ESSAYS IN MACROECONOMICS AND FINANCIAL FRICTIONS

by

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Abstract

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My dissertation is comprised of three papers on the causes and consequences of the U.S. Great Recession. The emphasis is on the role that financial frictions play in magnifying financial shocks, as well as in informing the effectiveness of potential policies.

Chapter 1, "Financial Frictions, Investment Delay and Asset Market Interventions," co-authored with Shouyong Shi, studies the role of investment delay in propagating different types of financial shocks, and how this role impacts the effectiveness of asset market interventions. The topic is motivated by the observation that, during the Great Recession, governments conducted large-scale asset market interventions. The aim was to increase the level of liquidity in the asset market and make it easier for firms to obtain financing. However, firms were observed to have delayed investment by hoarding liquid funds, part of which were obtained through the interventions. We construct a dynamic macro model to incorporate financial frictions and investment delay. Investment is undertaken by entrepreneurs who face liquidity frictions in the equity market and a collateral constraint in the debt market. After calibrating the model to the U.S. data, we quantitatively examine how aggregate activity is affected by two types of financial shocks: (i) a shock to equity liquidity, and (ii) a shock to entrepreneurs’ borrowing capacity. We then analyze the effectiveness of government interventions in the asset market after such financial shocks. In particular, we compare the effects of government purchases of private equity and of private debt in the open market. In addition, we examine how these effects of government interventions depend on the option to delay investment.

In Chapter 2, "Housing Liquidity and Unemployment: The Role of Firm Financial Frictions," I build upon the role that firms’ ability to obtain funding plays in the severity of the Great Recession. I focus specifically on how the housing crisis reduced the ability of firms to obtain funding, and the consequences for unemployment. An important feature I focus on is the role of housing liquidity, or how easy it is to sell or buy a house. I analyze how an initial fall in housing market liquidity, linked to rising foreclosure costs for banks, affects labor market outcomes, which can have further feedback effects. I focus on the role that firm financial frictions play in these feedback effects. To this end, I construct a dynamic macro model
that incorporates frictional housing and labor markets, as well as firm financial frictions. Mortgages are obtained from banks that incur foreclosure costs in the event of default. Foreclosure costs also affect the ease with which firms can borrow, and this influences their hiring decisions. I calibrate the model to U.S. data, and find that a rise in foreclosure costs that generates a 10% fall in the firm loan-to-output ratio results in a 3 percentage point rise in the unemployment rate. The rise in unemployment makes it more difficult for indebted owners to avoid defaulting on their mortgage. This rise in default, on the order of 20 percent, creates further slack in the housing market by both increasing the number of houses on the market and reducing the amount of buyers. Consequently, there are large drops in housing prices and in the size of mortgage loans. Notably, when firm financial frictions are absent, I observe a counter-factual fall in the unemployment rate, which mitigates the effects on the housing market, and even results in a fall in the mortgage default rate. The results highlight the importance of the impact of the housing market crisis on a firm’s willingness to hire, and how firms’ limited access to credit magnifies the initial housing shock.

In Chapter 3, "Housing Market Distress and Unemployment: A Dynamic Analysis," I add to the contributions of my second paper, and extend the analysis to determine the dynamic effects of the housing crisis on unemployment. In Chapter 2, I focused on comparing stationary equilibria when there is a rise in the foreclosure costs associated with mortgage default. However, a full analysis must also take into account the dynamic effects of the shock. In order to do the dynamic analysis, I modify the model in my job market paper to satisfy the conditions of block recursivity. I do this by incorporating Hedlund’s (2016) technique of introducing real estate agents in the housing market that match separately with buyers and sellers. Doing this makes the model’s endogenous variables independent of the distribution of households and firms. Rather, the impact of the distribution is summarized by the shadow value of housing. This greatly improves the tractability of the model, and allows me to compute the dynamic response to a fall in a bank’s ability to sell a foreclosed house, thus raising the costs of mortgage default. I find that the results are largely dependent on the size and persistence of the shock, as well as the level of firm financial frictions that are present. When firm financial frictions are high, as represented by the presence of an interest rate premium charged to firms, and the initial shock is large, the shock is transferred to firms via an endogenous rise in the cost of renting capital. Firms scale back on production and reduce employment. The rise in unemployment increases the debt burden for households with large mortgages. They can try and sell, but find it difficult to do so because they must sell at a high price to be able to pay off their debt. If they fail, they are forced to default, thus further raising the mortgage costs of banks, further reducing resources to firms, and propagating the initial shock. However, the extent of the propagation is limited; once the shock wears off, the economy recovers to its pre-crisis levels within
two quarters. I discuss the reasons why, and what elements would be needed for greater persistence.
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Contents

1 Financial Frictions, Investment Delay and Asset Market Interventions 1
   1.1 Introduction ........................................................................................................ 1
   1.2 Environment of the Model Economy ................................................................. 5
   1.3 Optimal Decisions and the Equilibrium ......................................................... 11
       1.3.1 A household’s decisions ........................................................................ 11
       1.3.2 Optimal investment and delay .............................................................. 15
       1.3.3 Definition of a recursive equilibrium ...................................................... 17
   1.4 Quantitative Analysis on the Effect of Financial Shocks ............................... 19
       1.4.1 Calibration .............................................................................................. 19
       1.4.2 Financial shocks and aggregate activities .............................................. 21
   1.5 Effects of Government Intervention in the Asset Market ............................... 24
       1.5.1 Government purchases of private assets in the open market ................. 24
       1.5.2 Effects of government interventions ..................................................... 25
       1.5.3 The role of investment delay ................................................................. 28
   1.6 Robustness of the Results .................................................................................. 30
   1.7 Conclusion ......................................................................................................... 31

Appendices .............................................................................................................. 33
   1.A The Non-stochastic Steady State and Calibration ........................................... 33
       1.A.1 Determining the non-stochastic steady state ........................................... 33
       1.A.2 Calibration procedure ............................................................................. 34
   1.B Computing the Dynamic Equilibrium and Responses .................................. 37
   1.C Tables .............................................................................................................. 40
   1.D Figures ............................................................................................................ 41
2 Housing Liquidity and Unemployment: The Role of Firm Financial Frictions 47

2.1 Introduction ................................................................. 47
2.2 Related Literature .......................................................... 52
2.3 Model Environment .......................................................... 55
  2.3.1 The labor market ......................................................... 56
  2.3.2 The housing market ..................................................... 57
2.4 Equilibrium in the Baseline Economy ..................................... 60
  2.4.1 Firm value functions .................................................... 60
  2.4.2 Household value functions ............................................. 62
  2.4.3 Construction firm ....................................................... 68
  2.4.4 Mortgage arm ............................................................ 68
  2.4.5 Laws of motion .......................................................... 69
  2.4.6 Equilibrium definition .................................................. 69
2.5 Calibration ................................................................. 70
2.6 Steady State ................................................................. 73
2.7 Rising Foreclosure Costs: Magnification Via Housing and Labor Market Interactions ... 76
  2.7.1 Baseline model ............................................................ 76
  2.7.2 The dependence of the firm’s cost of credit on $\chi$ ................ 78
  2.7.3 The role of labor market frictions ..................................... 79
  2.7.4 Changes in $\chi$ vs. changes in a firm’s marginal revenue ............. 79
2.8 Conclusion ................................................................. 81

Appendices 83

  2.A Household Value Functions ............................................ 83
  2.B Derivation of $P_t^\delta(m_0, n)$ ......................................... 86
  2.C Laws of Motion ........................................................... 87
  2.D Tables ................................................................. 92
  2.E Figures ................................................................. 94

3 Housing Market Distress and Unemployment: A Dynamic Analysis 100

3.1 Introduction ................................................................. 100
3.2 Related Literature .......................................................... 103
3.3 Model ................................................................. 105
  3.3.1 Households ............................................................ 105
Chapter 1

Financial Frictions, Investment Delay and Asset Market Interventions

Joint with Shouyong Shi

1.1 Introduction

The great recession in 2008-2009 has increased the interest in studying the importance of financial frictions. In that recession, governments in various countries intervened in asset markets on large scales. By selling relatively more liquid government assets for less liquid, or private, assets, the government intended to increase the overall liquidity in the asset market and, hence, to reduce the negative effect of financial frictions on firms’ financing ability. The effectiveness of such interventions is still being debated. One particular skepticism is that firms were observed in the recession to delay investment by hoarding liquid funds, part of which were injected by the government during the interventions. In this paper we construct a dynamic macro model to incorporate financial frictions and investment delay. We calibrate the model to the US data to examine the quantitative effects of financial shocks on aggregate activity and the effectiveness of government interventions in the asset market after such shocks.

Financial frictions in our model reside in the investment sector where entrepreneurs are endowed with investment projects and choose how many of these projects to implement in each period to produce new capital. The frictions appear in both the equity and the debt market. The frictions in the equity
market are modeled as in Kiyotaki and Moore (2012, KM henceforth). In any period, an entrepreneur is constrained to sell no more than a fraction $\phi \in (0, 1)$ of the holdings of existing equity and to issue new equity on no more than a fraction $\theta \in (0, 1)$ of new investment. The fraction $\phi$ is called equity liquidity and, hence, shocks to $\phi$ are called liquidity shocks. In the debt market, an entrepreneur can only borrow up to an amount that depends positively on the value of equity that the entrepreneur holds at the end of the period. Such equity holdings can be interpreted as collateral. The borrowing constraint can arise from the risk of the borrower’s default, as in Kiyotaki and Moore (1997). The borrowing capacity is subject to shocks, $\mu$. These financial frictions create a wedge between the values of an entrepreneur’s internal and external funds and thus, between the price of equity and the replacement cost of capital. They also imply that the composition of equity and debt matters to an entrepreneur; i.e., the Modigliani-Miller theorem breaks down in the environment.

An entrepreneur can choose to delay investment by choosing the number of projects to be implemented. The amount of new capital produced by an entrepreneur is assumed to be increasing in both the amount of the resource allocated to investment (i.e., investment expenditure) and the stock of investment projects available to the entrepreneur. Unimplemented projects add to the future stock of projects, subject to depreciation. The option value of delay creates an additional wedge between the price of equity and the replacement cost of capital.

After calibrating this model to the US data, we first examine the dynamic effects of two negative financial shocks separately in the absence of government interventions in the asset market. One is a negative shock to liquidity, $\phi$, and the other one to the borrowing capacity, $\mu$. Then, we introduce government purchases of private assets, the amount of which is proportional to the size of the reduction in liquidity or the borrowing capacity. These purchases are assumed to be conducted in the open market for assets rather than targeted to specific firms. They are financed immediately by increasing the issuance of government bonds, as in the Troubled Asset Relief Program in the U.S. in 2008, and ultimately by increasing taxes. We assess whether such interventions can significantly reduce the negative effect of financial shocks on aggregate activity. Moreover, because the composition of equity and debt is relevant to an entrepreneur’s decision, we compare how equity purchases may have quantitatively different effects from debt purchases. Finally, we compare the effects of government interventions in an economy with investment delay and in an economy where delay has no benefit.\(^1\)

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\(^1\)This exercise does not imply that we view the large fall in asset liquidity in 2008-2009 as exogenous. On the contrary, the fall was largely caused by changes in economic fundamentals, such as the realization that mortgage related assets had a much lower quality than expected. The purpose of our analysis is to get a sense of how much the changes in asset market conditions can affect aggregate activity and how effective government interventions in the asset market can be. See section 1.7 for further discussions.
With regards to financial shocks, we find that a negative liquidity shock can have large negative effects on aggregate activity. An unanticipated reduction in liquidity by 18% can reduce investment expenditure by 11.1%, employment by 2.9%, output by 1.9%, and aggregate consumption by 0.8%. It is remarkable that the liquidity shock alone can generate such positive comovement among investment, employment, output and aggregate consumption. Moreover, the negative liquidity shock induces a significant fraction of investment projects to be delayed. These negative effects are persistent if the liquidity shock is persistent. With debt financing, we calibrate the model so that debt issuance raises more funds than new equity sales in the steady state. However, unlike equity, existing debt does not raise additional funds or provide liquidity. For this reason, a negative shock to the borrowing capacity has only one twelfth of the effect of a negative liquidity shock on aggregate activity, given that the two types of shocks have the same magnitude in percentage terms.

With regards to government interventions in the asset market, we find that equity purchases by the government can have sizable effects. In the case of the negative liquidity shock mentioned above, equity purchases of one trillion dollars exacerbate the initial negative effect of the liquidity shock on aggregate activity when the tax is assumed to be fixed at the time of the initial purchases. In subsequent periods, however, the purchases induce aggregate activity to recover more quickly than without interventions. In the second period after the purchases, investment recovers by 54%, output by 32%, and employment by 36% of the initial fall in period one. These large effects of equity purchases arise even though they are conducted in the open market rather than being directed to specific firms in financial distress. In contrast, government purchases of private debt have only small effects on aggregate activity. This contrast between the two types of interventions is not specific to the case of a liquidity shock. Even when the shock occurs to the borrowing capacity, equity purchases are more effective than debt purchases in helping the economy to recover.

On investment delay, we find that a negative liquidity shock induces significant delay of investment even after the government intervenes in the asset market. With equity purchases in particular, delay reduces the fraction of implemented projects by sixteen to twenty-nine percentage points more than in the hypothetical economy where delay has no benefit. Moreover, the option to delay significantly reduces the effectiveness of asset market interventions. Specifically, relative to no interventions, investment expenditure with equity purchases falls by more in the first period when the purchases take place; it recovers by more in the second period; and it recovers more slowly from the third period onward.

Our paper is related to the large literature on financial frictions in macroeconomics. Some earlier references that focus on firms’ borrowing constraints are Hellwig (1977), Townsend (1979), Williamson
Chapter 1. Financial Frictions, Investment Delay and Asset Market Interventions

(1987), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999). A more recent reference is Jermann and Quadrini (2012). We model the collateral constraint and the frictions in the equity market in a similar fashion to Kiyotaki and Moore. The latter has also been used by Ajello (2016), Nezafat and Slavik (2010), Del Negro et al. (2017) and Shi (2012). Our paper follows Shi closely to construct a tractable model for aggregate dynamics and to incorporate financial frictions in both the equity and debt market. The main additions of the current paper to Shi are the incorporation of investment delay and the evaluation of government interventions in the asset market. Even with investment delay, our analysis confirms the result in Shi that liquidity shocks alone can have large significant effects on aggregate activity and can generate positive comovement among aggregate variables. Del Negro et al. also use the large household framework to quantitatively evaluate asset market interventions. The main contrasts are as follows: (i) our model has no nominal rigidity; (ii) we incorporate the frictions in both the equity market and the debt market; (iii) we compare the quantitative responses of the equilibrium to the two types of financial shocks; (iv) we evaluate both equity and debt purchases by the government; and (v) we introduce an investment technology that allows for investment delay. We will contrast our paper with this previous work in detail at the end of section 1.5.2.

There is also a large literature on investment delay. Pindyck (1991) and McDonald and Siegel (1986) emphasize the option value of investment delay when there is uncertainty in the economy and when investment is partially irreversible. Boyle and Guthrie (2003) incorporate this idea in a model where firms face financing or liquidity constraints. Stokey (2016) emphasizes uncertainty about future policy as a cause of investment delay. These papers are more on the micro side of the economy. On the macro side, some examples include Bernanke (1983), Khan and Thomas (2013), and Gilchrist, Sim and Zakrajsek (2012). In particular, Gilchrist, Sim and Zakrajsek emphasize the importance of uncertainty shocks for business cycle fluctuations in the presence of financial market frictions. The two main ingredients of all these models are (partial) irreversibility of investment and uncertainty. In the presence of irreversibility, aggregation of individuals' decisions is tractable only under strong assumptions on preferences and technology, which can undermine the quantitative analysis. Our model eliminates irreversibility and the quantitative analysis assumes all shocks to be realized in the first period so that there is no uncertainty in subsequent periods. Instead, investment delay in our model arises from the role of the stock of projects as an input in the investment technology. The stock of projects is an endogenous state variable whose level is reduced by investment. The shadow price of this stock in the next period reflects the option value of current investment and is a cause of investment delay. A similar modeling approach is adopted by Jovanovic (2009), but his model does not incorporate financial frictions (see sections 1.2 and 1.3.2
1.2 Environment of the Model Economy

Time is discrete and lasts forever. There is a continuum of identical households, with measure one, and we choose an arbitrary household as the representative household. A household has a large number of members, and the total measure of members is set to one.\(^2\) At the beginning of each period, all members are identical. During a period, the members go to the market and are separated from each other until the end of the period. While in the market, a member receives a shock whose realization determines whether the member is an entrepreneur or a worker. The probability with which a member is an entrepreneur is \(\pi \in (0, 1)\). These shocks are iid across the members and over time. An entrepreneur receives a number \(a\) of new investment projects, and can implement investment projects but has no labor endowment.\(^3\) A worker receives one unit of labor endowment, receives no new investment project and cannot implement investment projects. The members’ preferences are represented by the household’s utility function:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \{ \pi u(c^e_t) + (1 - \pi) [U(c^w_t) - h(\ell_t)] \}, \quad \beta \in (0, 1),
\]

where the expectation is taken over the aggregate state of the economy that will be described later. Here, \(\beta\) is the discount factor, \(c\) consumption and \(\ell\) labor supply. The superscript \(e\) indicates an entrepreneur and the superscript \(w\) a worker. The utility functions \(u\) and \(U\) are strictly increasing and strictly concave, with \(u'(0) = U'(0) = \infty\) and \(u'(\infty) = U'(\infty) = 0\). The disutility function of labor, \(h\), is strictly increasing and strictly convex, with \(h'(0) \geq 0\) and \(h'(1) = \infty\). It is useful to interpret “workers” and “entrepreneurs” in the model broadly: Relative to their investment opportunities, entrepreneurs are individuals in an actual economy who are more financially constrained than workers.

At the beginning of each period, the household has assets, liabilities and investment projects. Assets consist of a diversified portfolio of equity claims (i.e., claims on capital), \(s\), and government bonds, \(b\). Liabilities consist of the household’s debt, \(d\), and (lump-sum) taxes, \(\tau\). The stock of investment projects is \(n\). Because all members are identical at the beginning of the period and the household does not know

---

\(^2\)The structure of a large household enables us to analyze aggregate dynamics tractably in the presence of heterogeneity. It is an extension of the structure used by Lucas (1990) who assumes that each household consists of three members. A similar structure has been used in other fields, e.g., monetary theory (Shi, 1997). The structure used in the current paper is a modification of that in Shi (2015).

\(^3\)Endogenizing \(a\) requires a study of the innovation process. We abstract from it to focus on implementation.
which member will be an entrepreneur or a worker in the market, the household divides the assets and liabilities evenly among the members. After this division, the household cannot reshuffle assets and liabilities among the members during the period and, in particular, workers cannot return to the household before the end of the period. The household does not divide the stock of investment projects among all the members. Instead, the household keeps the projects until the roles of all members in the period are realized, at which time the household divides the stock of projects only among the entrepreneurs. During the period, the members undertake their activities, including consumption, separately from each other in the market. At the end of the period, the members return home and pool their assets, liabilities and unimplemented projects.

The assumptions on timing and the separation of the members during a period capture the misalignment of funds and investment projects that are important for financial frictions to affect investment and aggregate activity in reality. Although a member who becomes an entrepreneur will need more funds for investment than another member who becomes a worker, the two are given the same amount of funds by the household. The assumption that only entrepreneurs hold investment projects is not only realistic but also meant to widen the gap between investment needs and the availability of funds. If the household divided the stock of investment projects among all the members instead, the entrepreneurs as a group would have only a small fraction of these projects given any reasonable value of \( \pi \). In this case, the need for investment funds at the aggregate level would be significantly smaller than under the maintained assumption. Also, entrepreneurs’ decisions on how much to invest would have much smaller consequences on the future stock of projects.\(^4\)

Two types of goods are produced in the economy, according to different technologies. Final consumption goods are produced by competitive firms according to the function \( F(k^D, \ell^D) \), where \( k^D \) is the amount of capital and \( \ell^D \) the amount of labor employed by such a firm. The function \( F \) exhibits diminishing marginal productivity in each factor and constant returns to scale. In contrast, new capital goods can only be produced by entrepreneurs. Each implemented project yields \( \gamma \) units of new capital, where \( \gamma \) is assumed to be a constant for simplicity. The number of projects implemented by an entrepreneur, \( m \), depends on the input of final goods in the investment, \( x \), and the stock of potential projects available to the entrepreneur, \( n/\pi + a \). Formally, the level of investment undertaken by an entrepreneur is

\[
i = \gamma m(x, \frac{n}{\pi} + a),
\]

\(^4\)Note that precautionary savings do not arise in this setup. Because the shock that determines whether a member is an entrepreneur or worker is iid across the members and over time, a household cannot build precautionary savings conditional on whether a member will become an entrepreneur in a period.
where \( m \leq n/\pi + a \). The investment technology has constant returns to scale with the additional properties \( m_1 > 0, m_2 > 0, m_{11} < 0 \) and \( m_{22} < 0 \). We refer to \( i \) as an entrepreneur’s investment, \( x \) as an entrepreneur’s investment expenditure, and \( \gamma \) as the size of a project. Projects that are not implemented in the current period can be carried over to the next period, with a survival rate \( \sigma_n \in (0, 1) \).

Because each entrepreneur receives a number \( a \) of new projects and implements \( m \) projects in the period, the net change in the total stock of projects in the household is \( \pi(a - m) \). The household’s stock of projects at the beginning of the next period will be

\[
n_{t+1} = \sigma_n \left\{ n + \pi \left[ a - m(x, \frac{n}{\pi} + a) \right] \right\}.
\]

The investment technology can be interpreted as follows. Suppose that an entrepreneur can choose the number of projects to experiment, \( n_x \leq n/\pi + a \), and the resource \( x \) to spend on these projects. The probability with which an experimented project succeeds in the current period is an increasing function of the resource spent on the project, which is \( x/n_x \). Let this probability of success be \( \hat{m}(x/n_x) \), with the properties \( 0 < \hat{m}' < (n_x/x)\hat{m} \) and \( \hat{m}'' < 0 \). If a project does not succeed in the current period, it can be experimented again in the future. Then, it is optimal to choose \( n_x \) to maximize the expected number of successfully implemented projects, which is \( n_x \hat{m}(x/n_x) \). With the properties of \( \hat{m} \), it is easy to verify that this optimal choice of \( n_x \) is \( n_x = n/\pi + a \); that is, the entrepreneur will experiment all available projects. The expected number of successfully implemented projects by the entrepreneur is \( (n/\pi+a)\hat{m}(x/(n/\pi+a)) \), which is denoted as \( m(x, n/\pi + a) \). For simplicity, we eliminate the uncertainty in the number of successes by assuming that it is equal to the expected number of successes for each entrepreneur.\(^5\) Clearly, the function \( m \) derived has constant returns to scale. Moreover, the assumptions on \( \hat{m}' \) and \( \hat{m}'' \) ensure that \( m \) has strictly positive and diminishing marginal productivity of each input.

The presence of the stock of projects in the investment technology creates an option value of a project and, hence, the possibility of investment delay. More precisely, the assumption \( m_2 > 0 \), together with \( \sigma_n > 0 \), is necessary for unimplemented projects to yield a future benefit, by increasing investment in the next period. However, even in the special cases \( m_2 = 0 \) and \( \sigma_n = 0 \), implementing all available projects may drive down the marginal productivity of investment expenditure by too much to be optimal. In fact, we assume that the constraint, \( m \leq n/\pi + a \), is never binding.\(^6\) We delay the comparison of our modeling of investment with that of the literature to the end of this section.

\(^5\)This idiosyncratic uncertainty may be interesting for studying other issues such as consumption inequality among entrepreneurs. We abstract from this idiosyncratic uncertainty in order to focus on the aggregate behavior of asset price in the business cycles. Note that since each household has many members, the number of successfully implemented projects in a household is deterministic and given by \( \pi m \).

