Examining the Effectiveness of Lesson Study on Teachers’ Practice in Mathematics within a Secondary School Environment

By

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy

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Abstract

Professional development is an important tool for enhancing teachers’ practices, content and pedagogical knowledge, which, in turn, helps to improve student achievement. This study examined the effectiveness of lesson study as a form of professional development for teaching Calculus at the secondary school level. The study employs a case study model of a team of teachers including one Grade 12 teacher, a Head of Mathematics Department in a private school, and the researcher. The research utilizes Murata’s (2010) lesson study cycle to plan, observe, and reflect on a specific research lesson and consider goal setting. Qualitative data was collected in the form of teacher interviews, classroom observations, and documentary analyses. Throughout the three-and-a-half-month study, participants completed six cycles of lesson study.

This investigation found that lesson study shows promise as an effective form of professional development for teachers. The findings suggested that lesson study acted as a reflection for the teacher to examine his practice and student learning thus identifying the things that went well and overcoming the challenges within the classroom. It assisted in unraveling students’ thinking processes through the introduction of different teaching approaches leading to enhanced student learning and increased their engagement. The Lesson Study activities empowered students to become more engaged in the class and shifted the teacher’s attitude from
a traditional orientation into a more student-centered classroom. The teacher was involved in the decision making thus improving his metacognition and empowering him to try new ideas.
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Chapter One: Introduction

1.1 Introduction

Teaching quality, whether it be good or mediocre, is reflected in student learning and achievement not only in the current year, but also for years to come (Darling-Hammond, 2006). A plethora of factors must be considered when preparing teachers for the classroom, especially if we are to prepare them “for a changing world” and provide them with the skills needed to advance in their profession (Bransford, Darling-Hammond & LePage, 2005). Eisner (1985) remarks that quality of teaching is counted among the most important features of the educational enterprise.

Training teaching candidates is a cornerstone of the education system. In some respects, the teaching profession is similar to that of medicine; obtaining a medical degree marks only the beginning of a successful career. In the same regard, effective teachers continue to learn and evolve throughout their careers in order to become better practitioners. Teachers hone their skills by attending relevant conferences, keeping abreast of emergent research, and by continuing to acquire teaching experience.

For teachers, professional development (PD) is a “keystone to educational development” (Özer & Beycioglu, 2010, p. 4929) that should continue throughout the span of a teaching career, rather than ceasing when the individual receives their certification (Bransford, Darling-Hammond & LePage, 2005; Evans, 2008). Garet, Porter, Desimone, Briman and Yoon (2001) further observe that “the continual deepening of knowledge and skills is an integral part of any profession” including teaching (p. 196).
Professional development is a powerful vehicle for creating exceptional teachers who are able to improve student achievement and provide high quality teaching (Goldsmith, Doerr & Lewis, 2014; Lawrence & Chong, 2010; Yoon, Duncan, Lee, Scarloss & Shapley, 2007). Although it may seem logical or intuitive that professional development is reflected in student achievement, the link is difficult to quantify (Yoon et al, 2007). Nonetheless, effective professional development has been seen to improve and enhance teachers’ skills, knowledge, and capabilities through access to education and training opportunities.

Cookson (2005) observes that teachers often find themselves isolated in the work place due to lack of opportunities for interaction and compressed timetables that make it difficult for them to engage or collaborate with colleagues. Cookson adds, “One of the ironies of teaching is that it is one of the most social occupations, but it is also one of most isolating professions” (p. 16). Another benefit of professional development that provides opportunities for teachers to interact with their colleagues (Ostovar-Nameghi & Sheikhahmadi, 2016), which in turn improves academic success. In order to increase professional engagement among teaching staff, Ostovar-Nameghi and Sheikhahmadi (2016) recommends that schools “schedule classes in a way that maximizes professional interaction” (p. 203) and move away from “once-popular teaching courses” (p. 203) to schedules more conductive to professional development, facilitating collaboration and knowledge construction.

However, not all professional development activities claiming to bolster teachers’ pedagogical knowledge are effective. One reason for this is that they often embody traditional forms (Bolam, 2008; Lieberman & Mace, 2008; Lock, 2006), such as lecturing, or workshops initiated by stakeholders at the school or in the Ministry (Desimone, 2009) to support the teacher
as they assume that if teachers are left to themselves, they would not seek professional development (Beck & Kosnik, 2014, p. xxvi).

Second, professional development is usually driven by a desire for educational reform (Bolam, 2000), with a view to instilling teachers with the latest educational knowledge, and in turn lacks a connection to “the current classroom context in which the participants work” (Kennedy, 2014, p. 338). Finally, some of the activities described above fail to consider teachers’ needs and interests, both important considerations for professional teaching development (Beck & Kosnik, 2014).

Professional development can take place in an outside organization, or through workshops offered in schools (Buysee, 2009; Beck & Kosnik, 2014; Evans, 2007, 2008; OECD, 2009; Thompson, 2009). In their book “Pathway of Teacher Growth”, Beck and Kosnik (2014) observe that informal types of professional development initiated by teachers themselves are most effective, as they allow for the pursuit of self-learning opportunities.

There are several forms of PD that have proven effective for teaching mathematics, including Action Research and Professional Learning Communities. All of these forms of professional development provide active learning opportunities for teachers. They share the common characteristic of enabling collaboration among teachers during the design and implementation of lessons based on the curriculum. However, it should be noted that PD activities involving discussion of instruction tend to be removed from the reality of the classroom, “emphasiz[ing] information transfer” in a theoretical rather than applied context (Lewis et al., 2016, p. 539).

Lesson study (LS) owes its effectiveness to its “concrete embodiment” of the curriculum (Baba, & Kojima, 2004, p. 231). This process allows teachers “to confirm the significance of
teaching materials and to correct unclear points” (Baba, & Kojima, 2004, p. 228). It also helps teachers improve their “personal pedagogical and pedagogical content knowledge” through lesson planning and collaboration with the Lesson study team (Sibbald, 2009, p. 451), increasing teachers’ self-efficacy (Baba, & Kojima, 2004; Puncher & Taylor, 2006; Sibbald, 2009). Lesson study has aided in “supporting teachers’ motivation for the ongoing work of improving teaching” (Lewis, Perry, Friedkin & Roth, 2012, p. 373).

1.2 Research Questions

This research examines lesson study and its effectiveness as a professional development process within a secondary school setting. The following research questions were used to guide this investigation:

1. How can secondary school mathematic teachers use Lesson study to examine their practice and enhance students’ learning?

2. How will engaging in Lesson study affect secondary school teachers’ instruction?

3. What benefits and challenges did the team experience when implementing LS?

1.3 Rationale of the Study

Effective professional development is learning that occurs at the site of teachers’ work. Perhaps an even more important consideration than the environment in which the learning takes place is that the learning occurs in the company of others, who support, encourage, and learn in partnership (Zepeda, 2012, p. xxii). In her book “Professional Development: What Works”, Zepeda (2012) dedicates a chapter to highlighting the benefits of lesson study as a form of “collaboration with a very specific focus that allows the teachers to share their expertise, observation, and questions in order to generate a higher level of teaching and learning” (p. 230),
observing that it “offers a promising approach to advancing the learning and development of teachers” (p. 238).

Examining the process that lesson study undergoes as a form of professional development, we find that it aligns considerably with what Tomas and Ball (2010) concluded in their search for the type of knowledge mathematics teachers need to help their students:

Teaching is not merely about doing math oneself, but about helping students learn to do it... Examining practice itself - from planning lessons to using textbooks, leading a discussion, using the board carefully, and choosing examples- reveals the mathematical demands of the work, which are often overlooked. (p. 228)

Research has found that there are three main attributes that contribute to the effectiveness of the lesson study strategy. First, it does not create completely new lessons but instead guides teachers to perfect an existing lesson (Chokshi & Fernandez, 2004). Second, it includes key features of effective professional development programs (Desimone, 2009). Third, it is “located within the structure and cultures of [teachers] own school where teachers collaborated with their colleagues while focusing on their own students” (Ni Shuilleabhain, 2015, p. 31). Furthermore, there is tremendous body of literature on the implementation and effectiveness of lesson study at the elementary education level. However, to date, very little research has concentrated on implementing the framework at the secondary level, particularly in Grade 12 classrooms. The preponderance of lesson study research conducted in elementary and middle school classrooms suggest that the method shows great promise as a professional development tool (Cajkler, Wood, Norton & Pedder, 2014; Chong & Lawrence, 2010; Desimone, 2009; Lee & Madden, 2019; Lewis & Hurd, 2011; Verhoef, Coenders, Pieters, Smaalen & Tall, 2015). It had a marked influence on teachers’ teaching as they are directly involved in the lesson study which consequently affects students’ learning.
If mathematics teachers benefit from implementing lesson study, several changes can be made to teaching practices, content and pedagogical knowledge. These changes would occur through collaboration among teachers and brainstorming teaching ideas for the topics suggested. By improving the mathematics teaching strategies, teachers will gain insight into students’ learning and potential ways of engaging them.

The primary goal of this research is to better understand how the lesson study process can enhance secondary school mathematics teachers’ content and pedagogical content knowledge to improve student learning. This investigation aims to shed light on the importance of implementing lesson study at the secondary school level and uncover possible challenges faced on implementation, thus serving as a stepping stone for future research.

1.4 Significance of the Research

There is a large body of literature supporting the use of lesson study at the elementary and middle school levels in the United States, United Kingdom, and Japan. This body of research has provided ample evidence suggesting that the lesson study approach shows promise as an effective model for teacher professional development. However, there is a gap in research addressing the use of lesson study to support professional development at the secondary school level.

This investigation aims to help both teachers and school administrators garner experience and knowledge in the altering of lesson plans according to established goals in a manner that facilitates student learning. This study provides participating teachers with an opportunity to examine their practices through collaboration and engaging in lesson study. It is anticipated that the participation in the study will also help them to enhance student learning. For school administrators, this study will illustrate an effective model of school-based teacher professional
development that is supported by research studies and literature. Furthermore, the constraints and challenges that arise in this investigation of lesson study will help researchers, teachers and administrators to avoid potential pitfalls when implementing LS.

1.5 Background of the Researcher

When I was in secondary school, I enjoyed mathematics. I attended faculty of Science, Alexandria University in Egypt and graduated with distinction from mathematics and physics department. I began working at a private university as a TA teaching mathematics to the engineering students. During this time, I completed a master’s degree in Applied Mathematics. After finishing my master’s degree, I moved to Dubai (U.A.E), where I was offered a job teaching mathematics in a private secondary school.

I came to realize how much I enjoyed teaching mathematics and decided to pursue it as a lifelong career. At the same time, I found myself challenged by teaching students of different cultures, and this caused me to re-examine my teaching pedagogy. I began looking for ways to improve my students’ understanding of mathematics. I took additional qualification courses in Dubai that provided me with a variety of strategies for teaching mathematics, and attended several professional development courses and in-service workshops. One of the courses introduced the use of technology as an effective tool for teaching mathematics to visual learners. Drawing on the content of this workshop, I conducted several workshops for teachers on implementing the Casio graphical calculator in mathematics lessons at the secondary school level.

I started using a teaching style similar to that of my own high school teachers, which followed a teacher-centric approach. Employing this approach, I explained the lesson concept through direct instruction followed by examples for illustration. Over time, I began to encounter
two persistent issues. First, the method was not enjoyable for either me or the students. Accordingly, I adjusted my practice to increase engagement with students, and in this manner, my pedagogy shifted towards a more lively, student-centered approach. Second, I came to understand that not all learners benefited from one pedagogical method. In search of a solution, I attended a workshop on differentiated instructions in an attempt to broaden my understanding of the different types of learners. Applying this in practice, when teaching difficult concepts, I would include think-pair-share and jigsaw activities to observe students’ understanding of the material and assist in their thinking process.

At my school, I have held a diverse range of positions including classroom teacher, mathematics lead for grade 12 teachers, and mathematics coordinator for grade 9 to 12 teachers, all of which have required me to collaborate and provide instructional leadership. I have worked with a number of teachers in a variety of capacities to help them in their teaching strategies, planning, and assessment in ways that benefited both teachers and their students.

In 2008, I moved to Canada where I attended a year-long professional learning program at the University of Toronto. I worked at a private school teaching mathematics and physics to secondary school students. My broad experience teaching in several countries has helped me cater to different types of learners, a skill that is particularly useful in Canada’s multicultural environment. However, I was met with the new challenge of how to guide students to become critical thinkers in a way that could be assessed in the manner called for by the Ontario educational curriculum. According to the Ontario curriculum, students must be assessed based on four categories: Knowledge and Understanding, Application, Thinking and Inquiry, and Communication.
Until this point, my teaching strategies has emphasized the Knowledge and Understanding, Application, and Communication categories (Ministry of Education, 2005), so I had to adjust my teaching to incorporate more inquiry-based learning in order to help my students succeed in the inquiry and thinking category. I looked for online educational resources to guide me, as I believe that all teachers need to have an open mindset to cope with the challenges in this rapidly changing world. My own experience suggests that effective teachers do have a fixed or rigid teaching style; rather, they incorporate different teaching styles informed by the classroom dynamic and curriculum. For example, if the dynamic of the classroom was an active, I would facilitate their knowledge through guided questions that challenged them and kept them thinking about the ‘why part’ of that principle. If my class were more reserved, I would engage them in discussion and group activities. It is important to create an environment where students are both academically challenged and feel comfortable taking risks and making mistakes.

I am currently Head of Mathematics in the school, and I am working with teachers to create a sustainable program that would facilitate development of students’ critical thinking skills through planned lessons and activities. Although the teachers I supervise are well equipped with content knowledge, they often fall short of effectively applying this knowledge in the classroom. The teachers need support in learning and applying mathematical pedagogy across a spectrum of learning types to create equity and inclusiveness in classroom. Furthermore, continuous administrative guidelines and direction are required for the teachers to take part in professional development focusing on mathematical teaching strategies geared towards the secondary school learners.
1.6 Outline of the Thesis

This thesis is comprised of five separate chapters. Chapter One provides an overview of my proposed study, including the research context and rationale. In addition to the intended research questions, the significance of the study and the background of the researcher are also included.

Chapter Two consists of a literature review, which examines scholarly research investigating the criteria necessary for developing and evaluating effective professional development with the aim of improving both teachers’ skills and student achievement. In addition to describing the stages of lesson study and theoretical perspective used for this research, I highlight the benefits of lesson study as a job-embedded form of professional development and illustrate its usefulness in fulfilling the conditions necessary for effective professional development.

Chapter Three is dedicated to the methodology used to investigate the research questions. Here, I describe the research context in greater detail, and present the issue of access to teacher participants and the ethical considerations involved. I will conclude this chapter by outlining the qualitative data sources for my study and methods that will be used to analyse this data, in addition to discussing the study’s reliability, validity and limitations.

Chapter Four describes the findings from my two case studies and the data collected. It includes the four steps that comprise the lesson study: goal setting, planning for the lesson, class observation, and debriefing/reflecting.

Chapter Five revisits the research questions and interprets the findings, including a discussion of the benefits and challenges attained during the lesson study. I also examine how Lesson study affected teacher attitude and student learning experience.
Chapter Two: Literature Review

2.1 Introduction

Teachers, as individuals, may find it challenging to adjust their classroom practices when new policies are introduced. Several studies have found that teachers often reshape or ignore policies that are intended to influence their basic classroom routines (Cohen & Ball, 1990; Wilson & Corbett, 1990). According to McQualter (2016), this may occur because “every mathematics teacher has different views about how mathematics should be taught [and ways] of selecting, organizing and presenting mathematics to the pupils” (p. 7). In spite of this tendency, other scholars have found that high quality professional development has the potential to affect teachers’ practices (Desimone, Porter, Garet, Yoon & Birman, 2002; Goldsmith, Doerr & Lewis, 2014), attitudes, and perceptions (Guskey, 2002).

Understanding the goals of professional development is essential for achieving the desired results. According to Ross (2007), the goal of any professional development program is to introduce a new reform that can help the teacher develop his or her content knowledge (represented by the subject and curriculum), pedagogical knowledge (i.e., differentiated instructions), and/or pedagogical content knowledge (i.e., knowledge of educational ends, purposes and values, and their philosophical and historical bases collectively or individually). Seferoglu (2010) notes that “Literature on the improvement of the teaching profession suggests that professional development is a necessity for better teaching and better schools” (p. 547), while Desimone (2009) suggests that professional development “is a key to reforms in teaching and learning, making it essential that we use best practice to measure its effects” (p. 192). A number of other researchers have found that quality professional development has a strong influence on reinforcing a teachers’ growth in the profession (Desimone, 2009; Goldsmith et al,
increasing student achievement (Yoon et al, 2007) and removing teachers’ isolation (Ostovar-Nameghi & Sheikhamadi, 2016).

There are numerous research studies that investigate the ways in which professional development improves teachers’ practice and skills (Broad & Evans, 2006; Desimone, 2009; Evans, 2002; Goldsmith, 2014; Guskey, 2003; Huzicker, 2011; Lawrence & Chong, 2010; Luneta, 2012; Penuel et al, 2007; Whitworth & Chiu, 2015), student achievement (InPraxis Group Inc., 2006; Lawrence & Chong, 2010; Yoon et al, 2007), and by extension, school performance (Evans, 2002). The literature’s emphasis on the teachers’ role in education is unlikely to diminish as they are the primary facilitators of curriculum learning (McQualter, 2016). In the sections that follow, I will examine some of this research to illustrate how professional development affects teachers’ practice and student achievement.

2.2 Secondary School Mathematics Teaching Practices

Preparing students for a changing world is not an easy task, especially for teachers as they are the ones who provide means to this change through education (Ahuja, 2015; Jukes, McCain & Crockett, 2010). Teaching mathematics is a very complex activity that is influenced by several factors, often attributed to teachers’ mathematical knowledge and beliefs (McQualter, 2016; Yeping, 2014). There is no doubt that mathematical content knowledge (acquired at the undergraduate level and any further mathematical training or courses) is essential, but research found that it is insufficient for skillfully performing the mathematical tasks of teaching (Li et al., 2014; Thames & Ball, 2010; Zazkis & Leikin, 2010).

Several researches examined mathematics teaching practices hoping to guide teachers in their profession. Thames and Ball (2010) conducted a study over several years to uncover
mathematical issues that arise in practice and affect teachers’ instruction—and, by extension, student learning. The researchers observed and videotaped teaching in many different classrooms to uncover specific teaching needs. Based on their analysis of teachers’ classroom tasks, they noticed that content knowledge and pedagogical content knowledge consisted of subdomains that should be considered when teaching mathematics. These subdomains “combine knowledge of content with knowledge of students, teaching, and curriculum” (p. 228) that comprise pedagogical content knowledge. They concluded that mathematical content knowledge is an important consideration, but it is not sufficient for effective teaching; rather, effective teachers require “mathematical understanding, skill, and fluency used in the work of helping others learn mathematics” (p. 228).

Zazkis and Leikin (2010) investigated “secondary mathematics teachers’ perceptions of when, how, what for, and to what extent their [mathematical content knowledge] is used in their teaching practice” (p. 279). The study examined 52 practicing secondary school teachers teaching mathematics. The authors acknowledged that mathematical content knowledge in itself is not sufficient for effective teaching and student learning, but must also be accompanied by a variety of pedagogical mathematical knowledge and advanced mathematical content knowledge which include “proof, elegance of solution, and rigor of language” (p. 279). A great part of the pedagogical mathematics practice depends upon teachers’ instruction and the quality of this instruction.

Li et al (2014) suggested improving the quality of mathematics instruction using three approaches that target different potential avenues for change and improvement:

- Approaches that directly target innovative classroom instruction practices
- Approaches that seek to change instructional practice indirectly through the introduction of curriculum reforms that alter content or performance requirements
Approaches that seek to change instructional practice through professional development aimed at augmenting teachers’ mathematics pedagogical knowledge and proficiency. (p. 3)

This study will investigate the third approach as a means of improving teacher instruction, “as the teachers are the architects of the classroom, professional development and teacher education are the architects of the teacher” (Lucilio, 2009, p. 53).

2.3 Effects of Professional Development on Mathematics Teachers

Most of the literature supports the idea that professional development may have a positive impact on mathematics teachers if it effective embodies key features (Desimone, 2009; Goldsmith et al, 2014; Lucilio, 2009), focuses on students learning (Shulman, 1986), and teachers willingly volunteer to participate (Ni Shuilleabhain, 2015). Moreover, professional development affects teachers’ evolving knowledge and practice (Desimone et al., 2002; Kennedy, 2016), which influences students’ learning and understanding (Cole 1992; Fishman, Marx, Best & Tal, 2003; Hiebert & Grouws, 2007; Lawrence & Chong, 2010; McCutchen et al, 2002; Saxe et al, 2001; Supovitz, 2001; Tienken, 2003).

Shulman (1986) defined Pedagogical Content Knowledge as “the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others” (p. 9). Shulman notes that Pedagogical Content Knowledge can be acquired from two sources, professional development and experience:

Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice… teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates. (p. 9)
Professional development may have a variety of goals, ranging from increasing teacher pedagogical content knowledge to improving the quality of teaching in general, or as means of improving teachers’ instruction and supporting students’ learning. Investigating the effects of professional development on mathematics teachers’ content knowledge and pedagogical content knowledge, Desimone et al. (2002) examined professional development given to teachers in elementary, middle and secondary school by the Eisenhower Professional Development Program (Title II of the Elementary and Secondary Education Act (ESEA)). This study focused on developing teachers’ content knowledge and pedagogical content knowledge, collecting data from 207 participants.

The researchers ensured that the teachers were teaching the same subject over the course of the study in order to confirm that the change was due to the effect of the professional development. They considered six features for assessing quality of professional development, three of which are structurally related to the form of the activity, in addition to three core features related to the characteristic of the activity itself. They examined the structure of the professional development in terms of:

- the location where professional development took place, whether it was traditional (i.e. the classroom) or reform (e.g. teacher study groups, teacher collaborative, networks, or committees, mentoring, internships, and resource centers).
- contact time spent in professional development as well as time spent conducting a specific activity.
- “whether the activity in which the teacher participated was designed for all teachers in a school or set of schools or all teachers in the teachers’ department or grade level” (p. 86).

The other three criteria were related to the type of the professional development:
• whether the professional development entailed active learning which involved the teacher in the planning and review of student work,

• professional development alignment with the teachers’ goals, and

• its alignment with curriculum, state standards, assessment, and the content focus of the professional development (use of technology, use of higher order instructional methods, use of alternative assessment practices).

The results of this study showed that professional development is most effective when there is collaboration between participants from the same school or grade, in addition to active learning, coherence (i.e., activity that builds on teachers’ previous knowledge), and reform.

In a very recent research project, Tröbst et al. (2018) investigated the formation of pedagogical content knowledge from teachers’ content knowledge and pedagogical knowledge. The professional development involved three stages with a total of seven-hours of intervention and teacher assessment. The study entailed a pretest, intermediate test, posttest and follow-up test of teachers’ pedagogical content knowledge acquired through the intervention. The researchers concluded that there was a gain in teachers’ pedagogical content knowledge as instruction was geared directly towards that goal, and that, while acknowledging the continued importance of content knowledge, it “did not constitute a necessary precursor for learning from instruction on pedagogical content knowledge” (p. 1061).

To improve teaching quality, Gore, Lloyd, Smith, Bowe, Ellis and Lubans (2017) examined the effects of professional development on teachers’ practice and quality of teaching. The study included 192 teachers from 24 schools (8 in each school): 12 from the primary level, and 12 from the secondary level. The intervention was conducted over four-half days, with follow up data collection and the six-month and twelve-month marks, during which the research
team monitored teachers for the effects of the intervention. The teachers also completed an online fidelity checklist. The data analysis indicated that professional development with the intended goal demonstrated a strong “gain in teaching quality… such effects on the quality of teaching have rarely been reported in other studies” (p. 107).

