Understanding Math Anxiety in School-Aged Children

by

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Abstract

This study investigated the effectiveness of measuring math-anxiety in elementary school students using real-time facial expression software (emotient) and self-reported Math anxiety using the Achievement Emotions Questionnaire- Elementary Students (AEQ-ES). Students aged 11-14 were asked to complete the AEQ-ES. Students were then given 5 standardized math questions taken from the Education Quality and Accountability Office (EQAO) and asked to rate how difficult they found each question (1-4). It was hypothesized that math anxiety would be related to difficulty ratings of math questions and performance on the math task, while parent ratings of their child’s math abilities was expected to affect the development of child’s math anxiety and math performance. This study will inform research on development of math anxiety and possible areas for intervention.
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Chapter 1
Introduction

1 Introduction

1.1 Background

Emotions develop early on after birth, as infants learn to produce emotions in order to meet their needs, attract appropriate attention, express desires, and develop attachment with their caregivers (Gergely & Watson, 1996; Bridges, 1932). It is not until around the age of three that children are able to understand that there is a strong connection between facial expressions, thoughts, and feelings (Bornstein & Atteberry, 2013; Wellman, Cross & Watson, 2001). Children then begin to express emotions through facial expressions that match their feelings, and vice versa. Along with the understanding of their own thoughts and emotions, normally developing children at this age also gain an understanding for the emotions of other people. This phenomena coined Theory of mind (ToM), allows children to recognize their own emotions, emotions of others, and how those emotions of others may differ from their own (Astoning & Jenkins, 1995; Wellman, 1992; Barron-Cohen, 1985; Carlson, 2013). Around the same time, children begin to understand that feelings can come from their own thoughts and can be altered by their memories (Flavell, Flavell & Green, 2001; Lagattuta & Wellman, 2001).

As children develop, their emotions and cognitions start to interact and influence one another through a reciprocal relationship. Emotions guide attention and influence what information is noticed, how it is encoded and processed, and how this information is accessed later from memory (Core & Palmer, 2009). Cognitive processes, however, influence how we think about emotional events and our emotional reaction to these events through cognitive appraisal (Smith & Ellsworth, 1985; Webb, Miles & Sheeran, 2012). Cognitive appraisal is the interpretation that is made by the individual to the emotional event or stimuli (Lazarus, 1984). This relationship is evident in all aspects of a person’s everyday life, but it may be especially important for education researchers to consider in relation to learning. This relationships between emotion and cognition occurs when students are encoding and learning new information. As a result, it may be
influencing what students learn, as well as how they are able to encode and process information for future access and assessment of knowledge (Core & Palmer, 2009).

The classroom is thought to be a learning space in which students can learn, share, collaborate and flourish. In an ideal world the classroom is a space that promotes positive feelings in relation to learning. However, in the classroom, elementary school-aged children experience a large range of emotions that stretch much further than positivity, greatly implicating learning, achievement and mental health (Pekrun & Linnenbrink-Garcia, 2014). Emotions, including fear, anger, enjoyment, anxiety, boredom, and zeal, can impact student learning by influencing motivation, achievement, and attention (Pekrun, 2017). These emotions can have positive and negative effects on students, however the presence of negative emotions should stand as cause for worry among educators and researchers, as they can impact the quality of students learning and achievement (Goetz, Pekrun, Hall & Haag, 2006). Furthermore, these negative emotions have been found to negatively affect experiences in mathematics for students as early as elementary school (Jackson & Leffingwell, 1999).

Emotions can also direct students’ attention. Positively valanced emotions, such as enjoyment, can broaden attention; in contrast negatively valanced emotions, such as anxiety, can narrow the scope of attention (Pekrun & Linnenbrink-Garcia, 2014). Other consequences of the influence of emotion on attention involves its ability to affect the retrieval of stored information from long-term memory. When information is more emotionally salient, regardless of its objective importance to the task, it is more deeply encoded in memory (Janata, Tomic & Rakowski, 2007; Krauel, Duzel, Hinrichs, Santel & Baving, 2007). This may have both positive and negative influences on a learner, depending on the task, though it becomes more problematic when emotions are so strong that they prevent the encoding of other important information during learning (Talmi, 2013).

Negative emotions associated with learning can manifest across a multitude of different subjects, however math anxiety is the most prevalent type of anxiety affecting learning and performance in educational settings (Blazer, 2011). Math anxiety can be defined as a feeling of apprehension and amplified physiological reactivity when working on a mathematic task, or even just thinking about doing mathematics (Lyons & Beilock, 2012). Tasks that induce these feelings include
solving mathematic problems, manipulating numbers, or simply being exposed to evaluative situations related to math (Luttenberg, Wimmer, & Paetcher, 2018). It has been estimated that well over half of students suffer from math anxiety at some point in their schooling (Beilock & Willingham, 2014; Luttenberger, Wimmer & Paechter, 2018). Learners experiencing anxiety are highly likely to have negatively impacted performance on tasks due to interference in attention and memory (Eysenck, Derakshan, Santos & Calvo, 2007). According to the cognitive literature, for any arithmetic process beyond simple memory retrieval, math performance critically depends on working memory, specifically rehearsal through the phonological loop (Ashcraft & Krause, 2007; Baddeley, 2003; Friso-Van Den Bos, Van Der Ven, Kroesbergen & Van Luit, 2013).

A key aspect of working memory is the phonological loop which is responsible for the storage and rehearsal of information (Badley & Hitch 1974; Baddeley, 2000). Research by Ashcraft and Kirk (2001), indicated that a higher cognitive load in the form of a larger string of numbers was more detrimental to those individuals with higher levels of math anxiety. Working memory is needed to be able to focus attention on the task at hand, while inhibiting all information that is not relevant (Kane & Engle, 2002). The phonological loop is key for accessing long strains of numbers from memory, as well as problem solving (Swanson, 2004). Aschraft et al. (2001) showed that worries about math took over working memory resources, specifically the phonological loop functions, which could otherwise be put towards remembering a longer sequence of numbers. This can cause serious implication for students in the classroom and in evaluative situations. When working memory is depleted due to these intruding thoughts of anxiety, students’ math learning and performance is likely to be hindered. Thus, individuals’ anxieties are depleting working memory capacity and causing them to learn less than their low-anxious classmates, be awarded lower grades in math, and have less motivation to take math classes as elective courses (Ashcraft & Krause, 2014; Beilock, 2008).

This effect of anxiety on working memory capacity could be a driving factor as to why some children learn, understand, and complete mathematics tasks with such ease. There has been research conducted to examine working memory as an individual difference variable, where some individuals have a larger working memory capacity as compared to others (Engle, 2002; Ramirez, Gunderson, Levine & Beilock, 2013). When comparing those students with differing working memory capacity, we can expect those with larger working memory capacity to
overperform (Engle, 2002), experience and report overall fewer negative emotions (Schmeichel & Demaree; Suinn et al., 1988). In contrast, students with low working memory capacity, paired with feelings of anxiety may have what little working memory they have inhibited and their performance will suffer (Ramirez, Gunderson, Levine & Beilock, 2013).

It is possible, however, that students with low working memory capacity are relying on heuristics, or mental shortcuts, to overcome the problems associated with not being able to hold long strains of information in their memory, such as equations. Research Beilock and DeCaro (2007) illustrated that individuals with both high and low working memory had the same level of anxiety while math problem-solving. This may indicate that rather than relying on working memory to perform well on academic tasks, individuals rely more on mental shortcuts to may simply be unable to allocate resources in order to attend to mathematics because the perception of threat is redirecting the student’s thoughts.

However, Beilock and Carr (2005) examined that even students with high working memory may be harmed by pressure to perform. The researchers coin the term “choking under pressure” to describe the harmful effects pressure places on differing working memory demands. For individuals that rely greatly on their working memory capacity to perform well on a math task, their performance was shown to be greatly hindered when working memory capacity was not available for skill execution (Beilock & Carr, 2005). Thus, when there is a threat to attentional capacity due to pressure or demand, the advantages individuals with higher working memory seem to have had disappear. This may have serious implications when studying student’s performance on math tests and tasks that induce feelings of stress and demand, regardless of high working memory capacity.

The effect of negative emotions on mathematics, such as anxiety, fear, and disgust, has been previously studied and it has been shown that these emotions can have a strong influence on the perception of task difficulty, performance, and overall achievement. By integrating technology that allows researchers to track the real-time emotions experienced by students, they have been able to avoid relying solely on children’s self-reports to understand which methods are best for measuring emotions accurately in a population of elementary school-aged students (Littlewort, Bartlett, Salamanca & Reilly, 2011). These measurement methods can then be used to obtain a
clearer photo of the implication’s mathematics may have on emotions, working memory and performance.

1.1.1 The Current Study

This study examined the role of emotions on a math task and math related anxieties as a result of these emotions experienced. The emotional experience of students aged 11-14 was examined to determine the influence emotions may have on math performance. The overarching goal was to identify the most reliable method researchers can use to measure and identify math anxiety, in order to develop early detection methods of math anxiety. While many studies with adults completing self-report data have been validated, measuring self-reported emotional experience in children can be much more difficult. The difficulty occurs because children at younger ages lack the metacognitive awareness to correctly identify and label their emotions (Davis, Levine, Lench & Quas. 2010). Additionally, for individuals to properly complete self-report measures they must be able to identify emotions previously experienced during a task. However, there are issues concerning younger children capabilities to remember previously experienced affective states without the intrusion of current affective states (Lagatutta, Elrod & Kramer, 2016). These issues of validity must be combatted when measuring emotions in children and can solved by introducing non-intrusive cognitive measures of emotion, such as measuring real-time facial emotion expression using Emotient Software (imotions.com).

Cognitive theories of emotion are guiding this research project (Oatley & Johnson Laird’s, 2011; Pekrun & Perry, 2014; Schacter & Singer, 1962). According to these theories, cognitive appraisals occur once an emotion is experienced (Frijda & Parrot, 2011). This cognitive appraisal then guides the individual to act. The reciprocal relationship between cognition and emotion indicates that there is an appraisal of emotions based on the intensity of that emotion. (Oatley & Johnson-Laird, 2014; Webb et al., 2012). This relationship between cognition and emotion then allows thoughts to rearrange emotions. Appropriate actions are then taken based on this cognitive restructuring of emotions. The type of emotions experienced during a task can be detrimental to task performance, as more negative emotions can inhibit cognitive function and thus lead to worsened performance (Ashcraft & Krause, 2007; D’Mello, Kappas, Gratch, 2017; Hembree, 1990; Pekrun et al., 2012). Thus, performance on the math task can be hindered if negative
emotions are overtaking working memory capacity and disallowing students to properly allocate resources to focusing on the task at hand (Ashcraft & Kirk, 2001; Daneman & Merkle, 1996).

Overall performance on the math task, involving computation and problem solving, was examined as a result of possible negative emotions experienced by participants due to lower working memory capacity. Some students may be experiencing more positive emotions as related to the math task, while some students may experience more negative emotions as a result of lowered working memory capacity. Students who have a more difficult time with the math task may rate the difficulty higher, experience more negative emotions as related to the task, and thus have lower amounts of working memory capacity available to them during the math task.

1.2 Research Question & Hypotheses

RQ1 Do parents’ ratings of child’s math abilities and students self-reported emotions predict performance on standardized math task?

RQ2 Can students reported levels of anxiety and their difficulty ratings of math questions predict scores on a standardized math task?

RQ3 Can emotional expression, detected in real-time by Emotient software, predict performance on a math task?

Based on these four research questions, the following two hypotheses are expected to emerge:

H1 Parent reports of their child’s math capabilities and children’s self-reported emotions will predict performance on the math task. Parent reports of their child’s math capabilities will predict children’s actual math scores illustrating that the reports are good indicators of math ability.