\(^6\)This assumption is satisfied in the computed equilibrium.
Investment is impeded by financial frictions in both the equity and the debt market. To specify these frictions, consider an entrepreneur, who enters the market with equity claims $s$, holdings of government bonds $b$, and net debt in the private sector, $d$. The entrepreneur receives capital income from equity claims, after which a fraction $1 - \sigma_k$ of capital depreciates and so equity claims are rescaled by $\sigma_k$. To finance investment, the entrepreneur can use the newly received capital income, issue new equity on the investment, sell existing equity, and issue new debt. There are two frictions in the equity market as described by KM. First, an entrepreneur can issue equity in the market on only a fraction $\theta \in (0, 1)$ of investment. The equity on the remainder of new investment, $(1 - \theta)i$, must be retained in the current period by the entrepreneur’s household. The second friction is that at most a fraction $\phi \in (0, 1)$ of existing equity can be sold in the current period, and so the entrepreneur must retain the remaining amount $(1 - \phi)\sigma_k s$. These two frictions in the equity market place a lower bound on an entrepreneur’s equity holdings at the end of the period, $s_{t+1}^e$, as follows:

$$s_{t+1}^e \geq (1 - \theta)i + (1 - \phi)\sigma_k s.$$  \hspace{1cm} (1.2.3)

This lower bound constrains an entrepreneur’s financing ability because, if the equity market frictions did not exist, the entrepreneur would rather reduce equity holdings at the end of the period to zero and use the proceeds to finance investment projects. Although it is useful to endogenize $\theta$ and $\phi$ by specifying asset market frictions in detail, we follow KM and Shi (2015) by treating $\theta$ and $\phi$ as exogenous. Also, we fix $\theta$ and focus on the effect of changes in $\phi$. The shocks to $\phi$ are called liquidity shocks.

For (1.2.3) to be binding, there should also be frictions in the debt market that restrict an entrepreneur’s ability to borrow. In particular, because it is difficult to enforce debt repayment, a lender demands a borrower to submit collateral to back up the borrowing. Following the argument by Kiyotaki and Moore (1997), we assume that an entrepreneur can use equity holdings at the end of the period as collateral. To be precise, the face value of debt issued by an entrepreneur, denoted $d_{t+1}^e$, is bounded above by a multiplier $z$ of the value of the entrepreneur’s equity holdings at the end of a period:

$$d_{t+1}^e \leq z(\phi, \mu)qs_{t+1}^e,$$  \hspace{1cm} (1.2.4)

where $q$ is the price of equity claims and $0 < z < 1$. The element $\mu$ follows a Markov process in which the innovation represents a shock to an entrepreneur’s borrowing capacity that is not necessarily related to equity liquidity. We assume that the collateral multiplier $z$ is increasing in $\phi$ to capture the realistic feature that a lender who can sell the collateral more easily in the market is more willing to lend a higher
Let us summarize the timing of events in an arbitrary period. At the beginning of the period, the shocks to \( \phi \) and \( \mu \) are realized.\(^7\) The aggregate state in the period is \( A = (K, N, \phi, \mu) \), where \( K \) is the capital stock per household and \( N \) is the stock of investment projects per household at the beginning of the period. The household evenly distributes the assets and liabilities among the members. The household also chooses consumption, investment, labor supply, and the end-of-period portfolio holdings for each member, conditional on whether the member will be an entrepreneur or worker. Then the members enter the market and cannot share funds until the end of the period. The shocks are realized to determine whether an individual is an entrepreneur or a worker in the period. The household divides the stock of projects among the entrepreneurs, each of whom also receives a number \( a \) of new projects. Then, the producers of final goods rent capital and hire labor to produce consumption goods. After production, workers receive wage income, equity holders receive the rental income of capital, and a fraction \((1 - \sigma_k)\) of capital depreciates. Next, the asset market opens. Individuals repay private debt, redeem government bonds and pay taxes. An entrepreneur seeks funds to finance projects and carries out investment. Of the projects that are not implemented, a fraction \( \sigma_n \) survive. After consuming goods, individuals return to the household, where they pool the assets, liabilities and unimplemented projects. Time proceeds to the next period.

It is worthwhile repeating that the separation of the members during a period is important in the model. The separation captures the reality that entrepreneurs who need funds to finance investment have difficulty obtaining funds. If they were able to meet the workers in the household during a period, contrary to what we assume, then they would be able to circumvent the financial frictions by simply using the workers’ funds. This importance of separation during a period is similar to that in the models of limited participation (e.g., Lucas, 1990).

The government in each period spends \( g \) on final goods, collects lump-sum taxes \( \tau \), issues government bonds \( B_{t+1} \), and redeems outstanding government bonds \( B \). In addition, the government may intervene in the asset market by purchasing equity and lending to entrepreneurs. Let \( s^g \) be the amount of private equity and \( d^g \) the face value of private debt held by the government, both of which are measured in amounts per household. Let \( p_b \) denote the price of government bonds and \( p_d \) the price of private debt. The government budget constraint is:

\[
g = \tau + (p_b B_{t+1} - B) + [(r + \sigma_k q)s^g - qs^g_{t+1}] + (d^g - p_d d^g_{t+1}).
\]

\(^7\)By assuming that the shocks are realized at the beginning of the period, we simplify the analysis by eliminating the need for precautionary savings.
We assume that $g$ is constant over time, while other terms in the above constraint can be time varying. In the baseline model, we assume that the amount of government bonds is constant over time at $B^* > 0$ and government purchases of private equity and private debt are zero. In section 1.5 we will introduce government purchases of equity or private debt, accompanied by changes in the amount of government bonds issued. In both cases, $(B_{t+1}, s_{t+1}^R, d_{t+1}^R)$ are only functions of $(q, p_b, p_d, A)$.

Financial frictions put a wedge between private assets and government bonds. In particular, private debt is not a perfect substitute for government debt. While the debt issued by entrepreneurs requires collateral, government debt does not. It is possible that a household simultaneously lends to the government and borrows from the government through entrepreneurs. That is, $B$ and $d$ can both be positive in the equilibrium. Moreover, issuing government bonds to purchase private debt has real effects in general. Similarly, issuing government bonds to purchase equity has real effects. We will examine such government interventions in the asset market in section 1.5.

We end this section by comparing the investment technology and delay in our model with those in the literature. The majority of the literature on investment delay relies on uncertainty in the environment for future investment. Although such uncertainty is allowed in the general formulation of our model as innovations in future $(\phi, \mu)$, it will not be present in the quantitative analysis that will only examine one-time shocks to $(\phi, \mu)$. The literature on investment delay also emphasizes that investment is lumpy and partially irreversible, as captured by the assumptions that there is a fixed cost to investment and that capital is more productive for the original creator than for an outsider. The fixed cost to investment does not exist in our model, provided that $m(0, n/\pi + a) \geq 0$. Irreversibility does not exist in our model, either, despite the illiquidity of claims on capital. In fact, a producer of capital in our model (i.e., an entrepreneur) does not employ what he produces; instead, capital is employed by the producers of final goods and has the same productivity with all such producers. Even the illiquid claims on capital can be resold at the market price asymptotically. In lieu of the fixed cost and irreversibility, the role of the stock of projects in the investment technology, (1.2.1), generates delay in our model. As current investment reduces the stock of projects available in the next period, the option value of investment increases, and so does the benefit of delay. This modeling of delay induces a smooth tradeoff between current and future investment that simplifies aggregation. Also, since investment is a fixed multiplier $(\gamma)$ of the number of investment projects undertaken, our model retains the importance of the extensive margin of investment emphasized by the literature, without the complexity introduced by the fixed cost and irreversibility.

The role of the stock of projects as an endogenous state variable is closely related to the role of
“seeds” in Jovanovic (2009). In his model, investment in new capital (“trees”) is constrained by the amount of “seeds”, and new seeds are generated as a by-product of the production of consumption goods (“fruits”). In our notation, he assumes

\[ i = x \leq n/\pi + a, \quad a = \lambda F \quad \text{and} \quad F = k, \]

where \( \lambda > 0 \) is a constant. We fix the number of new projects and, instead, endogenize the use of the projects through the function \( m(x, n/\pi + a) \). This function can be plausibly interpreted as an outcome of experimentation. In addition, the interaction between the two inputs in this function provides a channel by which financial frictions affect investment delay and, hence, the price of capital (see section 1.3.2). In contrast, financial frictions are absent in Jovanovic’s model. Finally, in the special case \( m(x, n/\pi + a) = x \), the investment technology in our model becomes the one assumed by KM and Shi (2015) who also investigate the importance of equity illiquidity for macro dynamics.

### 1.3 Optimal Decisions and the Equilibrium

#### 1.3.1 A household’s decisions

A household makes the decisions for each member, conditional on whether the member will be an entrepreneur or a worker in the period. For an entrepreneur, the household chooses consumption \( c^e \), investment expenditure \( x \), the face value of debt issuance \( d^e_{t+1} \), and the holdings of equity and government bonds at the end of the period, \( (s^e_{t+1}, b^e_{t+1}) \). The implied investment is \( i = \gamma m \), where \( m \) is given by (1.2.1). Similarly, for a worker, the household chooses consumption \( c^w \), labor supply \( \ell \), debt \( d^w_{t+1} \), and the holdings of equity and government bonds at the end of the period, \( (s^w_{t+1}, b^w_{t+1}) \). For each entrepreneur, the household faces the equity liquidity constraint (1.2.3) and the following budget constraint:

\[
rs + (p_d d^e_{t+1} - d) + (b - p_b b^e_{t+1}) + q(i + \sigma_k s - s^e_{t+1}) \geq c^e + x + \tau, \tag{1.3.1}
\]

where \( r \) is the rental rate of capital and \( q \) the post-dividend price of a share of equity, measured in terms of the consumption good. The right-hand side of (1.3.1) represents an entrepreneur’s expenditures on consumption, investment expenditure and taxes. The left-hand side represents an entrepreneur’s resources. The first is the rental income on capital, \( rs \). The second is the value of new debt minus the repayment on outstanding debt, \( (p_d d^e_{t+1} - d) \). The third is net income from re-balancing the holdings of government bonds, \( (b - p_b b^e_{t+1}) \). The fourth is the net value of re-balancing equity holdings. Implemented projects create \( i \) units of new capital, the claims on which can either be sold to outsiders or retained by the household. After capital depreciates, the entrepreneur also holds \( \sigma_k s \) claims on existing equity.
Thus, the entrepreneur has in total $i + \sigma_N s$ of equity claims. Since the entrepreneur keeps $s_{x+1}^e$ claims at the end of the period, the rest must be sold in the asset market. The value of this sale is $q(i + \sigma_N s - s_{x+1}^e)$.

Consider the case where the liquidity constraint binds. An entrepreneur will optimally hold as little equity at the end of the period as possibly allowed by the constraint (1.3.1), set the amount of government bonds to be carried into the next period to zero, and borrow up to the bound allowed by (1.2.4). That is,

$$s_{x+1}^e = (1-\theta) i + (1-\phi) \alpha_N s, \quad b_{x+1}^e = 0, \quad d_{x+1}^e = z(\phi, \mu) q s_{x+1}^e. \tag{1.3.2}$$

Substituting these optimal choices of $(s_{x+1}^e, b_{x+1}^e, d_{x+1}^e)$ into (1.3.1), we obtain the following consolidated financing constraint on an entrepreneur:

$$(r + \phi z \sigma_N q) s + b - d \geq c^e + (x - \theta_z qi) + \tau, \tag{1.3.3}$$

where $\phi_z$ and $\theta_z$ are defined as

$$\phi_z = \phi + (1-\phi) z p_d, \quad \theta_z = \theta + (1-\theta) z p_d. \tag{1.3.4}$$

The quantity $\phi_z q$ is the amount of funds raised from each unit of existing equity, by re-selling $\phi$ fraction of the equity and using the remaining fraction as collateral in borrowing. Since $z < 1$ (and $p_d < 1$), the more reselable is equity (i.e. the higher is $\phi$), the more an entrepreneur is able to finance investment by selling existing equity. Similarly, $\theta_z q$ is the amount of funds raised from equity on a unit of new investment, by issuing new equity on a fraction $\theta$ of the investment and using the retained equity as collateral in borrowing. The amount of funds needed for $i$ units of investment is $x$. Since the amount raised from equity is $\theta_z qi$, the remaining amount, $(x - \theta_z qi)$, is the “downpayment” on investment that must come from other sources.

A worker enters the market with the same asset portfolio, $(s, b)$, and debt, $d$, as an entrepreneur does. In contrast, a worker earns labor income and does not have investment projects. The worker also earns income by re-balancing his portfolio of assets, but cannot sell new equity. Hence, a worker’s

---

8The price of new equity is the same as $q$, the post-dividend price of existing equity. The reason is that a share of existing equity after paying dividends and a share of new equity command the same stream of future dividends and are subject to the same frictions. Also, as in KM, we simplify the analysis by assuming that the claims on the household’s own capital and other households’ capital have the same liquidity, and so they have the same price.

9This is the case in the dynamic equilibrium computed under the parameter values calibrated later.

10Since $u(c^e)$ is strictly concave, (1.3.1) holds with equality. Thus, an entrepreneur’s liquidity constraint, (1.2.3), is binding if and only if (1.3.3) is binding.
budget constraint is:

\[ rs + w\ell + (p_d d_{+1}^w - d) + (b - p_b b_{+1}^w) + q(\sigma_k s - s_{+1}^w) \geq c^w + \tau. \]

Here, \( w \) is the real wage rate. Because a worker does not have investment to finance, a worker is a buyer of new and existing equity and a lender in the equilibrium. As a result, a worker at the end of the period will hold more equity than the lower bound imposed by equity market frictions (i.e., \( s_{+1}^w > (1 - \phi)\sigma_k s \)), lend to the government (i.e., \( b_{+1}^w > 0 \)), and lend to entrepreneurs (i.e., \( d_{+1}^w < 0 \)). This result supports the earlier statement that the workers in the model should be broadly interpreted as financially unconstrained individuals in an actual economy.

Denote average consumption per member in the household as \( c \) and the average holdings of the portfolio and debt per member, including projects, at the end of the period as \((n, s, b, d)\). Then

\[ c = \pi c^e + (1 - \pi)c^w, \]

and similar equations hold for \((n, s, b, d)\). The household’s budget constraint can be obtained by weighting the entrepreneur’s and the worker’s budget constraints by \( \pi \) and \( 1 - \pi \), respectively, and adding up:

\[ (1 - \pi)w\ell + (p_d d_{+1} - d) + (b - p_b b_{+1}) + (r + \sigma_k q)s - qs_{+1} \geq c + \tau + \pi(x - qi). \quad (1.3.5) \]

Recall that the aggregate state is \( A = (K, N, \phi, \mu) \).\(^{11}\) Equilibrium prices are functions of \( A \), which include equity price \( q(A) \), the price of government bonds \( p_b(A) \), the price of private bonds \( p_d(A) \), the rental rate of capital \( r(A) \) and the wage rate \( w(A) \). All prices are expressed in terms of the consumption good, which is the numeraire. The household’s value function is \( v(n, s, b, d; A) \), where \((n, s, b, d)\) are the individual household’s state variables. The household’s choices in a period are \((x, c^e, s_{+1}^e, b_{+1}^e, d_{+1}^e)\) for each entrepreneur, \( \ell \) for each worker, and \((c, n_{+1}, s_{+1}, b_{+1}, d_{+1})\) for the average quantities per member, which together imply the choices for each worker. As explained earlier, when the financing constraint (1.3.3) binds, the optimal choices of \((s_{+1}^e, b_{+1}^e, d_{+1}^e)\) are given by (1.3.2). The other choices, \((x, c^e, \ell, c, n_{+1}, s_{+1}, b_{+1}, d_{+1})\), solve:

\[
v(n, s, b, d; A) = \max \{ \pi u(c^e) + (1 - \pi)[U(c^w) - h(\ell)] \\
+ \beta E v(n_{+1}, s_{+1}, b_{+1}, d_{+1}; A_{+1}) \} \]

\(^{11}\)Strictly speaking, the aggregate state should also include the supply of government bonds, \( B \), and government purchases of private assets, \((s^g, d^g)\). We omit them from the list because they are assumed to be functions of \((q, p_b, p_d, K, \phi, \mu)\).
subject to (1.3.3), (1.3.5), and the following constraints:

\[ c^w = (c - \pi c^e)/(1 - \pi), \quad i = \gamma m(x, \frac{n}{\pi} + a), \]
\[ n_{+1} = \sigma_n \{ n + \pi [a - m(x, \frac{2}{\pi} + a)] \}, \]
\[ x \geq 0, \quad c^e \geq 0, \quad c^w \geq 0, \quad n_{+1} \geq 0, \quad s^w_{+1} \geq 0, \quad b^w_{+1} \geq 0. \]

The expectation in the objective function is taken over \( A_{+1} \).

Denote \( \lambda \) as the Lagrangian multiplier of the household’s budget constraint, (1.3.5). The optimal choice of \( c \) yields \( \lambda = U'(c^w) \). Let \( \lambda^e \pi U'(c^w) \) be the Lagrangian multiplier on the financing constraint, (1.3.3), so that \( \lambda^e \) is the shadow price of the financing constraint measured in a worker’s consumption units. The liquidity constraint (1.2.3) binds if and only if \( \lambda^e \) is positive. The optimal choices of \((\ell, c^e)\) yield:

\[ h'(\ell)/U'(c^w) = w, \quad (1.3.6) \]
\[ u'(c^e) = U'(c^w)(1 + \lambda^e). \quad (1.3.7) \]

Condition (1.3.6) is the familiar condition of optimal labor supply. Condition (1.3.7) shows that a marginal unit of the consumption good yields the additional value \( \lambda^e U'(c^w) \) to an entrepreneur relative to a worker by relaxing the financing constraint (1.3.3). Thus, \( \lambda^e \) is indeed the marginal value of liquid funds to an entrepreneur in terms of a worker’s consumption. We will exhibit and explain the condition of optimal investment and the value of an investment project in the next subsection.

In addition, the optimality conditions of asset holdings and debt, \((s_{+1}, b_{+1}, d_{+1})\), and the envelope conditions of \((s, b, d)\) yield the following asset-pricing equations:

\[ q = \beta E \left\{ \frac{U'(c^w)}{U'(c^w)} \left[ r_{+1} + \sigma_k q_{+1} + \pi \lambda^e_{+1} (r_{+1} + \phi_{z_{+1}} \sigma_k q_{+1}) \right] \right\} \quad (1.3.8) \]
\[ p_b = \beta E \left[ \frac{U'(c^w)}{U'(c^w)} (1 + \pi \lambda^e_{+1}) \right] \quad (1.3.9) \]
\[ p_d = p_b. \quad (1.3.10) \]

These asset pricing equations require the effective rate of return to an asset, evaluated with the marginal utility of consumption, to be equal to \( 1/\beta \). The effective return to an asset includes the direct return and liquidity services provided by the asset. For example, in (1.3.9), an additional unit of government bond in the hand of an entrepreneur enables the entrepreneur to reduce the extent to which the financing

---

12The constraints \( c^e \geq 0, c^w \geq 0, s^w_{+1} \geq 0, b^w_{+1} \geq 0 \) and \( n_{+1} \geq 0 \) do not bind. In particular, \( n_{+1} > 0 \) under the assumption that \( m_2(x, 0) \) is sufficiently large.
constraint (1.3.3) binds, which generates liquidity service in the amount \( \pi \lambda_{t+1} \). Moreover, the price of private debt is equal to the price of government bonds because a worker, who is a lender, is indifferent between lending to the government without the collateral requirement and lending to entrepreneurs with the collateral requirement (1.2.4). This equality between the two prices does not contradict the earlier statement that the debt issued by an entrepreneur is not a perfect substitute for government debt. The debt issued by an entrepreneur requires collateral according to the constraint (1.2.4), but government debt does not. This borrowing constraint (1.2.4) imposes an additional cost to an entrepreneur on issuing debt. An entrepreneur strictly prefers issuing government bonds to issuing private debt, but doing the former is not possible.

### 1.3.2 Optimal investment and delay

To characterize optimal investment, let us define the implicit price of an investment project in terms of a worker’s consumption as

\[
p_n = \frac{1}{U'(c^w)} \frac{\partial v}{\partial n}.
\]  

(1.3.11)

The optimal choice of \( x \) and the envelope condition of \( n \) in the household’s optimization problem yield:

\[
q - \frac{1}{\gamma m_1} \leq \lambda^c \left[ \frac{1}{\gamma m_1} - \theta_z q \right] + \frac{\sigma_n}{\gamma} \beta \mathbb{E} \left[ \frac{U'(c^w)}{U'(c^w)} p_{n+1} \right],
\]  

(1.3.12)

\[
p_n = \gamma m_2 (1 + \theta_z \lambda^c) q + (1 - m_2) \sigma_n \beta \mathbb{E} \left[ \frac{U'(c^w)}{U'(c^w)} p_{n+1} \right].
\]  

(1.3.13)

The inequality in (1.3.12) and the inequality \( x \geq 0 \) hold with complementary slackness.

Let us explain (1.3.12) first. Given the stock of investment projects available to an entrepreneur, the direct cost of producing one unit of new capital at the margin is \( 1/\gamma m_1 \). Let us refer to this cost as the replacement cost of capital. Because each unit of capital has value \( q \), then \( (q - 1/\gamma m_1) \) is the marginal benefit of one unit of new capital in excess of the replacement cost. For investment to be optimal, this excess benefit must be equal to the opportunity cost of investment, which consists of the two terms on the right-hand side of (1.3.12). One is the cost of downpayment on investment. Since the amount of funds that can be raised through equity on each unit of investment is \( \theta_z q \), the downpayment on a marginal unit of investment is \( (1/\gamma m_1 - \theta_z q) \), and so the cost of such a downpayment is \( \lambda^c (1/\gamma m_1 - \theta_z q) \). The other implicit cost of investment is the option value of delaying investment. Delaying one unit of investment saves \( 1/\gamma m_1 \) units of the resource and, hence, increases the number of unimplemented projects by \( m_1/\gamma m_1 = 1/\gamma \). As a result, the household’s stock of projects in the next period will increase by \( \sigma_n/\gamma \).
Because each project in the next period will have a value \( p_{n+1} \) in terms of a worker’s consumption in the next period, its expected value in terms of a worker’s current consumption is \( \beta E\{[U'(c_{w+1})/U'(c_w)]p_{n+1}\} \). The last term in (1.3.12) is the option value of one unit of delayed investment. Therefore, this model presents two forces that drive equity price above the replacement cost of capital. One is financial frictions that affect the amount and the cost of downpayment on investment, and the other is the option value of a project. In contrast, Jovanovic (2009) emphasizes the second force but abstracts from the first.

The price of an investment project obeys the intertemporal equation, (1.3.13). The two terms on the right-hand side are the values that a higher stock of investment projects today can generate in the current and the next period. A higher stock of investment projects increases current investment by \( \gamma m_2 \). The equity associated with a unit of new capital is \( q \). In addition, a unit of investment can be used to raise the amount \( \theta z q \) units of funds to relax an entrepreneur’s financing constraint, the implicit value of which is \( \theta z q \lambda^c \). Thus, the increased investment brought about by a higher stock of investment projects has the marginal value \( \gamma m_2 (1 + \theta z \lambda^c) q \) in the current period. Moreover, a higher stock of investment projects increases the stock in the next period by \( (1 - m_2) \sigma_n \). The future value of this higher stock of projects is given by the last term in (1.3.13) according to the above explanation for the option value of delayed investment.

As mentioned before, it is never optimal to implement all available projects because of diminishing marginal productivity of investment expenditure. This is true even when the financing constraint is not binding and unimplemented projects have zero option value, i.e., when \( \lambda^c = 0 \) and \( \sigma_n = 0 \). Thus, investment delay is not indicated by the mere existence of a positive gap, \( (n/\pi + a - m) \), but rather by how much of this gap is caused by financial frictions and how much by the option value of unimplemented projects.

