Comparing to commercial cadmium thiogallate based HWPs [62], the AAS HWP has a low efficiency but a more accurate phase retardation. The commercial HWP has an efficiency of 97% and a phase of 160° at the design frequency. Hence, AAS HWPs can be an attractive solution for replacing conventional HWPs. The AAS HWP can also achieve additional functionalities than the conventional or the grating based HWPs. As we will show in the next section, each unit cell of the AAS HWP can be locally rotated to synthesize compact refracting, focusing, and polarization discriminating devices for manipulating circularly polarized waves.

2.3 Wavefront Manipulation Using HWP AAS

2.3.1 Background

The conventional method of optical wavefront control and manipulation often relies on bulky dielectric lenses. For example, a glass lens that is used to focus light has a polished parabolic surface such that light rays propagating through it experiences spatially-dependent delay and arrive at the focal point in phase. However, even with current nano-fabrication technology, it is still difficult to create a dielectric lens with a parabolic or non-planar surface at the micron scale to integrate with nano-photonic systems. To overcome this challenge, novel techniques have been proposed in [23, 53, 63–67]. In [23, 63], a 2D patterned structure was demonstrated to control the wavefront of light by using 2D arrays of V-shaped...
gold nano-antennas deposited on a silicon wafer. As the linearly-polarized light impinges on the V-shaped antennas, symmetric and anti-symmetric currents are induced on the antennas such that the phase of the scattered light in the cross-polarization can be controlled while maintaining a constant scattering magnitude. Hence, the emerging wavefronts of the scattered light from the silicon interface are tailored into the desired pattern. The advantage of this concept is that it leads to a thin planar structure that can be easily realized with current fabrication technology. The disadvantage is though that the scattered light from these nano-antennas has too small intensity to be useful for practical applications. In [64], Babinet-inverted V-shaped slot antennas are introduced for focusing the transmitted light in the cross-polarization. The working principle is similar to these in [23, 63], but the efficiency is improved. However, according to [53], the fundamental upper limit of the power that can be coupled into the cross-polarization of the scattered light is 50%. Even if this cross-polarization coupling can be maximized, due to the symmetry of the design in [23, 63, 64], half the power is scattered into the forward direction and half into the backward direction. This implies that at most only 25% of the incident power can be coupled into the transmitted cross-polarization.

To improve the transmission efficiency, [53] introduced the concept of the meta-transmit-array which is similar to transmittarrays in the microwave regime. Three cascaded metasurfaces are used to provide full control of the transmission phase. The metasurface is composed of periodically arranged unit cells. Each unit cell consists of a block of a nonmetallic plasmonic material (aluminum-doped zinc oxide) and a block of a dielectric material (silicon). Tuning of the phase delay of each unit cell is achieved through changing the thicknesses of the two material blocks. The complexity and required accuracy of the design still remains a challenge for its realization with existing technologies. In [65], a more realizable lens was proposed. Each unit cell in this lens comprises 4 cascaded patterned metallic sheets with dimensions parameterized to obtain the required transmission phase and magnitude for two orthogonal polarizations. The structure can convert linearly-polarized incident light into circularly-polarized one and achieve focusing at the same time. The drawback is that to keep the lens functional, the linearly polarized incident light has to be oriented at 45° relative to the horizontal x-axis so that the X and Y components of the metasurface are equally excited. Moreover, the unit cell design has to be parametrized to achieve the required phase delays. In fact, the designs in [23, 53, 63–67] all use aperiodic unit-cell structures to tailor the phase component of the linearly-polarized light. If a different phase requirement is imposed, the unit cells have to be re-designed and optimized. Such a design process can be very cumbersome and inflexible.

In [68], a rotation technique has been introduced to simplify the phase tailoring process which is related to the concept presented earlier in [69]. The unit cell in [68] consists of a plasmonic dipole made of gold. By rotating each dipole, the incident circularly-polarized (CP) light is scattered into a right-hand circularly-polarized (RHCP) and a left-hand circularly-polarized (LHCP) light. In addition, the local phase delay for LHCP light is controllable through the local geometric rotation of the dipoles. This geometric phase is also known as the Pancharatnam-Berry (PB) phase [70] in optical communities. By tailoring the local phase delay profile, refracting and focusing functionalities can be realized for the incident circularly-polarized waves. This concept leads to a simpler phase control technique compared to the ones presented in [23, 53, 63–67]. However, there are two major drawbacks. First, as in any single layer lens, the scattering efficiency has an upper bound as mentioned before. In this particular case, the efficiency is only 5%, which deems the lens impractical for real world applications. Second, the transmitted light has both RHCP and LHCP light and only one of the polarizations is controllable, the
other one is not. Hence, with the uncontrollable polarization present, a QWP and a polarizer are needed for filtering, which ultimately undermines the potential for integration. Furthermore, the additional power loss introduced by the QWP and the polarizer is undesirable in imaging applications. In the microwave regime, an efficient transmitarray was designed based on the method proposed shown in [69]. However, it is difficult to directly translate these designs to the optical regime. First the fabrication can be very challenging. The unit cells consist of 5 layers of metallic structures. The top and bottom two layers are rectangular patches. The middle layer is a metallic sheet with orthogonal slots, which are used to couple the top and bottom patches. Second, the achievable fabrication resolution might not reach the required accuracy for the narrow slots for efficient aperture coupling, and this will have a great negative impact on the performance of the transmitarray.

By utilizing the geometric phase approach, we show that an efficient optical transmitarray that tailors the transmission phase of the incident CP light can be built with the AAS HWP unit cells presented in Section 2.2.3. Since the AAS HWP unit cell is efficient and the arbitrary phase delay can be achieved with just the rotation of unit cells, efficient optical transmitarrays can be designed with a simple and yet flexible approach. This concept is demonstrated by designing refracting, focusing, and polarization discriminating optical transmitarrays for circularly-polarized light at 30 THz.

2.3.2 Theory and Design

Suppose the incident field is LHCP propagating in the +Z direction, in an unprimed X-Y coordinate system, the incident field is given by (2.2) in time harmonic form with $e^{j\omega t}$ terms omitted.

$$\vec{E}_{\text{inc}} = \hat{x}E_x + j\hat{y}E_y$$

(2.2)

Once the incident field impinges on the unit cell, it may experience changes in both the amplitude and phase in the X and Y polarizations, then the transmitted field $\vec{E}_t$ is given by equation (2.3)

$$\vec{E}_t = \hat{x}E'_x + j\hat{y}E'_y e^{j\theta_y}$$

(2.3)

Let the unit cell (gray patch) as shown in Fig. 2.9 be rotated by an angle $\varphi$ with respect to the original (unprimed) coordinate system and aligned to the new (primed) coordinate system. The incident field can be written in the new coordinate system as in equation (2.4)

$$\vec{E}_{\text{inc}} = (\hat{x}'E'_x \cos \varphi - \hat{y}'E'_y \sin \varphi) + j(\hat{x}'E'_x \sin \varphi + \hat{y}'E'_y \cos \varphi)$$

(2.4)

Then the transmitted field simply becomes

$$\vec{E}_t = \hat{x}'E'_x e^{j\theta_x} e^{j\varphi} + j\hat{y}'E'_y e^{j\varphi}$$

(2.5)

We can further rewrite $\hat{x}'$ and $\hat{y}'$ in terms of $\hat{x}$ and $\hat{y}$, then $\vec{E}_t$ can be written as

$$\vec{E}_t = (\hat{x} \cos \varphi + \hat{y} \sin \varphi) E'_x e^{j\theta_x} e^{j\varphi} + j (\hat{x} \sin \varphi + \hat{y} \cos \varphi) E'_y e^{j\theta_y} e^{j\varphi}$$

(2.6)
After some algebra, we arrive at (2.7), in which we have written the transmitted wave as a summation of LHCP and RHCP waves whereas the incident wave is LHCP. Only the first term, i.e., the RHCP wave has a phase associated with the element rotation $\varphi$. In order to control the phase of the transmitted wave by directly rotating the element, the second term in equation (2.7) has to be eliminated. The required conditions are given by (2.8)

$$\vec{E}_t = \frac{1}{2} (\hat{x} - j\hat{y}) \left( E'_x e^{j\theta_x} - E'_y e^{j\theta_y} \right) e^{j2\varphi} + \frac{1}{2} (\hat{x} + j\hat{y}) \left( E'_x e^{j\theta_x} + E'_y e^{j\theta_y} \right) e^{j2\varphi}$$

(2.7)

$$\begin{cases} E'_x = E'_y \\ \theta_x = \theta_y + \pi \end{cases}$$

(2.8)

If the above amplitude and phase constraints are satisfied, the transmitted field is a LHCP wave as shown Eq. (2.9) (notice that the handedness of the polarization is converted). The transmission phase is directly proportional to the local rotation angle $\varphi$ and the uncontrollable RHCP wave is eliminated. If the incident wave is LHCP, then the transmitted wave would be RHCP and vice versa. The condition in Eq. (2.8) requires the transmitting unit cell to be a HWP.

$$\vec{E}_t = E'_x e^{j\theta_x} e^{j2\varphi} (\hat{x} - j\hat{y})$$

(2.9)

Figure 2.9: Rotated unit cell with respect to the original (unprimed) and the new (primed) coordinate system. The rotation angle is $\varphi$.

### 2.3.3 1D-Refraction

From the Huygens principle, the wavefront can be reconstructed from the spherical wavelets of the constituent Huygens’ sources. Refraction arises if there is a constant phase delay between successive Huygens’ sources. By treating each unit cell as a Huygens’ source and manipulating the phase delay of each individual unit cell, we are able to create a transmitarray that can refract a normally incident wave to an arbitrary angle. With the unit cell design presented in Section 2.2.3, the required phase delay can be achieved by rotating each unit cell. Fig. 2.10 shows the top view of such a transmitarray. As shown, the unit cells from left to right have an incremental rotation angle of $20^\circ$. Hence, each unit cell can introduce an additional $40^\circ$ phase shift compared to the adjacent unit cell on the left. The simulated result of this transmitarray is shown in Fig. 2.11a and Fig. 2.11b. The transmitted beam corresponds to a refraction angle of $17^\circ$. Fig. 2.12 shows the extracted transmission efficiency.
vs. frequency of the transmitarray. Above 70% efficiency can be maintained over a 14% bandwidth (28-32 THz). At the operating frequency, the transmission efficiency is 76%. It should be noted that the efficiency of the entire transmitarray agrees well with the efficiency of the single unit cell presented in Section 2.2.3. Therefore, once the efficiency of a single unit cell is maximized, the efficiency of the entire transmitarray is maximized as well. Compared to prior art, the design process for this refraction lens is greatly simplified and is more scalable.

Figure 2.10: Top view of the refraction transmitarray. Unit cells from left to right have an incremental rotation angle of 20°.

Figure 2.11: (a) Transmitted RHCP wave from a LHCP Gaussian beam at normal incidence. The transmitarray is outlined by the red dashed box. (b) Normalized far-field pattern of the refracted and reflected beams.
Figure 2.12: Over 70% transmission efficiency can be maintained over the frequency range of 28-32 THz. This is equivalent to 14% bandwidth.

2.3.4 1D-Focusing

A focusing transmitarray can be created by using the unit cells to compensate different free space path delays as shown in Fig. 2.13. By setting a desired focal length, each unit cell is rotated accordingly. The design of the 1D-focusing transmitarray is shown in Fig. 2.14. With a normal incident plane wave, the magnitude of the transmitted field is shown in Fig. 2.15(a). The focal spot of this transmitarray is designed to be $21 \mu m$ or $2\lambda$ away from the transmitarray. The simulated focal spot is taken to be the center of the beam spot that is $20.6 \mu m$ away. The simulated result is in a good agreement with our calculation. The complex magnitude of the E-field across the focal plane is plotted in Fig. 2.15(b). The incident field magnitude is $\sqrt{2}$ V/m. The peak magnitude of the focused field is 3.89 V/m. The transmitarray has a focusing power of 2.75 times. The half power beam width is $5.2 \mu m$, or half-wavelength.

Figure 2.13: The central element has a free space path delay of $\phi_0$. The adjacent elements have additional free space path delays of $\phi_1$ and $\phi_2$. By rotating the corresponding unit cells with phase delays of $-\phi_1$ and $-\phi_2$, the path delays can be compensated and the transmitted waves from each unit cell will arrive at the focal spot in phase.
Figure 2.14: Over 70% transmission efficiency can be maintained over the frequency range of 28 – 32 THz. This is equivalent to 14% bandwidth.

Figure 2.15: (a) Complex magnitude of the transmitted field. The simulated focal length is taken to be the center of the beam spot which is 20.6 µm away from the transmitarray and the designed focal length is 21 µm. b) The complex magnitude of the transmitted E-field across the focal plane. The incident field magnitude is $\sqrt{2}$ V/m and the peak magnitude of the focused field is 3.89 V/m. The transmitarray has a focusing power of 2.75 times.
2.3.5 2D-Focusing

A small 2D focusing transmitarray with unit cells arranged in a $7 \times 7$ square array is shown in Fig. 2.16. The full-wave simulation result is shown in Fig. 2.17 where the focusing effect is clearly observable. It was not possible to simulate larger transmitarray designs due to the enormous computational resources required. However, we can approximate the fields for large designs with the fields obtained from unit cell simulations. The total E-field and H-field of a single unit cell are sampled on two sampling planes as shown in Fig. 2.18. One sampling plane is above the top elliptical patch and one sampling plane is below the bottom elliptical patch. The sampling planes are parallel to the X-Y plane and reside in the near-field region of the radiating patches. By applying Love’s equivalence principle [71], equivalent electric and magnetic surface currents $J_s$ and $M_S$ can be computed from those sampled fields. The far-field radiation pattern of this single unit can then be computed from those equivalent surface currents. By stitching together the surface currents from all the unit cells, the field pattern of the entire array can be calculated. This procedure assumes the inhomogeneity in the transmitarray has no impact on the fields of each unit cell, which is not true. However, it can serve as a good approximation if the variation between adjacent unit cells is small. Fig. 2.19a and Fig. 2.19b show the complex magnitudes of the E-field in the Y-Z and X-Y planes for a square array with $81 \times 81$ unit cells. The focusing beam patterns match well those from the full-wave simulation of the smaller 2D array.

Figure 2.16: Full-wave simulation model of a 2D-focusing transmitarray lying in the X-Y plane.
Figure 2.17: (a) Complex magnitude of the electric field on the (a) Y-Z plane and (b) X-Y plane. Incident plane wave has a magnitude of $\sqrt{2}$ V/m.

Figure 2.18: Surface electric current $J_s$ and magnetic current $M_s$ are computed from the sampled E-field and H-field on the sampling planes that reside in the near field of the radiating patches. By stitching together the surface currents from all the unit cells in the array, the far-field pattern can be computed.
Figure 2.19: Normalized complex magnitude of E-field on the (a) Y-Z plane and (b) X-Y plane.
2.3.6 Polarization Discriminator

In addition to the refraction and focusing applications presented above, the refraction lens presented in Section 2.3.3 can be used to detect the orientation angle of a linearly polarized plane wave at normal incidence by simply measuring the spatial variation of the transmitted intensity. Such a concept can be very attractive for potential infrared polarimetric imaging applications [35]. To begin with, if the handedness of the incident CP wave changes, it is not difficult to see that the phase delay due to the rotation of the unit cell reverses its sign with equal magnitude. Hence, if the refraction lens introduces a positive phase gradient for a normally incident RHCP wave, it will introduce a negative phase gradient for a normally incident LHCP wave. As shown in Fig. 2.20, an incident LHCP wave is refracted into a RHCP wave at an angle of $-\theta$, and an incident RHCP wave is refracted into a LHCP wave at an angle of $+\theta$.

![Figure 2.20](image_url): For a 1D refraction lens with phase gradient in the X-direction, the refraction angles for the transmitted plane waves are equal but opposite in sign for LHCP and RHCP incident waves.

A normally incident linearly polarized wave can be decomposed into a summation of a LHCP and a RHCP wave. For example, an X-polarized and a Y-polarized wave with unity magnitude can be written as

\[
\hat{x} = \frac{1}{2} (\hat{x} - j \hat{y}) + \frac{1}{2} (\hat{x} + j \hat{y}) \quad (2.10)
\]

\[
\hat{y} = -j \left[ \frac{1}{2} (\hat{x} - j \hat{y}) - \frac{1}{2} (\hat{x} + j \hat{y}) \right] \quad (2.11)
\]

As a result, for X-polarized and Y-polarized waves, the refracted waves consist of both RHCP and LHCP waves, but there is a $180^\circ$ phase difference between the RHCP and LHCP waves depending on whether the incidence is oriented along the x or the y-axis. This phase difference results to different interference patterns of the transmitted waves.

To obtain an analytical expression of the interference pattern due to different orientations of a linearly polarized incident wave, we can assume that the two transmitted waves are two CP plane waves with propagation vectors $\vec{k}_L = k \sin \theta \hat{x} + k \cos \theta \hat{z}$ and $\vec{k}_R = -k \sin \theta \hat{x} + k \cos \theta \hat{z}$, where $\theta$ is the refraction angle with respect to the Z-axis as shown in Fig. 2.20. Then the relation between the incident and transmitted CP waves can be mapped as (2.12) and (2.13).
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\[(\hat{x} - j\hat{y}) \rightarrow (\cos \theta \hat{x} + j\hat{y} - \sin \theta \hat{z}) e^{-j(kx \sin \theta + k \cos \theta)} \]  
\[(\hat{x} + j\hat{y}) \rightarrow (\cos \theta \hat{x} - j\hat{y} + \sin \theta \hat{z}) e^{j(kx \sin \theta - k \cos \theta)} \]  
(2.12)  
(2.13)

With the help of equations (2.10) and (2.11), the transmitted waves for normally incident linearly polarized waves with fields polarized along the $\hat{x}$ and $\hat{y}$ directions can be mapped as in equations (2.14) and (2.15), assuming that the field magnitudes are unity and dropping the constant phase term $e^{-jk \cos \theta z}$.

\[
\hat{x} \rightarrow \hat{x} \cos \theta \cos (kx \sin \theta) + \hat{y} \sin (kx \sin \theta) + j \hat{z} \sin \theta \sin (kx \sin \theta) \]  
(2.14)

\[
\hat{y} \rightarrow \hat{x} \cos \theta \sin (kx \sin \theta) - \hat{y} \cos (kx \sin \theta) - j \hat{z} \sin \theta \cos (kx \sin \theta) \]  
(2.15)

Finally, for a linear polarization with any orientation and unity magnitude, we can decompose it as

\[
\vec{E}_{inc} = \hat{x} \cos \varphi + \hat{y} \sin \varphi, \]  
(2.16)

where $\varphi$ is the angle between the $x$-axis and orientation of the electric field. By substituting equation (2.14) and (2.15) into equation (2.16), the transmitted field $\vec{E}_t$ due to $\vec{E}_{inc}$ can be written as

\[
\vec{E}_t (\varphi, x, \theta) = \hat{x} [\cos \varphi \cos \theta \cos (kx \sin \theta) + \sin \varphi \cos \theta \sin (kx \sin \theta)] \\
+ \hat{y} [\cos \varphi \sin (kx \sin \theta) - \sin \varphi \cos (kx \sin \theta)] \\
+ j \hat{z} [\cos \varphi \sin \theta \sin (kx \sin \theta) - \sin \varphi \sin \theta \cos (kx \sin \theta)]. \]  
(2.17)

The full setup for the proposed polarization discriminator is shown in Fig. 2.21. If a normally-incident linearly-polarized wave with orientation angle $\varphi$ with respect to the $x$-axis passes through a refraction lens with a phase gradient in the X-direction, the transmitted intensity is $|\vec{E}_t|^2$ with spatial variations in the X-direction. The positions of the peaks and troughs of this intensity pattern depend on the orientation angle $\varphi$. By simply placing photo-detectors behind the lens to measure the spatial variation of the transmitted intensity, the orientation angle of the normally incident linearly polarized light can be determined.

We can illustrate this concept by using an example. The refraction lens design in Section 2.3.3 has a designed refraction angle $\theta$ of $-17^\circ$. Using this refraction angle, we can plot the intensity $|\vec{E}_t|^2$ vs. $x$ for various orientation angles $\varphi$ as shown in Fig. 2.22. The transmitted intensities have sinusoidally varying spatial distributions. As the orientation angle $\varphi$ of the incident linearly polarized wave changes, the positions of the peaks and troughs of the transmitted intensity distributions change correspondingly.

Full-wave simulated results using the refraction lens presented in section 2.3.3 are shown in Fig. 2.23. The intensity distributions are not pure sinusoids as in the theoretical case. This is because the transmitted wave is not a perfect plane wave; there are distortions due to imperfect CP plane-wave reconstruction. There are two major factors that affect the plane-wave reconstruction. The first is due to the loss of the unit cell. Upon normal incidence, the current induced along the longer axis of the elliptical patch experiences greater loss compared to the current induced along the shorter axis. Hence, the transmitted fields along the two axes of the elliptical patch have different magnitudes. This can be observed in Fig. 2.7. Due to the difference in transmission magnitudes, the transmitted plane wave is elliptically-polarized instead of circularly-polarized. Second, the transmission property of each unit cell is characterized under
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Figure 2.21: If a linearly polarized wave $E_{\text{inc}}$ with orientation angle $\varphi$ with respect to the $x$-axis impinges normally on a refraction lens having a phase gradient in the X-direction, the X-positions of the peaks and troughs of the transmitted intensity pattern will depend on the orientation angle $\varphi$.

Figure 2.22: The theoretical result of the transmitted intensity $|\vec{E}_t|^2$ vs. the spatial position $x$ for various orientation angles $\varphi$ of a linearly polarized incident plane wave. The spatial positions of the peaks and troughs vary correspondingly with the orientation angle.

Despite these factors, the simulated relative positions of the peaks and troughs with respect to the various orientations in $\varphi$ match well with the theoretical ones. Capturing these relative positions is the most important factor for the functionality of the proposed linear polarization discriminator. The detectors behind the lens only need to determine the relative positions of the highest and lowest received intensity in order to determine the angle of the orientation. The magnitude variation between the peaks and troughs is less important. Fig. 2.24 shows the theoretical and simulated positions for one peak and one trough with respect to the orientation angle $\varphi$. The positions are sampled every $20^\circ$ change in $\varphi$. 

the assumption of infinite periodicity. However, once the unit cells are sequentially rotated, the periodicity is lost. The mutual coupling between patches in a periodic arrangement is different than that in an aperiodic arrangement. Due to this variation in mutual coupling, the reconstructed plane wave experiences an inhomogeneous amplitude and phase delay along the lens. Hence, the intensity shape in Fig. 2.23 is not exactly sinusoidal.
It can be seen, that the simulated results follow the theoretical results fairly well.

This concept can result to a very simple and compact imaging system for polarimetry applications. The conventional methods for polarimetry application [72, 73] use a supercell consisting of 4 wire grid micropolarizers oriented at 0°, 45°, 90° and 135°. By using CCD sensors behind each micropolarizer to measure the transmitted intensity, the Stokes parameters are extracted to estimate the angle of the orientation. However, each micropolarizer suffers from different degrees of polarization mismatch which leads to a power loss of 50%. If we replace the wire grid micropolarizers in the supercell with our refraction lens, also oriented at 0°, 45°, 90° and 135°, we gain two important advantages. First, each refraction lens offers some degree of estimation of the orientation angle; hence, with 4 refraction lenses used in combination, we can have a much better estimation of the Stokes parameters. Second, the refraction lens has no polarization mismatch, almost around 70% of the incident power is transmitted to the detectors. Hence, the SNR of the system can be improved.

\[ |E_t|^2 = 0°, 45°, 90°, 135° \]

![Figure 2.23](image_url)  
**Figure 2.23:** The simulated result of the intensity $|E_t|^2$ vs. spatial position $x$ for various orientation angles $\varphi$. Since the reconstruction of the plane wave is not perfect, the intensity distribution is not sinusoidal as in the theoretical result. However, the relative positions of the peaks and troughs match well with those of the theoretical results.

![Figure 2.24](image_url)  
**Figure 2.24:** Theoretical and simulated peak and trough positions along the lens with respect to the orientation angle of a linearly polarized incident light.
2.4 Conclusion and Outlook

In this chapter, we have presented a thin quarter-wave plate (QWP) and a half-wave plate (HWP) based on multilayer antenna array sheets (AAS) that are designed based on microwave phase-shifter theory. The wave-plates operate in the long wavelength infrared region at 30 THz with above 80% transmission efficiency. Good axial ratios and phase differences can be maintained over bandwidths of 19% and 17% for the QWP and HWP respectively. The advantages of the proposed plates are that (a) they are compatible with current planar fabrication technology and potentially are scalable up to the near IR regime where metal remains reasonably opaque (b) they are typically less than one wavelength thick (c) they offer high transmission since they are well matched due to their multilayer structure and (d) they exhibit a good bandwidth. Comparing to the meander-line based QWP, the AAS QWP is less broadband, but it is more efficient and has a better axial ratio. Comparing to the diamond grating HWP or conventional cadmium thiogallate HWPs, the AAS HWP is less efficient but has better control on the phase retardation. In addition, the AAS HWP can be less costly and easier to integrate. It may be tempting to use more layers in the AAS waveplates to increase their bandwidth. However, adding more layers may lead to additional metal loss, which decreases the total efficiency. Thus, the AAS waveplates are best suited for integrated optical devices that don’t demand a huge bandwidth.

We further adapted the AAS HWP unit cells to tailor the wavefront of circularly-polarized waves. The transmission phase of light is only dependent on the rotation angle of the unit cell in a simple deterministic manner. Thus, by arranging the unit cells with particular orientations, a family of optical transmitarrays are presented to demonstrate refraction and focusing. The resulting structures are completely planar and offer a transmission efficiency above 70% over the frequency range of 28 – 32 THz, which is similar to the efficiency of the unit cell. Furthermore, by using a flat refraction lens made using this approach, the orientation angle of a normally incident linearly polarized light can be detected by simply measuring the transmitted intensity pattern. This compact polarization discriminator can be more efficient than the conventional ones with micropolarizers consisting wire grids, which reflect 50% of the incident light. Hence, with an efficient HWP unit cell, the geometric phase approach is a simple and flexible method for constructing efficient wavefront manipulating devices for circularly-polarized waves.

In the current work, the AAS waveplates are impedance matched from the supporting substrate to free space with a quarter-wave transformer deposited on top of the AAS. In the future, it is possible to eliminate this matching layer to simplify the fabrication process. The impedance matching functionality can be embedded into the AAS design, e.g., by designing different AAS for different layers.
Chapter 3

Metalenses for Short-Wave Infrared Focal Plane Arrays

3.1 Introduction

The short-wave infrared (SWIR) spectrum is typically defined as light within the wavelength range 0.9 µm–1.7 µm. Unlike Mid-Wave Infrared (MWIR) and Long-Wave Infrared (LWIR) light, which is emitted from the object itself, SWIR is similar to visible light in that photons are reflected or absorbed by an object, providing the strong contrast needed for high resolution imaging. Ambient star light and background radiance (nightglow) are natural emitters of SWIR and provide excellent illumination for outdoor, night time imaging. A simplified model of a typical SWIR imaging system is shown in Fig. 3.1. The object at a distance forms an image at the focal plane of the imaging lens. A focal plane array (FPA), which is an array of imaging pixels, is placed at the focal plane to capture the incoming photons and convert them to electrons. Based on the number of electrons collected from each pixel, an image of the object can be faithfully reproduced by the FPA.

Figure 3.1: A typical setup of the SWIR imaging system. The object at a distance forms an image at the focal plane of the imaging lens. A FPA is placed at the focal plane and collects the incident photons from the object. An image of the object is formed based on the amount of photons collected by each pixel.
Each pixel or photon sensor in the FPA is a photo diode junction which can be fabricated with a variety of materials such as germanium (Ge), indium gallium arsenide (InGaAs), indium antimonide (InSb), and mercury cadmium telluride (MCT or HgCdTe) [74]. Of these, InGaAs sensors have proved to be the most practical for imaging applications due to their high quantum efficiency and low dark current at room temperature. A typical InGaAs photo diode junction for SWIR applications can be fabricated with a mesa or a planar architecture as shown in Fig. 3.2 [75, 76]. The indium phosphate (InP) substrate offers a transparent window for SWIR frequencies. Hence, the incident photons pass through the substrate with minimal loss and impinge on the InGaAs material, which absorbs the photons and converts them to electrons based on the photo-electric effect. A read-out integrated circuit (ROIC) collects the electrons through the indium bumps and an image can be formed based on the number of electrons collected from each pixel.

The advantages and disadvantages associated with each type of architecture are illustrated with the help of Fig. 3.3. For the mesa architecture, there is a metal passivation layer surrounding the well-shaped InGaAs absorption region. The converted electrons in the absorption region are all contained inside the well and cannot drift to adjacent pixels. As a result, crosstalk between adjacent pixels is extremely low for this architecture. However, as shown in Fig. 3.3a, the top opening of the well usually is not equal to the pixel area. For incident photons near the edge of the pixel, they will usually reflect at the metal passivation and eventually be lost. The amount of photons collected by each pixel is defined by its fill factor (FF), which is the ratio between the photo-sensitive InGaAs area and the total pixel area. For a typical sensor with the mesa architecture, the FF is around 50%, which implies that around 50% of the incident photons are not collected. In comparison, the FF for planar architectures is nearly 100%. This is because the InGaAs absorption region in a planar architecture is an uniform layer. Theoretically, all the incident photons can be converted to electrons and be collected. However, due to random thermal processes, the electrons can drift in the InGaAs layer. As shown in Fig. 3.3b, the electrons generated in the middle pixel can freely drift to adjacent pixels. Hence, even though the planar architecture captures all the incident photons, the resulting large crosstalk can degrade the image quality. The goal of this chapter is to explore methods that can improve the photon collection efficiency while minimizing crosstalk between pixels. Specifically, we seek methods to improve the FF of the mesa architecture, since the crosstalk problem in the planar architectures is mostly related to semiconductor physics, which is out of the scope of this thesis.

![Figure 3.2](image-url): (a) Mesa and (b) planar architectures for SWIR FPAs with InGaAs material for absorbing photons and converting them to electrons.
Chapter 3. Metalenses for Short-Wave Infrared Focal Plane Arrays

InGaAs Absorption Region
InP Substrate
Indium bump
ROIC
ARC
metal passivation
N+InP
P+InP
e–
hypothetical pixel boundary

Figure 3.3: (a) For the mesa architecture, the electrons are contained inside each well and cannot drift to adjacent wells. The resulting cross talk between pixels is very low. However, incident photons can scatter at the metal passivation, resulting to a lower photon collection efficiency. (b) For planar architecture, the InGaAs absorption region is an uniform layer. The incident photons can be collected with nearly 100% efficiency. However, the electrons can drift inside the layer, resulting to a large crosstalk between adjacent pixels.

To improve the FF of the mesa architecture, refractive [77–90] and diffractive [87,88,91–97] microlens arrays are often employed as shown in Fig. 3.4 for camera sensors in the visible and IR spectra. The microlenses are able to refract the incident photons that originally fall on the passivation to the absorption region. As a result, the effective FF can be significantly increased. However, this implementation has its shortcomings. As shown in Fig. 3.4(c), when the incident photons come at a large oblique angle, which is often associated with high numerical aperture (NA) imaging lenses, a significant portion of the concentrated photons land on the metal passivation. In this scenario, the effective FF can be lower than that without the microlenses. Thus, microlenses can be less desirable for applications requiring high NA imaging lenses [98,99]. Even if the focusing capability of the microlens are matched to the high NA imaging lens, the fabrication can be extremely challenging. The microlenses are usually made by machining or etching the substrate, which poses restrictions on the minimum diameter of the microlenses. A typical IR microlens has a diameter of 50µm, which is too large for modern high-resolution FPAs with a pixel size around 25µm. Furthermore, the surface profile of the microlenses cannot be controlled precisely [84,85,100], which may result to a poor performance.

Figure 3.4: A microlens array is added to refract and focus the incident photons. From ray tracing illustrations, microlenses can successfully focus all the photons to the absorption regions for (a) normal incidence and (b) small oblique incidence. However, the focusing is less efficient for (c) large oblique incidence.

In the following sections, we explore microlens designs for improving the FF of a FPA with a small 25µm×25µm pixel area and a 18µm×18µm InGaAs absorption area. The FPA is coupled to a high...
NA imaging lens with $f/D = 1$, where $f$ is the focal length and $D$ is the diameter of the lens. We provide a synthesis procedure to synthesize an appropriate surface profile for the microlenses, such that they work properly at large oblique incidence associated with the high NA imaging lens. Since the pixels of the FPA has a relatively small size, fabricating the microlenses with accurate shape can be extremely challenging. To tackle this problem, we propose to transform the microlenses into planar metalenses using metamaterial concepts comprising periodically arranged nano-hole unit cells. Compared to conventional microlenses, these metalenses can be easily fabricated with standard reactive ion etching (RIE) [101] method. The performance of the developed metalenses are investigated through simulations and measurements.

