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New method to calculate the apparent phase velocity of open-ended pipe pile

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\textbf{Abstract}: The apparent phase velocity of open-ended pipe piles after installation is difficult to predict owing to the soil plug effect. This paper derives an analytical solution to calculate the apparent phase velocity of pipe pile segment with soil plug filling inside (APVPSP) based on the additional mass model. The rationality and accuracy of the developed solution have been confirmed through the comparison with the solution derived using the soil-plug Winkler model and experimental results. A parameter combination of the additional mass model that can be applied in most commonly used concrete pipe piles is recommended. The attenuation mechanism of the soil plug on the APVPSP has been clarified. The findings from this study demonstrate that the APVPSP decreases with the mass per unit length of the pile, but has nothing to do with the material longitudinal wave velocity of pipe pile. The APVPSP decreases significantly as the impulse width increases, however, for pipe piles without soil plug filling inside, the impulse width has negligible influence on the apparent phase velocity.

\textbf{Keywords}: Apparent phase velocity; soil plug; pipe pile; additional mass model; low strain test.

\textbf{Introduction}

Pipe pile is widely used all over the world for its apparent advantages such as good adaptability, high bearing capacity, and remarkable economic results. However, various defects may occur during pipe pile installation regardless of the driving method. Low strain integrity testing is a cost-efficient and straightforward method for evaluating pile integrity (Likins and Rausche 2000). For open-ended pipe piles, the interpretation of the measured signal faces more challenges due to the existence of soil plug, which
affects not only the reflected signal waveform but also the apparent phase velocity of pipe piles (Guo and Ke 2011). Finding a rational theoretical model to simulate the dynamic interaction between the soil plug and pipe pile during low strain testing, is of great importance in improving the testing accuracy and reducing miscalculations or misjudgments.

The ‘pile within a pile’ model pioneered by Heerema and De Jong (1979), which respectively discretizes the soil plug and the pipe pile volume into lumped nodes and springs, with frictional forces between the corresponding soil nodes and pile nodes, is often used in pile driving analysis, and estimating the static soil resistance distribution using the dynamic testing method (Randolph and Simons 1986; Matsumoto and Takei 1991; Liyanapathirana et al. 1998, 2001). However, this model is rarely utilized in the low strain testing of pipe piles, mainly because that the ‘pile within a pile’ model is based on the one dimensional stress wave theory (Smith 1960), whereas large diameter pipe piles exhibit serious three-dimensional characteristics during low strain test (Liao and Roesset 1997; Chow et al. 2003; Chai et al. 2010; Ding et al. 2011; Zheng et al. 2016a, 2016b, 2017; Li et al. 2017; Li and Gao 2019). The measured reflected signal of low strain testing of pipe piles may be misinterpreted using the one-dimensional stress wave theory. The plane strain model (Novak et al. 1978) and its simplified dynamic Winkler model (Lee et al. 1988; Wang et al. 2010) are two primary models to simulate the surrounding soil-pile interaction during low strain test. However, these two models cannot directly extrapolate to the soil plug-pipe pile interaction owing to the different boundary conditions, where the plane strain model assumes the surrounding soil to be an infinite medium, but indeed the soil plug is encircled inside the pipe pile, and the vibration energy cannot be propagated to the far-field (Randolph 2003). Another apparent deficiency of the two commonly used models is that the effect of soil plug mass is not taken into consideration, which in fact remarkably affects the energy transmission and the apparent phase velocity of pipe piles.

The apparent phase velocity of pipe piles, which plays an essential role in length estimation and defect detection, is conventionally taken as the material longitudinal wave velocity measured before the pile installation. However, the apparent phase velocity cannot be treated as a constant when considering the soil damping effect. Makris and Gazetas (1993) modeled the pile surrounding soil as dynamic Winkler medium and reported that the pile phase velocity varies with frequency. Wu et al. (2017a) observed that the apparent phase velocity for the pipe pile segment with the soil plug decreases by nearly 50%. Nevertheless, none of the existing soil plug models can reflect this phenomenon. Therefore, a new soil plug model, namely, the additional mass model was proposed by Wu et al. (2017a, 2017b) to solve this problem. In this model, the
soil plug is divided into small segments connected to the inner pile wall through the distributed Voigt model. Based on the additional mass model, Wu et al. (2017b) derived an analytical solution of the vertical dynamic velocity response at pipe pile head which matches well with the measured curve, and Liu et al. (2017) obtained an analytical solution of the torsional dynamic response at pipe pile head. Furthermore, Liu et al. (2018a) provided a more accurate solution with the consideration of stress wave propagation in both the vertical and circumferential directions, and proposed the double-velocity symmetrical superposition method to eliminate the high-frequency interference at pipe pile head without increasing the predominant period of impact pulse (Liu et al. 2018b). The merits and accuracy of the additional mass model have been confirmed through a series of model and field tests, but, still, two factors limit its application: (a) the apparent phase velocity is back analyzed from the dynamic velocity response, which is not straightforward and needs sophisticated derivation; (b) the selection of its parameter combination has not been clarified.

In light of this, the objective of this paper is to present an analytical solution to calculate the apparent phase velocity of the pipe pile segment with soil plug (APVPSP) based on the additional mass model. A parameter combination suitable for most commonly used concrete pipe piles is recommended. The rationality and accuracy of the developed solution have been confirmed through the comparison with the solution using the soil-plug Winkler model and experimental results. A parametric study is also conducted to investigate the influence of soil plug, pipe pile and incident impulse on the APVPSP.

**Theoretical model and derivations**

*Analytical solution of the APVPSP calculated using the additional mass model*

The conceptual model is shown in Fig. 1. \( r_1 \) and \( r_2 \) represent the outer and inner radii of the pipe pile, respectively, and \( h_p \) is the wall thickness of pipe pile and satisfies \( h_p = r_1 - r_2 \). The dynamic interaction between surrounding soil-pipe pile is simulated by the distributed Winkler model, of which \( k_1 \) and \( c_1 \) represent the spring constant and damping coefficient, respectively. The soil plug herein is divided into inner and outer two annular columns. \( r_3 \) and \( b_s \) denote the inner radius and wall thickness of the outer annular column, respectively, and \( b_s = r_2 - r_3 \). Assume only the outer soil-plug annular column taking part in the vibration of pipe pile shaft. The pipe pile and the outer soil-plug annular are further discretized into unit length segments along the vertical direction. Each of the unit soil-plug segments is connected to the inner wall of the corresponding pile segment through the distributed Voigt model with spring constant \( k_2 \) and damping coefficient \( c_2 \), where the dashpot represents the viscous damping due to relative velocity between
the pile and the soil adjacent to the pile. The mass per unit length of each additional soil-plug segment is
given by \( m_s = \pi (r_2^2 - r_1^2) \rho_{s2} \), where \( \rho_{s2} \) is the density of the soil plug.