Of particular interest is the extent of investment delay caused by financial frictions. To measure this, let us characterize optimal investment in the hypothetical economy where the financing constraint is not binding and an unimplemented project has the same option value as in the model economy. Precisely, in this hypothetical economy, \( (n, q, p_d, p_{n+1}, c_w, c_{w+1}) \) are the same as in the model economy but \( \lambda^c = 0 \). Then, an entrepreneur’s optimal investment expenditure, denoted as \( x^f \), satisfies (1.3.12) with \( \lambda^c = 0 \). That is,

\[
q - \frac{1}{\gamma m_1} = \frac{\beta \sigma_n}{\gamma} E \left[ \frac{U'(c_{w+1})}{U'(c_w)} p_{n+1} \right],
\]

where \( m_1 = m(x^f, n/\pi + a) \) and \( (m_1^f, m_2^f) \) denote the partial derivative of \( m(x^f, n/\pi + a) \). By con-

\[\text{Note that this term is not divided by } \gamma \text{ because all terms in (1.3.13) are measured in a worker’s current consumption, which contrasts with (1.3.12) where all terms are measured in current investment.}\]
struction, investment delay in this hypothetical economy is caused entirely by the option value of an investment project and not by the financing constraint. The additional delay caused by financial frictions can be measured by \(1 - f_{imp}\), where \(f_{imp}\) is defined as

\[ f_{imp} = \frac{m}{m^f}. \quad (1.3.15) \]

Subtracting (1.3.14) from the equality form of (1.3.12) yields:

\[ \frac{1}{\gamma m^1} - \frac{1}{\gamma m_1} = \lambda^e \left( \frac{1}{\gamma m_1} - \theta z q \right). \quad (1.3.16) \]

The left-hand side of (1.3.16) is increasing in \(x^f\) and decreasing in \(x\). Thus, intuitively, financial frictions are likely to cause a larger delay in investment if the current period has a tighter financing constraint (i.e., higher \(\lambda^e\)), a lower ability to use equity to raise funds (i.e., a lower \(\theta z q\)), or a lower productivity of investment expenditure (i.e., a lower \(\gamma m_1\)).

### 1.3.3 Definition of a recursive equilibrium

Let \(K \subset \mathbb{R}_+\) and \(N \subset \mathbb{R}_+\) be compact sets that have as their elements all possible values of \(K\) and \(N\), respectively. Let \(\Phi \subset [0,1]\) be a compact set that contains all possible values of \(\phi\) and let \(\mu\) lie in the set \([0,1]\). Denote \(A = K \times N \times \Phi \times [0,1]\). Let \(C_1\) be the set of all continuous functions that map \(A\) into \(\mathbb{R}_+\), \(C_2\) the set of all continuous functions that map \(N \times K \times [0,B] \times \mathbb{R} \times A\) into \(\mathbb{R}_+\) and \(C_3\) the set of all continuous functions that map \(N \times K \times [0,B] \times \mathbb{R} \times A\) into \(\mathbb{R}\). Asset and factor prices, \((q, p_b, p_d, r, w)\), are functions of the aggregate state \(A\) and, hence, lie in \(C_1\). The value function \(v\) is a function of the household’s own state variables \((n, s, b, d)\) and the aggregate state \(A\). So are the household’s policy functions for optimal choices, \((x, c^e, s^e_{+1}, b^e_{+1}, d^e_{+1}, \ell, c, s_{+1}, b_{+1}, d_{+1})\). Given government policies \((B, s^g, d^g)\), a recursive competitive equilibrium is a list of asset and factor price functions \((q, p_b, p_d, r, w) \in C_1\), a household’s policy functions \((x, c^e, s^e_{+1}, b^e_{+1}, d^e_{+1}, \ell, c, s_{+1}, b_{+1}, d_{+1}) \in C_2\), the value function \(v \in C_3\), the factor demand functions, \((k^D, \ell^D)\), and the laws of motion of the aggregate capital and project stocks that meet the following requirements:

(i) Given price functions and the aggregate state, a household’s policy and value functions solve a household’s maximization problem;

(ii) Given price functions and the aggregate state, \((k^D, \ell^D)\) maximize producers’ profit, i.e., \(r = F_1(k^D, \ell^D)\) and \(w = F_2(k^D, \ell^D)\);
(iii) Given the law of motion of the aggregate state, prices clear the markets:

\begin{align*}
\text{goods: } F(k^D, \ell^D) &= c + g + \pi x \\
\text{capital: } k^D &= K = s + s^g \\
\text{labor: } \ell^D &= (1 - \pi)\ell \\
\text{government bonds: } b_{+1} &= B_{+1} \\
\text{equity: } s_{+1} + s^g_{+1} &= \sigma_k(s + s^g) + \pi i \\
\text{private debt: } d_{+1} &= d^g_{+1}.
\end{align*}

(iv) Symmetry and aggregate consistency: \((s, n, b) = (K, N, B)\), and the laws of motion of the aggregate capital and project stocks are consistent with the aggregation of individual households' choices:

\begin{align*}
K_{+1} &= \sigma_k K + \pi i, \quad N_{+1} = \sigma_n[N + \pi(a - m)].
\end{align*}

In the above conditions, we have suppressed the arguments of the policy functions. Condition (1.3.18) says that all capital is claimed by the private sector and the government. To explain (1.3.19), recall that \(d_{+1}\) is the household’s average debt per member. Because all households are symmetric and each worker in a household lends to other households’ entrepreneurs, \(d_{+1}\) is a household’s debt position after netting out the liabilities with other households. This private debt, if positive, must be held by the government, which is what (1.3.19) requires. The consistency conditions in (iv) are required in order for households to compute expectations in their optimization problem. Note that since \(K = s + s^g\), the law of motion for the aggregate capital stock is identical to the equity market clearing condition.

Finding an equilibrium amounts to finding the asset price functions, \(q(A)\) and \(p_b(A)\), that solve (1.3.8) and (1.3.9), which are part of the requirements for optimality in (i) above. Once the asset price functions are solved, factor price functions, and the value and policy functions can be recovered from other equilibrium conditions. Appendix 1.B describes the procedure for computing the equilibrium.
1.4 Quantitative Analysis on the Effect of Financial Shocks

1.4.1 Calibration

To calibrate the model, we assume the following functional forms:

- production function: \[ F(K, (1 - \pi)\ell) = K^\alpha((1 - \pi)\ell)^{1-\alpha} \]
- worker’s utility function: \[ U(c^w) = (c^w)^{1-\rho} + 1 \]
- entrepreneur’s utility function: \[ u(c^e) = u_0U(c^e) \]
- disutility of labor: \[ h(\ell) = h_0\ell^\eta \]
- investment technology: \[ m(x, \frac{a}{\pi} + \alpha) = \left\{\frac{1}{2}(\delta x)\xi + \frac{1}{2}(\frac{a}{\pi} + \alpha)\xi\right\}^{\frac{1}{\xi}} \]
- collateral multiplier: \[ z(\phi, \mu) = \phi \mu. \]

In the investment technology, \( \delta \) converts investment expenditure into the same unit as the available stock of projects, and \( 1/(1 - \xi) \) is the elasticity of substitution between the two inputs. The functional form of the collateral multiplier has the maintained property that it is increasing in equity liquidity. The multiplicative form conveniently implies that a shock to \( \phi \) and a shock to \( \mu \) of the same percentage points affect the collateral multiplier in the same percentage points. Denote \( \tilde{\phi} = 1/\phi - 1 \) and \( \tilde{\mu} = 1/\mu - 1 \). We assume the following processes for \( \tilde{\phi} \) and \( \tilde{\mu} \):

\[
\begin{align*}
\log \tilde{\phi}_{t+1} &= (1 - \sigma_\phi) \log \tilde{\phi}^* + \sigma_\phi \log \tilde{\phi}_t - \varepsilon_{\phi, t+1} \\
\log \tilde{\mu}_{t+1} &= (1 - \sigma_\mu) \log \tilde{\mu}^* + \sigma_\mu \log \tilde{\mu}_t - \varepsilon_{\mu, t+1},
\end{align*}
\]

where \( \tilde{\phi}^* \) is the steady state level of \( \tilde{\phi} \), \( \tilde{\mu}^* \) the steady state level of \( \tilde{\mu} \), and \( \sigma_\phi, \sigma_\mu \in (0, 1) \).

The deterministic steady state is described in Appendix 1.A.1 for any given government policy \( (B^*, s^g, d^g) \). In the calibration, we set government purchases of equity and private debt to zero, i.e., \( s^g = d^g = 0 \). The length of a period is one quarter. Table 1 lists the parameters, their values, and the targets. Appendix 1.A.2 describes how to use the targets to solve the parameters. With these parameter values, the financing constraint (1.3.3) is binding.

The targets that determine \( (\beta, \rho, \eta, u_0, h_0, \alpha, \sigma_k, g) \) are standard in macro analyses.\(^{14}\) One exception might be the elasticity of labor supply, which is deliberately set to be relatively low in order to ensure that the response of employment to financial shocks does not come from very elastic labor supply. Note that \( u_0 \) affects investment undertaken by an entrepreneur and, hence, setting the ratio of the steady

\(^{14}\)A target may involve more than one parameter. To identify the parameters, we use several targets jointly to solve a number of parameters. However, to link the parameters to the targets intuitively, we describe the identification as if each target could identify a parameter separately.
The three parameters, \( (\theta, \mu^*, \phi^*) \), describe financial frictions. The target for \( \theta \) comes from the evidence in Nezafat and Slavik (2010). Using the U.S. Flow of Funds, these authors construct a time series for the ratio of funds raised in the market to investment expenditure by nonfarm nonfinancial corporate firms. They find that the mean of this ratio is 0.284. In their definition, the amount of funds raised in the market is equal to new equity issuance plus credit market instruments. In our model, this amount is the sum of the value of new equity issuance, \( q_\theta i \), and the amount borrowed in the market, \( pdzqs_{s+1} \). We set the ratio of this sum to investment expenditure \( x \) in the steady state to 0.284. The target for \( \mu^* \) comes from the evidence in Covas and den Haan (2011). Using the data COMPUSTAT, they report the ratio of debt issuance to assets and the ratio of stock sales to assets in each period for US nonfarm nonfinancial corporate firms. Dividing these two ratios yields the ratio of debt issuance to stock sales, denoted as \( DE \). This ratio exhibits large variation across firms and is increasing in firm size. We target the value for the bottom 50% of firms, 1.287, in order for the collateral constraint to be binding. Setting the equity premium as a target helps identifying \( \phi^* \) because the lower liquidity of equity relative to government bonds is the cause of equity premium in our model. Although the target, 0.02, is smaller than the equity premium in the US data, it may be justified by noting that it is the equity premium in a deterministic steady state. Even with this relatively low target on the equity premium, \( \phi^* \) is far below one. If the equity premium were set at a higher target, say 0.03, \( \phi^* \) would be very close to zero.

It is clear that the size of a project, \( \gamma \), cannot be identified separately from the number of projects received in each period, \( a \). We pin down \( a \) by normalizing the number of projects in the steady state, \( n^*/\pi + a \), to one, which amounts to choosing the unit of a project. The target on \( c^*/F^* \) enables us to determine steady state investment expenditure, \( x^* \), through the goods market clearing condition. Because \( i^* \) was identified earlier, the relationship \( i^* = \gamma m(x^*, n^*/\pi + a) \) helps us identifying \( \gamma \). With the chosen unit of a project, the value of \( \gamma \) suggests that each project is relatively large, and so the extensive margin of investment (i.e., the number of implemented projects) is important. The target on the annual rate of return to liquid assets comes from Del Negro et al. (2017). They report that the net rate of return to U.S. government liabilities is 1.72% for one-year maturities and 2.57% for 10-year maturities. The
value we choose, 0.02, lies in this range. This target enables us to solve the price of government bonds and, hence, the Lagrangian multiplier of the financing constraint in the steady state, \( \lambda^* \) (see (1.3.9)). Because the marginal cost of the downpayment on investment depends on \( \lambda^* \), knowing \( \lambda^* \) helps us in identifying the marginal replacement cost of capital that is needed for the investment decision to be optimal (see (1.3.12)). In turn, this helps identifying \( \delta \) since the replacement cost of capital depends on \( \delta \).

The target on the fraction of implemented projects is the ratio of the number of projects implemented in the steady state relative to the hypothetical case where the financing constraint is not binding. This target comes from Graham and Harvey (2013). In this quarterly survey of CFOs, the CFO’s are asked whether they postponed investment projects specifically because external financing was limited.\(^{15}\) With the responses in the US to this question, we calculate the average number of implemented projects with financial constraints as a fraction 0.57 of the number that would be implemented in the absence of financial constraints. In the model, this fraction is \( \text{fimp} \) defined by (1.3.15). However, because the survey was conducted in the great recession, this fraction is lower in the survey than in the long run. We set the target on this fraction in the steady state at \( \text{fimp}^* = 0.78 \) so that the fraction indeed drops to 0.57 after a negative shock to equity liquidity of a reasonable size (see section 1.4.2). This target gives a restriction on \((\xi, \sigma_n)\) (see Appendix 1.A.2). For investment delay to be important, unimplemented projects must be able to survive to the next period with high probability. We set \( \sigma_n \) to a high level and use the restriction just described to pin down \( \xi \). In subsection 1.5.3, we will contrast the results with those obtained in the case \( \sigma_n = 0 \) where delay yields no benefit to an entrepreneur.

Finally, for financial shocks to have persistent effects on aggregate activity, the shocks cannot be temporary. Thus, we set the persistence of the shocks as \( \sigma_\phi = \sigma_\mu = 0.9. \)\(^{16}\)

1.4.2 Financial shocks and aggregate activities

In this model, financial shocks are shocks to equity liquidity and entrepreneurs’ borrowing capacity, modeled by \( \varepsilon_\phi \) and \( \varepsilon_\mu \), respectively. Suppose that the economy has been in the deterministic steady state before period \( t = 1 \). At the beginning of period 1, there is an unanticipated realization of either \( \varepsilon_{\phi,1} < 0 \) or \( \varepsilon_{\mu,1} < 0 \). The size of the shock is chosen such that it reduces \( z = \phi \mu \) by 18% in period 1 from the

\(^{15}\)The question has been asked seven times since December 2008, and the exact question varies slightly each time it is asked. For example, in December 2008, the question was: “When external capital is limited, are your corporate investments postponed or canceled?” In September 2009, the question was: “Over the past 18 months, did you company pass up attractive investment project specifically because of the cost or availability of credit?”

\(^{16}\)We have also computed the responses of the model and evaluated government interventions in the asset market with \( \sigma_\phi = \sigma_\mu = 0.7 \). Predictably, the effects of the shocks are less persistent. However, the comparisons between different types of interventions and the importance of delay are robust.
steady state level. After the initial shock, the shock variable returns to the steady state asymptotically according to the process in (1.4.1). We examine how aggregate activities respond to the shock. The computation of equilibrium dynamics is described in Appendix 1.B.

Consider first a negative shock to equity liquidity $\phi$. Figure 1a depicts the dynamics of equity liquidity, investment expenditure $x$, and the marginal replacement cost of capital $1/(\gamma m_1)$. The negative liquidity shock has a large effect on investment. On impact, the shock reduces investment expenditure by about 11.1%, which is comparable to the magnitude of the fall in investment from the trend to the trough in 2009. Given the persistence of the shock, the effect of the liquidity shock on investment is also persistent. Two years after the shock, investment expenditure is still about 2.5% below the steady state. The fall in investment increases the marginal productivity of the resource in investment and, hence, reduces the replacement cost. As investment recovers, so does the replacement cost of capital. This is the case despite that the stock of projects rises after the shock due to delay, which tends to increase the marginal productivity of the resource in investment and depress the marginal replacement cost of capital.

Figure 1b depicts the dynamics of aggregate output, employment and consumption after the negative liquidity shock. On impact of the shock, output falls by about 1.9%, employment by 2.9% and aggregate consumption by 0.8%. All three variables take a significant amount of time to return to the steady state. Thus, liquidity shocks alone can generate significant aggregate fluctuations and induce positive comovement among aggregate quantities. This result extends the finding in Shi (2015) to an environment with investment delay. The result is remarkable, since there is no shock to total factor productivity or the existence of nominal rigidity. It is expected that the negative liquidity shock reduces investment and output. To explain the response of employment, note that workers do not face the liquidity constraint because they do not have the opportunity to invest. The negative liquidity shock shifts wealth from the liquidity constrained individuals (i.e., entrepreneurs) to the unconstrained individuals (i.e., workers). This increase in workers’ consumption induces them to reduce labor supply. Despite the increase in workers’ consumption, aggregate consumption falls because entrepreneurs’ consumption falls by a much larger magnitude than the increase in workers’ consumption.

Figure 1c shows the ratio of the number of implemented projects to that in the hypothetical case where the financial constraint is not binding. Note that it is the level of $fimp$ instead of its percentage change that is depicted in Figure 1c. On impact of the negative liquidity shock, the ratio $fimp$ drops from 0.78 to 0.57. The magnitude of the reduction is what we intended to achieve by setting the target $fimp^* = 0.78$ in the calibration.

Figure 1d shows the responses of the stock of projects available to an entrepreneur and the implicit
price of a project to the negative liquidity shock. The stock of projects does not change in period 1 because it is predetermined. Starting in period 2, the stock of projects increases until reaching the peak in the eighth period, which is about 7.3% higher than the steady state level. Such a long delay in reaching the peak and the magnitude of the peak reflect the significant amount and duration of investment delay. After the eighth period, the stock of projects starts to fall toward the steady state, as the reduction in investment has abated and the stock of projects has been accumulated sufficiently. The implicit price of a project follows a non-monotonic path after the shock. The immediate fall in this price reflects the feature that the stock of projects and investment expenditure are complementary with each other in the investment technology. As investment expenditure falls, the marginal productivity of the stock of projects falls, which reduces the price of this stock. The price of the stock of projects continues to fall until the third period after the shock. This response is caused by delay. As investment projects are delayed, the stock of projects increases, which reduces the marginal productivity of this stock. After the fourth period, the price of the stock of projects starts to rise to return the steady state gradually.

Consider next a negative shock to $\mu$ of the same size as the shock to $\phi$. For brevity, we only present the responses of investment expenditure, output and employment to the $\mu$ shock, as depicted in Figure 2. The responses of these variables are qualitatively similar to the responses to the negative liquidity shock depicted in Figures 1a and 1b, but the magnitude is twelve times smaller. The qualitative similarity between the responses to the two shocks reflects the fact that both shocks reduce entrepreneurs’ ability to finance investment. To understand why a shock to $\mu$ has much smaller effects than a shock to $\phi$, note that the calibration sets the funds raised from debt to be 1.287 times that from new equity in the steady state. Thus, the difference in the magnitude between the effects of the two shocks arises not because debt is insignificant relative to new equity in financing investment. Also, this difference does not depend much on the modeling that $\phi$ appears in the collateral multiplier $z$ (see section 1.6 for an explanation). Rather, the difference arises from the fact that a liquidity shock affects an entrepreneur’s financing ability in more ways than a $\mu$ shock does. A negative $\mu$ shock reduces the financing ability solely by reducing an entrepreneur’s borrowing capacity. A negative liquidity shock has this effect as well, because a lower liquidity of equity reduces the effectiveness of equity used as collateral in borrowing. In addition, a negative liquidity shock directly reduces the amount of existing equity that can be re-sold to raise funds for investment. This additional effect is much larger than the effect through the borrowing capacity.
1.5 Effects of Government Intervention in the Asset Market

In this section, we examine how government interventions in the asset market affect the response of the economy to financial shocks and how investment delay affects the impact of these interventions. The interventions are government purchases of either private equity or debt issued by entrepreneurs.

1.5.1 Government purchases of private assets in the open market

Let \( J \) denote the type of private assets that the government holds and \( p_J \) the price of this asset. If the assets are private equity, then \( J = s \) and \( p_J = q \); if the assets are private debt, then \( J = d \) and \( p_J = p_d \).

Assume that the government did not hold private assets before period \( t = 1 \); i.e., \( J_t^g = 0 \). After the realization of the shock in period 1 to either equity liquidity or the borrowing capacity, the government announces the following path of holdings of private assets:

\[
J_{t+1}^g = \psi_J K^* \left( 1 - \frac{z_t}{z^*} \right) \text{ for all } t \geq 1, \tag{1.5.1}
\]

where \( \psi_J > 0 \) is a constant. Given the size of the shock, the coefficient \( \psi \) determines the magnitude of the intervention. We set \( \psi_s = 0.115 \). To gauge the magnitude of these interventions, recall that the capital stock is 3.32 times as large as annual output in the steady state and the equity liquidity shock reduces \( z \) by 18% in period 1. The parameter value \( \psi_s = 0.115 \) implies that the initial purchase of private assets in period 1 is 6.87% of annual output, which amounts to about one trillion dollars in 2008. Given the persistence of the shock, (1.5.1) requires a sufficiently long time to unload this initial purchase. For comparison between purchases of equity and debt, we set \( \psi_d \) to be such that the two types of purchases have the same value in period 1 when they are evaluated with steady state asset prices. That is, \( q^* s_2^g = p_d^* d_2^g \).\(^{17}\) This amounts to

\[
\psi_d = \psi_s q^*/p_d^*. \tag{1.5.2}
\]

The path of asset purchases above can be financed by infinitely many combinations of government bond issuing and taxes. We consider two specific combinations that are indexed by \( \tau_{1,d} = 0 \) or 1. Taxes

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\(^{17}\) We use steady-state asset prices in this requirement in order to simplify the calculation of \( \psi_d \). If we use actual prices of assets in period 1, this requirement becomes \( q_1 s_2^g = p_{d,1} d_2^g \). This alternative requirement is much more difficult to use, because actual prices of assets in period 1 are equilibrium objects and, hence, are affected by the interventions. Finding the value of \( \psi_d \) that satisfies this alternative requirement involves solving a fixed-point problem.
Chapter 1. Financial Frictions, Investment Delay and Asset Market Interventions

satisfy the government budget constraint, (1.2.5), while the issuance of government bonds satisfies:

\[ p_{b,t} B_{t+1} = \tilde{p}_b B^* + p_{d,t} d_{t+1}, \quad t \geq 1, \]  

(1.5.3)

where \( \tilde{p}_b = p^*_b \) if \( \tau_{1_d} = 0 \) and \( \tilde{p}_b = p_b \) if \( \tau_{1_d} = 1 \). In both cases of \( \tau_{1_d} \), the tax \( \tau \) satisfies the government budget constraint (1.2.5). It is easy to verify that in the case \( \tau_{1_d} = 0 \), the tax in period 1 stays at the steady state level, which explains the meaning of \( \tau_{1_d} = 0 \). In the case \( \tau_{1_d} = 1 \), the tax in every period \( t \geq 1 \) can change. We focus on the case \( \tau_{1_d} = 0 \) because it resembles the Troubled Asset Relief Program in the U.S. in 2008: In period 1, government purchases of private assets are financed entirely by the change in the value of public debt issued and not by the change in the tax, although taxes in future periods may be adjusted.

Note the following features of the interventions specified above. First, the government is assumed to purchase private assets in the open market rather than directly from entrepreneurs. The latter resembles bailouts or lending through discount windows, and its effects are larger than those of the interventions described above. Second, government purchases of assets eventually return to zero as the effect of the shock dissipates, because \( z_t \) approaches the steady state asymptotically. Also, because \( z = \phi \mu \), the size of the intervention is the same under the shocks to \( \phi \) and \( \mu \). Finally, government policies \( (B_{t+1}, s_{t+1}^g, d_{t+1}^g) \) are indeed functions of only \( (q,p_b,p_d,A) \), as we have assumed.