2.4 Effects of Professional Development on Student Achievement in Mathematics

“Pedagogical content knowledge (PCK) and content knowledge (CK) are key components of teacher competence that affect student progress” (Kleickmann et al., 2013, p. 90). There is an immense body of research connecting teacher professional development and student achievement in mathematics (Cole 1992; Fishman, Marx, Best & Tal, 2003; Hiebert & Grouws, 2007; Lawrence & Chong, 2010; McCutchen et al, 2002; Saxe et al, 2001; Supovitz, 2001; Tienken, 2003), particularly in cases where the end goal was to increase student learning and understanding of mathematics.

Lawrence & Chong (2010) conducted a study in which teachers collaborated with an overarching goal to “help students attain better academic performance” (p. 569). The researchers found that teachers, through collaboration, had improved their content knowledge and continuous review of pedagogical content knowledge. Additionally, they noted that teachers gained “important knowledge about how to maximize student learning through better classroom management and appreciation of student learning styles” (p. 567).

However, questions remain concerning the evidence these studies to illustrate this correlation. Yoon et al. (2007) conducted a systematic and comprehensive review of more than 1,300 studies published between 1986 and 2003 investigating this potential correlation. Yoon and his team found that an average of 49 hours of high-quality professional development could
boost students’ achievement in mathematics by about 21 percent. This finding indicates the effectiveness of professional development.

A study by Telese (2012) examined the relation between mathematics teachers’ professional development and student achievement using a database from 2005 National Association of Educational Progress mathematics assessment (NAEP). A questionnaire was distributed to over 100,000 students and teachers. The teacher questionnaire included questions “about teacher characteristics, such as educational background, preparation to teach specific content, participation in professional development activities, and using a wide range of teaching strategies” (p. 105). The study revealed a positive correlation between Professional development and raising student achievement, as long as it “includes training in content standards, the available curriculum materials, instructional methods for teaching mathematics, and effective use of calculators in mathematics instruction” (p. 109).

There is no doubt that student achievement is related to teachers’ mathematical affinity, pedagogical knowledge and teacher perceptions, but the extent of this correlation remains to be determined. Patricia et al. (2014) examined teachers’ knowledge and perception from 259 upper-elementary and 184 middle-grade early-career mathematics teachers from 23 school districts cross three states. The researchers developed and administrated 20-item instrument measuring teachers’ beliefs about mathematics teaching and learning via Likert-format questionnaire. It also assessed teachers’ awareness of student disposition in addition to teachers’ professional background and instructional context. Teacher and linked-student data were analyzed according to grade.

This investigation by Patricia et al. (2014) provided empirical evidence addressing the “relationship between teachers’ knowledge content and pedagogical content knowledge and
student achievement is substantially stronger in the middle grades than in the upper elementary grades” (p. 453). The authors concluded that middle-grade teachers demonstrated a better understanding of mathematics, students’ thinking about key mathematical ideas, and “how to interpret students’ emerging mathematical explanation and interpretations” (p. 452).

2.5 Characteristics of Effective Professional Development in Mathematics

Researchers continue to struggle with developing a metric for assessing the effectiveness of professional development, as some experts observe that “[d]efining what makes professional development effective can be a subjective exercise at best” (InPraxis Group Inc. & Alberata, 2006, p. 4). However, practitioners may benefit from being able to turn to fixed criteria denoting various degrees of success or failure, as this may in turn help to expedite improvements associated with professional development (Desimone, 2009; InPraxis Group Inc. & Alberata, 2006).

2.5.1 Standards for Professional Development

There is a substantial body of literature concerning what constitutes effective professional development and how it should be designed to help teachers improve their instruction and student learning (Ahuja, 2015; Avalos, 2011; Broad & Evans, 2006; Desimone, 2002; Evans, 2002 & 2009; Garet et al, 2001; Goldsmith et al, 2014; Grierson & Gallagher, 2009; Hough, 2011; Hunzicker, 2011; Penuel et al, 2007; Simon, Campbell, Johnson & Stylianidou, 2011). Searching for a systematic process and set of standards, Guskey (2002) explored the characteristics of effective PD. The author analyzed 13 well known articles discussing the characteristics of effective professional development, published between 1995 to 2002. Guskey investigated whether overlap existed between frameworks employed in the different studies, and whether certain characteristics were common to all of the lists. He found that there was a low
level of consensus among researchers concerning the characteristics of effective PD. Some of the agreed-upon characteristics, however, included “carefully structured” time (p. 12), collaboration “guided by clear goals” (p. 12), that is was school-based, and derived from teachers’ identified needs.

Hunzicker (2011) offered a checklist to be consulted “when designing learning opportunities for teachers [that]… can be used to assess any professional development activity in terms of its alignment with the needs of adult learners” (p. 178). The first item on the checklist specifies that professional development should be “supportive”. This means that professional development must engage teachers, address their needs, and allow them to make choices.

Second, it needs to be “job-embedded”, connecting teachers to their daily lesson activities. Third, it must be “instructionally-focused” to increase the teacher content knowledge and content pedagogical knowledge. Fourth, it should be “collaborative” by engaging and connecting teachers “physically, cognitively, and emotionally’.

Finally, the checklist states that the professional development must be “ongoing” for many hours or even several months as the Hunzicker believes “PD is most effective when teachers have multiple opportunities to interact with information and ideas over several months” (p. 178).

In Ontario, the Ministry of Education’s Report of the Expert Panel on Mathematics (2004) outlined the main characteristics of effective professional development in mathematics. According to this report, effective professional development should:

- be focused on specific goals that are clearly connected to mathematics and the teaching of mathematics (Kennedy, 1998; Cohen & Hill, 2001);
- be supportive of the development of teachers’ knowledge of mathematics (Gadanidis, Hoogland, & Hill, 2002) and the development of teachers’ knowledge of how students
learn mathematics (Garet, Porter, Desimone, Birman, & Yoon, 2001);

▪ be active and give teachers the opportunity to try new ideas and discuss them (McGowen & Davis, 2001; Stipek, Givvin, Salmon, & McGyvers, 2001); and

▪ include support from scholars and value teachers as professionals (Fullan & Connelly, 1987).

All of these attributes should contribute to increasing teachers’ content and pedagogical content knowledge.

2.6 School Role in Professional Development

The stakeholders represented by the school play a critical role in supporting the teachers’ growth in their practices and the profession by providing time and opportunities. Simon et al. (2011) examined the role of stakeholders where schools introduced provisions for professional development. The researchers selected schools that have been identified as being good, very good or outstanding during official inspections. The schools were “successful in [their] concerted effort to secure suitable professional training and development opportunities as part of the school’s strategy for improvement” (p. 10).

Ten schools volunteered to participate in the Simon et al. (2011) research. In these schools, qualified teachers, subject mentors, head of departments, and senior management team or continuing professional development coordinator were interviewed. Six themes emerged from the interviews:

▪ broadening experience, in which the whole school encouraged teachers to be proactive and seek professional development outside school (p. 14);

▪ capacity building, in which the school invested in the staff by providing resources and contributed a percentage towards the fees for their graduate studies (p. 14);

▪ supportive systems and structures; denoted by a whole school initiative and coordinated teachers’ timetable to facilitate meeting;
▪ an open sharing culture, allowing time for peer observations, for shared practice and promoting a sense that teachers were part of a community of learners, where the culture was seen as non-judgmental, and teachers were part of a ‘genuine learning community’ (p. 15);

▪ mentoring and coaching for pre-service and early career teachers, providing new staff with professional development opportunities (p. 16); and

▪ critical roles for professional development, which involves having an effective subject mentor that is a good listener, collaborates with all staff, is reflective, offers constructive criticism accompanied by high expectations, and builds confidence and passion for teaching (p. 18).

The importance of these themes has also been emphasized by other researchers, including Lawrence and Chong (2010), Saito et al. (2012), and Lewis (2016).

Taking into consideration all of this research, it is important to reach a consensus regarding the core features of an effective professional development in order to improve our conceptualizations and evaluating strategies (Desimone, 2009) to determine the content that is most beneficial and will enable the teacher to grow in their profession. All of this should be undertaken with the support of schools for their teachers to improve their instruction, and ultimately, student learning (Lawrence & Chong, 2010).

2.7 Lesson Study

Lesson study is a form of teachers’ professional development that originated in Japan in the early 1990’s and it represents the main professional learning approach for elementary teachers there (Puchner & Taylor, 2006). It involves group of teachers collaboratively planning a research lesson, then teaching, observing and analyzing students’ understanding based on the lesson and specified goals to create changes in teachers’ knowledge and beliefs (Lewis, Perry & Hurd, 2009).
Although lesson study has been used for all subjects, it is most commonly applied in mathematics and science (Doig & Groves, 2011). Saito and Atencio (2013) mention that there are two types of lesson study. One type of lesson study is a top-down method initiated by external stakeholders to improve teachers’ pedagogical practices. Second type is “a more teacher-led involving intensive dissection of lesson content, structure, and practices” (p. 88) and this type is the most commonly used nowadays in different countries as the United States, United Kingdom, and Australia.

Japanese authors view lesson study as a bridge that gradually closes the gap between the curriculum and classroom lessons. This is accomplished through teachers’ collaboration as they add practical improvements to their teaching. It is “a methodology denoting collaborative action by teachers to improve the quality of lessons” (Baba & Kojima, 2004, p. 226). “One important aspect of LS, namely a teachers’ teaching ability, can be analyzed from three perspectives: the ability to form an insight into children; the ability to interpret teaching materials... [and] the ability to construct a lesson” (Baba & Kojima, 2004, p. 228).

Other researchers have defined lesson study differently. Lewis (2004) summarizes the concept as “a complex process, supported by collaborative goal setting, careful data collection on student learning, and protocols that enable productive discussion of sensitive issues” (p. 140), while Isoda (2010) states that it is “a reproductive science for teachers” (p. 19) that features Plan, Do, and See activities in collaboration with other teachers. Matanluk, Johari and Matanluk (2013), in contrast, describe lesson study as “a pedagogical approach that requires the teacher to provide a lesson plan that can be implemented and understood by students” (p. 245). Dotger (2015) defined lesson study as “a process that links standards, teacher learning, curriculum materials, and instructional enactment together to facilitate student learning” (p. 349).
Research has found lesson study to be a “critical pedagogy for teacher education” (Dotger, 2015, p. 350) and a tool for improving instruction and facilitating student learning at the primary and middle school levels. Subsequently, it is promoted as a successful tool at the secondary level. It was deemed so effective that lesson study moved “from school network to national policy” (Dudley, 2012, p. 85). Lesson study owes its effectiveness to its “concrete embodiment” of the curriculum (Baba, & Kojima, 2004, p. 231). Within this process, the teacher is able “to confirm the significance of teaching materials and to correct unclear points” (Baba, & Kojima, 2004, p. 228).

Lesson study can also help teachers to improve their personal pedagogical practice, their personalized views of teaching, and their pedagogical content knowledge (Sibbald, 2009, p. 451). It is suggested that the collaboration with others is accompanied by gains in teachers’ self-efficacy (Baba, & Kojima, 2004; Puchner & Taylor, 2006; Sibbald, 2009). Lesson study has aided in “supporting teachers’ motivation for the ongoing work of improving teaching” (Lewis, Perry, Friedkin & Roth, 2012, p. 373).

2.7.1 Theoretical Perspectives on Lesson Study

There are several researchers who worked on developing a framework for lesson study (Dudley, 2015; Lewis et al, 2009; Wake, Swan & Foster, 2016) that examined the entire process under various lenses. These researchers presented a theoretical model of lesson study based on cognitive learning that included four lesson study features: investigation, planning, research lesson, and reflection. Along with improving instruction through changes in teachers’ knowledge and beliefs, the framework also helped to create a professional community and added to teaching–learning resources. The framework was tested in US K-8 public schools by a team of six mathematics teachers from five different schools. They found that the suggested model
helped teachers use “lesson study to build their knowledge of mathematics and its teaching, their capacity for joint work, and the quality of the teaching materials” (p. 302).

Lesson study interpretations and practices have varied widely in the form and type of activities used across countries, and at times, within countries depending on their goals (Saito, 2012). The main framework adapted from Japan depends on four main stages: goal setting, planning, teaching and observation, and lastly reflection. Adding to these main stages, researchers suggested optional phases that they believed would enhance lesson study; revising the lesson plan (Saito, 2012; Fernandez & Yoshida, 2004); teaching the revised lesson (Fernandez & Yoshida, 2004; Lewis, Perry & Hurd, 2009); interviewing students to ascertain their perspective (Dudley, 2008).

In the context of my research study, I found that the framework presented by Lewis, Perry & Hurd (2009) is best suited to the investigation. The framework entails collaboration among a group of teachers or teachers and researcher(s) and runs through the four stages depicted in the schematic below (Figure 2.1).

Consider goals for student learning and development

Plan a research lesson based on these goals

Observe the research lesson and collect data on student learning and development

Use this data to reflect on the lesson and on instruction more broadly

If desired, revise and re-teach the research lesson to a new group of students
2.7.2 First Stage: Goal Setting

Unlike other types of PD, lesson study requires a significant amount of advance preparation. A large fraction of this preparation time is devoted to goal setting. Setting the goals and planning the lesson comprise the most important phase of lesson study. A useful metaphor underscoring the importance of this phase is to consider the phase as the “underwater”, which is needed to support an “iceberg” needed for planning the lesson (Doig & Groves, 2011). From my understanding of the literature, goal setting is the backbone that support the different stages of lesson study as it gives purpose to the professional development. There are four levels of goal setting: “goals specific to the lesson, goals specific to the unit, broad goals for the subject area and long-term goals for student development” (Lewis, 2004, p. 142).

The first two goals are usually connected to the curriculum, while the “long-term goals may be about behavior, attitude or learning” (Doig & Groves, 2011, p. 80). The selected goals are based on their suitability to student qualities/characteristics (Doig & Groves, 2011; Lewis, Perry & Hurd, 2009). Upon examination, these goals took teacher professional development into consideration as well as student development (Hurd & Licciardo-Musso, 2005). The goals are used to evaluate whether the lesson study is sustained after completion (Dotger, 2015).

2.7.3 Second Stage: Planning

After identifying the goals and the focus of the lesson study, participants meet to collaboratively plan the teaching approach along with any manipulatives/technology to be used. “[T]eachers verbally make proposals about their ideas, justifying their suggestions with anecdotes from their experience” (Dotger, 2015, p. 361), which helps to re-examine their pedagogical content knowledge and gain new insights to the curriculum topics (Baba & Kojima,
2004; Lewis, 2004; Murata, 2010). Teachers explore the lesson content in greater depth and anticipate student questions, difficulties and ideas based on their experiences; as Lewis (2004) observes, “you have to think about things from the student point of view, and that is a big change” (p. 141). Dotger (2015) similarly remarks, “[t]he lesson design must make student thinking visible” (p. 353). The team concludes with detailed lesson plans that include the goals and the research topic. The purpose of this step is not to assess the teachers’ ability to plan or design the perfect lesson, but rather to test the proposed approach and learn how students think.

2.7.4 Third Stage: Observation

After completing the planning stage, prior to beginning the observation stage, one teacher will volunteer to take the lead role in teaching the lesson to their class while the rest of the team observes his or her class. The participants who will be observing the lesson should decide on important aspects of the lesson and “points to notice” (Lewis, 2004, p. 144) and “should respect the classroom atmosphere by arriving and leaving on time, refraining from side conversations” (Lewis, 2004, p. 144). They should focus on student learning and unexpected outcomes, refraining from judging the teacher.

The participants are already familiar with the lesson being planned, so their focus should be on observing students’ thinking/reactions, how they interact with the lesson, and what they actually learned from the lesson, as “observation of students is a key skill/habit for building knowledge” (Lewis et al, 2009). The observers focus on different students’ learning “to provide a legitimate starting point for discussion about the pupils & [their] learning” (Dudley, 2012, p. 93) as “the planning team [identifies] specific data for the observers to collect and record, much of it focused on the students’ discussion” (Hurd & Licciardo-Musso, 2005, p. 393).
2.7.5 Fourth Stage: Reflection/Debriefing

After observing the lesson, the team members meet for approximately 20 minutes to review it. Ideally this is done immediately after the lesson, or later on the same day. The researcher acts as facilitator among teachers to follow a structured agenda with a protocol. The facilitator’s role is to keep track of time and make sure that each of the team members have time to comment on his/her observations. The facilitator also passes several questions to guide the discussion (Lewis, 2004) and helps the teachers to think more deeply about what was experienced and observed (Rock & Wilson, 2005).

Usually, the teacher who taught the lesson comments first; he or she highlights “the strength of the lesson, changes made to the original lesson plan, surprises, and evidence that the lesson met the instructional goals” (Hurd & Licciardo-Musso, 2005, p. 393). Then, each member of the team takes turns reporting his/her observations relevant to the lesson goals. Next, the team works on redesigning the lesson based on the reported observations and what was learned regarding the students’ thinking. Finally, the lesson will be taught to a different class.

Each cycle of lesson study creates a shared collaborative environment between participants working towards a successful endeavor (Dodger, 2015). The cycle is then repeated with the same lesson or moves to a different one (Flynn, Hedges, & Bruce, 2009; Lewis, 2004; Murata, 2010). Lesson study should be an ongoing process of professional development so that its benefits to students and teachers can become more prevalent.

2.8 Benefits of Implementing Lesson Study

Desimone (2009) proposed a core conceptual framework for studying the effects of professional development on teachers and students (Figure 2.2).
When comparing features of Desimones’ framework with lesson study, we find that it incorporates all of these core ideas in its implementation. Lesson study is focused on enhancing teachers’ instruction through collaborative participation and planning topics from the curriculum over an extended period of time, making it effective professional development for teachers, and hence, beneficial to their students. Lesson study allows teachers to understand how students learn through the observation of their work and from the feedback of team members attending the demonstration class.

Many scholars have found that lesson study can have a positive impact on teacher’s pedagogical content knowledge competencies (Dudley, 2015; Mon, Chiew Chin, Dali, Mohd Hasani, Sam, & Lim Chap, 2016), in addition to student’s achievement and problem solving (Lucenario, Yangco, Punzalan, & Espinosa, 2016).

Another benefit that has been realized by some teachers involved in lesson study is an increase in teachers’ professional confidence. Rock and Wilson (2005) investigated the effectiveness of lesson study on six upper-elementary teachers over a four-month period. The teachers’ goal was to meet students’ differentiated instructional needs in mathematics and reading. They found that lesson study helped teachers in differentiating their instruction through peer collaboration, thus meeting their intended goal. They also noticed an increased teachers’
professional confidence, which guided their professional development. Data further revealed that the lesson study model could serve as teacher professional development having a positive impact on teacher instructional practice.

A systematic review of lesson study was conducted by Cheung and Wong (2014) to reveal its impact on teachers and students. They examined 74 studies published between 2000 and 2010 on the effects of lesson study on teachers and students. The study found that lesson study was a powerful tool that enhanced student learning and enabled teachers to continually examine their practices.

Due to the benefits of lesson study for the teacher and the students, it has been widely adopted in several countries. Dudley, director of “The National Strategies Primary” Ministry of Education in the UK, introduced lesson study as a platform for in-school coaching. He believes that “lesson study works because it is a very clear deliberative process” (Dudley, 2008, p. 1). Dudley (2008) wrote a handbook for head teachers, leading teachers, and subject leaders outlining the key criteria for a successful LS. Dudley’s version of lesson study includes a student perspective, involving and interviewing particular students after the lesson study to acquire their perspective and input. These students should represent a group of diverse levels of achievement in the subject area of which the lesson is taught. Dudley believes students should be involved in their learning, “engaging with us, in helping us to help them to learn” (p. 10).

It is common a misconception that the primary goal of lesson study is to produce the perfect lesson (Chokshi & Fernandez, 2004; Lewis & Hurd, 2011; Yoshida, 2012); it is rather intended to help the teacher develop good “eyes” to notice and analyze students’ learning (Fernandez & Yoshida, 2004; Lewis & Hurd, 2011).
2.9 Lesson study and peer coaching

Lesson study is different from peer coaching or any other professional development due to the rigorous process lesson study entails that start with preplanning, classroom observation and debriefing. These processes are guided by set of goals tailored to the teachers’ and student needs and according to the classroom dynamics (Lewis, 2004; Isoda, 2010). In all of the stages, student learning is the focal point and how to make it visible in the classroom with the planned lesson (Murata, 2011). Even in classroom observation, the lesson study team focus their attention to observing student learning (Bjuland & Mosvold, 2015) while, in peer coaching, the core is on helping the teacher and sharpening their skills in the content of the lesson delivered (Robbins, 2015; Hagen, Bialek, & Peterson, 2017).

2.10 Lesson Study in the Secondary Level

Lesson study was implemented by several researches in different fields of study at the secondary school level. One of these studies involved the initial teacher education of a geography and languages teacher (Cajkler, Wood, Norton, Pedder, & Xu, 2015). They concluded that student-teacher engagement in lesson study helped in meeting the ‘qualifying to teach’ standards and offered opportunities leading to growth. Another study investigated principals and teachers as partners in lesson study to address the challenges of English and disciplinary literacy standards (Lee & Madden, 2019). The researchers found that lesson study encouraged principals to “move from evaluator to coach” (p. 60) and to have a better understanding of the teachers’ role. Lesson study guided teachers to effectively understand English literacy standards in regular and special education classrooms. Handayani, Wilujeng, Prasetyo and Triyanto (2019) examined the effect of lesson study on a science teacher. The study found that it had improved teachers’ content knowledge and developed a variety of teaching strategies.
In mathematics, Cajkler, Wood, Norton, and Pedder (2014) noted that lesson study is a useful vehicle for professional learning and collaborative exploration of pedagogy. The focus of the research was fractions, decimals and percentages, in addition to measuring surface area. Teachers reported that they developed approaches that were less teacher-centered which improved the quality of students’ learning and created a sense of teacher community.

For grade 12 Calculus, in particular, Verhoef, Tall, Coenders and Smaalen (2014) reported on two cycles of lesson study. One cycle focused on the concept of trigonometric functions and the other cycle on definition of derivatives using limits and rate of change concepts. The study resulted in a change of teachers’ practices through a “slow and idiosyncratic process” (p. 876). Verhoef, Coenders, Pieters, Smaalen and Tall (2015) also investigated the use of GeoGebra to visualize the concept of derivatives. The study helped the teachers learn how to use visualizations in their instruction and the importance of student interaction.

After searching the literature for the implementation of lesson study in the secondary level, it is evident that there is little research that address lesson study in secondary school mathematics (Lee & Madden, 2019).

2.11 Perceived Limitations of Lesson Study

Although there are obvious gains to implementing LS, there are also several barriers. One of the major challenges that limit teacher and administrator involvement in lesson study is the availability of time (Chassels & Melville, 2009; Mon, Dali & Sam, 2016; Norwich & Ylonen, 2013). Lesson study is time consuming, and entails four stages of goal setting, planning, class observation, and debriefing. Many teachers do not have flexibility of time needed for lesson study due to their workloads.
Another potential barrier is that teachers may be reluctant to being observe by a group of teacher peers (Chassels & Melville, 2009). Not every teacher is comfortable being observed by a team in a performance-related pay, which deters teachers from participation.

Tensions or conflicts among teachers on the team may represent another limitation in implementing lesson study (Dudley, 2015), as teachers tend to have different classroom cultures and employ different pedagogies.