Fig. 1 Conceptual model of pile-soil dynamic interaction

The vertical dynamic vibration of the pipe pile segment can be established as

\[
E_p A_p \frac{\partial^2 u_p(z,t)}{\partial z^2} + \delta_p A_p \frac{\partial^3 u_p(z,t)}{\partial t \partial z^2} - \rho_p A_p \frac{\partial^2 u_p(z,t)}{\partial t^2} - 2\pi r_1 [k_1 u_p(z,t) + c_1 \frac{\partial u_p(z,t)}{\partial t}] - 2\pi r_2 k_2 [u_p(z,t) - u_s(z,t)] - 2\pi r_2 c_2 \frac{\partial [u_p(z,t) - u_s(z,t)]}{\partial t} = 0
\]  

(1)

where \( u_p(z,t) \) and \( u_s(z,t) \) represent the vertical displacements of the pipe pile and soil plug segments, respectively; \( E_p, A_p, \rho_p \) and \( \delta_p \) denote the Young’s modulus, cross-sectional area, density and material damping coefficient of the pipe pile, respectively. \( k_1 \) and \( c_1 \) are derived from Novak’s elastodynamic solution for vibrating piles in a horizontally homogeneous soil medium (Novak et al. 1978; Lee et al. 1988), and can be expressed as

\[
k_1 = \frac{2.75 G_{s1}}{(2\pi r_1)}
\]

(2)

\[
c_1 = \sqrt{\rho_{s1} G_{s1}}
\]

(3)

where \( \rho_{s1}, G_{s1} \) are the density and shear modulus of the surrounding soil, respectively.

The vertical dynamic vibration of the soil-plug segment is given by

\[
m_s \frac{\partial^2 u_s(z,t)}{\partial t^2} - 2\pi r_2 k_2 [u_p(z,t) - u_s(z,t)] - 2\pi r_2 c_2 \frac{\partial [u_p(z,t) - u_s(z,t)]}{\partial t} = 0
\]

(4)

\[
u_p(z,t) = B \exp[i(\omega t - f_1 z)]
\]

(5)

\[
u_s(z,t) = C \exp[i(\omega t - f_2 z)]
\]

(6)

where \( \omega \) is the impulse frequency and satisfies \( \omega = 1/T \), \( T \) representing the impulse width of the impact force acting on the pipe pile head; \( i \) is the imaginary unit and satisfies \( i = \sqrt{-1} \); \( B, C, f_1 \) and \( f_2 \) are undetermined coefficients.

Substituting Eqs. (5) and (6) into Eq. (4) yields
\[ u_i(z,t) = \frac{2\pi r_i(k_i + i \cdot \omega \cdot c_i)}{2\pi r_i(k_i + i \cdot \omega \cdot c_i) - m_i \cdot \omega^2} u_p(z,t) \] (7)

Substituting Eq. (5), Eq. (6) and Eq. (7) into Eq. (1) gives

\[
(E_p A_p + i \cdot \omega \cdot \delta_p A_p) \frac{\partial^2 u_p(z,t)}{\partial z^2} + \rho_p A_p \omega^2 u_p(z,t) - 2\pi r_i(k_i + i \cdot \omega \cdot c_i) u_p(z,t) \\
+ \frac{2\pi r_c \cdot \omega^2}{2\pi r_i(k_i + i \cdot \omega \cdot c_i) - m_i \cdot \omega^2} u_p(z,t) = 0
\] (8)

Eq. (8) can be rewritten as

\[
(1 + \frac{i \cdot \omega \cdot \delta_p}{E_p}) \frac{\partial^2 u_p(z,t)}{\partial z^2} + \left[ \frac{\omega^2}{C_0^2} \frac{2\pi r_i(k_i + i \cdot \omega \cdot c_i)}{E_p A_p} + \frac{1}{E_p A_p} \frac{2\pi r_c \cdot \omega^2}{2\pi r_i(k_i + i \cdot \omega \cdot c_i) - m_i \cdot \omega^2} \right] u_p(z,t) = 0
\] (9)

where \( E_p = \rho_p C_0^2 \), \( C_0 \) is the material longitudinal wave velocity equal to the apparent phase velocity of pipe piles measured on the ground, where there is no surrounding soil or soil plug.

Combining Eq. (9) with Eq. (5) and taking the partial derivative with respect to \( z \) yield

\[-f_1^2 \left(1 + \frac{i \cdot \omega \cdot \delta_p}{E_p}\right) + \frac{\omega^2}{C_0^2} x(k_i + i \cdot \omega \cdot c_i) + \frac{\omega k_i + \omega c_i}{2\pi r_i(k_i + i \cdot \omega \cdot c_i) - m_i \cdot \omega^2} = 0\] (10)

where \( x = \frac{2\pi r_i}{E_p A_p} \), \( y = \frac{2\pi r_c \cdot \omega^2}{E_p A_p} \).

Assuming \( f_1 = h + ia \), where \( h \) and \( a \) are real variables, it is easy to obtain

\[-f_1^2 \left(1 + \frac{\delta_p}{E_p} \cdot i \cdot \omega\right) = -(h^2 - a^2 - 2ah \omega \delta_p E_p + i[2ah + (h^2 - a^2) \omega \delta_p E_p]) \] (11)

\[
\frac{2\pi r_c(k_i + i \cdot \omega \cdot c_i)}{2\pi r_i(k_i + i \cdot \omega \cdot c_i) - m_i \cdot \omega^2} = \left[ \frac{k_i(2\pi r_i k_2 - m_i \cdot \omega^2) + 2\pi r_c \cdot \omega^2}{(2\pi r_i k_2 - m_i \cdot \omega^2)^2 + (2\pi r_c \cdot \omega^2)^2} \right] + \frac{c_i \omega(2\pi r_i k_2 - m_i \cdot \omega^2) + 2\pi r_c \cdot \omega^2}{(2\pi r_i k_2 - m_i \cdot \omega^2)^2 + (2\pi r_c \cdot \omega^2)^2} \] (12)