1.5.2 Effects of government interventions

Consider the same negative shock to equity liquidity as examined in subsection 1.4.2. The left panel in Figure 3a depicts the response of investment expenditure with equity purchases, debt purchases and no intervention, and the right panel depicts the response of output in the three cases. The intervention by purchasing equity has large effects on investment and output. One period after the purchase (in period 2), investment recovers 54% and output recovers 32% of the initial loss. With equity purchases, the paths of investment expenditure and output from the second period onward are significantly above the ones with no intervention. The intervention also induces employment in period 2 to recover 36% of the initial loss (not depicted). These large effects of equity purchases are remarkable for two reasons. First, they cannot be explained by merely referring to the large size of the intervention. Even with the same size, debt purchases have only a small effect, as the responses with debt purchases are close to the ones with no intervention. Second, as mentioned before, the interventions are conducted in the open market and, hence, they are not specifically aimed at only taking illiquid assets out of entrepreneurs’ hands.
Instead, among the equity purchased by the government, the fraction that will become illiquid is the same as that held by households.

Equity purchases by the government generate the large effect on aggregate activity by creating liquid funds based on temporarily illiquid assets. To see this, let us contrast the values of one unit of equity held by an entrepreneur and by the government after the depreciation of capital. When a unit of equity is held by an entrepreneur, a fraction $\phi$ can be resold immediately, and the remaining fraction can be used as collateral to borrow $(1 - \phi)zp_d$ units of funds. The total amount of liquid funds raised from one unit of equity is $\phi z$, as defined in (1.3.4). If a unit of equity is held by the government, again, only a fraction $\phi$ can be resold immediately. However, the government can make use of all of the remaining fraction of equity. In particular, because equity can be resold eventually and the government does not face the borrowing constraint as entrepreneurs do, the government can borrow against the currently illiquid fraction $(1 - \phi)$ of equity and reduce current taxes. This reduction in taxes increases an entrepreneur’s liquid funds by the amount $(1 - \phi)zq_\sigma k$ per equity. Since $\phi_z$ is significantly below one as a result of the calibration, this increase in liquid funds is significant and leads to a large recovery of investment from the initial fall after the shock.

From the above explanation, it is also easy to see why purchases of private debt by the government do not affect aggregate activity by as much as equity purchases. Debt purchases increase the demand for private debt, but the benefit to an entrepreneur of borrowing from the government is limited for two reasons. First, any borrowing from the government by an entrepreneur is subject to the same collateral constraint (1.2.4) as borrowing from the private sector. Second, the household must repay the debt to the government next period, which reduces the scope of creating liquid funds based on temporarily illiquid assets.\footnote{It is then reasonable to conjecture that the effect of debt purchases can be increased if the maturity of private debt is increased.}

Figure 3a also shows that the interventions exacerbate the impact of the shock in period 1. In particular, investment expenditure falls by 13.9% in period 1 with equity purchases, in comparison with 11.1% with no intervention. Most of this difference is caused by the assumption that the tax in period 1 is fixed under the interventions. To see this, note that the negative liquidity shock increases the price of government bonds, because such bonds perform a greater role of providing liquidity when the financing constraint is tighter. The increase in the price of government bonds increases government revenue raised by issuing new bonds, which leads to a reduction in the tax in period 1 when there is no intervention. This lower tax mitigates the shortfall in an entrepreneur’s liquid funds and, hence, in investment. When the tax in period 1 is assumed to be fixed under interventions, the mitigating effect is absent. However,
this difference between the cases with and without interventions is temporary. From the second period onward, the tax revenue is allowed to change in all cases.

Figure 3b depicts the effects of government interventions on investment expenditure and output in the case of a negative shock to $\mu$. These effects are smaller than in the case of the liquidity shock. However, relative to the case of no interventions, the effects of equity purchases are more dramatic. With equity purchases by the government, investment and output do not merely recover from the initial fall – they overshoot the steady state in period 2 and return to the steady state from above. This overshooting indicates that the amount of liquid funds created by equity purchase, $(1 - \phi Z)q\sigma K$, is large relative to the negative effect of the $\mu$ shock on the amount borrowed by an entrepreneur. As in the case of the liquidity shock, debt purchases have only small effects on investment and output.

Let us contrast our analysis with Del Negro et al. (2017) who evaluate the extent to which asset market interventions reduced the recession in 2008-2009. They integrate the KM frictions in the equity market into a new Keynesian model that features nominal rigidities and a monetary policy rule prominently. Our paper and Del Negro et al. share the emphasis on equity liquidity. Moreover, both papers use the large household framework of Shi (1997) and both find that asset market interventions can have quantitatively large effects on aggregate activity. However, there are important differences between the two papers. On the modeling side, we incorporate frictions in both the equity and the debt market, while Del Negro et al. assume that entrepreneurs cannot borrow. Moreover, we introduce investment delay but abstract from nominal rigidities and monetary policy rules. On the quantitative importance of financial shocks, our model with no nominal rigidity shows that liquidity shocks can have large effects on aggregate activity and can induce significant positive comovement among aggregate variables. In contrast, Del Negro et al. find that such shocks have only small effects when there is no nominal rigidity. In addition, we find that the shock to the borrowing capacity has only small effects on aggregate variables. This quantitative comparison of debt shocks with liquidity shocks seems new relative to the literature on financial frictions in general. On the policy analysis, we evaluate both equity purchases and debt purchases by the government, while Del Negro et al. examine only equity purchases and only after equity liquidity shocks. Moreover, we assume that government purchases of private assets are initially financed entirely by selling liquid government assets, while Del Negro et al. assume that the tax adjusts immediately to the interventions. As a result, our analysis reveals that government interventions in the asset market can exacerbate the negative effect of a liquidity shock initially, although they help aggregate activity to recover in subsequent periods.
We now investigate the quantitative importance of investment delay for the responses of the equilibrium to financial shocks and to government interventions in the asset market. To this end, suppose counterfactually that investment delay does not yield any benefit to an entrepreneur, in the sense that an unimplemented project cannot survive to the next period. We refer to this economy with $\sigma_n = 0$ as an economy without delay. To recalibrate the model with the new value $\sigma_n = 0$, we discard the target on the fraction of implemented projects in the steady state, because the target was obtained from a survey in a real economy where delay has a positive option value. Since this target was used to pin down $\xi$ in the baseline calibration, we set $\xi$ in the economy without delay to the same value as in the baseline economy. Also, we discard the normalization $n^*/\pi + a = 1$ and, instead, set $a$ to the same value as in the baseline calibration. The reason for doing so is to maintain comparability between the two economies. Because $n^* = 0$ with $\sigma_n = 0$, the normalization would yield a much higher endowment of new projects ($a = 1$) in the economy without delay than in the economy with delay. Appendix 1.A.2 describes the recalibration, which yields the following new parameter values: $\sigma_n = 0$, $\gamma = 15.283$, and $\delta = 0.1225$. All other parameters are the same as in Table 1.

Financial frictions affect the number of implemented projects even when $\sigma_n = 0$, as explained in section 1.3.2. Thus, as in the baseline economy, we can calculate investment expenditure in the economy with $\sigma_n = 0$ in the hypothetical situation where the financing constraint is not binding. This is done by setting $\sigma_n = 0$ in (1.3.14). Then, we can calculate $f_{imp}$ for the economy with $\sigma_n = 0$, which is the ratio of the number of implemented projects to that in the case with $\lambda^e = 0$. The difference in this ratio between the baseline economy and the economy with $\sigma_n = 0$ indicates the effect of investment delay caused by the option value of investment. It is worth noting that $f_{imp} = 0.94$ in the steady state with $\sigma_n = 0$.

We compare equilibrium responses to financial shocks in the baseline economy and the economy with $\sigma_n = 0$. To economize on space, we present the results for only the liquidity shock and, when interventions are introduced, only equity purchases. The responses are much smaller with debt purchases or the shock to $\mu$. The left panel in Figure 4a depicts investment expenditure in the two economies after a negative liquidity shock, where the government purchases equity, and the right panel depicts the ratio $f_{imp}$. Even with the large equity purchase, there is still significant delay in investment, because the ratio $f_{imp}$ in the economy with delay is sixteen to twenty-nine percentage points below that in the economy without delay. This large delay indicates that the financing constraint is binding severely even with equity purchases. Moreover, delay plays a quantitatively important role in the response of the equilibrium to...
Chapter 1. Financial Frictions, Investment Delay and Asset Market Interventions

the liquidity shock. Relative to the economy without delay, investment expenditure in the economy with delay falls by more in period 1 and recovers more quickly in subsequent periods. After period 3, investment expenditure in the economy with delay is significantly above that in the economy without delay.

These effects of delay on investment dynamics are intuitive. Delay increases the initial reduction in investment because entrepreneurs postpone part of the investment until the financing constraint becomes less tight. Delay induces investment to recover more quickly in subsequent periods because investment expenditure is complementary with the stock of projects in investment. By increasing the stock of projects in the next period, delay increases the marginal productivity of investment expenditure in the future, thus inducing future investment to rise more quickly toward the steady state. The right panel in Figure 4a confirms this intuition. The ratio $f_{imp}$ is smaller in the economy with delay than without delay. Moreover, the ratio $f_{imp}$ falls by a larger amount in period 1 and increases more quickly in subsequent periods with delay than without delay.

Investment delay affects equilibrium responses to the liquidity shock both with and without the interventions in the asset market. To see the net effect of delay with interventions, we subtract equilibrium responses without delay from the ones with delay. This difference is computed separately with equity purchases and with no interventions, and then the differences in the two cases are put in the same figure. The left panel in Figure 4b depicts this difference in investment expenditure and the right panel in the ratio $f_{imp}$. In both cases, the difference in investment expenditure between delay and no delay is negative in period 1 and positive from period 3 onward. This means that investment expenditure falls by more in period 1 and recovers more quickly in subsequent periods with delay than without delay, regardless of whether there are equity purchases. In comparison with the case of no interventions, delay generates the following differences in the quantitative responses: (i) Investment expenditure falls by more in period 1 and recovers by a larger amount in period 2; (ii) Investment expenditure recovers less quickly from period 3 onward; (iii) There is less delay from period 2 onward, as depicted by the right panel in Figure 4b. Taken together, these differences suggest that the option to delay reduces the effectiveness of equity purchases in counteracting the negative liquidity shock.

We explain the above effects of delay as follows. First, delay reduces investment expenditure in period 1 by more with equity purchase than with no interventions because the tax in period 1 is assumed to be fixed with equity purchases. As explained before, this difference in the tax in period 1 implies that the negative liquidity constraint tightens the financing constraint to a greater extent in the first period when the government purchases equity purchases. As a result, entrepreneurs curtail investment by a
larger amount given the option to delay. Most of this difference in the tax between equity purchases and no interventions disappears from period 2 onward. This explains why investment expenditure recovers by a larger amount in period 2 in the case of equity purchases. Second, from period 2 onward, the main difference between the case of equity purchases and the case of no interventions lies in how quickly the financing constraint is relaxed over time as equity liquidity increases back to the steady state. In the case of equity purchases, the government gradually unloads private equity purchased in period 1, by selling equity for government bonds. Because this operation extracts part of liquid assets from the market, the financing constraint is relaxed relatively slowly over time. The benefit to delay is smaller in this case, since the option value of delay is to undertake investment at a future time when the financing constraint is less tight. This explains why there is less delay from period 2 onward in the case of equity purchases than in the case of no interventions. Third, less delay means that the stock of projects next period will be lower in the case of equity purchases. Because the stock of projects is complementary with investment expenditure, a lower stock of projects next period will reduce the marginal productivity of investment expenditure next period, thus inducing future investment expenditure to rise less quickly.

So far we have assumed $\tau_1d = 0$ so that the tax in period 1 is fixed at the steady state level when the government purchases private assets. To see whether the main quantitative results above depend on this particular assumption, let us change the assumption to $\tau_1d = 1$. Under this alternative assumption, the tax in period 1 can change with the shock under government interventions as under no intervention. Figure 5 depicts the responses of investment expenditure and the ratio $f_{imp}$ to the negative liquidity shock in the economy with delay and in the economy without delay. The two panels in Figure 5 are similar to their counterparts in Figure 4a. The only discernible difference is that the initial fall in investment expenditure with equity purchases is smaller in Figure 5 than in Figure 4a. Thus, the assumption $\tau_1d = 0$ is not critical for the main results on the effect of government interventions and the role of investment delay.

1.6 Robustness of the Results

Equity liquidity shocks have much larger effects on aggregate variables than debt shocks, and equity purchases by the government are more effective than debt purchases. In this subsection, we show that the comparisons between equity and debt are robust to changes in the modeling of the collateral constraint and in the calibration of financial frictions.

One robustness check is on the collateral multiplier, $z(\phi, \mu)$. Although it is intuitive that equity
liquidity $\phi$ affects the effectiveness of equity as collateral, one might think that its appearance in the collateral multiplier is the reason why equity shocks have much larger effects than debt shocks. Quantitatively, the additional effect of $\phi$ through the collateral multiplier is small. Most of the effects of equity liquidity shocks come from changes in the amount of funds raised through equity. This should be clear from the comparison between Figures 1b and 2, which show that the effects of equity shocks are twelve times as large as those of debt shocks. If we eliminate $\phi$ from the collateral multiplier, the effects of equity shocks will remain to be ten to eleven times as large as those of debt shocks. Similarly, equity purchases remain to have much larger effects than debt purchases even if the collateral multiplier does not depend on $\phi$.

Another robustness check is on the calibration of financial frictions and, specifically, the identification of $\mu^*$. In the calibration (see Table 1), we determined $\mu^*$ by setting the ratio of debt issuance to stock sales in the steady state to the one observed in the bottom 50% of firms in COMPUSTAT. If we restrict attention to COMPUSTAT and use smaller firms to calculate the ratio of debt issuance to stock sales, this ratio will decrease (see Covas and den Hann, 2011), and so the difference between the effects of equity shocks and debt shocks will be even larger than the one reported above. However, COMPUSTAT is biased toward relatively large firms, and even small firms in the dataset may be larger than many firms in the economy. To illustrate the robustness of our results, we increase the ratio of debt issuance to stock sales from 1.287 to 4.775 (which is the average in COMPUSTAT including all firms). Using this new target, we recalibrate the model to obtain the parameter values: $\mu^* = 0.1321$, $\theta = 0.0249$, and $\phi^* = 0.2829$.

With these new parameter values, the effects of the two shocks of the magnitude in subsection 1.4.2 are depicted in Figure 6a and Figure 6b, respectively. Comparing Figure 6a with 1b, and Figure 6b with Figure 2, we can see that the quantitative effects of the shocks with the new parameter values are similar to those with the baseline calibration. The effects of equity shocks are still much larger than those of debt shocks. Then, it should not be surprising that equity purchases by the government are still much more effective than debt purchases. To economize on space, we do not depict these effects of government interventions with the recalibration.

1.7 Conclusion

We construct a dynamic macro model to incorporate financial frictions and investment delay. Investment is undertaken by entrepreneurs who face liquidity frictions in the equity market and a collateral constraint
in the debt market. After calibrating the model to the US data, we find that shocks to equity liquidity can affect aggregate activity significantly and, in particular, can induce positive comovement among investment, employment, output and aggregate consumption. In contrast, a shock to entrepreneurs’ borrowing capacity has only small effects on aggregate activity. After financial shocks, if the government intervenes in the asset market by using its liquid assets to buy private assets, the effectiveness of the intervention depends on the type of private assets that the government buys. Equity purchases can speed up the recovery of the economy significantly, although they exacerbate the negative effect of the shock in the first period of the intervention. In contrast, debt purchases have only very small effects. Moreover, the option to delay investment reduces the effectiveness of government intervention through equity purchases.

The findings in this paper provide guidance to future research on the macro importance of financial frictions. First, it is important to build a microfoundation for the frictions in equity liquidity and the cause of their fluctuations. In this paper, we followed KM to parameterize equity liquidity by \( \phi \) and assumed that \( \phi \) follows an AR(1) process. Given the finding that equity liquidity can cause large fluctuations in aggregate activity, a microfoundation for such liquidity frictions will enhance the understanding of where these frictions come from and why they vary over time. It may also help resolving the puzzling response of equity price to changes in asset liquidity in this class of models (see Shi, 2012). Second, it is useful to conduct welfare analysis of asset market interventions. In this paper, we focused on the effects of asset market interventions on aggregate activity. The interventions also redistribute wealth from unconstrained individuals in the model (workers) to constrained individuals (entrepreneurs). The structure of a representative household is a convenient device for a welfare evaluation of the interventions. However, we refrained from evaluating the welfare consequence of the interventions because such an evaluation would be more credible if the primary frictions in the model were microfounded rather than being exogenously imposed. With a strong microfoundation of these frictions, the welfare analysis of the interventions will provide theoretical guidance to the policy debate on how often and how large the interventions should be carried out.
Appendix

1.A The Non-stochastic Steady State and Calibration

1.A.1 Determining the non-stochastic steady state

In the deterministic state, (1.3.20) implies \( i^* = (1 - \sigma_k)K^*/\pi \) and \( n^* = [\pi \sigma_n/(1 - \sigma_n)](a - m^*) \), where \( i^* = \gamma m(x^*, n^*/\pi + a) \). These results solve for unique \((n^*, x^*, i^*, m^*)\) for any given \( K^* > 0 \). Denote these solutions as \( n^* = n(K^*) \), \( x^* = x(K^*) \), \( i^* = i(K^*) \) and \( m^* = i(K^*)/\gamma \). Next, we express \((r^*, \lambda^*, q^*, p^*_b)\) as functions of \((K^*, \ell^*)\). The optimization by the producers of consumption goods yields \( r^* = r(K^*, \ell^*) \), where \( r(K, \ell) = F_1(K, (1 - \pi)\ell) \). In the steady state, (1.3.13) yields:

\[
p^*_n = \frac{\gamma m_2^*(1 + \theta_z^* \lambda^*)q^*}{1 - \beta \sigma_n(1 - m_2^*)}, \tag{1.A.1}
\]

where \( \theta_z^* = \theta + (1 - \theta)z^*p^*_d \) and \( z^* = \phi^* \mu^* \). Substituting the solution into (1.3.12) yields:

\[
q^* = \frac{1 + \lambda^*}{1 + \theta_z^* \lambda^*} \left( 1 + \frac{\beta \sigma_n m_2^*}{1 - \beta \sigma_n} \right) \frac{1}{\gamma m_1^*}, \tag{1.A.2}
\]

The asset pricing equations, (1.3.8)-(1.3.10), imply:

\[
q^* = \frac{\beta(1 + \pi \lambda^*)}{1 - \beta \sigma_k(1 + \pi \phi_z^* \lambda^*)} r^*, \tag{1.A.3}
\]

\[
p^*_b = p^*_d = \beta(1 + \pi \lambda^*), \tag{1.A.4}
\]

where \( \phi_z^* = \phi + (1 - \phi)z^*p^*_d \). With (1.A.4), we express \( \phi_z^* = \phi_z(\lambda^*) \) and \( \theta_z^* = \theta_z(\lambda^*) \). Substituting these functions and \((n^*, x^*, i^*, m^*)\) as functions of \( K^* \), (1.A.2) and (1.A.3) become equations involving only the variables \((q^*, \lambda^*, K^*, \ell^*)\). Thus, we can use these equations to solve \((q^*, \lambda^*)\) as \( q^* = q(K^*, \ell^*) \).
and \( \lambda^* = \lambda^r(K^*, \ell^*) \). \(^{19}\) With these solutions, we express \((p_n^*, p_d^*, \phi_z^*, \theta_z^*)\) as functions of \((K^*, \ell^*)\).

To solve \((K^*, \ell^*)\), we use the steady-state version of \((1.2.5)\) to solve \(\tau^*\). Substituting this solution, we write the steady-state version of \((1.2.5)\) and \((1.3.3)\), and \((1.3.7)\) as

\[
\begin{align*}
\tau^* &= r + \phi_z^* \sigma_k q^* K^* - \theta_z^* q^* i^* - g \\
+ p_b^* B^* - p_d^* d^* - [1 - \sigma_k (1 - \phi_z^*)] q^* s^* \\
u'(c^*) &= (1 + \lambda^*) U'(c^*)
\end{align*}
\]

where \( r^* = F_1(K^*, (1 - \pi) \ell^*) \). To obtain the first equation above, we have used the fact that \( s^* = K^* - s^g \), \( b^* = B^* \) and \( d^* = d^g \). Substituting \( \tau^* \) and the solutions for \((i^*, q^*, \lambda^*, \tau^*, p_b^*, p_d^*, \phi_z^*, \theta_z^*)\), we use the two equations above to solve \((c^*, c^w)\) as functions of \((K^*, \ell^*)\). Then, \( c^* = \pi c^e + (1 - \pi) c^w \) can be expressed as a function of \((K^*, \ell^*)\). Finally, in the steady state, \((1.3.6)\) and \((1.3.17)\) become:

\[
F_2(K^*, (1 - \pi) \ell^*) = \frac{h'(\ell^*)}{U'(c^w)}
\]

\[
F(K^*, (1 - \pi) \ell^*) = c^* + g + \pi x^*.
\]

These equations determine \((K^*, \ell^*)\).

### 1.A.2 Calibration procedure

We use the targets listed in Table 1 and the steady state characterized above to identify the parameters. The values \( \beta, \rho, \sigma_\alpha, \sigma_\mu, \alpha \) and \( \pi \) are determined directly either as their exogenously chosen values or their targets as explained in the main text. The other parameters are identified as follows:

**Part 1.** The parameters \((\eta, \sigma_\delta, g^*, B^*)\). Because the elasticity of labor supply is \(1/(\eta - 1)\), the target on this elasticity solves \(\eta\). In the steady state, the replacement of capital in a period is \(\pi i^* = (1 - \sigma_k) K^*\). The ratio of annual replacement to capital is \(4\pi i^*/K^* = 4(1 - \sigma_k)\). Equating this to the target, 0.076, yields \(\sigma_k\). The target on total hours of work in the steady state requires \((1 - \pi) \ell^* = 0.25\), and the target on the ratio of capital to annual output requires \(K^*/4F^* = 3.32\). These two requirements solve \((\ell^*, K^*)\). Then, we can solve \(\gamma m^* = i^* = (1 - \sigma_k) K^*/\pi, F^* = F(K^*, (1 - \pi) \ell^*), r^* = F_1, \) and \(w^* = F_2\). Subsequently, the target on the ratio of government spending to GDP solves \(g^* = 0.18 F^*\), the target on the ratio of private consumption to GDP solves \(c^* = 0.7 F^*\), and the goods market clearing condition solves \(x^* = (F^* - c^* - g^*)/\pi\). The target on the annualized net rate of return to liquid assets requires \((p_n^*)^{-4} - 1 = 0.02\), which solves for \(p_n^*\) and, hence, \(p_d^*\). Substitution into \((1.4)\) solves for \(\lambda^e\). The

\(^{19}\)For arbitrarily given \((K^*, \ell^*)\), these equations can have one or two pairs of solutions for \((q^*, \lambda^*)\). However, if there are two pairs, only the one with the larger value of \(q^*\) leads to admissible solutions for \((K^*, \ell^*)\) below.
target on the annual equity premium requires \((r^*/q^* + \sigma_k)^4 - (p_k^*)^{-4} = 0.02\). Because \(r^*\) and \(\sigma_k\) are already solved, this requirement solves for \(q^*\). Using the target on the share of liquid assets gives \(p_d^*B^*/(p_d^*B^* + q^*K^*) = 0.12\), which solves for \(B^*\) and, hence, \(\tau^* = g^* - (p_k^* - 1)B^*\). Notice that the solutions of \((\ell^*, K^*, c^*, p_d^*, q^*)\) used five targets but have not led to the solution for any parameter yet.

We will utilize the solutions of these five variables to identify five parameters in the parts below.