2.12 Summary

Responsibility for maintaining a high level of competency in any profession rests with the wider professional community. In the field of education, it requires the personal commitment of all individuals in every school, district, professional association and university. Well-designed professional development opportunities can enhance student learning by guiding teachers to both extend and renew their practice, skills and beliefs. Researchers have found that a professional development program is effective when it is collaborative, which enhances teachers’ practice and, ultimately, student achievement.

Professional learning communities, and lesson study all have qualities necessary for effective professional development for teachers. Lesson study, however, has proven to be a catalyst for teacher change as it places teachers at the centre of the learning process through collaborative work with experienced peers, reflection, and monitoring of student progress.

The influence of lesson study extends from teachers’ practice to student achievement. “While there are other professional development programs that incorporate many of the characteristics of lesson study (e.g. action research, professional learning communities and peer coaching), what sets it apart from others is the live research lesson … and opportunities for teachers compared to other forms of professional development” (Murata, 2010, p. 575).
Chapter Three: Methodology

The goal of my study is to examine the effectiveness of lesson study on teachers’ practice in a mathematics classroom within a secondary school environment. The purpose of this chapter is to provide an overview of the research design and methodology used. It will describe the procedure followed for data collection and construction. The chapter also addresses ethical considerations throughout the investigation, in addition to discussing the reliability and validity of the study.

3.1 Research Design

A qualitative case study design is appropriate for this study as, according to Harrison, Birks, Franklin, and Mills (2017), “case study research is consistently described as a versatile form of qualitative inquiry most suitable for a comprehensive, holistic, and in-depth investigation of a complex issue (phenomena, event, situation, organization, program individual or group) in context, where the boundary between the context and issue is unclear and contains many variables” (p. 12). Thus, the case study method is critical for gaining an in-depth understanding of lesson study and its effect on teachers’ practice in mathematics classrooms. This research study addresses the needs of teachers educating students at the senior level (grade 12). Data will be collected through teacher interviews, field notes, observations and teacher reflection journals.

3.1.1 Case Study

My research question is best investigated using an explanatory case study, which will aid in examining the effectiveness of lesson study in teachers’ instruction in a practical context. An explanatory case study seeks to answer questions that are “too complex for survey or experimental strategies” (p. 547) and “is associated with the comparison of a small number of variables across a few cases” (Blatter & Haverland, 2012, p. 2). It also seeks to explore situations
that “[do] not imply that the boundaries between the strategies or the occasions … are always clear and sharp (Yin, 2003, p. 4).

This qualitative case study design will implement the guidelines and procedures suggested by Merriam (1998). Merriam defines a case study “as a thing, a single entity, a unit around which there are boundaries” (p. 27). In her design, the emphasis is placed on the use of a literature review to examine any emerging framework, which helps to shape the research questions and points of emphasis, and which then guide the selection of a purposive sample. The researcher must “utilize three data collection techniques conducting interviews, observing, and analyzing documents” (Yazan, 2015, p. 149). When analyzing the data, Merriam (1998) follows the application of constructivist epistemology and suggests “consolidating, reducing, and interpreting what the people have said and what the researcher has seen and read” (Yazan, 2015, p. 145) to construct their knowledge about the phenomenon.

3.1.2 Participants

I visited public and private schools in September of 2017 to January of 2018 to explain my research and provide letter of consent (Appendix A- Teacher Consent) for teachers and administration outlining specific details of the lesson study and a proposed timeline for the research. I also contacted teachers and administrators that I worked with in the past to ask them if their school was willing to be part of the study. Participants were informed that participation in the study was voluntary and teachers could withdraw from the study at any point in time without repercussion. They were also reminded that their information would be kept confidential and that pseudonyms would be used to protect their anonymity.

Most of the schools I approached had other commitments and could not spare the time to be part of the study. Towards the end of January, I was contacted by one school in which I knew
the head of mathematics department. He explained that the school administration and the grade 12 teacher agreed to participate in my research, so I visited the school to meet the principal and receive an administrative approval.

I acted as a team member, an observer and facilitator for two reasons: there was only Liam and Ali, and I felt two people would not be able to provide enough diversity in ideas to implement lesson study, and it was not possible to involve another teacher. I tried to minimize my interference and gear the discussion towards Liam and Ali throughout the cycles. In the fourth cycle, I was only an observer in the planning as it was an assessment of learning and I wanted to see how Liam and Ali would incorporate the decided goals without my involvement.

3.1.2.1 Lesson Study Team

The lesson study team included the mathematics teacher, the head of mathematics department of the school, and me, the researcher. Both the teacher and the head of mathematics were asked to participate in the study and were given letter of consent to participate. The consent letter asked them to give permission for audio and video recordings of the classroom and during team meetings. After receiving the consent, it was necessary to engage in a pilot project to identify the goals and topics from the curriculum of the lesson study.

It was the desire of the teacher to implement lesson study in the Grade 12 Calculus curriculum. The teacher was interviewed after each lesson using open ended questions to capture his perception of the effects of his practices/instructions (if any) on student learning. The head of the department was also interviewed after classroom observation to identify any additional perceptions, which may differ from those of the lead teacher. The debriefing after each lesson was about 25 min to 40 min.
3.1.2.2 Students

There are 15 grade 12 students in the school and 11 of them are in the Calculus class. Within three weeks of the start of the study, two students dropped the course, leaving seven boys and two girls in the Calculus class. The lesson study team visited the class to explain the research and assure students that their mathematics learning would not be affected by the research and that we are to help them better understand the subject and achieve their goals. Some students had concerns that the lesson study team would judge them due to the classroom observation component, while others asked about the timeline and whether their participation would affect their lessons. We met with the students to assure them that disruption would be minimal, and to address any questions they had in a direct and professional manner.

A parent/guardian consent letter was given to each student and collected prior to beginning of the study (Appendix E: Student – Parent/Guardian Consent Form). The consent form was given to allow the lesson study team to observe and video record the class for data collection and analysis. All students’ names were encrypted, and no personal information of any student will be shared.

3.2 Procedure

Before the start of the LS, there was a preparation/explanation stage to summarize the steps of the lesson study cycle for participants. This step was important for ensuring that everyone understood what the lesson study is, the meticulous planning and time commitment required, and to emphasize the importance of the presence of all members of the group at all meetings (Appendix, Description of Lesson Study Plan). Since commitment and time are essential for the success of the lesson study, it was important to create an agreed-upon timeline for meetings and reflections with team members for the entire semester (Hurd & Licciardo-
Musso, 2005). Once the teacher and the head of mathematics department agreed to participate, consent forms were signed.

The teacher and the head of mathematics department were asked to participate in semi-structured interviews (Appendix B- Teacher’s interview). Some of the questions were about the participating teachers experience in teaching Calculus, any previous involvement in professional development and how the teacher supported student’s success in mathematics. The goal of the initial interview was to identify experience in teaching grade 12 Calculus, teachers’ attitudes towards professional development and mathematics, as well as the teachers’ content and pedagogical content knowledge.

The interviews, as well as all of our meetings for planning and debriefing, took place in the school in the head of mathematics department office. Lunch time was agreed upon as the regular meeting time as it directly followed the lesson and was convenient for both the teacher and head of the mathematics department.

After obtaining the consent of the participants, I needed to address the students. It was necessary to explain the presence of the lesson study team in the classroom (the lesson study team are studying the lesson to improve their own teaching and not to judge the students or the teacher (Lewis, 2004)). The lead teacher introduced me to the class, and I explained the research procedure, assuring students that their learning would not be interrupted by the presence of the lesson study team inside the classroom. I then distributed the student – parent/guardian consent form to be signed.

After receiving student – parent/guardian consent, the head of the mathematics department and I attended several lessons with the lead teacher in order to set the long-term and short-term goals for the lesson study based on our classroom observations and the lead teacher’s
vision for the semester. During this time, I made observational notes about the class environment, the teacher, and the students’ attitudes to understand the class mechanism and provide some planning ideas for the lead teacher.

Following the initial interview and the post observation of the class environment, the lesson study team met to set the goals and plan different lessons. The teacher had six years of experience teaching this course; he already had the lesson plan prepared from past years, so the lesson study teams’ main discussion related to redesigning his lesson plan and assessment in such a way that aligned with our goals.

3.3 Data Collection

My main study took place in the second semester of the 2017-2018 academic year, from February to May 2018, in a private school with a total of 69 secondary school students. Once consent was obtained, data collection included a brief individual interview with all of the participants on the team in order to identify their professional backgrounds in teaching mathematics to secondary school students (see Appendix B for the sample interview questions). Audio recordings were collected for all team meetings, starting with the planning stage and discussions, and ending with classroom observation and post lesson colloquia (see Appendix C for the sample interview questions).

Additional data took the form of artifacts produced by the teacher and participating team during lesson planning and participant contributions during lesson observation. Audio recording of all the discussions were taken during our meetings. At the end of the study, I conducted a recorded semi-structured interview with the teacher to gain insight into the effects of the lesson study on his teaching practices/beliefs (see Appendix D for the sample interview questions).
In summary, data was gathered from three main sources: 1) the lesson study team interviews; 2) classroom observations; and 3) artifacts produced by the teacher. The research lessons were also video-recorded but were not transcribed or coded. The reason for recording was to review the lessons and reflect on any missing observations, in addition to adding validity to the data.

### 3.3.1 Classroom observations

Classroom observations were used to monitor lessons for goal setting and were also used regularly throughout the study to observe both teachers’ and students’ reactions, attitudes and changes. We were in the class for the entire duration of the one hour and fifteen-minute teaching period. We sat at the back of the classroom, out of the way of the students and their classroom activities. We would engage with students if they initiated conversation; however, this rarely occurred. For the most part, the students seemed to ignore our presence in the classroom. Audio-visual recordings were used to provide me with evidence of students’ engagement with their teacher and other students, students’ understanding of the lesson, and an opportunity to review any unobserved events that I neglected to record.

### 3.3.2 Lesson Study Team Interviews and Planning

Interviews took place both before and after participation in the lesson study. There were two main semi-structured interviews. One interview took place with the lesson study team prior to the study. During this interview, participants were asked about their experience teaching math to secondary school students, how they perceived students’ engagement and create an environment that support students’ success in mathematics, in addition to their opinion on professional development. A second interview took place at the end of the semester. This
interview focused on the benefits (if any) and challenges of being part of the lesson study, in addition to recommendations that would enhance their experience.

Throughout the study, additional meetings took place to discuss lesson planning and assess our established goals. These meeting took place immediately after the observed lesson, as it was lunchtime at the secondary school. I interviewed both the teacher and the head of mathematics department. The interviews consisted of open-ended questions (Appendix C). Although I began the interview with some idea with what I wanted the teacher/ head of mathematics department to elaborate on, the direction of the interview reflected ideas expressed by the teacher/ head of mathematics department. There were specific questions for the teacher after teaching the lesson regarding ideas for improvement. There were also questions for the observer regarding students’ engagement and approaches to increase their participation. These questions aligned with our intended goals.

These sessions were audio-taped and transcribed. I transcribed each interview, and gave participants the opportunity to review transcripts of their interviews if they wished. Participants also had the opportunity to add their own comments, clarify, and elaborate on their thoughts if they wished. None of the participants chose to do this.

3.3.3 Lesson Study Timeline

Table 3.1 shows the amount of time the lesson study team spent planning in each cycle and time spent debriefing. It is worth noting that time spent for initial interview, explaining what the lesson study process is, and conducting the final interview of the research are not included in the table.
Table 3.1: Timeline in Planning and Debriefing for each Lesson Study Cycle.

### 3.4 Data Analysis

Qualitative analysis involved the lesson study team interview, direct observation, participants’ observations, and physical artifacts. The process of analyzing data followed that of Bloomberg and Volpe (2012), and encompassed:

1. Reviewing and exploring data; identifying “Big Ideas”
2. Re-reading and examining data, code data, and placing coded data in categories

From: Memo/journal, data summary tables, inter-rater reliability

Then: Revising coding scheme, assigning codes, eliminating, and/or collapsing
It should be noted that, in keeping with Bloomberg’s method, the coding process “should not be addressed with a set of hypotheses that you set out to prove or disprove. Rather, you need to approach your transcripts with an open mind” (p. 100).

Nvivo12, a qualitative software, was used to organize and perform data analysis on the transcribed interviews. Classroom observations and participants interviews regarding the lesson study was searched for major patterns and themes based on the investigated questions and divided into categories accordingly. Based on these categories, an appropriate key coding was developed that could be used in the inductive analysis. The categories were then interpreted and reflected upon to answer the research questions. To make the findings more reliable, it was also important to look for and reflect upon contradictions or conflicts within the patterns (Schwalbach, 2003), as this “will make the findings more accurate” (Mertler, 2009, p. 143).

Afterwards, the participants in the lesson study team were asked to review my analysis and, if questions arise, that particular episode can be jointly revisited on the tape to confirm or disconfirm my interpretation of the data and to consider any possible alternative interpretation (Lewis et al, 2009). But none of the participants wanted to do that. Based on the observations and discussion, recommendations are given to enhance the lesson experience for another class.

3.5 Ethical Considerations

I received approval from the Office of Research of Ethics of the University of Toronto prior to beginning any research or contacting any teachers. Pseudonyms were used for teachers and schools to ensure confidentiality and any specific details that could link the school, teacher, department head or students to this study were omitted. All data, including video recordings, audio recordings, and any transcripts were stored in files protected by passwords.
All teacher participants of the study were provided with a letter of consent that contained an overview of the study. The overview had information about the stages of the study, as well as the goal of the study, length and time it is expected to take place if they choose to participate. The letter of consent informed them that there were no apparent risks associated with participation in this research project and asked them to sign and return the form prior to the beginning of the study. The consent letter also informed teachers that they may withdraw from the study at any point in time.

Participants had the opportunity to benefit from reflection on their experiences, in addition to having the lesson study team listen to these views carefully and offer their reflections/planning/ideas. It is anticipated that this research will serve to inform secondary level classroom teachers, by providing insight into the advantages and challenges of the lesson study that can be used as a future point of reference.

3.6 Reliability, Validity and Limitations

The study was conducted in a private school with 9 students taking Calculus, which helped the observing team to more precisely monitor students’ engagement, and to design lessons that could meet their needs.

The study is limited by the size of the lesson study team, with only one teacher and the head of mathematics department, in addition to the researcher. This size constraint limits the influence of the lesson study on the participating teacher and the flow of ideas directed at the lesson. However, having a small lesson study team did allow me to develop meaningful connections with the team and to find a suitable meeting time that we all agree upon.

My participation in the lesson study team could be considered another limitation, but my role was restricted to that of a researcher and discussion facilitator. It was not possible to find
another teacher to participate in the study due to their other commitments. I tried to minimize my suggestions so that I was not influencing decisions made, and this is evident throughout the planning stage of the different cycles, as will be discussed at length in the coming chapter.

The amicable relationship that Ali, an administrator, had with Liam prior to the study, along with Liam’s open mindedness towards being observed helped in implementing lesson study. Other teachers may be reluctant to open themselves up to critical analysis or being observed by an administrator over a three-month period.

Another limitation of this study arises from the different beliefs and perceptions towards teaching held by the lesson study team, which made it difficult at some points to modify the teachers’ planning. This study also does not take the students’ perspective into consideration. Because of this, the results may be slightly skewed due to their dependence on participants’ abilities to reflect on the effectiveness of lesson study.

In spite of these limitations, this study provides a rich illustration of classroom observations and debriefing interviews as a means of correcting for some of these issues, enabling a comparison of lesson study team classroom observations with the teacher’s accounts. Thus, the use of multiple data collection methods strengthens the internal validity of this investigation.
Chapter Four: Findings

This chapter details the participation of Liam, a Grade 12 teacher, and Ali, the Head of Mathematics Department, in a lesson study. The head of mathematics observed Liam teaching the planned lesson for grade 12 Calculus class. I will commence with a brief description of the school followed by a description of the case study of participants involved in the research. I will include an overview of the academic background of each participant and their attitudes towards instructing mathematics at the secondary level. In addition, the chapter will encompass a detailed report about the lesson study cycles that include goal setting, planning, classroom observation and ending with debriefing.

4.1 Ambition School

Ambition School is a private school located in the Greater Area of Toronto that offers grades spanning from Kindergarten to Grade 12. The school is located in a culturally diverse community, with members comprised of differing and varying family structures, incomes, and educational background. Although it is a private school, few students receive bursaries for their tuition costs. The administrative team at Ambition School is comprised of a principal and a vice-principal, who is also the Head of the Mathematics Department. The school is inspected every two years by the Ontario Ministry of Education inspector. The inspector checks to see if the teachers are following the Ontario curriculum guidelines, conducting an assessment and evaluation which may involve attend classes to observe teacher instruction and to meet with students.

The secondary school enrolment was 69 students at the time of the research. The secondary school has one class for each grade. During the research period, Grade 9 had 15 students, Grade 10 had 20 students, Grade 11 had 19 students and Grade 12 had 15 students.
Among the 15 students registered in grade 12; nine of them are taking Calculus with Liam. The class is comprised of two girls and seven boys. All of the students have Calculus as a prerequisite for their program except for one student who is taking it to fulfil the number of hours required for his graduation. There is a designated room for each grade and the school follows a two-semester academic school year. The research took place in the second semester.

The school has department meetings at the beginning of the semester to review teaching expectations for each grade based on teacher feedback:

We will give feedback. For example, grade 9 teachers will probably talk to teachers from grade 6, 7 and 8 and tell them that this is the required skill in grade 9. So, when they teach grade 6, 7 and 8, those [what you should] concentrate on and I will also talk to the grade 9 teacher so we are on the same page. Now since I am the teacher of grade 10, 11 and 12 so in that case I basically count with myself. (Liam, interview, February 5)

They also meet once per month for professional development. On the professional development day, teachers share ideas about projects or assignment that went well with the students, and in some cases, the principal invites a guest speaker to conduct a workshop. The workshop usually takes around two to three hours about a specific topic that the principal feels the teachers are lacking and need support in. For example, in one instance, the workshop was on the topic of assessment and evaluation. Another workshop was about lesson planning and differentiated instructions. These workshops are divided between explanations of the topic and practice through samples, where the teachers implement their understanding and the trainer modifies and provides comments on their work. In another case, the professional development lasted two days and was conducted by a certified trainer to train the teachers to meet the requirements of Ministry inspection, including what should be included in their binders from the three forms of assessment: As, Of, and For learning.
4.2 Team member – Liam

Liam is the Grade 12 Calculus teacher at the school. He attended the University of Toronto and graduated with teaching subjects Mathematics and Chemistry. Currently, Liam is in his sixth-year teaching at Ambition School. During the time of the study, Liam taught one Grade 10 class, Grade 11 Academic Mathematics courses, and the Grade 12 Calculus class. When he was in grade 5 or 6, he would pretend to be a teacher and explain a lesson to an empty room, which helped him with his grades and in his presentation skills. In grade 8, Liam decided to pursue a career in teaching. His tutoring experience in university as a TA aligned with this plan. He describes his experience and decision to become a teacher as follows:

> I enjoy teaching because through teaching I get to help students to understand the material. It is always nice to see others learn something. It is mostly satisfying to see when students understand a concept after you explain to them. Furthermore, it is also very rewarding some they are able to achieve their fullest because of my contributions. (Liam, interview, February 1)

Liam attended several professional development meetings at the school that lasted for one to two hours but felt it did not contribute to his mathematical content knowledge, instead feeding his pedagogical content knowledge (“not subject-wise; more of the teaching methods” (Interview, February 5)). Due to the small size of the school, he did not have the chance to co-teach, but he wanted to: “we (Liam and Ali) wanted to do that for the longest time. It was in the work except we did not have the time to match our schedule together” (Liam, interview, February 5).

Liam’s Calculus classes were rather formulaic and traditional. He started off by giving his students a handout outlining definitions, concepts, and rules in the lesson followed by a couple of questions for practice. Occasionally, he would ask students to participate by raising
their hands; otherwise there was limited opportunity for student contribution. He assigned homework and in-class assignments to support their learning.

4.2.1 Teacher Mindset and Philosophy

Liam’s teaching was traditional however, he had an open mindset. During our discussions, he was observed to listen to suggestions and accept them when he felt it helped him in his class instruction; in cases where he disagreed with our suggestions, he would attempt to persuade the group of his opinion. When asked about his teaching philosophy, he replied: “I believe all students are capable of learning and we should train them to enjoy learning and to do something about it. I do also believe that students may be able to learn differently through different methods, but again, it is very difficult to manage that during a class setting with a given curriculum” (Liam, interview, January 19).

4.2.2 Teacher-Student Relationship

Liam believes that the relationship between students and the teacher is “super important”, as it helps to close the gap between the teacher and the students, hence allowing students to trust him or her. It also teaches the students respect and responsibility.

4.2.2.1 Social Relationship

In class, Liam likes to tell jokes from time to time so that the students feel comfortable and that they can approach him; however, he maintains some boundaries. He added:

At their age, they need some "friend". I am not asking the teacher to be student's best friends, but the students should feel the teacher is approachable so when they have questions, they are not afraid to ask. They should also feel that the teacher is open to discuss their progress or if they have some concerns about their future career. (Liam, interview, January 19)

When Liam knows that it is one of the student’s birthdays, he usually offers to buy them lunch as a celebration. Liam believes trying to befriend to the students is a way to train them to work with
a colleague who is senior. He added: “when they later enter a work place or university, they won't be socially awkward, and they are not afraid to ask questions to their seniors with respect. Only through working with their seniors, they will learn the past experience and learning from teacher is a good beginning”.

4.2.2.2 Academic Relationship

Liam tries to teach students more effectively by offering suggestions when opportunities arise and encourages group discussions - although copying answers is prohibited. He makes sure to recognize the students’ achievements. For the grade 12 students, once a student receives their university or college acceptance, he congratulates that student and announces it to the class. He dedicates most of his lunch and free time to answering students’ questions, whether they relate to social or academic matters. Liam always tries to help his students to achieve their goals and offer advice about university or college programs that he feels the students are interested in applying to.

When he notices that students did not do well on one of the tests, he tries to minimize the effect of that test so that the students are not afraid to try again. He assures them that it is the consistency in their work that counts. Failure of one test does not mean they will continue to fail, but rather they need to learn to face the failure and move on. He also encourages them to revisit their weaknesses and continue to try throughout the course. If he feels that a student is struggling, he speaks with them individually, tries to encourage them, and offers them help.

Liam shares:

…. at the end of the day, as teachers, we want them to not just learn the knowledge, but acquire the skills, which hopefully will benefit them in the long run. A good and effective study habit and work ethic goes a long way. I also constantly tell them that the world does not go around them, they have to adapt to the world, whether it is future study or at work place. They are worker in training, but we have to teach them the responsibility while teaching the subject. (Liam, interview, January 19)
4.2.3 Teacher Attitude

Liam was very willing and open to participating in this study. He felt that this was a good chance to share ideas. He taught most of the students previously when they were in grade 10 and 11 and taught all of them Advanced Functions course in the first semester. Liam has a good relationship with his students and plans lessons based on their feedback. He modifies his teaching methods to accommodate their needs in the hopes of increasing their engagement. He recognizes that just because a teaching method may work for a group in particular, it will not necessarily work for everyone.