Substituting Eq. (11) and Eq. (12) into Eq. (10), and letting the real and imaginary parts of Eq. (10) respectively equal to zero yield

\[-(h^2 - a^2 - 2ah \omega \delta_p E_p) + \frac{\omega^2}{C_0^2} xk_i + \frac{\omega k_i + \omega c_i}{(2\pi r_i k_2 - m_i \cdot \omega^2)^2 + (2\pi r_c \cdot \omega^2)^2} = 0\] (13)
\[-2ah+(\sqrt{a^2-h^2})\frac{\delta_p}{E_p} - x\omega \cdot c_1 + \frac{y[c_1 \omega (2\pi r_2 k_2 - m_s \cdot \omega^2) + 2\pi r_2 c_2 k_2 \cdot \omega^2]}{(2\pi r_2 k_2 - m_s \cdot \omega^2)^2 + (2\pi r_2 c_2 \cdot \omega^2)^2} = 0 \quad (14)\]

Combining Eq. (13) and Eq. (14) obtains

\[a = \frac{N - M \omega \frac{\delta_p}{E_p}}{2h[1 + (\omega \frac{\delta_p}{E_p})^2]} = \frac{L}{h} \quad (15)\]

where

\[M = \frac{\omega^2}{C_0^2} - xk_1 + \frac{y[c_1 \omega (2\pi r_2 k_2 - m_s \cdot \omega^2) + 2\pi r_2 c_2 \cdot \omega^2]}{(2\pi r_2 k_2 - m_s \cdot \omega^2)^2 + (2\pi r_2 c_2 \omega)^2} \quad (16)\]

\[N = -x\omega \cdot c_1 + \frac{y[c_1 \omega (2\pi r_2 k_2 - m_s \cdot \omega^2) + 2\pi r_2 c_2 k_2 \cdot \omega^2]}{(2\pi r_2 k_2 - m_s \cdot \omega^2)^2 + (2\pi r_2 c_2 \cdot \omega^2)^2} \quad (17)\]

\[L = \frac{N - M \omega \frac{\delta_p}{E_p}}{2[1 + (\omega \frac{\delta_p}{E_p})^2]} \quad (18)\]

Substituting Eq. (15) into Eq. (13) yields

\[h^4 - (2L \omega \frac{\delta_p}{E_p} + M)h^2 - L^2 = 0 \quad (19)\]

\[h^2 = \frac{\zeta \pm \sqrt{\zeta^2 + 4L^2}}{2} \quad (20)\]

\[\zeta = 2L \omega \frac{\delta_p}{E_p} + M \quad (21)\]

As \(h^2\) is a value greater than or equal to 0, therefore

\[h = \sqrt{\frac{\zeta + \sqrt{\zeta^2 + 4L^2}}{2}} \quad (22)\]

The apparent phase velocity of pipe pile is derived as

\[C = \frac{\omega}{h} = \frac{\omega \sqrt{\zeta}}{\sqrt{\zeta + \sqrt{\zeta^2 + 4L^2}}} \quad (23)\]

If the resistance of soil plug and surrounding soil, as well as the material damping coefficient, is set to 0, it can be obtained \(C=C_0\), indicating that the apparent phase velocity of the pipe pile before installation is
equal to its material longitudinal wave velocity.

**Analytical solution of the APVPSP calculated using the soil-plug Winkler model**

To evaluate the validity and accuracy of the developed solution, the APVPSP calculated using the additional mass model is compared with that computed using the soil-plug Winkler model and the experimental results. In the soil-plug Winkler model, the effect of soil plug on pipe pile is simulated by the distributed Winkler model whose spring constant and damping coefficient are represented by \( k_2 \) and \( c_2 \), respectively. Then the vertical dynamic vibration of the pipe pile segment becomes

\[
E_p A_p \frac{\partial^2 u_p(z,t)}{\partial z^2} + \rho_p A_p \frac{\partial^3 u_p(z,t)}{\partial t \partial z^2} - \rho_p A_p \frac{\partial^2 u_p(z,t)}{\partial t^2} - 2\pi_i [k_i u_p(z,t) + c_i \frac{\partial u_p(z,t)}{\partial t}] - 2\pi_i [k_j u_p(z,t) + c_j \frac{\partial u_p(z,t)}{\partial t}] = 0
\]  
\[
(24)
\]

Substituting Eq. (5) into Eq. (24) yields

\[
-f_i^2 (1 + \frac{\delta_p}{E_p} \cdot i \cdot \omega) + \frac{\omega^2}{C_0^2} - \frac{2\pi r_1 (k_i + i \cdot \omega \cdot c_i)}{E_p A_p} - \frac{2\pi r_2 (k_j + i \cdot \omega \cdot c_j)}{E_p A_p} = 0
\]  
\[
(25)
\]

Substituting Eq. (11) into Eq. (25), and letting the real and imaginary parts of Eq. (25) respectively equal to zero yield

\[
h^2 - a^2 - 2ah \omega \frac{\delta_p}{E_p} + xk_i + yk_j - \frac{\omega^2}{C_0^2} = 0
\]  
\[
(26)
\]

\[
2ah + (h^2 - a^2) \omega \frac{\delta_p}{E_p} + \omega (xc_i + yc_j) = 0
\]  
\[
(27)
\]

where \( x = \frac{2\pi r_1}{E_p A_p} \), \( y = \frac{2\pi r_2}{E_p A_p} \).

Combining Eq. (26) and Eq. (27) obtains

\[
a = \frac{L}{h}
\]  
\[
(28)
\]

\[
L = \frac{\omega^3 \frac{\delta_p}{E_p} + \omega (xc_i + yc_j) - \omega (xk_i + yk_j) \frac{\delta_p}{E_p}}{2[-(\omega \frac{\delta_p}{E_p})^2 - 1]}
\]  
\[
(29)
\]

Eq. (26) can be rewritten as

\[
h^4 - \zeta h^2 - L^2 = 0
\]  
\[
(30)
\]
\[ h^2 = \frac{\zeta \pm \sqrt{\zeta^2 + 4L^2}}{2} \]  

(31)

\[ \zeta = 2L\omega \frac{\delta_p}{E_p} - (xk_1 + yk_2) + \frac{\omega^2}{C_o} \]  

(32)

As \( h^2 \) is a non-negative value, it easily obtains

\[ h = \sqrt{\frac{\zeta + \sqrt{\zeta^2 + 4L^2}}{2}} \]  

(33)

The apparent phase velocity of pipe pile calculated using the soil-plug Winkler model is derived as

\[ C = \frac{\omega}{h} = \frac{\omega\sqrt{2}}{\sqrt{\zeta + \sqrt{\zeta^2 + 4L^2}}} \]  

(34)

It is worth noting that the final expressions of the apparent phase velocity calculated by the additional mass model (i.e. Eq. (23)) and soil-plug and Winkler model (i.e. Eq. (34)) are the same, but the intermediate parameters in Eq. (23) and Eq. (34) are much different.