**Part 2.** The parameters \((\phi^*, \theta^*, \mu^*, u_0, h_0)\). We use three targets to solve for \((\phi^*, \theta^*, z^*)\) and, hence, \(\mu^*\). The first target is the annual equity premium, which is equal to \((r^*/q^* + \sigma_k)^4 - (p_k^*)^{-4}\) in the steady state of the model. We already used this target to solve for \(q^*\). Using this solution for \(q^*\) in (1.A.3) solves

\[
\phi_z^* = \frac{1}{\pi x^*} \left( \frac{1}{3\sigma_k} - 1 \right) - \frac{r^*}{\sigma_k q^*} \left( \frac{1}{\pi x^*} + 1 \right). \tag{1.A.5}
\]

The second target is the ratio of funds raised in the market to fixed investment expenditure, which is 0.284 as estimated by Nezafat and Slavik (2010). In our model, the value of new equity issuance is \(q\theta i\) and the amount borrowed in the market is \(p_d z q s_{x^*}^{\ast}\). Thus, the target on the ratio of funds raised in the market to fixed investment expenditure in the steady state requires

\[
\frac{q^*}{x^*} [\theta i^* + p_d^* z^* s_{x^*}] \equiv 0.284. \tag{1.A.6}
\]

The third target for identifying \((\phi^*, \theta^*, z^*)\) is the ratio of debt issuance to stock sales, which requires \(p_d^* z^* s_{x^*}/\theta i^* = 1.287\). Using this requirement to express \(p_d^* z^* s_{x^*} = 1.287\theta i^*\) and substituting into (1.A.6), we solve \(\theta = 0.284x^*/2.287q^*i^*\). Using \(s_{x^*} = (1 - \theta)i^* + (1 - \phi^*)\sigma_k K^*\), we have

\[
z^* = \frac{1.287\theta i^*}{p_d^* [(1 - \theta)i^* + (1 - \phi^*)\sigma_k K^*]} \tag{1.A.7}
\]

This is a relationship between \(z^*\) and \(\phi^*\). Substituting the definition of \(\phi_z^*\) into (1.A.5) yields another relationship between \(z^*\) and \(\phi^*\). The two relationships together solve for \((\phi^*, z^*)\). Then, the definition \(z^* = \phi^*\mu^*\) leads to \(\mu^* = z^*/\phi^*\), and \((\phi_z^*, \theta_z^*)\) can be calculated from their definitions.

Substituting the solved parameters and steady state variables into the steady state version of (1.3.3), we solve for \(c^*\). The definition of \(c^*\) yields \(c_{u^*} = (c^* - \pi c^*)/(1 - \pi)\). Substitution into the steady state versions of (1.3.7) and (1.3.6) solves for \(u_0\) and \(h_0\).

Notice that the second target and the third target above are new ones in this part. The equity premium is not a new target because it was already used in Part 1. With the two new targets, we identified five new parameters. This was achieved by using the variables \((\ell^*, K^*, c^*, p_d^*, q^*)\) that were calculated in Part 1 with additional targets. We still have two restrictions from this list to be utilized.
Part 3. The parameters \((\sigma_n, \xi, \delta, \gamma, a)\). We utilize the target on \(f_{imp}^*\), the exogenously set value \(\sigma_n\), and the normalization of \(n^*/\pi + a\). The target \(f_{imp}^* = 0.78\) is the ratio of investment in the steady state to that in the hypothetical economy where the financing constraint does not bind. Rewrite (1.3.16) as
\[
\frac{m_1}{m_1^f} = 1 + \lambda^e(1 - \theta_z q\gamma m_1^f). \tag{1.A.8}
\]
Using the functional form of \(m\) to derive \(\gamma m_1\) and substituting \(\gamma = i/m\), we get \(m^c = 0.5i \delta x^{\xi - 1}/\gamma m_1\).

Inverting the \(m\) function, we get:
\[
(\delta x)^\xi = 2m^c - (\frac{n}{\pi} + a)^\xi, \quad (\delta x^f)^\xi = 2(f_{imp}^\xi)m^c - (\frac{n}{\pi} + a)^\xi.
\]
Subtracting these two equations, substituting \(m^c\) just derived, and re-arranging, we get: 
\[
(x^f)^\xi - x^\xi = (f_{imp}^{-\xi} - 1)ix^{\xi - 1}/\gamma m_1. \tag{1.A.9}
\]
Dividing the formula of \(m_1^f\) by \(m_1^f\), and substituting \(x^f/x\) from above, we get:
\[
\frac{m_1}{m_1^f} = \left[1 - f_{imp}^\xi \left(\frac{i}{x^f} + f_{imp}^\xi\right)\right]^{(1-\xi)/\xi}. \tag{1.A.10}
\]

Equating the two expressions for \(m_1/m_1^f\) in (1.A.8) and (1.A.9) yields:
\[
\gamma m_1 = \frac{1}{\lambda^e q\theta_z} \left\{1 + \lambda^e - \left[1 - f_{imp}^\xi \left(\frac{i}{x^f} + f_{imp}^\xi\right)\right]^{(1-\xi)/\xi}\right\}. \tag{1.A.10}
\]

The target on the fraction of implemented projects is \(f_{imp}^* = 0.75\). With this target, the steady state version of (1.A.10) involves only two unknowns, \((\gamma m_1^*, \xi)\). In addition, the steady state equation, (1.A.2), can be rewritten as
\[
\sigma_n = \frac{(sxz)\gamma m_1^* - 1}{\beta[(sxz)\gamma m_1^* - 1 + m_2^*]}, \quad \text{where} \quad sxz \equiv q\frac{1 + \theta_x^\ast \lambda^e}{1 + \lambda^e}. \tag{1.A.11}
\]
This equation involves only three unknowns, \((\sigma_n, \gamma m_1^*, m_2^*)\). Moreover, \(m_2^*\) can be expressed as a function of \((\gamma m_1^*, \xi)\), as shown below.

For any exogenously set value of \(\sigma_n > 0\), we can solve \((\gamma m_1^*, \xi)\) from (1.A.11) and the steady-state version of (1.A.10). It is convenient to reverse this process. We choose a value of \(\xi\), set the target \(f_{imp}^* = 0.78\), and use the steady-state version of (1.A.10) to solve \(\gamma m_1^*\). Using this solution and the
Chapter 1. Financial Frictions, Investment Delay and Asset Market Interventions

37

\[ \gamma m^*_2 = \frac{i^*-x^*\gamma m^*_1}{\pi} + a, \quad m^* = \left[ \frac{i^*}{2\gamma m^*_2} \left( \frac{n^*}{\pi} + a \right) \right]^{\frac{1}{\xi}}. \]

The first equation comes from the feature that \( m \) has constant returns to scale, and the second equation from differentiating \( m \) with respect to \( \frac{n}{\pi} + a \) and substituting \( \gamma = \frac{i^*}{m^*} \). Once \( (\gamma m^*_1, \gamma m^*_2, m) \) are solved, we can further solve:

\[ \gamma = \frac{i^*}{m^*}, \quad m^1 = \frac{\gamma m^*_1}{\gamma}, \quad m^2 = \frac{\gamma m^*_2}{\gamma}, \quad \delta = \frac{1}{x^*} \left[ 2(m^*)^\xi - \left( \frac{n^*}{\pi} + a \right) \right]^{\frac{1}{\xi}}. \]

The last equation comes from inverting the \( m \) function. Substituting the solutions for \( (\gamma m^*_1, m^*_2) \) into (1.A.11) yields a value of \( \sigma_n \). If this value of \( \sigma_n \) differs from the intended one, adjust the value of \( \xi \) and repeat the process until the intended value of \( \sigma_n \) is achieved.

With the solution of \( m^* \) and the normalized value of \( \frac{n^*}{\pi} + a \), we obtain:

\[ n^* = \sigma_n \pi \left( \frac{n^*}{\pi} + a - m^* \right), \quad a = \left( \frac{n^*}{\pi} + a \right) - \frac{n^*}{\pi}. \]

Notice that this part used three new targets, \((fimp^*, \sigma_n, \frac{n^*}{\pi} + a)\), but identified five parameters, \((\sigma_n, \xi, \delta, \gamma, a)\). The two additional restrictions used in this part are the two that remained in the list \((\ell^*, K^*, c^*, p^*_b, q^*)\) after Part 2.

In the special case \( \sigma_n = 0 \), we do not use the target \( fimp^* = 0.78 \), its associated equation, (1.A.10), or the normalization \( \frac{n^*}{\pi} + a = 1 \). Instead, we set \( \xi \) and \( a \) at their values identified in the case \( \sigma_n > 0 \) (see section 1.5.3 for the explanation). Then, (1.A.11) solves \( \gamma m^*_1 = 1/(sxc) \). The other variables and parameters, especially \((\gamma m^*_2, m^*, \gamma, m^*_1, m^*_2, \delta)\), are solved in the same way as above.

1.B Computing the Dynamic Equilibrium and Responses

We have assumed that government policies, \((B_{+1}, s^q_{+1}, d^q_{+1})\), are only functions of \((q, p_b, p_d, A)\), where \( A = (K, N, \phi, \mu) \) is the aggregate state. After substituting these policies, the value function \( v \), the policy functions and price functions can be expressed as functions of \( A \) in the equilibrium. We compute these functions and simulate the responses of the equilibrium to the shocks. Because the shocks are sizable, we do not linearize the equilibrium around the steady state.

The computation iterates on four functions, \((q,p_b,p_n,x)(A)\), where \(p_n\) and \(x\) are added to the list for convenience. To do so, we substitute all other variables. Note that \(p_d = p_b\), and so \((\phi_z, \theta_z)\) defined in \((1.3.4)\) are functions of only \((p_b, \phi)\). We use \((1.3.12)\) to solve:

\[
\lambda^e = \frac{q - p_n/\gamma - (1 - m_2)/(\gamma m_1)}{(1 - m_2)/(\gamma m_1) - \theta_z q}.
\] (1.B.1)

This is a function of \((N, \phi, \mu, q, p_n, x)\). With this, we use \((1.3.7), (1.3.6)\) and \(w = F_2(K, (1 - \pi)\ell)\) to solve \((c^e, \ell, w)\) as functions of \((c^w, A, q, p_n, x)\) by using the following equilibrium relationships:

\[
\begin{align*}
\lambda^e &= \frac{q - p_n/\gamma - (1 - m_2)/(\gamma m_1)}{(1 - m_2)/(\gamma m_1) - \theta_z q},
\end{align*}
\]

\[
(1 - m_2)/(\gamma m_1) - \theta_z q.
\]

Substituting \((Y, c, x)\) into the clearing condition of the goods market, \((1.3.17)\), we solve \(c^w\) as a function of \((A, q, p_n, x)\) and recover \((\lambda^e, c^e, \ell, w, r, Y, c)\) as functions of \((A, q, p_n, x)\).

Next, we construct equilibrium mappings on the functions, \((q,p_b,p_n,x)(A)\). Substituting \(\tau\) from \((1.2.5)\) into \((1.3.3)\) and using \(s = K - s^g, b = B\) and \(d = d^g\), we obtain:

\[
x = (r + \phi_z \sigma_k q)K + [\sigma_k(1 - \phi_z)s^g - s_{t+1}^g]q
\]

\[
- c^e + \theta_z qi - g + p_b B_{t+1} - p_d d^g_{t+1}.
\] (1.B.2)

Given government interventions specified in section 1.5.1, especially \((1.5.3)\), we can write this expression for \(x\) further as

\[
x = (r + \phi_z \sigma_k q)K - c^e + \theta_z qi - g + \tilde{p}_b B^* + \sigma_k(1 - \phi_z)q s^g.
\] (1.B.3)

This depends on \(s^g\). Rather than adding \(s^g\) as a state variable, we express \(s^g\) as a function of the current state \(A\). This is trivial if government interventions are purchases on private debt, in which case \(s^g = 0\).

When government purchases are on private equity, \(s^g_t = \psi_s K^*(1 - z_{t-1}/z^*)\) for all \(t \geq 2\) and \(s^g_1 = 0\).

Because there are no new shocks in any period \(t \geq 2\) (i.e., \(\varepsilon_{\phi,t} = \varepsilon_{\mu,t} = 0\) for all \(t \geq 2\)), we can use \((1.4.1)\) to get the following relationships for all \(t \geq 2\):

\[
\begin{align*}
\log \phi_{t-1} &= \log \phi_t - (\frac{1}{\sigma_\phi} - 1) \log \phi^* \\
\log \mu_{t-1} &= \log \mu_t - (\frac{1}{\sigma_\mu} - 1) \log \mu^*.
\end{align*}
\]
Using these relationships, we can express \( \phi_{t-1} \) as a function of \( \phi_t \) and \( \mu_{t-1} \) as a function of \( \mu_t \) for all \( t \geq 2 \). Then, we can express \( z_{t-1} \) as a function of \( (\phi_t, \mu_t) \) for all \( t \geq 2 \). Denote this function as \( z_{t-1} = \zeta(\phi_t, \mu_t) \) for some function \( \zeta \). For \( t = 1 \), we have \( s^q_1 = 0 \). Note that \( K_1 = K^* \). Let \( \chi(K = K^*) = 1 \) if \( K = K^* \), and \( \chi(K = K^*) = 0 \) otherwise. We put the case \( t = 1 \) and the case \( t \geq 2 \) together as

\[
s^g_t = \psi s K^* \left[ 1 - \frac{\zeta(\phi_t, \mu_t)}{z^*} \right] \chi(K_t = K^*) \text{ for all } t \geq 1. \tag{1.B.4}
\]

This expresses \( s^g \) as a function of \( (K, \phi, \mu) \).

Now, the right-hand sides of (1.3.8), (1.3.9), (1.3.13) and (1.B.3) form the mapping on the functions \( (q, p_b, p_n, x)(A) \). Denote this mapping as \( T : C_1 \to C_1 \), where \( C_1 \) is the set of all continuous functions that map \( A \) into \( \mathbb{R}_+ \). We iterate on the mapping \( T \) on a discretized state space of \( A \) until convergence. The result is a fixed point of \( T \) for the functions \( (q, p_b, p_n, x) \) on the discretized state space. We then use Chebyshev projection to approximate the functions \( (q, p_b, p_n, x) \) on the entire state space. Other equilibrium functions can be recovered accordingly.

**Part 2. Simulate dynamic responses to shocks**

Once a shock is realized at the beginning of period 1, \( \phi_1 \) and \( \mu_1 \) are known. The dynamics of \( A \) are described by (1.3.20) and (1.4.1). We simulate the equilibrium forward as follows. For any \( t \), starting with \( t = 1 \), compute \( A_t = (K_t, N_t, \phi_t, \mu_t) \) and evaluate the functions \( (q, p_b, p_n, x) \) to obtain \( (q_t, p_{b,t}, p_{n,t}, x_t) \). Obtain other equilibrium functions accordingly. Then, use (1.3.20) and (1.4.1) to compute \( A_{t+1} \). Repeat this process for as many periods as desirable.
### 1.C Tables

Table 1. Parameters and calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$: discount factor</td>
<td>0.9879</td>
<td>$\beta^{-4} = 1.05$</td>
</tr>
<tr>
<td>$\rho$: relative risk aversion</td>
<td>2</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\eta$: curvature in labor disutility</td>
<td>2</td>
<td>labor supply elasticity $\frac{1}{\eta-1} = 1$</td>
</tr>
<tr>
<td>$u_0$: constant in entrep. utility</td>
<td>75.757</td>
<td>capital stock/annual output = 3.32</td>
</tr>
<tr>
<td>$h_0$: constant in labor disutility</td>
<td>18.840</td>
<td>hours of work = 0.25</td>
</tr>
<tr>
<td>$\alpha$: capital share</td>
<td>0.36</td>
<td>capital income share = 0.36</td>
</tr>
<tr>
<td>$\sigma_k$: survival rate of capital</td>
<td>0.981</td>
<td>annual replacement of capital = 7.6%</td>
</tr>
<tr>
<td>$g$: government spending</td>
<td>0.1928</td>
<td>government spending/GDP = 0.18</td>
</tr>
<tr>
<td>$\pi$: fraction of entrepreneurs</td>
<td>0.06</td>
<td>annual fraction of investing firms = 0.24</td>
</tr>
<tr>
<td>$B^*$: steady state liquid assets</td>
<td>1.831</td>
<td>fraction of liquid assets = 0.12</td>
</tr>
</tbody>
</table>

Table 1. Parameters and calibration (continued)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$: fraction of new equity sold</td>
<td>0.0629</td>
<td>funds raised in markets/ investment expenditure = 0.284</td>
</tr>
<tr>
<td>$\mu^*$: steady state borrowing capacity</td>
<td>0.0891</td>
<td>issuance of debt/sale of stock = 1.287</td>
</tr>
<tr>
<td>$\phi^*$: steady state equity liquidity</td>
<td>0.2913</td>
<td>annual equity premium in the deterministic steady state = 0.02</td>
</tr>
<tr>
<td>$a$: endowment of new projects</td>
<td>0.3361</td>
<td>normalization $\frac{a^*}{\pi} + a = 1$</td>
</tr>
<tr>
<td>$\gamma$: converting $m$ into $i$</td>
<td>15.480</td>
<td>$c^<em>/F^</em> = 0.70$</td>
</tr>
<tr>
<td>$\delta$: efficiency unit of $x$ in $m$</td>
<td>0.0795</td>
<td>annual return to liquid assets=0.02</td>
</tr>
<tr>
<td>$\sigma_n$: survival rate of projects</td>
<td>0.9363</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\xi$: $\frac{1}{1-\xi} = \text{elasticity of sub. in } m$</td>
<td>-1.0</td>
<td>$fimp^* = 0.78$</td>
</tr>
<tr>
<td>$\sigma_{\phi}$: survival rate of $\phi$</td>
<td>0.9</td>
<td>exogenously chosen</td>
</tr>
<tr>
<td>$\sigma_{\mu}$: survival rate of $\mu$</td>
<td>0.9</td>
<td>exogenously chosen</td>
</tr>
</tbody>
</table>
1. D Figures

Figure 1a. Liquidity $\phi$, investment expenditure $x$ and the replacement cost of capital $rep_k$ after a negative shock to $\phi$

Figure 1b. Aggregate consumption $c$, employment $eL$ and output $Y$ after a negative shock to $\phi$
Figure 1c. The number of implemented projects relative to the unconstrained case, after a negative shock to $\phi$.

Figure 1d. The stock of investment projects, $N/\pi + a$, and the implicit price of this stock $p_n$ after a negative shock to $\phi$. 
Figure 2. Investment expenditure $x$, employment $eL$ and output $Y$ after a negative shock to $\mu$.

Figure 3a. Investment $x$ and output $Y$ under no intervention, equity purchases ($omeq$) and debt purchases ($omdt$): liquidity shock.
Chapter 1. Financial Frictions, Investment Delay and Asset Market Interventions

Figure 3b. Investment $x$ and output $Y$ under no intervention, equity purchases ($omeq$) and debt purchases ($omdt$): $\mu$ shock

Figure 4a. Delay versus no delay: investment $x$ and the ratio $fimp$ with equity purchases after a liquidity shock
Figure 4b. Net effects of delay with equity purchases and no interventions: investment expenditure $x$ and the ratio $fimp$ after a liquidity shock.

Figure 5. Delay versus no delay with $\tau_{1d} = 1$: investment $x$ and the ratio $fimp$ with equity purchases after a liquidity shock.
Chapter 1. Financial Frictions, Investment Delay and Asset Market Interventions 46

Figure 6a. Aggregate consumption $c$, employment $eL$ and output $Y$ after a negative shock to $\phi$ (with a larger $\mu^*$)

Figure 6b. Investment expenditure $x$, employment $eL$ and output $Y$ after a negative shock to $\mu$ (with a larger $\mu^*$)
Chapter 2

Housing Liquidity and Unemployment: The Role of Firm Financial Frictions

2.1 Introduction

I construct a dynamic general equilibrium model of housing and mortgages, firm lending, and labor to study the feedback between housing market liquidity, default, and unemployment. This paper is motivated by three salient features of the Great Recession: (i) the collapse of housing markets; (ii) the large fall in bank lending; and (iii) the rise in unemployment.

The Great Recession of 2007-2009 is largely believed to have been instigated by a rise in mortgage defaults, which resulted in the collapse of the prices of mortgage-backed securities. These securities made up a significant portion of the financial assets of some commercial banks. Finding their balance sheets deteriorated, these banks reduced their lending. The bank loan growth rate declined for 10 consecutive quarters following the start of the Great Recession, with total borrowing falling by 5 percent of GDP, more than in any other post-World War 2 recession. Firms with a high reliance on external funding found themselves borrowing constrained, which forced them to cut back on expenses, including hiring. Unemployment rose from 4.7% to its peak of 10%, and recovered at its slowest pace in the post-World War 2 period.

The broad objective of this paper is to analyze the interaction between the housing and labor markets
following an initial fall in housing liquidity, modeled as rising foreclosure costs to banks. Specifically three questions are addressed:

1. How does unemployment respond to an initial fall in housing liquidity?
2. What role do firm financial frictions play in propagating the initial shock to the labor market?
3. Do labor market outcomes feed back into the level of housing liquidity?

To answer these questions, I develop a model characterized by frictional housing and labor markets, and by firm financing constraints. When housing market distress affects the borrowing conditions of firms, they cut back on hiring. The increase in unemployment makes mortgage default more likely. As a result, the bank reduces the size of a mortgage at origination. This increases the difficulty of buying (and thus selling) a house, which further contributes to a rise in default for distressed and unemployed owners. In this way, the impact of the rise in foreclosure costs is magnified. Importantly, unless there is a direct impact of housing distress on firm decision making, the rise in unemployment observed in the Great Recession cannot be captured. In this paper, I focus on the impact of housing market distress on the ease with which firms can obtain external funding. Later on I contrast the findings with the more general case of an adverse shock to the marginal revenue of a firm.

This paper also has something to say about the nature of mortgage defaulters. Prior empirical research has found conflicting evidence on the importance of unemployment as a predictor of mortgage default (e.g. Gerardi et al. 2015). This brings into question whether default arises out of strategy or necessity. The model predicts that a large proportion of defaulters do so because they are unemployed and in financial distress. However, these conditions only result in default when the housing market is illiquid. When such owners cannot sell their house quickly, they have no choice but to default. However, the model also highlights the role of housing liquidity for sellers who don’t need to default. An illiquid market is characterized by long wait times to sell and by low prices. In such a situation, a seller with a high level of debt may find it optimal to default immediately.

The defining feature of the model is the presence of housing and labor markets that are both characterized by search frictions. This allows for the ease with which a house is traded to factor into the ease with which a job is found, and vice versa. This link is novel to the literature on housing and unemployment. The housing market is adapted from that of Head, Sun, and Zhou (2015) — hereafter HSZ. The key features of their housing market are directed search, long-term mortgages and limited commitment. Households live in a “city” where they can own at most one house. The housing market is divided into smaller submarkets, in which buyers and sellers are randomly matched. Each submarket is
characterized by its own housing price and matching probability. Participants choose which submarket to enter based on the trade-off between the transaction price and the matching probability. In this sense, search is directed.

A household takes out a mortgage from a perfectly competitive bank in order to buy a house. A mortgage is a finite-horizon long-term contract with a fixed interest rate. Indebted owners are subject to shocks that cause them to relocate outside of the city, and shocks that force them into financial distress. Based on their specific situations, households make decisions on house buying/selling, and on whether or not to default on mortgage debt. In the event of default, the owner’s house is seized by the bank, and a foreclosure flag is placed on her record, meaning that she is excluded from the housing market. Later on, the flag is lifted randomly.

There is ex-ante heterogeneity among house sellers; house sellers consist of home owners, construction firms, and banks that want to unload their inventory of foreclosed houses. Moreover, home owners may be unemployed, or employed at different wages. There is also ex-post heterogeneity among household sellers in that they differ in their outstanding debt. In order to maintain tractability, goods are non-storable and so households cannot save over time. This, combined with modeling assumptions in the labor market, render all house buyers homogeneous. Moreover, free-entry of buyers into the housing market gives rise to the trade-off between house prices and matching probabilities. The heterogeneous sellers optimally separate into different submarkets based on their individual states.