3.2 Microlens Design

3.2.1 Preliminary Analysis and Assumptions

To synthesize an appropriate surface profile for the microlenses, it is helpful to perform a preliminary analysis with simple ray tracing techniques for a 1-D lens, from which some assumptions can be made to help with the synthesis procedure. Fig. 3.5a shows the ray tracing diagram for the FPA system. Assume the point sources are in the far-field, then electromagnetic waves emanating from these point sources impinge on the imaging lens as plane waves, which can be described as ray bundles consisting of parallel rays. For a better visualization, the ray bundle for each point source is shown with a different color. Each ray bundle can be further characterized with a chief ray and two marginal rays. The chief rays are the rays emanating from the point sources and travel through the lens center, and the marginal rays are the ones that travel through the edges of the lens. For the imaging lens with $f/D = 1$, the angle between the marginal rays and the optical axis of the lens is approximately $\pm 25^\circ$. Consequently, the rays incident on the microlenses are approximately from $-25^\circ$ to $+25^\circ$, and this is the incident angular range that the microlenses should be designed for. From Fig. 3.5a, it seems that chief rays from each ray bundle impinge on the central microlens at different angles. This is due to the exaggerated scale; in reality, the microlenses are tiny (25$\mu$m \times 25$\mu$m) compared to the imaging lens (diameter on the order of 10 cm). Thus, we assume all the chief rays would strike the microlens at 0$^\circ$ (parallel to the optical axis) as shown in Fig. 3.5b. This assumption would simplify the lens surface profile synthesis. One more assumption made for the lens synthesis is that the microlenses are made from InP, which is the same material as the substrate. This is a valid assumption since many IR microlenses are directly machined or etched on the substrate as we mentioned previously. In general, the microlenses can be made from other materials. However, by having the microlenses to be of the same material as the substrate, reflections can be avoided at the substrate/lens interface.

3.2.2 Lens Surface Profile Synthesis

To begin with the synthesis, consider the central pixel shown in Fig. 3.5b, where the rays are bundled together according to different incident angles. Fig. 3.6 shows a different but equivalent perspective: instead of grouping the rays with the same incident angle, the rays from the same point source are grouped together, i.e, the rays with the same color in Fig. 3.5b. For each point source, even though only three rays are shown, in reality there can be an infinite number of rays, effectively forming a light cone with its boundary prescribed by the marginal rays. The advantage of this perspective is that
instead of analyzing all the rays in the light cone, only the behavior of the marginal rays (shown by solid arrows) are considered. For example, if the refracted marginal rays from the red light cone land inside the absorption region, then all the internal rays from the same cone are guaranteed to land inside the absorption region. Hence, the lens synthesis is simplified to analyzing the marginal rays at each point on the lens surface, ensuring the surface has the right local curvature such that the refracted marginal rays fall inside the absorption region. This type of marginal ray analysis was first presented in [83] for designing a 1-D lenticulated faceplate for a Schottky IR-CCD array. Assuming the lens surface is an arc from a circle, the optimum radius of curvature can be obtained from the analysis. Adapting from this method, we propose an iterative procedure which is more versatile since there is no assumption on the lens surface profile.

The synthesis procedure starts with geometric ray optics as shown in Fig. 3.7. The left and right marginal rays with incident angles of $\theta_L$ and $\theta_R$ impinge at an arbitrary location $x_i$ on the lens. Locally, the lens surface can be considered flat and the angle of the local surface normal with respect to the X-Y coordinate system is defined by $\theta_n$. Treating this as a local refraction problem, the refracted angles of the marginal rays are represented by $\theta'_L$ and $\theta'_R$. From these angles, the incident locations at the bottom of the pixel for both marginal rays can be solved. Thus, the goal is to make sure the incident locations are inside the absorption region. Using Snell’s law and basic trigonometry, the a system of equations of (3.1) to (3.4) can be established to solve for the optimal local $\theta_n$ and $h$ to meet the above condition.

\begin{align*}
h \tan \theta'_L &= x_i - \frac{W_{\text{sensor}}}{2} \quad (3.1) \\
h \tan \theta'_R &= x_i + \frac{W_{\text{sensor}}}{2} \quad (3.2) \\
\theta'_L &= \theta_n + \sin^{-1} \left( \frac{n_1}{n_2} \sin (\theta_L - \theta_n) \right) \quad (3.3)
\end{align*}
Chapter 3. Metalenses for Short-Wave Infrared Focal Plane Arrays

Figure 3.6: Instead of bundling rays according to the incident angle as shown in Fig. 3.5b, the rays are bundled together according to its point source. The rays emanating from the same point source have the same color and can be bundled together, effectively forming a light cone. However, it is not necessary to analyze each individual ray in a light cone, only the marginal rays are analyzed. If the two refracted marginal rays land inside the absorption region, then all the internal rays are guaranteed to land inside it. Hence, the problem is simplified to synthesizing a surface profile such that the marginal rays from each point source (that are incident on different positions on the lens) land inside the absorption region.

\[ \theta'_R = \theta_n + \sin^{-1} \left( \frac{n_1}{n_2} \sin (\theta_R - \theta_n) \right) \]

(3.4)

Figure 3.7: Geometric ray optics for marginal ray analysis. The width of the absorption region is \( W_{\text{sensor}} \). The refractive indices of air and the InP substrate is \( n_1 \) and \( n_2 \), respectively. The refractive index of InP is 3.14 at SWIR frequencies. The refracted left and right marginal rays should land on the edge or inside the absorption region. This can be achieved by solving the required local height \( h \) and local angle \( \theta_n \) at the position \( x_i \).

Once the local \( \theta_n \) and \( h \) at a particular incident location are determined, the same procedure can be repeated for the rest of the locations on the lens surface. This iterative procedure is shown in Fig. 3.8. The incident locations are discretized to an array points specified by \( x_i \) with a spacing of \( \Delta x \), which is typically assumed to be a hundredth of the pixel size. The starting point of the iterative procedure
is at the middle of the lens, where we assume the surface normal to the lens is aligned with the \( y \)-axis. By solving the system of equations at a starting point, the maximum substrate height \( h_{\text{max}} \) is determined. As shown in Fig. 3.8, at the maximum substrate thickness, both refracted marginal rays are at the boundaries of the absorption region. Once \( h_{\text{max}} \) is determined, \( \theta_{n,0} \) and \( h(x_0) \) are solved at the next incident location \( x_0 \) with an additional constraint of \( h(x_0) = h_{\text{max}} - \Delta x \tan \theta_{n,0} \). This process is repeated to solve for all the \( \theta_{n,i} \) and \( h(x_i) \). Once a set of \( h(x_i) \) is obtained, the surface of the lens can be constructed as shown in Fig. 3.9.

**Figure 3.8:** The marginal ray analysis is an iterative procedure. The lens is discretized by \( \Delta x \). Local \( \theta_n \) and \( h \) are solved for each \( x_i \). The red bar represents the absorption region.

**Figure 3.9:** The surface profile of a single microlens can be completely defined by \( h(x_i) \), which is the set of the local substrate heights. The red bar shows the absorption area.

With the above procedure, we construct the microlens for a single 25 \( \mu \)m pixel with a 18 \( \mu \)m absorption region. The marginal rays impinge at \( \pm 25^\circ \). Fig. 3.10 shows the constructed lens surface using this iterative algorithm. The maximum substrate thickness is calculated to be 63.2\( \mu \)m and the lens itself has a thickness of 1.75\( \mu \)m. In fact, the surface profile of the microlens is very close to a parabola described by (3.5). From this parabolic equation, the focal length of the microlens in air (illuminated from the InP side) and in an InP substrate (illuminated from air side) can be calculated from (3.6) and (3.7), respectively; where \( C = 0.0112 \). The numerical aperture of the microlens is approximated to be 0.56
as shown in (3.8). In the above synthesis, the iterative procedure starts at the center with maximum substrate thickness \( h_{\text{max}} \). In fact, it is possible to start the iterative procedure with a value less than \( h_{\text{max}} \). Appendix A shows some examples of the resulting lens shapes. Once the surface profile of a 1-D lens is obtained, we assume the 2-D lens can be constructed by revolving the 1-D surface profile around the optical axis.

\[
h(x) = -0.0112x^2 + 63.2 \quad (3.5)
\]

\[
f_{\text{air}} = \frac{1}{4C} = 22.3\,\mu\text{m} \quad (3.6)
\]

\[
f_{\text{InP}} = \frac{1}{4Cn_{\text{InP}}} = 70.8\,\mu\text{m} \quad (3.7)
\]

\[
NA = \frac{D}{2f_{\text{air}}} = 0.56 \quad (3.8)
\]

### 3.2.3 2D Ray-Tracing Simulations

To validate the synthesized lens profiles, COMSOL Multiphysics was used to perform ray-tracing simulations. Fig. 3.11 shows the simulation setup for three pixels. Each pixel unit cell is modeled as two parts: 61.5\,\mu\text{m} thick flat substrate and a 1.75\,\mu\text{m} thick lens. The refractive index of InP is 3.14 at SWIR frequencies. A 50° light cone is incident on the central unit cell from the top and the neighboring unit cells can be used to observe light collection and crosstalk effects. The simulated result is shown in Fig. 3.12. The incident cone is laterally shifted by 0\,\mu\text{m}, 5\,\mu\text{m}, and 10\,\mu\text{m} respectively. In all cases, the transmitted rays fall within the absorption region. As we have discussed before, the incident rays can be represented by either light cones or parallel rays from different angles (see Fig. 3.5). Fig. 3.13 shows the simulation with parallel rays that impinge at 0°, 12.5°, and 25°. At 0° (normal incidence), the rays are focused inside the absorption region instead of at the interface. This is because the focal length is 70.8\,\mu\text{m} from (3.7), which is larger than the maximum substrate thickness of 63.2\,\mu\text{m}. For a 25° angle of incidence, the rays are focused near the edge of the absorption region. By examining Fig. 3.14, which
shows the rays at bottom of the InP substrate for a 25° angle of incidence, it is clear that refracted rays are concentrated inside the absorption region, which spans from −9 μm to 9 μm.

**Figure 3.11:** The simulation contains three pixels. Each pixel unit cell is modeled as two parts: a 61.5 μm thick flat substrate and a 1.75 μm thick lens. The center pixel is illuminated with a light cone that can be moved laterally in the simulation.

**Figure 3.12:** The light cone is represented by a ray bundle and it is laterally shifted by 0 μm, 5 μm, and 10 μm away from the optical axis of the central pixel. It is clear that all the refracted rays land inside the absorption region.

**Figure 3.13:** The microlens array is illuminated with parallel rays from 0°, 12.5°, and 25°.
3.3 Transforming the Microlens into a Metalens

As was described previously, it is very challenging to fabricate such small microlenses, especially if an accurate surface profile is desired. Hence, we propose to transform the convex-shaped microlens into a flat metalens, in which the wavefront shaping capability is identical to conventional lenses. The concept of flat lenses is not new to the microwave community. In 1948, Kock [102] experimentally demonstrated an artificial material composed of metallic antennas embedded in a polystyrene foam host to build lightweight lenses in the microwave range. This lead to the development of metal based artificial dielectrics and flat graded-index (GRIN) lenses with these artificial dielectrics. In the optical regime, it is difficult to fabricate artificial dielectrics which usually require multiple metallic layers. Thus, single layer plasmonic metasurface lenses [103–109] generated a lot of interest. However, the metal used in these plasmonic lenses usually introduce large Ohmic losses. This loss is also apparent in the AAS waveplates presented in Chapter 2. At LWIR, the metal is reasonably opaque and the loss is tolerable. At SWIR, the metal loss can be very significant. As a result, a metalens without metal is preferable for maintaining high efficiency. In recent years, a significant effort has been made in the scientific community to design highly efficient lenses that are purely made of dielectric materials, which can have small losses in the optical regime [16, 110–112]. These lenses are designed with dielectric building blocks arranged on a lattice as in many metamaterial designs. Hence, these lenses are also termed metalenses.

The dielectric building blocks of these metalenses can be divided into 4 major groups as shown in Fig. 3.15: nano-beam [113–120], nano-fin [118, 121–129], nano-disk [130–136], and nano-pillar [137–155] structures. The nano-beams work similarly to dielectric gratings, which can be engineered to accept light at different incident angles and steer to desired directions. Typically, these nano-beam based metalenses are polarization-sensitive. The nano-fin designs achieve wavefront control with the Pancharatnam-Berry (PB) phase [70], also known as geometric phase. These PB phase based nano-fin metalenses are conceptually similar and are technically closely-related to the optical transmitarrays we presented in Chapter 2. In this approach, all the nano-fins have an identical size, and the local phase variation is achieved via rotations around each nano-fin’s vertical axis of symmetry. In addition, each nano-fin is engineered as a half-wave plate. Thus, the nano-fins are of rectangular shape to produce the required birefringence. Same as in the AAS based optical transmitarray, these metalenses only work for

![Figure 3.14: Rays at the bottom of the InP substrate with a 25° incidence. All the rays land inside the absorption region, which spans from −9 μm to 9 μm.](image)
circular polarization.

To enable polarization-insensitive wavefront manipulation, building blocks with circular or fourfold-symmetric cross sections such as nano-disk and nano-pillars are utilized. The nano-disks work as dielectric resonators and are designed so that the electric and magnetic dipole resonances overlap (thus meeting the first Kerker condition, in which reflection is inhibited by destructive interference [130,133]). The working principle of the nano-disks is closely related to the Huygens’ metasurface in the microwave regime [156,157]. By changing the diameter of the disk, a $2\pi$ phase shift range can be achieved with near unity transmission. Since nano-disks are resonance-based structures, and the unity transmission is achieved by overlapping the electric and magnetic resonances, the operation can be highly sensitive to the change in wavelength and incident angle. Hence, nano-disk based metalenses usually have very narrow bandwidth and angular range. However, the low aspect-ratio (height to diameter/width) of the disks makes them easier to fabricate.

In contrast to the nano-disks, the nano-pillars usually have larger aspect-ratios, and the working principle can be quite different. The nano-pillars can be treated as truncated waveguides, and the phase accumulation is achieved via propagation in the pillars. By adjusting the diameter of the pillar, the effective waveguide mode index, which governs the propagation phase, can be tailored. Since the nano-pillars are usually made of high-index dielectric material, the electric field is mostly confined inside the pillars [144]. In addition, Fabry-Perot effects are present due to reflections at the waveguide ends (which are caused by the refractive index mismatch between air and the dielectric material). Hence, the pillars are essentially weakly coupled low-quality factor resonators. It is possible to excite the resonant mode with the axial component of the electric field from a transverse-electric (TE) polarized wave [147]. In general, one desires to utilize the waveguide mode instead of the resonant mode, since it would reduce the transmission efficiency.

![Dielectric building blocks for metalenses. (a) nano-beam [116], (b) nano-fin [122], (c) nano-disk [133], and (d) nano-pillar [150] (©2016 ACS). These building blocks are low loss dielectrics sitting on optically transparent supporting substrates (shown in gray).](image)

In the following sections, we explore the possibility of designing a metalens with sub-wavelength sized
nano-holes etched into the InP substrate. For each nano-hole unit cell, the diameter of the nano-hole is adjusted such that the low-index material (air) and the high-index material (InP) are mixed in various proportions and locally produce the required effective index [158], which in turn produces the desired local phase delay. Same as the nano-pillars, the nano-holes possess circular symmetry; thus, the resulting metalens is polarization-insensitive. In addition, controlling the transmission phase is the same as in the nano-pillar designs. By varying the hole diameter, the phase can be tuned between 0 to 2\( \pi \). However, the nano-holes can be more broadband than the nano-pillars. As shown in Appendix B, the nano-pillars have an additional resonance beyond those presented in the nano-holes. This is because each pillar is an isolated high-index cylinder and can be easily excited as a dielectric resonator. In comparison, the high-index material in the nano-hole design is inter-connected and the electric field is not highly confined. Hence, it is more difficult to excite the resonator mode in nano-holes, making it more broadband than the nano-pillars.

### 3.3.1 Metalens Unit Cell Design

The design of the nano-hole based metalens starts from the unit cell characterization. The nano-holes are arranged in a hexagonal lattice as shown in Fig. 3.16. To avoid grating lobes, the lattice constant should be less than \( \lambda_0/2 \) (same as a square lattice) [159], where \( \lambda_0 \) is the wavelength in free space. If the shortest operation wavelength is 0.9 \( \mu \)m, then the nano-hole periodicity should be less 450 nm. To have a finer phase sampling, \( p \) is chosen be 400 nm. Due to fabrication constraints, the diameter of the nano-holes can be varied between 100 nm and 300 nm (a minimum gap between two adjacent hole edges has to be larger than 100 nm).

Fig. 3.17 shows the simulated transmission phase and magnitude of the nano-hole unit cells with respect to the hole diameter for different hole depths. It can be seen that a minimum of 2000nm hole depth is required to achieve the 2\( \pi \) phase range. Furthermore, the nano-holes are fairly transmissive; at least 85% efficiency is maintained for all diameters. The variation in the transmission efficiency can be attributed to the impedance mismatch due to the different effective indices for different hole diameters.
3.3.2 Modification of Unit Cells Based on the Preliminary Etching Results

From the idealized nano-hole model, at least 2000 nm depth is required to achieve a full $2\pi$ phase delay. This large depth leads to a very high aspect-ratio for the smallest nano-holes (2000 nm/100 nm). Thus, it was useful to do some preliminary fabrication to evaluate the feasibility of etching nano-holes in bulk InP with such large aspect ratios.

The nano-holes were fabricated with reactive ion etching performed at the National Research Center (NRC) in Ottawa, Canada. The etching process can be summarized as follows:

1. Deposit 300 nm of PECVD oxide on InP samples (deposition was done at 300°C).
2. Spin-coat the samples with ZEP520A e-beam resist.
3. Pattern arrays of nano-holes of different diameters. Patterning was done at 7 different exposures to determine the proper pattern transfer.
4. Dry etch oxide using ZEP520A as a mask.
5. Dry etch InP using SiO$_2$ as a mask.

Based on the experiments, at least 1 minute of etching time is required to achieve an etch depth of 2 µm. Table 3.1 to Table 3.6 show the etching results for nano-holes with six different diameters.
between 100 nm and 500 nm. It is clear that the etched nano-hole shapes are far from the ideal ones in the simulation models. In particular, the etched nano-hole side walls are not perfectly vertical; instead, the nano-holes are tapered and the side wall slope is approximately $1^\circ$. Furthermore, both the sidewall slope and the etch depth are dependent on the nominal hole diameter. Thus, the unit cell models have to be modified accordingly and corresponding transmission phase and magnitude have to be re-evaluated.

Table 3.1: 100 nm nano-hole characteristics

<table>
<thead>
<tr>
<th>Nominal $D_{\text{hole}}$</th>
<th>100nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole No.</td>
<td>1</td>
</tr>
<tr>
<td>Depth ($\mu$m)</td>
<td>2.30</td>
</tr>
<tr>
<td>Average depth ($\mu$m)</td>
<td></td>
</tr>
<tr>
<td>Average cross-section diameter ($\mu$m) at:</td>
<td>0 $\mu$m depth</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
</tr>
<tr>
<td>Approximate sidewall slope</td>
<td></td>
</tr>
</tbody>
</table>

FESM image

Table 3.2: 150 nm nano-hole characteristics

<table>
<thead>
<tr>
<th>Nominal $D_{\text{hole}}$</th>
<th>150nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole No.</td>
<td>1</td>
</tr>
<tr>
<td>Depth ($\mu$m)</td>
<td>2.80</td>
</tr>
<tr>
<td>Average depth ($\mu$m)</td>
<td></td>
</tr>
<tr>
<td>Average cross-section diameter ($\mu$m) at:</td>
<td>0 $\mu$m depth</td>
</tr>
<tr>
<td></td>
<td>0.17</td>
</tr>
<tr>
<td>Approximate sidewall slope</td>
<td></td>
</tr>
</tbody>
</table>

FESM image

In the preliminary etching, nano-holes are only etched for six different diameters. However, in the actual metalens design, the minimum diameter change can be as small as 5 nm. Thus, many nano-hole unit cells were not characterized in the preliminary test. To obtain the possible etch depth and side wall profile for all the nano-hole unit cells, interpolation is used based on existing data points. Figure 3.18 shows the interpolated hole depth versus nominal hole diameter. It can be seen that the hole depth follows an exponential curve closely.

In addition to the interpolation of the depth, the side wall profiles of the nano-holes are also interpolated. The sides walls may be modeled with a straight slope with the angles calculated in Table 3.1.
Table 3.3: 200 nm nano-hole characteristics

<table>
<thead>
<tr>
<th>Nominal (D_{\text{hole}})</th>
<th>200nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole No.</td>
<td>1</td>
</tr>
<tr>
<td>Depth ((\mu)m)</td>
<td>3.20</td>
</tr>
<tr>
<td>Average depth ((\mu)m)</td>
<td>3.07</td>
</tr>
<tr>
<td>Average cross-section diameter ((\mu)m) at:</td>
<td>0 (\mu)m depth</td>
</tr>
<tr>
<td></td>
<td>0.22</td>
</tr>
<tr>
<td>Approximate sidewall slope</td>
<td>0.57(^\circ)</td>
</tr>
</tbody>
</table>

FESM image

Table 3.4: 300 nm nano-hole characteristics

<table>
<thead>
<tr>
<th>Nominal (D_{\text{hole}})</th>
<th>300nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hole No.</td>
<td>1</td>
</tr>
<tr>
<td>Depth ((\mu)m)</td>
<td>3.70</td>
</tr>
<tr>
<td>Average depth ((\mu)m)</td>
<td>3.63</td>
</tr>
<tr>
<td>Average cross-section diameter ((\mu)m) at:</td>
<td>0 (\mu)m depth</td>
</tr>
<tr>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td>Approximate sidewall slope</td>
<td>0.57(^\circ)</td>
</tr>
</tbody>
</table>

FESM image

Figure 3.18: Interpolated etch depth versus hole diameter. The blue error bars indicate the minimum and maximum measured depths. The red dots indicate the average depths. It is clear that the hole depth is an exponential function of the hole diameter.
Table 3.5: 400 nm nano-hole characteristics

<table>
<thead>
<tr>
<th>Hole No.</th>
<th>Depth (µm)</th>
<th>Average depth (µm)</th>
<th>Average cross-section diameter (µm) at:</th>
<th>Approximate sidewall slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.00</td>
<td>3.90</td>
<td>4.00</td>
<td>1.43º</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td></td>
<td>1.37</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.90</td>
<td></td>
<td>0.30</td>
<td></td>
</tr>
</tbody>
</table>

FESM image

Table 3.6: 500 nm nano-hole characteristics

<table>
<thead>
<tr>
<th>Hole No.</th>
<th>Depth (µm)</th>
<th>Average depth (µm)</th>
<th>Average cross-section diameter (µm) at:</th>
<th>Approximate sidewall slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.10</td>
<td>4.03</td>
<td>4.00</td>
<td>1.15º</td>
</tr>
<tr>
<td>2</td>
<td>3.90</td>
<td></td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.10</td>
<td></td>
<td>0.33</td>
<td></td>
</tr>
</tbody>
</table>

FESM image

to Table 3.6. However, it can be more accurate to model the side wall profiles as fitted curves based on the measured cross-sectional diameters at different depths. In addition to the diameters measured at 0µm, 1µm, and 2µm depth, the diameter at the bottom of the nano-hole is approximated to be 50 nm by examining the SEM images. By utilizing the four data points for each measured nano-hole, the side wall profiles are fitted as a third order polynomial as shown in Fig. 3.19. This fitting technique is also applied to all the nano-hole unit cells that are going to be used in the metalens design. Fig. 3.20 shows the cross-section view for all the tapered nano-holes.

Based on the fitted side wall profiles in Fig. 3.20, Fig. 3.21 shows some of the tapered nano-hole models constructed in HFSS with diameters varying from 100 nm to 300 nm. Fig. 3.22 shows the simulated transmission and phase at the center wavelength of 1.3µm. Even though the etch depth is greater than 2µm, e.g., the depth of a 300 nm hole is 3.8 µm, the phase range is 330º, which is slightly less than the required full $2\pi$. This is due to the tapering of the nano-holes, which results in a lower effective index change. Remarkably though, due to the tapering, the transmission magnitude is close to unity, which is
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Figure 3.19: With the cross-sectional diameters measured in Table. 3.1 to Table. 3.6, and assuming the diameter at the bottom of the nano-holes is 50nm, 4 data points are available to fit the side wall profile to a 3rd order polynomial curve.

Figure 3.20: Cross-sectional view of all the nano-hole unit cells in the metalens design with interpolated side-wall profiles. The bottom side of each nano-hole is assumed to be flat and has a diameter of 50 nm.

an improvement over the regular nano-holes as shown in Fig. 3.17b. Thus, the tapered nano-holes act as an GRIN medium in the vertical direction, in which the impedance is gradually matched from InP to free space. With such high transmission, one may not need an anti-reflection layer, which would be
required for conventional microlenses.

Figure 3.21: (a) to (i) show the cross-sectional view of the tapered nano-hole unit cells with nominal diameters from 100 nm to 300 nm and with a 25 nm increments.

Figure 3.22: (a) Transmission phase and (b) magnitude of the tapered nano-hole unit cells as a function of the nominal hole diameter. Achieved phase range is $330^\circ$, and the average transmission magnitude is near unity. The unit cell simulation is performed at 1.3 $\mu$m.
3.4 Full-wave Simulation of the 2D Metalens

3.4.1 2D Metalens Design

In the previous lens synthesis procedure, the surface profile was obtained for a 1D microlens. We assume a corresponding 2D design can be obtained by simply revolving the 1D microlens around its optical axis. This 2D microlens can be further transformed into a 2D metalens. The 2D metalens has the same focal length as the microlens, which is 22.3 µm. Based on this focal length, the required phase profile \( \varphi_{ml} \) of the metalens can be calculated from

\[
\varphi_{ml}(x,y) = \frac{2\pi}{\lambda_d} \left( \sqrt{x^2 + y^2 + f^2} - f \right),
\]

(3.9)

where \( \lambda_d \) is the 1.3 µm design wavelength; and \( x \) and \( y \) are the coordinates on the metalens. Notice that the \( \varphi_{ml} \) is calculated at a discrete set of coordinates that specify the unit cell centers on a hexagonal lattice. The corresponding phase profile of the metalens is shown in Fig. 3.23. At each lattice point, the tapered nano-hole unit cell that has the closest phase to the required one is placed at that point. This nano-hole placement procedure is repeated at every lattice point, and the resulting 2D metalens is shown in Figure 3.24. Here only half of the constructed 25 µm x 25 µm metalens is displayed. Multiple phase-wraps, which are the regions with rapid transition from largest nano-holes to smallest nano-holes, can be observed.

![Figure 3.23: Calculated phase profile of the 2D metalens. Base](image)

3.4.2 Proposed Simulation Procedure

Due to the fine feature of the tapered nano-holes, it is not feasible to simulate the metalens with ray-tracing techniques. In addition, the periodicity of the nano-holes is close to a half-wavelength at 0.9 µm. Ray-tracing formulation may not be valid. Thus, the full-wave method is employed. The incident 50° light cone is approximated with a Gaussian beam with a divergence angle \( \theta_{beam} \) of 25°. In HFSS, by setting the Gaussian beam-waist \( w_0 \) to an appropriate value from (3.10), the Gaussian beam with correct divergence angle can be obtained.

\[
w_0 = \tan^{-1} \frac{\lambda}{\pi \theta_{beam}} = 0.89 \mu m
\]

(3.10)
Fig. 3.24: Half of the 25 μm × 25 μm metalens with a cross-sectional view. Multiple phase-wraps and the variation in the hole depths can be observed.

Fig. 3.25 shows the model for a single pixel that includes the metalens and the absorption region, which is 63.2 μm below the lens surface. The Gaussian beam should be simulated to impinge at different positions on the lens. From the beam shape profiles, the percentage of the energy concentrated in the absorption region can be calculated. However, the entire volume of the pixel model is 25 μm × 25 μm × 63.2 μm, which is too large to be simulated. Hence, techniques are employed to reduce the simulation space.

Fig. 3.25: A Gaussian beam is simulated to impinge at different positions on the metalens. The resulting beam shape at the bottom can be used to calculate the energy concentration efficiency.

Since the electric field of the incident Gaussian beam is mostly concentrated in a small region on the
lens, it is not necessary to simulate the entire lens. The required computational resource can be greatly reduced by only simulating a small lens section near the incident location as shown in Fig. 3.26. The simulation volume of a single lens section is $5 \mu m \times 5 \mu m \times 5 \mu m$ and the Gaussian beam impinges at the center. From the electric field plot, it is clear that the field intensity is very small outside the simulation volume. Hence, the full-size lens can be divided into numerous lens sections as shown in Fig. 3.27a. Due to the symmetry of the lens, ten lens sections are used to characterize the full-size lens. Lens section 7-10 are used to investigate the crosstalk at the sensor if the Gaussian beam impinges at the edge of the lens. Since the simulation volume is limited to $5 \mu m \times 5 \mu m \times 5 \mu m$, the absorption region, which is $63.2 \mu m$ below the lens surface, is not included in the simulation. To obtain the beam spot profile at the absorption region, a near-field to far-field transformation (NFFFT) algorithm [54] is used. Electric fields at the bottom face of the simulation volume in Fig. 3.26b can be sampled and transformed to fields at the absorption region. As a result, the required computational resource can be greatly reduced.

![Diagram](image)

**Figure 3.26:** (a) The simulation volume is limited to $5 \mu m \times 5 \mu m \times 5 \mu m$ to save the computational resource. The Gaussian beam is incident at the center of the section with the beam waist aligning with the top surface of the lens. To obtain the beam spot size at the absorption region, which is $63.2 \mu m$ below the lens, a near-field to far-field transformation algorithm is used. (b) From the simulated electric field result, it is apparent that the field outside the simulation space is negligible.

### 3.4.3 Gaussian Beam Incidence Simulation Results

Fig. 3.28 shows the electric field profiles obtained from NFFFT for each lens section. We can see that for lens sections 1-4, the resulting beam spot is fairly well contained within the absorption region. However, for lens sections 5 and 6, where the Gaussian beam impinges on the phase-wraps, the resulting beam spot is severely distorted. The energy is not well contained in the absorption region and there is strong crosstalk to adjacent pixels. For lens sections 7-10, where the Gaussian beam impinges at the edge of the pixel, the energy splits between two adjacent pixels. However, the energy that is collected by the absorption region is small. Fig. 3.29 shows the beam pattern for lens section 1 simulated with different incident Gaussian beam polarizations. It is clear that the lens is polarization-insensitive. This is because the nano-holes are circularly symmetric and do not introduce anisotropy in the metalens.

Due to the added distortion from the phase-wraps and the possible discrepancy in the focal length between the metalens and ideal microlens, placing the absorption region at $63.2 \mu m$ below the lens...
Figure 3.27: (a) The 25 $\mu$m $\times$ 25 $\mu$m metalens can be divided into many 5 $\mu$m $\times$ 5 $\mu$m lens sections. Due to the symmetry of the lens, 10 lens sections are selected to characterize the full-size lens. The Gaussian beam impinges at the center of each lens section and the resulting beam shape at the absorption region is investigated. Lens sections 7-10 are used to investigate the resulting beam shape if the Gaussian beam impinges at the edge of the lens. (b) Detailed nano-hole pattern for each lens section.
surface may not be the optimal position. In Fig. 3.30, the percentage of energy concentrated in the absorption region and in the pixel is plotted the $z$-axis (in the vertical direction), from which we can find the optimal depth to place the absorption region to maximize the energy concentration and minimize the crosstalk. 50 $\mu m$ seems to be this optimal depth, since the energy concentration in the absorption region peaks around this depth. Despite the effort, less than 50% of the power is concentrated in the absorption and more than 30% of the power leaks into adjacent pixels when the Gaussian beam impinges on the phase-wraps (lens sections 4 and 5). Thus, for energy concentration purposes, it is preferable to design a metalens without phase-wraps. In addition, phase-wraps should be eliminated for broadband performance. A phase-wrap free metalens is possible if the nano-holes can achieve a phase range that is more than $2\pi$. However, due to fabrication limitations, it is difficult to etch deeper holes without merging the adjacent holes. In addition, the tapering in the nano-holes makes deep etch has effective. Other methods require modification of the system, i.e., using smaller pixel size and/or reducing the numerical aperture of the imaging lens. For example, if the pixel size is reduced to $10 \mu m \times 10 \mu m$ and the $f/D$ ratio of the imaging lens is increased from 1 to 2, then it is possible to design a metalens free of phase-wraps and it can concentrate energy inside an $8 \mu m \times 8 \mu m$ absorption region. However, for the current system, the metalens will suffer from this kind of scattering at the phase-wrap regions.

Figure 3.28: The resulting beam shape for each lens section is obtained from NFFFT. The pixel area and the absorption region are outlined by the larger red and smaller blue dashed boxes, respectively.
Figure 3.29: The lens section 1 is illuminated with Gaussian beams with (a) X-polarization, (b) Y-polarization, and (c) diagonal-polarization. From the obtained beam shapes at the absorption region, it is apparent that the lens is polarization insensitive.

Figure 3.30: (a) Energy concentration inside a 18 µm x 18 µm square is plotted against the distance $z$ away from the lens surface. From (a), we can obtain an optimal position to place the absorption region such that the energy concentration is maximized in the absorption region under all incident conditions. The absorption region should be placed at 40 µm away from the lens surface. From (b), we can see that the energy is poorly contained in the pixel if the Gaussian beam is incident at the phase-wraps. In particular, if the absorption region is placed at the optimal position ($z = 40$ µm), the energy concentration is less than 70%. Under such a scenario, we expect a large crosstalk between adjacent pixels.
3.5 Metalens Focusing

From the simulation results from the previous section, it was found that the metalens is not very efficient for increasing the fill factor of the current FPA design, especially when the Gaussian beam impinges at the phase-wraps of the lens. However, due to the high transmission of the unit cells (see Fig. 3.22b), the metalens can be used as a highly efficient focusing device. In this section, we investigate the focusing capability of the metalens in air with a normally incident plane wave. In addition to the metalens presented in the last section, two more metalenses are designed to serve as references as shown in Fig. 3.31. Metalens-1 is the one we designed in the last section. Metalens-2 is a 25 µm × 25 µm lens without phase-wraps, and metalens-3 is a 10 µm × 10 µm lens without phase-wraps. For these two metalenses, the maximum phase variation along the diagonal is 2π. All the metalenses are designed at the center wavelength of the SWIR spectrum, which is 1.3 µm. Table 3.7 summarizes the characteristics of all the metalenses at the design frequency. Metalens-2 and metalens-3 have larger \( f/D \) in order to eliminate the phase wraps. Hence, these two metalenses should be ideally used for FPAs with small NA imaging lens or FPAs with smaller pixel size. The impact of phase-wraps and the lens size on the focusing capability can now be investigated. These metalenses will be characterized by both simulations and measurements in the following sections.