**Comparison of the additional mass model with the soil-plug Winkler model**

Since the boundary conditions and assumptions in the additional mass model and soil-plug Winkler model are inconsistent with those in the plane strain model, the spring constant and damping coefficient deduced from the plane strain model, cannot be directly used in the soil plug-pipe pile interaction model. Therefore, a series of model pipe pile experiments were conducted to get the parameter values for both the additional mass model and Winkler model. The model pipe pile used herein was a polypropylene-random (PP-R) pile with a length of 2.6 m. The outer diameter, wall thickness and density of the model pipe pile were 110 mm, 10 mm, and 900 kg/m$^3$, respectively. The pipe pile was laid on the ground, so the resistance caused by the pile surrounding soil was ignored. The soil plug was simulated using fine sand, and its density was controlled to 1800 kg/m$^3$; the inner radius of the outer annular columns is set to \( r_2 = 0 \), indicating that the whole soil-plug is vibrating with the pipe pile shaft.

The parameter values for these two models were back-analyzed from the dynamic velocity response measured at the pipe pile head. The model test proceeded as follows:

Step 1—An experiment of pipe pile without soil plug was performed. The material longitudinal wave velocity and material damping coefficient of the model pipe pile were determined using the parameter inversion method by comparing the calculated and measured dynamic velocity response at the pipe pile.
head, as shown in Fig. 2. They were found to be 1400 m/s and 29.848 kN·m⁻³·s, respectively. The impulse width was fitted as \( T = 0.69 \) ms.

Step 2—The sand was poured into the model pipe pile to a height of \( H_s = 0.5, 1.0, 1.5, 2.6 \) m, subsequently, with its density controlled as 1800 kg/m³ by compacting the soil plug in each 150 mm layers with the same compacting effort. The dynamic velocity response of the model pipe pile with the four different heights of soil plug was recorded as depicted in Fig. 4 and Fig. 6.

Step 3—Utilizing the settings obtained in step 1 and step 2, the parameters in the two soil-plug modes were respectively back-analyzed from the measured dynamic velocity response. First, the impulse width and peak amplitude of the incident signal in the fitted curves were set to be equal to those in the measured curve. Then adjust the parameter values of the two soil-plug models, until the peak amplitude and arrival time of the reflected signal in the fitted curve had reached a good agreement with those in the measured curve. The parameters that give a reasonable fit to the measurements would be determined as the parameter values of these two models.

The theoretical dynamic velocity response is calculated using the analytical solution developed by Liu et al. (2018), from which the analytical solution based on the soil-plug Winkler model can also be easily derived. To verify the effectiveness of the theoretical solution, the calculated dynamic velocity response based on these two soil-plug models is compared with the measured curve. First, the comparison for the case of pipe pile without soil plug filling inside is conducted, where the parameters in the two soil-plug models are set as \( k_2 = 0 \) kN·m⁻³, \( c_2 = 0 \) kN·m⁻³·s. Fig. 2 compares the dynamic velocity responses calculated using the two soil-plug models with the measured curve for the case \( H_s = 0 \) m. It shows that the dynamic velocity response computed employing the additional mass model is coincided with that using the soil-plug Winkler model. The amplitude peak and arrival time of the incident and the first reflected signals in the calculated curves are equal to the experimental results. The calculations match reasonably well with the measurements, indicating the analytical solution has high accuracy.

Fig. 2 Comparison of the calculated dynamic velocity responses with the measurement for the case \( H_s = 0 \)

Fig. 3 Parametric sensitivity analysis of the soil-plug Winkler model

The parametric sensitivity analysis of the two soil-plug models is implemented by comparing their amplitude peak and arrival time in the calculated dynamic velocity response with those in the measured
curve for case $H_s = 2.6$ m. Fig. 3 illustrates the parametric sensitivity analysis of the soil-plug Winkler model. Fig. 3(a) depicts the influence of damping coefficient on the dynamic velocity response of pipe pile, where the spring constant is set as $k_2 = 1000$ kN·m$^{-3}$. It can be seen from Fig. 3(a) that the amplitude peak of the calculated dynamic velocity response decreases with increasing damping coefficient. When the damping coefficient exceeds 15 kN·m$^{-3}$·s, the amplitude peak of the reflected signal has completely vanished. Compared with the measured curve, it can be concluded that the value of the damping coefficient in the soil-plug Winkler model lies in the range 5-15 kN·m$^{-3}$·s. Whereas as $c_2$ varies in this band, the arrival times of amplitude peak of the reflected signal keep invariant, obvious divergent from the experimental result, where the arrival time of the measured reflected signal is significantly delayed due to the existence of soil plug. Such arrival times in Fig. 3 seem to be consistent with that of measurement in Fig. 2 for no soil plug filling inside. Fig. 3(b) shows the influence of spring constant on the dynamic velocity response, where the damping coefficient is set as $c_2 = 5$ kN·m$^{-3}$·s. It is noting that the spring constant gives rise to negligible influence on the dynamic velocity response when $k_2$ falls in the range $10^2-10^3$ kN·m$^{-3}$. The amplitude value of the incident signal and the high-frequency interference may vary with the spring constant when $k$ exceeds $10^4$ kN·m$^{-3}$·s. However, the variation of spring constant gives rise to negligible influence on the reflected signal from pile toe and the apparent phase velocity of the pipe pile.

From the above analysis, one can conclude that the calculated dynamic velocity response using the soil-plug Winkler model has a noticeable divergence with the experimental result when considering the soil plug effect. The soil-plug Winkler model cannot reflect the attenuation effect of the soil plug on APVPSP.

Fig. 4 Parameter sensitivity analysis of the additional mass model

Fig. 4 depicts the parametric sensitivity analysis of the additional mass model. Fig. 4(a) shows the influence of damping coefficient on the dynamic velocity response, where the spring constant is set as $k_2 = 1000$ kN·m$^{-3}$. Note that the amplitude peak of the measured reflected signal falls between the amplitude peaks of the calculated reflected signals for $c_2 = 150$ kN·m$^{-3}$·s and 600 kN·m$^{-3}$·s. The arrival times of amplitude peak in the calculated reflected signals are generally consistent with the experimental result. Therefore the damping coefficient in the additional mass model is deduced falling in the range $c_2 = 150$-600 kN·m$^{-3}$·s. Fig. 4(b) demonstrates the influence of spring constant on the dynamic velocity response, where the damping coefficient is set as $c_2 = 600$ kN·m$^{-3}$·s. Also, the spring constant in the additional mass model has a gentle influence on the dynamic velocity response when $k_2$ varies within 10-1000 kN·m$^{-3}$, similar to
that in the soil-plug Winkler model in Fig. 3(b). The findings from the additional mass model are in accordance with the study presented by Randolph and Deeks (1992) that for typical accelerations levels the dashpot term will be at least a factor of 10 larger than the spring term during the initial response, and slip starts to occur after a pile displacement of 0.3 mm, when the dashpot accounts for 97% of the resistance. The dynamic velocity response calculated using the additional mass model matches much better than that based on the soil-plug Winkler model.