In addition to the housing market above, there is a frictional labor market, also characterized by directed search, in the spirit of Shi (2009), which is visited by firms and potential workers. Each submarket in the labor market is characterized by a wage and a matching probability. Firms and workers are randomly matched within each submarket. The choice of submarket in which to search is based on the trade-off between the wage and the matching probability. A worker has a higher probability of meeting a firm by searching in a submarket with a lower wage, and vice versa for a firm. A successful match lasts for one period.

Workers consist of all residents in the economy, and are heterogeneous in terms of their ownership and mortgage debt status. Firms are ex-ante identical. Free entry of homogeneous firms into the labor market gives rise to the trade-off between wages and matching probabilities. Heterogeneous workers optimally separate themselves into various submarkets based on their individual states.

A firm operates a project that requires one worker to complete. The output from the project is

---

1I deviate slightly from HSZ, in terms of modeling this shock. Here, a financial distress shock is an unexpected and relatively large expense that must be paid in the current period. In HSZ, a resident that is hit by a financial distress shock must terminate her mortgage immediately, either by repaying outstanding debt, or by defaulting.

2In the model, one period is a year. At this frequency, it become less important to model transitions into and out of unemployment. This will be discussed in greater detail in Section 3.
stochastic and follows a uniform distribution. Firms face a wage-in-advance constraint, in which they must pay workers prior to the realization of output.

There is a perfectly competitive bank that consists of two arms: the mortgage arm and the firm lending arm. Within each arm, the bank earns zero expected profit on each individual contract. The size of the mortgage that the bank originates depends on the default risk of a borrower, as well as the recovery costs associated with foreclosing on a house. In the model, these costs are both direct and indirect. They are direct in that the bank loses a fraction, $\chi$, of the sale price to costs associated with foreclosure. They are indirect in that they depend on the degree of housing market liquidity. Housing market liquidity is higher when houses can be sold quickly and at a relatively high price. A bank is willing to lend more when housing market liquidity is high; not only does default become less likely, but even when default occurs, the bank is able to sell the foreclosed house quickly, and at a higher price.

The firm lending arm of the bank grants loans to firms. The relationship between the firm and the bank is characterized by asymmetric information, limited commitment, and costly auditing. Firms observe the realization of output before the bank does, and can hide it away before they make the decision on whether to repay their debt or to default. The bank can choose to audit the firm in the event of default, but doing so is costly. This relationship between the firm and the bank resembles that in Williamson (1987). Similarly, the optimal contract that arises is a debt contract.

The cost to the bank of lending to a firm is the sum of the risk-free interest rate and a premium that depends on foreclosure costs. The premium is increasing in the level of foreclosure costs. This can be interpreted as follows. A shock to foreclosure costs directly affects a bank’s mortgage origination decisions. Such a shock also implies a deterioration of bank balance sheets, which often translates into a reduction of all lending, including commercial loans (e.g. Ivashina and Scharfstein, 2010). Thus, while nothing changes about the fundamentals of the firm, its cost of credit rises.

In order to capture the drying-up of the liquidity of mortgage-related securities that occurred during the Great Recession, I examine the effects of an exogenous rise in housing foreclosure costs. Later on, I contrast these effects with an exogenous fall in the marginal revenue of a firm. Both shocks decrease the value of a vacancy to the firm; however the specific mechanism at work varies slightly. In particular, the magnification effects of the former can be very important.

The experiment conducted is a stationary equilibrium analysis for different housing foreclosure costs. The results are informative. When the value of a firm’s land is affected by $\chi$, their cost of borrowing rises, and the value of a vacancy to the firm falls. The increase in unemployment worsens the impact of financial distress to indebted owners. Without a job, it becomes impossible to deal with financial distress. They must sell their house, and if they fail to do so, they must default. However, selling the
house becomes more difficult. First of all, since a greater measure of residents find themselves with no choice but to sell, the overall measure of sellers in the housing market rises. Secondly, the rise in foreclosure costs increases the downpayment required of potential buyers. This increases the lower bound on the wage that they are willing to accept, further exacerbating the unemployment level. Furthermore, fewer households enter the city, as the value of living there falls, which reduces the measure of potential buyers. The rise in the measure of sellers, combined with the fall in the measure of buyers, reduces housing market liquidity. Sellers are less likely to find a buyer, and thus more likely to default. A higher rate of default also means there are more residents with a foreclosure flag, which prevents them from participating in the housing market. Overall, the original rise in foreclosure costs reduces housing market liquidity even further.

These results depend crucially on the fact that a rise in $\chi$ directly impacts firm decision making. Let us consider the case in which the value of a firm’s land is unaffected by $\chi$. We then observe counterfactual movements in unemployment. A rise in $\chi$ reduces the size of a mortgage issued by the bank. Correspondingly, potential home buyers are required to provide a higher downpayment, and so owe less debt. This reduces the future risk of default for indebted owners, since there is no corresponding rise in the difficulty of finding a job. Additionally, they are less likely to search for a high-wage job, since mortgage payments are smaller. This increases the value of a vacancy to a firm; more matches are made, and unemployment falls.

Let us discuss the impact on wages in greater detail. One reason why the presence of labor market frictions matters is due to the behavior of wages. Consider for a moment, the case in which there are no labor market frictions, so that wages adjust to clear the market. In this case, the wage would be constant and equal to the marginal product of labor. Then, the rise in a firm’s cost of credit simply reduces the equilibrium wage by more than in the baseline model. This effect is still there in the case of labor market frictions, so that the upper bound on wages falls; however, there is now an impact on the wage for which residents search. That is, the lower bound on the wage for which residents search is rigid. In the case of indebted owners, those that are distressed seek to pay off their additional expenses in addition to their mortgage payments; the wage they search for must cover these. Potential new home buyers must also ensure that their wage is sufficient to afford the higher downpayment that must be made on a house. Thus, there exists a gap between the wages that firms are willing to offer and the wages that workers are willing to accept, and this gap is larger when there are labor market frictions. This contributes to a larger unemployment rate.

The remainder of this paper is organized as follows. Section 2.2 reviews the relevant literature. Section 2.3 introduces the environment of the baseline economy. Section 2.4 outlines the equilibrium
in the economy. Section 2.5 outlines the calibration strategy. Section 2.6 describes the steady state implications, and Section 2.7 discusses the implications of rising foreclosure costs. Section 2.8 concludes.

2.2 Related Literature

There is a vast literature that examines the role of the housing market in explaining unemployment during the Great Recession. A common focus has been the impact of falling house prices on aggregate demand (e.g. Mian et al. 2013 and Mian and Sufi, 2014). Another explanation focuses on the harsher credit conditions that firms face, either due to the fall in the value of their collateral (e.g. Liu et al., 2015), or due to a fall in the amount of credit supplied by banks (Ivashina and Scharfstein (2010)). It is the second strain of literature that is more closely related to this paper. I will first review the literature that connects housing market outcomes to firm lending. I will then move on to the literature relating firm financial frictions to employment decisions. I will end with reviewing the literature relating the housing market to employment outcomes.

Commercial and industry loans dropped significantly during the Great Recession, and data from the Federal Reserve Senior Loan Officer Opinion Survey show that more loan officers reported a tightening of commercial lending standards during the Great Recession than in past recessions. Ivashina and Scharfstein (2010) analyze how the lending behavior of banks varied with their exposure to Lehman Brothers, and find that banks with more co-syndicated credit lines with Lehman cut back on lending more substantially following the Lehman failure. Helbling and Terrones (2003) investigate asset price booms and busts in industrialized countries in general, and find that housing price busts reduce the capacity and willingness to lend to a greater extent than do equity busts. Huang and Stephens (2015) show that the housing crisis in the U.S. resulted in a modest drop in the availability of credit for small businesses, and further, banks that were more affected by the housing crisis cut back on lending by relatively more.

Financial frictions and employment outcomes are studied in the financial accelerator framework in the works of Kiyotaki and Moore (1997) and Bernanke et al. (1999). Collateral, in the form of land or real estate holdings, makes it easier for firms to borrow from a financial intermediary. When the economy experiences a shock that leads to a fall in the value of the collateral, firms face tighter borrowing constraints, and this leads them to cut back on investment and production, further exacerbating the effects of the original shock.

The increase in credit constraints for firms, whether they be tighter lending standards, or a reduced credit supply, seems to matter for unemployment. Duygan-Bump et al. (2010) investigate the link
between small business lending and unemployment during the Great Recession. They find that small businesses with a high degree of dependence on external finance experienced the largest cutback in lending, and also experienced the largest increase in unemployment. Greenstone and Mas (2014) purge their measure of the national change in lending from exposure to local markets, in an attempt to isolate the supply shocks in lending. They are able to place an upper bound on the impact of the fall in bank lending due to supply factors: For small businesses, less lending is responsible for a 1.4 percentage point decline in employment in firms with less than 20 employees, a 0.8 percentage point fall in total employment, and a 1.2 percentage point decline in aggregate wages. They also find that the relationship between lending supply and economic activity was not present in the decade prior to 2007.

Haltenhof et al. (2014) extend their analysis beyond small firms, and analyze the impact of restricted credit to both firms and consumers. They find that while restrictions in credit to both firms and consumers matter, it is the latter that translates into more job losses for large U.S. manufacturing firms. Bentolila et al. (2015) provide further evidence of the impact of bank stress on credit supply and unemployment, but for the case of Spain. They find that weak, or bailed-out banks, lent relatively more to the real estate industry and also curbed lending more than healthier banks following the housing market collapse. They also find that client firms of these weaker banks had larger employment losses, and estimate that 24 percent of job losses in firms exposed to weak banks was due to this attachment. It is important to note however, that Spanish firms are more reliant on bank credit than U.S. firms, which could account for the relatively stronger relationship between the bank lending and unemployment across all firm sizes.

The literature on the housing crisis and its impact on the real economy extends far beyond access to firm credit. Branch, Nadeau and Rocheteau (2014) develop a DSGE model to analyze the mechanisms through which housing-related financial frictions impact labor markets in the long-run. Specifically, they look at various financial innovations that increase the collateralization of housing, which is used to secure loans for consumption goods. The mechanism works as follows: An increase in the borrowing capacity of consumers raises a firm’s expected revenue, and they hire more workers. In addition, the demand for housing rises as well, increasing housing prices and construction, both of which amplify the shock to the household’s borrowing capacity. Their work differs from this paper, in that their focus is on the aggregate demand channel between housing and unemployment, while I focus on firm credit conditions.

Another related work is that of Corbae and Quintin (2015), who look at how much of the post-2006 rise in foreclosures is due to the increase in originations of high-leverage mortgages during the housing boom. They calibrate the model to match the pre-boom times, and then relax mortgage standards in a variety of ways. They find that, following an exogenous fall in home prices, meant to model a bubble
correction, that mortgage default increases by 275%, and that, without the aforementioned relaxation of approval standards, default would have risen by 105%. While my work is somewhat related, in that it involves the role of changes in housing market liquidity on mortgage standards, and the impact on housing default, it extends the analysis to look at the resulting impact on the labor market.

Perhaps the paper that is most related to the present work is that of HSZ. While I build on their model, the purpose of their work is different. Their model features a frictional housing market, house price distributions, and endogenous default. They aim to explain variation in house-selling behavior in the presence of a frictional housing market, and how mortgage standards vary with changes in housing liquidity brought on by income shocks. Frictional housing markets are important in explaining changes in default and housing market liquidity, and thus house prices. The goal of the present paper is to incorporate this model of housing into an economy that features a frictional labor market, and allows for feedback between the two markets. Specifically, changes in housing liquidity have the ability to impact labor search decisions, which can then further influence mortgage lending standards. Moreover, changes in housing liquidity can influence firms’ access to funds, which can also influence labor search decision, further impacting mortgage default, house-selling decisions, and lending standards. Although I incorporate an HSZ-style housing market, the mechanisms at work in their model alone are insufficient to explain a rise in unemployment. To that end, I also incorporate a firm lending market, the conditions of which are related to housing market liquidity.

The work of Liu et al. (2016) also shares similarities with this paper. They develop a DSGE model with both a housing and labor market, and with credit and labor search frictions. They find a strong transmission mechanism between land prices and unemployment. In an environment in which there are borrowing constraints, housing serves as collateral to finance firm investment. An exogenous fall in housing demand increases unemployment through two channels. With the credit channel, low housing demand decreases land prices, making collateral less valuable and tightening borrowing constraints. This lowers the value of a firm-worker match, leading to a rise in unemployment. With the labor channel effect: due to the non-separability of consumption and housing services in the utility function, there exist nominal wage rigidities, which would not be present otherwise. This further increases unemployment. The present paper differs from Liu et al. in several ways. Housing can only be obtained via an imperfectly liquid housing market, and only by incurring long-term debt in the form of a mortgage, on which debtors can default. While the credit channel works somewhat similarly in both models, in that the value of a match to the firm falls, the labor channel works somewhat differently, and has different amplification implications, as well as different implications on the role of default. In the present paper, an increase in unemployment increases default, which further tightens mortgage standards, and alters labor market
searching behavior.

2.3 Model Environment

Time is infinite and discrete. Each time period is indexed by \( t \). The economy consists of a city characterized by a housing market, a labor market, and a credit market, and the “rest of the world”. Many features of the housing market are modeled after HSZ. What follows is taken from them, unless otherwise noted. At the start of each period, infinitely-lived households in the rest of the world choose whether or not to enter the city. The measure of these ex-ante identical households in the world in period \( t \) is denoted by \( Q_t \), and grows exogenously at net rate \( \mu \). Households work, consume the consumption good, and buy housing. A household’s periodic utility is given by

\[
U_t = u(c_t) + z_t,
\]

where \( c_t \) denotes consumption, and \( z_t \) denotes the utility derived from home ownership: \( z_t = z^H \) if the household owns their home, and \( z_t = 0 \) otherwise. The function \( u(\cdot) \) exhibits the standard properties, and all households discount the future by factor \( \beta \). For tractability purposes, consumption goods cannot be stored, and there exists no technology that allows for intertemporal savings.

At the beginning of each period, a measure \( \mu Q_t \) of new agents from the rest of the world arrive in the economy and have the option to enter the city. Each agent has an outside option to entering the city, the value of which is denoted \( \epsilon_t \), which is i.i.d. across new agents according to a stationary distribution function \( G(\epsilon) \), with support \([0, \bar{\epsilon}]\). There exists a reservation outside option, \( \epsilon^c_t \), at which a new agent is indifferent between entering and not entering the city. A household that enters the city becomes either (i) a buyer who values home ownership, or (ii) a perpetual renter who does not value home ownership. Let \( \psi \) denote the probability of becoming a buyer. The reservation value is then given by

\[
\epsilon^c_t = \psi U^b_t + (1 - \psi)U^p_t,
\]

(2.3.1)

Here, \( U^b_t \) and \( U^p_t \) are measured at the opening of the labor market, which will be described shortly. \( U^b_t \) is the value of being a potential homeowner, and \( U^p_t \) is the value of being a perpetual renter.

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3Entry and exit into the city ensures a steady stream of supply and demand for housing.
2.3.1 The labor market

After new residents enter the city, the labor market opens, in which firms and households look to match in order to produce the consumption good. The model deviates here from HSZ, which does not contain a labor market. There is free entry of firms. The labor market is frictional and search is directed, as in Shi (2009). That is, the labor market is further divided into submarkets, each of which is indexed by \((\theta_\ell, x)\), where \(x = (w, A)\). Here, \(\theta_\ell\) denotes the market tightness, or the ratio of workers to firms \((\frac{U}{V})\), \(w\) is the wage, and \(A\) describes the housing status of the worker. Given the index, firms and workers choose a submarket to visit, within which they are randomly matched according to the function \(T(V, U)\), which is increasing in each argument and exhibits constant returns to scale. The matching probability of a worker, \(\gamma(\theta_\ell(x))\), and of a firm, \(\rho(\theta_\ell(x))\) are given by

\[
\gamma(\theta_\ell(x)) = \frac{T(V(x), U(x))}{U(x)} = T\left(\frac{1}{\theta_\ell(x)}, 1\right) \quad (2.3.2)
\]
\[
\rho(\theta_\ell(x)) = \frac{T(V(x), U(x))}{V(x)} = T\left(1, \theta_\ell(x)\right) \quad (2.3.3)
\]

For a given \(A\), free entry of firms implies that there exists a trade-off between the wage and the matching probability across submarkets. That is, higher-wage submarkets are less tight, so that a firm that is willing to pay a higher wage enjoys a higher likelihood of finding a worker. For a worker, the situation is reversed. Additionally, since workers vary according to their level of debt and financial distress, these characteristics may attenuate the relationship between tightness and wage; they are thus included in the index of a submarket. If the household fails to find a match, then it receives an unemployment benefit \(w^u\). A match lasts only for the duration of the period, after which search begins anew.

Although it is conventional in the labor search literature for a match to last over multiple periods, until endogenous or exogenous job separation occurs, relaxing this feature allows for the model to be simplified without altering the predictions in any meaningful manner.\(^4\) This is because the time period will be set to one year. Households are unable to save and can only finance the downpayment on a house with their periodic labor income. If the time period were set to the standard quarter, then the household’s income would be too small to afford a typical downpayment, which tends to be around 2 or 2.5 times the size of quarterly income. Empirically, there is a lot of churning in the labor market, and we would fail to capture movement into and out of unemployment at such a low frequency. For example, according to Sierminska and Takhtamanova (2011), the monthly job finding probability at the pre-recession peak was 40 percent. The trough during the Great Recession was 17 percent. This

\(^4\)This may not be the case under different assumptions.
implies that the probability of remaining unemployed for 12 consecutive months would be between 91 and 99 percent. Thus, the majority of those experiencing job separation within the year would fail to be captured. Moreover, Sieminska and Takhtamanova (2011) also find that a lack of hiring, rather than job separation, factored more prominently in the high unemployment of the Great Recession; a model in which jobs last only for one period would highlight the importance of job creation.

A successfully matched firm produces output. The firm is subject to a wage-in-advance constraint: it is required to pay workers before output is realized. They do this by borrowing the wage from a perfectly competitive bank. Firms are also subject to a limited commitment constraint, and may default on their loan. A firm can be thought of as the owner of a project that requires a worker to complete. The output from the project is stochastic and is independently and identically distributed across firms according to $F(y)$, with support $[\underline{y}, \overline{y}]$, $y > 0$. Although the distribution of output is known to everyone in the economy, the realization, $y$ of an individual project is the firm’s private information, and is only observable to the bank by incurring a monitoring cost $\alpha^c$. This way of modeling the project of a firm, and its relationship to a lender, is similar to the works of Diamond (1984) and Williamson (1987). Likewise, a debt contract is the optimal arrangement between the firm and the lender. Specifically, given an amount borrowed $w$, the firm is charged a constant interest rate $R(w)$ that is independent of the realization of the project. If it defaults on the debt, then the bank audits with probability 1, and seizes all of the output. Thus, the default state can be interpreted as a state of bankruptcy, and $\alpha^c$ as a cost of bankruptcy. A feature of this contract is that the interest rate rises with the amount lent. Thus, the probability that monitoring occurs, as well as the bank’s expected cost of monitoring also rise. Given the competitive nature of banks, this places an upper limit on the amount that firms can borrow.

### 2.3.2 The housing market

Once the labor market closes, the housing market opens. A household can own at most one house. Houses are identical and indivisible and can be old or new. New houses are built by competitive construction firms, and require only a unit of land to build. Let $N_t$ denote the amount of new houses built in period $t$. A construction firm purchases land in a competitive market at price $q(N_t)$, and also incurs construction costs $k_t = K(N_t)$. Houses constructed in period $t$ are available for sale in period $t + 1$. In order to prevent depreciation over time, a maintenance cost $m$ is incurred each period, regardless of the vacancy status of the house.

As in the labor market, the housing market is characterized by directed search, in which potential buyers and sellers look to match in order to trade a house. Potential buyers consist of employed house-
holds who value housing but are not yet owners. Sellers in the housing market consist of household sellers, construction firms, and the mortgage arm of the competitive bank (to be specified shortly). A submarket is indexed by \((\theta_h, p)\), where \(\theta_h\) denotes the market tightness, or the ratio of buyers to sellers \(\left(\frac{B}{S}\right)\), and \(p\) is the price of the house. Given the submarket index, buyers and sellers choose which submarket to visit. They are then matched randomly according to a matching process defined by the function \(M(B, S)\), which is increasing in each argument, and exhibits constant returns to scale. The matching probability of a buyer, \(\nu(\theta_h)\), and of a seller, \(\tau(\theta_h)\) are given by

\[
\nu(\theta_h) = \frac{M(B, S)}{B} \quad \text{and} \quad \tau(\theta_h) = \frac{M(B, S)}{S} = M(\theta_h, 1) = \theta_h \nu(\theta_h)
\]

There is no cost to entering a submarket. Free entry creates a trade-off between the house price and the matching probability across submarkets. All else equal, a submarket in which a seller posts a high price is visited by fewer buyers, so that each buyer has a higher probability of matching with a seller. Once a match is formed, the buyer moves into his house, and receives utility from housing and incurs the maintenance cost.

In general, wages are not high enough to finance the entire payment of a house; combined with a lack of saving, this means that a buyer must obtain a mortgage to finance the purchase. Mortgages are for a fixed long-term period of time and are obtained from the mortgage arm of a perfectly competitive bank. To finance its loans, the bank trades one-period risk-free bonds at an exogenous interest rate \(i\) in the international bond market. The bank also incurs service costs of rate \(\phi\) per period, which represents the opportunity cost of using the funds to lend to households for housing purposes.

The mortgage contract can be described by the tuple \((m_{0,t}; r_m, T)\), where \(m_{0,t}\) is the size of the loan at origination in period \(t\), \(r_m\) is the fixed interest rate, and \(T\) is the finite maturity term, the latter two of which are exogenous and fixed throughout time. The mortgage rate \(r_m\) is given by \(r_m = i + \phi + \varrho\), where \(\varrho\) is a risk premium. Each period, a payment \(x(\cdot)\) must be made, which can be calculated as

\[
x(m_0) = \frac{r_m}{1 - (1 + r_m)^{-T}} m_0.
\]

The principal balance \(d\) after \(n \in \{0, T - 1\}\) completed payments is then

\[
d(m_0, n + 1) = (1 + r_m) d(m_0, n) - x(m_0),
\]

---

5For simplicity, I allow only employed non-owners to buy a house. This increases tractability, but is also realistic, as employment is usually a necessary condition of obtaining a mortgage.
and \( d(m_0, 0) = m_0 \). Note that both \( x(\cdot) \) and \( d(\cdot, \cdot) \) are independent of the date of mortgage issuance. An ongoing mortgage can thus be represented by \((m_0, n)\).

In each period, an indebted owner has the option of defaulting on a mortgage, which is followed by foreclosure. Adverse financial shocks, such as accidents or unexpected illness, may hit households at the beginning of each period with probability \( \pi_d \). The adverse financial shock manifests itself in the form of a lump-sum payment that a household incurs in the current period. In order to finance this payment, they can change their wage searching behavior, decrease their consumption, or sell their house. If they fail to find a sufficient wage and fail to sell their house, then they cannot afford to make both the mortgage payment and the additional financial expenses, and so they have no choice but to default. Following default, the bank repossesses the defaulter’s house, and puts it in its real-estate-owned (REO) inventory, setting it up for sale in period \( t + 1 \). The defaulter receives any positive difference between the value of a foreclosed house and the outstanding mortgage balance. Once the bank sells the house, it loses a fraction \( \chi \in (0, 1) \) of the revenue to foreclosure costs, such as legal fees. The defaulter has a foreclosure flag placed on his credit record, and loses access to the mortgage market (and thus housing). Beginning in the following period, the foreclosure flag remains on her record with probability \( \pi_f \in (0, 1) \). Once it is lifted, the household regains access to the mortgage market.