**Table 3.7:** Characteristics of three metalenses designed at 1.3 µm

<table>
<thead>
<tr>
<th>Size</th>
<th>Maximum phase variation</th>
<th>Focal length</th>
<th>( f/D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metalens-1</td>
<td>25 µm × 25 µm</td>
<td>1700°</td>
<td>22.3 µm</td>
</tr>
<tr>
<td>Metalens-2</td>
<td>25 µm × 25 µm</td>
<td>360°</td>
<td>119.5 µm</td>
</tr>
<tr>
<td>Metalens-3</td>
<td>10 µm × 10 µm</td>
<td>360°</td>
<td>18.6 µm</td>
</tr>
</tbody>
</table>

**Figure 3.31:** (a) metalens-1 is the 25 µm × 25 µm metalens designed in the last section. (b) metalens-2 is 25 µm × 25 µm a lens without phase-wraps. (c) metalens-3 is a 10 µm × 10 µm lens without phase-wraps.

3.5.1 Simulation Strategy

In the experimental characterization, the designed metalens are etched on the top surface of a 500 µm thick InP substrate as shown in Fig. 3.32. A weakly focused Gaussian beam that approximates a plane-wave is illuminated from the back surface of the substrate at normal incidence. The focal spot is formed
in the air above the metalens. The simulation model should emulate the measurement setup. However, it is impractical to include both the 500 μm thick substrate and large air region above the metalens in the simulation. Fig. 3.33 shows the model with a reduced simulation space of 12.5 μm × 12.5 μm × 5 μm. With a plane wave excitation at normal incidence, symmetry boundary conditions can be used such that only a quarter of the lens needs to be simulated. The InP region is to limited 4 μm, which is slightly thicker than the deepest nano-hole in the metalens. By setting the bottom face of the InP region to be a radiation boundary, the InP region is effectively a semi-infinite space, which can be used to approximate the 500 μm thick substrate. A plane wave propagates upward from the bottom of the simulation domain and impinges on the metalens at normal incidence. A 1 μm air region above the lens is included in the simulation and the electric field profile is sampled at the top radiation boundary. The far-field results are obtained from the NFFFT as before. Despite all the techniques that are applied to reduce the computational resources, this is still an extremely large model. Hence, it is simulated on the SciNet [160] super-computing cluster at the University of Toronto with a total memory usage of 1 terabyte.

![Diagram of metalens simulation model](image)

**Figure 3.32:** Metalenses are simulated in a way that corresponds to the experimental characterization. The metalens is patterned on the top surface of the 500 μm thick InP substrate. A weakly focused Gaussian beam that approximates a plane-wave illuminates the lens from the back surface of the substrate. A focal spot is formed above the lens.

### 3.5.2 Simulation Results

The plane-wave simulations are performed at 0.9 μm, 1.1 μm, 1.3 μm, 1.5 μm, and 1.7 μm to characterize the focusing performance as a function of the wavelength. Fig. 3.34 to Fig. 3.38 plot the electric field in the X-Z plane and the focal plane for the metalens-1 at different wavelengths. Clear and circular focal spots can be observed at all wavelengths, indicating the broadband nature of the metalens. The focal length or the focal plane (shown by the red dashed line) is defined as the plane which has the highest field intensity. At the design wavelength of 1.3 μm, the simulated focal length is 21 μm. This is very close to the nominal focal length of 22.3 μm obtained in (3.6). Table 3.8 summarizes the focal lengths and focal spot sizes at different wavelengths.
Figure 3.33: (a) The reduced simulation model with a total volume of $12.5 \, \mu m \times 12.5 \, \mu m \times 5 \, \mu m$. The InP region is limited to $4 \, \mu m$ thick, and the air region is limited to $1 \, \mu m$ thick. (b) The boundaries used in the boundary for reducing the computational demand. (c) The electric field is sampled at the top surface in (a), and NFFFT is used to find fields in the X-Z plane and the focal plane.

Figure 3.34: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-1 is illuminated with a $0.9 \, \mu m$ plane wave. The focal plane is at $z = 33 \, \mu m$. The gray rectangle represents $1 \, \mu m$ air above the metalens. The red dashed lines shown the focal plane.

Figure 3.35: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-1 is illuminated with a $1.1 \, \mu m$ plane wave. The focal plane is at $z = 26 \, \mu m$.

The simulation results for the metalens-2 are shown from Fig. 3.39 to Fig. 3.43. Since metalens-2 has smaller phase range (a maximum of $2\pi$), the numerical aperture is smaller than that of the metalens-1.
Naturally, the focal length is longer and the focal spot size is bigger. The simulation results for metalens-3 are shown in Fig. 3.44 to Fig. 3.48. Even though the phase range is the same as the metalens-2, the size of the lens is smaller. Hence, the numerical aperture is larger; resulting in a shorter focal length and a smaller spot size.
The phase-wrap effect can be observed by comparing Fig. 3.34a, Fig. 3.39a, and Fig. 3.44. From the X-Z plane electric field plot in Fig. 3.34a, there is a second focal spot near \( z = 15 \mu m \), which is absent in both the metalens-2 and metalens-3. The second focal spot is a result of the diffraction due to the phase-wraps, which makes the lens behave as a blazed grating at \( 0.9 \mu m \). As the wavelength increases, the grating period starts to mismatch with the wavelength, resulting to a weaker diffraction. This effect can be observed through Fig. 3.35 to Fig. 3.38. The implication is that the phase-wraps decrease the focusing efficiency, since part of the energy is diverted from the main focal spot.

![Figure 3.39: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-2 is illuminated with a 0.9 \( \mu m \) plane wave. The focal plane is at \( z = 94 \mu m \). The gray rectangle represents 1 \( \mu m \) air above the metalens.](image1)

![Figure 3.40: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-2 is illuminated with a 1.1 \( \mu m \) plane wave. The focal plane is at \( z = 74 \mu m \).](image2)

<table>
<thead>
<tr>
<th>Wavelength</th>
<th>Metalens-1</th>
<th>Metalens-2</th>
<th>Metalens-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9 ( \mu m )</td>
<td>Focal length: 33 ( \mu m )</td>
<td>Focal length: 94 ( \mu m )</td>
<td>Focal length: 17.5 ( \mu m )</td>
</tr>
<tr>
<td>1.1 ( \mu m )</td>
<td>Spot size: 1.1 ( \mu m )</td>
<td>Spot size: 3.3 ( \mu m )</td>
<td>Spot size: 1.5 ( \mu m )</td>
</tr>
<tr>
<td>1.3 ( \mu m )</td>
<td>Focal length: 21 ( \mu m )</td>
<td>Focal length: 64 ( \mu m )</td>
<td>Focal length: 12 ( \mu m )</td>
</tr>
<tr>
<td>1.5 ( \mu m )</td>
<td>Spot size: 1.3 ( \mu m )</td>
<td>Spot size: 3.5 ( \mu m )</td>
<td>Spot size: 1.7 ( \mu m )</td>
</tr>
<tr>
<td>1.7 ( \mu m )</td>
<td>Spot size: 1.5 ( \mu m )</td>
<td>Spot size: 3.7 ( \mu m )</td>
<td>Spot size: 1.8 ( \mu m )</td>
</tr>
</tbody>
</table>

The transmission efficiencies of the metalenses, which is defined as the ratio between the output power
Figure 3.41: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-2 is illuminated with a 1.3 µm plane wave. The focal plane is at z = 64 µm.

Figure 3.42: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-2 is illuminated with a 1.5 µm plane wave. The focal plane is at z = 59 µm.

Figure 3.43: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-2 is illuminated with a 1.7 µm plane wave. The focal plane is at z = 53 µm.

from the top surface of the metalenses and the incident power from the bottom, are shown in Fig. 3.49. The metalenses maintain near unity transmission from 0.9 µm to 1.7 µm. These results correspond well to the unit cell results in Fig. 3.22b as all the unit cells have near unity transmission. In fact, for the metalens-2 design, the efficiency is slightly over unity. This can be attributed to the accuracy
Figure 3.44: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-3 is illuminated with a 0.9 µm plane wave. The focal plane is at \( z = 18.5 \) µm. The gray rectangle represents 1 µm air above the metalens.

Figure 3.45: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-3 is illuminated with a 1.1 µm plane wave. The focal plane is at \( z = 14.5 \) µm.

Figure 3.46: Electric field in (a) X-Z plane and (b) focal plane at when the metalens-3 is illuminated with a 1.3 µm plane wave. The focal plane is at \( z = 13 \) µm.

of the extremely large simulation. For the smaller metalens-3 simulation with a better accuracy, the transmission stays close to unity as expected. The phase-wraps in metalens-1 result in a small decrease in the transmission; however, it is still over 90% for the entire bandwidth. It is worth pointing out that for conventional microlens designs, an anti-reflection coating is required; otherwise, 27% of the
incident power is reflected at the air/InP interface. However, the simplest anti-reflection coating, i.e., the quarter-wave transformer can only provide matching at a single wavelength. Matching from 0.9 µm to 1.7 µm requires complex multi-layer coatings, and achieving near unity transmission can still be very challenging. In Table 3.9, our proposed metalenses are compared to state-of-the-art metalenses reported in the literature. Our metalenses have the highest efficiency and they remain efficient over the entire SWIR spectrum. They have good NA and they are polarization-insensitive. Hence, the metalenses presented in this chapter can have superior performance over conventional microlenses and other metalenses in the literature.
Figure 3.49: Transmission efficiency of the metalens designs. Due to the tapered unit cells, the transmission efficiency is near unity over the entire bandwidth.

Table 3.9: Comparison of state-of-the-art metalenses

<table>
<thead>
<tr>
<th>Design</th>
<th>Year</th>
<th>Unit cell type</th>
<th>Polarization</th>
<th>NA</th>
<th>Frequency range</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metalens-1</td>
<td>2019</td>
<td>nano-hole</td>
<td>insensitive</td>
<td>0.622</td>
<td>0.9 µm to 1.7 µm</td>
<td>&gt; 90%</td>
</tr>
<tr>
<td>Metalens-2</td>
<td>2019</td>
<td>nano-hole</td>
<td>insensitive</td>
<td>0.146</td>
<td>0.9 µm to 1.7 µm</td>
<td>≈ 100%</td>
</tr>
<tr>
<td>Metalens-3</td>
<td>2019</td>
<td>nano-hole</td>
<td>insensitive</td>
<td>0.354</td>
<td>0.9 µm to 1.7 µm</td>
<td>≈ 100%</td>
</tr>
<tr>
<td>[128]</td>
<td>2019</td>
<td>nano-fin</td>
<td>CP only</td>
<td>0.2</td>
<td>460 nm to 700 nm</td>
<td>≈ 35%</td>
</tr>
<tr>
<td>[129]</td>
<td>2019</td>
<td>nano-fin</td>
<td>CP only</td>
<td>0.157</td>
<td>400 nm to 660 nm</td>
<td>≈ 39.1%</td>
</tr>
<tr>
<td>[135]</td>
<td>2018</td>
<td>nano-disk</td>
<td>sensitive</td>
<td>0.944</td>
<td>715 nm</td>
<td>≈ 37%</td>
</tr>
<tr>
<td>[155]</td>
<td>2018</td>
<td>nano-pillar</td>
<td>insensitive</td>
<td>0.35</td>
<td>3 µm to 5 µm</td>
<td>≈ 70%</td>
</tr>
<tr>
<td>[134]</td>
<td>2016</td>
<td>nano-disk</td>
<td>insensitive</td>
<td>0.75</td>
<td>633 nm</td>
<td>88%</td>
</tr>
<tr>
<td>[149]</td>
<td>2016</td>
<td>nano-pillar</td>
<td>insensitive</td>
<td>0.46</td>
<td>915 nm, 1550 nm</td>
<td>22% @915 nm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>65% @1550 nm</td>
</tr>
<tr>
<td>[122]</td>
<td>2016</td>
<td>nano-fin</td>
<td>CP only</td>
<td>0.8</td>
<td>405 nm, 532 nm, 660 nm</td>
<td>86% @405 nm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>73% @532 nm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>66% @660 nm</td>
</tr>
<tr>
<td>[161]</td>
<td>2016</td>
<td>nano-grating</td>
<td>sensitive</td>
<td>&lt; 0.166</td>
<td>495 nm, 540 nm, 630 nm</td>
<td>&lt; 15%</td>
</tr>
<tr>
<td>[132]</td>
<td>2015</td>
<td>nano-disk</td>
<td>sensitive</td>
<td>N/A</td>
<td>1550 nm</td>
<td>&lt; 45%</td>
</tr>
</tbody>
</table>
3.5.3 Measurement Results

The fabricated metalenses are shown in Fig. 3.50. The measurement setup for characterizing the metalenses is shown in Fig. 3.51 (courtesy of Dr. Flueraru at NRC Ottawa). A 1.3 μm SWIR laser is used as the source. Laser power is around 55 mW and it is attenuated to 550 μW through the variable attenuator. The laser beam is then passed through a focusing lens and a 5X illuminating objective. The collimated beam passes through a mask, which has a 25 μm × 25 μm square aperture, and then illuminates the metalens. The focal spot is formed in the air with a spot size of 1.3 μm to 3.4 μm based on the simulation results. Since the focal spot size is much smaller than the 40 μm pixel size of the camera, it is not possible to measure the focal spot size through direct imaging. Hence, a 100X imaging objective is used to magnify the image. The imaging objective can be moved along the z-axis and the image in the X-Y plane can be captured. From these images, the intensity along the z-axis can be measured. Due to the limited tunability of the laser, the characterization is carried out only for 1.3 μm.

![Figure 3.50: Fabricated (a) metalens-1, (b) metalens-2, and (c) metalens-3.](image)

For comparison purposes, a sample without the metalens is measured. Images at different z values are shown in Fig. 3.52. The shape of the 25 μm × 25 μm square aperture remains the same and no focusing effect is observed. Fig. 3.53 shows the captured image at different z values at metalens-1. As z increases, the square aperture evolves into a focal spot. Fig. 3.54 plots the simulated and measured intensity along the optical axis of metalens-1. The simulated focal length is 21 μm and the measured one is 20 μm. Fig. 3.55 shows the measured result for metalens-2. A tight focus can be observed around 60 μm. From Fig. 3.56, the simulated focal length is around 64 μm and the measured one is at 58 μm. The ripples in the simulated results could be attributed to simulation inaccuracies. Fig. 3.57 shows the measured result for metalens-3. Again, a clear focal spot can be observed. There are some discrepancies between the simulated and measured focal length as shown in Fig. 3.58. The measured focal length is 8 μm longer than the simulated one. We speculate the discrepancy could be a result of a smaller sample, which has less tolerance to dimension variations. Furthermore, the mask opening remains unchanged from 25 μm × 25 μm when measuring a 10 μm × 10 μm lens. The spill-over light could result in an inaccurate intensity measurement.

Fig. 3.59 shows the captured images at the focal planes for the metalenses. For comparison purposes, the image for a reference sample without metalenses is also shown. From the intensity profile, Metalens-1 has the smallest focal spot as expected. It is worth pointing out that the camera used has a 40 μm pixel
The focal spot is around 1.5 μm-3.4 μm, which is much smaller than the 40 μm pixel size of the camera. Hence, a 100X magnifying imaging objective is used to characterize the spot size.

The mask has a 25 μm by 25 μm square aperture.

**Figure 3.51:** (a) Schematic diagram of the measurement setup (b) actual measurement setup (c) mask pattern (courtesy of Dr. Fleraru at NRC Ottawa)

size, which may not be accurate enough for metalens-1 and metalens-3 even after 100X magnification. Their focuses are so small and fall onto only one or two pixels as shown in Fig. 3.59b and Fig. 3.59d. Hence, it can be difficult to obtain accurate spot sizes for these two metalenses. Fig. 3.60 shows the simulated and measured intensity profiles in the focal planes along the x-axis. From the intensity lines, the spot size (FWHP) can be obtained. The simulated and measured results are in a very good
Figure 3.52: Intensity profile measured at different image planes along the z-axis for a sample without metalenses. As expected, no focusing effect is observed for the 25 µm × 25 µm square aperture.

Figure 3.53: Intensity profile measured at different image planes along the z-axis for metalens-1. As z increases, the square aperture evolves to a focal spot around z = 20 µm.

Figure 3.54: Simulated and measured intensity along the optical axis of metalens-1. The simulated and measured focal lengths are 21 µm and 20 µm, respectively. There is slight difference in the intensity distribution. We speculate this is caused by a slight misalignment of the sample.

Figure 3.55: Intensity profile measured at different image planes along the z-axis for metalens-2. As z increases, the square aperture evolves to a focal spot around z = 58 µm.
Figure 3.56: Simulated and measured intensity along the optical axis of metalens-2. The simulated intensity shows a lot of ripples, which could be a result of simulation inaccuracies. By smoothing the simulated curve, the focal length is found to be 64 $\mu$m. The measured focal length is 58 $\mu$m.

Figure 3.57: Intensity profile measured at different image planes along the z-axis for metalens-3. As $z$ increases, the square aperture evolves to a focal spot around $z = 20 \mu m$.

Figure 3.58: Simulated and measured intensity along the optical axis of metalens-3. The simulated and measured focal lengths are 12 $\mu$m and 20 $\mu$m, respectively. We speculate the discrepancies is due to the smaller sample size.
agreement. The measured results can be improved further if the camera has a smaller pixel size. The simulated measured focal length and spot size for each metalens are summarized in Table 3.10.

Figure 3.59: Captured images for (a) a reference sample without metalenses, (b) metalens-1 at focal plane, (c) metalens-2 at focal plane, and (d) metalens-3 at focal plane.

Figure 3.60: Simulated and measured intensity profiles along the x-axis at the focal plane for (a) metalens-1, (b) metalens-2, and (c) metalens-3. Due to the large camera pixel size, the small spot sizes for metalens-1 and metalens-3 cannot be measured very accurately. The simulated and measured spot sizes are summarized in Table 3.10.
Table 3.10: Simulated and measured focal lengths and focal spot sizes for the three metalenses

<table>
<thead>
<tr>
<th></th>
<th>Metalens-1</th>
<th></th>
<th>Metalens-2</th>
<th></th>
<th>Metalens-3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Focal length</td>
<td>Spot size</td>
<td>Focal length</td>
<td>Spot size</td>
<td>Focal length</td>
<td>Spot size</td>
</tr>
<tr>
<td>Simulated</td>
<td>21 µm</td>
<td>1.3 µm</td>
<td>64 µm</td>
<td>3.4 µm</td>
<td>12 µm</td>
<td>1.6 µm</td>
</tr>
<tr>
<td>Measured</td>
<td>20 µm</td>
<td>1.7 µm</td>
<td>58 µm</td>
<td>3.2 µm</td>
<td>20 µm</td>
<td>1.9 µm</td>
</tr>
</tbody>
</table>
3.6 Conclusion and Outlook

In this chapter, we investigated the feasibility of improving the fill-factor of an infrared focal-plane-array. We synthesized a microlens with a specific surface profile to concentrate the incident light into a 18 $\mu$m $\times$ 18 $\mu$m absorption region for a 25 $\mu$m $\times$ 25 $\mu$m pixel. From ray tracing simulations, the synthesized microlens array works properly for the imaging lens with a f/D ratio of 1. Since it is difficult to fabricate microlenses with such a small size, a metamaterial-based lens or a metalens was proposed to replace the microlens, but achieving the same phase delay profile or numerical aperture. The metalens comprises of tapered nano-holes arranged on a hexagonal lattice with a lattice constant of 400 nm. The phase delay profile is achieved by varying the nano-hole diameter according to the position of the nano-hole on the lens. The designed metalens is simulated with a Gaussian beam incidence that mimics the light cone generated by the imaging lens. By varying the incident position of the Gaussian beam, the resulting beam shape profile and the energy concentration in the absorption region can be found. By examine the simulation results, we conclude that the metalens has a shortcoming for the energy concentration applications. Specifically, due the phase-wraps present in the metalens, the resulting beam is distorted, leading to poor concentration and large crosstalk. The phase-wraps can be eliminated if the nano-holes can cover phase range more than 2$\pi$. However, due the tapering in the nano-holes and fabrication limitations, it is very difficult to achieve a larger phase range. Alternatively these planar metalenses can be used with smaller numerical aperture lenses in which case the phase-wraps can be eliminated even with 2$\pi$ phase range.

Even though the metalens is not very suitable for energy concentration purposes with the intended f/D = 1 imaging lens, we found the metalens can be very efficient for focusing applications with a plane wave incidence. The tapering in the nano-holes achieves impedance matching from InP to air as a graded-index medium. A total of three metalenses were designed for focusing applications. One is a 25 $\mu$m $\times$ 25 $\mu$m metalens with phase-wraps, another is a 25 $\mu$m $\times$ 25 $\mu$m metalens without phase-wraps, and a third one is 10 $\mu$m $\times$ 10 $\mu$m metalens without phase-wraps. Through simulation, we found the metalenses have near unity transmission efficiency from 0.9 $\mu$m to 1.7 $\mu$m. The phase-wraps reduce the efficiency by a little for metalens-1, but it is still over 90\% over the entire bandwidth. Compared to other state-of-the-art metalenses reported in the literature, the nano-hole based metalenses have the highest efficiency and broadest bandwidth. Compared to conventional microlenses, the nano-hole metalenses have the advantage of not requiring anti-reflection coatings. A complex multi-layer anti-reflection coating is required to achieve good transmission over the entire SWIR bandwidth. With a good numerical aperture, polarization-insensitivity, broad bandwidth, and near unity transmission, the metalenses presented in this chapter are excellent candidates to replace conventional microlenses or other metalenses for imaging applications. All three metalenses were experimentally characterized at 1.3 $\mu$m. The simulated and measured focal lengths and focal spot sizes have a good agreement. However, we were not able to measure the transmission efficiency of these metalenses. In the future, we expect to experimentally characterize the transmission efficiency at different wavelengths in the SWIR spectrum.
Chapter 4

Matched Graded-index Lenses at Millimeter-wave Frequencies

4.1 Introduction

In Chapter 3, we presented a nano-hole based metalens that is low-loss and nearly reflection-less. In this chapter, we apply the hole array concept to design a reflection-less graded-index (GRIN) lens in the millimeter-wave regime. This type of lens can be used for 5th Generation (5G) mobile technology [162] or automotive radars [163]. Quasi-optical beam-steering with the dielectric lens can be an attractive option since the overall system is simple, robust, and less expensive than more complex phased arrays. One may design a conventional dielectric lens for this purpose, but it can be difficult to meet the fabrication tolerance for the curved surface profile required for mm-wave frequencies. In addition, conventional lenses usually employ a low permittivity material such as Teflon to minimize reflections. The resulting lenses can be thick, making them less ideal for integration. Some thin and flat lenses were developed based on artificial dielectrics and metamaterials [164–168]. However, at mm-wave frequencies, the metallic structures used in these lenses can lead to significant Ohmic losses. For example, in [168], due to the metal loss, the overall system gain with the metamaterial lens is lower than without the lens.

In this aspect, GRIN lenses purely made with dielectrics [169–179] become attractive as the dielectric losses can be very small in the mm-wave regime. Similar to the design of the metalens, a GRIN lens can be made by perforating a piece of dielectric substrate. However, in the metalens, the matching problem is solved elegantly with the tapered nano-holes. It is very difficult to adapt such concept to the GRIN lens designed for mm-waves, since the holes are usually drilled with constant diameters. Hence, the GRIN lens is inherently reflective, and the reflections varies spatially due to the inhomogeneity of the GRIN lens. As a result, matching the GRIN lens can be challenging. To avoid complicated matching layer designs, authors in [169] resort to using a low-index substrate for the GRIN lens. Since the index variation in the lens does not deviate much from the index of air, reflections can still be small without matching layers. However, a small index variation may lead to a thick lens as the electromagnetic wave has to propagate a sufficient distance in the lens to achieve at least a $2\pi$ phase variation across the lens. In general, the lower the index of the substrate, the thicker the GRIN lens will be [170]. Perforating such a thick lens can be problematic since the required aspect-ratios of the drill bits are large (deep drill depths combined with small drill diameters). Most often, such drill bits are not available. To make fabrication easier, the
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GRIN lens has to be made thinner by using a high-index substrate [173, 176, 177]. Using a high-index substrate can lead to strong reflections due to the large index mismatch at the air/substrate interface. Isotropic matching layers similar to QWTs were introduced in [178], but the reflection is reduced only for the central portion of the lens. To match for the entire lens, an inhomogeneous matching layer is numerically investigated in [180]. The matching strategy is based on the conventional QWT; both the index and the thickness of the matching layer are varying spatially. This concept may work theoretically, but it can be very difficult to implement in practice.

In this chapter, we present a practical approach to match a GRIN lens with GRIN matching layers. The matching layers have a fixed thickness, making them feasible for easy fabrication, i.e., perforation. The required indices in the layers for perfect matching are numerically calculated using the transfer matrix method. The matched GRIN lens is designed at 34.3 GHz to collimate a commercial pyramidal horn. Both full-wave simulations and far-field measurements are used to validate the concept.

4.2 Unit Cell Design

4.2.1 Lens Unit Cell Design

During the design of the GRIN lens unit cells, a few aspects have to be considered simultaneously. First, the unit cells should be realizable within the fabrication tolerances, i.e., holes should be designed according to the available drill bit sizes. Second, the unit cells should provide a full $2\pi$ phase shift, and the incremental steps in the phase delay provided by each unit cell should be fine enough for accurate phase sampling. Third, the periodicity should be sub-wavelength such that the effective medium assumption is valid. However, the periodicity should not be too small; otherwise, the phase sampling will be coarse. This is because drill bits have a minimum increment in the diameter, which can lead to a large increment in phase if the periodicity is too small. In addition, a sufficient gap should be kept between adjacent hole edges to avoid cracking during the perforation process.

Bearing those considerations in mind, the proposed unit cell of the GRIN lens is shown in Fig. 4.1. The substrate is Rogers Duroid RO6010 with a permittivity of 13.5 at 34.3 GHz. The lens thickness is set 5 mm. The periodicity should not be too large or too small. If the periodicity is too large, grating lobes may appear and the bandwidth may be narrowed. For a hexagonal lattice, grating lobes can appear if the lattice constant is larger than $\lambda_0/2$, which is 4.37 mm. [159]. On the other hand, if the periodicity is too small, then the hole diameter can be varied in a very limited range. This is because a minimum gap should be maintained between the edges of two adjacent holes for avoiding cracking the substrate when drilling densely packed holes. For a safe practice, the minimum edge spacing is 0.3 mm, which can severely limit the range of hole diameters if the periodicity is too small. Consequently, a full $2\pi$ phase range may not be achievable for a certain lens thickness. In addition, the typical CNC drill bit diameter increment is 0.05 mm to 0.1 mm. A 0.1 mm increment may result to a coarse phase sampling for a small periodicity.

A periodicity of 2.5 mm ($0.284\lambda_0$) is a good balance. The hole diameter can be varied from 0 mm to 2.2 mm, which is just enough to achieve a full $2\pi$ phase range with a sufficient phase sampling resolution as shown in Fig. 4.2. Fig. 4.3 shows the effective permittivity as a function of the hole diameter, which is extracted using the method outlined in Appendix C. The effective permittivity can also be approximated using the effective medium theory [158]; however, we found this approach to be less accurate than the
extraction method (see Appendix D). For \( d_{\text{tens}} = 0.9 \text{ mm} \) and 1.9 mm, near unity transmissions are observed in Fig. 4.2. The extracted effective permittivities at these diameters are 12 and 7 respectively. The corresponding electrical thicknesses of a 5 mm thick substrate are \( 2\lambda \) and \( 1.5\lambda \), where \( \lambda \) is the wavelength in the substrate. If the electrical thickness of the lossless substrate is equal to an integer number of half-wavelengths, the reflections at the two substrate/air interfaces out of phase and cancel each other. In this case, the substrate becomes reflection-less and anti-reflection layers are not required. However, in the actual lens implementation, high transmission unit cells occupy only a small portion of the lens. For unit cells with smaller diameters, the transmitted power is around 40%. In particular, these unit cells are placed in the middle of the lens where the incident power density is highest. Without proper matching layers, the total reflection loss will be significant.

From Fig. 4.3, one can notice that the minimum achievable effective permittivity is 30% (= \( 4/13.5 \)) of the substrate permittivity based on the available hole diameter variation. This limitation should be kept in mind when designing the GRIN matching layers in the next section.

\[ \begin{bmatrix} a_4 \\ b_4 \end{bmatrix} = \frac{T_{01}P_1T_{12}P_2T_{21}P_1T_{10}}{t_{01}t_{12}t_{21}t_{10}} \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \frac{[M]}{t_{01}t_{12}t_{21}t_{10}} \begin{bmatrix} a_4 \\ b_4 \end{bmatrix} \]

Figure 4.1: Unit cell design of the GRIN lens. The holes are arranged on a hexagonal lattice with a periodicity of 2.5 mm. The hole diameter can be varied from 0 mm to 2.2 mm. A triangular (which is equivalent to a hexagonal) lattice is chosen over the square lattice as it provides a greater index variation [170]. Hence, a thinner lens can be designed with a hexagonal lattice while achieving a full \( 2\pi \) phase range.

4.2.2 GRIN Matching Layer Unit Cell Design

To reduce reflections, two identical GRIN matching layers are used to match both the top and the bottom surfaces of the GRIN lens as shown in Fig. 4.4. To find the optimal local index of the matching layer, the transfer matrix method [181] is used in conjunction with the unit cell analysis. With a TEM wave incident normally from the top of the unit cell, the transfer matrix can be constructed as
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Figure 4.2: (a) The transmitted power and (b) transmission phase of the unit cell are plotted as a function of the hole diameter $d_{\text{lens}}$. Here the minimum hole diameter increment is 0.1 mm.

Figure 4.3: The effective relative permittivity is extracted as a function of $d_{\text{lens}}$. The extraction is performed at normal incidence and the method is outlined in Appendix C.

Figure 4.4: Two identical GRIN matching layers are added on both sides of the lens to reduce lens reflection. Transfer matrix method is used to find the optimal index (as a function of lens position) of the matching layers.

where $a_i$ and $b_i$ are the wave amplitudes in medium $i$. $T_{ij}$ and $P_k$ are defined as

$$T_{ij} = \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix} \quad (4.2)$$
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\[ P_k = \begin{bmatrix} e^{j\phi_k} & 0 \\ 0 & e^{-j\phi_k} \end{bmatrix} \]

(4.3)

\[ r_{ij} \text{ and } t_{ij} \text{ are the Fresnel coefficients at each layer interface, which are give by} \]

\[ r_{ij} = \frac{1}{\sqrt{\varepsilon_j}} - \frac{1}{\sqrt{\varepsilon_i}} \]

\[ 1/\sqrt{\varepsilon_j} + 1/\sqrt{\varepsilon_i} \]

(4.4)

and

\[ t_{ij} = 1 + r_{ij} \]

(4.5)

\[ \phi_k \text{ is the phase delay in each layer, which is given by} \]

\[ \phi_k = k_0 h_k \sqrt{\varepsilon_k} \]

(4.6)

The total reflection of the entire structure is given by (4.7) and the full expression is given by (4.8)

\[ r = \frac{M_{21}}{M_{11}} \]

(4.7)

\[ r = \frac{2j \left[ r_{12} \sin \phi_2 \left( 1 + r_{01}^2 \right) + r_{01} \sin (2\phi_1 - \phi_2) + r_{01}^2 r_{12} \sin (2\phi_1 - \phi_2) + r_{01}^2 r_{12} \sin \phi_2 \right]}{4j r_{01} r_{12} \sin \phi_2 + e^{j(2\phi_1 + \phi_2)} - r_{01}^2 e^{-j(2\phi_1 + \phi_2)} - r_{12}^2 e^{-j(2\phi_1 - \phi_2)} + r_{01} r_{12}^2 e^{-j(2\phi_1 - \phi_2)}} \]

(4.8)

(4.4)–(4.6) can be substituted into (4.8) to numerically solve for the zero reflection condition. Based on the extracted \( \varepsilon_{\text{lens}} \) in Fig. 4.3, the required \( \varepsilon_{\text{ML}} \) for zero reflection is plotted in Fig. 4.5 for three standard PCB thicknesses of 0.787 mm, 1.27 mm, and 1.52 mm. If the GRIN matching layer is fabricated with the same periodicity and the same range of drill diameters as with the GRIN lens, then the minimum realizable permittivity is around 30% of the substrate permittivity as mentioned in Sec. 4.2.1. The red dotted lines in Fig. 4.5 indicate the achievable permittivity ranges. For \( h_{\text{ML}} = 0.787 \) mm, the required permittivity is outside the achievable range. For \( h_{\text{ML}} = 1.27 \) mm and 1.52 mm, the required \( \varepsilon_{\text{ML}} \) is complex for certain range of \( \varepsilon_{\text{lens}} \).