Fig. 5 Influence of damping coefficient on the APVPSP

The approximate range of the parameter values of the additional mass model and the soil-plug Winkler model has been deduced from the dynamic velocity response. Then, the analytical developed in this study can be employed to back-analyze the adequate parameter values of these two soil-plug models. Given previous analysis have identified that the spring constants in the two soil-plug models have little influence on the APVPSP; therefore only the effect of damping coefficient on APVPSP is taken into consideration, as shown in Fig 5. For the soil-plug Winkler model, when damping coefficient varies within 5-15 kN·m⁻³·s, the APVPSP remains invariable, approximately equal to the material longitudinal wave velocity, which is consistent with the calculated dynamic velocity response in Fig. 3(b), but obvious conflicting with the experimental results. Based on the above analysis, we can find that the soil-plug Winkler model is not suitable for simulating the soil plug-pipe pile interaction during low strain testing.

It can be observed that the APVPSP, calculated using the additional mass model, decreases as the damping coefficient increases. Another interesting finding is that the APVPSP is not a constant, but decreases gradually as the impulse width increases (frequency decreases). In order to examine these two findings, the APVPSP calculated using the additional mass model is compared with the experimental result. The measured APVPSP is obtained from the Eq. (35).

$$\frac{H - H_s}{C_0} + \frac{H_s}{C} = \frac{H}{C_a}$$  \hspace{1cm} (35)

where $H$ and $H_s$ are the pile length and soil plug height, respectively. $C_a$ is the average apparent phase velocity of the whole pipe pile, defined as the pile length divided by the time difference of the amplitude peak between the reflected wave and the incident wave; $C$ represents the apparent phase velocity of pipe pile segment with soil plug filling inside (APVPSP). $C_0$ was determined in the model pipe pile test without soil plug filling inside as shown in Fig. 2, while $C_0 = 1400$ m/s.
Fig. 6 depicts the comparison of the measured dynamic velocity response with those calculated using the additional mass model for different soil plug heights. It can be seen from the Fig. 6 that the theoretical curves match reasonably well with the measured curves. The soil plug heights in Fig. 6 (a), (b), (c) were \( H_s = 0.5, 1.0, 1.5 \) m, respectively; and their corresponding impulse widths were fitted to be \( T = 0.515, 0.535, 0.54 \) ms. The measured APVPSP in Fig. 6 (a), (b), (c) and Fig. 4 were found to be 763, 655, 749, 729 m/s, respectively, which appear to be very close to the theoretical APVPSP calculated using the additional mass model for \( c_2 = 600 \text{ kN} \cdot \text{m}^{-3} \cdot \text{s} \). Therefore an attempt of using \( c_2 = 600 \text{ kN} \cdot \text{m}^{-3} \cdot \text{s} \) as the damping coefficient of the additional mass model is made, and its calculated APVPSP are compared with the measured results for different impulse widths, as illustrated in Table 1. The comparison shows that the theoretical predictions agree pretty well with the measured results, with a calculation error of less than 0.5%. The APVPSP has been confirmed to decrease with increasing impulse widths.

Table 1 Comparison of the calculated APVPSP with the measured results for different impulse widths

From the above analyses, one can conclude the additional mass model has distinct advantages over the soil-plug Winkler model. The theoretical solution derived from the additional mass model has high accuracy.

**Parametric sensitivity analysis of the additional mass model**

Although the model pipe pile experiment has identified that the solution developed in this study can provide a quick and direct method to estimate the APVPSP, the parameters of the additional mass model are obtained through the parameter inversion method, which needs sophisticated derivation and limits its application in the practical engineering. If we can find a relatively easy way to determine these parameters, this problem will be solved. Therefore the parametric sensitivity analysis of the additional mass model is conducted in this section. Spring constant, damping coefficient, and soil plug mass are the three primary parameters of the additional mass model. As the influence of spring constant and damping coefficient on the apparent phase velocity has been clarified, the soil plug mass is the only factor required to be considered. The soil plug mass is varied by adjusting the wall thickness of the outer soil-plug annual. The
parameters of the pipe pile used herein are the same as those in Fig. 4. The damping coefficient of the additional mass model is set to be $c_2=300 \text{kN} \cdot \text{m}^{-3} \cdot \text{s}$; the wall thickness of the outer soil-plug annular is assumed to be $b_s = 0.045, 0.035, 0.025, 0.015, 0.005 \text{ m}$. It can be seen from Fig. 7 that the APVPSP decreases with the increasing soil plug mass, besides, such reduction becomes more significant as the impulse width increases. When the soil plug mass exceeds a particular value, say, $b_s >0.005 \text{ m}$, for the impulse width less than 0.55 ms, the APVPSP for different soil plug masses coincide. It can be observed that whatever the soil plug mass changes, the APVPSP calculated with $c_2=300 \text{kN} \cdot \text{m}^{-3} \cdot \text{s}$ is still much larger than that for $c_2=600 \text{kN} \cdot \text{m}^{-3} \cdot \text{s}$ in Fig. 5. It can be concluded that the damping coefficient of additional mass model plays a dominant role in determining the APVPSP, whereas soil plug mass also affects the computational accuracy of the APVPSP.