H2 It is hypothesized that students with high self-reported math anxiety (as measured by the AEQ-ES) will express more negative academic emotions, receive lower scores on the math task, and will report higher difficulty on the math task as compared to their peers with low math anxiety. Emotional expressions reflecting negative emotions (anxiety), will be more reliably
measured and identified by real-time emotion software, compared to self reports from students and parents.

H3 Emotional expressions that are of high intensity and negative valence, such as anger, frustration, and confusion, will be negatively related to performance on the math task. The relationship will show that students who experience negative emotions during the math task will have lower scores on the math test.

1.2.1 Rationale

The hypothesis stated are based on previous research findings that have indicated how high intensity and valence of emotions can impact performance on academic tasks, such as mathematics (Gernsbacher, Goldsmith & Roberston, 1992; Lichtenfeld, Pekrun, Stupinsky, Reiss & Murayama, 2012). These effects are especially detrimental if the students are experiencing negative emotions, as compared to positive emotions. In a study by Aschraft and Kirk (2001), it was found that individuals experiencing higher levels of negative emotions during a math task (i.e., anxiety and tension) had poor performance on the task, as compared to those experiencing fewer negative emotions. Previous research by Pekrun et al. (2006) indicates that negative deactivating emotions, such as boredom, has also led to lower motivation and poor self-regulation of learning. Poor motivation and fewer self-regulated approaches then lead to shallow information processing due to depleted working memory capacity, and lower performance on tasks (Pekrun, 2006). If working memory is depleted, students are unable to access retrospective memory in order to appropriately answer math questions (Baddeley, 2000, 2003; Krumm, Ziegler & Buehner, 2008; London, Bruck, Poole & Melnyk, 2011).

Self-report measures have been highly used and well established in the literature (Spielberg, 1996; Watson, Clark & Tellegen, 1988). However, there is a strong focus on adult self-reports, prompting individuals to retrospectively recount their emotional experiences during a specific situation. Researchers tend to question the validity of using self-report measure for children, as they are more likely to misattribute and have increased misinformation than that of adults (London, Bruck, Poole & Melnyk, 2011; Litchenfeld, Pekrun, Stupinsky, Reiss & Murayama & Pekrun, 2012). However, the questionnaire used in this given study had high external validity, where all three emotions ratings (anxiety, boredom, and enjoyment) were valid in a sample of
grade two students, and external validity was even higher in a sample of grade three students. The researchers suggested that the validity may increase as students become older (Lichtenfeld, Pekrun, Stupinsky, Reiss & Murayama, 2012).

However, to combat the chance of methodological limitations, this study incorporated multiple channels of technological advances to measure student’s emotional expression in real-time. This was used to determine whether this method would better identify the emotional experience among students during a math task. The use of real-time measurement of facial expression while engaging in the math task adds high value to research being conducted in the field of emotions and education because researchers will better be able to identify the source of negative emotions during mathematics and specific areas for appropriate intervention. Despite the potential bidirectionality of math performance and math anxiety, it is hypothesized that emotional experiences during the math task will contribute to their performance on the task itself. If the child is experiencing high negatively valanced emotions, such as anxiety, they are expected to perform worse at the math task.

The rationale for this study relies on the cognitive theories of emotion and control-value theory (Pekrun 2006; 2009) to guide this research. The control-value theory (Pekrun, 2006) states that emotions are multidimensional constructs that are made up of affective, cognitive, expressive, motivational and physiological factors (Pekrun 2006; 2009). Central to Pekrun’s theory are control and value appraisals; determinants of achievement emotions, causing the sense of “being in control” or “being out of control”. These feelings of control are what leads to the experience of different emotions (Pekrun, 2007; 2000). Positive emotions are found to be positively related to control and value appraisals in students, while in contrast negative emotions are related to lack of control and value. We would expect the presence and intensity of negative emotions to be negatively related to performance on the math task. Thus, students who have high levels of anxiety would lead to a negative appraisal and lower performance on the task. This study will examine the interaction of cognitive theories of emotion (Oatley & Johnson- Laird, 2014; Schacter & Singer, 1962), Pekrun’s (2006) Control-value Theory, and emotional experiences to understand math-task performance in students aged 11-14. This research is extremely important to the field of education as students who have differing appraisals of their learning environment due to emotions experienced may have vastly different learning and academic experiences.
2 Literature Review

2.1 Emotions and Mathematics

Emotions are constantly emerging in students during the learning process. When specific emotions emerge within classrooms, they are related to appraisal of the task itself. This appraisal leads students to determine whether they can control the outcome of a task and how much value they place on the task outcome (Pekrun & Stephens, 2010). Negative emotions, such as anger or confusion, are elicited in many different environments, by many different factors. However, in all cases they signal a type of danger or threat which are not the typical emotions expected to be experienced in the classroom. This may negatively affect cognition and learning among students. When a threat is sensed, attention becomes assiduously focused on the stimulus which is causing these negative emotions; the student is appraising the negative stimuli. In the context of mathematics, when a difficult math question elicits a negative emotion from a student during a test, attention is drawn to this question and that student is not able to attend to a much broader array of information available to them (Meinhardt & Pekrun, 2003; Pekrun, 2017). If these emotions are not properly regulated, students can continue to ruminate on negative emotions, disengage from the task, and begin thinking of task-irrelevant information (Pekrun et al., 2012; Pekrun, Maier & Elliot, 2006).

2.2 Cognitive Theories of Emotion

It has been noted increasingly that different emotions influence cognitive processes in different ways (Vogl, Pekrun, Murayama & Loderer, 2019; Angie, Connelly, Waples, & Klgyte, 2011; Lench, Flores, & Bench, 2011). Cognitive theories of emotion posit that emotions involve thoughts and cognitions, allowing individuals to solve problems and react to situations in specific ways. There are multiple theories that aim to describe emotions and the process that elicits these emotions. The Two-Factor Theory of Emotions, proposed by Schacter and Singer (1962), for
which the first factor is an involuntary physical arousal leading to an emotional reaction, and the second factor is the cognitive label or conscious interpretation of the physiological arousal, leading an individual to understand what caused the emotional response. In terms of mathematics, this theory may indicate that a physiological arousal to a math test may lead to a negative emotional reaction, causing the student to label mathematics with this negative emotion and continue to view mathematics as negative or threatening.

Oatley and Johnson Laird’s (2011) Communicative Theory of emotions illustrates that there are a limited amount of basic emotions that each serve as a solution to a species-specific problem of adaptation. It speculates that humans have evolved from a specific strain of mammals that are better able to handle novel situations, due to the fact that humans are uniquely propelled to connect with other humans (Keltner, Oatley & Jenkins, 2014). Basic emotions serve as heuristics that cause people to enlist cognitive structures to allow for functional and adaptive appraisal to the environment in order for individuals to take proper action (Oatley & Johnson Laird, 2011).

The Control Value theory of Emotion separates emotions into distinct categories. First, emotions are separated into two main types of emotions; epistemic emotions and achievement emotions. Epistemic emotions refer to emotions that lead to inquiry and critical reflection (Vogl, Pekrun, Murayama & Loderer, 2010). These emotions develop as a result of contradictory information, leading to incongruent cognition as this information impedes goal attainment when thoughts and beliefs are found to be incorrect. When an individual is given information that does not align with their beliefs and thoughts, their cognition is interrupted and certain emotions are elicited (D’Mello, Taylor & Grasser, 2007). Achievement emotions, in contrast, are emotions that have success and failure as their objects. When a student feels frustration resulting from personal failure, this would be an achievement emotion (Pekrun, Vogl, Muis & Sinatra, 2017). Achievement emotions are extremely important for students’ performance on tasks, their engagement on tasks, and their well-being (Pekrun and Stephens, 2010).
2.2.1 The Relationship Between Emotions, Cognition & Self-Regulated Learning

The relationship between emotions and cognition is one of bidirectionality, starting first with an appraisal that guides the individual to take suitable action. This appraisal then determines the intensity and valence of the specific emotion that is experienced (Siemer, Mauss & Gross, 2007). Cognitions are thus restructuring emotions so that appropriate changes and action can be taken to fit the situation (Webb et al., 2012). However, if there is an interruption in learning and information gathering, the individual may be at risk. According to theories of self-regulated learning, individuals use information from their surroundings to create a plan and understand possible difficulties (Pintrich, 2000). A key aspect in Pintrich’s (2000) model for self-regulation includes cognitive control. Cognitive control allows students to modify their cognition, problem solve, and reason. Without the ability to engage in these self-regulated learning strategies and access their cognitive control, students are at high risk for shallow learning and poor memory (Pekrun, 2011).

2.3 Measuring Emotions

There have been many different methods for measuring emotions, while self-report measures have been long considered as the most used method for to determine the intensity at which emotions are felt. Participants are, in most cases, asked to report their experience of specific emotions on a numeric scale. Specific emotion scales, such as anger and fear (Spielberger & Gorsuch, 1983) have been designed and used to assess the presence and intensity of one emotion at a time. More general measures of emotion have been used to measure both positive and negative emotional states in adults (Watson, Clark & Spielberger, 1983). In children, the primary method for measuring emotions has also been through self-report or facial expression coding. However, pictorial assessments are also widely used to combat the differing reading capabilities across child participants (Bradley & Lang, 1994; Coan & Allen, 2007; Pekrun, Lichtenfeld, Killi & Reiss, 2007). As students get older, the validity and reliability of their self-reported answers are much stronger. In a study by Litchenfeld & Pekrun (2012) grade three students self-reports were much more valid and reliable than that of graded one and two students
(Litchenfeld & Pekrun, 2012). However, there may be threats to the validity of self-report measures given to children as their ability to remember previous affective states relies on working memory. Young children who are completing a specific task and then asked to report on these previous emotions experienced may have a lack of working memory capacity and may have their judgements of past emotional states clouted by current emotional states evoked by the task (Coan & Allen, 2017; Lagatutta, Elrod & Kramer, 2016).

2.3.1 Facial Action Coding System (FACS)

The Facial Action Coding System (FACS) was created by Ekman and Friesen (1976) to code and measure facial expressions elicited by children and adults. The coding scheme was created based on well-established theories of emotion, relying on combinations of specific muscular movements in the face to form facial expressions for each emotion. An Action Unit (AU) was used to label each specific element and facial muscle movements. Together, each combination of specific AUs create specific emotional expressions. The FACS system is also created to distinguish between voluntary and involuntary facial expressions. Specific facial muscle combinations create the profile for identifying whether facial expressions are involuntary or voluntary. Thus, based on the difference of muscle activation for involuntary fear and voluntary fear, researchers can distinguish between which type of facial expression has been exhibited in each individual (Coan & Allen, 2017; Cohn, Ambadar & Ekman, 2007; Dimberg, Thunberg & Grunedal, 2002). This instrument widely used in research of emotions, cognitive science, and behavior and has been validated in numerous studies across many cultures (Cohn, Ambadar & Ekman, 2007; Ekman & Friesen, 1972; Ekman & Rosenberg, 2005).

2.3.2 Real-Time FACS Processing

Advances in technology have allowed researchers to combat the limitations of using the FACS system. While the FACS system is widely used, it is quite vulnerable to human error as emotional expressions last only a few seconds (Ekman, 1993). Technology has now been developed to replace human raters and recognize each of the AU combinations identified by the FACS program developed by Ekman & Friesen. The technology is an online system that codes students experience of emotions through their muscle movements on their face. The software
codes the facial expression and compares it with the human ratings of those specific facial emotional expressions (Bartlett et al., 2009). This program is non-invasive and allows for measurement of emotional expressions while students complete academic tasks and research participation. This tool is extremely useful for the understanding of emotions and development of real-time academic tools that may lead to increased academic achievement across students.