Figure 4.5: The required \( \Re\{\varepsilon_{\text{ML}}\} \) (solid blue lines) and \( \Im\{\varepsilon_{\text{ML}}\} \) (dashed blue lines) for zero reflection is plotted against \( \varepsilon_{\text{lens}} \) for matching layer thicknesses of (a) 0.787 mm, (b) 1.27 mm, and (c) 1.52 mm. The red dotted lines highlight the achievable range of permittivity based on the fabrication tolerances. The minimum achievable permittivity is 30% of the maximum permittivity. For \( h_{\text{ML}} = 0.787 \) mm, the required permittivity is outside the achievable range. For \( h_{\text{ML}} = 1.27 \) mm and 1.52 mm, the required \( \varepsilon_{\text{ML}} \) is complex for certain range of \( \varepsilon_{\text{lens}} \).
\( \varepsilon_{ML} \). For \( h_{ML} = 1.27 \text{ mm} \) or \( h_{ML} = 1.52 \text{ mm} \), \( \Re \{ \varepsilon_{ML} \} \) is within the achievable range, but a complex \( \varepsilon_{ML} \) is required for a certain range of \( \varepsilon_{lens} \). Since the perforation process only tunes the real part of \( \varepsilon_{ML} \), realizing the \( \Im \{ \varepsilon_{ML} \} \) requires incorporating some lossy metallic structures. However, this is counter-productive to design a highly efficient lens, since the goal is not only minimizing the reflection, but also maximizing the transmission of the lens. In addition, a very lossy structure can be difficult to synthesize at mm-wave frequencies using regular PCB materials. A resonant structure is usually required to introduce a large loss and it can be quite narrowband. As a result, one can choose to realize \( \Re \{ \varepsilon_{ML} \} \) only and suffer some reflections.

Fig. 4.6 shows the total reflection \( r \) if only the \( \Re \{ \varepsilon_{ML} \} \) is realized. For \( h_{ML} = 1.52 \text{ mm} \), the reflection is quite strong. However, for \( h_{ML} = 1.27 \text{ mm} \), the maximum reflection is 0.085, which is equivalent to a reflected power of less than 1%. In particular, since these unit cells only occupy a small portion of the lens, the total reflection introduced is even smaller. On the contrary, if the matching layer is designed with the required complex \( \varepsilon_{ML} \), then the transmission loss is close to 40%. Based on the 1.27 mm thickness, the maximum required \( \Re \{ \varepsilon_{ML} \} \) is 3.5. A Rogers TMM3 substrate with \( \varepsilon_r = 3.45 \) is an appropriate choice. It is worth emphasizing at this point that the thickness of the matching layers is constrained to the standard substrates thicknesses purely based on fabrication considerations. Matching layers with real and positive permittivities can be designed based on a non-standard substrate thickness. Realizability of the required permittivity range can also be improved by having better fabrication tolerances.

![Figure 4.6](image)

**Figure 4.6:** Total reflection \( r \) is calculated using \( \Re \{ \varepsilon_{ML} \} \) in Fig. 4.5 for (a) \( h_{ML} = 1.27 \text{ mm} \) and (b) \( h_{ML} = 1.52 \text{ mm} \).

Fig. 4.7 shows a realistic unit cell design of the GRIN lens with GRIN matching layers. The 5 mm thick RO6010 substrate for the GRIN lens is not readily available. Hence, two 2.5 mm thick substrates are bonded together with a Rogers 3001 bonding film to achieve the required thickness. The matching layers are then bonded to the lens body using the same bonding films. The required drill diameter \( d_{ML} \) in the matching layers can be calculated based on the \( \Re \{ \varepsilon_{ML} \} \) in Fig. 4.5. However, it was found that the three thin bonding films introduce small reflections in the numerical study. The \( d_{ML} \) has to be further adjusted to compensate for these additional reflections. For each \( d_{lens} \), a simple Quasi-Newton optimization routine is used to find the optimal \( d_{ML} \) that gives the highest transmission.

Fig. 4.8 and Table. 4.1 show the optimal unit cell geometries and the corresponding characteristics. A sudden increase in reflectance is observed for unit cells 12–14. This increased reflection is expected from the prior discussions on Fig. 4.6a. From the phase plot, these unit cells exhibit a nearly flat phase response with phase delays comparable to that of unit cell 11. As a result, these unit cells can be
discarded without impacting the overall phase sampling. By discarding these unit cells, the average reflectance ($|S_{11}|^2$), the average transmittance ($|S_{21}|^2$), and the average dielectric loss are 0.06%, 96.7%, and 3.3% respectively.
Figure 4.8: (a) The optimal hole diameter $d_{ML}$, (b) reflectance, (c) transmittance, (d) transmission phase, and (e) dielectric loss are plotted for each unit cell # (see Table 4.1). Unit cells shown by the red circles can be discarded in the final design due to their flat phase response and higher reflectance.
Table 4.1: Unit cell geometries and characteristics

| Unit cell # | \( d_{lens} \) (mm) | \( d_{ML} \) (mm) | \(|S_{11}|^2\) | \(|S_{21}|^2\) | \( \angle S_{21} \) | Dielectric loss |
|------------|---------------------|-----------------|-----------------|-----------------|-----------------|----------------|
| 1          | 0                   | 0.40            | 2.9 \times 10^{-6} | 0.9622         | 0°              | 3.78%          |
| 2          | 0.2                 | 0.50            | 3.2 \times 10^{-6} | 0.9626         | 4.63°           | 3.75%          |
| 3          | 0.3                 | 0.60            | 2.1 \times 10^{-5} | 0.9628         | 9.98°           | 3.72%          |
| 4          | 0.4                 | 0.70            | 6.5 \times 10^{-5} | 0.9629         | 16.48°          | 3.70%          |
| 5          | 0.5                 | 0.80            | 9.8 \times 10^{-5} | 0.9631         | 24.19°          | 3.67%          |
| 6          | 0.6                 | 0.90            | 1.9 \times 10^{-4} | 0.9633         | 32.80°          | 3.65%          |
| 7          | 0.7                 | 1.00            | 1.9 \times 10^{-4} | 0.9635         | 42.36°          | 3.62%          |
| 8          | 0.8                 | 1.10            | 1.6 \times 10^{-4} | 0.9637         | 53.02°          | 3.60%          |
| 9          | 0.9                 | 1.20            | 2.2 \times 10^{-5} | 0.9641         | 64.91°          | 3.59%          |
| 10         | 1.0                 | 1.40            | 3.8 \times 10^{-5} | 0.9641         | 81.76°          | 3.58%          |
| 11         | 1.1                 | 1.70            | 9.5 \times 10^{-4} | 0.9626         | 105.7°          | 3.65%          |
| 12         | 1.2                 | 1.50            | 1.2 \times 10^{-2} | 0.9520         | 109.6°          | 3.53%          |
| 13         | 1.3                 | 1.10            | 2.2 \times 10^{-2} | 0.9426         | 107.6°          | 3.49%          |
| 14         | 1.4                 | 0.40            | 1.7 \times 10^{-2} | 0.9471         | 106.4°          | 3.63%          |
| 15         | 1.5                 | 0.10            | 2.2 \times 10^{-3} | 0.9614         | 121.6°          | 3.65%          |
| 16         | 1.55                | 0.30            | 2.3 \times 10^{-5} | 0.9641         | 132.9°          | 3.58%          |
| 17         | 1.60                | 0.70            | 7.3 \times 10^{-6} | 0.9653         | 149.2°          | 3.48%          |
| 18         | 1.65                | 0.90            | 4.0 \times 10^{-6} | 0.9661         | 164.7°          | 3.39%          |
| 19         | 1.70                | 1.10            | 1.1 \times 10^{-5} | 0.9669         | 179.2°          | 3.32%          |
| 20         | 1.75                | 1.10            | 1.1 \times 10^{-5} | 0.9675         | 194.6°          | 3.25%          |
| 21         | 1.80                | 1.20            | 3.5 \times 10^{-4} | 0.9679         | 211.1°          | 3.17%          |
| 22         | 1.85                | 1.20            | 4.0 \times 10^{-6} | 0.9688         | 225.4°          | 3.11%          |
| 23         | 1.90                | 1.20            | 1.7 \times 10^{-5} | 0.9696         | 240.6°          | 3.03%          |
| 24         | 1.95                | 1.20            | 9.4 \times 10^{-5} | 0.9704         | 256.8°          | 2.95%          |
| 25         | 2.00                | 1.50            | 3.3 \times 10^{-3} | 0.9690         | 285.0°          | 2.76%          |
| 26         | 2.05                | 1.70            | 7.2 \times 10^{-3} | 0.9736         | 312.1°          | 2.61%          |
| 27         | 2.10                | 1.70            | 3.6 \times 10^{-6} | 0.9752         | 331.7°          | 2.48%          |
| 28         | 2.15                | 1.70            | 3.6 \times 10^{-7} | 0.9767         | 352.5°          | 2.33%          |
| 29         | 2.20                | 1.70            | 6.8 \times 10^{-7} | 0.9783         | 374.5°          | 2.17%          |
4.3 GRIN Lens Design for Horn Antenna Collimation

As a proof of concept, a matched GRIN lens is designed to collimate a commercial Quinstar QWH-APRS00 pyramidal horn with an aperture of 53.8 mm by 69.3 mm as shown in Fig. 4.9. The size of the lens is limited to the size of the largest available sample substrate that can be obtained, which is 127 mm × 127 mm (14.5λ₀ × 14.5λ₀). The lens is placed in a plane that is parallel to the horn aperture. For best collimation, the phase of the lens should compensate the phase of the incident field, which is a function of the spacing between the lens and horn aperture.

![Figure 4.9: The matched GRIN lens is used to collimate a pyramidal horn.](image)

Fig. 4.10 shows the simulated magnitude and phase of the radiated field at a plane that is 1 cm, 2 cm or 3 cm away from the horn aperture. Two factors are considered for determining the appropriate distance between the lens and the horn aperture. First, the lens should be placed close enough such that spill-over loss is minimized. Second, the phase of the incident field should not vary too quickly. A slower phase variation indicates a smaller hole diameter variation between the adjacent unit cells. Thus, the unit cells will satisfy the quasi-periodic condition and the results obtained from the periodic analysis would remain valid. In addition, a slower phase variation also leads to a smaller number of phase wraps, which are the regions with a rapid transition from the largest holes to the smallest holes. This rapid transition breaks the assumption of quasi-periodicity, and can lead to scattering that results to increased side lobe levels (SLL) and lens reflection. As we have seen in Fig. 3.49, the metalens-1 with phase wraps has the lowest transmission. Hence, to minimize both the spill-over loss and the number of phase wraps, a 3 cm spacing between the lens and the horn aperture is deemed to be a good compromise.

Based on the phase in Fig. 4.10c, a unit cell with an appropriate phase delay is used to compensate the input phase at each location on this phase plane. Since the phase of each unit cell (in Fig. 4.8d) is the total phase delay accumulated in both the GRIN matching layers and the GRIN lens, the phase-compensating procedure ensures that the wave exiting the matched GRIN lens will have a flat phase distribution across the entire lens. By stitching all the unit cells together, the lens is subsequently formed. Fig. 4.11 shows the perforation patterns in the lens body and the matching layers.
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Figure 4.10: The simulated magnitudes and phases of the electric field are plotted for 34.3 GHz at planes that are (a) 1 cm, (b) 2 cm and (c) 3 cm away from the horn aperture. The $E$-field of the horn is aligned with the $y$-axis. The black dashed boxes outline the aperture of the horn and the red dashed boxes outline the 127 mm by 127 mm lens.

4.4 Results

4.4.1 Simulation Results

The lens performance is analyzed by a full-wave simulator and the simulation model is shown in Fig. 4.9. The lens is placed 3 cm away from the horn aperture. The incident field on the lens is shown in Fig. 4.10c. To reduce the computational requirement, symmetry boundaries (perfect E-plane and H-plane) are used.
such that only a quarter of the model is simulated. Due to the large electrical size and the thin bonding films in the model, an extremely large number of tetrahedra are required to achieve a good convergence. Such computations were performed on the Sandy supercomputer of the SciNet HPC Consortium [160]. Fig. 4.12 shows the scattered electric field of the GRIN lenses with and without the GRIN matching layers. It is clear that the reflected field is much stronger without the matching layers. Based on the scatter fields, the reflectances are calculated to be 3% and 27% with and without the matching layers, respectively.

To gauge the effectiveness of the GRIN matching layers, the GRIN lens with the GRIN matching layers is compared to the same GRIN lens matched by the QWTs, the GRIN lens without matching layers, and a conventional hyperbolic lens. The QWTs are designed to match the center of the lens and they are placed on both sides of it. The required index and the thickness of the QWTs are 3.67 and 1.15 mm respectively. It is just 0.12 mm thinner than the GRIN matching layers. The hyperbolic lens is made from Teflon, a low-index material with $\varepsilon_r = 2.1$ to minimize the lens reflections. The hyperbolic lens is designed to have the same focal length and diameter as the GRIN lens and its surface profile is calculated using (4.9) [182]

$$r = \sqrt{(n_{lens}^2 - 1) h^2 + 2 (n_{lens} - 1) f h}$$  \hspace{1cm} (4.9)

where $f$ is the focal length of the lens, which is the distance between the apex of the lens and the phase center of the horn; $r$ is the distance in the radial direction; and $h$ is the local thickness of the lens. The maximum thickness of the hyperbolic lens is calculated to be 23 mm, which is three times the thickness of the GRIN lens with the GRIN matching layers.

Fig. 4.13 compares the gain patterns for each design. The GRIN lens with GRIN matching layers has the highest gain of 29 dB. It is 0.7 dB higher than the gain of the GRIN lens with the QWTs or the hyperbolic lens; it is also 2 dB higher than the gain of the GRIN lens without matching layers. This clearly suggests matching a high-index GRIN lens can significantly improve gain. The low-index hyperbolic has a slightly lower gain to the matched GRIN lens due to small reflections at the lens surface. Unlike the matched GRIN lens, the hyperbolic lens is not designed to perfectly compensate the phase of the incident field (Fig. 4.10c); hence, it has the highest SLL.
Figure 4.12: The scattered fields of the GRIN lenses with (top figure) and without (bottom figure) the GRIN matching layers are plotted at 34.3 GHz. Identical color scales have been used in both figures. A plane wave impinges from the bottom of the lens. With the symmetry boundary condition, only half of the lenses are shown. Strong reflections can be observed underneath the unmatched lens. In comparison, the reflections from the matched lens are negligible. The reflected power with and without the matching layers is 3% and 27% respectively.
Figure 4.13: Simulated gain patterns are plotted at 34.3 GHz in the (a) H-plane and the (b) E-plane for the GRIN lens with the GRIN matching layers, the GRIN lens with the QWTs, the GRIN lens without matching layers, and a conventional hyperbolic lens. The inset figures show the main beams and the first side-lobes. The GRIN lens with the GRIN matching layers has the highest gain and the smallest side lobes in the principle plane.
4.4.2 Measurement Results

Fig. 4.14 shows the experimental setup of the gain measurement and the inset picture shows the fabricated prototype. Fig. 4.15 compare the simulated and measured gain patterns in the E-plane and the H-plane at 34.3 GHz. Excellent agreement is observed between the simulated and measured results. The measured peak gains for the horn in the E-plane and H-plane are 23.3 dB and 23.4 dB, respectively. The measured peak gains with the matched GRIN lens in the E-plane and H-plane are 28.9 dB and 28.4 dB, respectively; these values are very close to the simulated peak gain of 29 dB. The slight difference in the peak gain values could be a result of alignment errors during the measurements. On average, the lens increases the peak gain by at least 5 dB and reduces the HPBW from $10^\circ$ to $6^\circ$.

From the measured gain patterns, one can notice that there is no significant increase in gain in the back region ($\phi = -90^\circ - 180^\circ$ and $\phi = 90^\circ - 180^\circ$) after adding the lens. This already suggests that the lens introduces very small reflections. To estimate the reflection of the lens, the $S_{11}$ of the horn is measured with and without the lens. As shown in Fig. 4.16, there is no significant change in $S_{11}$ at the design frequency. The measured $S_{11}$ is $-26.4$ dB and $-26.5$ dB with and without the lens respectively. The reflection is calculated to be $0.7\% - 43.1$ dB), which is equivalent to a reflectance of $0.005\%$. This estimation is in line with the average reflectance of $0.06\%$ ($-32.4$ dB) obtained from the unit cell analysis.

Since the matched GRIN lens neither stores power nor has large reflections, the radiation efficiency (due to dielectric losses) $e_d$ of the lens can be approximated from the directivity and the measured gain of the lens using (4.10). The directivity $D_0$ can be estimated from (4.11) or (4.12) [54]

$$e_d = \frac{G_0}{D_0} \quad (4.10)$$

$$D_0 \simeq \frac{32400}{\Theta_E \Theta_H} \quad (4.11)$$

$$D_0 \simeq \frac{72815}{\Theta_E^2 + \Theta_H^2} \quad (4.12)$$

where $\Theta_E$ and $\Theta_H$ are the HPBW in the E-plane and H-plane. Specifically, $\Theta_E = 6.2^\circ$ and $\Theta_E = 6.5^\circ$. 
Figure 4.15: Simulated and measured gain patterns are plotted at 34.3 GHz in the (a) H-plane and (b) E-plane. Excellent agreement is observed between the simulated and measured results. The lens increases the peak gain by 5.5 dB and reduces the HPBW from 10° to 6°. The SLL in the H-plane and E-plane are −31 dB and −20 dB respectively.

Using the measured gain of 28.9 dB, the radiation efficiency is estimated to be either 97% or 86%, resulting to an average efficiency of 92%. This estimated average efficiency is in line with the average efficiency of 96.7% obtained from the unit-cell analysis.

Even though the GRIN lens is only designed for 34.3 GHz, the matching performance is quite wideband. Less than 1% of reflected power (equivalent to $S_{11}$ of −20 dB) is maintained from 30–35 GHz as shown in Fig. 4.16. The measured peak gains with and without the matched GRIN lens are plotted against frequency in Fig. 4.17. The lens increases the gain of the horn by an average of 5 dB throughout the entire frequency range of the horn. 1 dB gain bandwidth is about 30% from 30 GHz to 39.7 GHz. Since the highest measured frequency is 39.7 GHz, we speculate that the 1 dB gain bandwidth can be larger than 30%. Fig. 4.18 shows the measured gain patterns with the matched GRIN lens in the H-plane at a few discrete frequencies. The HPBW slightly widens at lower frequencies. The SLL is −31 dB at 34.3 GHz and stays below −19 dB for the entire frequency range of 26.6–39.4 GHz. Table 4.2 compares the main features between the matched GRIN lens and other dielectric lenses reported in literature. The matched GRIN lens has the smallest overall thickness while achieving the largest 1 dB gain bandwidth.
Figure 4.16: $S_{11}$ as measured at the throat of the horn with and without the lens. The black dashed line is the reference line for 34.3 GHz. At the design frequency, there is no significant change in $S_{11}$, which indicates the reflection from the lens is extremely small.

Figure 4.17: The measured peak gains of the horn with and without the GRIN lens are plotted against frequency. Around 5 dB gain is maintained throughout the entire frequency range of the horn.

The aperture efficiency of the matched GRIN lens is not as high as some of the lens designs. This is because the horn is placed very close to the lens to avoid spill-over loss. As a result, the outer part of the lens is not well illuminated. In addition, the aperture efficiency of the horn is only 35%. By placing the horn further away, the aperture efficiency of the lens can be improved.

4.4.3 Discussion

The simulated and measured reflected power of the matched GRIN lens is $-32.4$ dB (0.06%) and $-43.1$ dB (0.005%) respectively; either is much less than the reflected power of $-15.2$ dB (3%) under plane-wave illumination (refer to Fig. 4.12). This difference is a result of the different illuminations on the phase wraps of the lens, which are known to cause scattering and introduce reflections. In the lens/horn combination, the energy from the horn is concentrated near the center of the lens. Electromagnetic waves impinging on the phase wraps will have a much lower energy density under the horn illumination than under the plane-wave illumination. Consequently, the reflected power is smaller under
Chapter 4. Matched Graded-index Lenses at Millimeter-wave Frequencies

Figure 4.18: Measured gain patterns with the matched GRIN lens in the principle plane (H-plane). Worst SLL of $-19$ dB is observed at 39 GHz.

Table 4.2: Comparison between the matched GRIN lens and other dielectric lenses reported in literature

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Year</th>
<th>Freq (GHz)</th>
<th>$f/D$</th>
<th>Overall thickness ($\lambda_0$)</th>
<th>Size ($\lambda_0^2$)</th>
<th>Gain (dB)</th>
<th>HPBW (°)</th>
<th>Aperture eff. (%)</th>
<th>1-dB Gain BW (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[183]</td>
<td>2018</td>
<td>34.3</td>
<td>1.236</td>
<td>0.875</td>
<td>14.5 $\times$ 14.5</td>
<td>29</td>
<td>6</td>
<td>30%</td>
<td>30%</td>
</tr>
<tr>
<td>this work</td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
<td> </td>
</tr>
<tr>
<td>[184] (TA4)</td>
<td>2018</td>
<td>30</td>
<td>1</td>
<td>3.3</td>
<td>15.6 $\times$ 15.6</td>
<td>30.7</td>
<td>4</td>
<td>38.6%</td>
<td>21.5%</td>
</tr>
<tr>
<td>[185]</td>
<td>2016</td>
<td>60</td>
<td>0.42</td>
<td>2</td>
<td>9.5 $\times$ 9.5</td>
<td>23.5</td>
<td>N/A</td>
<td>20%</td>
<td>17%</td>
</tr>
<tr>
<td>[177]</td>
<td>2015</td>
<td>60</td>
<td>0.25</td>
<td>1.4</td>
<td>$\pi(2.5)^2$</td>
<td>18.3</td>
<td>18</td>
<td>6.8%</td>
<td>10%</td>
</tr>
<tr>
<td>[179]</td>
<td>2014</td>
<td>94</td>
<td>0.77</td>
<td>1.31</td>
<td>18.2 $\times$ 18.2</td>
<td>30</td>
<td>N/A</td>
<td>25%</td>
<td>15.9%</td>
</tr>
<tr>
<td>[173]</td>
<td>2014</td>
<td>73.5</td>
<td>0.857</td>
<td>1.6</td>
<td>$\pi(4.3)^2$</td>
<td>25.1</td>
<td>N/A</td>
<td>44%</td>
<td>6.5%</td>
</tr>
<tr>
<td>[170]</td>
<td>2003</td>
<td>30</td>
<td>1</td>
<td>0.9</td>
<td>$\pi(7.5)^2$</td>
<td>28</td>
<td>4</td>
<td>33%</td>
<td>13%</td>
</tr>
</tbody>
</table>

the horn illumination. This also suggests that it is preferable to minimize the number of phase wraps in the lens. This can be done by having a greater index variation across the lens, which can be achieved by either increasing the lens thickness or having better fabrication tolerances, i.e., reducing the minimum edge to edge distance between the holes.

As the frequency deviates from the design frequency, ripples in the $S_{11}$ or the gain can be observed in Fig. 4.16 or Fig. 4.17. This is because the matching becomes imperfect and reflections arise as the frequency deviates from the design frequency. Due to the inhomogeneity in both the lens and the matching layers, the reflections from different unit cells will have different magnitudes and phases. In addition, the magnitudes and the phases of these unit-cell reflections vary with frequency. As a result, the total reflection from the entire GRIN lens, which is an ensemble of these unit-cell reflections, can constructively interfere at one frequency and destructively interfere at another. Thus, the ripple effect is a manifestation of the underlying complex interference phenomenon.

The matched GRIN lens is also quite broadband. 1 dB gain BW is more than 30%. The gain can be maintained over such a broad bandwidth is because the lens is quite well matched in Ka-band as shown in Fig. 4.16. The matching performance can be understood from the matching network theory [60].
The matching layer/lens/matching layer structure can be treated as a three-segment transmission line matched to transmission lines with free space impedance. The bandwidth of the matching network is usually proportional to the number the transmission-line segments. In this work, we did not try optimize the bandwidth using the matching network theory. We believe the bandwidth can be further increased if the thickness and index of each layer is optimized while maintaining the required transmission magnitude and phase. Multiple layers can also be used to further increase the matching bandwidth. However, a multiple layer matching structure may introduce extra thickness and fabrication complexity.

It is worth noting that after the publication of this work [183], there has been a subsequent paper [184] by Massaccesi et al., where they showed a 3D-printed lens with tapered microholes as the matching layers. The fabricated sample has an overall thickness of $3.3\lambda_0$ and a 1 dB gain bandwidth of 21.5%. In comparison, the matched GRIN is three times thinner and has a wider bandwidth. The performance of this 3D-printed lens and our matched GRIN lens is summarized in Table 4.2. The main challenge of the 3D-printed lens is the material availability. The lens has to be printed with a dielectric material with a low permittivity of 2.77, resulting to a large overall thickness. In addition, the loss tangent is 0.021, which is much higher than the loss tangent of the Rogers substrates used in our design. The total loss of the 3D-printed lens is not characterized in [184], but we expect it to be higher than the loss in our lens, as the electromagnetic wave has to propagate over a thicker and more lossy medium. However, with improvements on the material availability in the future, 3D-printed lenses can become very attractive.

### 4.5 Conclusion and outlook

In this chapter, we presented a thin graded-index (GRIN) lens based on a high-index substrate. This GRIN lens is matched by GRIN matching layers with a fixed thickness. The required indices in the matching layers are calculated through the transfer matrix method. The graded-index in the lens and the matching layers are achieved by perforating Rogers RO6010 and TMM3 substrates with holes arranged on a hexagonal lattice that have varying diameters. Excellent agreement is observed between the simulated and measured results. The resulting reflectance, transmittance, and loss of the lens at the design frequency (34.3 GHz) are 0.06% (-32.2 dB), 96.7% (-0.14 dB), and 3.3% (-14.8 dB) respectively. When coupled to the Quinstar QWH-APRS00 pyramidal horn, the GRIN lens increases the peak gain by 5.5 dB and reduces the HPBW from 10° to 6°. The matched GRIN lens is also wideband. Less than 1% reflectance (equivalent to $S_{11}$ of $-20$ dB) is maintained between 30–35 GHz. In the principal plane (H-plane), a gain improvement of 5 dB and a SLL below $-19$ dB is maintained for the entire frequency range of 26.6–39.7 GHz. Compared to other lenses, the matched GRIN lens is not only more compact, but also more broadband with a 1 dB gain bandwidth more than 30%. The aperture efficiency of the matched GRIN lens is not as high as we expected. This is because only the central portion of lens is illuminated. By placing the horn further away, the aperture efficiency can be improved. With low reflections, low losses, and wide bandwidth, the compact matched GRIN lens is an excellent candidate for applications such as increasing the gain or the scan range of mm-wave phased arrays.
Chapter 5

Sub-wavelength Anisotropic Metamaterial Anti-reflection Layers

5.1 Introduction

In previous chapters, we presented efficient metamaterial-based wave plates and lenses. In these designs, it is imperative to consider anti-reflection (AR) for achieving high transmission. The phenomenon of anti-reflection (AR) was discovered by Lord Rayleigh in the 19th century when he observed that a thin film of tarnish on the surface of glass can reduce the reflectivity. Since then, various types of AR layer were developed over the years [186]. The simplest AR layer is the quarter-wave transformer, which is used to match the wave plates in Chapter 2. It consists of a single layer of isotropic material with the refractive index of \( n = \sqrt{n_s} \), where \( n_s \) is the index of the substrate to be matched. The thickness of the matching layer is \( \lambda/4 \) where \( \lambda \) is the wavelength inside the matching layer. Despite its simplicity, the quarter-wave transformer has two main drawbacks: 1. The material with the required index is not always available; 2. It is designed for normal incidence and a single frequency. To extend the angular and frequency range, multilayer and graded-index (GRIN) AR layers are developed [187]. The multilayer structure uses stacked layers with alternating high index and low index. The thickness and the index of each layer are optimized to maximize the frequency and angular range. The GRIN AR layer reduces reflectance by gradually reducing the refractive index of the layer from the index of the substrate (to be matched) to the index of air. If the index tapering is gradual enough, the GRIN AR layer is extremely broadband and can match over a wide angular range. This broadband behavior can be directly observed from the metalenses presented in Chapter 3 (Fig. 3.49), in which the tapered nano-holes constitute a GRIN medium. Even though GRIN layers provide superior performance in bandwidth and angular range, they are difficult to fabricate, not very durable, and have appreciable thicknesses. For these reasons, GRIN layers are rarely used in optical devices. However, this concept is employed in one of the most common radar absorbers [188] used in anechoic chambers. It has a pyramidal or conical shape such that the tapering presents a gradual transition in impedance from air to that of the absorber.

The holy grail of anti-reflection is perhaps a layer that can achieve all angle matching for both polarizations. Even though GRIN layers have a very wide angular range, reflectance is still high near grazing angles. All angle matching is possible by the perfectly matched layer (PML) [189, 190] that was first introduced in computational electromagnetics. It has uniaxial permittivity and permeabilities.
tensors and it is matched to free space at all angles. However, PML is designed to only match a single interface (free space/PML interface). If there is a third medium (substrate) with a different permittivity to that of free space, then the additional interface cannot be matched by PML. In fact, from our investigation in Appendix E, we show that it is not possible to achieve all angle matching even with a fully anisotropic layer, which has different permittivities and permeabilities along all three axis.

Even though all angle matching is not possible, we show that with the additional degrees of freedom provided by the anisotropy in the layer, matching at extreme incident angles, which is very difficult for conventional matching structures, becomes possible. In this chapter, we show that near perfect matching can be achieved up to 88°. To our best knowledge, this is most extreme angle that is matched. One of the best results reported a 5% reflectance at 85° of incidence [191], and it is just a numerical investigation.

The purpose of this chapter is threefold. First, we develop a matching theory based on an anisotropic metamaterial anti-reflection layer (AMAL), and derive the necessary material parameters for matching an arbitrary substrate from free space to an arbitrary angle (in this case, 88°). Second, we provide a synthesis procedure to design realistic unit cells that can achieve the required material parameters. This synthesis process relies on the parameter extraction method we developed in Appendix C. Hence, the third purpose of this chapter is to demonstrate the validity of the parameter extraction method at extreme angles. Moreover in this chapter, AMAL is developed to match for TM or TE polarizations separately. This is a practical consideration since there are only three material parameters needed to be considered during the design process. It is possible to synthesize a layer to match both TE and TM polarizations simultaneously, and we termed this layer as magneto-electric uniaxial matching layer (MEUML). The synthesis of MEUML is much more challenging than AMAL; hence, we defer the discussion of it till Chapter 6.

5.2 Theory

As shown in Fig. 5.1, a plane wave with TE or TM polarization is incident on an AMAL from the Y-Z plane at an angle of θ₁. The AMAL sits on an isotropic non-magnetic semi-infinite medium with arbitrary permittivity ε₃. Suppose the permittivity and permeability of the AMAL are characterized by:

\[ \varepsilon_2 = \begin{bmatrix} \varepsilon_{2x} & 0 & 0 \\ 0 & \varepsilon_{2y} & 0 \\ 0 & 0 & \varepsilon_{2z} \end{bmatrix} \]  

\[ \mu_2 = \begin{bmatrix} \mu_{2x} & 0 & 0 \\ 0 & \mu_{2y} & 0 \\ 0 & 0 & \mu_{2z} \end{bmatrix} \]  

(5.1)

(5.2)

where all the matrix elements are assumed to be real. A diagonal form is assumed for the permittivity and permeability to simplify the synthesis and fabrication. The total reflection is given by (5.3) [181]

\[ r = \frac{r_{12} + r_{23}e^{-i2\phi}}{1 - r_{12}r_{23}e^{-i2\phi}}, \]  

(5.3)
where \( r_{ij} \) is the reflection coefficient at the \( i, j \) media interface with incidence from medium \( i \). The phase \( \phi \) is the total phase delay accumulated in the AMAL along the normal of the layer surface (\( z \)-axis). Specifically,

\[
 r_{ij} = \frac{Z_j - Z_i}{Z_j + Z_i}, \quad (5.4)
\]

\[
 \phi = k_{2z}d, \quad (5.5)
\]

where \( Z_i \) is the wave impedance in medium \( i \), \( k_{2z} \) is the wave number along the \( z \)-axis and \( d \) is the layer thickness. \( Z_i \) and \( k_{2z} \) depend on the polarization state, so we treat TM and TE polarizations separately. Nonetheless, to achieve zero reflection, the numerator of (5.3) should equal to zero. Explicitly,

\[
 r_{12} + r_{23}e^{-i2\phi} = 0. \quad (5.6)
\]

With the assumption that the permittivity and the permeability of all media are real, then \( r_{12} \) and \( r_{23} \) are both real. To satisfy (5.6), \( e^{-i2\phi} \) should be equal to \( \pm 1 \). With \( e^{-i2\phi} = 1 \), \( Z_1 \) has to equal to \( Z_3 \) with the substitution of (5.4) into (5.6). Thus, this is a trivial case. With \( e^{-i2\phi} = -1 \), we have

\[
 k_{2z} = \frac{\pi}{2d}, \quad (5.7)
\]

\[
 Z_2^2 = Z_1Z_3 \quad (5.8)
\]

(5.7) stipulates the required phase relation for destructive interference and (5.8) stipulates the required condition for impedance matching. Notice that these are also the conditions for the quarter-wave transformer. They have to be satisfied simultaneously to achieve zero reflection. We derive the necessary conditions for TM and TE polarizations separately in the following.

Referring to Fig. 5.1, for TM polarization, only \( \varepsilon_{2y}, \varepsilon_{2z} \) and \( \mu_{2x} \) matter. Thus, the wave impedances and the normal component of the wave vector can be expressed as

\[
 Z_1^{TM} = \eta_0 \cos \theta_1, \quad (5.9)
\]

\[
 Z_2^{TM} = \eta_0 \sqrt{\frac{\mu_{2x}}{\varepsilon_{2y}} - \sin^2 \theta_1 \frac{1}{\varepsilon_{2y}\varepsilon_{2z}}}, \quad (5.10)
\]

\[
 Z_3^{TM} = \eta_0 \sqrt{\frac{1 - \sin^2 \theta_1}{\varepsilon_3}}, \quad (5.11)
\]

\[
 k_{2z}^{TM} = k_0 \sqrt{\mu_{2x}\varepsilon_{2y} - \sin^2 \theta_1 \frac{\varepsilon_{2y}}{\varepsilon_{2z}}} = k_0n_{2z}^{TM}. \quad (5.12)
\]

Substituting (5.12) into (5.7) and (5.9) to (5.11) into (5.8), we obtain the following two equations:

\[
 \varepsilon_{2y} \left( \mu_{2x} - \sin^2 \theta_1 \frac{1}{\varepsilon_{2z}} \right) = \left( \frac{\lambda_0}{4d} \right)^2, \quad (5.13)
\]

\[
 \frac{1}{\varepsilon_{2y}} \left( \mu_{2x} - \sin^2 \theta_1 \frac{1}{\varepsilon_{2z}} \right) = \frac{\cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}{\varepsilon_3}. \quad (5.14)
\]
Chapter 5. Sub-wavelength Anisotropic Metamaterial Anti-reflection Layers

Figure 5.1: A plane wave with TE/TM polarization is incident on an AMAL at $\theta_1$. The AMAL has thickness $d$ and sits on a semi-infinite isotropic medium that is to be matched.