Grade 12 students choose courses based on the program they are interested in; some programs require prerequisite courses for enrolment. Liam understands that students are taking Calculus because it is a prerequisite for their program; at the same time, he acknowledges that mathematics may not be their strength, so he makes an effort to motivate and encourage them:

Some of the students for example if they were into arts or into law it is hard to connect that with math, so because everyone is coming from different background they want to go different places, it is hard to find a universal one to say that math is important even though you are going to law. It is kind a hard to convince…. I will research material and see how math can be related to whatever field they are going to… so through that I will get them hopefully interested and continue. (Liam, interview, February 5)

Liam is dedicated to helping students in his free time:

They stop me whenever usually lunch time, but today I am with you. Most of the time I am in one of the classrooms so if they have a question they come to me, so they will know where I am. Almost every single lunch I am helping in one way or another. This semester might be slightly different because I will be doing more of the math contests and science fair projects with them, so I have less available time. Usually I am in one of the classrooms, so lunch time is my office hours I eat, and I wait for them. (Liam, interview, February 5)

Liam does not like to follow any specific text book. He thinks that the textbook should be used as a guideline only. He uses many online resources in addition to Nelson and McGraw Hill
Calculus books. He seeks out a variety of resources to support his teaching rather than following a particular textbook, and alters his instruction according to student feedback.

Through his experience of teaching, Liam shared some concerns regarding student attitudes in secondary school in general, as well as their lack of responsibility:

I feel the student are not taking responsibility, you have a lot of student now they do not do anything until the end of the semester and you cannot do anything, you cannot take marks off, you are supposed to judge on their work not their work ethic. (Liam, interview, February 5)

He feels that these student attitudes come about as a result of the Growing Success document (Ontario, 2010), which forbids teachers from giving zeros to students if they have handed in the work.

In 2010, The Ontario Ministry of Education published a document “Growing Success” that outlines guidelines for teachers on assessment, evaluation, and reporting. The document discusses divergent opinions regarding dealing with late and missed assignments, but emphasizes that “because every assignment – whether submitted on time or late – provides evidence of learning, deducting marks for late assignments could misrepresent the student’s true level of achievement ... supporting non-performing students by helping them develop these skills and habits, rather than using punitive measures, is a matter of meeting individual students’ needs and should not be considered a form of unwarranted “special treatment” (Ontario, 2010, p. 46). Liam, however, feels the policy is unfair to students who submit their work on time:

It is unfair to the teacher because I already spent time to mark other people work periodically and I give it back to them within a week or sometimes two days if it is smaller. Now, at the end of the year I have like two or three piles and I have to go back and remember, you know even though I know how to do it, but I still have to see which criteria how many marks are for each part. (Liam, interview, February 27)

He believes that the policy may have some merit up to Grade 10, but should be eliminated for students in higher grades:
11, 12 have to be separate, you have to be able to give them zero, you have to be able to give them something punishment or whatever it be because otherwise they never learn …. we are not teaching them anything good. (Liam, interview, February 27)

His concerns arise from the need to prepare grade 12 students for university:

… everyone says when they go to university. They feel that we throw them into water to sink, right because we are not training them well in responsibility. We are not training them well in terms of the material because all the courses are now easier compared to 10 years from now. Math [has become] so easy barely that there anything is there. When they go to university, [mathematics] is much higher level and they are less responsible. That is bad combination. Back in the days, when we were there we were responsible because we do no hand it in, it is a zero and they do not accept any late [assignments] and the courses are harder so more responsibility. The courses are hard, so when we go to university, we are fine but now days they have less responsibility and easy curriculum that’s a horrible deadly combination for the student. (Liam, interview, February 15)

The other concern Liam cited was students’ lack of participation. This was also noted by the lesson study team during the post observation, who witnessed very low engagement from students:

I have taught them for two years some for three, in general they are not very verbal …. Taha (one of the students) knows how to answer but Taha have some difficulty in terms of understanding the language because he has only been here for two years so if he is not sure he will not answer. (Liam, interview, February 12)

This is similar to what Ali and I had observed: “as if you are pulling it from their mouth, you see, they do not have that initiative” (Ali, interview, February 13)

4.3 Team member – Ali

Ali is a member of the lesson study team and part of the lesson planning, helping to create the researched lesson and observing the class environment. Ali was not available for the initial interview, so I emailed him the Semi-Interview Teacher Questions (Appendix B) and he responded with his answers via email. Because he was unable to understand some of my questions over email, I interviewed him with these questions at a later date.
Currently, Ali is the vice principal and the head of mathematics department at Ambition School, where he has worked for four years. Ali graduated from University of Jordan, Amman, with two teaching subjects: Physics and Mathematics. He has a master’s degree in physics, which encouraged him to become a teacher. He worked in the UAE for 28 years; 13 of which he taught Physics, Precalculus, Calculus, and University level Physics. Following this, he worked as a vice principal for 15 years before coming to Canada. When he came to Canada, he worked at Ambition School for one year as a Physics teacher, then moved to becoming a fulltime vice principal and head of mathematics department. When I asked him why he chose to become a teacher, he replied that he enjoys teaching: “I visualized myself as a teacher and did not think of any other career.” He said that he misses being a teacher and jumps at the opportunity of teaching whenever one of the teachers is absent.

When he was a teacher, he had the opportunity to attend many professional development activities and became interested in expanding his knowledge and helping other teachers. Ali felt that professional development was very important as it helped him improve his content and pedagogical knowledge (and thus, his teaching) throughout his career.

4.3.1 Teacher Mindset and Philosophy

Ali has an open mindset, as he is “constantly hunting for new ideas, new methods, and new ways of thinking”. He believes that using technology in the classroom is important as it provides a visual aspect to math and helps the students to develop a better understanding. He “keeps on searching for methods to integrating technology in education”. Ali describes himself as an innovator as he always in search of new ideas or projects for the students and is willing to “innovate to provide dynamic and effective instruction” to guide the teachers in his school.

When I asked him about his teaching philosophy, he replied:
all children are unique and must have different talents. It is my desire to create this type of atmosphere where students can meet their full potential. I will provide different ways of teaching to meet all students’ abilities and approach all students by the most possible way to learn to share their ideas. (Ali, interview, January 19)

Ali also believes that it is important to engage students in the learning process, and that teachers are tasked with being facilitators of this knowledge. He emphasizes that it is also important that teachers inform students that “student-teacher are partners for learning”, as some students mistakenly believe that it is the duty of the teacher to explain and teach without a need for their active involvement.

4.3.2 Teacher-Student Relationship

Ali is the vice principal at the school and maintains strong relationships with the students. He believes that “students who have good relationships with their teachers show more engagement in the academic content presented, display better classroom behavior, and achieve at higher level academically” (Ali, interview, January 19).

4.3.2.1 Social Relationship

In one situation, Ali recalls, “one student was thinking to donate part of his “lever” to a child in need. I encourage him, gave him support and helped him academically when he missed many days in the school. Finally, he did it and finished his courses and he achieved higher results”. (Ali, interview, January 19).

Ali maintains this relationship during summer holidays, as he is used to keeping in contact and meeting with his students from time to time. Even with students who have already graduated from the school, Ali feels that although they have moved on to university, they should continue to receive support and advice as needed (whether social or academic), which he never hesitates to provide.
4.3.2.2 Academic Relationship

Ali is always willing to help students with math and physics, and takes advantage of any opportunity to do so. When he has time, he supply for teachers in math or physics classes—something he misses from his early career. On one occasion, he heard that a few students were planning to meet at the public library to study for a trigonometric functions test on the weekend. Knowing from his teaching experience that the unit was difficult, he jumped in and offered to meet them at the library: “I helped them by solving and practicing a lot of questions” (Ali, interview, January 19). He expressed to the students that he was happy to help them and thanked them for their questions.

4.3.3 Teacher Attitude

Considering his past experience with professional development activities, Ali observes:

Part of them are useless but few were very helpful, especially the ones that are related to implementing technology in the teaching process. Furthermore, I participated in many International conferences and all of them were helpful. (Ali, interview, February 4)

He recalls having professional development in the following areas: “lesson planning; changing from the traditional way of teaching (lecturing) to a facilitator; forming rubrics for projects and labs; using descriptive feedback”. Ali’s experience with professional development also extends to facilitator roles. For example, he was so passionate about using technology in his teaching that he conducted several workshops in Japan in which he acted as the trainer. He demonstrated how to use GeoGebra in Grade 9 and 10 math classes, in addition to Casio graphing calculators as a tool for enhancing learning.

When he was vice principal in Dubai, he conducted several professional development sessions for teachers in his school. He noticed that most of the time, the implementation is so brief and “…. in some cases, he had to repeat the training all over again”. So, from his point of
view, there had to be continuous follow up from the administration with teachers after conducting professional development sessions.

He became aware of the importance of collaboration going through the exercise of preparing assessments with another teacher teaching the same grade and using the same curriculum: “it is very beneficial and important; exposing the student to different style of teaching and different way to approach the problems thus improving the learning process”.

Ali also understands the challenges that come with collaboration. He mentions three main challenges that he has encountered when collaborating with colleagues:

First challenge, some teachers are against change were they like to teach the same way as they were doing since long time and what they are comfortable with. Second challenge is shortage of time, this is due to the duties that the teacher has apart from teaching as morning or lunch supervision, arranging trips, or attending school activities, so it become harder for teachers to meet and collaborate during school time. Third challenge, teachers wants to see the benefits of collaboration or implementing a new idea instantly and this in some cases is hard. (Ali, interview, February 4)

Although Ali was a teacher in the past, he currently occupies an administrator role, so I asked him how he dealt with these challenges overseeing teachers from an administrative perspective.

He replied:

From year to year, I found that teachers engage in change that they initiate where often resist change suggested by others. I realized that administrator cannot force a teacher to change but have a role of helping and guiding them for the better of the students. When a teacher feels this, they are more willing to change but it needed time and patience.

I then asked him how his perspective changed towards teachers after becoming an administrator.

He remarked that in the administrator role, he developed a new appreciation for the teaching profession:

Since I was a teacher and promoted to a vice principal, I realize the following towards teachers: first that teachers’ job is much easier than administration’s job since the teacher has to look after his students learning where administration need to look after all teachers, students, parents and other components related to school like government related. Second, teachers are the busiest people among other profession, so they need to
manage their time effectively. Third, teachers are asked to do a lot of extra things besides teaching like participating in trips, supervision, sport activities and extra curriculum activities. (Ali, interview, February 4)

4.4 Lesson Study Process

Prior to starting the lesson study cycles, two important steps were taken to simplify the process. First, Liam had some idea of what lesson study was, but was unaware of the process it followed. He welcomed the idea of being part of the team and collaborating for the benefit of his students. Because Ali had no prior knowledge of lesson study, I had to dedicate some time to explaining the concept to both Liam and Ali. An hour and a half was spent explaining all of the details regarding lesson study. This explanation included a description of plan and their rights and responsibilities as participants, to ensure that everyone was aware of their role and the procedure that need to be followed. Participants were given an approximate timeline for the investigation’s completion.

In the second step, the lesson study team wanted to attend a class with Liam prior to the planning sessions for two reasons. The first reason was to observe the class environment focusing on the students’ interaction with the teacher for setting the long-term and overarching goals for our researched lesson. The second reason was to familiarize the students with our presence in the classroom. This was done after ensuring all the students had returned their consent forms.

During this observation, Liam gave the students a handout for the lesson that included all of the concepts that would be taught during the lesson and several practice questions. The lesson study team noticed that Liam was a traditional teacher who started with a teacher-directed lesson, explained the concept, provided one or two examples, and then asked the students to complete the practice questions. He checked the students’ work, indicating whether it was correct or
incorrect, and when necessary, provided additional guidelines to help students correct their answers.

After the observation, we met in the office of the head of mathematics department, our designated meeting location for all stages of the lesson study cycles. During the meeting, we discussed goal setting.

4.4.1 Goal Setting for Mathematics

Lewis (2004) outlines four levels of goal setting. The first two goals are implemented at the unit and lesson levels and are informed by the Ontario curriculum guidelines, while the remaining two, long-term goals and broad goals, address bigger picture or overarching interests.

The long-term goal (Lewis, 2004), based on the lesson study team observation and the teachers’ input, was to increase students’ engagement through planned activities while challenging them to improve their critical thinking skills using inquiry questions. This was Liam’s long-term goal that he first introduced with the grade 10 students, and eventually shifted to the same strategy for his grade 12 students as well. He said: “we need to train the students that, in the real world, you get hired to do a job because we do not have a solution for it, if you already done it and other people have done it too so why would they want to hire you (Liam, interview, February 5)”.

The broad goal for the subject area (Lewis, 2004) was to increase students’ interest in mathematics, particularly for those who wished to pursue different fields of study. This goal was fostered through consideration of student interests in the design of the problem-solving assignments. Liam has been using this technique for the past two years with his grade 12 classes:

For people who are going to medical school, I will give questions that relates to the medical stuff. For people going to engineering, I would give questions related to engineering. People going to environmental science, I will give a question with temperature or carbon dioxide emission etc. So, they are towards their own special
Liam feels it is important to prepare Grade 12 students to transition to university:

So, to make that easier first of all work ethic, so I need to strengthen their work ethic, handing things on time. You got to be able to do the test I [have] given you in 80 minutes, you got to finish in 80 minutes, no extension… I train students based on university standards… so my goal basically getting them faster work, more accurate, handing things on time and then to think like grade 12, think like pre-university student, try to inspire them [and] try to force them in a way. (Liam, interview, February 13)

And he added that, within process of preparing them for post-secondary study, he wanted them to:

Trust yourself, to be more a little confident, but then again … I want them to be able to face failure. I want them to say “yes, I failed how do I become better next one”, “yes I do poorly on this how do I become better in next one”. I want them to build more like a character issue so that when they go to university [where] there is a lot of pressure, they are able to sort of lay it out somehow or know how to adjust it. That’s basically my goal setting for grade 12 in general. (Liam, interview, February 14)

The other two goals were “goals specific to the unit [and] goals specific for the lesson” (Lewis, 2004, p. 142), which are derived from the curriculum expectations. For Calculus, we decided to do the lessons study on Differentiation/ Rate of Change (unit 1) and its Applications (unit 2) for two reasons. The first reason is because two thirds of the curriculum is comprised of these two units; and the second reason is because these topics are best suited for the broad goal described above. The main goal for the first unit, Rate of Change in Ontario, is for students to:

Demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit; [and] graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative; verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems. (The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 101)
For this goal, the lesson study team decided to create four cycles of lesson study connected to that unit:

1. Rules of Derivatives (three lessons),
2. Derivative of Trigonometric, Logarithmic and exponential Functions,
3. Implicit Differentiation, and
4. Preparing a Discussion Task.

Each topic has its own specific goal connected to the lesson that will be described in the pages that follow.

In addition to the four cycles, the lesson study team decided to incorporate two more cycles on another unit: Derivatives and their Applications. The main goal described in the Ontario curriculum for this unit is for students to:

solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models. (The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 105)

The topics selected in this unit were:

1. Related Rates, and
2. Optimization.

The topics involve solving problems using mathematical models and derivatives. The application aspect of the unit aligns with both the broad goal and long-term goal. We completed six lesson study cycles.

**4.4.2 First Cycle of Lesson Study - Rules of Derivatives**

The first cycle concerned the rules of derivatives, including power, constant, constant multiple product and quotient rule. The lesson was held on February 21.

**4.4.2.1 Planning of the Cycle**
We met to plan the lesson and derive the goals for this cycle. The specific goals for rules of derivatives are:

Verify the power rule, constant, constant multiple, sum, and difference rules graphically and numerically. [Also,] Determine algebraically the derivatives of polynomial functions, and use these derivatives to determine the instantaneous rate of change at a point and to determine point(s) at which a given rate of change occurs. [Beside] solve problems, using the product and chain rules, involving the derivatives of polynomial functions. (The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 103)

This cycle represented the foundation for Calculus, so we had to plan it at the same time as we detailed and designed each part. As can be seen in the curriculum guidelines included above, the specific goals for the rules of derivatives are comprehensive, so we needed to divide the topic into three separate classes.

During planning process, I assumed both the role of a lesson study team member and also the role of the facilitator. Although I lacked personal experience in lesson study, I did have background knowledge from my coursework, in addition to having conducted a thorough review of relevant literature, which aided me preparing my study.

The planning for the lesson started on February 12. The lesson study team met on seven school days, and each meeting lasted approximately 45 minutes each day. We started the meeting by discussing the goal connected to the specific subject area (“apply rules of derivative to determine the derivative functions”) and determined how the long-term and broad goals could be implemented. The planning was partitioned between reviewing ideas for lesson delivery (including potential practice questions for facilitating student learning), and preparing the in-class assignment to assess students’ understanding.

The first class focused on the main four rules of derivative: power rule, constant rule, constant multiple rule, and sum and difference rule. The second class focused on the product and quotient rule, and lastly, the third class focused on the chain rule. A fourth class was reserved for
completing the in-class assignment. All of these lessons were studied under the same specific goal.

4.4.2.1.1 First Class - Planning

In previous years, Liam structured his lesson to explain each rule in a separate class followed by a class reserved for an in-class assignment that combined all the rules to assess student understanding and to clarify any missed concepts. The lesson study team did not want to change the way Liam organized his unit, but wanted to focus on small changes that would change teachers’ traditional way of delivering the concepts through instructions that would support students’ learning, refine the questions given in class, and prepare a cohesive in-class assignment that explicitly linked all lessons with all of the intended goals.

Liam appeared engaged and open to new ideas during the planning process. Since he has taught the course for the past five years, he shared his previous year’s lesson plan. The lesson study team used this plan as a starting point for the planning. For the first class, we discussed the possibility of using a deductive method to build on previous learning goals on Limits to come up with the power and constant multiple rule rather than using a formula or an abstract approach. To help the students derive the rules, we suggested giving examples where the students use their previous knowledge to solve the problem and then compare these examples, looking for patterns from which the desired rule could be deduced. We anticipated that this would help to engage and challenge the students at the same time.

For the constant rule, I suggested using the definition of derivative to formulate the rule because this was simple to derive. For the rule of sum and difference lesson, we discussed using the concept of conjecture and testing this claim, as it would have taken longer to use deductive method. Next, we moved on to refining and selecting the examples that will be introduced in the
lesson and added more practice questions for students to work on, as a means of encouraging participation.

### 4.4.2.1.2 First Class – Observation

All lessons were video-recorded to provide opportunities for the teacher and the lesson study team to debrief the lesson. The class for the first part of rules of derivatives was delivered on February 21 with eight students present and one student absent. The students sat in pairs, and the lesson study team sat at the back of the class. Liam started by distributing a lesson handout that included an outline for the lesson rules, concept and practice questions that were to be discussed in class. There were no answers in the handout, but they included an empty space for students to insert their answers. He preferred this method to having students copying notes, as it ensured that they followed up with him to fill in the missing parts. The handout also included a derivative worksheet that Liam prepared using online resources, which contained questions from each class in this cycle. This was homework for students to practice, and the lesson study team did not take part in preparing it.

Based on our discussion and feedback, Liam followed the lesson plan as intended, straying from the plan only to meet student need for clarification. He began the lesson by addressing the class as a whole with a question “How do I find a derivative of a function if given \( y = f(x) \)?”, and a student asked him to clarify what he meant. Following his clarification, one of the students was able to connect his response to what they had previously learned in the class covering Limits. Liam wrote the student’s observations on the board, and this was used as a gateway for introducing the rules of derivatives. The purpose of the question was for students to reconstruct their knowledge and give them time to think and derive the power rule. Liam followed with other simple questions, during which students began participating and sharing
answers as he moved between the tables. If the student was wrong in his answer, Liam would ask the question in a different way, providing them with a second chance to arrive at the correct response.

Liam started with a simple example. As he solved it on the board, he repeatedly asked the students “so what do we do next?” or “…and this gives me?” He paused from time to time to give the students time to copy the information. He followed with another example, and then he asked the students if they detected a pattern. His questioning techniques assessed the students in their ability to deduce the power rule using previous their understanding and learned strategies related to rate of change. To emphasize the power rule, he provided extra examples for illustration using the same questioning technique. In this way, he ensured that students were engaged and following up throughout the class.

Although the students correctly deduced the rule, Liam explained to the students that this was a guess and that they still need to prove that the formula worked for any case using an abstract approach. He added “the reason for that, this is a university preparation course and you should know some derivation of rules”. During the abstract proof, Liam was observed going through each step, giving students enough time to follow up with him and ask questions before moving to the next step. One student asked “would we get this in a test” Liam replied: “no, but I may put it as a bonus question”.

Liam followed the same procedure to derive the constant and constant multiple rule. He continuously gave examples for illustration, asking students to try to solve them. He wrote their answers on the board. The examples gradually increased in level of difficulty to challenge students. Liam asked his students if they had any questions or if they needed clarification as he moved on. For one of the challenging examples, Liam asked one student to explain how to solve
it on the board as he was not sure if the student understood the concept. The first lesson was taught in one period.

Before introducing the last rule of the lesson, he told the students “one more rule before we call it for the day”. For the sum and difference rule, he used a new strategy. He asked the students to guess what the rule was, and when one of the students guessed correctly, Liam acknowledged the student’s success by saying “very good guess, so now we need to prove it”. Liam used an abstract approach to derive this rule as well, using a concept that students had learned in the previous semester. As he was going on with the proof, he asked the students if they had any questions. Liam followed with examples that combined all the rules to ensure that students were able to apply them in the same question. He used the same questioning technique as he was going through the examples and corrected students if they used the wrong rule, explaining the reason for his choice and their misunderstanding. At end of class, Liam gave the students a worksheet for practice.

4.4.2.1.3 First Class – Debriefing

We discussed the class, identifying which goals were not being met, and made recommendations for the upcoming segment. The lesson study was an ongoing process involving observing the class, debriefing and reflecting. After the lesson observation for the first class, the lesson study team met in the head of mathematics department office for debriefing. Both Liam and Ali were interviewed after classroom observation (Appendix C), which was audio recorded.

The questions for Liam focused on his perception of how he thought the lesson went. He felt “overall the lesson we had today was actually pretty good for them” and that students were most engaged when he asked them to make an educated guess about the rule of sum and difference, as it helped them think more critically “I want them to go further because I want to
say here you go, you guessed this, you got to prove it. That’s how we do in math, prove to show it works”. Liam thought that it would be beneficial for the students to include abstract parts in the explanation: “if we continue parts of the harder theory and some of the easy stuff and we combine them”.

Ali’s questions were geared towards meeting our goals and succeeded in achieving them. Asked about student engagement throughout the lesson, Ali said, “engagement with the students [was] very low [in spite] they get the idea, they understood the lesson [and] they know how to solve questions after that. But, their motivation [to participate] is not that high”. Ali felt that the teacher “should engage them more, push them more to solve [and] come to the board”.

Being an active observer, I added my observation to the lesson. I felt that student engagement was low. The same two to three students repeatedly responded to the teacher’s questions, while the rest of the students appeared unwilling to participate. They were copying notes and appeared attentive during the lesson, but it was not clear whether they understood the lesson. It was important to use our observations to reflect on the goals that were not being met, and discuss strategies for achieving them in the next class.

**4.4.2.1.2.1 Second Class - Planning**

For the second class on the rules of derivatives, we decided to use a different approach for illustrating the product and quotient rule. The lesson study team suggested asking the students to solve a question that would link the first lesson with this one. The purpose of the question was to check students’ understanding of the previous lesson, and challenge them to expand their understanding in the new lesson to include the product and quotient rules.

From our experience, we knew that it would be hard for the students to deduce the product rule, so after discussion, we agreed to modify the approach to use conjecture followed by
investigating the validity of the suggested rule using several of examples. Then, the teacher proceeded with verifying the product rule using an abstract approach that was connected to concepts they learned earlier in the semester. The same strategy was repeated for the quotient rule. The practice questions were refined to help the students grasp the concept.

4.4.2.1.2.2 Second Class – Observation

The second class of the rules of derivatives (product and quotient rule) took place on February 23. All of the students were present. Liam started by congratulating two students on their acceptance to universities they applied to, while reminding them that this “offer might be conditional, that means you have to check your conditional acceptance” and continue to work hard to maintain the average grade for pre-requisite subjects the program requires.