2.4 Anxiety and Mathematics

In educational settings, anxiety is one of the most commonly experienced emotions among students (Ashcraft & Kirk, 2001; Fiedler & Beier, 2014). Math anxiety can be defined as an unpleasant emotional response to math or the prospect of doing math (Hembree, 1990). Learners experiencing anxiety are highly likely to have negatively impacted math performance in school (Beilock & Willingham, 2014; Eysenck, Derakshan, Santos & Calvo, 2007). It has been estimated that most students have had at least one negative experience with math at some point during their schooling (Luttenberger, Wimmer & Paechter, 2018). Research has indicated a peak in math anxiety around middle to high school age, while the development of math linked anxieties start as early as the fourth grade (Blazer, 2011). The need for early intervention is clear, while targeting teachers of mathematics for young children is a necessity.

Math anxiety is impeding individuals’ ability to regulate their emotions during math tasks. Emotions and cognitions have bi-directional relationship that starts with an initial appraisal which directs the individual to react appropriately. This appraisal of an emotion is what determines the valence and intensity of how the emotion will be experienced and cause the individual to act appropriately (Siemer, Mauss & Gross, 2007; Webb et al., 2012). In order for an individual to act appropriately, they must have access to their working memory resources so they can assess the task, plan a course of action, and adapt if their goal is impeded. However, if this process is in fact impeded by interfering emotions, individuals completing math tasks may experience negative effects and decreased math performance as a result of math anxiety (Aschraft & Krause, 2007).

Math anxiety is likely an issue of working memory. Mathematics problems that are more difficult, and require the use of working memory, show considerable math anxiety effects across individuals (Ashcraft, 2002). The harder the question, the more resources an individual must
devote to solving the problem. However, if anxiety levels are high, it is more than likely that these resources are depleted by feelings of anxiety and working memory cannot be properly allocated to solving the mathematic problem (Marttarella, Mateo, Kozak, Foster & Beilock, 2011). Remediation techniques will ideally target misconceptions early on and prevent development of math anxiety, thus lowering levels of confusion and frustration with more difficult math concepts and increased availability of working memory (Ojose, 2015).

2.4.1 The History of Math Anxiety

Math anxiety can be defined as a feeling of tension, apprehension or fear that interferes with math performance (Aschraft & Kirk, 2001; Levine & Zimmerman, 1995). Over time researchers have acquired a much deeper understanding of the phenomenon of math anxiety. Math anxiety research can be traced all the way back to the 1950s. The earliest paper concerning math anxiety was written by Gough (1954), a classroom teacher, indicating emotional difficulties in math and labeling them as “mathephobias”. Around the same time, researchers Dreger and Aiken (1957), studied math anxiety in college students and included the term “numerical anxiety” in their paper describing students’ math performance. In Dreger and Aiken’s paper, math anxiety was, for the first time, distinguished as distinct from general anxiety; predicting that highly anxious students would have worse performance on academic work (Ashcraft & Ridley, 2004).

However, it wasn’t until 1972 that Richardson and Suinn developed the Math Anxiety Rating Scale (MARS) (Richardson & Suinn, 1972). This 98-item Likert-format rating scale is concerned with how anxious respondents reported feeling in certain situations regarding math, in both educational and informal settings. The MARS has been widely used in math anxiety research due to the high level of reliability with the measure. The MARS has been shortened by Alexander and Martray (1989), becoming the shortened MARS (sMARS), including 25-items that correlate .97 with the MARS. More recently, Hopko and colleagues (2003) constructed the Abbreviated Math Anxiety Scale (AMAS), as a much shorter, but highly reliable, alternative to the 98-item MARS. The AMAS has gained recognition for its two-factor structure that includes learning math anxiety and math evaluation anxiety, which accounts for 70% of the overall variance in test scores and contains 2-week test-retest reliability of .85 (Aschraft & Ridley, 2004).

Questionnaires using pictorial scales have been developed for use with primary and elementary school students, such as the Achievement Emotions Questionnaire- Elementary School (AEQ-E),
where students are asked to rate how they feel about math class, math tests, and math homework (Pekrun, Lichtenfeld, Killi & Reiss, 2007).

### 2.5 The Cause of Math Anxiety

The cause of math anxiety may not yet be fully understood by researchers. However, there have been many theories as to what factors influence the development of math anxiety. Factors such as age, sex, and culture have been highly investigated in the literature (Beilock, Gunderson, Ramirez & Levine, 2010; Woodard, 2004; Wigfield & Meece, 1998).

Genetic influences of math anxiety have been postulated as reason for math anxiety development. Few studies have found a link between the two. A study by Wang et al. (2014) used a sample of 514 twelve-year old twins to understand the behavioural genetics of mathematics anxiety. It was noted that math anxiety was highly correlated with general anxiety and genetics accounted for about 40% of the variance in math anxiety (Dowker, Sakar & Looi, 2016).

Most likely, however, is that students who suffer from math anxiety do not have good number sense and often have misconceptions associated with how to do certain math problems (Ashcraft, 2002; Dehane, 2011, 1997; Lyons, 2012; Skagerlund, Östergren, Västfjäll & Träff, 2019). Misconceptions developed early on may be hindering their performance later in their academic careers, causing them anxiety and lowering their willingness to continue taking mathematics courses. Research by Mestre (1989) indicated that once students have these misconceptions, they become emotionally and intellectually attached to them and these misconceptions interfere with new learning. Thus, it is extremely important to identify misconceptions early on and develop interventions that eradicate these misconceptions.

Additionally, math anxiety is likely an issue of working memory. Mathematics problems that are more difficult, and require the use of working memory, show considerable math anxiety effects across individuals (Ashcraft, 2002). The harder the question, the more resources an individual must devote to solving the problem. However, if anxiety levels are high, it is more than likely that these resources are depleted by feelings of anxiety and working memory cannot be properly allocated to solving the mathematic problem (Marttarella, Mateo, Kozak, Foster & Beilock,
2011). Remediation techniques will ideally target misconceptions early on and prevent development of math anxiety, thus lowering levels of confusion and frustration with more difficult math concepts and increased availability of working memory (Ojose, 2015).

Another possibility is the math anxiety is impeding individuals’ ability to regulate their emotions during math tasks. Emotions and cognitions have bi-directional relationship that starts with an initial appraisal which directs the individual to react appropriately. This appraisal of an emotion is what determines the valence and intensity of how the emotion will be experienced and cause the individual to act appropriately (Siemer, Mauss & Gross, 2007; Webb, Miles & Sheeran, 2012). In order for an individual to act appropriately, they must have access to their working memory resources so they can assess the task, plan a course of action, and adapt if their goal is impeded. However, if this process is in fact impeded by interfering emotions, individuals completing math tasks may experience negative effects and decreased math performance as a result of math anxiety (Aschraft & Krause, 2007).

Another postulated reason poor performance may be attributed to the high level of attention a perceived threat is receiving. This perceived threat may be redirecting attention towards self-regulatory approaches, thus stopping the individual from accessing whatever information is available to them in their working memory. Further directions for research should attempt to parse emotions and cognitions into two distinct categories that function separately (Baddley, 1992). Only then can research understand if working memory is used during appraisal or if it is not a part cognition.

2.6 Influence of Important Adults

The influence of important adults (i.e. teachers and parents), highly influence children’s beliefs of their own math abilities. Teachers stereotypes and beliefs about which students are good at math subsequently influence students’ own beliefs of their math abilities. This influence of teacher’s biases in turn lowers math achievement among students and increases feelings associated with math anxiety (Beilock, Gunderson, Ramirez & Levine, 2010). However, teacher biases may not be the key component that is interacting with students’ math anxiety levels. Research has pointed to teacher anxiety as possible influence on student subsequent anxiety
development (Geist, 2010). A gap in the literature exists regarding the positive relationship between student and teacher anxiety, as well the negative relationship with student achievement.

Research has indicated a considerable influence of parents on their children’s attitudes towards mathematics and school in general. A study by Davies & Kandel (1981) of 55 grade seven students indicated that adolescents found their parents had a much bigger influence on their attitudes, behaviours and performance in schools, as compared to their friends. This may be in part due to the pressure parents put on their child for specific standards of performance in school and their future. This study will add to the existing literature and help understand how parent perceptions of their children’s math abilities is influencing student’s math achievement scores. However, it is not only parents with high expectations of their children who influence their children’s math performance. It has been shown that children of math-anxious parents learn significantly less math in a school year and have more math anxiety at the end of a school year. This effect held even when attempting to control for development of math anxiety with a non-math anxious teacher. Additionally, the reasoning for anxiety in children of math anxious parents may be attributed to poor homework help, (Maloney, Ramirez, Gunderson, Levine & Beilock, 2015). Thus, specific research questions addressed in this paper will add to the existing literature on the influence math anxious parents may have on their children.

Teachers attitudes about teaching mathematics also plays a large role in shaping the attitudes of students towards learning mathematics (Olatonde, 2009). Through developing curriculum that enhances student’s participation, interest and understanding, teachers can play a direct role in decreasing feelings of anxiety surrounding mathematics (Al-Qaisi, 2010). The process of teaching is a highly important aspect to the development of anxieties in the classroom, as there is a critical link between student achievement and teacher practices within the classroom (Smith, 1987). Research by Sommers (1992) found that quality of teaching was a significant predictor of student achievement when controlling for the effects of student’s characteristics. Quality of teaching can be understood in terms of teaching style, as this has a direct effect on students’ achievement (Smith, 1987). Thus, a teaching style that promotes autonomy and incorporates the interests of students while engaging them in real-time math tasks for conceptual practice (Buchler, 2013), would be beneficial for students in order to have their anxiety levels low and keep them engaged in classroom activities.
Teacher’s math anxieties, however, have been found to influence the subsequent development of math anxiety in their students (Luttenberger, Wimmer & Paechter, 2018; Geist, 2010). This high level of math anxiety teachers experience has been found to be highly correlated with their students’ achievements and fears of math (Hadley & Dorward, 2011), despite autonomy and engaging educational experiences. This is problematic as previous studies have demonstrated that math anxiety affects working memory, math test performance and performance-based situations (Ashcraft & Kirk, 2001; Buelow & Frakey, 2013), demonstrating that developing math anxiety can be quite detrimental to students’ learning and achievement. It has been stated that math teachers’ own levels of anxiety have been shown to implicate students’ math achievement levels (Beilock & Willingham, 2014) and several studies have indicated a negative relationship between teacher math anxiety and student math achievement (Ramirez et al., 2018), however the way in which this relationship has developed has not been studied in depth. What unfortunately is not yet understood is in what way teacher anxiety is affecting poor mathematic achievement in students and what path may lead to the development of math anxiety in students.

### 2.6.1 Measuring Math Anxiety

While math anxiety can be measured using rating scales, researchers have commonly indicated that self-report measures may influence how accurate and truthful participants may be while responding. Thus, several researchers include physiological measures to increase the validity of their research findings. For example measures of cortisol (Maloney & Beilock, 2012), heart rate and skin conductance (Dew, Galassi & Galassi, 1984; Hopko et al., 2003; Faust, 1993), and brain imaging measures, such as Functional Magnetic Resonance Imaging (fMRI) (Lyons & Beilock, 2011) and Electroencephalogram (EEG) (Diego et al., 1996), have been used throughout the literature. By using these measures in conjunction with self-report, researchers have attempted to circumvent possible issues of reliability that may arise when depending solely on self-report measures to assess individuals’ levels of math anxiety.