Dividing (5.13) by (5.14), we can obtain $\varepsilon_{2y}$ directly,

$$
\varepsilon_{2y} = \frac{\lambda_0}{4d} \sqrt{\varepsilon_3 \frac{\varepsilon_3}{\cos \theta_1 \sqrt{\varepsilon_3} - \sin^2 \theta_1}}
$$

(5.15)

By rewriting (5.14), we can obtain an expression for $\mu_{2x}$,

$$
\mu_{2x} = \left( \frac{\lambda_0}{4d} \right)^2 \frac{1}{\varepsilon_{2y}} \sin^2 \theta_1 \frac{1}{\varepsilon_{2z}^{\mu_{2x}}}.
$$

(5.16)

For TE polarization, the wave impedances and the normal component of the wave vector can be expressed as

$$
Z_{1}^{TE} = \frac{\eta_0}{\cos \theta_1},
$$

(5.17)

$$
Z_{2}^{TE} = \frac{\eta_0}{\sqrt{\varepsilon_{2y} \mu_{2x} - \sin^2 \theta_1 \mu_{2y} \mu_{2z}}},
$$

(5.18)

$$
Z_{3}^{TE} = \frac{\eta_0}{\sqrt{\varepsilon_3 \sqrt{1 - \sin^2 \theta_1}}},
$$

(5.19)

$$
k_{2z}^{TE} = k_0 \sqrt{\mu_{2y} \varepsilon_{2x} - \sin^2 \theta_1 \frac{\mu_{2y}}{\mu_{2z}} \varepsilon_{2z}^{\mu_{2z}}} = k_0 n_{2z}^{TE}.
$$

(5.20)

Following the same procedure as for the TM polarization, we obtain the following two equations for the
material parameters

\[
\mu_{2y} = \frac{\lambda_0}{4d} \sqrt{\frac{1}{\cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}},
\] (5.21)

\[
\varepsilon_{2x} = \left(\frac{\lambda_0}{4d}\right)^2 \frac{1}{\mu_{2y} \sin^2 \theta_1} + \frac{1}{\mu_{2z}},
\] (5.22)

Note that \(\varepsilon_{2y}\) and \(\mu_{2y}\) in (5.15) and (5.21) are stipulated directly from the layer thickness, the incident angle and the permittivity of the substrate to be matched. Fig. 5.2 and Fig. 5.3 plot the required \(\varepsilon_{2y}\) and \(\mu_{2y}\) versus the incident angle and substrate permittivity for various layer thicknesses. The general trend can be observed clearly: 1. Both \(\varepsilon_{2y}\) and \(\mu_{2y}\) increase with \(\theta_1\), but the increase is more dramatic near grazing angles. 2. \(\varepsilon_{2y}\) increases with \(\varepsilon_3\) but \(\mu_{2y}\) decreases with it. Overall, \(\varepsilon_{2y}\) and \(\mu_{2y}\) are not a very sensitive function of \(\varepsilon_3\). 3. Both \(\varepsilon_{2y}\) and \(\mu_{2y}\) decrease with increasing layer thickness \(d\). The permeability \(\mu_{2x}\) in (5.16) depends on both \(\varepsilon_{2y}\) and \(\varepsilon_{2z}\). However, \(\varepsilon_{2z}\) is not defined and can be of any value. Thus, one is free to choose the desired \(\varepsilon_{2z}\) in an actual synthesis process and subsequently stipulate \(\mu_{2x}\). Nonetheless, we can still predict the trend of \(\mu_{2x}\). As one can see, \(\mu_{2x}\) is inversely proportional to \(\varepsilon_{2y}\) and \(\varepsilon_{2z}\). Permittivity \(\varepsilon_{2y}\) can assume a very large value near the grazing angle as in Fig. 5.2, hence, \(\mu_{2x}\) can be a very small value. Similarly, one is free to choose the desired \(\mu_{2x}\) in (5.22) to stipulate \(\varepsilon_{2x}\). Near the grazing angle, \(\varepsilon_{2x}\) can be very small due to the large \(\mu_{2y}\).

Figure 5.2: Required \(\varepsilon_{2y}\) versus incident angle \(\theta_1\) and substrate permittivity \(\varepsilon_3\) for various layer thicknesses. Notice the difference in the color scale.
5.3 Structure Synthesis

To design the AMAL to achieve matching at a particular incident angle for a particular substrate, one can follow the synthesis algorithm summarized in the following steps:

1. Define the desired incident angle $\theta_1$ to achieve perfect matching for a substrate with $\varepsilon_3$.

2. Make an assumption on the possible values for $\varepsilon_2$ or $\mu_2$. By doing so, $\mu_2$ or $\varepsilon_2$ are stipulated by (5.16) or (5.22).

3. Define an initial layer thickness $d$ and the required parameter values for perfect matching can be calculated from (5.15), (5.16), (5.21) and (5.22).

4. From the calculated parameter values, propose an initial structure and extract the actual parameter values from the modified parameter extraction method outlined in Appendix C.

5. Based on the extracted parameter values, one has the following options:
   - If the extracted values are close to the desired ones, one can optimize the structure geometries such that the extracted values approach the desired ones.
   - If the extracted values are not close to the desired ones, one can either revise the assumption made in step (2) or the layer thickness in step (3) and repeat the subsequent steps.
For a proof of concept, we demonstrate matching at 10 GHz at an extreme angle of 88°. The substrate to be matched is Rogers RO3010 with $\varepsilon_3 = 10.2$. The required matching structures are synthesized separately for TM and TE polarizations in the following sections.

### 5.3.1 TM Polarization

The first step is to assume some possible values of $\varepsilon_{2z}$ as mentioned in the last section. One possible choice is making $\varepsilon_{2z} = \varepsilon_{2y}$. This implies the synthesized structure possesses symmetry in the y-z plane, i.e., the matching layer is uniaxial. With this assumption, we obtain an expression for $\mu_{2x}$

$$\mu_{2x} = \frac{\lambda_0}{4d} \sqrt{\frac{\varepsilon_3}{\cos\theta_1 \sqrt{\varepsilon_3} - \sin^2\theta_1}}$$

(5.23)

To achieve zero reflection at $\theta_1$, $\varepsilon_{2y}$ and $\mu_{2x}$ have to satisfy (5.15) and (5.23) for a given layer thickness $d$. Fig. 5.4 shows the $\varepsilon_{2y}$ and $\mu_{2x}$ values with three different layer thicknesses. One can see that $\varepsilon_{2y}$ increases rapidly to a large value and $\mu_{2x}$ approaches zero near the grazing angle. In the microwave regime, large $\varepsilon$ can be achieved by arranging a large number of identical conducting objects in a regular three-dimensional pattern. Such structure is usually termed an artificial dielectric [181]. For example, small conducting strips are used to obtain very high permittivity. The strips produce a capacitive loading of the medium and lead to an increased $\varepsilon$. Similarly, small $\mu$ can be obtained by loading the host medium with diamagnetic structures such as conducting rings or split rings [4, 5] that can even achieve a negative $\mu$ value. One may wonder what kind of a structure can achieve a capacitive loading for large $\varepsilon$ and produce a diamagnetic effect simultaneously. The answer is surprisingly simple: an array of sub-wavelength sized closed conducting rings.

![Figure 5.4: Permittivity ($\varepsilon_{2y}$) and permeability ($\mu_{2x}$) values versus incident angle $\theta_1$ for various layer thicknesses $d$.](image)

As shown in Fig. 5.5, the conducting rings are arranged in a rectangular grid with the rings lying in the Y-Z plane. The host medium for the rings is a Rogers RO3006 substrate with a relative permittivity of 6.15. The spacing of the rings in the Y-direction is judiciously chosen to be the same as the layer thickness $d$. This is because we made an assumption earlier that the structure possesses symmetry in the Y-Z plane such that $\varepsilon_{2y} = \varepsilon_{2z}$. The ring spacing in the X-direction, the ring diameter and the layer thickness can be parameterized to yield the desired $\varepsilon_{2y}$ and $\mu_{2x}$ for matching at $\theta_1$. Fig. 5.6
Chapter 5. Sub-wavelength Anisotropic Metamaterial Anti-reflection Layers

Figure 5.5: The anisotropic matching layer consists of closed conducting rings arranged in a rectangular grid. The rings lie in the y-z plane. The host medium for the rings is a Rogers RO3006 substrate with a relative permittivity of 6.15. The substrate to be matched is Rogers RO3010 with $\varepsilon_3 = 10.2$. The ring spacing $d_x$, the ring diameter $d_{\text{ring}}$ and the layer thickness $d$ can all be parametrized to yield the desired $\varepsilon_{2y}$ and $\mu_{2x}$ such that matching is achieved at $\theta_1$.

shows extracted the permittivity and permeability values. The layer thickness $d$ is fixed at 2 mm. The diameter of the ring varies from 0.9 mm to 1.9 mm and the spacing $d_x$ varies from 0.25 mm to 1.25 mm. One can see that with an increasing ring radius, $\varepsilon_{2y}$ and $\varepsilon_{2z}$ increase with it and $\mu_{2x}$ decreases with it. Furthermore, we notice that $\varepsilon_{2y}$ and $\varepsilon_{2z}$ are very close. Thus, our initial assumption of $\varepsilon_{2y} = \varepsilon_{2z}$ holds. The imaginary parts of all parameters remain near but smaller than zero such that the passivity requirement is satisfied (assuming the time convention is $e^{j\omega t}$). There are no sharp peaks in the real or the imaginary parts of the parameter values, which indicates that there is no resonance [192]. Furthermore, the closed rings exhibit no bi-anisotropy as in the conventional split rings. If there is magneto-electric coupling, the imaginary parts of the permittivity and permeability would have opposite signs [192]. The reason behind the large permittivity and low permeability of the array of rings can be understood intuitively from Fig. 5.7. As shown in the figure, an incident plane wave traveling downwards has its $E$ field pointing to the left and its $H$-field pointing into the page. The electric field induces a surface current following from the right to the left on both the upper half and the lower half of the ring. In other words, the electric field excites an even-mode current on the ring. This even-mode current produces capacitive loading as in the conventional artificial dielectric. The incident $H$-field induces an opposite flowing current in the counter-clockwise direction (it is well known that small solid loops are diamagnetic due to their inductive nature and Lenz’ law), which is an odd-mode current. Thus, the total current is a superposition of the even-mode and the odd-mode which ultimately leads to a current flowing only on the upper half of the ring. Fig. 5.7b shows the simulated surface current of the ring. This current
profile matches the theoretical total current from Fig. 5.7a. Since the $E$ field and the $H$-field excites orthogonal current modes on the ring, the zero magneto-electric coupling of the rings can be understood immediately.

Now suppose we want to match at $\theta_1 = 88^\circ$ with a layer thickness around 2 mm. From Fig. 5.4, the required parameter values are: $\varepsilon_{2y} = 36.8$ and $\mu_{2x} = 0.41$. From Fig. 5.6, we can see that the rings with diameter of 1.9 mm and 0.25 mm spacing can produce parameters close to these values. Hence, we can use this result as a starting point for optimization. We are at the liberty of optimizing the layer thickness $d$, ring spacing $d_x$ and the ring diameter. Table 5.1 shows the optimized geometry for the rings and the corresponding extracted parameter values. The extracted values are very close to the ideal ones.

Fig. 5.8 plots reflectance versus incident angle $\theta_1$. The reflectance at $88^\circ$ is 0.0086%, which is a remarkable result compared to any other conventional matching technique. Thus, this proves not only

---

**Figure 5.6**: (a) $\varepsilon_{2y}$, (b) $\mu_{2x}$, and (c) $\varepsilon_{2z}$ are plotted against the diameter of the ring for three different $d_x$. The general trend can be observed clearly: $\Re \{\varepsilon_{2y}\}$ and $\Re \{\varepsilon_{2z}\}$ increase with ring diameter and $\Re \{\mu_{2x}\}$ decreases with ring diameter. In addition, $\Re \{\varepsilon_{2z}\}$ is very close $\Re \{\varepsilon_{2y}\}$, which indicates our initial assumption is valid. $\Im \{\varepsilon_{2y}\}$ and $\Im \{\mu_{2x}\}$ remain negative and close to zero. This satisfies the passivity requirement and suggests that there is neither a resonance nor a magneto-electric coupling.
Figure 5.7: (a) The electric field induces a surface current following from the right to the left on both the upper half and the lower half of the ring. This is an even-mode current. This even mode current produces capacitive loading as in the conventional artificial dielectric. The incident $H$-field induces an opposite flowing (counter-clockwise) current, or an odd-mode current which leads to the diamagnetic effect. Thus, the total current is a superposition of the even-mode and the odd-mode that leads to a current flowing only on the upper half of the ring. (b) The simulated total surface current on the ring. Both the current direction and magnitude match the theoretical current profile.

Table 5.1: Optimized ring geometry and extracted parameter values for TM polarization

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>2.36 mm</td>
</tr>
<tr>
<td>$d_x$</td>
<td>0.106 mm</td>
</tr>
<tr>
<td>$d_{ring}$</td>
<td>2.08 mm</td>
</tr>
<tr>
<td>$\varepsilon_{2y}^{\text{theoretical}}$</td>
<td>31.19</td>
</tr>
<tr>
<td>$\varepsilon_{2y}^{\text{extracted}}$</td>
<td>31.23</td>
</tr>
<tr>
<td>$\mu_{2x}^{\text{theoretical}}$</td>
<td>0.355</td>
</tr>
<tr>
<td>$\mu_{2x}^{\text{extracted}}$</td>
<td>0.34</td>
</tr>
</tbody>
</table>

the theory, but also the extraction method are accurate. By adjusting the geometry of the rings, matching beyond 89° is also achievable. We would also like to remind the reader that the total layer thickness is only 2.36 mm, which is less than 1/10th of the free space wavelength. In comparison, the numerical study performed in [191] showed a multilayer design with thickness over 2 wavelengths and can at best achieve 5% reflectance at 85°. Fig. 5.9 plots the magnitude of the electric field at 88° incidence. From the total field, we can see that the planar wavefront is well preserved with the AMAL. From the scattered field, we observe no reflection as expected. The reflectance versus frequency of the AMAL at the design angle of 88° is shown in Fig. 5.10. For 5% reflectance, the bandwidth is from 8.95GHz to 10.95GHz. Thus, the fractional bandwidth is 20%.
Figure 5.8: At the design angle of 88°, the reflectance is 0.0086% and the transmittance is 98.1%. The Ohmic loss is around 1.9%.

Figure 5.9: Here we plot the total and scattered $|\vec{E}|$ (same color scale). From the total field, we can see that the AMAL preserves the planar wavefront. The field magnitude in the substrate is weaker because the power density along the normal to the interface should be conserved at the interface. Due to the difference in the propagation angles, the wave in the substrate has a larger projection onto the interface normal than the wave in the air. Thus, the required field magnitude is smaller in the substrate to conserve the normal power density. From the scattered field, we observe no reflected wave with the AMAL. In comparison, the reflected wave is clearly observable without the AMAL.
Figure 5.10: At the design angle of 88°, the bandwidths for 1%, 2% and 3% reflectance are 9%, 12% and 20% respectively.
5.3.2 TE Polarization

Same as before, the first step is to make an assumption on $\mu_{2z}$ to stipulate $\varepsilon_{2x}$. If we use the same approach as in the TM polarization by assuming $\mu_{2y} = \mu_{2z}$, then this suggests the synthesized structure involves orthogonal rings. In particular, $\mu_{2z}$ involves some sort of a ring structure in the X-Y plane. However, the rings in the X-Y plane in turn affect the $\varepsilon_{2x}$ due to the polarizability of the rings. As a result, the coupling between $\mu_{2z}$ and $\varepsilon_{2x}$ complicates the synthesis process. However, we are at liberty to assume any value for $\mu_{2z}$. In particular, we can assume $\mu_{2z} = 1$. The logic behind this assumption is that if we choose to not use any rings in the X-Y plane, then $\mu_{2z}$ should just be (or remain very close to) unity. By doing so, $\varepsilon_{2x}$ is also decoupled from $\mu_{2z}$ and we can use some other structures to tune $\varepsilon_{2x}$ independently. With this assumption, $\varepsilon_{2x}$ in (5.22) becomes

$$\varepsilon_{2x} = \left(\frac{\lambda_0}{4d}\right)^2 \frac{1}{\mu_{2y}} \sin^2 \theta_1$$  \hspace{0.5cm} (5.24)

Fig. 5.11 shows the parameter values for a varying layer thickness $d$. Near the grazing angle, $\mu_{2y}$ increases to a large value and $\varepsilon_{2x}$ approaches 1. Large permeability can be achieved by using the split rings [192] near resonance. However, large $\Re\{\mu\}$ is typically associated with large $\Im\{\mu\}$. A complex $\mu$ does not satisfy (5.21). As a result, we should keep $\Im\{\mu\}$ as close to zero as possible while maintaining a large $\Re\{\mu\}$.

![Figure 5.11: $\varepsilon_{2x}$ and $\mu_{2y}$ are plotted against the incident angle $\theta_1$ for layer thicknesses of 3 mm, 5 mm and 7 mm. To match near the grazing angle, $\mu_{2y}$ is quite large, which can be quite difficult to synthesize. However, by increasing the layer thickness, the required $\mu_{2y}$ can be lowered.](image)

The split ring as shown in Fig. 5.12 is analyzed by parametric study. The permeability of the split ring is plotted against $l_x$ in Fig. 5.13. Near the edge of the resonance, $\Re\{\mu_{2y}\}$ typically is no higher than 3.5 given that $\Im\{\mu_{2y}\}$ should be sufficiently close to zero. However, to match at 88°, the required $\mu_{2y}$ can be quite large as shown in Fig. 5.11. Only until the layer is sufficiently thick, i.e, 7 mm, the required $\Re\{\mu_{2y}\}$ can be lowered to 3.4. As a result, we choose a 7 mm layer thickness in our split ring design. With this thickness, the required $\varepsilon_{2x}$ is 1.35, which is usually lower than the permittivity of most common microwave substrates. (For example, a typical low index substrate is Rogers Duroid 5870 with $\varepsilon_r = 2.3$). Even though one can use foam with permittivity around 1 as the host medium, the split rings can lead to a significant increase of the permittivity beyond 1 due to the polarizability of the rings.
In addition, for practical purposes, it is not easy to fabricate the split rings on foam. As a result, we need a method to reduce the permittivity to 1.35 with a regular low-index microwave substrate. This can be done by combining a wire array with the split rings to form a super-cell as in [5]. The wire array itself has a negative permittivity [2] and near unity permeability. Thus, by combining the wire array with the split rings, we can lower the permittivity without lowering the permeability. Fig. 5.14 shows the geometry of the split ring wire super-cell. The wires are sandwiched between successive rows of split rings with a spacing of \(d_y/2\). Fig. 5.15 shows the extracted \(\mu_{2y}, \mu_{2z}\) and \(\varepsilon_{2x}\) versus \(l_x\) with \(d_x = 7.5\) mm, \(d_y = 2\) mm and \(d = 7\) mm. With \(l_x = 4.3\) mm, the extracted parameters are fairly close the required \(\mu_{2y}\) and \(\varepsilon_{2x}\).

Using this geometry as a starting point, we can optimize our structure. Table 5.2 summarizes the geometries and the extracted parameter values. Fig. 5.16 shows the reflectance of the optimized structure. At 88°, the reflectance is 0.67%. However, the transmittance is only 66% due to losses present in the system. The dashed lines show the reflectance and transmittance when the losses are removed. The transmittance rises to 98% and the reflectance is around 2%. We speculate that the loss is due to the thin metal strips and the split rings that are near the resonance. Near-resonant split rings can induce large Ohmic loss. By using wider strips and less resonant split rings, it is possible to reduce the Ohmic loss. In addition, if the AMAL is designed for less extreme angles, the required permeability value is also less extreme and hence leads to less resonant split rings. Thus, we expect the AMAL to have better
Figure 5.13: $\mu_{2y}$ is plotted against $l_x$ for three $d_y$ values. Layer thickness $d$ is 7 mm. Spacing $d_x$ is 7.5 mm. The gap is 0.1 mm. The solid and the dashed curves are the real and imaginary parts of the $\mu_{2y}$. The magnetic resonances can be observed from the sharp peaks in the real and imaginary parts of the permeability. We can avoid the large imaginary component of the permeability by operating near the edge of the resonance, but this limits the $\Re\{\mu_{2y}\}$ that can be synthesized. Typically this value is around 3.

Figure 5.14: A super-cell is formed by the split rings and the wires (strips). The strips are sandwiched between successive rows of split rings with a spacing of $d_y/2$. The wire array provides a negative permittivity which can be used to lower the effective permittivity of the super-cell.
Figure 5.15: (a) $\mu_{2y}$, (b) $\mu_{2z}$ and (c) $\varepsilon_{2x}$ are plotted as a function of $l_x$ with $d_x = 7.5$ mm, $d_y = 2$ mm and $d = 7$ mm. With $l_x = 4.3$ mm, the extracted $\mu_{2y}$ and $\varepsilon_{2x}$ are close to the desired values of 3.3 and 1.35. The extracted $\Re\{\mu_{2z}\}$ is 1.05 which validates our assumption of $\mu_{2z} \approx 1$.

transmission efficiency for less extreme angles. Fig. 5.17 shows the bandwidth of the AMAL at 88°. Due to the resonant nature of the split ring, the AMAL is fairly narrowband. The 5% reflectance bandwidth is 0.06%. Nevertheless, we proved that a very low reflectance can be achieved at the extreme angle for TE polarization. In Table 5.3, matching performance is compared between AMAL and other AR coatings that are numerically investigated in [191]. AMAL achieves near perfect matching with a very small layer thickness. To our best knowledge, AMAL is the only matching layer that is designed beyond 85°.
Table 5.2: Optimized ring geometry and extracted parameter values for TE polarization

<p>| | |</p>
<table>
<thead>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>7mm</td>
</tr>
<tr>
<td>$d_x$</td>
<td>7.5mm</td>
</tr>
<tr>
<td>$d_y$</td>
<td>1.98mm</td>
</tr>
<tr>
<td>$l_x$</td>
<td>4.35mm</td>
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</tr>
<tr>
<td>$\mu_{2y}$</td>
<td>theoretical</td>
</tr>
<tr>
<td>$\mu_{2y}$</td>
<td>extracted</td>
</tr>
<tr>
<td>$\mu_{2z}$</td>
<td>theoretical</td>
</tr>
<tr>
<td>$\mu_{2z}$</td>
<td>extracted</td>
</tr>
</tbody>
</table>

Figure 5.16: Reflectance versus incident angle $\theta_i$ for TE polarization. The reflectance at 88° is 0.67%. However, the transmittance is only 66%. This is due to metal and dielectric losses. The dashed lines show the reflectance and transmittance for the same unit cell but with the losses removed. The transmittance rises to 98% and the reflectance is 2%.

Figure 5.17: Due to the resonant nature of the split ring, the TE AMAL is fairly narrowband. The 5% reflectance bandwidth is 0.06%.
Table 5.3: The performance of AMAL is compared to that of numerically investigated AR coatings

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Polarization</th>
<th>Design angle</th>
<th>Layer thickness ($\lambda_0$)</th>
<th>Reflectance @88°</th>
<th>Angular range ($R &lt; 10%$)</th>
<th>Bandwidth ($R &lt; 10%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AMAL</td>
<td>TM</td>
<td>88°</td>
<td>0.0787</td>
<td>0.0086%</td>
<td>86° − 88.8°</td>
<td>30%</td>
</tr>
<tr>
<td>AMAL [191]</td>
<td>TE</td>
<td>88°</td>
<td>0.2333</td>
<td>0.67%</td>
<td>85.5° − 88.7°</td>
<td>0.8%</td>
</tr>
<tr>
<td></td>
<td>TM</td>
<td>80°</td>
<td>1.4</td>
<td>≈ 50%</td>
<td>75° − 85°</td>
<td>23%</td>
</tr>
<tr>
<td>Multilayer [191]</td>
<td>TE</td>
<td>80°</td>
<td>2.01</td>
<td>≈ 40%</td>
<td>55° − 85°</td>
<td>6%</td>
</tr>
<tr>
<td>Multilayer [191]</td>
<td>TE, TM</td>
<td>N/A</td>
<td>57</td>
<td>≈ 40%</td>
<td>0° − 85°</td>
<td>&gt; 46%</td>
</tr>
</tbody>
</table>

Graded-index
5.4 Experimental Results

For experimental purposes, we choose to only fabricate the AMAL consisting of closed rings for TM polarization due to the low loss and the non-resonant nature of the closed rings. In addition, the closed ring's behavior is less sensitive to fabrication inaccuracies. A quasi-optical setup is used to characterize the AMAL as shown in Fig. 5.18. It consists of two dual-polarized X-band horns and two Rexolite ($\varepsilon_r = 2.1$) lenses. The electromagnetic radiation from the horns is collimated by Rexolite lenses. A plane-wave incidence on the AMAL is emulated by placing it at the collimated Gaussian beam-waist.

![Figure 5.18](image_url)

Figure 5.18: A Quasi-optical setup is used for characterizing the transmission of the AMAL-RO3010-AMAL stack versus incident angle. The AMAL is designed for $60^\circ$, thus, maximum transmission should be observed at $60^\circ$.

Since it is not possible to have a semi-infinite substrate in the microwave regime, we will characterize the substrate with a finite thickness. Here we choose a standard thickness of 1.27 mm. If we place the AMAL on only one side of the substrate, then we need to measure both reflection and transmission to characterize the AMAL. However, our existing measurement setup is not suited for measuring reflections directly; hence the transmission measurement is preferred. To overcome this difficulty, we place AMALs on both sides of the substrate to form an AMAL-RO3010-AMAL stack. The transmission is measured as the entire stack rotates. Maximum transmission can be observed at the design angle for the stack even though we have designed the AMAL with a semi-infinite substrate. Due to the finite size of the stack, the spillover will be significant at extreme angles. This means the experimental result will be inaccurate if the AMAL is designed for $88^\circ$.

To alleviate this problem, the AMAL is re-designed to match at a less extreme angle. However, an un-matched RO3010 substrate has unity transmission at $72^\circ$ due to the Brewster angle. To avoid ambiguity, we designed the AMAL to match at $60^\circ$. At this angle, the transmittance is $85\%$ for an un-matched RO3010 substrate in comparison to the near unity transmittance for a matched case. Thus, we believe such difference in transmittance is sufficient for the experimental validation. Fig. 5.19 shows the unit cell of the entire stack. The AMAL is 4.75mm thick, consisting of one piece of 3.175mm Rogers 5870 and two pieces of 0.787mm Rogers 5870 substrates bonded together. The rings are fabricated on the 3.175mm Rogers 5870 substrate using strips and vias.

The simulation and measurement of the transmission result for this AMAL-substrate-AMAL stack are shown in Fig. 5.20. The trend of transmittance versus incidence angle matches well between the simulation and the measurement. Maximum transmittance around $99\%$ is observed at $60^\circ$. From the simulation, the AMAL has a great bandwidth as shown in Fig. 5.21. At the design angle, the bandwidth
for 99% transmittance and 98% transmittance are 22% and 39% respectively. The bandwidth is not characterized experimentally because as the frequency changes, the distances between the horns and lenses and the distances between the lenses and the stack have to change accordingly to ensure that the collimated Gaussian beam-waist coincides with the stack. Each re-positioning of the instruments may introduce slight alignment errors and require a separate calibration. In addition, the horn itself has a bandwidth of 8-12GHz. The variation of the transmittance inside this frequency range is too small (1% variation in transmittance corresponds to 0.5% in transmission) to distinguish it from the calibration or the alignment errors.

**Figure 5.19:** The AMAL is placed on both sides of the RO3010 substrate to form an AMAL-RO3010-AMAL stack. The AMAL is 4.75mm thick (one piece of 3.175mm Rogers 5870 substrate is bonded to 2 pieces of 0.787mm Rogers 5870 substrates). The ring is fabricated using copper traces connected by vias.

**Figure 5.20:** Simulated and measured transmittance of the AMAL-RO3010-AMAL stack. Maximum transmittance around 99% is observed in both the simulation and the measurement. The result is less reliable over 75° due to spillover loss. The transmittance of an un-matched RO3010 substrate is plotted for comparison. The error bars are obtained by measuring the alignment error and also by estimating the spill-over loss at large angles.
Figure 5.21: At the design angle, the simulated bandwidth for 99% transmittance and 98% transmittance is 22% and 39% respectively.
5.5 Conclusion and Outlook

In this chapter, we proposed anti-reflection theory based on an anisotropic metamaterial layer. The synthesized layers can achieve matching at near grazing angle ($88^\circ$) with a deep sub-wavelength thickness. The reflectance is less than 1% for TE or TM polarization. In comparison, conventional matching structures can only achieve 5% reflectance at $85^\circ$ and require multiple layers making the total thickness over a few wavelengths. Different structures were used for matching the TM and TE polarizations, which is the result of the different assumptions we made on the possible values of $\varepsilon_{2z}$ and $\mu_{2z}$. By no means the structures we used are the only possible choices. If different assumptions are made, then different structures might be synthesized. One can generalize this approach to match to less extreme angles or to substrates with a lower index. In fact, it is more challenging to match to a high index substrate at extreme angles as we have demonstrated due to the large parameter contrast, i.e., the required permittivity is very large and the required permeability is very small. Such parameter contrast decreases with incident angle and substrate permittivity. Furthermore, we also showed that the proposed synthesis procedure and the parameter extraction technique are a powerful combination for realizing anisotropic materials with extreme properties, e.g., we synthesized a layer with a permittivity more than 30 and a permeability less than 0.5 simultaneously. The validity of the parameter extraction method is also proved at extreme angles. This combination can be used to design structures not only for anti-reflection purposes, but also for other exotic functionalities. Last but not least, for practical purposes, simultaneous matching for both TE and TM polarizations can be desirable. This is achieved by a magneto-electric uniaxial matching layer, which is addressed in Chapter 6.
Chapter 6

Magneto-electric Uniaxial Metamaterial Layer

6.1 Introduction

In Chapter 5, an anisotropic metamaterial anti-reflection layer (AMAL) was developed for matching an arbitrary substrate at an arbitrary angle of incidence for either TE or TM polarization. Realistic structures were synthesized to demonstrate matching at an extreme angle of 88°. At such an angle, matching with conventional methods becomes difficult if not impossible. The shortcoming of the AMAL is that it cannot match TE and TM polarizations simultaneously. This is because the wave impedances for the two polarizations diverge as the incidence angle increases. Thus, it is difficult to design matching structures to match for both polarizations. Conventional AR structures are often prioritized to match one of the two polarizations.

In this chapter, we extend the matching theory to achieve polarization-insensitive matching at an arbitrary angle for an arbitrary substrate. We show that this can be achieved with a magneto-electric uniaxial metamaterial layer (MEUML), which possesses specific uniaxial permittivity and permeability tensors. A novel metamaterial unit cell is proposed to realize the MEUML. By carefully controlling the transversal and longitudinal electric and magnetic coupling in the unit cell, we can simultaneously tune the uniaxial permittivity and permeability tensors to the desired values. The MEUML is designed to match to a high-index substrate at 45° incidence and at a design frequency of 10 GHz. Furthermore, we adapt the MEUML to a radome design at X-band. The proposed MEUML radome is polarization-insensitive and works over an extremely wide angular range.

6.2 MEUML Matching Theory

The derivation process is similar to the one presented in Chapter 5. As shown in Fig. 6.1, a plane wave with TE or TM polarization from air (medium 1) impinges on the MEUML (medium 2) at an angle of θ1. The MEUML sits on an isotropic non-magnetic semi-infinite substrate (medium 3) with an arbitrary permittivity ε3. The permittivity and permeability tensors of the uniaxial layer assume the form
Figure 6.1: A plane wave impinges on the MEUML at $\theta_1$ with an arbitrary polarization. The MEUML has thickness $d$ and sits on a semi-infinite isotropic medium that is to be matched.

$\bar{\varepsilon}_2 = \begin{bmatrix} \varepsilon_{2t} & 0 & 0 \\ 0 & \varepsilon_{2t} & 0 \\ 0 & 0 & \varepsilon_{2n} \end{bmatrix}, \quad (6.1)$

$\bar{\mu}_2 = \begin{bmatrix} \mu_{2t} & 0 & 0 \\ 0 & \mu_{2t} & 0 \\ 0 & 0 & \mu_{2n} \end{bmatrix}, \quad (6.2)$

where subscript $t$ denotes the tangential material parameters in the X and Y direction, and subscript $n$ denotes the longitudinal material parameters along the Z direction. In addition, all the matrix elements are assumed to be real. The wave impedances in all three media are expressed by (6.3) to (6.8). The wave numbers in the MEUML are give by (6.9) and (6.10).