Fig. 7 Influence of soil plug mass on the APVPSP

It can be observed from Fig. 7 that there exists a coupling relationship between the damping coefficient and additional soil plug mass. The lambda coefficient $\lambda$, defined as the ratio of sectional damping per unit length (the damping coefficient multiplying the inside perimeter of pipe pile), to the mass per unit length of the soil-plug, is introduced to get a better understanding of such coupling relationship

$$\lambda = \frac{2 \pi r_s c_2}{\pi \left( r_2^2 - (r_2 - b_s)^2 \right) \rho_{s_2}} \quad (36)$$

Eq. (36) can be simplified as

$$\lambda = \frac{2 c_2}{\rho_{s_2} \left( 2 r_2 b_s - b_s^2 \right)} \quad (37)$$

Eq. (36) shows that $\lambda$ is a function of $\rho_{s_2}$, $c_2$, $r_2$ and $b_s$. Previous analyses have identified that the damping coefficient of additional mass model dominates the minimal APVPSP, and its value range is relatively easy to be deduced. If the damping coefficient of additional mass model is assumed as an approximately specific value, the soil-plug mass will become the crucial factor in determining the APVPSP. Since the density of the soil plug varies in a narrow range, $\rho_s$ can be treated as constant. Keeping $b_s$ and $c_2$ as constants, then $\lambda$ becomes only a function with variable $r_2$. If we can find suitable value of $b_s$ that makes $\lambda$ fall in a relatively narrow range for most commonly used pipe piles, the possibility to find a corresponding damping coefficient may be realized. For convenient comparison, the normalized $\lambda$ is
introduced herein, defined as $\lambda' = \lambda / \lambda_0$, where $\lambda_0$ is the value of $\lambda$ for $r_2 = 0.12$ m. Fig. 8 demonstrates the influence of inner radius $r_2$ on the variation of $\lambda'$. It is worth noting that for most commonly used pipe piles with an inner radius varying within 0.12–0.5 m, when $b_s$ is given as 0.10 m, $\lambda'$ falls in a narrow band as 0.65–1. Therefore the wall thickness of the outer soil-plug annular is assumed to be $b_s = 0.1$ m in the subsequent analysis. If there exists a damping coefficient of additional mass model that suitable for all pipe piles with $\lambda'$ falling in this narrow band, then the developed theoretical formula can be used to calculate the APVPSP of most commonly used pipe piles quickly.

Fig. 8 Influence of inner radius $r_2$ on the variation of $\lambda'$

In order to find a suitable parameter combination for most often used concrete pipe piles, field tests of pipe piles with $\lambda' = 1$, $\lambda' = 0.67$ were conducted, where the wall thickness of the outer soil-plug annular keeps uncharged as $b_s = 0.1$ m. The parameters used in the field tests are demonstrated in Table 2. The parameter combination obtained in the model pipe pile test, $k_2 = 1000$ kN·m$^{-3}$, $c_2 = 600$ kN·m$^{-3}$·s, is employed as the initial values for the back-analysis of the measured APVPSP. Coincidentally, the theoretical predictions calculated using this parameter combination match well with the measured results of both $\lambda' = 1$ and $\lambda' = 0.67$. Table 3 compares the calculated and measured APVPSP for $\lambda' = 1$, 0.67. As shown in Table 3, theoretical predictions have high accuracy with an error less than 4%, indicating that the parameter combination, $k_2 = 1000$ kN·m$^{-3}$, $c_2 = 600$ kN·m$^{-3}$·s, $b_s = 0.1$ m, can be used for concrete pipe piles with an inner radius varying within 0.12–0.5 m.

Table 2 Parameters of the field pipe pile test

Table 3 Comparison of the calculated and measured APVPSP

Fig. 9 presents the calculated APVPSP curve for $\lambda' = 1$ and $\lambda' = 0.67$. It can be found in Fig. 9, when the impulse width is very short, like $T = 0.5$ ms, the APVPSP is close to the material longitudinal wave velocity of the pipe pile. Although using a relatively shorter impulse can reduce the testing error attributed to the variation of the APVPSP, the short impulse width may arouse serious high-frequency interference at pipe pile head, which may in return compromise the interpretation of the dynamic velocity response. Therefore most widely used impulse widths lie in the range $T = 1–2$ ms, where the calculated APVPSP curve displays a steady decline.

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Fig. 9 Calculated APVPSP curve of the field pipe pile test: (a) $\lambda'=1$; (b) $\lambda'=0.67$

Based on above analysis, it can be concluded that the parameter combination, $k_2=1000$ kN·m$^{-3}$, $c_2=600$ kN·m$^{-3}$·s, $b_s=0.1$ m, can be used for the calculating the APVPSP of concrete pipe piles with an inner radius varying within 0.12~0.5 m. For the pipe pile with an inner radius less than 0.1 m, $b_s$ is set equal to its radius, and then the proposed parameter combination can still be satisfied. Therefore, the proposed parameter combination is used in the following analysis.

**Parametric study of the APVPSP**

Owing to the soil plug effect, the APVPSP in the model pipe pile test decreases by nearly 50%; however such decrease in the field test is only about 10%. The influence of soil plug on the reduction of the APVPSP varies with different pipe piles. In this section, a parametric study is carried out to clarify the major factors affecting the APVPSP. First, the influence of soil plug density on the APVPSP is investigated. Parameters used herein are set as: the density, material longitudinal wave velocity, material damping coefficient, inner and outer radii of the pipe pile are $\rho_p=2400$ kg·m$^{-3}$, $C_0=3500$ m/s, $\delta_p=30$ kN·m$^{-3}$·s, $r_2=0.3$ m, and $r_1=0.4$ m, respectively. The density and shear wave velocity of the pile surrounding soil are $\rho_{s1}=1800$ kg·m$^{-3}$, $v_s=120$ m/s, respectively; the corresponding spring constant and damping coefficient of the Winkler model of surrounding soil are calculated from Eq. (2) and Eq. (3) as $k_1=5.7\times10^4$ kN·m$^{-3}$, $c_1=22$ kN·m$^{-3}$·s, respectively. Parameters of the additional mass model are set to the recommended parameter combination as $k_2=1000$ kN·m$^{-3}$, $c_2=600$ kN·m$^{-3}$·s, $b_s=0.1$ m; the density of soil plug is set to $\rho_{s2}=0, 1600, 1800, 2000, 2200$ kg·m$^{-3}$, where $\rho_{s2}=0$ kg·m$^{-3}$ indicates pipe pile without soil plug filling inside. Fig. 10 displays the influence of soil plug density on the APVPSP. When the soil plug density varies within 1600~2200 kg·m$^{-3}$, the maximum difference of the APVPSP is less than 50 m/s. Therefore, soil plug density gives rise to a relatively small influence on the APVPSP. For the pipe pile without soil plug filling inside, the APVPSP is approximately equal to the material longitudinal wave velocity of the pipe pile.

Fig. 10 Influence of soil plug density on the APVPSP

Fig. 11 shows the influence of pile wall thickness on the APVPSP. The soil plug density is set to $\rho_{s2}=1800$ kg·m$^{-3}$; the inner radius of pipe pile remains constant, as $r_2=0.3$ m, the pile wall thickness is set to
To intuitionally and quantitatively investigate the influence of soil plug effect on the decrease of the APVPSP, the APVPSP reduction ratio is introduced, defined as \( e = \frac{(C_0 - C)}{C_0} \). It can be seen from Fig. 11 that the APVPSP decreases with the pile wall thickness owing to the soil plug effect. This has been verified by the comparison of the reduction ratio of the calculated APVPSP for \( b_p = 0.05 \) with those of measurements, as demonstrated in Table 4.