A more recent avenue that researchers have taken is studying individuals’ real-time emotional responses to mathematics (Goetz, Bieg, Ludtke, Pekrun & Hall, 2013). This can be done using non-invasive technology that can extract all facial expressions data from video recordings of human faces to analyze and aggregate the emotions expressed on the face during a task (imotions.com). Some research on learning and education has implemented these technologies to
analyze facial emotions (Baker, D’Mello, Rodrigo, Graesser & D’Mello, 2010; D’Mello, Lehman, Pekrun, Graesser, 2014). To form a clearer understanding of how math anxiety is presented in individuals this research project will combine all three: facial-emotional data, physiological data, and self-reports to see whether these correlate and which methods are more reliable.

2.6.2 Consequences of Math Anxiety

Consequences of Math anxiety can be extensive, ranging from poor performance on mathematics to low grades in math classes, while the tendency to avoid taking further mathematics courses is the most pervasive (Ashcraft, 2002; Hembree, 1990). The bulk of the literature concerning emotions and anxiety to date has focussed on how both positive and negative emotions affect learning achievement (Frenzel et al., 2007; Pekrun, Elliot & Maier, 2006). Research concerning emotion and cognition indicate that anxious learners experience more impaired attention and memory, causing negative effects on performance (Oatley, Parrot, Smith & Watts, 2011). However, it is still unclear what specific emotions are present in math-anxious learners and how these emotions are affected by varying mathematical tasks.

By understanding these emotional underpinnings, specific physiological states, and possible cause for development of math anxiety, early interventions can be created to attenuate the development of math anxiety in young learners.
Chapter 3

Method

3 Method

3.1 Participants

Invitations to participate in the study were sent to parents of students in grades six, seven, and eight at an independent school in Toronto, Ontario. A complete list of participating grades and classes is provided in Table 1. Within the school, 264 invitations were sent out, 44 consent forms were returned, and 44 students participated in the study. The sample consisted of 44 students between the ages of 11 and 14 years old \([M=12.9, \ SD=0.92]\), 68% female \((n=44)\). Of the 44 parents that agreed to participate, 36 parents completed the demographic questionnaire that was sent to them (Table 2). Of the 6 math teachers that consented to participate, all six teachers completed the questionnaires.

Table 1. Distribution of Student Participation

<table>
<thead>
<tr>
<th>Grade/Class</th>
<th>Consent Distributed</th>
<th>Consent Returned</th>
<th>Participants (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6A</td>
<td>22</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6B</td>
<td>22</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>6C</td>
<td>22</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>6D</td>
<td>22</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7A</td>
<td>22</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>7B</td>
<td>22</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7C</td>
<td>22</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>7D</td>
<td>22</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8A</td>
<td>22</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>8B</td>
<td>22</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8C</td>
<td>22</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>8D</td>
<td>22</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td><strong>264</strong></td>
<td><strong>44</strong></td>
<td><strong>44</strong></td>
</tr>
</tbody>
</table>
Table 2. Distribution of Parent Participation

<table>
<thead>
<tr>
<th>Consent Distributed</th>
<th>Consent Returned</th>
<th>Participants (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>44</td>
<td>36</td>
<td>36</td>
</tr>
</tbody>
</table>

3.2 Measures

The Parent Demographic Questionnaire was sent to parents who completed the consent forms for both themselves and their child (Appendix E). The survey included 17-items that measured parent-reports about the child, family, household environment, education levels, child’s knowledge and motivation with mathematics, and parent’s attitudes towards mathematics. Parents were asked to report their child’s age, birthdate, classroom, number of people living in the household, number of siblings the child participating in the study has, and highest education obtained by both guardians. Parents were then asked to rate their child’s ability in math on a scale ranging from very low (1) to very high (5). Parents were asked to report how often their child practices math outside of the classroom on a scale ranging from None (1) to Everyday (5). Parents were asked to report if, and how many times per week, their child attends extra mathematics classes outside of the classroom (i.e. extra help, tutoring, etc.). Parents were also asked to rate their child’s motivation to practice math on a scale ranging from very low (1) to very high (5). Parents were then asked to rank their own comfort and capability with helping their child with their math homework on a scale of very low (1) to very high (5). Finally parents were asked to rate how anxious they feel about helping their child with their math homework on a scale ranging from very low (1) to very high (5), and if yes, they were asked to describe why they felt anxious about helping their child. The coding process used to evaluate the responses on the parent demographic questionnaire depended on the question and response options. Questions eight, nine, ten, eleven, twelve, thirteen, sixteen, seventeen and eighteen were scored on a scale of one-to-five. Questions eight, nine, ten, eleven, fourteen, and sixteen were scored as positive. Question sixteen was negatively scored. Question twelve was binary score of (1- yes, or 0- no).

Achievement Emotions Questionnaire- Elementary Students (AEQ- ES) is a 28-item (nine enjoyment, 12 anxiety, and seven boredom items) self-report instrument used to assess elementary school student’s achievement emotions. It was adapted from the AEQ, a
multidimensional self-report used to examine college students’ achievement emotional experience in situations of academic achievement (Appendix G; Appendix H) (Pekrun et al., 2012; 2007). In order to cater to the cognitive and language ability of elementary school students, the questionnaire was adapted and uses photos of both boys’ and girls’ faces to ensure both males and females could identify themselves. The instrument was designed to assess emotions of enjoyment, anxiety and boredom during mathematics in elementary school students. Enjoyment and anxiety were measured using three scales relating to experiences in the classroom, doing homework, and writing tests. Boredom was measured on two scales relating to experiences during class time and with homework. All items are answered on a 5-point Likert scale using these male and female faces, respectively (Litchfield, Pekrun, Stupnisky, Reiss & Murayama, 2012).

*Education Quality and Accountability Office (EQAO) Math Questions.* The EQAO is used to assess how well Ontario’s public education system is developing students’ reading, writing, and math skills. This test is administered by the government and allows for a standardized measure of achievement across students in the Ontario School System (Education Quality and Accountability Office, 2018). Five math questions, of varying difficulty, were taken from the grade six and grade nine EQAO (Appendix I; Appendix J). Grade six and seven students were given questions from the grade six EQAO and students in grade 8 were given questions from the grade nine EQAO, as this sample of students take mathematics at an accelerated rate, one year in advanced. The five math questions were answered in multiple choice format. The question answers were scored as incorrect/correct (1/0). Scores were then totaled across each participant. Each of the five questions was first coded either (1- correct) or (0-incorrect) from four possible answers (Appendix I; Appendix J).

*Math Question Difficulty Ratings.* Following each math question, students were asked to rate the difficulty of the math question from (1- not difficult) to (4- very difficult). Each of the five math questions was rated for difficulty and total rating scores were totaled to get a score out of 20 possible points.

*Emotient* (imotions.com) is a facial expression coding software that was used to measure emotions on the face during the task. The software uses Facial Action Coding System (FACS;
Eckman & Freisen, 1979) to measure musculature movement in the face at 19 different Action Units (AU) to code into scores of the nine basic emotions (anger, sadness, fear, frustration, confusion, joy, surprise, disgust, contempt). The software has been widely used in the literature with both children and adults (Bartlett et al., 2009).

3.3 Procedure

This study received ethical approval from the University of Toronto Ethics Review Committee prior to beginning data collection (Appendix A). The Principal Investigator provided packages containing Parent Information Letters (Appendix B), Infographic about the study (Appendix F), and Parent Consent forms (Appendix C). All packages were sent both electronically to parents and home as physical copies with students. Following parental proxy consent, each child provided individual assent to participate in the study (Appendix D). During this assent process the Principal Investigator explained the purpose of the study, what their involvement would entail, that they could stop participation at any time, and that their parents knew about their involvement in the study.

Once parent consent was received, a date and time for their child’s participation was sent to them by email along with a secure link containing the parent demographic questionnaire (Appendix E). Each parent was given a random identification number (ID) that they were to use to input their answers into the online demographic questionnaire.

If the time and date were approved by both parent and teacher of the student, the child was scheduled to participate in the study. The study took place in the participant’s school, during school hours, for one 15-minute session. The child first was given the E4 bracelet (empatica.ca) to wear for the duration of the study. The student was to sit across from the computer, facing the webcam that would be recording the child as they perform the task. Each student was given a random identification number for confidentiality purposes. The study was created on E-prime Software (Psychology Software Tools, Inc., 2016) and all aspects of the research were gathered through the computer. The researcher sat next to the participant for the duration of the study. Each set of instructions and questions was read aloud to the student in order to prevent any influence of reading-level on study results.
First, each student completed the Achievement Emotions Questionnaire- Elementary School (AEQ-ES). Each question of the AEQ-ES was presented on the screen individually. The students used the mouse to select their rating for each question. The purpose of this task was to see students’ self-reports measures of anxiety, boredom, and enjoyment during math class, math homework, and math tests was correlated with their math test scores, math test difficulty ratings, and their real-time emotional experience.

Following the AEQ-ES self-report measure, the students were given a baseline task to measure their resting facial-emotions prior to performing the math test. A ten second baseline was taken by showing the children a neutral video.

After the baseline period, students were given five math questions taken from the EQAO. Following each question, students were asked to rate the difficulty of the given question on a scale from one to four. The purpose for this was to see if students could accurately identify what questions they had difficulty with and whether this would relate to their math scores and emotions experienced for the duration of the question. When the study was complete a message appeared on the screen and was read to each student indicating that they were finished the task. The computer program then automatically closed and saved all responses.
Chapter 4

Results

4 Results

4.1 Sample Characteristics

The sample consisted of 44 students between the ages of 11 and 14 years old \( M=12.19, \ SD=0.92, \) 68% female \((n=30)\). This sample included students in grade six \((n=19)\), grade seven \((n=12)\), and grade eight \((n=12)\). Thirty-six parents filled out the parent demographic questionnaire. The majority of parents in the sample obtained a masters level degree \((45.7\%) \ (n=36)\) or professional degree \((36.1\%) \ (n=35)\). Of the thirty-six parents, one did not report education levels for the second parent/guardian of the child.

4.1.1 Characteristics of the Family

Parents were asked to answer a demographic questionnaire to obtain a clear picture of the child’s home environment and parent’s attitudes towards mathematics. The analysis of parent demographic questionnaire indicated that 62.5\% \((n=20)\) lived in a four-person household, 15.6\% \((n=5)\) lived in a three-person household, 12.5\% \((n=4)\) lived in a five-person household, and 9.4\% \((n=3)\) live in a six-person household. Most participants lived in a home with one sibling as parents reported 53.1\% \((n=17)\) had one older sibling.

To measure attitudes towards math and student’s work on math outside of the classroom, parents rated their child’s motivation to practice math, how often they practice math, and if they attend extra math classes outside of school. Of the parents sampled, 73.5\% \((n=25)\) indicated that their child practices math outside of the classroom one to three times per week and 26.4\% \((n=9)\) indicated that their child practices math outside of the classroom more than four times per week. The majority of parents \((59.1\%, \ n=26)\) rated their children’s motivation to do math as above average, 59.1\% \((n=26)\), and rated their ability to do math as above average, 62.5\% \((n=19)\). Parents were also asked to rate how comfortable and capable they felt helping their child with
their math work and how anxious they feel about helping their child with their math work. Of the parents sampled, 72.7% (n=24) felt above-average comfort with helping their child and 63.7% (n= 28) reported low to some anxiety.

Parent-reported motivation of their child to do math and parent-reported ability to do math were moderately positively correlated (r=0.495, p.004).