He began by telling the students that we are continuing with two more rules. As an introduction, he asked the students to solve a question using concepts learned in the previous lesson. One student offered an incorrect response, so Liam guided the student with questions that gradually helped him solve it. Next, he had a discussion with the students about the possibility of solving the same problem in a different way. The students were confused as to why they needed to learn another rule if they could simply use the previous ones. Liam explained that not every case is that simple, and that there are more concepts in the course for which first three rules of derivative cannot be used. He provided an example on the board for the students to see what he meant.

He asked them to make a guess about the product rule, and then verify its validity using the rules from the first lesson. The students were not able to come up with the formula, so Liam explained the product rule and used an abstract method to prove it. He paused between steps to ensure that students were following him, and asked them how he should proceed. None of the
students responded, but instead simply smiled, so Liam made a joke “is it because it is Friday, Friday you cannot think anymore?” It was obvious that most of the students struggled to understand the abstract proof in the way that Liam explained it.

Liam gave students time to understand the procedure of the proof and to copy the notes. He mentioned the rule in a rhyme that would help the students remember the rule. Similar to the first class, one of the students asked, “could you ask us to prove it in a test?” and Liam replied “remember, anything I show in class is fair to put in a test or exam”. It was obvious that they struggled to understand the abstract proof. He gave them a couple of minutes and asked the students if they had any questions. One of the students asked for clarification, so he approached her table to explain. Two students were taking longer to copy so he took this opportunity to warn them about the university setting, where most professors use slides and do not wait for everyone to copy before moving on.

Waiting for students to finish copying, he suggested for the rest of the students to do an example in their handout. During that time, he was moving around the tables to review students’ solutions. Liam noticed a mistake in one of the student’s notes when copying the question, so he pointed it out.

After that, Liam went over the question he assigned, asking the students for the steps that should be taken in solving the question and having them repeat the rhyming the rule as he was writing it, so that the students could memorize it. He emphasized that his expectation was that students simplify after using the rules by providing the final answer in the most factored form; he told them “remember that” and wrote it on the board. A student raised her hand asking for clarification about the simplification and taking common factor. He approached her table to help her one on one. Liam kept on asking students if they had any questions, and told them that
Calculus involves a lot of process and that it is not only about applying rules, but also about understanding the abstract derivation and simplifying the final answer.

He told students to try another question on their own before he solved it on the board. A couple of students asked him if their solution was correct and he checked it for them. He allowed students enough time to attempt the examples before he solved them on the board. The questions were knowledge- and application-based but gradually increased in difficulty. When he felt some students had problems simplifying the final answer, he said “it looks complicated now, but it will become easier the more questions we do”.

For the quotient rule, Liam explained that he would not be deriving it the same way as the product rule because it is longer and more cumbersome. Instead, he used the product rule to prove it in a simpler fashion. Again, he stated the rule as a rhyme so that students could remember it more easily. One of the students asked him to explain again the rule so he reviewed it, pausing after every sentence to ensure the student understood. He provided one final example, and asked the students to rhyme the rule with him as he applied it.

4.4.2.1.2.3 Second Class – Debriefing

We held our second debriefing immediately after the second classroom observation. I began by asking Ali what he concluded from observing the students. Ali felt that student engagement was not noticeable, and that they “have some weakness from the previous lessons something like factoring [or] getting common factor, [and] simplifying long expressions”. Liam also felt that student engagement was low but he expected that “because they had a test this morning, for me it is always that, when they have something early in the morning, or they have a test, their mind is all tense for that test and once they are finished they feel relaxed and they think about that today is just a lesson so we do not have to engage in a lot of learning”. In addition,
Liam observed that “Friday is always the worse day, they know it is a shorter day and they just had a test, so their mind is not really focused”.

Because student engagement was our long-term goal, we had to suggest ideas that would help in achieving this goal and encourage students to participate. We suggested choosing particular students to do certain questions and allowing more thinking time. When Liam was asked what he would do differently if he had the opportunity to reteach the lesson, he replied:

I would have simplified the question down a little bit, I would probably start with something simpler like \((2x + 1)x\) and then would do a couple of those and then move to another one [problem], that is a little bit harder where it is required a little bit factoring same as division…. I would probably do that. (Liam, interview, February 23)

4.4.2.1 Third Class - Planning

For the last class in this cycle, we discussed different ways of explaining the chain rule. All three of us provided input regarding how it could be best explained to the students. Ali observed, “I like to divide the rules into something let’s say called rule for derivative that is the rule and I divide some other rule I can derive it and call it corollary, which is result [of that rule]”. In Ali’s view, the chain rule could be considered a corollary from the previous derivatives rules. Liam agreed to some of this, but voice some concerns regarding the method :“I agree with you on some level, but I do not like the way that we are doing that because I do not want them to memorize too many rules then they become confused”.

After a long discussion, we felt that the best approach was using the deduction method, which asks the students deduce the chain rule. The teacher would give them tips on how to use their previous knowledge about quotient rule and composite functions, which they learned in Advanced Functions and the previous class, and ask them to relate this knowledge to the rules of derivative learned thus far. Using several examples, the students should be able to deduce the
chain rule. Afterwards, Liam would proceed with giving more practice questions to ensure students’ grasp of the rule.

Based on our last class debriefing, we decided that Liam would engage students by taking turns calling on students to solve the question and allowing more thinking time, so that students could try to solve the questions themselves before Liam reviewed them.

4.4.2.1.2.3.2 Third Class - Observation

The last class of rules of derivatives (chain rule) was given on February 27. Two students were absent at the beginning of the class until 20 minutes into the lesson, when one of the absent students came to class. Liam started the lesson by explaining that, “in high school level, a lot of time we have to derive [rules].” In previous lessons he had used this strategy, and he intended to use it again in this lesson as well. He told the students that “the final rule for the basic operations is the chain rule”. To prove the rule, he used students’ knowledge of quotient rule to derive it, asking students for input at every step and waiting for them to respond before moving to the next step.

Liam gave an example, and as we agreed upon during the planning stage, he called on individual students to answer, rather than having the same students answer every question. If he felt that the student was hesitant or unsure of the answer, he guided him or her with comments intended to trigger their memory. He emphasized the importance of using proper notation when solving these problems, as using improper notation is a common mistake among students.

He followed the lesson plan, but changed it where further clarification was needed. While doing examples, he repeatedly asked students if there were any questions before moving on to the next point. Liam paused in between practice questions to give the students a chance to think about the questions on their own before asking them for solution. He discussed their responses
and clarified when there was misunderstanding. In one of the examples, Liam asked the students if there was a faster way of solving the question. The students were not sure what he meant, so he gave them the first step to guide them. He provided enough time for students to think about it and work out the final answer. Liam moved around the students’ tables to review their work. One student asked for clarification, so he approached his table and helped him by pointing out the mistake. He wanted to give the student a second chance to figure it out before he corrected it. Afterwards, Liam went over the example on the board to ensure everyone grasped the concept.

Liam summarized the basic operations rules of derivatives for the students in a recap. He also gave them a timeline for the topics and assessment in the coming days, noting that he would finish this unit before the March break. After they came back, he would begin the next unit, “Derivatives and Applications”. He also reminded the students that tomorrow’s class would be reserved for the in-class assignment. Finally, Liam solved another example, asking students “how would I proceed from this?” to ensure that they understood faster process he explained earlier in the lesson.

However, Ali, as an observer, felt that students were not understanding this process and were finding it hard to respond to Liam’s questions. He intervened by suggesting Liam use a different approach, such as assuming that one of the functions to be $U(x)$. Liam understood what Ali was saying and started solving the example again, this time using Ali’s suggested approach. As he was solving the question, Liam asked the students which rules he should follow, and when a student responded correctly, he said “good” then continued to next step. It was observed that students asked for clarification as he continued solving the question. The student had both answers on the board for them to compare and choose the one that they understood better.
Liam ended the class with another example that incorporated a different concept. He posed questions, but the students were unresponsive. He gave them clues and reworded the question, which prompted one student to volunteer a response. He praised the student for their correct answer. Whenever a student answered one of his questions, he always made sure to acknowledge them for it. He also reminded the students of the worksheet that he gave them earlier and asked them to complete it at home to practice what they have learned so far.

4.4.2.1.2.3.3 Third Class - Debriefing

For the third class, and before debriefing, Liam started by informing me that the students thought they were too old to continue doing questions on the board. The students also felt I might judge them if they made a mistake, so it was determined that I would address this issue in the next class by assuring them that I would not judge them, and that my role was to support the teacher for their learning and benefit.

For the debriefing, Liam started reflecting on the lesson by saying:

I think today’s lesson was not too bad. I think overall it’s very good and I think they are more engaging, and the concept is actually easier to convey to them than I anticipated. I thought it is going to be harder to explain the chain rule but apparently, they understood it pretty good. (Liam, interview, February 27)

Ali felt that students started to participate more, but “they do not have that initiative to answer” unless Liam called upon a specific student.

From my observation, I felt that they were a little bit shy even when they were answering the questions. They were answering in a very low voice as if they are afraid to make a mistake in front of the lesson study team, but Liam said that this is not the case and that generally this group of students was not verbal. Also, “it is very hard to pull things out of them, so this is actually pretty good they are actually engaging more than last year” (Liam, interview, February 27) as he taught most of them in grade 11 also.
4.4.2.1.2.4.1 Fourth Class - Planning

After debriefing the third lesson, it was time to prepare for the in-class assignment. The in-class assignment is a cohesive worksheet that includes all that has been learned so far in this cycle. The lesson study team worked on the in-class assignment, in which the teacher is a facilitator and the students work in groups to solve the questions. As a way to apply our broad goal (transition to university) in the assignment, I suggested using multiple choice questions, as most assessments in first year university courses are multiple choice, but Liam argued:

I cannot see what they are doing because all they could be doing is just guessing, they guess it is right. It does not mean they understand. If they guess wrong, it does not mean they do not understand so that is the problem I have. As a teacher, the inspector could come in and start criticizing our test and questions; the inspector comes in and asks why multiple choice is used in math class. (Liam, interview, February 27)

Ali added that the inspector would say “it is not your job [and that] your job is to cover the curriculum, that is it”. Accordingly, the assignment included questions with short and long answers from each lesson covered in the previous days.

The students needed to demonstrate their understanding by selecting the appropriate rules of derivatives. The questions covered different approaches of finding the derivative that varied in level of difficulty and included the four categories: knowledge, application, inquiry, and communication. In our discussion, we had to make sure we employed our goals in planning the assignment to increase students’ learning experiences, in addition to using it as an appropriate indicator of student learning.

The class activities and the in-class assignment were aligned with the intended goals. Liam applied these suggestions and modifications to both his lesson plan and to the assignment so that they would reflect the desired goals.
4.4.2.1.2.4.1 Fourth Class - Observation

The next class was on February 28. Students were assigned to reflect on their learning from the previous days through the in-class assignment. There were eight students present and one absent. During the in-class assignment, Liam did not specify the groups, so students formed their own groups. Two students preferred to work individually, but one of them asked Liam when she encountered a difficulty. The other student asked one of the groups for help and compared his final answers with them to check. There were two other groups with three students each.

Liam was observed moving amongst the students, reviewing their work. When he noticed a mistake, he would not correct it for the student, but would instead ask for clarification to probe their thinking. In this strategy, the student reflects on his or her work and corrects their own misunderstanding. It was also noticed that a student would leave the group and ask Liam for clarification, and then return to their group to share Liam’s explanation. Every student was observed to be focused on the task at hand, and if they could not figure the answer then they would ask a member in the group before asking Liam.

During the eighty-five-minute class, Liam was observed circulating between the groups offering encouragement, listening to their conversation, and providing hints when necessary. The students could not finish in the same day, so Liam gave them an extension until the next day; his idea was to ensure that they were not rushing through it and “[to] have more relaxed mentalities to get it done”. He collected the in-class assignment from the students as they were not allowed to work on it at home.

In the next class, Ali was busy during class time and was only able to attend the last thirty-five minutes of the class. Liam started the lesson by reminding the students of all the
derivatives rules studied so far. He called upon a different student to state each rule. Then he redistributed the in-class assignment to the students. Students reformed their group from previous class, and worked eagerly to finish the assignment.

You could hear a student explaining his answer to the other two group members and why he thought it was correct. Another student asked Liam to check his answer, and instead of pointing out the mistake, Liam asked him guiding questions until he realized his mistake. In response to another student’s question, he explained the importance of using brackets when applying the product rule as it would affect her simplification. At end of the class, Liam collected the assignments to mark them.

4.4.2.2 End of Cycle Debriefing

On last day of the lesson study cycle “Rules of Derivatives”, it was important to reflect on this cycle and assess our progress in implementing our intended goals. Debriefing took place on the following day due to other commitments of lesson study team members. After finishing the three lessons, the students had an in-class assignment, which included questions related to this unit. When debriefing asked Liam about student engagement, he replied:

It was really good. They got a chance to talk and work with each other briefly… you will see that showing up in terms of their learning. I find compared to other days. I think there was more energy as they know what they were doing, [and] were able to ask questions. I was surprised to see some students, who I thought they were a bit weaker, but they were able to answer the questions…. The only thing, [I need to] remind [them to] keep on practicing not to take oh this is easy. (Liam, interview, February 28)

For the in-class assignment, the lesson study team did not anticipate that students would be allowed to work in groups, so I asked Liam for his opinion. He said that he wanted them to learn from one another and foster communication among the students. He also encouraged them to work in groups and see each other as a resource, warning them at the same time not to be “like a leech… you need to contribute to the group discussion”. Liam explained that he is preparing
them for university: “This is how I learned when I was in university because you form a study
group, you do assignments and then you compare your answers, oh you have different answer
from me and you start learning from them as well”.

I asked Ali about his observation and he said that he likes these assignments as they allow
students to learn from each other, and added that “it is a positive collaboration”. From my
observation, it was obvious that students were collaborating and engaging with each other to
discuss answers, as one student was explaining to another, and another group was discussing the
questions and procedure they should follow. Every student was engaged in demonstrating their
understanding, and the lesson study team felt that the desired goals had been largely achieved by
the end of the cycle.

4.4.3 Second Cycle of Lesson Study - Derivatives of Trigonometric, Logarithmic,
and Exponential Functions

In this cycle, we discuss the rules for differentiating Trigonometric, Logarithmic, and
exponential functions. The lesson was held on March 2.

4.4.3.1 Planning

The curriculum goals for Derivatives of Trigonometric, Logarithmic, and Exponential
Functions in Ontario are:

Determine, through investigation using technology, the graph of the derivative \( f'(x) \) or
\( \frac{df}{dx} \) of a given sinusoidal function [i.e., \( f(x) = \sin x, f(x) = \cos x \)] [and] exponential
function [beside] recognize that the natural logarithmic function \( f(x) = \log_e x \), also
written as \( f(x) = \ln x \), is the inverse of the exponential function \( f(x) = e^x \), and make
connections between \( f(x) = \ln x \) and \( f(x) = e^x \). [Also] verify using technology, that
the derivative of the exponential function \( f(x) = a^x \) is \( f'(x) = a \ln a^x \) for various
values of \( a \). (The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 102)

The curriculum also includes another specific goal for the in-class exercise, which is:

Solve problems, using the product and chain rules, involving the derivatives of
polynomial functions, sinusoidal functions, exponential functions, rational functions.
(The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 104)
Both learning goals need to be incorporated when planning for the lesson and the in-class exercise. The planning sessions for this cycle started on February 27 to March 1 and the intended day for delivering the lesson was March 2. The lesson study team met for three school days; each meeting lasted for 45 minutes to one hour, and was used to discuss ideas for delivering the lesson and refining examples that support student’s learning while infusing our intended goals. It is important to note that not all of this time was spent planning, as a portion of time was reserved for debriefing the observed lesson on that day as well.

The lesson study team felt that the previous lesson study cycle had a lot of content in it. So, in this cycle, we decided to avoid that. We decided to divide this lesson into two classes, one class for Derivative of Trigonometric Functions and the second class for Derivatives of Logarithmic and Exponential Functions. At the end of these classes, we planned an in-class exercise that covered all the learned rules so far and including a variety of questions.

4.3.1.1 First Class – Planning

For the first class, there was a difference of opinion with respect to how Derivative of Trigonometric Functions, namely $\sin x$ and $\cos x$, should be taught. I suggested using a graphing tool where the students could use their previous knowledge of averages and instantaneous rate of change, which they had learned in the last semester (Advanced Functions) to illustrate the rule; this was a visual strategy.

Liam, however, suggested using The First Principle of Derivatives taught this semester. This was an abstract strategy. Ali approved of both methods but did not favour one over the other because, in the end, both ways used the deductive method.

In both methods, students should be able to come up with the correct rule. Liam decided on using the abstract method as this would incorporate the broad goal “transition to university”.
He explained that, in university, there is a lot of theory, so it is important that students become more familiar with it. For the remaining of Trigonometric Functions, students should be able to state all the rules using identities they learned in the previous semester.

After planning the lesson, we started thinking about how to increase student engagement. One suggestion was to assign each student a question from the practice to solve on the board, but it was remarked that students were still not confident in doing this. Liam suggested “maybe I will put one question on the board and say here try this, so maybe we should bring that a little bit of the thinking process” and use a more student-centered approach to engage them. Afterward, he would circulate the room to review their work and guide them through the process.

4.4.3.1.2 First Class - Observation

The first lesson for derivatives of trigonometric functions was delivered on March 2. The lesson study team could not be present on that day due to other commitments, and Liam was unable to make a video recording of the lesson. For the second day of lesson, March 5, the lesson study team was present, and the lesson was video recorded. There were four students absent on that day. Liam had explained the derivation of the trigonometric rules in the first class, so part of this class was a continuation for using these rules and the derivative rules to solve a variety of planned practice questions. The other part of class was intended for derivatives of exponential and logarithmic functions.

Liam started with an example from the previous day that involved trigonometric functions. He asked for the derivative of tan of an angle. One of the students answered but it was incorrect, so Liam paused, reminding students that they needed to memorize the rules. When he repeated the question, another student answered correctly. He asked students questions as he solved the question on the board. When he asked a question and received no response from the
students, he reworded the question or gave them a hint. He made sure to give students enough
time to copy notes from the board.

In the next example, Liam started with a question: “which rule would you use?” to
remind them that all rules are connected and that the focus in this lesson was on the derivative of
trigonometric functions. However, the students still needed to implement the derivatives rules as
well. Liam intentionally made a mistake in the product rule and asked, “is this correct” and one
student said no, so he asked, “then what do I do” and he wrote as the student told him, but it was
still not correct. Another student said product rule and Liam said “thank you” as he continued
solving the question on the board. He reminded the students in the case of products between
functions, we must use product rule.

Liam wanted the students to select the appropriate rule when solving problems, so he
intentionally wrote the wrong solutions to probe their thinking. At certain times, Liam addressed
two students rather than the whole class as they were engaged in the lesson and responded to
most of his questions. He was also noticed circulating around the class to ensure students were
focused and copying notes.

4.4.3.1.3 First Class - Debriefing

For this lesson study cycle, debriefing was conducted two days after the implementation
of the lesson. The lesson study team was familiar with the interview questions that were audio-
recorded. I started by asking Liam what went well in the lesson. He replied:

For trigonometry, it was pretty good. I think it is a good idea to start with the sine and
then we did that already in the beginning so then once you have that, you continue with
the cosine. For them, they know what to do with the cosine then we have the two
fundamental trig ratios and then we go to the derivative of the other trig ratios. I think in
their heads they already know what to do with them, … I think it is good to use the
quotient rule, the various rules … I am actually pretty happy that they actually
remembered some of the identities, so they know how to apply them. The only thing is, I
think they need more practice because for example we had one student, I think Danny,
who sees the quotient but still thinking that is the derivative of the top and derivative of the bottom, so they need to practice more. (Liam, interview, March 5)

When asked about how he would evaluate the lesson and student understanding, he said:

It is by student response, especially because I have a smaller class, so if one third to maybe about half of them gets it, it is pretty good because some people requires practice… I would be able to assess them during the time they are doing the [in-class] assignment. I will see how fast they do and how they are able to apply the concept; that is how I would know that they get it. That is why we have assignments so that I have a first raw mark saying okay I think they are okay now if they have difficulty with those then we start thinking into it then usually I spent an extra day to point out a few things. You have to do this, you have to do that but most of the time, I think they are pretty straight forward. I do not see them having much of difficulty. (Liam, interview, March 5)

He added that he tries to give them tips for processing questions without the need to solve them, and implements this strategy in class before solving a question:

So, one of things I have taught them that when you see the question you have to be able to do it mentally. You have to see the question [and understand], okay I know what to do so I do not have to actually write it down and if I do not know, I take a piece of paper and write it so that is how you study for university. You have the whole textbook, you cannot do every single question, you cannot do number 1 to 58 for that one section, so they have to be able to skim through it, read the question, quickly process in their heads oh okay I know how to do. I know the step [but] if something you are not sure, you do the question to see if you get the same answer within the text book. That is something we have to train them to figure out, see the question by inspection. Knowing what to do right away and you do not have to actually try it because that is going to help them with every other type of problem like integration, [and] differential equation in the future. (Liam, interview, March 5)

Liam started to notice that students’ engagement is improving gradually, and he liked the idea of posting a question on the board and allowing the students to discuss because it encouraged participation.

As an observer, I liked the idea of writing wrong solutions on the board because it shows which students are focused and tests their knowledge of the rules. The mistake that Liam made intentionally was a common mistake that students make when doing derivatives. Ali agreed with this as well.
4.4.3.2.1 Second Class - Planning

In the second class, we agreed to use the same strategy as the first class. The first principle of derivatives was used to show the rules for derivatives of exponential and logarithmic functions. The team refined the examples used to explain the rules and to reflect on the intended goals. This was done to support student learning.

For the in-class exercise, Liam discussed the practice questions that would be used, and we agreed that it should include a variety of ideas from this lesson in addition to those from previous lessons. It should be a comprehensive practice for all the rules learned thus far, and should cover the four categories of knowledge, application, inquiry, and communication with varying levels of difficulty.

Ali suggested that Liam prepare a practice sheet for the students to complete during the March break. The sheet should only have one question “just find the derivative, do not do anything else, all mixed up”, so that the students are familiarized with choosing the correct Derivative rule. Liam added “recognize quotient rule, recognize product rule, recognize this is composite function, recognize you have to do a chain rule before first before you do product rule or quotient rule, those mixed things is very very important. That’s why I will be doing it”. Liam then mentioned that he will be using this method with his students while doing the examples: “first I ask them what you see here, do you see this as a product rule, do you see this as quotient rule, do you see this as a function of a function?”

4.4.3.2.2 Second Class - Observation

Liam started the second class for the derivatives of exponential and logarithmic functions by asking the students “how can I find the Derivative of this \( y = a^x \)” and one of the students quickly answered “\( xa^{x-1} \)” using the Rule for Power Function. Liam responded with: “does it
work on that, do you think?’. The student replied “of course”, to which Liam replied, “you see, I am not sure, this is you guessing?” Liam tried to lead the student using questions to direct his thinking towards a method that would allow him to find the rule. He continued with, “if I want to find derivative and I have no rule like for $sin \ x$, we had no rule to check your answer, what would we do?” and then the students said, “the First Principle of Derivatives”. This was the intended strategy to prove the rule.

Liam followed the lesson plan as discussed. He paused where necessary to ask questions and to allow students to copy his notes. Liam gave practice questions that covered all of the rules studied so far in addition to the newly introduced rules. We wanted to ensure the students’ ability to choose the appropriate rules of derivatives when solving problems. Four students were given a question to solve on the board and to explain the procedure they followed to the rest of the class. When the student made a mistake, Liam refrained from giving the correct answer, but rather told them to revise their work. When the student used the wrong derivative rule, he would ask which rule was used to guide them towards discovering their mistake. At the end of the class, Liam asked the students to finish the practice questions before the next class so that they could check their work when he reviewed the responses.