$Z_{1\text{TM}} = \eta_0 \cos \theta_1 \quad (6.3)$

$Z_{2\text{TM}} = \eta_0 \sqrt{\frac{\mu_{2t}}{\varepsilon_{2t}} - \sin^2 \theta_1 \frac{1}{\varepsilon_{2t}\varepsilon_{2n}}} \quad (6.4)$

$Z_{3\text{TM}} = \frac{\eta_0 \sqrt{1 - \sin^2 \theta_1}}{\sqrt{\varepsilon_3}} \quad (6.5)$

$Z_{1\text{TE}} = \frac{\eta_0}{\cos \theta_1} \quad (6.6)$

$Z_{2\text{TE}} = \frac{\eta_0}{\sqrt{\varepsilon_3} \sqrt{1 - \sin^2 \theta_1}} \quad (6.7)$

$Z_{3\text{TE}} = \frac{\eta_0}{\sqrt{\varepsilon_3} \sqrt{1 - \sin^2 \theta_1}} \quad (6.8)$

$k_{2t}^{\text{TM}} = k_0 \sqrt{\mu_{2t} \varepsilon_{2t} - \sin^2 \theta_1 \frac{\varepsilon_{2t}}{\varepsilon_{2n}}} = k_0 n_{2t}^{\text{TM}} \quad (6.9)$
\[ k_{2t}^{\text{TE}} = k_0 \sqrt{\mu_{2t} \varepsilon_{2t} - \sin^2 \theta_1 \frac{\mu_{2t}}{\mu_{2n}}} = k_0 n_{2t}^{\text{TE}} \] (6.10)

Following the same steps in Section 5.2, (6.3) to (6.10) are substituted into (5.7) and (5.8), and we have a total of four equations with four unknown material parameters. Hence, the tangential and longitudinal permittivities and permeabilities can be solved exactly and they are given by (6.11) to (6.14).

\[
\varepsilon_{2t} = \frac{\lambda_0}{4d} \sqrt{\frac{\varepsilon_3}{\cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}} 
\] (6.11)

\[
\mu_{2t} = \frac{\lambda_0}{4d} \frac{1}{\sqrt{\cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}} 
\] (6.12)

\[
\varepsilon_{2n} = \frac{4d \sqrt{\varepsilon_3 \sin^2 \theta_1 \sqrt{\cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}}}{\lambda_0 \sqrt{\varepsilon_3 - \cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}} 
\] (6.13)

\[
\mu_{2n} = \frac{4d \sin^2 \theta_1 \sqrt{\cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}}{\lambda_0 \sqrt{\varepsilon_3 - \cos \theta_1 \sqrt{\varepsilon_3 - \sin^2 \theta_1}}} 
\] (6.14)

Notice that

\[
\frac{\varepsilon_{2t}}{\mu_{2t}} = \frac{\varepsilon_{2n}}{\mu_{2n}} = \sqrt{\varepsilon_3} 
\] (6.15)

Using (6.11)-(6.14), we can calculate the required MEUML parameters to achieve perfect matching for a particular substrate at a particular incident angle for both TE and TM polarizations. The general trends of the MEUML parameters vs. \(\theta_1\), \(\varepsilon_3\), and \(d\) are shown in Fig. 6.2 and are summarized in Table 6.1.

**Table 6.1:** General trends of the layer parameters vs. increasing \(\theta_1\), \(\varepsilon_3\), and \(d\)

<table>
<thead>
<tr>
<th>(\theta_1)</th>
<th>(\varepsilon_2)</th>
<th>(\mu_2)</th>
<th>(\varepsilon_3)</th>
<th>(d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
<td>↓</td>
</tr>
</tbody>
</table>

6.3 MEUML synthesis

6.3.1 Design considerations

Based on the calculated permittivity and permeability tensors, one can design a MEUML to perfectly match for an arbitrary substrate at a particular angle for both polarization. However, the synthesis of a MEUML unit cell that simultaneously achieves the four material parameter values is not a trivial task. This is significantly more challenging than the synthesis of AMAL in Chapter 5, where only two material parameters are required to be realized. In fact, synthesizing metamaterial structures with tensorial properties is particularly challenging. For example, a true PML has not been physically realized with metamaterials. The implementations are always an approximate form. Previously reported metamaterial-based absorbers do not work exactly as the theoretical PML. Absorbers in [193–195] are resonance based; thus, the absorption can be narrowband, angle-sensitive, and polarization-sensitive.
Figure 6.2: Required (a) $\varepsilon_2$, (b) $\varepsilon_2\eta$, (c) $\mu_2$, and (d) $\mu_2\eta$ for perfect matching are plotted as functions of $\varepsilon_3$ and incident angle $\theta_1$ for three MEUML thicknesses of $0.1\lambda_0$, $0.3\lambda_0$, and $0.5\lambda_0$. The values for the material parameters are plotted in logarithmic scale for better visualization.

Absorbers in [194–197] only demonstrated control of the transversal parameters while the control of the longitudinal parameters is absent. For the absorber in [198], the longitudinal permittivity can be tuned for an absorber based on a nano-wire array, but the control on the magnetic parts is absent. Comparing to PML with identical $\varepsilon$ and $\mu$ tensors, realizing a MEUML with non-identical $\varepsilon$ and $\mu$ tensors is a much more daunting task. To tackle the synthesis problem of the MEUML, we follow the same iterative procedure outline in Section 5.3. However, the initial guess of a possible unit cell is
particularly important and requires one to have a good understanding of the achievable electric and magnetic responses produced by different structures.

As a proof of concept, matching a Rogers RO3010 substrate ($\varepsilon_r = 10.2$) at $45^\circ$ is demonstrated. In Fig. 6.3, the required material parameters are plotted as a function of $\theta_1$ for a few substrate layer thicknesses using (6.11) to (6.14). Based on Fig. 6.3, the first step is determining a good layer thickness.

![Figure 6.3:](image)

Figure 6.3: To match to a high-index substrate with $\varepsilon_3 = 10.2$, the required (a) $\varepsilon_{2t}$, (b) $\varepsilon_{2n}$, (c) $\mu_{2t}$, and (d) $\mu_{2n}$ of MEUML are plotted against $\theta_1$ for three layer thicknesses.

One should consider loss, sensitivity and ease of synthesis of the unit cell during this process. In terms of the synthesis, it is generally easier to realize material parameters with less contrast, i.e., the longitudinal and tangential permittivities or permeabilities do not differ too much from each other. In terms of loss, notice that a thinner layer corresponds to a higher $\varepsilon_{2t}$. Since the effective permittivity of typical metamaterials can be characterized by the Drude model, the electrical response of such materials can be described by the Kramer-Kronig relation, which states that the real and imaginary parts of the permittivity are not independent from each other [199]. A material with a larger $\Re\{\varepsilon\}$ is often associated with a more negative $\Im\{\varepsilon\}$, suggesting the material is more dispersive and dissipates more energy. In this aspect, a thinner layer can be more lossy. In terms of sensitivity, the structure should not operate near its resonance. As we have discussed in Chapter 5, it was found that realizing a layer with a strong para-magnetic property (large $\mu$) relies on the resonance of the metamaterial structure. Such a layer is not only more lossy, but also more sensitive than a layer with a weak para-magnetic property ($\mu \approx 1$) or a diamagnetic property ($\mu < 1$). This is also why we choose to match to $45^\circ$ instead of an extreme angle of $88^\circ$ (as in the last chapter), as the required $\mu_{2t}$ is a much larger value at $88^\circ$ than at $45^\circ$. Even though one can increase the layer thickness further to decrease the required $\mu_{2t}$ at $88^\circ$, the resulting structure would be too thick to fabricate. Bearing those considerations in mind, a layer thickness of 5mm is a good starting point in the iterative procedure. The required permittivities are not too high and the permeabilities are close to or smaller than unity.
6.3.2 MEUML unit cell design and simulation results

The proposed MEUML unit cell is shown in Fig. 6.4. The final thickness of the MEUML is revised to 4.75mm, which is an easier thickness to realize in fabrication. The required material parameters are: \(\varepsilon_{2t} = 3.40, \varepsilon_{2n} = 1.51, \mu_{2t} = 1.06, \) and \(\mu_{2n} = 0.62\). To achieve those parameter values, we propose to use two copper rings lying in the X-Y plane that are separated by an air hole. The working principle of the proposed unit cell can be explained with the help of Fig. 6.5. The electric and magnetic fields of the incident TE and TM polarizations can be decomposed into their tangential and longitudinal components. The tangential electric field will induce current \(J_E\) flowing on both rings. The induced currents are dipolar in nature and will produce a capacitive loading as in conventional artificial dielectrics [181], which results in an increase in the effective tangential permittivity. This enables us to increase the effective \(\varepsilon_{2t}\) from the host medium permittivity of 2.3 to the required permittivity of 3.4. From the longitudinal electric field point of view, the top and bottom rings constitute a capacitor and produce a capacitive loading. By changing the outer and inner ring radii, and the ring spacing, \(\varepsilon_{2n}\) can be tuned. The longitudinal magnetic field will induce currents flowing around the rings [200]. Since the rings are electrically small, they are diamagnetic according to Lenz’s law. This diamagnetic effect is also enhanced by the mutual inductance between the rings. As a result, the effective longitudinal permeability is reduced. By controlling the size of the rings, \(\mu_{2n}\) can be tuned. The tangential magnetic field induces counter-flowing currents on the top and bottom rings [200,201], which form a partial current loop. This partial current loop can induce a weak magnetic response and causes a change in the effective tangential permeability. The effect generally gets weaker as the rings are spaced further apart. Thus, by controlling the spacing between the rings, \(\mu_{2t}\) can be tuned slightly from unity. Lastly, notice that the required \(\varepsilon_{2n}\) is 1.51, which is lower than the substrate permittivity of 2.3. Due to the capacitive coupling between the top and bottom rings, \(\varepsilon_{2n}\) is always larger than the host medium permittivity. Thus, a hole is drilled into the substrate to dilute the host medium permittivity, which in turn lowers \(\varepsilon_{2n}\). Varying the air hole diameter has minimal impact on the permeabilities, but it does change \(\varepsilon_{2t}\). Thus, the copper ring sizes have to be adjusted accordingly. In general, the four material parameters are coupled and varying one geometric parameter can change all four material parameters simultaneously. To find optimal geometries, extensive parametric sweeps and optimizations were performed.

The extracted wave impedances and the permittivities and permeabilities for the proposed unit cell are plotted against the incident angle in Fig. 6.6 and Fig. 6.7, respectively. The corresponding parameter extraction technique is described in Appendix C. Table 6.2 compares the extracted values and the required theoretical values for perfect matching at 45°. The extracted parameter values are in good agreement with the theoretical ones. Fig. 6.8 shows the simulated reflection, transmission, and loss of the proposed unit cell. Minimum reflections for both TE and TM polarizations are observed near 45° as expected. Due to the low-loss nature of the unit cell design, the transmission is near unity at the design angle. Since the extracted material parameters deviate slightly from the theoretical values, the minimum reflection for TM polarization is at 40°. We would like to point out that in this unit cell design, the spacing between the rings is constrained to a standard thickness of 3.175mm. The results can be further improved if one is not limited to standard substrate thicknesses. In addition, the basic optimization routines in HFSS often do not output optimal results for this complexly coupled structure. A more sophisticated optimization routine can be employed in future designs.

The reflections for TE and TM-polarized waves are plotted against \(\theta_1\) and frequency in Fig. 6.9. Since the synthesized MEUML is not resonant, the matching performance is wideband. At the design
Figure 6.4: (a) In the unit cell view, the MEUML is sandwiched between a semi-infinite air region and a semi-infinite substrate. The host medium of the MEUML is a Rogers RO5870 ($\varepsilon_r = 2.3$) slab with a total thickness of 4.75mm. (b) The physical realization of the MEUML. The 4.75mm slab is realized by bonding a 3.175 mm substrate to two 0.787 mm ones as shown in the exploded view. Two copper rings are patterned on the inner surfaces of the 0.787 mm substrates and an air hole is drilled through the 3.175 mm one. The geometries of the final structure are: $r_i = 1.04$ mm, $r_o = 1.94$ mm, and $r_{drill} = 2.3$ mm.

Figure 6.5: (a) The electric field of a TM-polarized wave at an oblique incidence can be decomposed into tangential and longitudinal components. The tangential electrical field induces dipolar currents flowing on the rings, which result to a capacitive loading and lead to an increase in $\varepsilon_{2t}$. The top and bottom rings can be treated as a capacitor; thus, $\varepsilon_{2n}$ can be tuned by controlling the spacing and the area of the rings. (b) The tangential magnetic field from the TE polarized component induces counter-flowing currents on the top and the bottom rings, which form a partial current loop. This induces a weak magnetic effect and can be used to tune $\mu_{2t}$ slightly from unity. On the other hand, the longitudinal magnetic field will induce current loops flowing around the rings. According to Lenz’s law, the induced currents generate an opposing magnetic field to the incident one. Due to this diamagnetic effect, $\mu_{2n}$ is lowered. Since the required $\varepsilon_{2n}$ of 1.5 is lower than the host substrate permittivity, an air hole is used to lower the effective host permittivity and this in turn lowers the $\varepsilon_{2n}$ to the desired value.

Table 6.2: Theoretical vs. extracted MEUML parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Theoretical</th>
<th>Extracted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{2t}$</td>
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<td>3.20</td>
</tr>
<tr>
<td>$\mu_{2t}$</td>
<td>1.06</td>
<td>1.09</td>
</tr>
<tr>
<td>$\varepsilon_{2n}$</td>
<td>1.51</td>
<td>1.40</td>
</tr>
<tr>
<td>$\mu_{2n}$</td>
<td>0.47</td>
<td>0.62</td>
</tr>
</tbody>
</table>
Figure 6.6: The wave impedances of the MEUML are plotted as a function of angle.

Figure 6.7: Extracted MEUML parameters as a function of the incident angle $\theta_1$. The real parts (solid lines) and the imaginary parts (dashed lines) are shown for (a) $\varepsilon_{2t}$, (b) $\varepsilon_{2n}$, (c) $\mu_{2t}$, and (d) $\mu_{2n}$. The imaginary parts of all four parameters are near zero, indicating the MEUML having a small loss. At normal incidence, the electromagnetic wave is independent from the longitudinal material parameters are undefined and cannot be extracted (See Appendix C).

angle, the fractional bandwidths for a -20 dB reflection level are 15% and 29% for TE and TM polarization, respectively. From Fig. 6.10, it is also clear that the matching performance is independent of $\phi$, which is the azimuthal angle in the X-Y plane. This is a result of the small unit cell size ($\lambda_0/6$) and the symmetrical geometries (circular rings and holes). The MEUML behaves as a well-homogenized slab with little material parameter variations in the tangential directions. The independence from $\phi$ is important for practical applications since the incoming signal may impinge at an arbitrary azimuthal angle. Matching variations in $\phi$ may lead to performance degradation. Fig. 6.11 plots the losses of the MEUML as a function of frequency. It is clear that the dielectric or the metal loss is quite small.
Figure 6.8: The simulated (a) reflection $r$, (b) transmission $t$, and (c) loss of the MEUML are plotted against $\theta_1$. Minimum reflections for both TE and TM polarizations are observed near 45°. Since the matching layer exhibits very small loss, the transmission is near unity at the design angle.

Figure 6.9: Reflections are plotted against $\theta_1$ and frequency. (a) TE polarization, (b) TM polarization. Due to the non-resonant nature of the MEUML, the matching performance is very wideband. At the design angle, the fractional bandwidths for a -20 dB reflection level are 15% and 29% for the TE and TM polarizations, respectively.

Figure 6.10: The azimuthal response of the MEUML. (a) $r_{TE}$ and (b) $r_{TM}$ are plotted against $\theta_1$ and $\phi$, where $\phi$ is the azimuthal angle in the X-Y plane. Due to the symmetry of the unit cell, $\phi$ is only plotted from 0° − 45°. It is clear that the matching performance is independent of $\phi$. 
Figure 6.11: The losses of the MEUML are plotted as a function of frequency.
6.4 MEUML based radome

To further demonstrate the new matching possibilities enabled by the MEUML, we adapt the MEUML for impedance matching in the microwave regime, which has a slightly different design philosophy than in the optical regime. For optical applications, the matching layer is usually designed to match from air to a dielectric substrate, which is usually optically thick (thousands of wavelengths). Thus, when designing the matching layer, it is assumed to be sandwiched between a semi-infinite air region and a semi-infinite substrate. In contrast, the assumption of a semi-finite substrate is typically not valid in the microwave regime since the wavelength is much longer. Hence, the matching layer is designed to match from air to air. This kind of matching layer is essentially a radome. In this section, we show that a high-performance radome can be constructed from the MEUML.

6.4.1 Background on radomes

Radomes are mechanically robust and act as RF (radio frequency) transparent enclosures that protect expensive radars or telecommunication apparatus from environmental effects. They have been widely used in weather and airborne radars. The most common radome designs are single-wall and sandwich constructions [202, 203]. A single-wall radome is shown in Fig. 6.12a. It is usually a piece of low-loss dielectric with a half-wavelength electrical thickness. At normal incidence, the reflectance from the radome is zero. Despite its simplicity, a single-wall radome is matched to a single frequency and the reflectance quickly rises as the incidence becomes more oblique, as shown in Fig. 6.13. Consequently, single-wall radomes typically have a spherical or hemispherical shape such that as the radars scan inside the radomes, the beams always impinge on the radome at normal incidence. For radars used on the nose of a high speed aircraft, aerodynamic consideration demand the radomes to have more tapered shapes such as cones or ogives as shown in Fig. 6.12b. Radar beams usually impinge on these radomes at large oblique angles. In order achieve a good transmission over a wide angular range for both polarizations, a sandwich radome is chosen over the simple single-wall radome. A sandwich radome usually consists of two high/low-index thin dielectric skin layers separated by a low/high index dielectric core layer. Minimization of the total reflection can be achieved by the mutual cancellation of the reflections between the skin layers. The indices of refraction and the thicknesses of the skin and core layers are extensively optimized to achieve the best angular performance for both polarizations. However, since the materials used are generally isotropic and non-magnetic, it is difficult to achieve the same performance for both polarizations. The performance for TM-polarizations is usually better due to the existence of a Brewster’s angle in the dielectric materials. We can show that with the MEUML as the skin layer, and a RO3010 substrate as the core layer in a sandwich radome, performance for TE and TM polarizations can be improved simultaneously.

6.4.2 MEUML radome design

The total reflection of the MEUML sandwich radome can be analyzed with the transfer matrix method (TMM). The transfer matrix is constructed as in (6.16) with reference to Fig. 6.14.

\[
\begin{bmatrix}
a_0 \\
b_0
\end{bmatrix} = \frac{T_{01}P_1T_{12}P_2T_{21}P_1T_{10}}{t_{01}t_{12}t_{21}t_{10}} \begin{bmatrix}
a_4 \\
b_4
\end{bmatrix} = [\mathcal{M}] \begin{bmatrix}
a_4 \\
b_4
\end{bmatrix}
\]  

(6.16)
Figure 6.12: (a) A single-wall radome is a dielectric substrate with a half-wavelength electrical thickness. (b) A sandwich radome is a multilayer structure with a low-index/high-index core layer sandwiched by high-index/low-index skin layers.

Figure 6.13: Reflections as a function of incident angle are plotted for a single-wall radome with different permittivities. Solid and dashed lines show the reflections for the transverse-electric (TE) and transverse-magnetic (TM) polarizations, respectively. The reflection is zero at normal incidence, but it rises quickly with increasing $\theta$, especially for the TE polarization. For TM polarization, the reflection is also zero at the Brewster’s angle, which is a function of the dielectric permittivity.

Figure 6.14: The MEUML sandwich radome can be analyzed with the transfer matrix method.
where \( a_i \) and \( b_i \) are the wave amplitudes in medium \( i \). \( T_{ij} \) and \( P_k \) are defined as

\[
T_{ij} = \begin{bmatrix} 1 & r_{ij} \\ r_{ij} & 1 \end{bmatrix}
\]

(6.17)

\[
P_k = \begin{bmatrix} e^{j\phi_k} & 0 \\ 0 & e^{-j\phi_k} \end{bmatrix}
\]

(6.18)

where \( r_{ij} \) and \( t_{ij} \) are the Fresnel coefficients at each layer interface, and \( \phi_k \) is the phase delay in each layer. The terms in (6.16) can be grouped as in (6.19) to gain insight in the total reflection of the sandwich structure.

\[
\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} = \frac{T_{01}P_1T_{12}}{t_{01}t_{12}} \frac{T_{21}P_1T_{10}}{t_{21}t_{10}}
\]

(6.19)

With (6.19), the total reflection of the MEUML radome can be calculated:

\[
r_{\text{sandwich}} = \frac{M_{21}}{M_{11}} = \frac{r_{02}e^{j\phi_2} - r_{02}e^{-j\phi_2}(t_{20}t_{12} - r_{02}r_{20})}{e^{j\phi_2} - e^{-j\phi_2}r_{02}r_{20}}
\]

(6.20)

From (6.20), we immediately notice that if the MEUML achieves perfect matching between the semi-infinite air and the semi-infinite substrate \((r_{02} = 0)\), the total reflection of the sandwich structure is immediately zero and it is independent of the thickness (which is related to phase \( \phi_2 \)) of the RO3010 substrate. Furthermore, if \( r_{02} \) is small, then \( r_{\text{sandwich}} \) can be approximated as (6.21). This implies that \( r_{\text{sandwich}} \) is small as long as \( r_{02} \) is small.

\[
r_{\text{sandwich}} \approx r_{02}\left( \frac{e^{j\phi_2} - t_{02}t_{20}e^{-j\phi_2}}{e^{j\phi_2}} \right)
\]

(6.21)

It is immediately clear why the MEUML sandwich radome would outperform conventional ones from Fig. 6.15. In conventional radomes, \( r_{02} \) for the TM polarization usually decreases first due to the Brewster’s angles, and \( r_{02} \) for the TE polarization usually increase monotonically. Hence, the angular range over which \( r_{02} \) remains small is usually smaller for the TE polarization than for the TM polarization. In comparison, with the MEUML as the skin layers, it is possible to minimize \( r_{02} \) at an oblique angle for both polarizations as shown in Fig. 6.8a. Hence, the angular range can be significantly increased for the TE polarization. From Fig. 6.8a, we can infer that the MEUML sandwich will have a small reflection between \( 0^\circ - 60^\circ \) as \( r_{02} \) is very small in this angular range. To reduce \( r_{\text{sandwich}} \) beyond \( 60^\circ \), the thickness of the RO3010 core layer can be optimized. Due to the weak dependence on the substrate thickness for angles between \( 0^\circ - 60^\circ \), the optimization process will not introduce too much performance degradation.
in this angular range.

![Diagram showing trends of \( r_{02} \) for different polarizations and substrate thicknesses.](image)

**Figure 6.15:** Trends of \( r_{02} \) for (a) a conventional dielectric skin, and (b) the MEUML skin

Using (6.20), we can calculate the maximum reflection within the angular range of \( 0^\circ - 85^\circ \) as a function of the RO3010 substrate thickness. The results is plotted in Fig. 6.16. With an optimal thickness of 1.25mm, the maximum reflections for both TE and TM polarizations are -14.6 dB, which correspond to a reflected power of 3.5%. For fabrication purposes, the RO3010 substrate is chosen with a standard thickness of 1.27mm. The resulting radome is very compact and has an overall thickness of 10.8mm, which is about \( \lambda_0/3 \). Fig. 6.17 shows the total reflection of the MEUML radome as a function of the incident angle. Very good agreement is observed between the calculated and simulated results. To our best knowledge, the performance of maintaining less than 5% reflected power over \( 0^\circ - 85^\circ \) for both TE and TM polarizations is one of the best results reported so far in the literature. In addition, the radome is wideband as shown in Fig. 6.18. For TE-polarized waves, the bandwidth is 9.95 – 10.25 GHz such that the reflection stays below -10 dB for the entire angular range of \( 0^\circ - 85^\circ \). For TM-polarized waves, the -10 dB bandwidth is 9.1 – 10.2 GHz. For applications that require a smaller angular range, e.g., \( 0^\circ - 60^\circ \), the -10 dB reflection bandwidths become 2.5 GHz and 3.6 GHz for TE and TM polarizations, respectively.
Figure 6.17: The total reflection of the MEUML radome is plotted against $\theta_1$ at 10 GHz. Excellent agreement is observed between the TMM calculation and the simulation results. The reflections for both polarizations stay below -14 dB for the entire angular range of $0^\circ - 85^\circ$.

Figure 6.18: The simulated total reflections of the MEUML radome are plotted against $\theta_1$ and frequency for (b) TE polarization and (c) TM polarization. The matching performance is wideband as expected. The bandwidths for maintaining less than -10 dB reflection for the entire range of $0^\circ - 85^\circ$ are 0.3 GHz and 1.1 GHz for the TE and TM polarizations, respectively.

6.5 MEUML radome measurements

The MEUML radome is fabricated with standard PCB techniques. As shown in Fig. 6.19a, the MEUML consists of three layers: two 0.787 mm and one 3.175 mm Rogers RO5870 substrate. The copper rings are patterned on the 0.787 mm substrates and the holes are drilled on the 3.175 mm one. After the patterning and drilling processes, the three substrates are bonded together with the Rogers 3001 bonding film to form the MEUML. The fabricated MEUML is cut in half and bonded to two sides of the 1.27 mm thick Rogers 3010 substrate. The resulting structure is the MEUML radome. The patterning, drilling and alignment errors are better than 0.1 mm. The detailed feature of the fabricated radome is shown in Fig. 6.19b.

To characterize the radome, we first performed a wideband far-field measurement with the setup
shown in Fig. 6.20a. A small slot antenna was placed behind the MEUML radome with a 1.5 cm thick foam as a spacer. Two gain measurements were conducted with and without the radome in front of the antenna. The transmission of the radome can be extracted from the difference of the gains. As the pedestal rotated, the transmission of the radome was measured against the incident angle. The transmitting pyramidal horn can be rotated such that the radiated electric field is polarized either horizontally or vertically. Thus, the transmission of the radome for a TE or TM-polarized wave can be measured accordingly. The transmitting pyramidal horn has a bandwidth of 8.2 – 12.4 GHz, so it is difficult to use a single antenna receiver to cover the entire bandwidth. Three slot antennas as shown in Fig. 6.20b were designed and each covers a different band. By stitching the results from each band, the radome can be characterized over the entire bandwidth.

Figure 6.19: (a) shows the expanded view of the MEUML radome. (b) shows the detailed features of the fabricated prototype.

Figure 6.20: (a) shows the far-field measurement setup. A receiving slot antenna is placed behind the MEUML radome with a 1.5 cm thick foam as a spacer. Two gain measurements are conducted with and without the radome in front of the antenna. The transmission of the radome can be extracted from the measured gain difference. As the pedestal rotates, the transmission of the radome can be measured against the incident angle. The transmitting pyramidal horn can be rotated such that the radiated electric field is polarized either horizontally or vertically. Thus, the transmission of the radome for TE or TM-polarized waves can be measured accordingly. (b) Since the transmitting horn has a wide bandwidth of 8.2 – 12.4 GHz, three slot antennas were used to cover the entire bandwidth. Slot antennas 1, 2, and 3 have -10 dB reflections between 8.6 – 10.2 GHz, 9.2 – 10.6 GHz, and 10 – 12.2 GHz, respectively.
Fig. 6.21 compares the simulated and measured radome transmission. The simulated and measured results are in good agreement. The radome is wideband; it maintains near unity transmission between 8.2 – 12.4 GHz for angles up to 80°. However, the accurate transmission of the radome cannot be obtained for the following reasons: First, the far-field measurement setup has an accuracy no better than 0.5 dB. Second, the radome was placed in the near field of the small receiving antenna, which may distort the radiation pattern of the antenna. Thus, when calculating the transmission of the radome from the measured gain difference, the transmission can be inaccurate and sometimes exceed unity. Third, the radome has a finite size; thus, diffraction occurring at the edges of the MEUML may introduce additional inaccuracies, especially at extreme angles.

To obtain a more accurate radome transmission, we conducted a quasi-optical measurement as shown in Fig. 6.22a using the same setup presented in Chapter 6. The quasi-optical setup is narrowband; hence, the radome was measured only at 10 GHz. The simulated and measured transmissions are shown in Fig. 6.23. A very good agreement between the simulated and measured results can be observed for angles between 0° – 65°. The measured results drop more sharply beyond 65°. This is due to the spill-over loss.
As illustrated in Fig. 6.22b, at 70°, the projection of the finite-size (30cm long) sandwich structure onto the beam-waist plane is smaller than the Gaussian beam-waist. Thus, near 70°, diffraction occurring at the edges of the radome leads to spill-over loss and results in a decrease in the measured transmission. Nevertheless, the good agreement between the simulated and measured results for less oblique angles validates the theory and the design of the MEUML based radome.

Table. 6.3 summarizes the performance of the MEUML radome and the conventional sandwich radomes reported in [202]. It is clear that the MEUML radome has very transmission from normal incidence to large oblique angles. In comparison, conventional sandwich radomes have to optimize for normal incidence at the expense of the transmission at large oblique angles, or vice versa.

![Figure 6.22: (a) shows the quasi-optical setup. Since this is a narrow-band system, the measurement is performed only at 10 GHz. (b) illustrates the quasi-optical characterization of the MEUML radome. Instead of measuring the total reflection, the total transmission is measured as the radome rotates. The radome is placed at the beam-waist plane such that the wavefront impinging on the radome is planar.](image)

6.6 Discussion and Conclusion

Until the recent developments in metamaterials, the anti-reflection structures developed for either optical or microwave applications were often limited to the use of isotropic dielectrics since the discovery of the anti-reflection phenomenon by Lord Rayleigh in the 19th century. However, most of the metamaterial based matching layers have similar structures to frequency selective surfaces, which often employ metallic strip or patch arrays. From the effective medium point of view, the metallic elements usually only increase the tangential effective permittivity due to the capacitive coupling between them. In this aspect, these metamaterial layers are not so different than conventional dielectric matching layers, which usually have a worse matching performance for the TE polarization due to the lack of control of the effective permeability. Although some metamaterial layers do achieve control of the permeability through the use of split-ring resonators (SRR), these resonant-based structures are often lossy and narrowband.
In this regard, unlike previous works, the proposed MEUML is designed by judiciously controlling the tangential and longitudinal permittivities, and permeabilities. Consequently, the MEUML is able to simultaneously match TE and TM-polarized waves at oblique incident angles. We demonstrated this concept by synthesizing a physical MEUML which achieves a near perfect matching for the high-index RO3010 substrate ($\varepsilon_r = 10.2$) at 45°. The resulting reflection levels are around -30 dB for both polarizations. In addition, since we have greater degrees of freedom in the design process than before, it is possible to relax some constraints such that extreme or negative material parameters are not required. Thus, we do not have to rely on resonances, which otherwise would make the structure sensitive, narrowband and lossy, to achieve those extreme material parameters. In the simulations, we showed that the MEUML is low-loss and broadband, which makes it very attractive for practical applications.

We have further demonstrated the practicality of the MEUML by designing a novel sandwich type radome for microwave applications. We were able to show that a $\lambda_0/3$ thick radome can maintain reflection levels below -14 dB over the entire angular range of 0° − 85° for both polarizations, and over
a wide bandwidth. To our best knowledge, this is the widest angular range that has been reported in the literature for such a compact radome. The transmission of the fabricated radome is characterized by a wideband far-field measurement and a narrowband quasi-optical measurement. From the far-field measurements, we can conclude that the radome is indeed wideband and wide angle albeit the measured transmissions is less accurate. More accurate transmissions were obtained from the narrowband quasi-optical measurement. We observed an excellent agreement between the simulated and measured results for angles between $0^\circ - 65^\circ$. For angles beyond $65^\circ$, the measured results are less accurate due to spill-over loss. Even thought the quasi-optical measurement was conducted only at 10 GHz, with the combination of the wideband far-field measurements, the radome is indeed working as it was proposed.

In summary, we have proposed a novel matching theory based on magneto-electric uniaxial metamaterials. This theory has been validated by both simulations and measurements. Although the simulations and measurements were performed in the microwave regime, the theory is general and can be applied in the optical and terahertz regimes. Furthermore, we showed that even though the MEUML has complex material parameter tensors, it is possible to realize it with a simple unit cell. With the help from the provided synthesis procedure and the proposed parameter retrieval technique presented in Appendix C, we can tune those parameters precisely. We believe the MEUML concept not only opens new routes for impedance matching applications, but also opens new possibilities for designing more sophisticated metamaterials that provide further exotic functionalities.
Chapter 7

Double-mesh Based Metamaterial Radome

7.1 Introduction

Fifth-generation (5G) telecommunication equipment [162] or autonomous driving radars [204] can operate between 28 GHz to 94 GHz to achieve high data rate or high resolution for obstacle detection. In this frequency range, the wavelength is on the order of millimetres, which can lead to a significant free space path loss. To compensate for this, high-gain antennas such as phased arrays become popular. For practical applications, there is an upper bound on the antenna gain due to limited physical size of the phased arrays. Hence, minimizing losses becomes important. One of the losses that is often overlooked is the one due to the reflections of enclosures, which the antennas are embedded behind. For many practical applications, the enclosure is on the order of a millimetre (mm), e.g., the thickness of plastic enclosures on laptops and cellphones is usually around 1 mm. These enclosures can be considered as electrically-thin substrates at 2.4 GHz (a typical ISM band for cellular and Wi-Fi communications), and the corresponding power reflection coefficients (reflectance) are negligible. However, in the mm-wave regime, the same enclosure has a thickness that is comparable to the wavelength, and can result in a strong reflectance as shown in Fig. 7.1. As one can see, the reflectance is maximized when the enclosure thickness is an odd number of multiples of a quarter wavelength, and it is minimized when the thickness is an even number of multiples of a quarter wavelength. To get a sense of possible reflectance introduced by the enclosure, a simple calculation is made for 30 GHz. Assuming the enclosure has a permittivity of 4 (a typical value for many plastics), a 1 mm thickness is equal to $0.19\lambda_e$ ($\lambda_e$ is the wavelength inside the enclosure), and the corresponding reflectance is 33%. In comparison, the reflectance of the same enclosure at 2.4 GHz is only 0.56%. Hence, in the mm-wave regime, the enclosures should be designed as radomes to minimize reflections.