4.2 Parent Contributions to Math Anxiety

To test the first hypothesis, that parents’ reports of their child’s math abilities and children’s self-reported emotions will predict performance on math task, an initial Pearson’s correlation analysis was run (Table 3). Child reported anxiety was negatively correlated with parent ratings of the child’s math ability (r=-.387, p=.001) and math test scores (r=-.486, p=.001). The hypothesis that parents’ ratings of their child math ability, the children’s anxiety scores and the math scores would be related was confirmed. Parent math ability ratings were also found to be correlated with children’s math scores (r=.480, p=.001). Parents report of their child’s math capabilities predict actual math scores.

Table 3. Correlation Matrix of Self-Reported Emotions and Parent Ratings of Child’s Math Ability

<table>
<thead>
<tr>
<th></th>
<th>Total Math Scores</th>
<th>Math Anxiety</th>
<th>Math Boredom</th>
<th>Math Score Total</th>
<th>Parent Ratings of Child’s Math Ability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Math Scores</td>
<td>1</td>
<td>-.402**</td>
<td>-.486**</td>
<td>-.319*</td>
<td>.480**</td>
</tr>
<tr>
<td>Math Anxiety</td>
<td>.402**</td>
<td>1</td>
<td>-.457**</td>
<td>-.601**</td>
<td>0.324</td>
</tr>
<tr>
<td>Math Enjoyment</td>
<td>-.486**</td>
<td>-.457**</td>
<td>1</td>
<td>.562**</td>
<td>-.387*</td>
</tr>
<tr>
<td>Math Boredom</td>
<td>-.319*</td>
<td>-.601**</td>
<td>.562**</td>
<td>1</td>
<td>-.178</td>
</tr>
<tr>
<td>Parent Ratings of Child’s Math Ability</td>
<td>.480**</td>
<td>0.324</td>
<td>-.387*</td>
<td>-0.178</td>
<td>1</td>
</tr>
</tbody>
</table>

**, Correlation is significant at the 0.01 level (2-tailed). 
*, Correlation is significant at the 0.05 level (2-tailed). 
N=44

To test the hypothesis that parent ratings of their child’s math abilities would mediate the relationship between the child’s math-anxiety and math test scores, a mediation analysis was run. All assumptions for performing a regression analysis were met. The mediation analysis was conducted using the PROCESS macro for SPSS (Hayes, 2013). Parent ratings of their child’s
math abilities was included as the mediator, math anxiety was included as the dependent variable, and math test scores were included as the dependent variable (see Figure 1). Confidence intervals were used to determine significance, intervals were set to 95%, 5000 bootstrapping samples were used, and the bias corrected confidence intervals were used to determine significance of the test of the indirect effect.

The result indicates a full mediated pathway between child math anxiety, parent ratings of child math ability, and math test performance. All individual pathways are illustrated in Figure 2. Note the pathway is considered significant when the confidence intervals for the effect do not cross zero (Hayes, 2013). The A pathway, from children’s math anxiety to parents ratings of child math abilities was significant $b=-.033$, $t(40)=-2.83$, $p=.0072$, 95% CI [-.0561,-.0094]. The B pathway from parent ratings to math scores was not significant $b=.3785$, $t(39)=1.53$, $p=.1332$, 95% CI [-.1207,.8777]. The mediated effect of parent ratings of child math abilities were significant $b=-.0522$, $t(39)=-2.64$, $p=.0119$, 95% CI[-.0922, -.0122] (Table 4; Table 1; Figure 1).

The model accounted for 28% of the variance in math test scores, as the proportion of variance in $y$ explained by the mediator (mediator) and the independent ($x$) ($R^2_{Y, MX}=.2800$). The estimate standard errors and confidence intervals are shown in Table 4. Pathways are demonstrated in Figure 1.

**Table 4. Mediation Analysis**

<table>
<thead>
<tr>
<th>Model</th>
<th>Estimate</th>
<th>SE</th>
<th>$p$</th>
<th>CI (lower)</th>
<th>CI (upper)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model without Mediator</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>4.7569</td>
<td>.3228</td>
<td>.0000</td>
<td>4.1045</td>
<td>5.4094</td>
</tr>
<tr>
<td>Pathway c</td>
<td>-0.0646</td>
<td>.0183</td>
<td>.0011</td>
<td>-.1017</td>
<td>-.0275</td>
</tr>
<tr>
<td>Model with Mediator</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pathway a</td>
<td>-0.0327</td>
<td>.0116</td>
<td>.0072</td>
<td>-.0561</td>
<td>-.0094</td>
</tr>
<tr>
<td>Pathway b</td>
<td>.3785</td>
<td>.2468</td>
<td>.1332</td>
<td>-.1207</td>
<td>.8777</td>
</tr>
<tr>
<td>Pathway $c'$</td>
<td>-.0522</td>
<td>.0198</td>
<td>.0119</td>
<td>-.0922</td>
<td>-.0122</td>
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<tr>
<td>Indirect effect (a x b)</td>
<td>-.0124</td>
<td></td>
<td></td>
<td>-.0293</td>
<td>.0009</td>
</tr>
<tr>
<td>$R^2_{M,Y}$</td>
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<td></td>
<td>.1671</td>
<td></td>
</tr>
<tr>
<td>$R^2_{Y,MX}$</td>
<td></td>
<td></td>
<td></td>
<td>.2800</td>
<td></td>
</tr>
</tbody>
</table>
4.3 Child Perceptions of Math

To test the hypothesis that students self-reported anxiety, boredom and enjoyment (as measured by the AEQ-ES) would influence scores on the math task and difficulty ratings of the math task, a hierarchical regression was performed. Correlations revealed that math anxiety was positively related to math difficulty ratings (.694, \( p<.000 \)) and negatively related to math scores (-.486, \( p=.001 \)). Math test difficulty ratings were negatively related to math test scores (-.633, \( p<.000 \)). Child math enjoyment was positively related to math test scores (.402, \( p=.008 \)). Child boredom was negatively related to math scores (-.319, \( p=.40 \)) and positively related to math difficulty ratings (.306, \( p=0.049 \))(Table 5).
Table 5. Correlation Matrix of Math Scores Related to Self-reported Emotions and Math Difficulty

<table>
<thead>
<tr>
<th></th>
<th>Math Enjoyment</th>
<th>Math Anxiety</th>
<th>Math Boredom</th>
<th>Math Score Total</th>
<th>Math Test Difficulty Ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Enjoyment</td>
<td>1</td>
<td>-.457**</td>
<td>-.601**</td>
<td>.402**</td>
<td>-.256</td>
</tr>
<tr>
<td>Math Anxiety</td>
<td>-.457**</td>
<td>1</td>
<td>.562**</td>
<td>-.486**</td>
<td>.694**</td>
</tr>
<tr>
<td>Math Boredom</td>
<td>-.601**</td>
<td>.562**</td>
<td>1</td>
<td>-.319*</td>
<td>.306*</td>
</tr>
<tr>
<td>Math Score Total</td>
<td>.402**</td>
<td>-.486**</td>
<td>-.319*</td>
<td>1</td>
<td>-.633**</td>
</tr>
<tr>
<td>Math Test Difficulty Ratings</td>
<td>-.256</td>
<td>.694**</td>
<td>.306*</td>
<td>-.633**</td>
<td>1</td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).
N=44

A hierarchical regression model was constructed with math boredom, and anxiety, difficulty ratings, and math test scores. Math enjoyment was not significantly related to math difficulty ratings and thus was excluded from the model. Prior to performing the regression, assumptions of linearity were confirmed. In the first step of the regression model, the anxiety, enjoyment, and boredom ratings were included. In the second step the difficulty ratings were included in the model. The outcome variable included was math test scores. As is illustrated in Figure 2, the results demonstrated that ratings of difficulty on the math test predict overall math scores ($b=-2.69$, $t=-3.444$, $p=.001$) when accounting for math anxiety ($b=.003$, $t=.106$, $p=.916$) and boredom ($b=-.003$, $t=-.077$, $p=.939$) (Table 6).
Table 6. Coefficients, Standard Errors, and Confidence Intervals for Regression Analysis

<table>
<thead>
<tr>
<th></th>
<th>Unstandardized Coefficients</th>
<th>Standardized Coefficients</th>
<th>t</th>
<th>Sig.</th>
<th>95.0% Confidence Interval for B</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B</td>
<td>Std. Error</td>
<td>Beta</td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>(Constant)</td>
<td>4.929</td>
<td>0.662</td>
<td>7.441</td>
<td>0</td>
<td>3.589</td>
</tr>
<tr>
<td>Math Anxiety</td>
<td>-0.057</td>
<td>0.021</td>
<td>-0.433</td>
<td>-2.738</td>
<td>0.009</td>
</tr>
<tr>
<td>Math Boredom</td>
<td>-0.028</td>
<td>0.04</td>
<td>-0.112</td>
<td>-0.71</td>
<td>0.482</td>
</tr>
<tr>
<td>(Constant)</td>
<td>6.287</td>
<td>0.716</td>
<td>8.779</td>
<td>0</td>
<td>4.837</td>
</tr>
<tr>
<td>Math Anxiety</td>
<td>-0.003</td>
<td>0.025</td>
<td>-0.025</td>
<td>-0.135</td>
<td>0.893</td>
</tr>
<tr>
<td>Math Boredom</td>
<td>-0.033</td>
<td>0.036</td>
<td>-0.131</td>
<td>-0.927</td>
<td>0.36</td>
</tr>
<tr>
<td>Math Difficulty</td>
<td>-0.268</td>
<td>0.08</td>
<td>-0.575</td>
<td>-3.344</td>
<td>0.002</td>
</tr>
</tbody>
</table>

a. Dependent Variable: Total Math Score

Figure 2. Diagram of Hierarchical Regression Analysis

4.4 Math Scores and Emotions

To test the hypothesis that math anxiety and real-time experience of negative emotions would contribute to lower math scores, a linear regression was performed. Due to the high correlations in emotions across all five math questions, each of the four emotion scores was condensed into an individual factor, to avoid multicollinearity (Table 7). All negative emotions scores for the five math questions (i.e. Anger 7-11, Confusion 7-11, Frustration 7-11, Disgust 7-11, Surprise 7-11, Fear 7-11) were condensed into one negative emotion score for all five math questions. Principal Factor Analysis was conducted in order to identify any common variance shared by variables. The principal Axis Factoring accounted for 89% of the variance and a linear regression was performed to extract factor scores per participant. This factor score for each
emotion was used to represent all six negative emotions over the course of five questions. None of the six emotions were correlated with math scores; Frustration (-0.009, \( p=.995 \)), Confusion (0.091, \( p=.575 \)), Fear (-0.028, \( p=.865 \)), Anger (0.001, \( p=.993 \)), Disgust (-0.186, \( p=.250 \)), Surprise (-0.178, \( p=.271 \)). As none of the emotions scores were significantly related to math test scores the regression analysis was not performed.