The next class on March 6 was a continuation lesson during which students worked on the practice questions. Liam started by solving several of questions on the board, repeatedly asking the students, “what would I do with this?”, giving them the opportunity to suggest a rule that could be used. He followed with questions regarding how to proceed from one step to the next, sometimes offering two possible solutions to help the students make the connection between the current lesson and previous learned concepts.
For the rest of the period (45 minutes), Liam instructed the students to work on the in-class exercise in the handout. He circulated the room to check their progress. After a few minutes, he assigned questions for each student to solve on the board. Students were observed working individually and asking Liam for clarification. He refrained from providing direct answers, but rather tried to guide students in discovering their own mistakes. In cases where students struggled to identify their mistake, he asked them to explain what they did, attempting to lead them to the correct response.

Later in the class, Liam asked the students to challenge him or the lesson study team with a question with condition that it was related to the class topic. He meant it as a joke, but one of the students took it seriously and started searching for a question online. The question he found related to what they were currently learning in addition to a different concept from an upcoming lesson. Liam took the opportunity to provide an early introduction to that lesson. The student turned to Ali for direction instead of asking Liam, so Ali started solving the question on the board and the students followed along with the steps shown in the online answer. I felt that this played out as an effective icebreaker activity for the students and the lesson study team.

**4.4.3.2.3 Second Class - Debriefing**

Reflecting on the students’ understanding, Liam felt that the abstract approach of proving the derivatives of exponential functions was unnecessary and agreed to use the graphing strategy next semester: “I think even in university they do not really care about limit (which is the first principle of derivatives) too much for the exponential, they care more about the derivative part of it. For the logarithm and exponential, I think it is fine I mean just given them the limit …… they do not really [need] to do any work on that, I think that is good.”
I asked Ali about the students’ engagement throughout this class and the previous class, and he observed that he noticed an improvement in their engagement: “today was very good they went to the board to solve questions [although] we pushed them, it was wonderful”. He liked that students were conferring with each other to solve the questions. Ali also enjoyed when students tried to challenge the lesson study team:

I liked it, the way they challenge us and we challenge them, they gave me let’s say more opportunity, more wide angle to think about. They were thinking which question I should challenge them, this way they were searching a question, they know the style of question, different questions even though he gave me the question, which is not in the current lesson, but they are going to take it. So, at least they get an idea, so the engagement was beautiful all of them participated. (Ali, interview, March 5)

He suggested that, for the next cycle:

Instead of each student do one question, each student can do two questions or next time, let’s say, we can ask student A to give student B a question, student B will give student C a question, student C will give student D [and so on], so something like this, let them play with the question as if… it is a game. (Ali, interview, March 5)

Liam agreed that it is important to engage students prior to giving an assessment, as it allows him to gauge their skill level and guide them accordingly.

I noticed that there were two students sitting at the back who rarely participated when Liam addressed the class with a question, so I wondered whether they were simply shy. Liam and Ali agreed that they were the weakest in the class. I pointed out the possibility of moving each of them next to a stronger student who could support them when they encounter difficulties. Liam noted that he has tried this strategy in the past, to no avail. Instead, he resolved to try and take them out of their comfort zone by calling on them directly.

After we were finished debriefing, we started discussing the next cycle of lesson study, “Implicit Differentiation”. 
**4.4.4 Third Cycle of Lesson Study – Implicit Differentiation**

In the previous cycles, rules were used to find the derivatives of explicit equations. In this cycle, students are using all rules they have learned thus far to solve a different type of problems, implicit equations.

**4.4.4.1 Planning**

The specific goals for Implicit Differentiation are:

State the difference between explicit and implicit equations; investigate the derivatives of implicit equations; make connection between rules of derivatives and implicit differentiation.

Students are expected to:

Solve problems, using the product and chain rules, involving the derivatives of polynomial functions, sinusoidal functions, exponential functions, rational functions.

(The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 104)

The planning for this cycle took two days, and lesson was delivered on March 8. This lesson was considered an extension of derivatives, as all of the previously taught rules are used in a different context. The lesson study team thought it was important to teach it to students. It falls under the broad goal of helping students transition to university, since 8 out of 9 students were taking Calculus as a prerequisite for their intended university program. The students had an idea of implicit differentiation, as it was a component of the challenging question that they challenged the lesson study team with in the previous lesson cycle. At that time, Liam explained the difference between explicit and implicit equations.

The lesson study team started discussing the strategy for introducing the lesson. Liam suggested “first explain… what is the difference between explicit and implicit…. then I am going to say when we do implicit, I am going to review the chain rule one more time”. He planned on following by explaining the process of finding the derivative for implicit equations.
The lesson study team agreed that this was the best approach for this lesson. We suggested that Liam solve several questions for illustration, and then devote the remainder of the period to allowing student to practice as a means of increasing engagement. We worked on refining the examples that should be used for illustration. The practice questions needed to be comprehensive and include all of the rules of derivatives and derivatives of trigonometric and logarithmic functions.

4.4.4.2 Lesson Observation

The lesson was delivered on March 8th. Two students were absent. Liam began the lesson by explaining the difference between explicit and implicit equations, and used examples for illustration and comparison. He reminded the students about chain rule, as it is considered an application to implicit differentiation, and he followed by explaining the procedure for finding the derivative of implicit equations. Liam posted some questions on the board and started asking students “what do I do next?” to ensure that they were focused and understood the concept.

The rest of the period was designated for the in-class exercise. Usually, Liam allows the student to work individually or in a group, however, this time he divided the students into pairs and assigned a question to each group. One student was left to work on her own as her partner was absent. Liam was observed circulating between tables and looking at student’s work. He insisted that the students work in pairs and use each other as a resource for checking each other’s work. He refrained from giving answers away, instead questioning the students to lead them to the correct answer. Liam tried to support their learning experience to allow them to become individual learners.

When Liam was busy with a group, I noticed one pair asking another student to look over their work. The student pointed out which steps he thought were incorrect. Afterwards, Liam
checked their work, and the students asked him several times for clarification. When Liam noticed a couple of groups had already finished, he called a member from one of the groups to present the solution to their problem. The first pair solved the question on the board. By that time, all students had nearly finished solving the question themselves, and started following the solution being written on the board. One of the students commented, “take your time”, as he was not in favour of writing on the board.

Liam was observed helping the student who worked individually. He was asking her leading questions and giving hints to guide towards the correct answer, responding to her question while reminding her of the rules. There was not enough time for all pairs to do their questions on the board, so Liam got chart paper and markers and asked one of the pairs to write their answer on the chart paper, which he posted on the walls so that other students could see their solution.

4.4.4.3 Lesson Debriefing

I started by asking Liam about the day’s lesson on Implicit Differentiation. He felt that the students understood the lesson. He liked the idea of putting them in pairs as it helped them “discuss among themselves and try to do the work and then I can go around and check, see what their processes is”. He agreed that pairing the students allowed them to discover each other’s mistakes, and that this in itself was a learning experience. Looking over the students’ work alerted Liam to their lack of practice: “I think they still need to work on the different, to differentiate between chain rule and product rule and how to use product rule. I am still kind a disappointed that Danny still has a problem with product rule, he remembers but when he do the question he forgets”.
Ali replied to Liam’s comment by pointing out that “it is the beginning [and] those are common mistakes”, and Liam agreed. Afterwards, I asked Ali how he found the students’ engagement. He said “the engagement I think it is good, excellent the activity at the end, each [pair] to write his own solution and display it, yeah, it is very nice idea… it saves time, a lot of time this way and all of them get to know everything and they work on it”. Reflecting on the day’s lesson, I think it was successful in implementing all of our four intended goals; increasing students’ engagement, helping ease students transition to university, and achieving goals specific to the curriculum and the lesson.

4.4.5 Fourth Cycle of Lesson Study – Preparing Discussion Task

4.4.5.1 Planning

Liam came up with the idea of Discussion Task as a type of assessment. Students are assigned questions to work on in class and at home then they do a presentation explaining the procedure to solve their question. The questions usually cover all the ideas in a specific unit. The Discussion Task is a student-teacher conference for recording student achievement in that unit “Rate of Change”. In addition to our intended goals, there was another goal behind having the Discussion Task in particular, which was to empower students through their presentation and to provide insight into students’ understanding at the end of the unit.

My role was to moderate between Liam and Ali, as this was an assessment of learning. I encouraged them to include all of the specific goals, in addition to long-term and broad goal when they were selecting questions. The planning for this cycle took was completed during a 45-minute session, as Liam had several Discussion Tasks from the previous year.

Liam and Ali agreed to choose from these questions, and to add some new questions for this year’s Discussion Task. The questions were designed to match the desired goals and to shed
light on students’ thinking and understanding. In this cycle, the questions varied in level of difficulty from medium to difficult. Liam and Ali decided to give one question to each student to work on individually. Instead of students asking each other questions, Liam suggested the students audit each other’s work, in order to ensure that the answer is correct to the best of their knowledge.

4.4.5.2 Lesson observation

It was originally intended that the Discussion Task would take place on March 26, but Liam gave it on March 20 instead, in order allow students sufficient time to prepare. The students had the opportunity to collaborate with each other next day in class to, and took the weekend to work on the problems. They had the chance to ask Liam for guidance in solving the questions before presenting their findings. He distributed the questions for the Discussion Task asked the students to select a question from the list. Liam wanted to help the students choose the problem that would best suit their skills. He was allowing them to work at their own level, developing their skills and aligning with their understanding. If two students chose the same question, he would choose which student to give it to or ask one of them to change.

Liam selected a different strategy. Instead of working in pairs, he decided that students should work individually, and then allow students to look over one another’s work to correct and refine their answers. Liam addressed the class: “so you will do the first part of judging and then I will do the second part of judging”. He also instructed them to write the answer on chart paper to save time during their presentations, and then posted them to the classroom wall so that the rest of the students could review them.

On the next day, students were working on their questions individually. Liam was available to take student questions. He encouraged them to collaborate and to learn from each
other. On presentation day, the team noticed students working attentively as they wrote their presentation on the chart paper. Other students were seen reviewing their classmate’s work. Most of the students were not able to finish on that day, so Liam postponed the presentation until the next day. Liam chose one student at a time to present. Students began their presentations by posting their chart paper that included the outline of their solution. Liam listened to their presentation asking for clarification and details to make sure they understood the concepts. They wrote on the board when Liam asked them for a more detailed solution.

For one presenter, Liam suggested supporting the presentation with a graph, as this would make it easier to see the different cases in the question, and then asked the student to graph it on the board. He tried to guide another presenter to discover his mistake using leading questions. Not all of the students were able to present on the same day, as some presentations lasted for up to 20 minutes. Two students were left to present the next day.

4.4.5.3 Debriefing

I asked Liam about the Discussion Task and level of difficulty of the questions. He mentioned that he is looking for the students’ process when solving the questions: “when I judge it is not only based on what I see but also during the class when they are working”. So, he takes into account the questions they ask, and considers whether they involve concepts taught in this course or in past courses. Regarding one particular student, he commented:

He will get marks off probably on the application because he has trouble in factoring. Even though, we were guiding him, but he had most of the concept correct, so he knows to take derivative, he knows. He already had a b-intercept in the notes, he knows how to take derivative and he knows how to make the slope the same, … [the] concepts are there. (Liam, interview, March 26)

For another student, he noted:

He actually had the idea, he keep on talking about tangent, but he did not really make that connection that the tangent is 90° but he knows… when he draws, he knows it is a 90°,
but he could not really verbalize it, so in that case I would just take a little part off for communication. (Liam, interview, March 26)

Then he added “so, that how I usually deal with my discussion task. Even though there is a range of difference, I think it is pretty good based on that”.

Afterwards, I asked Ali about his observation for the Discussion Task. He replied that he encourages this type of assessment, as the students are able to learn from each other and present their findings. He added that as discussion and collaboration between students increases, their confident in mathematics increases as well and they learn how to engage with their classmates.

4.4.6 Fifth Cycle of Lesson Study – Related Rates

This cycle is about using the rules of derivatives in real-world applications, where the questions entail finding the rate of change of moving objects relative to time, as time represents the main factor of change.

4.4.6.1 Planning

This cycle is part of the second unit in the curriculum “Derivatives and their Applications”. The Ontario curriculum goals for this cycle are as follows:

Make connections between the concept of motion (i.e., displacement, velocity, acceleration) and the concept of the derivative in a variety of ways; [in addition to] solve problems, using the derivative, that involve instantaneous rates of change, including problems arising from real-world applications, given the equation of a function; [and] solve problems arising from real-world applications by applying a mathematical model and the concepts and procedures associated with the derivative to determine mathematical results, and interpret and communicate the results. (The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 106)

4.4.6.1.1 First Class - Planning

This cycle involved two lessons: one that involved an application about velocity and acceleration and another that involved more complex problem solving. Planning for the first class took place over two 40-minute sessions held over two days, beginning on March 27. It is
important to note that the audio recorded in this cycle was lost. I accidentally recorded over it, but both the artifact produced, and the video recording made during classroom observation were available.

The lesson took place on April 5. This lesson represents the application of the first unit “Rate of Change”, and it involves many physical concepts regarding motion of an object. Liam suggested making the connection between their previous knowledge of Physics learned in grade 11 and derivative as a concept. Ali agreed that this would also assist students in reconstructing their knowledge into a new concept. The lesson study team started working on creating examples and practice questions that included a variety of problem solving.

Some of the problems involved calculating the rate of change of radius as a balloon is inflated using air pump. Another example involved calculating the rate of angle of elevation as a toy rocket is launched, and another the rate at which the distance increased between the runner and the home plate at a softball game. The plan was for Liam to explain two examples, then give the students the rest of the period to work independently.

4.4.6.1.2 First Class - Observation

The lesson took place as intended on April 5. Liam started discussing with the students that this unit has a lot of physical concepts and began the lesson by asking students to show hands who took Physics in grade 11 or 12. He wanted to make sure that students had enough knowledge about velocity and acceleration and their connection with displacement that he could use the concepts as a starting point for the application of derivatives. Two students did not take Physics in grade 11, so Liam had to adjust his explanation to include more details.

He started by defining common symbols that would be used as “s” for position, “v” for velocity, “a” for acceleration and “t” for time and the unit for each symbol. Liam followed that
by explaining the relation between velocity, acceleration and displacement. Then he explained that “math and physics are very different… math is abstract, does not have a physical meaning to it unlike physics, it is linked to a physical world”. However, he continued, we are now learning about related rates, which are connected to physics, so the math will take on a physical meaning.

Liam began with a very simple example about a moving car and explained the difference between average and instantaneous rate of change and their relationship with derivatives. He then explained the relation between position, velocity, and acceleration. He also clarified that, in these problems, solving the question will entail determining “how fast” something is moving, or “rate at which something is changing”, and there would be no need to derive as in previous lessons.

As he proceeded with the lesson, Liam asked students about different physical concepts and their connection with rate of change. The goal was for students to construct their knowledge by developing new concepts. Liam paused from time to time, allowing students a chance to copy notes and ask questions. One student asked for clarification about regarding how velocity could be positive at the same time as acceleration was negative. Liam explained that when an object is slowing down, “it is like friction, and it will be against the motion”. The team noticed that some of the students, who have not taken physics previously, were a bit confused and needed more time to understand the physical aspects. For this reason, Liam did a couple of more examples than we originally planned, as he felt it was necessary to adapt to student needs. As he spent more time clarifying these concepts, there was no time left for students to attempt questions on their own. He assigned two as homework.

The next day, two students were absent. Liam reviewed the questions he assigned as homework, asking students for each step to ensure they understood the concepts and followed
up with him; this took 15 minutes of class time. Then, he proceeded with the second class in this cycle.

**4.4.6.1.3 First Class - Debriefing**

There was one debriefing for both classes and it was conducted after the second class.

**4.4.6.2.1 Second Class - Planning**

Planning for the second class took approximately three 30-minute periods scheduled over three days starting on March 29. The intended day for the lesson was April 6. This lesson is a continuation of the applications on rate of change. The challenging part in this cycle was how to introduce the lesson, because each question would have a different approach depending on the application. Unlike the other units, there was one rule and all of the questions were focused on how to implement this rule.

The only variable common to previous lessons that students needed to keep in mind when solving questions of related rates is time, which is an unseen factor, but plays a role in every motion question. We decided that Liam would start by defining “related rates”, and then provide an example for illustration, highlighting important ideas within the process of solving. Liam would then provide the students with a variety of questions to find rate of change with more complex questions involving volume, radius, and angle of elevation.

**4.4.6.2.2 Second Class - Observation**

The second lesson of this cycle started 15 minutes after the class on April 6. Liam started by distributing the handout for related rates questions. He commenced by defining related rates. He asked two students to stand up and move, explaining that each had a rate (velocity) and when they are moving the distance between them changes as a result of their rates. He was hoping that
this visual demonstration would remind them that time is always a factor in these application questions.

He emphasized that, by stating “when Asem walking, distance with respect to “$t$” when Danny walking with respect to “$t$” so the distance between them changing is also with respect to “$t$”, they are all changing with respect to time, keep that in mind, everything is in respect to time”. As planned, he proceeded to solve an example on the board. Liam could be observed addressing the whole class as he explained the application question, listened to their responses, and occasionally challenged them to explain the physical meaning of their findings.

Based on the observation, the lesson study team felt that was too soon to give students questions to work on independently, as we sensed that they were encountering difficulties answering the questions. For this reason, another day was allocated for Liam to solve more examples before students would be asked to attempt on their own.

**4.4.6.2.3 End of Cycle - Debriefing**

The debriefing was done for both the first and second class at the same meeting. Ali started by saying today’s lesson was interesting, as he felt that:

students were participating because it is connected between reality and math sometimes. Most of the times, they are studying math like x and y and whatever it is, abstract values but here you connect to real life situations, going forward, going backward and students were able to answer most of the questions. (Ali, interview, April 6)

Liam agreed with Ali:

They are able to answer questions, I think overall today the lesson is not too bad it was not really rushed, we had a lot of time to discuss some basics and then I was able to convey them my idea of math and reality in a way so they understand math can be unlimited but physics you are kind of limited the physical aspect of it. (Liam, interview, April 6)

However, Liam expressed concerns regarding three students:
You got people in the front who always answer … but there are still a few students in the back who does not participate as much and I actually have to find a way … we cannot force them but they can have an answer and the whole class just dragging on, you got people in the front who always answer … to figure out a way to make them more up there. (Liam, interview, April 6)

I suggested that maybe they lacked motivation and we needed to encourage them more, but Liam disagreed with me, speculating that they may simply not like math. He followed by saying that one student, A, only required a grade of 70% in this course, which may contribute to their poor engagement. Ali mentioned that student B “is new to the country … his English is still weak that is his problem with any conversation”. Nonetheless, the lesson study team agreed that we needed to help them participate more in class and with the math activities.

When I asked Liam what he would do differently next time he teaches the course, he said:

I might want to give more of a physical example for example bring a spring in or bring in some kind of prop, so they can see you know moving right and left, maybe even include a video into that. At this moment in their head, they can think physically what a moving car is, moving things. I think it is not a big deal, but it would be nice to have that. (Liam, interview, April 6)

After finishing all the lesson study cycles we agreed upon, I interviewed Liam and Ali to examine their participation and identify any challenges they encountered as members of the lesson study team.

4.4.7 Sixth Cycle of Lesson Study – Optimization

This cycle was also about applying the rules of derivatives in real-world application questions by finding the optimum (maximum or minimum) cost, profit, amount of material used, … etc. It is part of the second unit in the curriculum “Derivatives and their Applications”. The specific goals in the Ontario curriculum for this cycle read as follows:
Solve optimization problems involving polynomial, simple rational, and exponential functions drawn from a variety of applications, including those arising from real-world situations. In addition to, solve problems arising from real-world applications by applying a mathematical model and the concepts and procedures associated with the derivative to determine mathematical results and interpret and communicate the results. (The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007, p. 106)

4.4.7.1 Lesson Planning

This cycle involved many applications and every question was different. Unlike other parts of the course, but similar to the related rates component, there is no specific format of solving the questions. Every question introduces a different scenario and hence a different equation. Planning for the first class started on April 16 and took place over three 30-minute periods held on consecutive days. The intended day of the lesson was April 23. Liam started by mentioning that this was not the first-time students were finding a maximum or a minimum. In grade 9, 10 and 11, for example, they were asked to find the largest area, or maximum height reached (vertex) or maximum revenue, so they were somewhat familiar with the concept. At that time, they used an algebraic approach to find the answer, whereas now they needed to use derivatives to find the maximum or minimum. They were being introduced to a different and faster approach to solve the question.

We agreed that this was the best way to start the class then follow this with a question about the importance of optimization and their field of study, thus integrating our broad goal to get students interested in mathematics. From our experience, students tend to encounter difficulties in this lesson for two reasons. First, unlike in grade 10 and 11, the mathematical model is not provided, and they are asked to make the connection between the given information to produce the equation. Second, there are no fixed steps for solving the questions.

It is difficult to plan a specific procedure for the students to follow, so we agreed on solving two to three examples on the board to model how to develop a strategy for solving
optimization problems and make connections between the given components and the required components. It was critical to emphasize the importance of listing the given conditions, which Liam called the constraint function, and those required to optimize, which Liam called the objective function, as a first step to solving the questions.

Liam had a bank of questions as he had taught the course before, so the lesson study team worked on selecting the in-class questions that meet our intended broad goal through applications geared towards student interest or university program they had applied to. The questions ranged from engineering (as minimizing material used or stress experienced in a building) to economics (as maximizing revenue or minimizing cost) to medical applications (as maximizing drug delivery after injection). The lesson was planned to cover two to three classes depending on the performance of the students, one class for the illustration and developing strategy to solving optimization problems and the other two classes for the in-class practice for the students to work on individually or in a group.

4.4.7.2 Lesson Observation

The lesson was demonstrated on April 23. Liam started by reminding the students that, if they had studied mathematics in a Canadian secondary school, then they have already explored simple optimization in grades 9 to grade 11. He mentioned the different applications that they may have studied in an attempt to jog their memories, and he asked why it is important to employ optimization. The goal for the question was to engage the students and help them make connection between mathematics and different fields in life. The students began offering different examples about what could be maximized and minimized and for what reasons while Liam listed them on the board. A couple of students were not engaging, so he asked them directly about their field and what they would optimize in business and computer science.
Liam followed with a simple example to guide the students in developing a strategy to solving these types of problems. He tried to remind them of previous methods they learned in grade 10 and 11 to find the maximum (vertex). Then he made connection between rate of change and finding the maximum. As he proceeded, he was observed repeatedly asking students about the given and the equation that connects them to what is needed to maximize.

Liam asked strategic questions to guide the students in solving the question. He commented that as he is asking them, the students need to be asking themselves the same questions when solving optimization problems on their own. He explained that the difficulty in optimization arises from finding that the equation will differ from question to another depending on the application. Sometimes there would be more than two variables in the question, but the students needed to reduce them to two variables using the constraint function.

To engage the students, Liam assigned an example for students to attempt on their own and asked them to find the constraint function based on the given and the objective function based on the required. He allowed time for students to reflect on the question before discussing their findings and the solution. Liam followed the same procedure for another question. Class time allowed for solving only three applications, so the next class was for the students do the prepared in-class questions handed to them at the beginning of class.