![Fig. 11 Influence of pile wall thickness on the APVPSP](image)

**Table 4 Comparison of the reduction ratio of the calculated APVPSP with measurements**

Fig. 12 displays the influence of material longitudinal wave velocity of the pipe pile on the APVPSP. Parameters used herein are given as: inner and outer radii of pipe pile are 0.30 m and 0.35 m, respectively; the material longitudinal wave velocity of pipe pile is set to \( v_p = 3000, 3500, 4000 \) m/s; while the other parameters are kept constant. As shown in Fig. 12, the APVPSP of the three conditions decrease as the impulse width increases, with a similar tendency. Fig. 13 compares the APVPSP reduction ratio of pipe piles with different material longitudinal wave velocities. It can be seen from Fig. 13 that reduction ratio curves for the three different conditions coincide, indicating that the material longitudinal wave velocity of pipe piles does not affect the APVPSP reduction ratio.

![Fig. 12 Influence of the material longitudinal wave velocity of pipe piles on the APVPSP](image)

**Fig. 13 Comparison of the APVPSP reduction ratio of pipe piles with different material longitudinal wave velocities**

In this section, the influence of pipe pile density on the APVPSP reduction ratio is investigated. Two different types of pipe piles are considered, namely steel pipe pile and concrete pipe pile. The density and material longitudinal wave velocity of the steel pipe pile are given as 7500 kg·m\(^{-3}\), 5100 m/s, respectively. The density and material longitudinal wave velocity of the concrete pipe pile are 2500 kg·m\(^{-3}\), 4000 m/s, respectively. The inner and outer radii of both pipe piles are kept the same, as 0.30 m, 0.35 m, respectively.
Other parameters used herein are the same as those used Fig. 10. The influence of pipe pile density on the apparent phase velocity cannot be compared directly due to the different material longitudinal wave velocities. Therefore the comparison is conducted in the term of the APVPSP reduction ratio. Fig. 14 shows the influence of pipe pile density on the APVPSP reduction ratio. It is noting that the APVPSP reduction ratio of the steel pipe pile is much smaller than that of the concrete pipe pile. Given that the material longitudinal wave velocity of the pipe pile has been confirmed not to affect the APVPSP reduction ratio, the pile density is the main factor to change the APVPSP reduction ratio. It can be found that the APVPSP reduction ratio increases with the decreasing pile density.

**Fig. 14 Influence of pipe pile density on the APVPSP reduction ratio**

Above analyses suggest that the APVPSP reduction ratio increases as the wall thickness and density of pipe pile decrease. The wall thickness and density are both associated with the mass per unit length of the pile. Therefore, the wall thickness and density are generalized as the mass per unit length to clarify the influence mechanism of pipe pile parameters on the variation of the APVPSP. The steel and concrete pipe piles used in Fig. 14 are also introduced herein. However, to guarantee the two kinds of pipe piles having a same mass per unit length, the inner and outer radii of the concrete pipe pile are set to 0.3 m and 0.35 m, respectively; while the inner and outer radii of the steel pipe pile are given as 0.3 m and 0.318 m, respectively; other parameters are kept constants.

**Fig. 15 Influence of the mass per unit length of the pile on the APVPSP reduction ratio**

The reduction ratio of the steel pipe pile is coincided with that of the concrete pipe pile. It can be concluded that the mass per unit length of the pile is the key factor to affect the APVPSP. The APVPSP reduction ratio increases as the mass per unit length of the pile decreases.

**Conclusions**

This study derives an analytical solution to calculate the apparent phase velocity of the pipe pile segment with soil plug filling inside (APVPSP) based on the additional mass model. The rationality and accuracy of the developed solution have been confirmed through the comparison with the solution derived using the
soil-plug Winkler model and experimental results. A parametric study is conducted to investigate the major factors to influence the APVPSP. The main conclusions can be drawn as follows:

(1) There exists a coupling relationship between the damping coefficient and the additional soil plug mass. The damping coefficient plays a dominant role in determining the APVPSP, whereas soil plug mass also affects the computational accuracy of the APVPSP.

(2) The parameter combination of the additional mass model, \( k_2=1000 \, \text{kN} \cdot \text{m}^{-3}, \ c_2=600 \, \text{kN} \cdot \text{m}^{-3} \cdot \text{s}, \ b_s=0.1 \, \text{m}, \) can be used to calculate the APVPSP of concrete pipe piles with an inner radius varying within 0.12–0.5 m.

(3) The attenuation degree of the soil plug on the APVPSP is controlled by the mass per unit length of the pile but has nothing to do with the material longitudinal wave velocity of pipe piles. The lower mass per unit length is, the more considerable attenuation degree of the APVPSP is.

(4) The impulse width is an essential factor to affect the APVPSP. The APVPSP decreases significantly as the impulse width increases. However, for the pipe pile without soil plug filling inside, the impulse width gives rise to negligible influence on the apparent phase velocity of the pipe pile.

**Acknowledgements**

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**List of symbol**

APVPSP apparent phase velocity of pipe pile segment with soil plug filling inside

\( r_1, r_2 \) outer and inner radii of the pipe pile

\( b_p \) wall thickness of pipe pile

\( k_1, c_1 \) spring constant and damping coefficient of the distributed Winkler model of the surrounding soil

\( r_3, b_s \) inner radius and wall thickness of the outer annular column of the soil plug
\( k_2, c_2 \) spring constant and damping coefficient of the distributed Voigt model

\( \rho_{k2} \) density of soil plug

\( m_s \) mass per unit length of each additional soil-plug segment

\( u_p(z,t) \) vertical displacement of the pipe pile segment

\( u_s(z,t) \) vertical displacement of the soil plug segment

\( E_p \) Young's modulus of the pipe pile

\( A_p \) cross-sectional area of the pipe pile

\( \rho_p \) density of the pipe pile

\( \delta_p \) material damping coefficient of the pipe pile

\( \rho_{s1} \) density of the surrounding soil

\( G_{s1} \) shear modulus of the surrounding soil

\( \omega \) impulse frequency

\( T \) impulse width of the impact force acting on the pipe pile head

\( i \) imaginary unit

\( B, C, f_1, f_2 \) undetermined coefficients

\( C_0 \) material longitudinal wave velocity of the pipe pile

\( h, a \) real variables

\( H_s \) soil plug height

\( H \) pipe pile length

\( C_a \) average apparent phase velocity of the whole pipe pile

\( C \) apparent phase velocity of pipe pile segment with soil plug filling inside

\( \lambda \) the ratio of sectional damping per unit length to the mass per unit length of the soil-plug.