Table 7. Correlation Matrix of Math Scores Related to Emotion Scores Across Five Questions

<table>
<thead>
<tr>
<th></th>
<th>Total Math Scores</th>
<th>Frustration</th>
<th>Confusion</th>
<th>Fear</th>
<th>Anger</th>
<th>Disgust</th>
<th>Surprise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Math Score</td>
<td>1</td>
<td>-0.009</td>
<td>0.091</td>
<td>-0.028</td>
<td>0.001</td>
<td>-0.186</td>
<td>-0.178</td>
</tr>
<tr>
<td>Frustration</td>
<td>-0.009</td>
<td>1</td>
<td>0.548**</td>
<td>0.451**</td>
<td>0.766**</td>
<td>0.283</td>
<td>0.234</td>
</tr>
<tr>
<td>Confusion</td>
<td>0.091</td>
<td>0.548**</td>
<td>1</td>
<td>0.264</td>
<td>0.597**</td>
<td>0.288</td>
<td>0.394**</td>
</tr>
<tr>
<td>Fear</td>
<td>-0.028</td>
<td>0.451**</td>
<td>0.264</td>
<td>1</td>
<td>0.672**</td>
<td>0.615**</td>
<td>0.672**</td>
</tr>
<tr>
<td>Anger</td>
<td>0.001</td>
<td>0.766**</td>
<td>0.597**</td>
<td>0.672**</td>
<td>1</td>
<td>0.525**</td>
<td>0.652**</td>
</tr>
<tr>
<td>Disgust</td>
<td>-0.186</td>
<td>0.283</td>
<td>0.288</td>
<td>0.615**</td>
<td>0.525**</td>
<td>1</td>
<td>0.737**</td>
</tr>
<tr>
<td>Surprise</td>
<td>-0.178</td>
<td>0.234</td>
<td>0.394**</td>
<td>0.672**</td>
<td>0.652**</td>
<td>0.737**</td>
<td>1</td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed).
N=44
Chapter 5

Discussion

5 Discussion

5.1 Summary of Findings

Parent ratings of children’s math ability was examined in relation to child math anxiety reports and math test scores. It was hypothesized that parents rating of their child’s math abilities would contribute to the child’s self-reported emotions and their math scores. A mediation analysis was conducted to examine the relationship between parents’ ratings of child’s math ability, math anxiety, and math test scores. It was hypothesized that parent ratings of their child’s math abilities would mediate the relationship between child math anxiety and child math scores as previous research has indicated a strong influence of parent’s expectations on their children (Maloney, Ramriez, Gunderson, Levine & Beilock, 2015). There was a main effect of math anxiety on math scores and the overall model was significant. This indicates that the negative relationship between children’s math anxiety and math test scores operates through parents’ perceptions of their child’s math ability.

It was also hypothesized that children’s math anxiety and the difficulty of math question was related to math performance, as previous research has indicated that worldwide, math anxiety has been associated with lower math achievement (Lee, 2009). Math anxiety and math boredom were negatively related to math question difficulty and math scores, indicating that those who experienced more boredom and anxiety also perceived the questions as more challenging. In turn, children’s ratings of math enjoyment were positively related only to math scores but not related significantly to math difficulty ratings. A model indicated that students’ perception of difficulty on the math test predicted math test scores. These results indicate that students can reliably report the emotions they experience during mathematics and how difficult they are finding questions. These reported emotions, specifically anxiety about math, and the difficulty ratings of math questions was directly related to how well they did on the math task. These
findings may indicate that students aged 11-14 have accurate perceptions of their own math abilities as they are able to give reliable self-reports of math difficulty ratings and the emotions they experience with mathematics (Pekrun, 2014).

It was further hypothesized that students’ emotional expression, as detected in real-time by *Emotient* software, could predict performance on a math task. It was hypothesized that negative emotions, which were measured by *Emotient* software, would be negatively affect math test performance. The emotions experienced during the math test were not correlated with math performance, thus the hypothesis that real-time emotions experienced during the math task influence overall math performance was not confirmed.

### 5.2 Parent Ratings of Child’s Math Ability

The first hypothesis in this study looked at whether parents’ ratings of their child’s math abilities would contribute to the child’s self-reported emotions and their math scores. There was a significant relationship between parent ratings, child math anxiety and math scores. This relationship indicates that parent perceptions and expectations may have a strong influence on their children.

These results add to the existing literature which point to parents as influencing the development of student math anxieties and lowering their performance in math (Beilock, Gunderson, Ramirez & Levine, 2010).

A large body of research has been conducted to show that parents own beliefs have a strong influence on the subsequent development of math anxiety in their children (Froiland & Davidson, 2014; Maloney, Ramirez, Gunderson Levine & Beilock, 2015; Berkowitz et al., 2015; Vukovic, Roberts & Green Wright, 2013). However, this research has not been as well studied in younger children. Numerous studies have been conducted with high school students and their parents, indicating that parents’ expectations for their child’s long-term educational attainment predict better achievement among adolescents (Froiland & Davidson, 2016; 2014; 2013). A meta-analysis by Jeynes (2012) indicated that parent expectations had the largest effect on student performance out of any parent involvement variable (ex: reading together, checking homework, communications between parents and teacher, etc.).
The influence of these parent expectations, ex: “my child’s math ability is very high”, may have a direct influence on student self-concept (ex: self-reported math anxiety and math performance) (Bandura, Barbarinelli, Caprara, and Pastorelli, 2001; Foidland & Davison, 2016). Many studies concerning child achievement are guided by the expectancy-value theory, measuring domain specific expectations such as, “I will do well in math this year”. By taking this theory into consideration we can expect that parental expectations are placed on children and the child is internalizing these expectations and in turn performing well on their math work. Thus, it is possible that the children of parents who believe their child is “good at math”, may internalize these views, have lower math anxiety and higher math scores. On the other hand, children of parents who believe their own math ability is low may develop more math related anxieties and perform worse on the math task.

By fostering a growth mindset in our children; teaching them their intelligence can be developed with hard work (Dweck, 2015), parents may be able to positively impact their child’s learning, motivation, self-concept and overall wellbeing. It has been found that how students perceive their own abilities; their mindset, boosts their academic achievement. More specifically, students who took part in a structured program teaching that it was possible to “grow your brain” developed a growth mindset and had better performance as compared to students who had a fixed mindset who did not believe their brain and intelligence were malleable (Dweck, 2015). Thus, by encouraging children to view their brains as something that can constantly be changed, grown, and developed, students may adapt a growth mindset and increase performance on academic tasks.

This finding is an important addition to the current literature. This research indicates that parents can skew their child’s self-concept by simply asserting their expectations or opinions about their child’s ability. It may be important to educate parents about how easily they can influence their child’s self-concept through simple conversation, and thus how their child performs on math.

Future research should focus on an even younger population and use this study as a foundation for creating educational programs catered towards adjusting language in dialogues between parents and their children in order to foster growth mindsets among students in attempt to increase performance in mathematics.
5.3 Math Difficulty Ratings, Self-Reported Emotions, and Math Performance

The next hypothesis postulated whether students self-reported emotions and ratings of math question difficulty accounted for unique variance in math score outcomes. Self-reports of negative academic emotions were correlated with lower scores on the math test. This may indicate that student’s levels of anxiety during math is related to math test scores; students who are more anxious do not do as well in math as compared to their non math-anxious classmates. Students’ difficulty ratings were also negatively related to math test scores, which demonstrates that students who found questions more difficult performed worse on the math test.

Research has indicated that when task difficulties cross above the difficulty threshold and are above a student perceived ability, students must use self-regulatory strategies to manage the perceived difficulty of the task. If students successfully use these strategies, they are more likely to do well on the task. These self-regulatory strategies restructure cognitive appraisals which can be beneficial to task performance. However, if students cannot successfully incorporate self-regulatory strategies, they are much less likely to do well on the task (Calvo & D’Mello, 2011). This may indicate that when students place a negative appraisal on the task (i.e. rate the task difficulty as high) they may be lacking in the necessary self-regulatory strategies to allow them to overcome the associated negative emotions (Eysenck, Derakshan, Santos, & Calvo, 2007). Thus, students who have rated the task as difficult may not be able to overcome their resulting negative emotions and do not perform well on the math test.

There are different goals that are associated with self-regulation strategies. A meta-analysis conducted by Webb, Miles and Sheeran (2012) indicated that cognitive restructuring, by directing attention away from or towards specific aspects of a situation and changing the interpretation of the situation so the emotional impact can be altered, was one of the strongest contributors to performance. For example, participants were told that the experience of certain emotions was normal and that they should accept the emotion or imagine negative events having positive outcomes. By incorporating these types of self-regulation strategies, participants were better able to perform on this task (Webb et al., 2012). This may be indicative of the process some students in this sample are using to minimize the influence experienced emotions may have
on task-performance. Self-regulatory strategies have been found to positively influence students learning and development, if they are experiencing an array of negative academic emotions (D’Mello, Lehman, Pekrun & Graesser, 2014). Future research studies should assess the efficacy of implementing or prompting self-regulatory behaviors in populations of students with self-reported anxiety, in attempt to increase task performance and learning.

Additionally, working memory capacities may be influencing the differing performance among participants on the math test (Engle, 2002; Ramirez, Gunderson, Levine & Beilock, 2013). Students who have stronger working memory may be performing worse on the task, as the perception of threat associated with performing a math task may be “choking” certain students’ working memory capabilities. This “choking” causes the students to report more difficulty on the task, perform worse, and experience higher levels of anxiety during math related events (Beilock & Carr, 2005). It is possible that students who reported higher enjoyment of the task may have larger working memory capacities and thus overperform on the task and while reporting fewer negative emotions (Engle, 2002; Schmeichel & Demaree; Suinn et al., 1988). However, it is unclear whether students’ working memory capacity is affecting their performance. It may be possible that students who perform well on the task and report lower anxiety have lower working memory capacity but are relying on mental shortcuts during the math task (Beilock & DeCaro, 2007). These mental shortcuts may be allowing the students to avoid relying on their working memory to answer questions. In fact, future research should focus on pinpointing which cognitive strategies can help students free up working memory space and provide appropriate coping strategies to lower “choking” effects and anxiety development during stressful and pressure inducing evaluative educational tasks.

While researchers have critiqued the use of self-report measures in children due to problems of working-memory and lack of metacognitive awareness, indicating that student’s memory becomes inhibited by their current affective states (Lagutta, Elrod & Kramer, 2016), this research uses a measure that has high reliability with an even younger population of students (Litchenfeld. Pekrun, Stupinsky, Reiss, Murayama, 2017). The self-reports of anxiety and boredom among students in this sample were found to be highly negatively correlated with performance, while enjoyment was found to be positively correlated with performance on the math task. Additionally, student’s math question difficulty ratings were also correlated with math
performance. This finding can add to the existing literature indicating that older elementary school students’ emotions are related to how well they do in mathematics. For this specific age group, students’ reports of anxiety and boredom are related to their math test scores.

5.4 Real-Time Emotions and Math Performance

It was hypothesized that emotions elicited in real-time during a math task, as measured by *Emotient*, would influence math test performance. However, the relationship between emotions on the math test were not significantly correlated with math performance. Based on the hypothesis, it was expected that real-time negative emotions would be highly correlated with math performance. Past research implementing real-time emotion data, as measured by emotion coding software, has been implemented in studies to measure and adapt tasks to improve student learning experiences. The creation of online learning programs and intelligent tutoring systems that use this interactive technology can identify students’ emotions and accommodate the task in order to improve learning experiences for students (Sazzad, AlZoubi, Calvo, & D’Mello, 2011). The facial emotion data was found to be highly reliable and had extremely useful practical implications for educational research (Sazzad et al., 2011). However, regardless of reliability of the software, the findings in this study indicated that the emotion data gathered was not related to the outcome variable.

These results may be indicative of many things. Firstly, it is possible that there is an issue of outliers. Since many emotions are simultaneous coded, there is a unique problem for emotion expression data because the outliers may indicate micro-expressions (Olderback, Hildebrandt, Pinkpank, Sommers & Whilhelm, 2014). These micro-expressions are small subtle changes in the face that may be indicative of emotional reactions in some individuals. However, due to the subtleness of these changes in expression, the software may not be able to identify micro-expressions as actual facial emotions (Olderback et al., 2014). Thus, the software may exclude this from facial expression data and sort them as outliers.