For the second class, the LS team members were not able to attend the classroom observation. LS team members were present for the third class, held on April 25. Liam started discussing a question that was left from the previous class and that students encountered difficulties in finding the objective equation to maximize. He was observed asking the students leading questions to aid them in making the connection for finding the objective function. One of the students asked if he could use a different equation to solve the question rather than the
equation Liam provided. Liam clarified that, while the equation the student suggested is correct, it would not help them find the answer. He explained that the key is finding the equation that connects the constraint with the objective function (required) and not just any equation that works. This was the last in-class practice question.

4.4.7.3 Lesson Debriefing

The debriefing for this cycle was completed on April 25. Liam felt that the students had “grasped the concept” and were able to identify the constraint function, but had problems with the objective function:

the only problem they are having until now is, how to relate them if they were simple cases. Some are easy but if it is a harder case like the one we did today, the geometry one, they start with x and y, they could not relate the x and y. They had to switch to \( \theta \), so I think that comes from practice. The more questions that they do, the better they become, but the process is there. They know that you have to take the derivative for extreme, critical number and they know to do the first derivative test whether it is a maximum or a minimum. So, I think most of them, I am not saying everyone, but I think most of them are getting it. (Liam, interview, April 25)

Ali noticed that students understood the concepts but needed more practice. He observed that some of them were having difficulties because they did not remember the mathematical content learned previously. He explained that mathematics in particular may differ from other subjects as the curriculum is built on previous knowledge. As a way to remind them, he suggested giving the students 30 more practice questions to do at home that include different ideas. Liam said that it will take a long time for him to prepare the questions and that students are unlikely to complete them at home, as no marks can be awarded for completing homework according to the Ministry guidelines. Liam agreed that they needed more practice. However, instead of giving 30 questions to complete at home, he would prepare an in-class assignment with 10 questions. In this case:
[It] is a positive learning experience for them which I encourage. We got student A, B, C, and they will work individually then they will correct each other. If they are not sure, they come and ask me, this is perfect. But, there are other students who work in groups because they do not have enough confidence and if they are unable to solve the question, they ask student A, B or C or me. (Liam, interview, April 25)

Ali agreed that this was better than his idea, as it allowed Liam to identify students’ weaknesses on the spot, correcting and guiding them as they work.

4.5 Liam’s Reflections

At the end of the study, I interviewed Liam (Appendix D) to reflect on his participation in lesson study. He mentioned several benefits of planning lessons through collaboration and classroom observations that helped him to be reflective on his teaching through constructive criticism. He said it “spiked more ideas, more trying different things, trying to put them into perspective or putting to logistic way of delivering a sequence of materials, it was actually very interesting”. He also said it helped him in implementing different ideas and delivery instruction:

I was able to explore different method of teaching, engaging students during the class in learning. Also, we shared the ideas on teaching. It is a good idea to get the teachers together to plan out a long-term and short-term goals, and to develop a better assessment plans so the school will have a fixed system to ensure the learning continuity from different teachers of same courses or different level of courses. (Liam, interview, May 15)

Liam felt that being part of the lesson study team is beneficial as he explains:

I mean a lot of constructive criticism, so sometimes I have to look into it and then I have to sort of make a decision whether I should change it or not because as a teacher sometimes we resist the way of teaching. Oh, I am happy with the way I teach - why would I change it, but sometimes when we do this extra step, you actually see the difference in student learning. So, the challenge I have to change the way I think about things, change the way I teach, it is still benefiting students. Also, I never really tried the poster thing but this time I am trying it. The good thing about it, I can actually do a judgement on that as well you know, like evaluation based on this, I think there is a lot of benefits, changes the way I look at things slightly differently. (Liam, interview, May 15)

But challenging at the same time as it requires from him to change the way he thinks and teaches certain topics, which he recognized not to be an easy thing.
When asked whether the observer remarks helped him to “see the unseen” when busy explaining or attending other students, he replied “it sorts of guided me in terms of focusing on different needs of the students like what is lacking, put my lesson in different perspective”.

Liam understands that being part of a team is challenging for the teacher:

Because, we look at things differently. So, Ali will talk about for example the rules, he will tell them to take the chain rule but, I do directly with the \( x \) and I teach them chain rule based on that because I want them to understand the fundamental instead of memorizing the formula so that is a little bit different. I do not know which is more effective. I think both are okay depending on how students [learn] but the thing is when I teach them I did not say you cannot use [that] rule. You can use it, if you can use it well, [or] you can use mine, that is the only tiny things. (Liam, interview, May 15)

He said part of the challenge was that lesson study is time consuming:

On top of that the final thing, I want to say just simply whether the teacher follows an expectation that alone can be a full year process, when you teach do you teach towards the curriculum or do you skip some or you do not even mention this due to time constrain, how to balance the time well. I think all of those are part of the lesson study group. (Liam, interview, May 15)

It was also challenging for the students. Liam told me that the students did not like the idea of writing on the board. He said:

They do not want to be that active, they feel that they are grade 12 already, they say going after the board doing the questions seems like the younger grade would do, grade 9, 8 or 7… so they are like “I am learning why do I have to do all this”, “I am answering question so why I have to go on the board and do the question”. That is the feedback from them. (Liam, interview, May 15)

Liam appreciated that our team consisted of three participants only:

If we have many people, a little simple instruction can be over complicated then in the end of the day becomes not how to effectively to deliver. [It] becomes who teaches better and I do not really like that. I had some discussion with other people before and it always turn out to be that. (Liam, interview, May 15)

When I asked Liam about any recommendations that he would give to better enhance his experience, he said that he enjoyed being part of the lesson study team and made two suggestions. He thought we should implement lesson study school wide:
I think this collaboration [LS] should happen at least once every two to three years and I think each class should do that once a year to sort of repeat it. For example, this year we did year grade 12, next year we do grade 11, and the year after that grade 10 and then grade 9 and every three to four years repeat back to grade 12 to review if there is anything we can add or modify. I think it is actually pretty good. I think the first time usually it takes a while but the second or third time might not be bad. For example, we did grade 12 this year pretty thorough and then next year perhaps we can just get a one teacher to go into class a couple of times in the year and just to see if the way of teaching changed, if it has changed then that is good. If it has not then we have to figure out where it is what is the problem to implement it. I think it is a good idea to have this kind of review every once or often. (Liam, interview, May 15)

He also thought that we should include more discussion about evaluation:

I think also on top of that, when we meet we talk about the teaching style because that is the thing we are working on this time, but it should be even more. It should be on the evaluation as well. It should be the lesson material so that we can have a complete picture in terms of looking in this course. (Liam, interview, May 15)

4.6 Ali’s Reflections

At the end of the study, I interviewed Ali (Appendix D) to reflect on his participation. I asked Ali about the benefits of being part of the lesson study team. He mentioned four main points. The first benefit was the huge improvement that was reflected in the students’ understanding, as the change of instructions helped them to better understand the lesson:

The benefits for the students [was that] before the lesson, we discuss the material. Three of us were adding comments to the lesson, so we improve the course content. The way we approach the students is different now. Each one will have an idea then we decide what idea to go through plus the way to deliver the material this effect the students positively. (Ali, interview, May 15)

Ali felt that the second benefit was classroom observation, as the team is able to notice students responding to different ideas thus guiding the teacher with ideas in the coming lessons. The third benefit is the pre-planning of the lesson with set goals that improve the teacher’s experience by practicing new ideas and adjusting his ordinary practices to develop a novel approach:

It is helpful for Liam, adds more ideas for him… if he is going to teach this course latter, he will use the whole advantage of what we did, … he will push the students to collaborate more, engage them more. Maybe he will use the same material and try to give extra exercise, which will not be counted towards marks, in order to practice more for
each lesson. The discussion task, those are advanced problems are very nice. I would recommend for everyone using such a style of teaching. (Ali, interview, May 15)

Lastly, debriefing is useful, as it gives “immediate feedback and useful remarks from an experienced teacher”.

During lesson planning, Ali suggested using a particular way of expressing the trigonometric rules, but Liam disagreed with it, so I asked Ali to reflect on this. Ali did not feel it was a big deal as both concepts were correct and simply employed different approaches:

This is from my experience; I got this idea from students while teaching them. It is not that Liam’s way is wrong, but students tend to forget to use chain rule so, in this way, they are using a rule. Now, Liam is teaching the class, so it is up to him. (Ali, interview, May 15)

When asked about the challenges that he may have faced being part of the Lesson Study, he replied “the challenge all of it is on the teacher, Liam. For me, the challenge was simple it was the timing, to afford the time to attend the classes”.

I asked Ali about any recommendations he may have, he said:

First of all, to start at the beginning of the semester, it is better to have more classes to attend. The other thing is let the teachers involved in the study to try some of the classes, to teach them. This is okay but maybe the visiting teacher as you to teach one, two, or three classes. I will teach one or two classes, let’s say. So, all of the team is involved in the teaching. (Ali, interview, May 15)

Ali also agreed with Liam that lesson study should be conducted once a year for one of the courses taught by each teacher, and that the administrator should have a continued role in facilitating such a professional development. He felt that lesson study promotes one of the best practices that he has seen. He felt that the administrator should help teachers in conducting lesson study by “facilitating a location for meetings; allocating time for the study in the busy
schedule; reducing the teachers’ duties; being part of the team and giving feedback to the teacher after class observation; and most importantly giving time to follow up”.

4.7 Summary

The lesson study team established broad and long-term goals. We completed six cycles in approximately three months that included our agreed upon goals. The topics were chosen from two units in the Calculus curriculum. Each lesson cycle encountered three stages that started with lesson planning followed by classroom observation and ending with debriefing. In the debriefing, the team was able to reflect on the classroom observation and incorporate suggestions while moving forward to the next cycle. After finishing the cycles, the lesson study team was interviewed to shed light on their experience, challenges that they may have encountered during their participation, and their recommendations for enhancing the study.
Chapter Five: Discussion

In this chapter, I will discuss the findings of this research using the research questions presented in Chapter One to explore the lesson study and the value that participants placed on the study. Next, I will discuss the challenges that the lesson study team faced during implementation and any limitations that may have affected this study. In addition to that, I will discuss recommendations for further research.

5.1 Research Questions

The thesis focuses on three main research questions:

1. How can secondary school mathematics teachers use lesson study to examine their practice and enhance students’ learning?
2. How will engaging in lesson study affect secondary school teachers’ instruction?
3. What benefits and obstacles did the team experience when implementing LS?

5.2 Question 1: How can secondary school mathematics teachers use Lesson Study to examine their practice and enhance students’ learning?

During the course of the study, there were several key ways in which the teachers used lesson study to examine their practice: collaboration; classroom observation; and debriefing, which led to changes in teachers’ attitude, empowered teachers, and enhanced teachers’ pedagogical content knowledge. There were also several key ways in which student learning in mathematics was enhanced: increased learning time; increased student engagement; increased student confidence.

5.2.1 Teacher Practice

The lesson study is one of the different types of professional development that is widely used in many countries. It is structured in a collaborative way that helps teachers reflect on their teaching methods through the stages it entails. It is this specific type of collaboration experienced
in lesson study that translates to instructional improvement because it guides the teacher through every stage; starting with pre-establishing goals followed by planning that corresponds with these goals, followed by observation of the effects on the teacher and students, and ending with reflection and a debriefing of what needs to be improved. This aligns with other research:

Lesson study employs “Best Practice” because the teachers are being truly collaborative. They research the topic, discuss and question others’ opinions and suggestions and are involved in an in-depth systematic examination of their practice as their focus shifts away from what is presented to what is learned. Teachers develop habits of mind of research to investigate teaching and learning. (McMahon, 2008, p. 34)

Classroom observation of the lesson study helped both the teacher and the students. The teacher examined his practice through the lesson study team classroom observations. The lesson study team remarks allowed the teacher to identify areas of improvement to the benefit of the students. The lesson study team suggested different teaching strategies that the teacher employed. This happened during each cycle, which, in turn, impacted the students’ learning. It was observed as time passed that the students started to be more engaged in class. It was clear in two ways: first, through questions posed. Second, through planned activities such as the in-class assignment and discussion tasks.

During the pre-observation in the classroom, the lesson study team attended several classes to examine teacher practice and set the goals based on the class environment. This is helpful because it is hard for a teacher to have full awareness of his or her class (Dudley, 2015).

The lesson study team acted as a second eye to examine Liam’s practice; using a different lens, they observed two main points. First, the teacher tended to use a teacher-centered approach, where he explained the lesson, completed several questions for illustration purposes, and then moved on to the next idea—a model that is often seen in a university setting. After having explained the less ideas coherently, he dedicated a separate class for student classwork, which
resembled the concept of a tutorial. In doing so, he wanted the students to have an easy transition to university by employing similar settings.

Second, students were not active, and their engagement was minimal during lesson delivery until the classwork period. A couple of students appeared interested and were answering questions when addressed by the teacher, but the rest of the class showed little interest.

Based on these observations, the lesson study team started working with the teacher to gradually shift his instruction to a more student-centered approach, increasing student engagement throughout every class, while maintaining his goal for making university an easy transition for the grade 12 students. The teacher was willing to work with the lesson study team to enhance student learning. He was enthusiastic in using different strategies and methods for increasing student engagement. He followed suggestions made by the lesson study team and made the necessary modifications to produce the researched lesson plan.

5.2.2 Enhance Student Learning

The teacher instruction shifted from a teacher-centered classroom into a student-centered one. Rather than giving them formula directly, Liam started asking leading questions for the students to scaffold their knowledge and arrive at the formula themselves, which enhanced student learning and understanding of concepts. During class instruction, he was continuously observed engaging students using questions. It was observed that the amount of discussion and questions asked during class time had increased. Students spent more time collaborating during class time, in-class assignments and discussion tasks. This suggests that student attitudes towards Calculus had improved.

The lesson study cycle increased the time students had to discuss concepts which led to increased student engagement and empowerment, as will be illustrated in the coming sections.
5.2.3 Increased Learning Time

Identifying the four goals for the lesson study at the beginning of each cycle helped the lesson study team to be more focused on the planning and lesson design that best fit these goals. Liam implemented teaching strategies discussed during the planning stage in his classroom practice. As the lesson was pre-established by the lesson study team along with all the necessary resources, Liam was provided with a clear foundation for his planning and teaching ahead of time. This allowed for a more effective use of time during the lesson and more focus on students’ engagement. It minimized his daily planning of student activities, as it was all pre-planned and discussed with the lesson study team in advance.

With this detailed plan, Liam had more time each day to reflect on the class through the debriefing with the lesson study team that followed the lesson. This appeared to support his teaching practice. Liam had become more focused on the strategies he used to deliver a lesson and the kinds of questions he asked became more student-centered, while meeting the desired goals. He felt that the pre-lesson planning was beneficial in generating more ideas and attempting new strategies that he would not have necessarily considered otherwise.

5.2.4 Increased Student Engagement

McMahon (2008) believes that a lesson study “classroom is learner centered with students learning from classmates” (p. 35), which help in increasing student engagement. Our long-term goal was to increase students’ engagement. Having a defined plan, Liam was able to reflect on the students’ engagement and, with the debriefing afterwards, Liam was able to listen to the lesson study team’s observation that he may have missed. Liam felt that the lesson study team remarks were useful in helping him “see the unseen” when he was explaining concepts or attending to other students. This aligned with the findings of Dudley (2015), who observed:
“lesson study group helps these teachers to see their pupils with new eyes and to make changes to teaching and curriculum which would not otherwise have been possible” (p. 13).

Ali felt that attending the class was beneficial for the teacher, as the team observed students’ educational behavior and thus were able to contribute more usefully to the teaching process. During the research lesson, Liam would ask specific students to answer the questions while, in the past, he would ask questions addressing the whole class, which resulted in the same students answering every time. The lesson study team suggested that he should engage all students by calling out names to ensure everyone’s participation and involvement in the learning process.

Liam was accustomed to working through a couple of examples on the board for a particular lesson; then, on the next day, he would move on to a different lesson and follow the same process. After finishing several lessons, he would give an in-class assignment that covered the concepts taught in these lessons. This strategy did not allow him to assess whether the students understood the concept at the time of the lesson delivery. With lesson planning help, he changed his routine as he started giving one or two examples for illustration of the concept and reserving the rest of the period for students to practice. Liam assigned different practice questions for the students to complete either as pairs or individually, thus focusing on students’ engagement and helping them address their misconceptions and understanding immediately.

Another strategy that proved useful was to maximize the use of time in classes reserved for working on in-class assignments. Instead of waiting for every student to write the solution on the board, students wrote the answers on chart paper and then posted them on the walls so that all students could see the solutions and record/ note them during class or afterwards, as this class
was designated for the Grade 12 courses. This, in turn, contributed to students’ engagement, as time was used more effectively and more purposefully.

Based on McMahon (2008) research, “The benefits for students are that the lesson study lessons are very engaging. They are inquiry-based, have input from many teachers and target specific learning needs” (p. 35). With the purpose of engaging the students, Liam’s instruction shifted to be more student-centered, as it involved posing specific questions which were asked in a sequence designed to guide students’ learning, and to give them enough confidence to try to solve the problem on their own. He also included varying levels of difficulties in the assessments and tasks so that the students could showcase their understanding. During lesson explanation, the questions were more knowledge-based and entailed communicating this knowledge, whereas, in the discussion task and in-class assignments, questions were geared towards applying the knowledge in different contexts as problem solving and critical thinking exercises.

In the pre-observation, students were not enthusiastic about answering questions posed by the teachers. The lesson study team noticed that the same two or three students answered in every class, the rest of the students were not eager to participate. This could have been for a number of reasons: lack of knowledge, lack of confidence, or being afraid of making a mistake in front of the lesson study team. It was important to create an environment where mistakes are allowed and discussed without the fear of being judged.

Instead of addressing the class overall for questions, Liam started asking different students and he would choose the students based on the questions and their level to give them more confidence. Initially, he started calling on students during class individually while ensuring that everyone took turns answering questions. Then, during the in-class assignment time, he requested that students work in pairs, to share ideas, and asked that one student from each group
write their findings on the board or chart paper, and the teacher would ask for further explanation if needed. In doing so, the students’ confidence was bolstered.

Students became more familiar with working together and collaborating to solve questions in a stress-free environment, without being embarrassed to make mistakes. In the last cycle, Discussion Tasks questions were assigned individually for students to demonstrate their understanding and to increase their confidence. Each student was to review a classmate’s work and identify any mistakes before they presented their work. This helped students to perceive each other as a resource, in addition to increasing their self-confidence. Liam wanted the students to build the notion of “trusting yourself” by increasing their confidence through activities and sharing ideas.

5.2.5 Change in Teachers’ Attitude

Based on the initial classroom observation of the lesson study team, the teacher followed the traditional method of teaching. Liam explained a concept followed by examples, and then posed questions to the students as he demonstrated the steps. However, he did not actually give students enough time to think about the question or solve it themselves.

It was obvious that the teacher’s attitude was changed based on the observation of the lesson study team. His practice became more student-centerered. This finding is supported by Cajkler et al. (2014), who observed that “that collaboration helped [the teachers] to develop less-teacher-centered approaches” (p. 511). He considered student engagement in his lesson plan and planned activities that emphasized that goal. Based on classroom observations, it was clear that Liam spent less time on lecturing. As a result, he became more aware of individual learning needs and this helped him to have an in-depth understanding of their knowledge. Accordingly, he
was able to adopt the lesson plan to better meet their needs while providing support in a timely manner, which aligns with the findings of Ylonen and Norwich (2012) and Cajkler et al. (2014).

He began giving students 40 minutes of class time to do practice questions and discuss them together; then, he would ask them to explain the answer on the board or write the steps out on chart paper. In one instance, he extended the discussion task one more day as he realized that the students needed more time to finish their work then review their peers work. In another activity, he understood the importance of changing his instruction to include technology to support student learning in addition to using abstract examples.

The lesson study team attended classes as a follow up that was not part of the lesson study cycle, and noticed that Liam’s attitude had continued in those classes. He seemed to have a clearer understanding of student engagement. He implemented ideas produced during the lesson study cycle. It is reasonable to infer that the collaboration experienced during the process of lesson study had a positive impact on his instruction.

Cajkler et al. (2014) found that teachers’ participation in the lesson study team led to “greater confidence to make changes and willingness to take risks” (p. 526). It was also observed that lesson study has given Liam the confidence to continue using the same instructions suggested by the lesson study team in other classes. He planned and delivered subsequent lessons more productively, and he noticed the positive impact on the students’ learning.

**5.2.6 Empowering the Teacher**

Lesson study is considered a form of professional development that is devised with the teacher and for the teacher. lesson study resembles a bottom-up approach as it is teacher-directed (Lewis, 2002). According to Dudley (2015), lesson study “creates conditions where [the teacher] feels safe to take risk” (p. 16) and try new approaches suggested by the team. Liam was involved
in lesson study and was part of every discussion and decision made. Liam had his own goals for the professional development: increase student confidence and help them to become resilient, in addition to the goals suggested by the lesson study team. The team worked together to plan lessons and discuss strategies for delivery that best suited the students and the re-established goals.

Puchner and Taylor (2006) reported that lesson study impacted the teacher confidence with respect to practicing new ideas when he or she believes it can make a difference in student learning. Liam believed that he acquired new teaching ideas and approaches that empowered him to try some of these ideas that he would not necessarily do before participating in the lesson study. In one instance, he used exploration for the students to deduce the Rules of Derivatives by asking them sequential questions that led them to the rule. In another instance, he used peer collaboration for students to review each other’s work, although he was not initially in favor of this method. It is reasonable to infer that Liam’s’ confidence level had increased. As a result, he implemented changes suggested by the lesson study team without hesitation, as the end goal was to help the students. In classroom observation, it was evident that he adjusted his strategies to align with a more student-centered approach.

5.3 Question 2: How will engaging in Lesson Study affect secondary school teachers’ instruction?

Lewis et al. (2006) remark that “lesson study improves instruction through the refinement of lesson plans” (p. 5). Other literature found lesson study to be a vehicle to teachers’ professional learning (Cajkler et al, 2014; Fernandez, 2004; Lewis et al., 2012) in which the principle aim is students’ engagement (Dudley, 2015). Further, Dudley (2015) suggested that lesson study is a dynamic approach and a key component of professional development and
learning for teachers. He argued that lesson study unearths what he calls the “black box” of the classroom, thereby improving teaching quality.

As part of the lesson study stages, the team worked with the teacher to improve his planning skills through discussion about the best approach to be taken. Illustrating examples and practice questions were constructed to support moving learners towards these goals. In lesson study, teachers meet to plan lessons based on pre-established goals. There are four goals: two of the goals are targeted towards student development and two targeted curriculum expectations. The success of implementing these goals can be attributed in part to the lesson study team, but more importantly to the teacher, because his instruction delivered the planned lesson.

The lesson study team worked with Liam to plan and refine practice questions to meet our intended goals. His lessons became more student-centered. Liam was observed frequently talking to individual students, guiding them to the answer through leading questions without actually providing the answer. He encouraged students to use collaboration and to view each other as a resource. In one case, Liam assigned particular students to review their peers’ work and make suggestions where needed before he evaluated their process. In every unit of the course, Liam gave in-class assignment and discussions task in which he encouraged and allowed students to work together. Being part of the lesson study team had helped Liam try new ideas such as using chart paper or posters to maximize learning time.

5.4 Questions 3: What benefits and obstacles did the team experience when implementing LS?

During the research, the lesson study team identified benefits associated with the study along with constraints and challenges. There were three key challenges while implementing
lesson study: difference of opinions among the team members; time for one of the participants; resistance from the students to change.

5.4.1 Benefits

As identified by Murata (2011), “different forms of lesson study provide different learning opportunities for teachers. Different formats of lesson study meet different needs and interests of the teachers” (p. 4). This research sought to implement changes based on pre-established goals that address the needs of this particular set of teacher and students, thus empowering the teacher to change their instruction and help the students gain a better understanding of the curriculum expectations.