The MEUML sandwich radome introduced in Chapter 6 has a very good bandwidth and angular range, but the fabrication is already challenging at 10 GHz, making the MEUML radome unsuitable to adapt to mm-wave frequencies. For practical reasons, single-wall radomes are preferable as they are thinner, lighter, and easier to fabricate. However, as we have discussed in Section 6.4.2, single-wall radomes achieve zero reflectance only at a single frequency and at normal incidence. Reflectance quickly rises as the incident angle becomes more oblique, especially for the transverse-electric (TE) polarization...
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Figure 7.1: The power reflection coefficient (reflectance) is plotted as a function of the normalized enclosure thickness (d/λe). εe and λe are the enclosure permittivity and the wavelength inside the enclosure, respectively.

(Fig. 6.13). Furthermore, it may be very challenging to meet the fabrication tolerance for single-wall radomes. Reflectance can increase rapidly as the thickness deviates away from half-wavelength, especially if the material has a high permittivity (a steeper slope in Fig. 7.1). As a result, mm-wave applications call for unconventional radome designs that not only can provide good performance, but also can be fabricated easily and economically.

In light of this, single layer metamaterial based radomes were investigated [205–207]. In [205], an array of parallel ribs were milled directly on a piece of a substrate. Those ribs act as a matching layer such that the reflection from the entire structure can be minimized. This design has two drawbacks: 1. It is polarization sensitive since the effective indices are different in the directions parallel and perpendicular to the ribs; 2. It is very sensitive to the fabrication accuracy. A 0.1 mm difference in milling can significantly affect the matching performance. In [206, 207], radomes based on the fishnet structure were studied. Near the resonance of the fishnet structure, the effective permittivity and permeability approach −1 simultaneously. As a result, the wave impedance of the structure is matched to that of free space, resulting in minimal reflections. However, since the radome is operating near the resonance of the fishnet, it is inherently narrowband and lossy. The minimum transmission loss in [206] is over 1dB and the transmission is highly sensitive to the incident angle. Inspired by [206], we propose a thin metamaterial-based radome (meta-radome) in this chapter. The enclosure, which is assumed to be a flat dielectric substrate, is sandwiched by two metallic meshes on both sides, effectively transforming the original enclosure into a homogenized metamaterial-slab. With an appropriate mesh geometry, the effective wave impedance of the slab can be matched to that of free space. In contrary to the work in [206,207], the structure we propose is not resonant and it is not required to achieve a negative index. As a result, the proposed meta-radome can be low-loss, broadband and insensitive to incident angles.

7.2 Theory and design

As shown in Fig. 7.2, a plane wave with TE or TM polarization is incident on a free-standing meta-radome from the Y-Z plane at an angle of θ1. The total reflection of this thin radome is given by
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Figure 7.2: A plane wave with TE/TM polarization is incident on a free standing meta-radome at \( \theta_1 \). The radome has a sub-wavelength thickness of \( d \).

(7.1)

\[ r = \frac{r_{12} + r_{23}e^{-i2\phi}}{1 - r_{21}r_{23}e^{-i2\phi}}, \]

where \( r_{ij} \) is the Fresnel reflection coefficient at the \( i, j \) media interface with incidence from medium \( i \). In this specific case, \( r_{23} = r_{21} = -r_{12} \). The phase \( \phi \) is the total phase delay accumulated in the radome along the normal of the layer surface (\( z \)-axis). Specifically,

(7.2)

\[ r_{ij} = \frac{Z_j - Z_i}{Z_j + Z_i}, \]

(7.3)

\[ \phi = k_2 z d, \]

where \( Z_i \) is the wave impedance in medium \( i \), \( k_{2z} \) is the wave number along the \( z \)-axis and \( d \) is the radome thickness. Zero reflection can be achieved if \( r_{12} \) or \( r_{21} \) is zero, which implies that the wave impedance of the meta-radome should be equal to the wave impedance of free space from (7.2). At normal incidence, the wave impedance of the meta-radome is \( Z_{mr} = \eta_0 \sqrt{\mu_t/\varepsilon_t} \), where \( \eta_0 \) is free space intrinsic impedance; \( \mu_t \) and \( \varepsilon_t \) are the tangential effective permittivity and permeability of the meta-radome. When \( \varepsilon_t \) is equal to \( \mu_t \), the meta-radome is matched to free space (at normal incidence).

One possible unit cell design for the meta-radome is shown in Fig. 7.3. Metallic mesh grids are patterned on both sides of the dielectric substrate. The design philosophy can be explained as follows. For a regular dielectric, the permittivity is larger than the permeability. To bring the effective permittivity of the meta-radome equal to the effective permeability, it is generally easier to decrease the effective permittivity than increase the effective permeability as we have discussed in Chapter 5. In addition, increasing the permeability usually invokes some structural resonance, which should be avoided for broadband performance. Since wire arrays can be treated as a medium with a negative permittivity \([2, 208]\), combining them with the dielectric substrate would lower the effective tangential permittivity \( \varepsilon_t \). By having orthogonal wire arrays, which would form a mesh grid, the resulting structure would be polarization insensitive. When two mesh grids are patterned on both sides of the substrate, there is a magnetic coupling between the top and both meshes, which can be used to tune the tangential permeability \( \mu_t \). Thus, by optimizing the mesh geometry, it is possible to make \( \varepsilon_t \) equal to \( \mu_t \). This tuning procedure is carried out by parametric studies, and the effective material parameters are extracted through the method outlined in Appendix C.
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As an example, we design a meta-radome with a 0.64 mm RO3006 substrate with $\varepsilon_r = 6.15$ that emulates the enclosure material. The design frequency is 34.3 GHz. Fig. 7.4 illustrates the dependence of $\varepsilon_t$ and $\mu_t$ on the geometric parameters $p$ while $w$ is fixed at 0.3 mm. As expected, the effective permittivity decreases as the wire grid gets denser, i.e., with a smaller $p$. On the other hand, $\mu_t$ depends less on $p$. Rather, $\mu_t$ depends more on the mutual coupling between the top mesh and the bottom mesh. In this case, the constant spacing between the meshes results in an almost constant $\mu_t$. By tuning $p$, it is possible to make the real parts of $\varepsilon_t$ and $\mu_t$ equal as shown in Fig. 7.4a. The imaginary parts of the extracted parameters are plotted in Fig. 7.4b, and they are slightly negative and close to zero. This implies the total loss (including both the conductor and dielectric losses) of the meta-radome can be very small, which is indeed the case as shown in Fig. 7.4c. When $\varepsilon_t = \mu_t = 0.72$, reflection is minimized and transmission is near unity. The total loss (including both the conductor and dielectric losses) is around 2.1% from the simulated results. The low-loss characteristic of the meta-radome can be attributed to its non-resonant nature. It is worth pointing out that the implementation of the meta-radome is not unique. It is possible to select other values of $p$ and $w$ for the meshes. For example, $p = 2.13$ mm and $w = 0.1$ mm, and $p = 2.96$ mm and $w = 0.5$ mm are the other two possible combinations for designing high-transmission meta-radomes on the 0.64 mm thick RO3006 substrate. In addition, the host medium is not limited to a 0.64 mm thick RO3006 substrate. It is possible to design the meta-radome based on other low-index or high-index substrates with different thicknesses.

7.3 Characteristics of the meta-radome

7.3.1 Transmission magnitude response

Fig. 7.5 shows the transmission versus incident angle $\theta_1$ for both TE and TM polarizations at the design frequency. The meta-radome is benchmarked against a regular low-index Rogers 5870 substrate ($\varepsilon = 2.3$) and Rogers RO3006 substrate, which is the host medium for the meta-radome. The meta-radome exhibits nearly flat angular responses up to 70° for both polarizations. Especially for TE polarization at oblique
angles, the meta-radome demonstrates a significant improvement over the regular substrates. For TM polarization, regular substrates can achieve a good transmission near their Brewster angles ($\approx 60^\circ - 70^\circ$). However, the transmission at normal incidence is compromised. Specifically, at normal incidence, the meta-radome has 0.25 dB and 2.7 dB higher transmission than the RO5870 and RO3006 substrates, respectively. If the transmissions for TE and TM polarizations are averaged, the meta-radome outperforms the regular substrates for all incident angles. Furthermore, the differences in the transmission magnitudes between the two polarizations are smaller in the meta-radome. A small magnitude difference
is important for maintaining polarization fidelity, i.e., one would want to avoid turning a circularly-polarized wave into an elliptically-polarized wave. From Fig. 7.5, one can see that at an angle around \(70^\circ\), the meta-radome has significantly smaller magnitude difference than the regular substrates.

![Figure 7.5](image)

**Figure 7.5:** The angular performance of the meta-radome is compared with a bare 0.64mm thick Rogers RO5870 \((\varepsilon_r = 2.3)\) and RO3006 \((\varepsilon_r = 6.15)\) substrates. From the average transmission (bottom left plot), one can observe that the meta-radome outperform the regular substrates for all angles. From the transmission difference between the TE and TM polarizations (the bottom right figure), one can see that the RO3006 substrate exhibits the largest difference whereas the meta-radome with \(w = 0.3\)mm exhibits the least. A large transmission difference can deteriorate polarization fidelity of the radome when it is considered in conjunction with the phase response.

Fig. 7.6 plots the extracted wave impedances of the meta-radome \((Z_{MR}^{TE/TM})\) at the design frequency, which are normalized to the free space wave impedances using (7.4)–(7.7),

\[
Z_{\text{vacuum}}^{TE} = \frac{\eta_0}{\cos \theta_1}, \tag{7.4}
\]
\[
Z_{\text{vacuum}}^{TM} = \eta_0 \cos \theta_1, \tag{7.5}
\]
\[
Z_{\text{norm}}^{TE} = \frac{Z_{MR}^{TE}}{Z_{\text{vacuum}}^{TE}}, \tag{7.6}
\]
\[
Z_{\text{norm}}^{TM} = \frac{Z_{MR}^{TM}}{Z_{\text{vacuum}}^{TM}}. \tag{7.7}
\]

The matched condition for TE or TM polarization is equivalent to \(Z_{\text{norm}}^{TE}\) or \(Z_{\text{norm}}^{TM}\) being equal to unity. Since the normalized wave impedances for the meta-radome stay closer to unity over a wider angular range, they achieve better matching than the regular substrates.
7.3.2 Transmission phase response

The phase response of the radome is another important design consideration. When a plane wave passes through the radome at an oblique incidence, it suffers phase retardations that are different for the TE and TM polarizations. In general, a linearly-polarized plane wave striking the radome contains both polarization components, which undergo different phase shifts when passing through the radome. In conjunction with the difference in the transmission magnitudes between the two polarizations, the resultant emerging wave is elliptically-polarized. Consequently, if the receiving antenna is also linearly polarized, then the received signal may be weakened. Furthermore, for transmitting applications, if the antenna array is dual-polarized or circularly-polarized, the radome with the least difference in transmission magnitudes and phases can preserve the polarization state better. Fig. 7.7 compares the transmission phases between the meta-radome and the regular Rogers substrates. For the substrates, the maximum phase differences $\delta$ between the two polarizations are around $20^\circ$. In comparison, the maximum phase difference of the meta-radome remains below $4.3^\circ$ for the entire angular range of $0^\circ - 85^\circ$.

To understand what governs the phase difference, we examine the total transmission of the meta-radome or the regular substrates, which is given by (7.8)

$$
\frac{t_{TE/TM}}{e^{i\phi_{TE/TM}}} = 1 - \left(\frac{1}{r_{12}^{TE/TM}}\right)^2 e^{-i\phi_{TE/TM}} - e^{-i\phi_{TE/TM}} \left(\frac{1}{r_{12}^{TE/TM}}\right)^2
$$

(7.8)

From (7.8), both $\phi_{TE/TM}$ and $r_{12}^{TE/TM}$ affect the transmission phase. However, $r_{12}^{TE/TM}$ is the dominating factor in determining the phase difference. To better illustrate this point, Fig. 7.8 shows the graphical representation of the TE and TM transmission in the complex plane for a regular substrate. The red and blue arrows indicate the numerator and the denominator in (7.8) respectively. The numerator $1 - \left(\frac{1}{r_{12}^{TE/TM}}\right)^2$ lies on the real axis. The denominator can be constructed from the green arrows. The angle or the phase of the total transmission is simply the negative of the angle between the denominator and the positive real axis. For a regular substrate, $\phi_{TE}$ and $\phi_{TM}$ are equal. Hence, the difference in the angles of the denominator is proportional to the magnitude difference between $(r_{12}^{TE})^2$ and $(r_{12}^{TM})^2$. Even though the transmission phases for the meta-radome are different for the two polarizations as shown in Fig. 7.9, but $\phi_{TE}$, $\phi_{TM}$, and the phase difference $|\phi_{TE} - \phi_{TM}|$ are generally very small. Hence, the total transmission phase difference is still predominantly dictated by the difference in the transmission magnitudes.
between $(r_{12}^{TE})^2$ and $(r_{12}^{TM})^2$. The differences for the meta-radome and regular substrates are plotted in Fig. 7.10, and as one can see, the meta-radome has a smaller magnitude difference. Thus, we observe a smaller phase difference $\delta$ between the two polarizations for the meta-radome in Fig. 7.7. This smaller difference can be attributed to the fact that the two wave-impedances of the meta-radome diverge less from unity than those of the regular substrates as shown in Fig. 7.6. Hence, to design a radome with good polarization-fidelity over a wide angular range, one should design the wave impedances (for both polarizations) of the radome to be as close as possible to the free space wave impedances as the incident angle increases.
Figure 7.8: Vectorial representation of the (a) $t^{TE}$, and (b) $t^{TM}$ in the complex plane using the numerator (red arrow) and denominator (blue arrow) of (7.8). The blue arrow can be constructed each term (green arrow) in the denominator. A smaller difference between $(r_{12}^{TE})^2$ and $(r_{12}^{TM})^2$ would lead to a smaller difference between the red arrows and blue arrows, which consequently leads to a smaller transmission phase difference $\delta$ between the TE and TM polarizations.

Figure 7.9: $\phi^{TE}$, $\phi^{TM}$, and the phase difference $|\phi^{TE} - \phi^{TM}|$ are plotted against the incident angle.
Figure 7.10: The magnitude differences between $(r_{12}^{TE})^2$ and $(r_{12}^{TM})^2$ are plotted for the meta-radome and regular substrates. The meta-radome exhibits a smaller difference than the regular substrates, which consequently leads to a smaller transmission phase difference.
7.3.3 Frequency response

Fig. 7.11 shows the bandwidth of the meta-radome at normal incidence. It is fairly broadband with a -10 dB reflection bandwidth around 20%. Fig. 7.12 shows the extracted $\varepsilon_t$ and $\mu_t$ versus frequency at normal incidence. Notice that $\varepsilon_t$ is strongly dependent on the frequency whereas $\mu_t$ is not. In fact, $\varepsilon_t$ becomes negative below 33 GHz. This phenomenon can be understood intuitively from effective medium theory. Since the permittivity of the host medium is frequency independent, the thinner the wires are, the less contribution they make to the effective permittivity of the composite structure and the less the frequency dependence of the effective permittivity.

![Figure 7.11](image1)

**Figure 7.11:** The reflection and transmission at normal incidence are plotted against frequency for the meta-radome. Due to the non-resonant nature of the meshes, the meta-radome is fairly broadband.

![Figure 7.12](image2)

**Figure 7.12:** The effective tangential permittivity and permeability of the meta-radome are plotted against frequency. The tangential permittivity exhibits strong frequency dependence. This is intuitive because as the frequency lowers, the mesh effectively becomes denser, and consequently the effective permittivity becomes lower. The tangential permeability is rather stable, exhibiting a small frequency variation.

One may notice that when $\varepsilon_t$ becomes negative below 33 GHz; the effective index and the wave impedance become complex as shown in Fig. 7.13. One may infer from the complex index and wave impedance that the meta-radome should become more reflective and lossy below 33 GHz. However, this is not the case. In fact, the reflection and the loss strongly depend on the thickness of the meta-radome. Fig. 7.14 plots the reflection, transmission, and loss of a hypothetical slab with the same permittivity and permeability shown in Fig. 7.12. It is clear that a thin slab with complex index and impedance is
not necessarily highly reflective and lossy. Fig. 7.15 plots the losses of the meta-radome as a function of frequency. Both the dielectric and the metal losses are around 1%.

**Figure 7.13:** The normalized wave impedance and the effective index \( n_t \) \((n_t = \sqrt{\varepsilon_t \mu_t})\) are plotted against frequency. Since \( \varepsilon_t \) is negative and \( \mu_t \) is positive below 33 GHz, the wave impedance and index become complex.

**Figure 7.14:** The reflection, transmission, and loss are shown for a hypothetical slab with various thicknesses \( d \). The permittivity and permeability of the slab are the same as ones shown in Fig. 7.12. The reflection and transmission are calculated using (7.1) and (7.8), respectively. It is clear that the reflection, transmission, and loss dependent strongly on the thickness. Even though the effective index and the wave impedance are complex below 33 GHz, a thin slab can still achieve high transmission and low loss.
7.3.4 Simulated field profiles

Fig. 7.16 shows the simulated electric field and surface currents on the meshes under normal incidence, TE-incidence at 30°, and TM-incidence at 30°. This provides us some insights on the working mechanisms of the meta-radome. From the electric field profiles, we can see that the meta-radome can preserve the planar wavefronts in all scenarios. This indicates the meta-radome behaves as a well-homogenized slab and the semi-analytical parameter extraction method is a valid way to characterize the meta-radome. From the surface current profiles, we can observe the transverse and longitudinal electric and magnetic responses of the meta-radome. Under normal incidence, the incident electric field excites surface currents on both the top and bottom meshes, flowing in the same direction along the transverse $x$-axis. On the other hand, the incident magnetic field excites a partial current loop, which consists of currents on the top and bottom meshes flowing along the $x$-axis, but in the opposite direction. The total current shown in the figure is the superposition of the induced currents from the electric and magnetic fields. Thus, we observe an asymmetry in the current magnitudes between the top and bottom meshes. Under a TE incidence at 30°, in addition to the aforementioned tangential electric and magnetic responses, we have a longitudinal magnetic response due to the longitudinal magnetic field. Such effect can be observed from the surface current as well. Due to the longitudinal magnetic field, surface currents are induced to flow around the square loops in both the top and bottom meshes. When these surface currents are superimposed with the surface currents from the tangential fields, they would break the symmetry of the surface currents in the square loop (notice that in the absence of the longitudinal magnetic field under normal incidence, the surface currents in the square loops exhibits symmetry along the $x-z$ plane). Indeed, we observe such an asymmetry in the total surface current within the square loops (such effect is more pronounced in the top square loops in the figure). Similarly, under a TM incidence at 30°, the longitudinal electric field is present. However, it is difficult to observe the longitudinal electric response from the induced surface currents (notice that the surface currents are symmetric along the $y-z$ plane). Nevertheless, such response can be clearly observed from the vectorial electric field as shown in the inset figure of Fig. 7.16. According to the boundary conditions, the tangential electric field is conserved at the interface and the normal electric field in the meta-radome is equal to $\frac{\varepsilon_n}{\varepsilon_0} E_{\text{inc}}$. Since the electric field in the meta-radome points nearly in the tangential direction, we can conclude that the $\varepsilon_n$ is much larger than the free-space permittivity. The large $\varepsilon_n$ is expected due to the capacitive coupling between the top and bottom meshes.
Normal incidence  30° TE  30° TM

Figure 7.16: The simulated electric field (top row) and surface currents (bottom row) are shown for (a) normal incidence, (b) TE incidence at 30° and (c) TM incidence at 30°. The inset figure in (c) shows the vectorial electric field near the free-space/meta-radome interfaces. From the surface currents and the electric fields, we can observe the tangential and longitudinal electric and magnetic responses of the meta-radome.

7.3.5 Comparison

Table 7.1 compares the simulated performance of the meta-radome to the single layer fishnet [206] and multi-layer FSS [209–211] radomes. The meta-radome has the smallest insertion loss and the best angular range. In addition, the meta-radome is a single layer design, making it easier to be fabricated than the multi-layer FSS radomes.
Table 7.1: Simulated performance of the meta-radome is compared to that of the fishnet or FSS based radomes. The insertion losses at different incident angles are obtained at the design frequency. The 3 dB bandwidth is obtained at the normal incidence.

<table>
<thead>
<tr>
<th>Ref.</th>
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<th>[209]</th>
<th>[210]</th>
<th>[206]</th>
<th>[211]</th>
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<td>3</td>
<td>1</td>
<td>2</td>
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<td>$\approx$ 0.2</td>
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<td>1.8</td>
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<tr>
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<td>$\approx$ 0.2</td>
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<td>$\approx$ 1.8</td>
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<td>20.7%</td>
<td>37%</td>
<td>13.9%</td>
<td>35.8%</td>
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7.4 Experimental results and discussion

The fabricated meta-radome is measured by a Ka-band quasi-optical setup as shown in Fig. 7.17. It is characterized from 26.6 GHz to 39.7 GHz with incident angles from $0^\circ - 85^\circ$. Since the meta-radome is very thin, it is very flexible. Thus, a wooden frame has to be used to support the radome during the measurements. The wooden support can introduce measurement errors at extreme incident angles. As a result, measurements beyond $80^\circ$ can be less accurate.

Fig. 7.18 compares the simulated and measured $t_{TE}$, $t_{TM}$, and phase difference $\delta$. The radome achieves maximum transmissions at 31.7 GHz, which is slightly lower than the design frequency of 34.3 GHz. This could be due to the tolerance in the substrate thickness and errors in the fabricated trace width and mesh alignment. Despite the slight frequency shift, the transmission of the fabricated radome is above 90% from $0^\circ - 70^\circ$ for both polarizations. This is in a good agreement with the simulated transmission, which is above 90% from $0^\circ - 80^\circ$ at the design frequency. At the operating frequency of 31.7 GHz, the maximum measured transmission is 97.9% ($-0.1848$ dB); the maximum measured phase difference within the angular range of $0^\circ - 75^\circ$ is $7^\circ$ with an error of $\pm 3^\circ$ (the measured phase is less accurate beyond $80^\circ$). From the measured results, we can conclude that the meta-radome is indeed broadband, wide angle, low-loss and polarization-insensitive.

We observe a sharp decrease in transmission near $80^\circ$ around 30 GHz in the simulated TM transmission. This sharp decrease is due to the onset of a trapped mode [51, 212, 213], which is a result of the coupling between the high order Floquet mode of a periodic structure and the surface wave supported by the dielectric substrate. For an electrically thin substrate, only one trapped mode exists and it is associated with the TM polarization [51]. This is indeed the case as shown by the simulation and
Figure 7.18: Simulated $t_{TE}$, $t_{TM}$, and the phase difference $\delta$ are plotted in (a), (c), and (e), respectively. Measured $t_{TE}$, $t_{TM}$, and $\delta$ are plotted in (b), (d), and (f), respectively. Excellent agreements can be observed between the simulated and measured results. The maximum measured power transmission is 97.9%, indicating an Ohmic loss around 2%. For angles between $0^\circ$ and $45^\circ$, the measured transmission is above 80% for the entire Ka-band and for both polarizations. The measured phase difference is also very small, indicating the meta-radome can preserve the polarization fidelity. The simulated results for TM polarization show an excitation of a trapped mode. However, such mode is weakly excited in the measurements.
measurement results. The onset of the trapped mode can be approximated from (7.9) [51,214]

\[ \lambda_c = p \sqrt{\varepsilon_r + \sin \theta_1} \]  

(7.9)

where \( \lambda_c \) is the wavelength of the first high-order Floquet harmonic, \( p \) is the periodicity of the mesh, \( \varepsilon_r \) is the permittivity of the dielectric substrate, and \( \theta_1 \) is the incident angle. For wavelengths shorter than \( \lambda_c \) (or frequencies higher than \( c/\lambda_c \)), a trapped mode can be excited. At an incident angle of 80°, (7.9) predicts a trapped mode above 32 GHz, which is in a good agreement with the simulation results. In the experiment, the trapped mode is observed near 29 GHz around 80°. However, the response is not as sharp as in the simulated result. This could be due to the fact that the fabricated radome is slightly warped instead of perfectly planar. The trapped mode, which is a form of surface wave, can be sensitive to the warping. As a result, it may not be well excited in the measurements. In addition, the meta-radome in the simulation is assumed to be infinite, whereas the fabricated meta-radome has a finite size. The surface waves can bounce at the edges and interfere with each other, and they are reinforced or resonate better as the structure size increases (approaching infinity), resulting to a sharper response [215,216].

To avoid the trapped mode, one can use a substrate with lower permittivity to push the cut-off frequency higher. As an example, the meta-radome is implemented on a 1.575mm Rogers 5870 substrate with a permittivity of 2.3. Fig. 7.19 shows the simulated transmission for both TE and TM polarizations. It is clear that the trapped mode is not excited within the frequency range of interest.

\[ \begin{array}{l}
\text{Simulated } |t_{TE}| \\
\text{Simulated } |t_{TM}| \\
\end{array} \]

Figure 7.19: The simulated (a) \( t_{TE} \) and (b) \( t_{TM} \) for a meta-radome designed with the RO5870 substrate (\( \varepsilon_r = 2.3 \)). The mesh is designed with \( p = 2.8\text{mm} \) and \( w = 0.2\text{mm} \). It is clear that the trapped mode is not excited.

7.5 Conclusion

In this chapter, we introduced a metamaterial radome (meta-radome) based on coupled metallic meshes patterned on a thin dielectric substrate. By judiciously controlling the geometry of the meshes, the wave impedance of the composite medium can be tuned to match to the wave impedance of free space. The resulting meta-radome can be reflection-less. We fabricated the proposed meta-radome and demonstrated an excellent agreement between the simulated and measured results. The proposed radome can work
up to 80° for both TE and TM polarizations at the design frequency. For a narrower angular range of 0° – 45°, the meta-radome can maintain transmission above 80% for the entire Ka-band and for both polarizations. Due to the non-resonant nature of the meshes, the measured loss of the meta-radome is around 2%. Furthermore, the meta-radome can preserve the polarization state as the transmission magnitude and phase differences between the two polarizations are small. In summary, the proposed meta-radome is wideband, low-loss, polarization-insensitive, angularly stable, and easy to fabricate. We believe the proposed meta-radome can be very attractive at mm-wave frequencies and beyond.
Chapter 8

Summary and Conclusion

In this thesis, we presented efficient metamaterial devices for efficient wavefront manipulation. The designed metamaterial devices can achieve refraction, focusing, polarization control, and impedance matching with high transmission in the optical or the microwave regime.

In Chapter 2, a thin quarter-wave plate (QWP) and a half-wave plate (HWP) based on multilayer elliptical antenna array sheets (AAS) were designed at long-wave infrared (LWIR) frequencies. The wave plates are impedance matched to free space with quarter-wave transformers, and transmission efficiency is above 80%. Comparing to the meander-line based QWP, the AAS QWP is less broadband, but it is more efficient and has a better axial ratio. Comparing to the diamond grating HWP or conventional cadmium thiogallate HWP, the AAS HWP is less efficient but has better control on the phase retardation. It is possible to further increase the bandwidth of the AAS waveplates, but it comes at a cost of total transmission efficiency. Hence, these compact AAS waveplates are excellent candidates for integrated opto-electronics that don’t demand a huge bandwidth. Furthermore, the efficient HWP unit cells were used for general wavefront manipulation for circularly-polarized waves. By utilizing the concept of Pancharatnam-Berry (PB) phase, which is also known as geometric phase, the local transmission phase delay is stipulated by the geometric rotation of the local unit cell. This is a simple and flexible method for constructing efficient wavefront manipulating devices for circularly-polarized waves. Using this concept, efficient refraction and focusing lenses were constructed. By using the refraction lens, a polarization discriminating device can be constructed. The orientation angle of a normally incident linearly polarized light can be detected by simply measuring the transmitted intensity pattern. This compact polarization discriminator can be more efficient than the conventional ones with micropolarizers consisting wire grids, which reflect 50% of the incident light.

In Chapter 3, we synthesized a microlens array that can be used to improve light collection efficiency for a short-wave infrared (SWIR) focal plane array that has a $25\mu m \times 25\mu m$ pixel size. Each microlens was designed to focus the incident light into a $18\mu m \times 18\mu m$ absorption region under a plane-wave incidence from $-25^\circ$ to $25^\circ$, or equivalent Gaussian beam incidences with a beam angle of $25^\circ$. The performance of the microlens was confirmed by ray-tracing simulations. Since the curved surfaces of the microlenses are difficult to fabricate, the microlenses were implemented as flat metalenses, which comprise arrays of nano-holes etched into an InP substrate. A full $2\pi$ phase delay is achieved by varying the diameter of the nano-holes. This is a all-dielectric design that has a very small loss compared to designs that incorporate metal, which can be very lossy at SWIR frequencies. The designed metalens was
Chapter 8. Summary and Conclusion

characterized with full-wave simulations that have Gaussian beams impinging at different positions on the lens. By examining the field at the absorption, the collection efficiency were obtained. It was found that when the Gaussian beam impinges at the phase wrap regions of the microlens, the refracted beam is severely distorted leading to poor light concentration and large crosstalk. Hence, the metalens has a shortcoming for light collection applications, but the performance can be improved if the phase wraps are eliminated. This is possible if the pixel size is reduced or the microlenses are designed for narrow angular ranges. Despite the shortcoming, it was found that metalenses can be very efficient focusing devices for a normally incident plane wave. The nano-holes have a tapered shape due to the RIE process used to fabricate them, which act as a graded-index medium that is impedance matched. Near unity transmission can be maintained from 0.9 µm to 1.7 µm. Compared to other state-of-the-art metalenses reported in the literature, the nano-hole based metalenses have the highest efficiency and broadest bandwidth. Compared to conventional microlenses, the nano-hole metalenses have the advantage of not requiring anti-reflection coatings. It is very challenging to design anti-reflection coatings for the entire SWIR spectrum. With a good numerical aperture, polarization-insensitivity, broad bandwidth, and near unity transmission, the metalenses presented in this chapter are excellent candidates to replace conventional microlenses or other metalenses in the literature. Three metalenses were designed, simulated, and measured to verify the focusing concept. Excellent agreement was observed between the simulated and measured focal lengths and focal spots. In the future, we expect to experimentally characterize the transmission efficiency of these metalenses at different wavelengths in the SWIR spectrum.

In Chapter 4, a hole-array based grade-index (GRIN) lens was designed for millimeter-wave (mm-wave) applications. Unlike the metalenses, which are intrinsically impedance-matched, GRIN lenses can be highly reflective near their central regions. We proposed to match the GRIN lens with GRIN matching layers. The optimal permittivity in the GRIN matching layers was obtained by using the transfer-matrix method. A matched GRIN lens was designed to collimate a Ka-band pyramidal horn, and the performance was verified with both simulations and measurements. Excellent agreement is observed the simulated and measured radiation patterns. Since the only loss is the dielectric loss, which is usually much smaller than the metal loss, and the lens is well-matched, the measured radiation efficiency is about 92%. Even though GRIN matching layers were designed only for 34.3 GHz, the reflection of the matched GRIN lens remains small and the gain of the horn is improved by about 5 dB for the entire Ka-band. The maximum side lobe level in the Ka-band remains below -19 dB. Compared to other dielectric lenses reported in the literature, the matched GRIN lens is not only more compact, but also more broadband with a 1 dB gain bandwidth of more than 30%. The aperture efficiency of the matched GRIN lens is not as high as we expected. This is because only the central portion of the lens is illuminated. By placing the horn further away, the aperture efficiency should be improved. With low reflections, low losses, and wide bandwidth, the compact matched GRIN lens is an excellent candidate for applications such as increasing the gain or the scan range of mm-wave phased arrays.

In Chapter 5, we explored the possibility of designing anti-reflection or impedance matching layers with anisotropic metamaterials. We showed that all-angle matching is not possible with a fully anisotropic layer or a perfectly matched layer. However, we demonstrated that by exploiting anisotropy, matching at a near grazing angle of 88° is possible with an anisotropic anti-reflection layer (AMAL). Due to the different wave impedances for the TE and TM polarizations at oblique angles, matching is performed separately. Required permittivity and permeability tensors of AMAL were derived for each polarizations. To synthesize an AMAL with realistic structure, we provided a synthesis technique and
parameter extraction technique, which can retrieve material parameter values accurately even at near grazing angles. Using this combination, AMAL was realized with an array of circular rings arranged in the vertical plane for the TM polarization. For the TE polarization, AMAL is realized with an array of split rings interleaved with an array wire strips. Both structures are sub-wavelength thin, and the simulated reflectances are less than 1%. The performance of AMAL is compared to other AR coatings that are numerically investigated in [191]. AMAL achieves near perfect matching at the design angle with a fraction of their thicknesses. To our best knowledge, AMAL is the only matching layer that is designed beyond 85°. Since AMAL designed for the TE polarizations utilized near-resonant split rings to achieve the required permeability, it is more lossy and narrowband than the one designed for the TM polarization, and more susceptible to fabrication inaccuracies. Hence, AMAL designed for TM polarization is experimentally characterized at 10 GHz. The layer was re-designed to match at 60° to avoid measurement inaccuracy caused by the spill-over loss during the quasi-optical measurements. Maximum transmittance was indeed observed at 60°.