Unmatched buyers and perpetual renters live in rental units provided by landlords. As in HSZ, I simplify the analysis to focus on house trading and so assume that landlords sell house services at an exogenously given price \( RR \). Rental units are not counted in the city’s housing stock.\(^6\)

At the end of each period, residents may receive a moving shock, in which case they immediately leave the city. For perpetual renters, this occurs with probability \( \pi_p \in (0, 1) \), whereas for resident owners and potential home buyers, it occurs with probability \( \pi_h \in (0, 1) \). Movers without housing receive continuation value \( L \). Moving owners will have a vacant house that they will want to sell. They may also have outstanding debt, and must choose if they will continue to make payments or if they will default. Their continuation value will depend on the level of debt and on the default decision.

**Timing**

The timing of the model is illustrated below. There are three subperiods. At the beginning of subperiod 1, new entrants with \( \epsilon \leq \epsilon^*_t \) enter the city. Shocks to the foreclosure cost \( \chi \), financial distress shocks, and shocks to the foreclosure flag are realized. The labor market then opens. Firms and households decide in which submarket to search. Successfully matched firms obtain a loan from the bank in order to pay

\(^{6}\)Relaxing this assumption does not alter the results of the model in any meaningful way, and allows for the simplification of an already complicated model.
workers their wages. Employed workers receive their wage immediately. Unemployed households receive benefit \( w^u \).

In subperiod 2, the housing market opens. Buyers and sellers decide in which submarket \((p, \theta_h)\) to search. The mortgage sector becomes active after the housing market closes. Borrowers make their default decision. New owners make their downpayment and take on mortgages to finance their purchases.

In subperiod 3, a firm’s output is realized, and it makes its default decision. Households make periodic payments (maintenance, mortgage payments, or rents), and consume the remainder of their income. At the end of the period, moving shocks are revealed for all households. Those who receive the shock leave the city immediately.

2.4 Equilibrium in the Baseline Economy

2.4.1 Firm value functions

There is free entry of firms in the labor market. Firms decide in which submarket to search for a potential worker. Potential workers consist of current households in the city, as well as newly entering households. Workers are heterogeneous in their home ownership status. Workers can be described by the tuple \( A = (h, b, f, (m_0, n), d) \), where

- \( h \in \{0, 1\} \), \( h = 1 \) if the worker does not value housing and \( h = 0 \) otherwise;
- \( b \in \{0, 1\} \), \( b = 1 \) if the worker is a buyer, and \( b = 0 \) if she is a homeowner;
- \( f \in \{0, 1\} \), \( f = 1 \) if the worker has a foreclosure flag, and \( f = 0 \) otherwise;
- \( (m_0, n) \), where \( m_0 \in \mathbb{R}_+ \) describes the original mortgage loan of the owner, and \( n \in \{0, 1, ..., T - 1\} \) is the number of mortgage payments made (if the worker has no debt, then \( (m_0, n) = (0, 0) \));
- \( d \in \{0, 1\} \), \( d = 1 \) if the worker is in financial distress, and \( d = 0 \) otherwise.
Firms incur cost $j^c$ to create a vacancy. Free entry of firms drives the value of a vacancy down to zero. The quantity of output yielded by a firm-worker match is stochastic and uniformly iid across matches, with support $[y, \bar{y}]$. The realization of output is the firm’s private information. Lending to firms is risky for the bank since the realization of output is the firm’s private information; however, it can be verified by incurring cost $\alpha^c$. A matched firm is subject to a wage-in-advance constraint, and so it borrows the wage from the bank. The bank is a modified version of that in Williamson (1987). In order to finance its loans, it trades one-period bonds in the international bond market. The rate at which it does this is composed of the risk-free rate $i$, and a premium, $\varepsilon(\chi)$ which depends on foreclosure costs, $\chi$, in the mortgage market. This is meant to capture the idea that, when foreclosure costs are high, bank balance sheets are relatively weaker, which translates into a reduction of all lending, including commercial loans (e.g. Ivashina and Scharfstein, 2010). That is, it may face a higher cost of borrowing from international markets. The premium $\varepsilon(\chi)$ is increasing in $\chi$.

The bank offers a debt contract to the firm. For an amount $w$ lent in the beginning of period $t$, the firm must return $R(\omega)$ at the end of the period, or default on the loan. If the firm defaults, the bank commits to observe and seize realized output $y$, at cost $\alpha^c$. As observed in Williamson (1987), this debt contract is optimal, and implies that firms default if and only if $y < R(\omega)$. Thus, the bank gets $R$ if $y > R$ and $y$ otherwise, and incurs the auditing cost only under default. Let $F(\cdot)$ denote the (uniform) distribution of output. Banks are perfectly competitive, so that each contract renders zero profit in equilibrium:

$$[i + \varepsilon(\chi)]w = \int_{R(w)}^{\bar{y}} R(w)dF(y) - \alpha^c F(R(w)) + \int_{y}^{R(w)} ydF(y)$$ (2.4.1)

Let $J(w)$ denote the value of a match in submarket $w$. If $y > R(\omega)$, the firm realizes a profit of $y - R(\omega)$. Otherwise, the firm defaults, is audited by the bank, and all output is seized. In this case, the value to the firm is zero:

$$J(w) = \int_{R(w)}^{\bar{y}} (y - R(w))dF(y)$$ (2.4.2)

Now, let $J_V(x)$ denote the value of a vacancy in submarket $(\theta_t, x)$. The firm incurs vacancy cost $j^c$ and finds a worker with probability $\rho(\theta_t(x))$, in which case the value of the match is $J(w)$:

$$J_V(x) = -j^c + \rho(\theta_t(x))J(w).$$ (2.4.3)

Free entry by firms drives the value of a vacancy down to zero in all submarkets: $J_V = 0$. This yields a relationship between $\theta_t(x)$ and $w$ in any submarket:
\[
\theta_t(x) = \rho^{-1} \left[ \frac{j^c}{\int_{R(w)}^y (y - R(w)) dF(y)} \right] \equiv \Gamma(w)
\]  

\[2.4.4\]

2.4.2 Household value functions

The following is a description of household value functions for a typical time period \(t\). Value functions at the opening of the labor market are denoted as \(U^j_t(\cdot)\), where \(j\) denotes the particular type of household. Similarly, \(V^j_t(\cdot)\) and \(W^j_t(\cdot)\) denote the value functions at the opening of the housing market and in subperiod 3, respectively.

Subperiod 1

All residents search for a job when the labor market opens. Let \(\mathcal{A}\) denote the housing status of the resident. A resident can be a perpetual renter, a buyer with or without a foreclosure flag, an indebted seller with or without financial distress, or a seller with no debt. The value of a resident with housing status \(\mathcal{A}\) at the opening of the labor market is then

\[
U_t(\mathcal{A}) = \max_{\theta_t, w} \gamma(\theta_t) V_t(\mathcal{A}, w) + (1 - \gamma(\theta_t)) V_t(\mathcal{A}, w^u) \text{ subject to } \theta_t = \Gamma(w).
\]  

\[2.4.5\]

If she is successful at finding a match, she earns wage \(w\), which yields value \(V_t(\mathcal{A}, w)\) in subperiod 2. Otherwise, she gets the unemployment value of \(V_t(\mathcal{A}, w^u)\).

The value functions for each specific type of resident appear in Appendix A.

Substituting \(\theta_t = \Gamma(w)\) into the value function yields the following general form of the first order condition:

\[
\gamma(\theta_t) \frac{\partial V_t}{\partial w} = -\frac{\partial \gamma}{\partial w} [V_t(\mathcal{A}, w) - V_t(\mathcal{A}, w^u)]
\]  

\[2.4.6\]

The left hand side of (2.4.6) is the marginal benefit of searching for a higher wage, which is the marginal utility associated with the higher wage in subperiod 2, adjusted for the probability of successfully finding a match. The marginal cost is the net value lost by failing to find a match, \(V_t(\mathcal{A}, w) - V_t(\mathcal{A}, w^u)\), adjusted for the change in probability of searching for the higher wage. Thus, there is a tradeoff between the value of searching for the higher wage, and the probability of successfully finding a match.

The choice of submarket is influenced by the strength of the tradeoff and by the housing status of the resident. All else equal, if the probability of finding a job falls for a given wage, the marginal benefit of the wage falls, and the marginal cost rises. This incentivizes the resident to search in a submarket...
with a lower wage. Now consider a resident’s housing status. Suppose, all else equal, the value of being employed in subperiod 2 rises. The resident is less likely to risk unemployment in this case, and will lower the wage for which she searches. This is also the case for which the value of being unemployed falls.

**Subperiod 2**

House trading decisions and mortgage default decisions are made in this period. We will discuss the relevant problems for each type of household that is active in the market. In what follows, let $w$ denote the household’s current income, where $w = w^u$ if unemployed.

An employed buyer without the foreclosure flag on her credit record will search to buy a house. The value of such a buyer who enters submarket $(\theta_h(w), p)$ is

$$V_t(b(w)) = \nu(\theta_h(w))W_t^b(p, m_0, w^b) + (1 - \nu(\theta_h(w)))W_t(b(0, w^b)). \quad (2.4.7)$$

The buyer gets matched with a seller with probability $\nu(\theta_h(w))$, in which case she will proceed as a new owner with value $W_t^b(p, m_0, w^b)$. The loan volume $m_0$ is determined by the mortgage company and will be specified later. With probability $1 - \nu(\theta_h(w))$, the buyer fails to get a match and will proceed as a buyer without the foreclosure flag, with value $W_t(0, w^b)$. With free-entry of buyers, any active submarket must offer the exact same level of $V_t(b(w))$ to the buyers. Rewriting (2.4.7) yields

$$\theta_h = \nu^{-1} \left( \frac{V_t^b(w^b) - W_t^b(0, w^b)}{W_t^b(p, m_0, w^b) - W_t^b(0, w^b)} \right) \equiv \Omega(p), \quad (2.4.8)$$

which is the free-entry condition of buyers that pins down the relationship between the transaction price and the market tightness.

Let us move on to the selling decision of resident owners. The housing status of a resident owner can be described by the tuple $(s, m_0, n)$, where $s$ is the financial distress status of the owner:

$$s = \begin{cases} 
0 & \text{if no financial distress} \\
1 & \text{if financial distress}
\end{cases}$$

Here, $(m_0, n)$ is the debt status of the owner. For owners without debt, $(s, m_0, n) = (0, 0, 0)$.

A resident owner’s housing market decision depends on the value of defaulting on the mortgage in the event that a house is not sold. Let $V_t^{def}(s, m_0, n, w)$ denote the value of default for a resident owner.
with wage $w$:

$$V_t^{def}(s,m_0,n,w) = \max_{D_t \in \{0,1\}} (1 - D_t)W_t^s(s,m_0,n,w) + D_t W_t^f(\max[0, \beta \mathbb{E}t^{REO}_{t+1} - d(m_0,n)], s, w)$$

(2.4.9)

If the owner chooses not to default, she continues onto subperiod 3 as an owner, and receives continuation value $W_t^s(s,m_0,n,w)$. If she chooses to default, she loses her house and a foreclosure flag is placed on her record. If the value of her debt, $d(m_0,n)$, is less than the expected value of a vacant house in the bank’s REO inventory, she receives the difference. Here, the expectation is taken over shocks such as aggregate shocks, adverse shocks, foreclosure flag shocks and moving shocks. Otherwise, the defaulter gets nothing. The value of entering subperiod 3 as a defaulter is then $W_t^f(\max[0, \beta \mathbb{E}t^{REO}_{t+1} - d(m_0,n)], s, w)$. A resident owner without debt will never default, so that $V_t^{def}(0,0,0,w) = W_t^s(0,0,0,w)$.

Let us consider how the housing and labor status of a resident owner influence the optimality of default. A key determinant of default is what I call “post-foreclosure equity,” or $\beta \mathbb{E}t^{REO}_{t+1} - d(m_0,n)$. All else equal, when this value is positive, the incentive to default is greater. Although the resident will be shut out of the mortgage market in the future for some time, she still receives a positive payoff in the current period. Post-foreclosure equity rises as the owner’s outstanding debt, $d(m_0,n)$, falls. Outstanding debt falls with a decrease in $m_0$, the size of the mortgage, and an increase in $n$, the number of payments already made. However, a fall in $m_0$ and rise in $n$ also reduce the value of default. This is because these movements increase the value of remaining a seller. With a smaller mortgage, and thus smaller payments, an owner has more income left over for consumption. As more payments are made, the number of future payments falls, and the owner can look forward to the future increase in consumption, in addition to the utility she gains from home ownership.

The effect of financial distress and having a low wage (or receiving the unemployment benefit) work in the same way to influence the decision to default. Both financial distress and a low wage lower current-period consumption, which not only increases the incentive to default, but may also require default, if current-period income is insufficient to pay all expenses.

We can use $V_t^{def}(s,m_0,n,w)$ to define the value of being a resident owner, $V_t(s,m_0,n,w)$ at the opening of the housing market:

$$V_t(s,m_0,n,w) = \max_{p,\theta_h} \tau(\theta_h)W_t^b(\max[0, p - d(m_0,n)], s, w) + (1 - \tau(\theta_h))V_t^{def}(s,m_0,n,w)$$

s.t. $\theta_h = \Omega(p)$

(2.4.10)
The resident owner must decide whether or not to put her house up for sale. If so, she chooses in which submarket \((p, \theta_h)\) to sell. This choice is subject to the buyers’ free-entry condition \((2.4.8)\). With probability \(\tau(\theta_h)\), the owner successfully sells the house. If so, she pays off the outstanding debt with the proceeds of the sale, and keeps any remaining profit \(a = \max[0, p - d(m_0, n)]\), and moves on as a buyer with no foreclosure flag, with value \(W^b_t(a, s, w)\). If she chooses not to sell the house, or has failed to sell, the owner makes a default decision, yielding \(V^\text{def}_t(s, m_0, n, w)\).

Let us analyze a resident owner’s decision to sell. A resident chooses not to participate in the housing market if the value of remaining on owner is higher than selling in any submarket that is visited by buyers. This is more likely to be the case under certain conditions. All else equal, a resident owner with a small mortgage \(m_0\) or with little time before the house is fully paid (high \(n\)) is less likely to sell her house. The value of remaining a seller is high, and in the case of low \(m_0\), the owner gets to enjoy relatively higher consumption. Similarly, if the owner’s wage is high and if she is not financially distressed, the housing-consumption trade-off is lower.

Suppose the resident owner decides to sell her house. Substituting the buyers’ free-entry condition into \((2.4.10)\) yields the following optimality condition:

\[
\tau(\theta_h(p)) \frac{\partial W^b_t}{\partial p} = -\frac{\partial \tau}{\partial p} \left[ W^b_t(\max[0, p - d(m_0, n)], s, w) - V^\text{def}_t(s, m_0, n, w) \right]
\]

The right hand side of \((2.4.11)\) is the marginal benefit of selling at a higher price, which is the marginal utility associated with the higher price in subperiod 3, i.e. the marginal utility of consumption. This is adjusted for the probability of successfully making a trade. The marginal cost is the net value lost by failing to trade, \(W^b_t(\max[0, p - d(m_0, n)], s, w) - V^\text{def}_t(s, m_0, n, w)\), adjusted for the change in the probability of a successful sale. As in the labor market, there is a trade-off between the value of searching for the higher price, and the probability of successfully selling the house.

As before, the choice of submarket is influenced by the strength of the trade-off and by the resident owner’s individual state. All else equal, if the probability of selling a house falls for a given price, the marginal benefit of the price falls, and the marginal cost rises. This incentivizes the resident to enter a low-price submarket. The impact of a resident owner’s individual state on her choice of submarket is not always clear analytically. Consider the number of payments, \(n\), that she has already made. This reduces the value of her debt, which increases both the value of a successful sale and the value of default. These have opposing effects on the marginal cost of selling at price \(p\). The same can be said for a smaller mortgage, \(m_0\). In this case however, there is an additional effect on the marginal benefit of \(p\). When the mortgage is small, the corresponding rise in the marginal utility of consumption is also smaller, which
reduces the marginal benefit of $p$. A higher wage and the absence of financial distress have the same
effect on the marginal benefit of $p$; that is, the additional utility from increased consumption due to
a higher price is smaller, and so is the marginal benefit. However, they also increase $W^b$ and $V^{def}$ as
well, and so the effect on the marginal cost of $p$ is uncertain. A quantitative analysis is needed in order
to determine the relative importance of the variables that make up the individual state of the resident
owner.

Now consider an owner who has been hit by a moving shock. She does not participate in the labor
market, as she relocates immediately. At the new location, she earns income $w_t^R$ and pays rent $RR_t^R$.
Her continuation value in the new city, once she is no longer an owner, is $L$. Let $V^{def,R}(m_0, n)$ denote
her value of defaulting:

$$V^{def,R}(m_0, n) = \max_{D_t \in \{0, 1\}} \left(1 - D_t\right) \left[u(w_t^R - RR_t^R - x(m_0) - m) + \beta \mathbb{E}_t V_t^{R}(m_0, n + 1)\right] + D_t \left[u(\max(0, \beta \mathbb{E}_t V_{t+1}^{REO} - d(m_0, n)) + w_t^R - RR_t^R) + \beta L\right]$$

(2.4.12)

If a relocated owner does not default, she continues to make mortgage payments and consumes the rest
of her income. She tries again next period to sell her house, the discounted expected value of which
is $\beta \mathbb{E}_t V_{t+1}^{R}(m_0, n + 1)$. If a relocated owner defaults, she consumes the expected value of the house in
the REO inventory, net of her debt, $\beta \mathbb{E}_t V_{t+1}^{REO} - d(m_0, n)$. A relocated owner with no debt will never
default, so that $V^{def,R}(0, 0) = u(w_t^R - RR_t^R - m) + \beta \mathbb{E}_t V_t^{R}(0, 0)$. The analysis of the decision of a
relocated owner to default is analogous to that of residents, with the exception that relocated owners
are not subject to additional financial distress, and no longer value home ownership.

At the opening of the housing market, a relocated owner will always want to sell her house. If she
succeeds, she keeps any remaining profit from the sale, and receives continuation value $L$ starting in the
next period. If the owner is unable to sell her house, she must make a default decision. The value of
such a household is given by

$$V_t^R(m_0, n) = \max_{p, \theta_h} \left[u(\max(0, p - d(m_0, n)) + w_t^R - RR_t^R) + \beta L\right] + (1 - \tau(\theta_h)) V_t^{def,R}(m_0, n)$$

s.t $\theta = \Omega(p)$.

(2.4.13)
For a construction firm, the value of a house in its inventory $V^c_t$ is given by

$$V^c_t = \max_{p, \theta_h} \left[ \tau(\theta_h)p + (1 - \tau(\theta_h))[-m + \beta \mathbb{E}V^c_{t+1}] \right] \text{ s.t. } \theta = \Omega(p)$$

(2.4.14)

The value of a foreclosed house, $V^{REO}_t$, is similar, except that the mortgage company loses a fraction $\chi$ of the sale proceeds as foreclosure costs:

$$V^{REO}_t = \max_{p, \theta_h} \left[ \tau(\theta_h)(1 - \chi)p + (1 - \tau(\theta_h))[-m + \beta \mathbb{E}V^{REO}_{t+1}] \right] \text{ s.t. } \theta_h = \Omega(p)$$

(2.4.15)

The role of $\chi$ in housing and labor market decisions

The impact of a household’s individual state on their labor and housing choices has already been discussed. In this section, I discuss the role of the aggregate state, that is $\chi$, on a household’s choices. This is especially important, since the goal of this work is to determine how a rise in foreclosure costs can impact and magnify housing market outcomes. Let us begin with the housing market, since the decisions there will affect the labor submarket in which a household decides to search.

An increase in $\chi$ lowers $V^{REO}$, the value of a foreclosed house in the REO inventory of the bank. All else equal, this lowers the value of post-foreclosure equity, which makes default a less appealing option. Consider the impact of this on an owner who decides to sell her house. If she fails to do so, her payoff is now lower. All else equal, this incentivizes her to post her house in a lower-wage submarket. With a lower default value and an overall lower price at which sellers sell, there is downward pressure on subperiod 2 value functions. However, consider the general equilibrium effects on $m_0$ as well. The fall in $V^{REO}$ reduces the value of mortgage origination to the bank, and they lend less. As seen in the previous analysis, a fall in $m_0$ has uncertain effects on housing market decisions and thus value functions. A quantitative analysis is required to determine the overall effect in general equilibrium.

Let us move on to the labor market. As we have seen, a rise in foreclosure costs has an indeterminate effect on households. Suppose the value of being a seller at the opening of the housing market falls. Then all else equal, an owner will search for a lower wage. This is because there is a relatively larger fall in the value of being unemployed vs. employed at the opening of the housing market. Due to diminishing marginal utility of consumption, the seller is less likely to risk unemployment, and so searches for a lower wage. For those in financial distress, the fall in consumption is even starker if they fail to find a job, putting more downward pressure on the wage for which they search. Of course, if the opposite occurs, and sellers are actually better off at the opening of the housing market, they have the luxury of searching for a higher wage in the labor market.
However, there is an additional effect on the firm’s cost of credit, which is positively related to $\chi$. This puts downward pressure on the wage that it posts. The trade-off between wage and the probability of finding a job for the household steepens; that is, for a given wage, the probability that a household finds a match falls. This decreases the marginal benefit, and increases the marginal cost, of searching for a given wage. This puts further downward pressure on the wage.

As shown above, the overall effects of a rise in $\chi$ cannot be determined explicitly in general equilibrium. In Section 2.5, I calibrate the model in order to quantitatively compute the impact of a change in $\chi$. The results are presented in Section 2.7.

**Subperiod 3**

At the beginning of subperiod 3, firm output is realized. If able, the firm repays its debt to the bank. Otherwise, it defaults and the bank seizes all output. Regardless, the match between the worker and the firm is completed. No decisions are made in this period – households simply consume their income net of housing, rental, and distress-related expenses. As such, the value functions at the beginning of subperiod 3 of period $t$ for the households are straightforward and relegated to Appendix A.

### 2.4.3 Construction firm

The construction industry has a large number of competitive firms. Let $N_t$ be the measure of new houses built in period $t$ and available at $t+1$. Building a new house requires one unit of land at price $q_t = Q(N_t)$ and construction costs $k_t = K(N_t)$.

A firm’s zero-profit condition requires that $N_t$ is such that the cost of building a house is equal to the expected value of a vacant house for sale in period $t+1$. That is,

$$Q(N_t) + K(N_t) = \beta E_t V_{t+1}^c.$$  \hfill (2.4.16)

### 2.4.4 Mortgage arm

Since the bank is competitive, it earns zero profit on each loan contract. This implies that the expected return, net of foreclosure costs, is equal to the cost of funds, which is the interest rate $i$ on international bonds plus the servicing premium $\phi$. New borrowers start to repay their mortgage in the period following origination. Since jobs last for only one period, they all search anew at the beginning of the following period. Thus, when the bank first issues the mortgage, all borrowers are identical. This implies that they all receive the same loan $m_{0,t}$ in period $t$, independent of their individual house price.

---

7 As in HSZ, I allow $q$ to depend on $N$ in a non-linear way to reflect the findings of Saiz (2010) that the price elasticity of housing supply (and thus the amount of land used by the construction firm) was elastic in the U.S. in the period 1975-2010.
Denote by $\delta = (m_0; r_m)$ the mortgage held by a resident owner and $P^\delta_t(m_0, n)$ the mortgage’s present value at the end of subperiod 2 of period $t$ after $n \geq 0$ payments have been made up to period $t$. Then, the zero profit condition of the bank is given by

$$P^\delta_t(m_0, 0) - m_0 = 0.$$  \hspace{1cm} (2.4.17)

Thus the problem solved by the bank is to find the value of $m_{0,t}$ that satisfies (2.4.17) in period $t$.

Due to the complicated nature of the exact form of $P^\delta_t(m_0, 0)$, its derivation can be found in Appendix B.

### 2.4.5 Laws of motion

The evolution of the stock of households, all measured at the opening of the housing market in period $t$, can be found in Appendix C.