Liam felt that being part of the lesson study team was beneficial and challenging at the same time. It was beneficial in that it helped him to reflect on his teaching through the team classroom observations, thus removing him from isolation. It was challenging because he realized that teachers tend to resist change, especially when they do not notice any problems in their teaching.

Another benefit was having a small lesson study team that consisted of three teachers, as Levine and Marcus (2010) argued the positive influence of collaboration in a small group. The small group made it easy to schedule meetings, and it reduced the time needed for planning and debriefing. However, teaching and assessment ideas were limited by the experience that each member had. Liam felt that having a small team was beneficial as it limited the ideas introduced and the discussion for lesson planning or debriefing. He added that, based on his previous experience, many ideas could underpin the collaboration and a simple instruction could become overwhelming.
Ali agreed with the benefits that Liam mentioned and added that, for him, he felt that the huge gain was reflected on the students because of the collaboration that took place in the lesson planning: adding comments to improve the course content; approaching students in a different way based on classroom observation; and modifying the way in which the lesson was delivered. All of this contributed to a positive change in the classroom dynamic. Ali also emphasized the importance of classroom observation, as it provides the teacher with useful remarks about student learning that may otherwise go unnoticed, as the teacher is focused on delivering the lesson.

Another benefit for Liam was the support provided by the school administrators. Ali, the Vice-Principal, arranged for his office to be used for interviews. Liam was released from some duties to provide him with sufficient time to concentrate on the lesson study process.

Administration support is essential for the success of any professional development as Moller and Pankake (2007) stated:

> Professional learning models are tools to be used, but the real learning happens in the cycle of conversations, actions, evaluation, and new actions that is supported through intentional leadership that gently pressures and nurtures teachers. This inquiry process must be organizationally embedded rather than externally imposed to build teachers’ knowledge and skills or increase human capital, within the school’s social networks. (p. 128)

### 5.4.2 Challenges

The lesson study team met with some challenges that were perceived in the literature as well: time (Cajkler et al, 2014; Chassels & Melville, 2009; Mon, Dali & Sam, 2016; Norwich & Ylonen, 2013), difference of opinions (Dudley, 2015) and with the students’ orientation that resisted change (Jagacinski & Nicholls, 1989).
5.4.2.1 Difference of opinions

Consistent with research conducted by Dudley (2015), conflict among teachers may occur due to difference of opinions. We were a group of three teachers with varied experience, and this variation was reflected in our opinions towards certain approaches to delivering the lesson or activities for students. One of the challenges that Liam recalled later was having difference of opinion about explaining a rule, adding a rule, or methods of delivering the lesson. This happened in three different occasions. The first time this occurred was when the team was discussing whether to provide students with a short cut rule, or allow the students deduce the rule every time they are differentiating with chain rule.

The second time was when I suggested in the second cycle to use a graphing approach for the students to deduce Derivatives of Trigonometric Functions, and Liam conveyed that he preferred the abstract process as it aligned towards our long-term goal.

The third situation occurred when the lesson study team suggested using peer assessment. Liam was convinced that peer assessment had no benefits: “we have tried that several times before, it never worked” because some students do not have the enough basis to do so or the level of maturity at that time. The lesson study team understood his perspective regarding this matter, and in the end we decided not to use it.

5.4.2.2 Time

Time plays a significant role in because it involves four complex stages: goal setting; planning; classroom observation; and debriefing. This was one of the preeminent challenges mentioned in lesson study literature (Cajkler et al, 2014; Chassels & Melville, 2009; Mon, Dali & Sam, 2016; Norwich & Ylonen, 2013). One reason for this is that the greater the number of teachers involved in the study, the more difficult it becomes for everyone to meet at the same
time. A significant amount of time is required for each member to meet to plan the research lesson, attend the class for the observation, and to adequately debrief after each lesson.

From the table in chapter 3, it is obvious that the first cycle took more time than other cycles, and this was the case for three reasons. One reason was that the concept of lesson study was not fully understood by the participants despite spending three hours in advance outlining its process. There were still many clarifications to be made as we proceeded in the first cycle. Second, as a team, we needed to identify our broad and long-term goals for the students. Third, this cycle was the foundation for the course, and we had to examine every detail of it with the consideration of our chosen goals.

Liam did not find time to be a challenge as this was done during lunchtime, which was very convenient for him. However, this affected the time he spent with his students, as in the past, students would meet with him over lunch for additional help. Ali felt the challenge was “to afford the time to attend the classes” for observation. For the planning and debriefing, he was available as all our meetings were conducted during lunch as well.

5.4.2.3 Resistance from the students to change

Jagacinski and Nicholls (1989) divided student attitudes in mathematics classroom into two categories: ego orientation and task orientation. Ego-oriented students are not willing to take part in classroom participation or try to solve a problem, as they are afraid from embarrassment in the eyes of the students or the teachers. While task-oriented, students are more into solving the task itself. In Liam’s class, the majority of the students were ego-oriented except for a couple of students who seemed task-oriented as they were continuously taking part in the discussion or answering questions.
Liam had taught most of the students previously in the Advanced Functions class and they were accustomed to his way of doing things: he explained the lesson, gave a couple of examples for illustration during which he asked a couple of questions. They noticed that Liam was no longer following this routine and that he had started to ask them to participate more in class. On the third part of the first cycle (February 27) and during debriefing, Liam told me that the students did not like the idea of writing on the board.

I had to address the class to emphasize that lesson study team role was not to judge them and accordingly, we had to think of doing things in a different way to engage them and fulfill the intended goals. We attributed their resistance to lack of self-confidence and the presence of self-doubt. After attending a couple of lessons, they got used to the presence of the lesson study team in the classroom, and did not mind answering questions on the board.

5.5 Major Findings

This study investigated the effectiveness of lesson study as a framework presented by Lewis et al (2009) to improve teachers’ instruction in the secondary level through intended goals. Through data analysis some findings were identified, these findings aligned with several studies investigating lesson study (Cajkler et al, 2014; McMahon, 2008; Dudley, 2015; Moller & Pankake, 2007). The findings can be summarized as follows:

1. Increased student engagement

The involvement of the teacher in the lesson study appears to have resulted in an increase in student engagement due to the detailed planned lessons and activities that were geared towards achieving that goal. This was evident from observations that students’ participation increased in answering questions posed by the teacher.
2. Increased student confidence

At the beginning of the study, students were hesitant to participate due to the presence of the lesson study team. They were afraid to be judged if they did not answer correctly. Afterwards, they came to understand that the lesson study team was there to assist the teacher. Although there was no formal measure of student engagement, classroom observations supported this claim. The students were more willing to answer questions posed by Liam. In the discussion task, the lesson study team observed students beginning to view each other as a resource and asking each other for help before asking the teacher.

3. Change in teacher instruction

The lesson study process stimulated considerable changes in the lesson planning and its implementation of the teacher. Such changes included specifying learning goals for students, refining questions, and designing a discussion task in a rigorous and meaningful way, all of which was aimed at maximizing student engagement. The teacher showed subtle changes in his teaching instruction during involvement in lesson study (Verhoef, Tall, Coenders, & Smaalen, 2014). Rather than having teacher-centered class where he explains the lesson followed by examples for illustration, he shifted his classroom to be more student-centered through more involvement of the students in solving questions rather than solving them himself. He started asking more leading questions to find the solution to the given example.

4. Enhanced teachers’ pedagogical content knowledge

Prior to participating in lesson study, Liam was working in isolation as he could not collaborate with another teacher; he was the only one teaching Calculus. Most teachers teach in isolation without a chance to collaborate with others or reflect on their own teachings (Remillard, 2005). Lesson study gave the teacher an educational community as he engaged in active
discussion and self-reflection that was geared towards improving students learning and understanding. The lesson study process - lesson planning, classroom observation and reflection on teaching - has enhanced his pedagogical content knowledge. Liam incorporated the strategies suggested by the lesson study team in planning and explaining lessons, such as allowing the students to construct and derive rules rather than simply stating them, and using new activities to encourage the students to review each other’s work.

5. Improved teacher’s metacognition

Metacognition is the thinking process by which teachers think and reflect on their teaching practices so that it meets the needs of their students (Jiang & Gao, 2016). Liam became more aware of students’ need and started to engage them during classroom explanation and assessment that show their thinking process and understanding as in-class assignment and discussion tasks.

6. Time

Lesson study changed the planning time for each lesson and related assignment due to the pre- and post-lesson discussions. It also increased the time for preparing the assignment, as the lesson study team collaborated to select questions based on our intended goals for the in-class assignment and discussion tasks.

7. Difference of opinions

In any type of professional development activity, difference of opinions tends to arise (Dudley, 2015). Liam was open to the changes discussed with the lesson study team, and agreed with some suggestions, modifying others to suit his teaching style. However, on one occasion, he disagreed with the team. He explained his reasoning for wanting the students to follow his method, and the lesson study team understood his point of view and agreed to using it.
8. Administration support

The participation of Ali, the vice principal and Head of Mathematics Department, and the support of the school principal had a positive influence on the study’s success. They supported Liam by removing his lunch duties, and Ali gave his office space for all of our meetings. Ali’s constructive feedback made it easier for Liam to continue in the study, a finding that was consistent with other research (Lewis, 2016; Saito et al. 2012), which found that strong support from the school leaders is crucial for the success of lesson study.

It is important to note the power differential between Ali as an administrator and Liam as a teacher exists, although it was not visible. It is the role of the facilitator to mediate between them and remind the administrator that he is a team member and not an evaluator to avoid any possible tension.

5.6 Recommendations Suggested by The Lesson Study Team

There are several recommendations that both Liam and Ali felt were important after being part of the lesson study process. Liam’s experience with lesson study was positive overall, and he felt that it empowered him to change and explore new ideas that he would not necessarily implement on his own. He would like the lesson study to be implemented throughout the school, and suggested they do it for a different mathematics course each year, before cycling back to his calculus class.

Another suggestion he made was to have one of the lesson study team members attend a couple of his classes to determine whether his teaching has changed in accordance with what was agreed upon. If the changes do not continue, then they could debrief the class observation to offer suggestions. He felt the lesson study process helped him to be a better teacher. It had a positive effect on him and also on the students. He felt that lesson study kept the level of
teaching about the same, and created consistency among teachers involved in the team because of the discussions in the planning and debriefing. This is consistent with what Sibbald (2009) found that, “over time, lesson study should slowly alleviate issues of discrepant content knowledge between classrooms” (p. 451).

Ali spoke positively about his experience and enjoyed being part of the lesson study team. He appreciated the positive changes that were observed with both the teacher and the students. The research did not commence at the start of the semester, so one of Ali’s recommendations was to start at the beginning and cover more topics in the Calculus curriculum rather than just parts of it. The second recommendation was to allow teachers in the lesson study team to take part in explaining the research lesson. Instead of having Liam teach the researched lesson in every cycle, Ali would also explain a lesson in some of the cycles. In this case, they are alternating. In that context, the focus is not on one teacher as it was in this case.

Ali believes that school administrators have a stake in helping the teacher progress in the profession, and one means of doing so is through facilitating professional development. From his perspective as a teacher and an administrator, he recommended implementing lesson study through grades 9 to 12. In this manner, every teacher would have the opportunity to participate in lesson study either the first or second semester for one of their courses. The school should also reduce the teachers’ duties, making it easy for them to participate.

He also recommended that an administrator be part of the lesson study team, as it would be easier for them to follow-up with the teacher and ensure that the lesson study is being transferred to the practice even after the study concludes. For this to take place, it is important for schools to create a non-judgemental atmosphere (Lawrence & Chong, 2010) to encourage teachers to participate in lesson study, especially novice teachers.
5.7 Impact of Lesson Study on the Researcher

Being part of the lesson study team allowed me to reflect on my teaching practices, especially given that I was teaching Calculus during the same time in my school. I agree with Liam that teachers sometimes teach in isolation, and do not have the opportunity to observe other teachers or have the time to pause and reflect on their own practices. This happens not because they choose isolation, but because of the complexity of their daily work of duties, marking assessment, and curriculum that must be covered.

Taking time off for the research gave me the chance to reflect on my teaching practices; I recognized the importance of students’ engagement in the classroom and creating assessments that would increase their collaboration and understanding. Classroom observation and reflection helped me understand how students learn in different ways, as I noticed that some students learn more by explaining to their peers, while others prefer to be independent learners in which they depend on themselves to figure things out.

I began to apply concepts and strategies that were discussed in the lesson study. One example that I found was very beneficial and positively affected students in Liam’s class was discussion task, so I started implementing discussion tasks as an assessment for students in my classroom. The students liked the idea and appreciated the opportunity for collaborating and working at home, and then coming to class to discuss their findings in a less stressful environment. They even asked to have a discussion task for each unit as some of them during written assessment are so stressed that sometimes they forget or get confused while in this case they have prepared their answer and thought about their question thoroughly. In addition, I started encouraging my students to see each other as a useful resource and to seek help from one another before asking me.
Teachers in my department have enquired about my research progress, which provided an opportunity to initiate a conversation about lesson study and some of the useful assessments that could be implemented in our school as well. Being the head of math department in my school, the responsibility rests on me to mentor and guide teachers. I spoke with the administrator in my school to arrange a department meeting in which I reviewed my research and the lesson study team’s accomplishments. The teachers liked the idea so much that the following semester they began implementing discussion tasks.

5.8 Implications for Future Research

This study involved a limited number of participants on the lesson study team; however, leading researchers found that professional development is more effective when limited to fewer teacher participants (Garet el al., 2001). The study also had limited time frame. It would be beneficial to investigate extending the length of the study to cover the entire semester, allowing for more lesson study cycles to be completed. Future research could also investigate the effectiveness of lesson study if more participants were involved.

Based on the findings of this study, it is possible that the smaller number of students contributed to its effectiveness. Future research is needed to explore the effectiveness of lesson study with more students involved. In future studies, students’ achievement data could be collected so that the effect on student achievement could be quantified. This quantitative study would provide another perspective on lesson study in secondary school mathematics.

5.9 Limitations

In general, a possible limitation that could have influenced the study is class size. There were only nine students in the class, so it was potentially easier to engage them all in every class, and to have class discussions that promoted their learning. It is reasonable to conclude that the
class size has an impact on the success of achieving the pre-established goals. As the number of students increases, the teacher would have less time to spend with each individual student.

The lesson study team was able to complete only six cycles in a three-month period. It would be advantageous to explore how successful lesson study would be if it was employed throughout several more lessons. In addition, future studies might explore implementing lesson study in at other grade levels and with a higher number of participating teachers.

All of the teachers participating in the lesson study had several years of experience teaching Mathematics at secondary level and Calculus in particular, which sped up the process when planning lessons for the desired goals. If one of the teachers was a novice, I think the planning would have taken longer and ended after two or three cycles rather than five.

The presence of an administrator in the lesson study team could be seen as a limitation. Ali being the head of mathematics and the vice principal could have created an influence of power differential on Liam during planning and the students during classroom observation. The amicable relationship that existed between Liam and Ali prior to the start of the study and their open mindset to suggestions and feedback contributed to the successfulness of the research. This may not be the case with other teachers and relationship dynamics.

Although I was both the researcher and a participant, I made an effort to limit my involvement in the planning to listening to the suggestions of Ali and Liam. This was supported by detailed description of the planning stage, classroom observations, and teacher interviews. I was also the facilitator who made sure that the planning aligns with the set goals.

As I was not a teacher at the school were the research took place, I was unable to incorporate the reteaching stage of the revised lesson. On the other hand, it would have been harder for the lesson study team to attend my school for the revised lesson because of their work.
Due to the limited time I had with the lesson study team, I also faced limitations involving students in the process and interviewing them after each lesson.

One final limitation to note is that this study did not use the students’ achievement in the course to examine how increasing students’ engagement affected their learning. The focus of this study is on lessons and teacher instruction, with the ultimate goal of enhancing student learning (Yoshida, 2012).

5.10 Conclusions

It is important to understand that lesson study is considered a form of professional development that aims to change participant attitudes with an end goal of enhancing teacher instruction, and consequently, the student learning experience. Similar to any professional development, there are challenges and constraints, but the overall outcomes were positive and encouraging.

This research documents the effectiveness of lesson study as a professional development tool. This was evident from the qualitative data collected using interviews and classroom observations. It was conducted and driven by a set of goals, some of which were directly related to the current classroom environment, while others reflected the curriculum expectations. The team analyzed the placement of their learners in relation to these set goals and developed a research lesson designed to move them closer to the goal. Classroom observations guided the team towards measuring the effects of the research lesson, and reflecting on those observations to modify and move closer to the set goals. Lesson study uses the classroom as its context through every step of the cycle.

The participants did derive value from the opportunity to participate in a meaningful collaboration, as they found their experience to be valuable. Lesson study cycles supported the teacher through the sharing of ideas with different perspectives for planning lessons and
reflecting on those lessons to “see the unseen”, as the method employs a model of collaboration that focuses directly on classroom instruction.

This study is different from other studies in that it was tailored based on the needs of the students and the classroom environment. The long-term goal for the study was focused on increasing student engagement through developing a variety of teaching strategies while challenging them at the same time to improve their critical thinking skills using Inquiry questions. Classroom observations imply subtle change occurring in students’ engagement and learning as it increased towards the end of the study.

The study highlights the importance of lesson planning guided by clear set of goals for the students. These desired goals had helped the teacher emerge from teaching in isolation through meaningful collaboration, allowing him to examine his practice from a different perspective. It improved the teachers’ metacognition, and this can be attributed to the detailed process embedded in lesson study that allows participants to embrace more effective strategies after analyzing the strengths and weaknesses of particular teaching approaches. These strategies serve the purpose of enhancing teachers’ pedagogical content knowledge and increasing student engagement.

Lesson study is a model of professional development, which values direct involvement of the teacher in the process. It had great potential that led to empowering the teacher to change their class instruction and reflect on their practice, and empowering the students to be more engaged in the process.
References


Appendix A

Letter of Consent

Dear _____________________,

I am a PhD student at the Ontario Institute for Studies in Education, University of Toronto, and I am investigating how mathematic teachers use Lesson Study to examine their practice and enhance students learning for the purpose of my thesis. I think that your participation and experience will provide insight into this topic. I believe that engaging in Lesson Study will enhance teachers’ content and pedagogical knowledge and hence increase student’s achievement.

My research data collection consists of four stages within each topic that will be taped and video recorded at all times: 1) A preliminary interview; 2) Goal setting/Planning; 3) Classroom observations; and 4) Debriefing.

1) Preliminary interview – This is 45-minute interview. I will ask you a series of questions about your teaching background, beliefs, goals and ideas about student engagement and success.

2) Goal setting/Planning – This is 45-minute. The team will decide on the math topics to investigate. Manipulatives, strategies, instructions and any assessment that will be used in the classroom.

3) Classroom observations – The team will observe the suggested lessons focusing on student’s engagement and understanding.

4) Debriefing – This is a 60-minute reflection. I will ask the teacher and the members who attended for clarification and reflections on the lesson that was taught. Suggested improvement for the lesson.

5) I will ask the Lesson Study team a series of questions about their involvement in the study, reflections on participating in Lesson Study and final thoughts about student engagement and success.

I will not use your name or anything else that might identify you in my written work, oral presentations, or publications. This information remains confidential. You are free to change your mind at any time, and to withdraw even after you have consented to participate. You may decline to answer any specific questions. I will destroy the tape and video recording after the research has been presented and/or published which may take up to five years after the data has been collected. There are no known risks or benefits to you for assisting in the project, and I will share with you a copy of my notes to ensure accuracy.

If you have any questions or concerns about your rights as a research participant, please contact the Office of Research Ethics (ethics.review@utoronto.ca, 416-946-3273).

Please sign the attached form, if you agree to be part of the study. The second copy is for your records. Thank you very much for your help.

Yours sincerely,
Consent Form

I acknowledge that the topic of this research has been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time without penalty.

I have read the letter provided to me by Eiman Zeini and agree to participate in the study according to the purposes described.

Signature: ______________________________________

Name (printed): ________________________________

Date: ________________________________
Appendix B

Semi-interview – Teacher Questions

Background questions

1. What is your name?
2. What courses are you currently teaching and what is your role in the school?
3. How long have you worked at this school?
4. Where did you teach before and what courses have you taught?
5. How many years have you been teaching?
6. Which university did you attend?
7. Why did you become a teacher?

Student engagement

1. How would you define student engagement?
2. To what extent do you consider student engagement when planning your lessons?

School culture

1. How do you create an environment, which supports success in mathematics?
2. What challenges have you faced in trying to create a culture that supports student achievement in mathematics?
3. How do you work with staff and administration to develop the goals/vision of the school?
4. Have you had the chance to co-teach? If yes, what benefits did it add to your classroom and what challenges did you encounter?
5. Were you involved in any other types of collaboration/collaboration groups? If yes, what benefits did it add to your classroom and what challenges did you encounter?

*Overall*

1- In your opinion, what programs or resources support success in mathematics outside of the classroom?

2- Have you ever attended workshops/conferences as a professional development? If yes, what is your input about it?

3- Have you heard before about lesson study as a professional development? If yes, what was your understanding of lesson study?
Appendix C

Debriefing after Classroom Observation

For the teacher teaching the lesson:

1. How did today’s lesson go?

2. Do you believe that the students were engaged throughout the lesson? (At what point do you believe they were the most engaged? Least engaged?)

3. What would you do differently next time?

For the observing members:

1. How were students engaged throughout the lesson?

2. What should be done differently to have a better lesson? What do you conclude from student observation throughout the lesson?
Appendix D

Semi-interview at the End of the Semester - Teacher Interview

1- What benefits (if any) of planning lessons through collaboration?

2- Were the observer remarks useful in helping you see the unseen when busy explaining or attending other students?

3- How did being part of the Lesson Study group impact your teaching?

4- What challenges did you find when participating in Lesson Study?

5- What recommendations would you give to better enhance your experience?
Appendix E
Student – Parent/Guardian Letter of Consent

Dear Parent/Guardian:
As you may be aware, your son’s/daughter’s teacher has volunteered to participate in a mathematics study aimed at improving mathematics education and instruction, beginning as early as February. This study will take place during the school day and will replace or supplement the mathematics instruction that they are currently receiving. Thus, your child may notice that some of the activities in the mathematics classes may be different. Although participation from your child is mandatory as it is part of the instructional day, it is based on the Ontario Ministry of Education’s Grade 12 Mathematics Curriculum.

Your son/daughter’s class is expected to participate in the study for no less than 8 weeks, and possibly longer depending on the teacher’s discretion.

During the study, the Investigator and other teachers will be present in your child’s class for some of the mathematics lessons. This is to provide feedback to the teacher on their implementation. Although participation in the study is not optional, the degree to which the Investigator works with your child and records information is optional. Attached is a consent form indicating the different levels and types of consent you may provide to the investigator regarding your child. The level and type of consent you provide can change at any point or time throughout the study by contacting the Investigator directly. PLEASE NOTE: All information collected is strictly confidential and your son/daughter will NOT be identified in any capacity. The reason for collecting this information is for data analysis and educational.

If you have any questions, concerns, or require more information about the study, please do not hesitate to contact the Investigator of the study, Eiman Zeini via email at eiman.darwishzeini@mail.utoronto.ca. You are also welcome to contact the researcher’s Faculty Advisor, Dr. Doug McDougall at 416-978-0056 or via email at doug.mcdougall@utoronto.ca, if you have any additional concerns or concerns that you feel have not been appropriately addressed. If you have any questions regarding your son’s/daughter’s rights as a research participant, please contact the University of Toronto’s Office of Research Ethics at 416-946-3273 or via email at ethics.review@utoronto.ca.

Sincerely,

Ms. Eiman Zeini
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