These issues with detection of slight musculature movements may have a direct negative effect in the classroom. Teachers may not be able to notice students who are struggling and anxious about math, as they are unable to notice subtle nuances in facial emotions of their students,
similarly to the issue the software is having. Math anxiety presents itself as “feelings of tension or apprehension” (Ashcraft, 2002), however if software or teacher cannot pick up on these feelings based on small or no facial expression changes, there must be a discrepancy between how the child is feeling and what they are presenting as facial emotions (Sallquist, Eisenberg, Spinrad, Eggum, & Gaerter, 2009). Further pedagogical classroom training should focus on supporting students that are math-anxious through methods other than simply observation. Possible solutions would be to ask students to rate how they anxious they are feeling about certain topics throughout the school day and directly make changes based on self-reports of emotions.

The software also may not be able to differentiate between emotions. Gross (2002) indicated that there is a phenomenon called “faking an emotion”. This includes individuals showing a typical facial expression without feeling that emotion. Based on the two-factor theory of emotion (Schacter & Singer, 1962) an outside stimulus first causes physiological responses, then leading to a psychological attribution of that responses, and finally the experience of that emotion. Students who are experiencing a negative emotion internally may be first experiencing a strong physiological response such as sweating and attributing it to the stimuli (i.e. math question). However, it has been found that anxiety does not present itself the same way across many students (Weisbrot, Gadow, DeVincent & Pomeroy, 2005). While most students do experience physiological responses to an anxiety provoking stimulus, they are likely to vary in how they deal with this. Students have been found to engage in avoidance mechanisms by sitting quietly and being disengaged to avoid showing their true feelings about the topic at hand in an academic situation (Bledsoe & Baskin, 2014). Thus, students taking part in the math study may be feeling a physiological response to the math questions during the study but chose to engage in avoidance mechanisms or self-regulatory behaviors to prevent exhibiting how they are truly feeling about the material, in fear of failure or ridicule (Gross, 1998). Future research should consider the use of physiological measures when trying to understand the cause for math anxiety development.

As children get older, they learn that emotions are a key social communication tool that can be used when interacting with others to portray emotions and understand the emotions of others in these interactions (Sallquist et al., 2009). In a study performed by Sallquist et al. (2009), it was found that children in grades one to three become aware of social and cultural norms and the
implications their facial expressions may have on interactions with others. This allows children to learn how to recognize emotions of others and to inhibit certain emotions that they believe may yield undesirable outcomes during social interactions. Based on these findings, it may be plausible to assume that students are inhibiting their facial expression of emotions in fear of exhibiting an undesired emotion, in attempt to facilitate a more positive image of themselves and their emotions during social interaction (Sallquist et al., 2009). Young students may benefit from emotional intelligence training programs that help teach about differing emotions and emotional reactions. This may help normalize the experience of certain emotions and allow students to feel more comfortable to express their true emotions.

Additionally, the reason the software may be having trouble detecting changes in muscle movements in the face is that this task was not designed to overtly induce negative emotions. The EQAO is a standardized test that is designed to measure math performance in students in Ontario (Education Quality and Accountability Office, 2018). The sample in this study consisted of students in grades six, seven, and eight. The school at which the study was performed provides math instruction one-year in advanced to their students, meaning that students in grade five would have been taught grade six mathematics and so forth. The grade six and seven students were each given the grade six EQAO. The math task may simply not have been difficult enough to induce negative emotions. Though we did expect students who rated themselves high on math anxiety to also exhibit negative academic emotions when performing on math tests, we did not find this result. We also expected students who rate the math questions as difficult to exhibit higher levels of negative emotions, though we did not find this result either.

5.5 Limitations and Future Directions

A limitation of this study is the homogeneity of the sample. The data for this study was collected from an independent school in Toronto. As such the generalizability of the research findings may be compromised, as the sample is not representative of the population due to higher SES of this population. Thus, the sample of higher SES individuals may have skewed the results (Esben, Miller, Taylor & Freng, 1999; Maloney, Ramirez, Gunderson, Levine, Beilock, 2015). The addition of students from a more diverse economic background may benefit the validity of the results.
A second possible limitation of the study is that students were taken into a separate room to participate in the study. The inherent anxiety associated with taking a math test and working in a separate room with a stranger may have influenced student emotional and physiological responses to the task (Leary, 2013; Paulos & Goodman, 2004; Spangler, 1997). This was minimized as much as possible by telling participants that their responses were confidential and anonymous, however the possibility for fear of evaluation and stranger-anxiety still exists.

Future research should consider the distinction between emotions and cognition. For example, including education about cognitive restructuring and other self-regulatory behaviours. Research must also understand how to effectively measure both executive functioning and working memory during differing emotional states in order to understand what underlying mechanisms are contributing to lower performance in students. Possible next steps should assess working memory capacity in tasks provoking differing emotional valence (Beilock & Carr, 2005).

5.6 Conclusions

This study explored emotion expressions and self-reported emotions contribution to performance on a mathematics task in elementary-school students. It was found that parent ratings of child math ability were significantly related to child’s math anxiety and math performance, indicating that parental expectations are highly influential on student performance. Additionally, there was a negative relationship between math anxiety and math boredom with difficulty ratings and math performance. This association indicated that the AEQ was a good measure of student emotions and that anxiety and boredom was related to math scores for this sample of grade six to eight students. Finally, there was no significant relationship between any of the negative emotions as measured by the imotions software and performance on the math task.

The implications of this study will inform research on the causes and development mathematics anxiety and self-regulation of emotions. This study suggests that further research on self-reported emotions may be imperative to measuring development of math related anxieties in elementary-school-aged children. The results also indicate that further research on self-regulatory mechanisms may be imperative in order to improve math performance. The methodological paradigm can be replicated to address all limitations of the study, including a more diverse sample and including different questionnaires for different age groups.
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Appendix A

Ethical Approval

Dear Ana Zdravkovic:

Re: Your research protocol application entitled, “Understanding Math Anxiety in School Aged Children”

The Social Sciences, Humanities & Education REB has conducted a Delegated review of your application and has granted approval to the attached protocol for the period 2019-03-20 to 2020-03-10.

Please be reminded of the following points:

- An Amendment must be submitted to the REB for any proposed changes to the approved protocol. The amended protocol must be reviewed and approved by the REB prior to implementation of the changes.

- An annual Renewal must be submitted for ongoing research. You may submit up to 6 renewals for a maximum total span of 7 years. Renewals should be submitted between 15 and 30 days prior to the current expiry date.

- A Protocol Deviation Report (PDR) should be submitted when there is any departure from the REB-approved ethics review application form that has occurred without prior approval from the REB (e.g., changes to the study procedures, consent process, data protection measures). The submission of this form does not necessarily indicate wrong-doing; however, follow-up procedures may be required.

- An Adverse Events Report (AER) must be submitted when adverse or unanticipated events occur to participants in the course of the research process.

- A Protocol Completion Report (PCR) is required when research using the protocol has been completed. For ongoing research, a PCR on the protocol will be required after 7 years, (Original and 6 Renewals). A continuation of work beyond 7 years will require the creation of a new protocol.

- If your research is funded by a third party, please contact the assigned Research Funding Officer in Research Services to ensure that your funds are released.

Best wishes for the successful completion of your research.
Appendix B
Parent Information Letter

Title of Project: Understanding Math Anxiety in School Aged Children

Principal Investigator: Ana Zdravkovic, BA., MA. (Candidate)  Supervisor: Dr. Earl Woodruff

Dear Parents,

I am an alumna of the Bayview Glen Graduating class of 2013. I am currently a graduate student in the Developmental Psychology & Education Program at the University of Toronto, working under the supervision of Dr. Earl Woodruff. I have been invited back to Bayview Glen to conduct research for my master’s thesis, studying the pattern and severity of math anxiety in school aged children. Please consult the information below regarding my research goals and what your child’s participation will entail. At your request, you can receive a summary report following your participation. The report will summarize research findings and possible recommendations or implications of the importance of real-time emotion identification in student’s math engagement and performance. I appreciate you taking the time to review the enclosed information and hope you will consider consenting your child to participate in this study.

What will this study be looking at?
This study will be a descriptive analysis examining children’s emotions experienced while completing a math task. The emotions children experience while completing these questions may be indicative of the types of emotions that are elicited when children are feeling anxious about certain types of math questions. Additionally, we will be looking at the specific emotions and feelings teachers have towards math and how this may correlate to emotions students experience during math tasks.

Why is this study important?
This study will provide a comprehensive understanding of the prevalence, severity, and risk associated with the development of math related anxieties. This may allow for a better understanding of what emotions students will display prior to the development of math related anxieties. Thus, research may be able to better develop interventions to lower any emotions associated with the development of math anxiety.

What will my child’s participation entail?
If you consent to allow your child to participate in this study, your child will engage in a math task that will take approximately 25 minutes. Your child will be under no time constraint. These questions will be taken from the Education Quality and Accountability Office (EQAO) math questions, which will be at grade level. During the task, I will be collecting audio and video recordings of your child. These video recordings will be used to analyze emotions displayed on your child’s face while completing the math questions. The software used to conduct this analysis, Emotient (emotions.com), measures intensity and valence of emotions displayed on an individual face by tracking the musculature movement in real time. There is no safety or security risk for your child. Your child’s video and audio file will be kept completely confidential. This software is widely used in multiple research contexts and projects.

Your child will also be asked to wear a small bracelet called the E4 wristband (empatica.com) that will be used to measure physiological processes (ex: heart rate) on the wrist. This is a validated, medical grade tool used across many psychological studies with children. All physiological data will be kept completely confidential.
Your child will also be asked to complete a series of self-report questionnaires of emotions experienced during math-related tasks, based on their age. Students in grades 2-5 will complete the Elementary School Students (AEQ-ES). The participants in grades 6-8 will complete both the Elementary School Students (AEQ-ES) and the AEQ- Mathematics (AEQ-M), a second self-report measure of emotions experienced during math tasks.

How will confidentiality be upheld?
All audio and video recordings collected during this study will be kept completely confidential, as only the principal investigator and supervisor will have access to them. Neither your child’s name, nor your name, will be used or connected in any way with this study. An identification number will be assigned to your child and they will be identified using this number. All results will be reported at the group level.

What are the risks associated?
There are no known risks to participating in this study. The mathematic questions presented to the child will be at their grade level and are not intended to elicit any intense emotions. All surveys are validated and well-used questionnaires. The questions presented in the survey are not intended to elicit any intense emotions. If your child feels uncomfortable at any time, he/she is free to withdraw from the study without any penalty from myself, the teachers, or the principal of the school.

What are the benefits?
By having your child participate in the study, research will greatly benefit from understanding the emotional experience of students doing mathematics. This work hopes to identify cognitive and emotional profiles of students struggling with math in order to create proper interventions in the future.

How can I agree to have my child participate?
If you consent to allow your child to participate in this study, please sign the consent form and fill out the survey. All this information has been sent home in a package with your child. Once the consent form is signed and the survey has been filled out, please place the documents into the envelope included within the package. Please seal the envelope and send it back to school with your child. Please ensure you do not write any identifying information on the outside of the package.

Thank you very much for considering to have your child participate. I am extremely grateful for the time you and your child are contributing to my research. If you have any questions, request any additional information regarding the findings of this study, published results or additional research that is being conducted in our lab, please contact myself or my supervisor.

If you have any questions or concerns regarding your rights as a participant of this study, please contact the University of Toronto, Office of Research Ethics at ethics.review@utoronto.ca or (416) 946-3273

Protocol Number: 00037230

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Appendix C

Informed Consent

Appendix C- Informed Consent Letter

Title of Project: Understanding the Pattern and Severity of Math Anxiety in School Aged Children

Before agreeing to participate in this study, we encourage you to read the following description of the purpose of this study, the procedures, possible benefits of participating, and your right to withdraw from the study at any time.

Background & Purpose of this Research. The purpose of this study is to identify emotions children demonstrate on their faces while they complete a mathematic task. Your child's participation in this study will help us to gain a better understanding of the development of math related anxieties children may face while completing math questions.