\( \lambda' \) normalized \( \lambda \)

\( \lambda_0 \) value of \( \lambda \) for \( r_2 = 0.12 \text{ m} \)

\( e \) APVPSP reduction ratio
References


Figure Captions:

Fig. 1 Conceptual model of pile-soil dynamic interaction

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Table 1 Comparison of the calculated APVPSP with the measured results for different impulse widths

<table>
<thead>
<tr>
<th>Soil plug Height (m)</th>
<th>Impulse width (ms)</th>
<th>Measured APVPSP (m/s)</th>
<th>Calculated APVPSP (m/s)</th>
<th>Error</th>
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<tr>
<td>0.5</td>
<td>0.515</td>
<td>763.4</td>
<td>763</td>
<td>0.05%</td>
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<tr>
<td>1.0</td>
<td>0.535</td>
<td>754.3</td>
<td>755</td>
<td>0.09%</td>
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<tr>
<td>1.5</td>
<td>0.540</td>
<td>745.7</td>
<td>749</td>
<td>0.44%</td>
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<tr>
<td>2.6</td>
<td>0.600</td>
<td>726.4</td>
<td>729</td>
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Table 2 Parameters of the field pipe pile test

<table>
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<tr>
<th>$\lambda'$</th>
<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
<th>$\rho_r$ (kg·m$^{-3}$)</th>
<th>$\rho_s$ (kg·m$^{-3}$)</th>
<th>$v_s$ (m/s)</th>
<th>$k_1$ (kN·m$^{-3}$)</th>
<th>$c_1$ (kN·m$^{-3}$·s)</th>
<th>$C_0$ (m/s)</th>
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<tr>
<td>1.0</td>
<td>0.2</td>
<td>0.12</td>
<td>2500</td>
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<td>0.67</td>
<td>0.5</td>
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<td>1600</td>
<td>100</td>
<td>$3.5 \times 10^4$</td>
<td>16</td>
<td>3160</td>
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Table 3 Comparison of the calculated and measured APVPSP

<table>
<thead>
<tr>
<th>$\lambda'$</th>
<th>$k_2$ (kN·m⁻³)</th>
<th>$c_2$ (kN·m⁻³·s)</th>
<th>$h_s$ (m)</th>
<th>$T$ (ms)</th>
<th>Measured APVPSP (m/s)</th>
<th>Calculated APVPSP (m/s)</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>$10^1$</td>
<td>600</td>
<td>0.1</td>
<td>1.0</td>
<td>3610</td>
<td>3638</td>
<td>0.78%</td>
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<tr>
<td>0.67</td>
<td>$10^1$</td>
<td>600</td>
<td>0.1</td>
<td>2.0</td>
<td>2700</td>
<td>2790</td>
<td>3.33%</td>
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Table 4 Comparison of the reduction ratio of the calculated APVPSP with measurements

<table>
<thead>
<tr>
<th></th>
<th>$T$ (ms)</th>
<th>$r_1$ (m)</th>
<th>$r_2$ (m)</th>
<th>$h_p$ (m)</th>
<th>$C_0$ (m/s)</th>
<th>$C$ (m/s)</th>
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<td>Model pipe pile test</td>
<td>2.0</td>
<td>0.055</td>
<td>0.045</td>
<td>0.01</td>
<td>1400</td>
<td>634</td>
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<td>Calculated</td>
<td>2.0</td>
<td>0.35</td>
<td>0.30</td>
<td>0.05</td>
<td>3500</td>
<td>2672</td>
<td>23.66%</td>
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<td>Ding’s test[1]</td>
<td>2.0</td>
<td>0.50</td>
<td>0.38</td>
<td>0.12</td>
<td>3160</td>
<td>2700</td>
<td>14.56%</td>
</tr>
</tbody>
</table>
Fig. 1 Conceptual model of pile-soil dynamic interaction
Fig. 2 Comparison of the calculated dynamic velocity responses with the measurement for the case $H_s = 0$
Fig. 3 Parametric sensitivity analysis of the soil-plug Winkler model

(a)

(b)
Fig. 4 Parameter sensitivity analysis of the additional mass model

(a) $H_s = 2.6 \text{ m}$
$T = 0.60 \text{ ms}$

(b) $H_s = 2.6 \text{ m}$
$T = 0.60 \text{ ms}$

- Head wave
- First reflected signal of pile toe

Measured
- Additional mass $c_2 = 150 \text{ kN} \cdot \text{m}^{-3} \cdot \text{s}$
- Additional mass $c_2 = 600 \text{ kN} \cdot \text{m}^{-3} \cdot \text{s}$

- Additional mass $k = 10^3 \text{ kN} \cdot \text{m}^{-3}$
- Additional mass $k = 10^4 \text{ kN} \cdot \text{m}^{-3}$
- Additional mass $k = 10^5 \text{ kN} \cdot \text{m}^{-3}$
Additional mass $c_2 = 150 \text{kN} \cdot \text{m}^3 \cdot \text{s}$

Additional mass $c_2 = 600 \text{kN} \cdot \text{m}^3 \cdot \text{s}$

Winkler $c_2 = 5 \text{kN} \cdot \text{m}^3 \cdot \text{s}$

Winkler $c_2 = 15 \text{kN} \cdot \text{m}^3 \cdot \text{s}$

Fig. 5 Influence of damping coefficient on the APVPSP
First reflected signal of soil plug top

Head wave

First reflected signal of pile toe

$V$ (mm/s)

$t$ (ms)

Measured

Additional mass $c_1 = 150 \text{ kN} \cdot \text{m}^{-3} \cdot \text{s}$

Additional mass $c_2 = 600 \text{ kN} \cdot \text{m}^{-3} \cdot \text{s}$

(a)

First reflected signal of soil plug top

Head wave

First reflected signal of pile toe

$H_s = 1.0 \text{ m}$

$T = 0.535 \text{ ms}$

(b)
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Fig. 13 Comparison of the APVSP reduction ratio of pipe piles with different material longitudinal wave velocities
Fig. 14 Influence of pipe pile density on the APVPSP reduction ratio
Fig. 15 Influence of the mass per unit length of the pile on the APVPSP reduction ratio