In Chapter 6, we extended the anisotropic matching layer to achieve polarization-insensitive matching with a magneto-electric uniaxial matching layer (MEUML). We calculated the required permittivity and permeability tensors of the MEUML for matching to a high-index substrate at 45°. We proposed two coupled rings as the unit cell of the MEUML. The transversal electric and magnetic dipole moments, and the longitudinal capacitive and diamagnetic coupling between the rings are judiciously controlled to achieve the required permittivity and permeability tensors. The resulting reflection levels are around -30 dB for both polarizations at 45°. By realizing this matching capability of the MEUML, we adapted it to a sandwich radome design at 10 GHz. Exceptional matching performance was obtained both in the simulations and measurements. The reflectance remains below 5% from normal incidence (0°) to near grazing angle (85°) for both polarizations and over a wide bandwidth. To our best knowledge, this is the widest angular range that has been reported in the literature for such a compact radome.

In Chapter 7, we presented a simpler single layer meta-radome design for mm-wave applications. For many practical applications, it is desirable to have a single layer instead of a multilayer design. For a single layer, reflections arise at the interfaces due to the wave impedance mismatch, unless its electrical thickness is a multiple of half-wavelength. We realized that if we can turn a single layer into a metamaterial slab, we have the freedom of tuning its effective permittivity and permeability. When these two quantities are equal, the slab is impedance matched to free space, regardless of its electrical thickness. This idea was implemented by patterning two metallic meshes on a regular dielectric substrate. By utilizing the electric and magnetic coupling between the meshes, the effective permittivity and permeability can be equal. The meta-radome has good transmission (> 90%) over an extremely wide angle of incidence (0° – 80°) for both transverse-electric (TE) and transverse-magnetic (TM) polarizations. The total loss of meta-radome is about 2% as a result of the non-resonant nature of the meshes. The proposed meta-radome is very easy to fabricate, which can be very attractive for many practical microwave and mm-wave applications.

### 8.1 Contributions

As stated above, the work in this thesis lead to the development of various metamaterial devices for refraction, focusing, and impedance matching with high efficiencies, and has lead to the following scientific contributions.
8.1.1 Journal papers


8.1.2 Conference papers


Appendix A

Additional microlens designs

In the microlens synthesis, the iterative procedure starts at the lens center and determines $h_{\text{max}}$, which is the maximum substrate thickness such that the refracted marginal rays hit the edge of the absorption region. However, one can start the iteration procedure with a value less than $h_{\text{max}}$. This will result to bi-focal lenses as shown in Fig. A.1 to Fig. A.3.

![Figure A.1](image1)

**Figure A.1:** The initial thickness for the iterative process can start with a thickness less than $h_{\text{max}}$, which is 50 µm. The resulting lens thickness is 1.64 µm. The 18 µm absorption regions are shown with the blue bars.

![Figure A.2](image2)

**Figure A.2:** The initial thickness for the iterative process starts with 40 µm. The resulting lens thickness is 1.56 µm.

Fig. A.4 to Fig. A.6 show the synthesized lens shape for smaller absorption region widths. As the width decreases, the maximum substrate thickness decreases and the lens thickness increases. This is an expected result since stronger lenses are needed to concentrate the energy into smaller areas.
Appendix A. Additional microlens designs

Figure A.3: The initial thickness for the iterative process starts with 30 μm. The resulting lens thickness is 1.58 μm.

Figure A.4: The synthesized lens profiles for a 15 μm absorption region width (shown with blue bars).

Figure A.5: The synthesized lens profiles for a 10 μm absorption region width.

Figure A.6: The synthesized lens profiles for a 5 μm absorption region width.
Appendix B

Nano-hole vs. nano-pillar lenses

B.1 Unit cell designs

To make an apple to apple comparison, the hole array and the pillar array are complimentary to each other as shown in Fig. B.1. The periodicity of the unit cell is 500 nm and the height of the hole or the pillar is 1000 nm. The diameter of the hole or the pillar varies from 50 nm to 450 nm with an increment of 50 nm. Both the hole array and the pillar arrays are supported by isotropic InP substrates. The illumination is a plane wave traveling in the $-z$ direction from air above the array. The unit cells do not model any imperfections from fabrication.

![Figure B.1: The unit cell view of the hole (left) and the pillar (right) array.](image)

B.2 Transmission Responses

The transmission magnitudes and phases of the hole and the pillar arrays are investigated at a wavelength of 1.3 $\mu$m or an equivalent frequency of 230.7 THz. Fig. B.2 shows the simulated transmission responses as a function of the diameter. The hole and the pillar arrays exhibit comparable transmission magnitudes. However, the pillar array achieves an additional phase range of 100°.
B.3 Frequency Responses

Fig. B.3 plots the transmission magnitudes of the nano-holes and nano-pillars. One can see that for a certain diameter near 245 THz, the transmissions reduces for both the nano-hole and nano-pillar designs. This is because the effective refractive indices of these structures are in a range which would cause a large impedance mismatch to air. However, unlike the nano-pillars, in which the electric field is highly concentrated in each individual pillar, the high-index material in the nano-hole design is interconnected; and the electric field will not be highly confined. Thus, one can observe an additional drop in transmission near 227 THz for the nano-pillars, which is a result of the excitation of a resonant mode in the pillars due to the high field containment. This drop in transmission is absent in the nano-holes since it is more difficult to excite the resonant mode in them. As a result, the nano-holes can have a wider bandwidth than the nano-pillars.
Figure B.3: Transmission magnitudes as a function of frequency and the nano-hole/nano-pillar diameter. The nano-pillar has an addition resonance.
Appendix C

Parameter retrieval for anisotropic metamaterial layers

The most standard way of extracting material parameters from a metamaterial layer is the S-parameter retrieval method \([27,217]\). As shown in Fig. C.1, the metamaterial layer is situated in free space, and it is illuminated with a plane wave at normal incidence using Floquet ports. \(S_{11}\) and \(S_{21}\) are de-embedded to the top and bottom surfaces of the layer, and they can be used to extract the normalized wave-impedance and the refractive index of the layer from (C.2) and (C.1). The normalized wave-impedance is the wave-impedance of the layer divided by the free space wave space. The permittivity and permeability can be found from (C.3) and (C.4).

\[
Z_{2,\text{norm}} = \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} \tag{C.1}
\]

\[
n_2 = \frac{1}{kd} \cos^{-1} \left( \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} \right) + m\pi, \quad m \in I \tag{C.2}
\]

\[
\varepsilon_2 = \frac{n_{ML}}{Z_{ML}} \tag{C.3}
\]

\[
\mu_2 = n_{ML}Z_{ML} \tag{C.4}
\]

Since the standard retrieval method is performed at normal incidence, the permittivity and permeability extracted are essentially the tangential ones. Hence, this method cannot extract longitudinal (along Z-axis) material parameters. In \([218,219]\), the standard method is extended to extract the both tangential and longitudinal parameters for anisotropic metamaterials. However, these methods assume the material parameters are angularly independent, which may not be true. For example, it was found that artificial dielectrics based on patch arrays \([181]\) and wire arrays \([208]\) exhibit spatial dispersion, which lead to angular dependent properties. Furthermore, if the metamaterial layer is sandwiched between two different materials, i.e, free space and an arbitrary substrate, then the metamaterial layer is affected by the substrate and may give rise to different material parameters \([220]\). To address this issue, an approximation is made by using a geometric averaging of \(S_{12}\) and \(S_{21}\). Such method has been applied to inhomogeneous metamaterials \([221]\), but we found that this approximation can lead to inaccurate results when the index contrast between the substrate and the air is large (10.2 to 1). As a result, current parameter methods are not sufficient for the design of the anisotropic matching layer presented.
in Chapter 5. In this section, we provide a retrieval method to extract an anisotropic metamaterial layer that is sandwiched by free space and an arbitrary isotropic substrate with a relative permittivity of $\varepsilon_3$. The retrieval method makes no approximation and no assumption on the angular property of the layer. The material parameters tensors of the anisotropic metamaterial layer can be extracted exactly at an arbitrary angle.

### C.1 Parameter retrieval equations

The derivation is based on the Nicolson-Ross method [222], but here we assumed one of the semi-infinite space is an arbitrary substrate with $\varepsilon_3$ and the incident angle is an arbitrary value of $\theta_1$. The normalized wave impedances and the refractive index (along Z-axis) of the metamaterial layer can be expressed as (C.5) to (C.7) for both TE and TM polarizations.

$$Z_{2,norm}^{TM} = \frac{\sqrt{\varepsilon_3 - \sin^2 \theta_1}}{\varepsilon_3 \cos \theta_1} \sqrt{\frac{(1 + S^{TM}_{11})(1 + S^{TM}_{22}) - S^{TM}_{21}S^{TM}_{12}}{(1 - S^{TM}_{11})(1 - S^{TM}_{22}) - S^{TM}_{21}S^{TM}_{12}}}$$

(C.5)
Appendix C. Parameter retrieval for anisotropic metamaterial layers

\[ Z_{2,\text{norm}}^{\text{TE}} = \sqrt{\frac{\cos \theta_1}{\varepsilon_3 - \sin^2 \theta_1}} \left( \sqrt{\frac{(1 + S_{11}^{\text{TE}})(1 + S_{22}^{\text{TE}}) - S_{21}^{\text{TE}}S_{12}^{\text{TE}}}{(1 - S_{11}^{\text{TE}})(1 - S_{22}^{\text{TE}}) - S_{21}^{\text{TE}}S_{12}^{\text{TE}}}} \right) \]  
(C.6)

\[ n_{2z}^{\text{TE}/\text{TM}} = \frac{1}{k_0 d} \cos^{-1} \left( \frac{1 - S_{11} S_{22} + S_{21} S_{12}}{S_{21} + S_{12}} \right)^{\text{TE}/\text{TM}} + m\pi, \quad m \in I \]  
(C.7)

\( \theta_1 \) is the incident angle from free space, and \( \theta_3 \) is the refraction angle in the substrate. It can be calculated from the Snell’s Law as in (C.8)

\[ \cos \theta_3 = \sqrt{1 - \frac{\sin^2 \theta_1}{\varepsilon_3}} \]  
(C.8)

Note that \( Z_2^{\text{TM}} \) and \( Z_2^{\text{TE}} \) have slightly different form. Readers can verify that at normal incidence with \( \varepsilon_3 = 1 \) and \( S_{12} = S_{21} \), (C.5) to (C.7) reduce to the standard equations of (C.1) and (C.2). One has to be careful when using the generalized S-parameters from commercial full-wave simulators, e.g., HFSS [28]. The S-parameters are usually normalized using power, i.e., \( S_{12}^{\text{HFSS}} = S_{21}^{\text{HFSS}} \). This guarantees the power transmission from either side of the passive metamaterial layer are equal to avoid violating the reciprocity theorem. In (C.5) to (C.7), the \( S_{12} \) and \( S_{21} \) refer to the voltages or the wave amplitudes instead of the power, thus, they are not necessarily equal. One can un-normalize the \( S_{12}^{\text{HFSS}} \) and \( S_{21}^{\text{HFSS}} \) using the following equations.

\[ S_{12}^{\text{TM}} = S_{12}^{\text{HFSS,TM}} \sqrt{\frac{\sqrt{\varepsilon_3} \cos \theta_1}{\cos \theta_3}} \]  
(C.9)

\[ S_{21}^{\text{TM}} = S_{21}^{\text{HFSS,TM}} \sqrt{\frac{\sqrt{\varepsilon_3} \cos \theta_1}{\cos \theta_3}} \]  
(C.10)

\[ S_{12}^{\text{TE}} = S_{12}^{\text{HFSS,TE}} \sqrt{\frac{\sqrt{\varepsilon_3} \cos \theta_3}{\cos \theta_1}} \]  
(C.11)

\[ S_{21}^{\text{TE}} = S_{21}^{\text{HFSS,TE}} \sqrt{\frac{\sqrt{\varepsilon_3} \cos \theta_3}{\cos \theta_1}} \]  
(C.12)

Assume the permittivity and permeability tensors of the anisotropic metamaterial layer have the following forms:

\[ \varepsilon_2 (\theta_1, \varepsilon_3) = \begin{bmatrix} \varepsilon_{2x} & 0 & 0 \\ 0 & \varepsilon_{2y} & 0 \\ 0 & 0 & \varepsilon_{2z} \end{bmatrix} \]  
(C.13)

\[ \mu_2 (\theta_1, \varepsilon_3) = \begin{bmatrix} \mu_{2x} & 0 & 0 \\ 0 & \mu_{2y} & 0 \\ 0 & 0 & \mu_{2z} \end{bmatrix} \]  
(C.14)

\( \varepsilon_{2y} \) can be extracted by illuminating the layer at \( \theta_1 \) in the Y-Z plane with a TM-polarization (azimuth angle \( \phi = 90^\circ \)) as shown in Fig. C.2a. By extracting (C.5) and (C.7) at \( \theta_1 \) in the Y-Z plane, \( \varepsilon_{2y} \) can be obtained as (C.15). This relationship is obtained by using (5.9),(5.10), and (5.12). Similarly, \( \mu_{2y} \) can be extracted by using a TE-polarization in the Y-Z plane as shown in Fig. C.2b. \( \mu_{2y} \) is given by (C.16).
To extract $\varepsilon_{2x}$ and $\mu_{2x}$, the metamaterial layer is illuminated with TM and TE-polarizations as shown in Fig. C.3, and they are given by (C.17) and (C.18).

Figure C.2: The anisotropic layer is illuminated with a (a) TM-polarized or (b) TE-polarized plane wave in the Y-Z plane (azimuth angle $\phi = 90^\circ$) to extract $\varepsilon_{2y}$ or $\mu_{2y}$.

Figure C.3: The anisotropic layer is illuminated with a (a) TM-polarized or (b) TE-polarized plane wave in the X-Z plane (azimuth angle $\phi = 0^\circ$) to extract $\varepsilon_{2x}$ or $\mu_{2x}$.

$$
\varepsilon_{2y} = \frac{n_{2y}^{TM}}{Z_{2}^{TM} \cos \theta_{1}} \bigg|_{\phi=90^\circ} \\
\mu_{2y} = \frac{n_{2y}^{TE}}{Z_{2}^{TE} \cos \theta_{1}} \bigg|_{\phi=90^\circ} \\
\varepsilon_{2x} = \frac{n_{2x}^{TM}}{Z_{2}^{TM} \cos \theta_{1}} \bigg|_{\phi=0^\circ} \\
\mu_{2x} = \frac{n_{2x}^{TE}}{Z_{2}^{TE} \cos \theta_{1}} \bigg|_{\phi=0^\circ}
$$

(C.15) (C.16) (C.17) (C.18)

By substituting extracted $n_{TM}$, $\varepsilon_{2y}$, and $\mu_{2x}$ into (5.12), $\varepsilon_{2z}$ can be obtained as in (C.19). Similarly, $\mu_{2z}$ is obtained as in (C.20). $\varepsilon_{2z}$ and $\mu_{2z}$ can be obtained equivalently as in (C.21) and (C.22).
Appendix C. Parameter retrieval for anisotropic metamaterial layers

\[ \varepsilon_{2z} = \frac{\sin^2 \theta_1 \varepsilon_{2y}}{\mu_{2z} \varepsilon_{2y} - \left( n_{2z}^{TM} \right)^2_{\phi=90^\circ}} \]  
\[ \mu_{2z} = \frac{\sin^2 \theta_1 \mu_{2y}}{\mu_{2y} \varepsilon_{2x} - \left( n_{2z}^{TM} \right)^2_{\phi=90^\circ}} \]  
\[ \varepsilon_{2z} = \frac{\sin^2 \theta_1 \varepsilon_{2x}}{\mu_{2y} \varepsilon_{2x} - \left( n_{2z}^{TM} \right)^2_{\phi=0^\circ}} \]  
\[ \mu_{2z} = \frac{\sin^2 \theta_1 \mu_{2x}}{\mu_{2x} \varepsilon_{2y} - \left( n_{2z}^{TE} \right)^2_{\phi=0^\circ}} \]  

To validate the extraction method, a hypothetical anisotropic layer with the following permittivity and permeability tensors is implemented in HFSS. The layer is 2.5 mm thick, and material parameters are extracted as a function of \( \theta_1 \) at 10 GHz. Fig. C.4 to Fig. C.9 compare the theoretical and extracted parameters. Excellent agreement can be observed in all the figures. For the longitudinal parameters \( \varepsilon_{2z} \) and \( \mu_{2z} \), values cannot be extracted at normal incidence. This is because these parameters will not impact the wave impedances and indices at this angle as one can see from (5.10), (5.12), (5.18), and (5.20).

\[ \varepsilon_{2} = \begin{bmatrix} (1 - 0.02i) (1.5 + 2 \sin \theta_1) & 0 & 0 \\ 0 & (1 - 0.04i) (2 + \sin \theta_1) & 0 \\ 0 & 0 & (1 - 0.06i) (1 - 0.25 \sin \theta_1) \end{bmatrix} \]  
\[ \mu_{2} = \begin{bmatrix} (1 - 0.01i) (0.5 + 0.5 \cos 2\theta_1) & 0 & 0 \\ 0 & (1 - 0.03i) (0.75 + 0.25 \cos 2\theta_1) & 0 \\ 0 & 0 & (1 - 0.05i) (1.25 - \cos 3\theta_1) \end{bmatrix} \]

Figure C.4: Theoretical and extracted real and imaginary parts of \( \varepsilon_{2x} \).
Figure C.5: Theoretical and extracted real and imaginary parts of $\varepsilon_{2y}$.

Figure C.6: Theoretical and extracted real and imaginary parts of $\varepsilon_{2z}$.

Figure C.7: Theoretical and extracted real and imaginary parts of $\mu_{2x}$. 
Figure C.8: Theoretical and extracted real and imaginary parts of $\mu_{2y}$.

Figure C.9: Theoretical and extracted real and imaginary parts of $\mu_{2z}$. 
C.2 Some examples

In this section, we show extracted parameters for some typical metamaterial unit cells as shown in Fig. C.10. The extracted material parameters are shown from Fig. C.11 to Fig. C.14. The longitudinal components show some fluctuation near normal incidence because they have very weak dependence \((\sin^2 \theta_1)\) on the extracted wave impedances and indices. The results can be improved by improving the convergence of the simulations. For the strip array that is aligned with the X-axis, \(\varepsilon_{2x}\) is negative and shows strong spatial dispersion. This result is expected from the discussion of the wire medium in [208]. The capacitive coupling between the strips in the Y-axis should lead to an increased \(\varepsilon_{2y}\), which is indeed the case. There is no capacitive coupling along the Z-axis; hence, \(\varepsilon_{2z}\) has the same value as the substrate permittivity, which is 2.2. The strip array also exhibits some weak magnetic response, which is a result of the current flowing along the strips. For a rectangular ring array lying in the X-Y plane, \(\varepsilon_{2x}\) and \(\varepsilon_{2y}\) are increased due to the capacitive coupling between the rings. Again, since there is no capacitive coupling along the Z-axis, \(\varepsilon_{2z}\) remains unchanged from the substrate permittivity. Due to the current flowing on the rings, there is a weak magnetic response along the X and Y axis, resulting in a very small decrease of \(\mu_{2x}\) and \(\mu_{2y}\) from unity. However, since the ring is in the X-Y plane, we expect a strong magnetic response along the Z-axis. This is indeed the case since the extracted \(\mu_{2z}\) is around 0.8. Similar reasoning can be applied to both the unit cells in Fig. C.10c and Fig. C.10d. Electric and magnetic responses can be observed along all three axis.

\(\text{Figure C.10: Metamaterial unit cells are shown for arrays of (a) copper strip, (b) rectangular copper rings, (c) three orthogonal dipoles, and (d) six rings. The host medium for all unit cells is Rogers RO5880 with } \varepsilon_r = 2.2\)
Figure C.11: Extracted material parameters for the strip array shown in Fig. C.10a. Real and imaginary parts of the extracted parameters are shown by solid and dashed lines, respectively.

Figure C.12: Extracted material parameters for the rectangular ring array shown in Fig. C.10b. Real and imaginary parts of the extracted parameters are shown by solid and dashed lines, respectively.
Figure C.13: Extracted material parameters for the three orthogonal dipole array shown in Fig. C.10c. Real and imaginary parts of the extracted parameters are shown by solid and dashed lines, respectively.

Figure C.14: Extracted material parameters for the 6 rings array shown in Fig. C.10d. Real and imaginary parts of the extracted parameters are shown by solid and dashed lines, respectively.
The substrate effect can be observed in Fig. C.15 and Fig. C.16. Fig. C.15 shows the extracted $\mu_{2y}$ for a closed copper ring array lying in the X-Z plane, which has a free space region or a high-index substrate below. The extracted $\mu_{2y}$ exhibits very small angular dependence and it is independent of the substrate. This is clearly not the case for the split-ring array as shown in Fig. C.16. With a free space region, the permeability is relatively angle independent. With a substrate, not only the permeability value is quite different, but also it becomes angle dependent. This is because the electric field distribution in the layer is different for a free space region and a substrate (since the boundary conditions are changed). The split rings with gaps in the X-Z plane, are sensitive to change in the longitudinal electric field. This could induce different currents following on the rings, which give rise to different magnetic responses.

Figure C.15: Extracted $\mu_{2y}$ for solid copper rings with and without the substrate. The imaginary part of the permeability indicates that the solid ring is not resonant and passivity is satisfied. The inset shows the unit cell of the split ring with $d_x=3\text{mm}$, $d_y=2\text{mm}$, $d_z=3\text{mm}$ and $d_{\text{ring}}=2.6\text{mm}$. The host medium for the split ring and the substrate are Rogers RO3006 and Rogers RO3010. The closed rings exhibit negligible angular dependence and substrate effect.
Figure C.16: Extracted $\mu_{2y}$ for split rings with and without the substrate. The imaginary part of the permeability indicates that the split ring is not resonant and the passivity is satisfied. The inset shows the unit cell of the split ring with $d_x=7.5\,\text{mm}$, $d_y=2\,\text{mm}$, $d_z=3\,\text{mm}$ and $l_x=4\,\text{mm}$. The host medium for the split ring and the substrate are Rogers RO3006 and Rogers RO3010. Angular dependence and the substrate effect can be observed.
C.3 Limitations

There are limitations if the unit cell has bianisotropic or chiral properties. For example, Fig. C.17 shows the unit cell of a split-ring lying in the X-Y plane and its corresponding material parameters. If the electric field is along the Y-axis, then the asymmetric current is going to be induced on the ring due to the gap. As a result of the asymmetric current, a magnetic response along Z-axis is induced. This bianisotropic effect can be observed in $\varepsilon_{2y}$ and $\mu_{2z}$. $\varepsilon_{2y}$ has a large negative imaginary part and $\mu_{2z}$ has a large positive one. This implies some electric energy is converted to magnetic energy. Thus, the extracted parameters become less meaningful for bianisotropic structures. The same argument can be made for chiral structures. Nevertheless, the proposed parameter retrieval method is a significant extension to the conventional ones. It can retrieve permittivity and permeability tensors as a function of the incident angle. Specifically, it enables one to retrieve longitudinal parameters, which cannot be

\[ \begin{align*}
\varepsilon_{2x} & \quad 8 \\
\varepsilon_{2y} & \quad 0 \\
\varepsilon_{2z} & \quad 3 \\
\mu_{2x} & \quad 1.5 \\
\mu_{2y} & \quad 1 \\
\mu_{2z} & \quad 2 \\
\end{align*} \]

Figure C.17: (a) Unit cell model of the split ring array. Host medium is RO5880. (b) Extracted material parameters for the split ring array. Real and imaginary parts of the extracted parameters are shown by solid and dashed lines, respectively.
obtained before. This not only provides more physical insights, but also enables designers to utilize
structures such as vertical vias (cylinders) or rings in the X-Y plane to provide the required longitudinal
responses to achieve exotic functionalities. Last but not least, this retrieval method may be extended to
include bianisotropic and chiral properties in the future.
Appendix D

Effective Medium Approximation

The effective tangential and longitudinal permittivity of the graded-index (GRIN) lens can be approximate with the Marwell-Garnet mixing formulas [158] as

\[
\varepsilon_{t,\text{eff}} = \varepsilon_{t,\text{sub}} \left( \frac{1 + f}{1 + f} \right) + \varepsilon_{\text{hole}},
\]

(D.1)

\[
\varepsilon_{n,\text{eff}} = f \varepsilon_{\text{hole}} + (1 - f) \varepsilon_{t,\text{sub}},
\]

(D.2)

where \( f \) is the filling ratio of the air hole, which is the ratio between the hole area and the unit cell area; \( \varepsilon_{\text{hole}} \) is the relative permittivity of the air hole; \( \varepsilon_{t,\text{sub}} \) and \( \varepsilon_{n,\text{sub}} \) is the tangential and longitudinal relative permittivity of the substrate. For the Rogers 6010 substrate used in the GRIN lens, it is anisotropic with \( \varepsilon_{t,\text{eff}} = 13.5 \) and \( \varepsilon_{n,\text{eff}} = 10.8 \) at 34.3 GHz. With (D.1) and (D.2), the effective tangential and longitudinal permittivity can be calculated as a function of the hole diameter.

Fig. D.1 compares the calculated effective permittivity and the extracted permittivity obtained from the unit cell simulation. The longitudinal permittivity is extracted at an incident angle of 45° with the method outlined in Appendix C. It is apparent that the discrepancy gets larger as the hole radius increases. There could be two reasons for this discrepancy. First, the periodicity is quite large. At 2.5 mm, the periodicity is close to a third of a wavelength, making the effective medium approach borderline valid. Second, the substrate index is very high. The large index contrast between the air hole and the substrate can make the effective medium approach, which is a spatial averaging approximation less accurate. As a result, phase delay calculated based on the effective medium approach will be inaccurate for the GRIN lens. However, if both the periodicity and the substrate index is decreased, then calculated and extracted permittivities have a much better agreement as shown in Fig. D.2.
Appendix D. Effective Medium Approximation

Figure D.1: Calculated and extracted permittivities are plotted as a function of the hole radius. The periodicity is 2.5 mm. The extraction is performed with a unit cell analysis at 34.3 GHz.

Figure D.2: Calculated and extracted permittivities are plotted as a function of the hole radius. The periodicity is 1.25 mm. The tangential and longitudinal relative permittivities for the substrate are 3.2 and 2.95, respectively.
Appendix E

All Angle Matching with a Fully Anisotropic Matching Layer

Perfectly matched layer was first introduced in computational electromagnetics [189,190]. With uniaxial permittivity permeability tensors, it is impedance matched to free space for all angles of incidence, and for both TE and TM polarizations. If losses are introduced in the PML, the incident electromagnetic wave can be absorbed without reflection for all incident angles. This unique property of the PML making it very popular in electromagnetic full-wave simulators. In this section, we investigate whether a fully anisotropic layer (with no bi-anisotropy or chirality) can be used to match between two different media at all angles. To address this question, we first derive the all angle matching condition at a single interface between two media.

In this derivation, we assume medium 1 is free space, and medium 2 is the fully anisotropic medium with the following permittivity and permeability tensors:

\[ \varepsilon_2 = \begin{bmatrix} 
\varepsilon_{2x} & 0 & 0 \\
0 & \varepsilon_{2y} & 0 \\
0 & 0 & \varepsilon_{2z} 
\end{bmatrix}, \quad (E.1) \]

\[ \mu_2 = \begin{bmatrix} 
\mu_{2x} & 0 & 0 \\
0 & \mu_{2y} & 0 \\
0 & 0 & \mu_{2z} 
\end{bmatrix}. \quad (E.2) \]

The reflection coefficient at the free space/PML interface for either TE or TM polarization is given by

\[ r_{12}^{\text{TE/TM}} = \frac{Z_2^{\text{TE/TM}} - Z_1^{\text{TE/TM}}}{Z_2^{\text{TE/TM}} + Z_1^{\text{TE/TM}}}, \quad (E.3) \]

where \( Z_i^{\text{TE/TM}} \) is wave impedance for TE or TM polarization in the \( i \)th medium. Assuming the incident angle is \( \theta_1 \) and the incident plane is the \( y-z \) plane, the wave impedances for TE and TM polarizations can be expressed as

\[ Z_1^{\text{TM}} = \eta_0 \cos \theta_1 = \eta_0 \sqrt{1 - \sin^2 \theta_1}, \quad (E.4) \]
Appendix E. All Angle Matching with a Fully Anisotropic Matching Layer

\[ Z_{2}^{TM} = \eta_{0} \sqrt{\frac{\mu_{2x}}{\varepsilon_{2y}} - \sin^{2} \theta_{1} - \frac{1}{\varepsilon_{2y} \varepsilon_{2z}}} \]  
(E.5)

\[ Z_{1}^{TE} = \frac{\eta_{0}}{\cos \theta_{1}} = \frac{\eta_{0}}{\sqrt{1 - \sin^{2} \theta_{1}}} \]  
(E.6)

\[ Z_{2}^{TE} = \frac{\eta_{0}}{\sqrt{\frac{\varepsilon_{2x}}{\mu_{2y}} - \sin^{2} \theta_{1} \mu_{2x} \mu_{2y}}} \]  
(E.7)

For zero reflection, we require \( Z_{2}^{TE/TM} = Z_{1}^{TE/TM} \), and to achieve this, we arrive at (E.8) for the TM polarization and (E.9) for the TE polarization.

\[ \eta_{0} \sqrt{1 - \sin^{2} \theta_{1}} = \eta_{0} \sqrt{\frac{\mu_{2x}}{\varepsilon_{2y}} - \sin^{2} \theta_{1} \frac{1}{\varepsilon_{2y} \varepsilon_{2z}}} \]  
(E.8)

\[ \frac{\eta_{0}}{\sqrt{1 - \sin^{2} \theta_{1}}} = \frac{\eta_{0}}{\sqrt{\frac{\varepsilon_{2x}}{\mu_{2y}} - \sin^{2} \theta_{1} \mu_{2x} \mu_{2y}}} \]  
(E.9)

To achieve perfect matching at all angles for the TM polarization, (E.8) has to be valid at very angle. This is only possible if \( \mu_{2x} = \varepsilon_{2y} \) and \( \varepsilon_{2y} = \varepsilon_{2z}^{-1} \). Similarly, for TE polarization, (E.9) has to be valid at every angle. This is only possible \( \mu_{2y} = \varepsilon_{2x} \) and \( \mu_{2x} = \mu_{2z}^{-1} \).

If \( \mu_{2x} = \mu_{2y} \) and \( \varepsilon_{2x} = \varepsilon_{2y} \), then the permittivity and permeability tensors have an uniaxial form

\[ \begin{pmatrix} \varepsilon_{2} = \mu_{2} = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & \frac{1}{\varepsilon_{3}} \end{bmatrix} \end{pmatrix} \]  
(E.10)

where \( a \) is an arbitrary number. These are identical the PML tensors derived in [190]. By choosing \( a \) to be complex number, we can implement an absorbing PML that is matched to free space for all angles.

Suppose now there is third medium, which is the substrate (with \( \varepsilon_{3} \neq \varepsilon_{1} \)) that is to be matched, we want to derive the necessary conditions for all angle matching, if possible. Since a second interface is introduced between the anisotropic layer and medium 3, the required matching condition is given by (5.6), which we rewrite here

\[ r_{12} + r_{23} e^{-i2\phi} = 0, \]  
(E.11)

where \( \phi \) is the phase in the matching layer. The first method to achieve zero reflection is by making \( r_{12} = r_{23} = 0 \). Using (E.3), we have \( Z_{1}^{TE/TM} = Z_{2}^{TE/TM} = Z_{3}^{TE/TM} \), where \( Z_{3}^{TE/TM} \) are given by

\[ Z_{3}^{TM} = \frac{\eta_{0}}{\sqrt{\varepsilon_{3}}} \frac{1 - \sin^{2} \theta_{1}}{\varepsilon_{3}}, \]  
(E.12)

\[ Z_{3}^{TE} = \frac{\eta_{0}}{\sqrt{\varepsilon_{3}}} \frac{1}{\sqrt{1 - \sin^{2} \theta_{1}}}. \]  
(E.13)

It is impossible to satisfy the impedance equality condition because \( Z_{1}^{TE/TM} \neq Z_{3}^{TE/TM} \) for all angles. As a result, \( r_{12} \) and \( r_{23} \) will never be zero at the same time. This means a PML or the fully anisotropic layer that is designed to perfectly match for one of the interfaces will not match the other one.
The second method to achieve zero reflection is the method we used to design the AMAL in Chapter 5 (see Section 5.2), which is by making $r_{12} = r_{23}$ and $e^{-i2\phi} = -1$. These two equations lead to the following requirements:

\[(Z_2^{\text{TE/TM}})^2 = Z_1^{\text{TE/TM}} Z_3^{\text{TE/TM}},\]  
\[k_{2z}^{\text{TE/TM}} = \frac{\pi}{2d},\]  
(E.14)  
(E.15)

where $k_{2z}^{\text{TE/TM}}$ is wave number along the $z$-axis and $d$ is the thickness of the anisotropic medium.

By Substituting (E.4), (E.5), and (E.12) into (E.14), we obtain the following equation for TM polarization

\[\frac{\mu_{2x}}{\varepsilon_{2y}} - \sin^2 \theta_1 \frac{1}{\varepsilon_{2y}\varepsilon_{2z}} = \sqrt{1 - \sin^2 \theta_1} \sqrt{\varepsilon_3 - \frac{\sin^2 \theta_1}{\varepsilon_3^2}}.\]  
(E.16)

It is clear that with constant material parameters, it is impossible to match the left side to the right side of the equation for all angles. A similar statement can be made for the TE polarization as well. As a result, it is not possible to use a fully anisotropic layer to match between two different media for all angles. For the PML, all angle matching is possible only if medium 3 is the same as medium 1. If medium 1 and medium 3 are different, then the PML has to be either lossy or infinitely thick such that there is no reflection from the second interface. This because fundamentally the PML is designed to match at a single interface.

Even though it is not possible to achieve all angle matching with an anisotropic layer, we show that it can be used to match at a large oblique angle for either TE or TM polarization in Chapter 5, or for both polarizations simultaneously in Chapter 6. In the future, it would be of interest to investigate bi-anisotropic, chiral, or spatially dispersive (which leads to angle dependent material parameters) medium for all angle matching.
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