You will be asked to fill out a short demographic questionnaire, as a part of this study. Your child will be asked to give assent to participate. If the child grants assent, he/she will be asked to participate in a computer math activity and fill out a series of self-report questionnaires. These questionnaires ask about the emotions experienced during math-related tasks, based on student's age. Students in grades 3-5 will complete the Elementary School Students (AEQ-ES). The participants in grades 6-8 will complete both the Elementary School Students (AEQ-ES) and the AEQ - Mathematics, a second self-report measure of emotions experienced during math tasks.

During the computer math activity, the researcher will present your child with a series of math questions on the computer and ask them to complete each question. While the child is completing the questions, an audio recorder will record his/her voice and a video recording software (FACET) will record the emotional expressions displayed on his/her face. The video files will be analyzed within the FACET software. Based on muscular movement on the face, the software will code the emotions experienced during the task as one of the seven basic emotions (happy, sad, angry, fearful, surprise, disgust, contempt).

During the computer activity, your child will also be asked to wear a small bracelet called the E4 wristband (emotica.com) that will be used to measure physiological processes (e.g., heart rate) on the wrist. This is a validated, medical grade tool used across many psychological studies with children. All physiological data will be kept completely confidential.

Confidentiality. Participation in this study is voluntary and consent can be withdrawn at any point during the study without any consequence. If you wish to withdraw from the study, any collected data will be destroyed and permanently deleted from computers and all other storage devices. All your information will be kept completely confidential. Information collected from questionnaires will be anonymous and data collected from in-person sessions will not be shared beyond the researchers listed below. All text, audio, and video data will be saved on the researcher's lab computer, which can only be accessed by the researchers listed below. To meet the University of Toronto's data security and encryption standards, data files will be encrypted using a software that has comprehensive functions to protect and secure the data. All physical data will be kept in a locked drawer in our lab and will only be accessible to the researchers listed below. The information collected for this study will be stored for seven years before being destroyed and will not be used for any other purpose than informing the current study.
Potential Risks. There are no known risks associated with participating in this study. You can chose to terminate your participation at any point during the study if you or your child feels uncomfortable.

Potential Benefits. Your participation in this study will help us further understand the pattern and development of math related anxieties in school aged children in order to create appropriate interventions.

At your request, you can receive a summary report following your participation. This report will summarize research findings and possible recommendations or implications of the importance of real-time emotion identification to student’s maths engagement and performance.

You are welcome to contact the researchers with any questions that you have during completion of the survey. If you have further questions once the interview is completed, you are encouraged to contact the researcher using the information listed below. If you have any questions regarding your rights as a participant of this study, please contact The University of Toronto, Office of Research Ethics at ethics.review@utoronto.ca or call (416) 946-3273. Protocol number: 00037230.

I, ___________________________(please print full name), have read the information above and freely agree to participate in this study. I understand that I am free to refuse to answer any questions and to withdraw participation from the study at any time. I understand all the information collected for the purpose of this study will be kept completely confidential.

_____ I consent to participation

(Participant Signature) (Date)

Principal Investigators

Ana Zdravkovic, B.A. M.A. (Candidate)  Dr. Earl Woodruff
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Appendix D
Child Assent

Title of Project: Understanding the Pattern and Severity of Math Anxiety in School Aged Children

Principal Investigator:
Ana Zdavkovic, B.A., M.A. (Candidate)
The Ontario Institute of Studies in Education at U of T

Supervisor:
Dr. Earl Woodruff
The Ontario Institute of Studies in Education at U of T

Why are we doing this study?
We want to understand how students do math questions and how those math questions make students feel. We want to understand if those feelings affect how well they do other math questions.

What will happen during this study?
If you and your parent(s) agree to participate in this study, a researcher will come to your classroom and ask you to answer some math questions on the computer. While you answer these math questions, the computer will record you and read the emotions on your face. You will also fill out a survey that will ask you about how you feel while you do math. While you are doing the study you will be wearing a small bracelet to measure physiological processes like heart rate on the wrist.

What are the good things that will happen if I am part of this study?
If you decide to participate in our study, you will be helping us understand more about how students feel while they are answering math questions. This could help us figure out better ways to teach math to kids and think of ways to help kids who have trouble with math.

What are the bad things that will happen if I am part of this study?
Nothing bad can happen to you if you participate in our study. If you feel upset about any of the math questions you are given, you can skip the question.

Who will know about what I did for this study?
Only two people running the study will see your videos and math answers.

Can I decide if I want/don’t want to be a part of the study?
Yes. You can choose if you want to participate in the study, or not. You can also change your mind and stop participating at any time during the study.

Would you like to participate (Please circle one)? Yes No

Name of Child (please print full name): ____________________________

Name of Witness (please print full name): ____________________________

Date: ____________________________ Signature of Witness: ____________________________
Appendix E

Parent Demographic Questionnaire

Participant ID: _______ Child’s Age: _______
Child’s Birth date (MM/DD/YYYY): _____/____/_____
Name of Child’s Teacher: _______________________

1. How many people currently live in your household? ______________

2. How many older siblings does the child participating in the study have? ______________

3. How many younger siblings does the child participating in the study have? ______________

4. What is the highest level of education attained by the parent/guardian of the household?
   a. High School
   b. College
   c. Undergraduate Degree
   d. Master’s Degree (MA, MSc, MEd, MBA)
   e. Doctorate/Professional Degree (MD, DDS, DC, JD, PhD)

5. If applicable, what is the highest level of education attained by the second parent/guardian of the household?
   a. High School
   b. College
   c. Undergraduate Degree
   d. Master’s Degree (MA, MSc, MEd, MBA)
   e. Doctorate/Professional Degree (MD, DDS, DC, JD, PhD)
   f. Not Applicable

6. Please rate your child’s math ability at this time:
   a. Very Low
   b. Below Average
   c. Average
   d. Above Average
   e. Very High

7. How many times per week does your child practice math outside of the classroom (i.e. homework, practice questions)?
   a. None
   b. 1-2
   c. 3-4
   d. 5-6
   e. Everyday
8. Does your child attend any mathematics classes outside of school (i.e., tutoring, extra-help)?

Yes  No

If Yes, how many times per week ______.

9. Please rate your child’s motivation to practice mathematics:
   a. Very Low
   b. Below Average
   c. Average
   d. Above Average
   e. Very High

10. Please rate your overall comfort level and capability of helping your child with their math work:
    a. Very Low
    b. Below Average
    c. Average
    d. Above Average
    e. Very High

11. How anxious do you feel about helping your child with their math work:
    a. Low Anxiety
    b. Some Anxiety
    c. Moderate Anxiety
    d. Quite a bit of Anxiety
    e. High Anxiety

If yes, please describe why:

________________________________________________________________________

________________________________________________________________________
Appendix F

Study Infographic

UNDERSTANDING MATH ANXIETY IN SCHOOL AGED CHILDREN
OISE at The University of Toronto

Principal Investigator:
Ana Zdavkovic

WHAT WILL THIS STUDY BE LOOKING AT?

- The types of emotions students experience during a math task
- This may include emotions students experience when they feel anxious about mathematics

WHY IS THIS STUDY IMPORTANT?

- This study will provide comprehensive understanding of the prevalence, severity & risk associated with the development of math related anxieties

WHAT WILL MY CHILD’S PARTICIPATION ENTAIL?

Your Child will:

- Answer 5 math questions from the EQAO
- Fill out a self-report questionnaire
- Be visually recorded during task to analyze emotions on the face
- Wear an E4 wristband to measure physiological processes (e.g., heart rate)

CONFIDENTIALITY & RISK

- All data, audio & video recordings will be kept completely confidential
- There is no known risk to participating in this study. All surveys & measures validated and widely used
- Neither your child’s name, nor your name will be used
- If your child feels uncomfortable at any time, he/she is free to withdraw at any time

Thank you for considering to have your child participate.
I am very grateful for you and your child’s time.

If you have any question, or require any additional information, please feel free to contact me.

Contact:
ana.zdavkovic@mail.utoronto.ca
(416) 589-9310
Appendix G

Achievement Emotions Questionnaire – Elementary School Boys

Math-Class
Please color in the circle that best fits your answer.

1. I enjoy math class.
   - 1: not at all
   - 2: a little
   - 3: somewhat
   - 4: a lot
   - 5: very much

2. Math class bores me.
   - 1: not at all
   - 2: a little
   - 3: somewhat
   - 4: a lot
   - 5: very much

3. Math scares me.
   - 1: not at all
   - 2: a little
   - 3: somewhat
   - 4: a lot
   - 5: very much
Appendix H

Achievement Emotions Questionnaire – Elementary School Girls

Math-Class
Please color in the circle that best fits your answer.

1. I enjoy math class.
   - 1 not at all
   - 2 a little
   - 3 somewhat
   - 4 a lot
   - 5 very much

2. Math class bores me.
   - 1 not at all
   - 2 a little
   - 3 somewhat
   - 4 a lot
   - 5 very much

3. Math scares me.
   - 1 not at all
   - 2 a little
   - 3 somewhat
   - 4 a lot
   - 5 very much
Appendix I

Grade 6 EQAO Math Questions

16. What value of $m$ makes this equation true?
   $72 - 16 = 8 \times m$
   - 7
   - 9
   - 11
   - 56

16. Trang earns money babysitting for four nights. She earns $23.45$ for the first night, $25.65$ for the second and $32.35$ for the third. She earns $100$ in total.
   Which of these is the \textbf{best estimate} of how much Trang earns on the fourth night?
   - $15$
   - $20$
   - $75$
   - $80$

5. A bean plant grows 0.7 cm each day.
   Which length is the closest to the total growth of the plant in one month?
   - 28 cm
   - 21 cm
   - 4.9 cm
   - 2.8 cm

12. What will the 9th term of this pattern be if it continues to decrease by the same amount?
   - 96, 84, 72, 60 ...
   - 0
   - 5
   - 12
   - 24

12. Which of the following polygons has sides that are all equal in length, and no right angles?
   - square
   - rectangle
   - isosceles trapezoid
   - equilateral triangle
Appendix J

Grade 8 EQAO Math Questions

3. Which of the following is a simplified form of $(-2m + 3) - (5m - 6)$?
   a. $3m - 3$
   b. $3m + 9$
   c. $-7m - 3$
   d. $-7m + 9$

16. What are the slope and the $y$-intercept of the line represented by $3x - 2y + 6 = 0$?
   a. $\frac{3}{2}, 3$
   b. $\frac{3}{2}, 6$
   c. $\frac{2}{3}, 2$
   d. $\frac{2}{3}, 3$

18. Fresh Springs Water Company delivers bottled water.
   The total cost of the water, $C$, in dollars, is represented by $C = 8 + 1.5n$, where $n$ is the number of litres.
   Which of the following statements could be true?
   a. $1$ for every $9.5$ L of water.
   b. $9.50$ for each litre of water.
   c. an $8$ delivery charge and $1.50$ per litre of water.
   d. a $1.50$ delivery charge and $8.00$ per litre of water.

19. Which of the following dimensions produces a rectangle with the smallest perimeter?
   a. $10\ m \times 120\ m$
   b. $30\ m \times 40\ m$
   c. $50\ m \times 24\ m$
   d. $60\ m \times 20\ m$

8. The total cost of yearbooks for a school is made up of a $375$ set-up fee and $25$ for each yearbook purchased.
   There is a linear relationship between the total cost and the number of yearbooks purchased.
   What type of variation is this relationship, and what is its initial value?
   a. direct variation, $375$
   b. direct variation, $25$
   c. partial variation, $375$
   d. partial variation, $25$