### 2.4.6 Equilibrium definition

**Definition.** An *equilibrium* is a collection of subperiod 1 and subperiod 2 value functions defined by

$$\left\{ \text{(2.4.5), (2.4.7), (2.4.10), (2.4.13), (2.4.14), (2.4.15)} \right\}$$  \hspace{1cm} (2.4.18)

the associated labor policy functions

$$\left\{ w_t(A), \theta_{t,t}(A) \right\},$$  \hspace{1cm} (2.4.19)

the associated housing policy functions

$$\left\{ p_t^k(A), p_t^R(A), \left\{ p_t^k \right\}_{j \in \{c, REO\}}, \theta_{h,t}(A), \theta_{h,t}^R(A), \theta_{h,t}^{k}(\cdot, \cdot); \left\{ \theta_{h,t}^{k}(\cdot, \cdot) \right\}_{j \in \{c, REO\}}, D_t^k(A), D_t^R(A) \right\},$$  \hspace{1cm} (2.4.20)

the entry cut-off value, the mortgage contract, rents

$$\left\{ c_t^e, m_{0,t}, RR_t \right\},$$  \hspace{1cm} (2.4.21)

and *per capita* measures of households and inventories.
such that, given the mortgage rate $r_m$ and the aggregate state, the above functions and values satisfy the following requirements:

1. New households enter the city optimally so that (2.3.1) holds;

2. All resident agents optimize such that the labor value and policy functions satisfy $J_V = 0$ and (2.1) to (2.4);

3. All sellers optimize such that the housing and associate policy functions satisfy ((2.4.7)) and (2.4.10) to (2.4.15);

4. Free entry of construction firms: $N_t$ satisfies (2.4.16);

5. Free entry of mortgage companies: $m_{0,t}$ satisfies (2.4.17);

6. The stocks of households and inventories $\{F_t, B_t, B_t^F, S_t(\cdot, \cdot, \cdot), S_t^R(\cdot, \cdot), S_t^R, S_t^{REO}, N_t, H_t, B_t^{sum}\}$ evolve according to (2.C.1) to (2.C.13);

7. Consistency: $B_t = B_t^{sum}$.

Requirements 1-6 in the above definition are self-explanatory. Requirement 7 states that in equilibrium, the measure of buyers without the foreclosure flag must be consistent with the total measure of buyers actively participating in the housing market.

### 2.5 Calibration

I calibrate the steady state of the economy to match the same selected facts of the U.S. economy as in HSZ. In order to calibrate the additional parameters in my model, I target selected labor market facts as well.

The functional forms used are as follows:

$$u(c_t) = \ln(c_t)$$

$$M(B, S) = \omega_h B^{\eta_h} S^{1-\eta_h}$$

$$T(U, V) = \omega_k U^{\eta_u} V^{1-\eta_v}$$

$$k_t = \frac{1}{\kappa} N_t^\delta$$
Chapter 2. Housing Liquidity and Unemployment: The Role of Firm Financial Frictions

\[ q_t = \tilde{q} N_t^{\xi} \]
\[ E_t = \chi^v - jE \]
\[ G(\varepsilon) = \left( \frac{\varepsilon}{A} \right)^{\alpha_p} \]
\[ F(y) = \frac{y - y}{y - \bar{y}} \]

Here, \( \eta_b \) and \( \eta_e \) are the elasticities of the measure of matches with respect to the measure of buyers and the unemployed, respectively. \( \xi \) represents the price elasticity of land supply. The function \( E(\cdot) \) captures the way in which the value of a firm’s borrowing costs is related to \( \chi \).

Tables 1 and 2 list the parameter values for the baseline economy. The first table covers the parameters that are also in HSZ. The same calibration targets are used. Table 2 covers the additional parameters in the model presented here. In both cases, some parameters are directly set to match certain targets, whereas others are determined jointly. The former are presented first in each table.

Let us first describe the calibration of the parameters in common with HSZ. The time period is one year, and so the discount factor \( \beta \) is set to match an annual interest rate of 4%. The annual yield on international bonds, \( i \), is also set at 4%.

The following parameters and targets come from Head et al. (2014). The rate \( \mu \) is set to target the annual population growth during the 1990s. The values for \( \pi_p \) and \( \pi_h \) are set to match the 1970-2000 annual fraction of renters and homeowners that move between counties (according to the Census Bureau), respectively. The price elasticity of land supply, \( \xi \), is set to 1.75, which is the population-weighted average measured by Saiz (2010). The steady-state unit price of land, \( \bar{q} \) is set to match the share of land in the price of housing, which is estimated to be around 30% (e.g. Saiz (2010)). The price elasticity of new construction, \( \zeta \), is taken from Green et al. (2005), who study 45 cities, and estimate its median value to be 5. The maintenance cost of housing, \( m \), is estimated by Harding et al. (2007) to be around 2.5% of the steady-state housing price. The value of \( \psi \) is set so that the ownership rate in the city is around two-thirds, which is the rate reported by the Census Bureau for the households whose head is between the ages of 35 and 44.

In order to target an average waiting period of five years following foreclosure, \( \pi_f \) is set to be 0.8. Based on empirical findings of the Lincoln Institute of Land Policy, the rent is set so that the average rent-to-housing price ratio is 5%. The average downpayment ratio is targeted at 20%. Using data from the National Delinquency Survey by the Mortgage Bankers Association, the average annual foreclosure rate in the 1990s, prior to the housing boom, is around 1.6%, so this serves as the target for the default rate in the model.
There are transaction and time costs associated with foreclosure, which in the model, are captured by $\chi$. Hayre and Saraf (2008) estimate that these foreclosure costs range from 35% to 60% of the original loan amount, depending on when the mortgage was issued. The target for this “loss severity rate” is set at 46%.

Different papers find different rates of repossession depending on the loan-to-value ratios (e.g. Pennington-Cross (2010), Phillips and Vanderhoff (2004), Ambrose and Capone (1996, 1998)), which can range from 30% and 50%. Since repossession rates at the upper end of this interval tend to be associated with very high LTVs, I choose one at the lower end of this range, around 38.5%.

The mortgage rate $r_m$ is set to 7.2%, which is the average fixed-rate on conventional contracts between 1995 and 2004, as measured using Federal Housing Finance Board data. Recall that $r_m = i + \varphi + \phi$, so that $\varphi + \phi = 3.2\%$. They are chosen in part to target the steady-state loan-to-income ratio at origination to be 2.72, which is consistent with pre-2003 data from the American Housing Survey.

The value for $\alpha_p$ is taken from the work of Head et al. (2014). The elasticity of the matching function with respect to the number of buyers, $\eta_h$ is calibrated to match the selling rate of new one-family homes, the data for which comes from the U.S. Census Bureau. The rate varies from 11 to 18 percent between 1985 and 2000. I target an average of 14.5%.

The parameters that remain to be calibrated are those that are unique to the present model. The value for $\upsilon$ is targeted using results from Flannery and Lin (2015), who find that a one percent increase in real estate prices brings about a 0.93 percent increase in small business lending. Because all firms are the same size in the model, I conservatively target a 0.7 percent increase in overall firm lending. Moreover, for simplicity, I assume that the change in real estate price is completely driven by a fall in $\chi$. The value for $j_E$ is set so that $E(\chi) = 0$ in steady state.

Some of the remaining parameters are directly chosen to ensure the model retains necessary features, rather than to match established targets. The value of the financial distress payment, $\lambda$, is chosen to be large enough so that an indebted owner cannot afford to make both the payment and pay mortgage and housing costs simultaneously, yet small enough that an employed worker has the possibility of doing so. The unemployment benefit $w_b$ is chosen to be small enough to make $\lambda$ difficult to pay, but large enough so that households are incentivized to look for jobs and thus match the unemployment rate. To set the value for $\eta_l$, I normalize the job-finding probability at the minimum wage to be 99.9%. I allow the output distribution to be uniformly distributed. I normalize the minimum realization of output, $y$ to 1.0609.

The remaining parameters are determined jointly. First, I target the share of labor income in GDP to be 0.70, which is consistent with data from the Bureau of Economic Analysis for the period prior to
2007. Using FRED data, the firm loan default rate, as measured by the charge-off and delinquency rate on U.S. commercial and industrial (C&I) loans, ranged from 2% to 6% during the 1990s. I target a value of 4%. The firm loan risk premium is also targeted. I measure this using the interest rate spread of C&I loans over the intended federal funds rate in the 1990s, which varied between 1.5% and 3%. I target 2%. Finally, I target the average post World War 2 unemployment rate, which is around 5%. Together, these targets are used to set values for \( \{ \bar{y}, j^c, \omega, \eta, \alpha^c \} \).

Finally, the income of a non-resident, \( y_L \) is equal to the average output multiplied by the share of labor income in output for the U.S. economy (0.70, as just described). The continuation value of leaving the city, \( L \), is equal to the present discounted lifetime value of utility of consuming the average income, net of rent.

### 2.6 Steady State

I now present the features of the steady state for the baseline economy. In the steady state, \( \chi \) remains constant over time. The definition of the steady state is the same definition of the equilibrium previously established, with the additional requirement that all functions and values listed in (2.4.18) to (2.4.22) are invariant over time.

All owners have strictly positive home equity. First, since there is a positive probability of housing default, and foreclosure costs are non-negative (i.e. \( \chi > 0 \)), the bank will never lend the full price of a house. Moreover, in steady state, nothing happens that would change the supply and demand for housing within the city, and thus the price distribution. Thus, an indebted owner could always sell their house and repay their debt, at even the minimum price in the market.\(^8\)

Resident owners who are not hit by the financial distress shock do not trade in the housing market, regardless of their amount of debt or employment status. Consequently they have no desire to default. For those in financial distress, housing market participation and default depend on employment status: those who are employed can afford the additional expense associated with distress, and choose not to sell or default. However, the unemployed always participate in the housing market; this is because they cannot afford to make both their mortgage payments and pay the costs associated with financial distress. If unsuccessful in selling their house, they must default. In what follows, I refer to an owner as a “foreseller” if she is both distressed and unemployed at the opening of the housing market.

The housing market behavior of relocated owners depends on the magnitude of their outstanding mortgage. All relocated owners participate in the housing market. However, those with a high level of

\(^8\)Although, due to search frictions, they cannot sell with certainty, the minimum price is chosen such that the probability of a successful sale is 1.
debt find it optimal to default if they fail to sell. Those with lower levels of debt, upon failing to sell, find it optimal to continue to make mortgage payments and to wait and try again in the next period. The results are similar to that in HSZ, with two exceptions. The first is that HSZ has no unemployment, so all owners in financial distress are forced to default upon failing to sell their houses. The second is that no relocated owners default.

Figure 1 shows the measure of owners and relocaters by mortgage tenure. The distribution of owners is entirely driven by the exogenous moving and distress shocks. That is, during each period of the mortgage, an indebted owner has some probability of being struck with either shock. As more time passes, there are more opportunities to be hit by the shocks, and so the measure of owners falls with tenure. The distribution of relocated owners by mortgage tenure is hump-shaped, since, for sufficiently low levels of debt, if they fail to sell their houses in one-period, they wait to try again in the next period.

Figure 2 illustrates how a foreseller’s trading decisions (i.e. selling price and matching probability) change with her debt position. Notably, this relationship is not monotonic over the length of a mortgage. Initially, those with high levels of debt (i.e. who are early on in their mortgage) choose to list their house at a higher price. As their level of debt falls, they reduce the price at which they sell. However, when they reach a sufficiently low level of debt, they increase the price at which they sell their house. This relationship can be explained as follows. A foreseller with high debt, all else equal, is more concerned with the gain of a house trade than with ensuring that a trade occurs. Suppose she fails in selling her house and so she defaults. She either gets nothing, or she gets the value of the house in the REO inventory, net of what she owes. Let us refer to this difference as the level of home equity conditional on foreclosure, or more briefly, “post-foreclosure equity”. For high levels of debt, post-foreclosure equity is negative. Unless she sells her house at a high price, there is not much difference between selling and defaulting. The only difference is in avoiding the foreclosure flag on her record. As the level of debt falls, so does the selling price post-foreclosure equity is non-negative. As the loan to value ratio continues to fall, post-foreclosure equity continues to rise. The gain from trade is more muted at this point. Although she continues to have access to the housing market in the second period if she sells, she also has less to lose is she fails to sell. Thus, this foreseller is willing to sacrifice a trade by selling at a higher price.

Let us compare the model’s mortgage default predictions with those studied by Gerardi et al. (2015), who use microlevel data from the Panel Study of Income Dynamics (PSID) to see how important job losses and adverse financial shocks are in predicting default. They also compare the default behavior of PSID households with the popular “double trigger” theory, which posits that both household level shocks and negative equity are present for most of the mortgage defaults that occur. Firstly, they find that, all else equal, the probability of default increases by around 5 percentage points if the head of
the household is unemployed. They also look at other “financial distress” factors, namely low liquid assets, high hospital and medical bills, recent divorces, and high unsecured debt. Accounting for these, as well as unemployment, can raise the probability of default by as much as 22 percentage points. This is consistent with the finding here that financial distress beyond job loss seems necessary to incentivize default.

What does the baseline model here have to say about the double trigger hypothesis, that is that both financial distress and negative home equity seem to be necessary for default? Note that in steady state, since all owners have positive home equity prior to foreclosure, direct comparison with this hypothesis is not possible. However, we can say something when we look at post-foreclosure equity. Recall that foresellers with negative post-foreclosure equity are more likely to sell their houses in high-price sub-markets, which reduces the probability of a sale, and increases the likelihood of default. In this sense, the model seems to support the double trigger hypothesis. What’s more, it goes beyond the presence of two triggers to examine the role of housing market frictions: it is not simply that the triggers must be present; how easily a foreseller can sell her house matters as well.

Gerardi et al. (2015) also find support for the double trigger hypothesis. However, they also find, somewhat surprisingly, that two thirds of households with negative home equity and a seeming inability to make mortgage payments, somehow still manage to do so, and avoid default. While a much richer model would be necessary to fully explain this, the default behavior of relocated owners may be a first step in matching this. First, consider the default choice of an indebted relocated owner. Note that, when debt is high, or when there are many periods remaining in the mortgage, she chooses to default upon failure to sell her house. However, with around 21 out of 30 payments remaining, she chooses to stay and continue to make payments even though her level of post-foreclosure equity is still negative. Although she is not subject to financial distress, like a resident owner, a moving shock can be seen as an ad-hoc way of capturing a different type of distress. For example, a recent divorce would be consistent with a moving shock, or perhaps needing to move due to a closing of a particular industry in the city. Or, one might need to move for medical treatment, if located in a small town. Moreover, in the way the model is calibrated, the lifetime value of leaving the city, \( L \), is chosen to match the value of being a resident perpetual renter. This value is significantly lower than being a resident owner, even with debt, so that a relocated owner is not as well off than if she had not had to move. Thus, the behavior of relocated owners is a first step to explaining the phenomenon observed in Gerardi et al. Clearly, being able to model financial distress in a more endogenous manner, and allow for greater interactions between income, savings, and home equity, as recommended by Gerardi et al., would be ideal. However, despite the many simplifications used in the model, it is still able to capture some of the empirical stylized facts...
observed in the PSID data.

Next, let us examine how a household’s labor market behavior is affected by their ownership status in the steady state. For indebted resident owners without distress, there is little variation in wage search behavior. This can be explained as follows. Firstly, without the expenses associated with financial distress, these owners continue to make payments on their mortgage and choose not to participate in the housing market. Thus, the wage has no impact on house selling or default decisions. Furthermore, a job lasts for only one period. Thus, the wage only impacts current consumption. Of course, the present discounted value of continuing to be an owner rises with the tenure of the mortgage, but only serves to shift the current value function of being an owner, rather than the shape, and thus has very little impact on the worker’s choice of wage.

However, this is not the case for indebted resident owners who find themselves in financial distress. Figure 3 shows how a distressed worker’s search decision (i.e. wage and job-finding probability) changes with her debt position. Importantly, the wage searched for falls with the worker’s debt position. The reasons for this are as follows: All indebted owners at the early stages of their mortgage are facing relatively low consumption and high housing costs for the foreseeable future. The utility of homeownership makes owning worthwhile; however, when these high-debt owners are hit with the distress shock, the resulting one-time fall in consumption decreases utility enough to merit searching for a higher wage. The fall in consumption offsets the value of remaining an owner and thus they search for a higher wage. Moreover, if they fail to find a job, they still have the option to sell their house when the housing market opens. However, when a resident owner is closer to full repayment of their mortgage, the present discounted value of remaining an owner is much higher, and the fall in the current-period utility associated with a one-time fall in consumption due to financial distress is not sufficient to merit searching for a higher wage and risking unemployment and thus selling the house. Consequently, they search for a low wage in order to increase their job-finding probability.

2.7 Rising Foreclosure Costs: Magnification Via Housing and Labor Market Interactions

2.7.1 Baseline model

In the baseline model, the cost of borrowing for a firm depends positively on $\chi$. The analysis below analyzes the impact of an increase in $\chi$. Recall that $\chi$ is the fraction of the sale price that the mortgage arm of the bank loses as foreclosure costs.
Consider the impact of $\chi$ on the change in mortgage lending standards. Figure 4 illustrates how the loan-to-income (LTI) ratio and the downpayment ratio vary with $\chi$. All else equal, a rise in $\chi$ increases the foreclosure costs to the bank, which leads them to issue smaller mortgages. Correspondingly, the downpayment ratio rises. For example, a 5% increase in $\chi$ lowers the LTI ratio by 12% and raises the downpayment ratio by 9%.

The fall in the LTI ratio and the corresponding rise in the downpayment ratio can be explained by the impact of $\chi$ on the mortgage default rate and on the unemployment rate. Let us start with the unemployment rate. A rise in $\chi$ increases $E$, so that borrowing a given amount is more difficult and costly for the firm. That is, since output is stochastic, and auditing is costly for the bank, the risk of lending to the firm rises. The interest rate that the firm must pay rises, which reduces the value of a vacancy to the firm. Job creation falls, and unemployment rises. This positive relationship between $\chi$ and the unemployment rate is illustrated in the right panel of Figure 5. For example, a 5% rise in $\chi$ increases the unemployment rate by 58%.

The rise in unemployment increases the rate of mortgage default. There are now fewer eligible buyers in the housing market, since employment is a condition for obtaining a mortgage. Moreover, the value of living in the city falls, which discourages the entry of new residents (and thus buyers). As a result, the overall demand for housing falls. Additionally, there are more sellers. Recall that indebted residents in distress must put their house on the market if they fail to find a job. When fewer distressed owners find jobs, more houses are on the market. Thus, there is an overall rise in housing supply. The fall in demand and rise in supply reduce liquidity in the housing market, which raises mortgage default, and thus tightens mortgage lending standards even further. Overall, a 5% rise in $\chi$ increases the rate of mortgage default by almost 20%.\footnote{In reality, home repossession went up by a factor of 5. However, it is still significant that a 5% rise in $\chi$ can still elicit a large increase in mortgage foreclosures.} The fall in housing market liquidity is also evident in the behavior of the price-to-income (PTI) ratio, which is illustrated in the left panel of Figure 6. The overall effect of $\chi$ on mortgage lending standards can be summarized by the salvage rate of foreclosed houses for banks. This is illustrated in the right panel of Figure 6. There are both direct and indirect impacts of $\chi$ on the salvage rate. Higher foreclosure costs directly lower the value of a house in the REO inventory of the bank. However, the further general equilibrium effects on mortgage lending standards that stem from a rise in the rate of mortgage default indirectly lower the value of the house to the bank and thus the salvage rate.
2.7.2 The dependence of the firm’s cost of credit on $\chi$

The results described above stem critically from the impact of $\chi$ on the rate of unemployment. It is the rise in unemployment that incentivizes mortgage default, and magnifies the effect of a rise in foreclosure costs on the tightening of mortgage lending standards. Unemployment rises because of a rise in borrowing costs for firms. Without this fall, how would the results differ? In order to answer this question, I perform the same exercise as above, with the exception that a firm’s borrowing costs are independent of $\chi$. The results of this “fixed E, lmf” model are illustrated in Figures 7 to 9.

The analysis is informative. The fall in the LTI ratio and the rise in the downpayment ratio, shown in Figure 9, are muted relative to the baseline model. That is, in the baseline model, the LTI ratio fell by 12% with a 5% rise in $\chi$; in the “fixed E, lmf” case, the fall is around 2%. Just as before, the reason for this can be explained by the response of the rate of mortgage default to a rise in foreclosure costs. Recall that, in the baseline model, default rose with a rise in foreclosure costs primarily because of the rise in unemployment. However, as shown in Figure 8, this no longer occurs; rather unemployment falls, although the magnitude is small, around 0.02%.

The counter-factual fall in the unemployment rate can be explained as follows. A rise in foreclosure costs causes banks to cut back on the size of the mortgages offered at origination. A smaller loan reduces the mortgage payment each period. When a resident owner is faced with financial distress, she can afford to search for a lower wage, which increases her probability of employment. Recall that foresellers and highly indebted relocated owners are the only defaulters in steady state. Distressed owners are now less likely to be unemployed. For relocated owners, the lower periodic payments reduces the carrying cost of a mortgage. Thus, both groups are less likely to default, and the overall rate of mortgage default falls, as shown in Figure 8. A 5% rise in $\chi$ in this environment results in a 22% fall in the rate of mortgage default.

The left panel of Figure 9 illustrates the response of the PTI ratio. As with the LTI and downpayment ratios, the fall is muted relative to the baseline model. Firstly, housing demand falls less than in the baseline model. Although there are fewer entrants into the city, those that do are still eligible to buy a house, since finding a job is easier. Furthermore, housing supply does not rise, since distressed owners are less likely to be unemployed relative to the baseline model. Overall, a 5% rise in $\chi$ results in less than a 0.01% fall in the PTI ratio.
2.7.3 The role of labor market frictions

Both the analysis of the baseline model and the analysis omitting the dependency of a firm’s borrowing costs on $\chi$ highlight the role that employment plays in amplifying the impact of a rise in foreclosure costs. It is also worthwhile then to see what effect labor search frictions have on the magnitude of this amplification. In order to do so, I replace the DMP-style labor market with a competitive market that clears every period. In this case, labor supply is perfectly inelastic and there is no unemployment. Since the production technology is linear, this implies that the wage adjusts to equal the expected output for the firm, net of the cost of borrowing. The results of this “variable E, no lmf” model are also shown in Figures 7 to 9.

The behavior of most of the moments illustrated in these figures is qualitatively similar to that in the baseline model; however the magnitudes are attenuated. The exception both qualitatively and quantitatively is the rate of mortgage default, which falls as $\chi$ rises. The rise in foreclosure costs increases borrowing costs for firms. This reduces the expected net revenue of the firm, and they lower the wage as a result. That is, all adjustment occurs through wages. Since there is no effect on unemployment, there is no rise in the rate of mortgage default, which falls for the same reasons described as in the “fixed E, lmf” case. Thus, the presence of a frictional labor market, that is, the presence of unemployment, is responsible for amplifying the effect a rise in foreclosure costs on housing market outcomes.

In summary, an exogenous rise in foreclosure costs is magnified in the baseline model due to the feedback effect of the impact of $E$ on unemployment. Tighter borrowing standards for firms reduce the value of a vacancy and increase unemployment. This increases the rate of mortgage default, which is absent otherwise. The rise in default reduces housing market liquidity. This feedback between the housing and labor markets is further exacerbated by the behavior of wages, as shown in Figure 10. Let us start by considering the case in which there are no labor market frictions, and then analyze the impact of a rise in $\chi$ when a firm’s cost of borrowing is affected, compared to when it is not. Due to tighter borrowing standards, wages fall considerably more in the former case. However, if we do the same analysis, this time when labor market frictions are present, we see that the wage falls by less. This suggests that labor market frictions generate some wage rigidity. Firms offer lower wages, but households are less willing to accept such low wages in the full model.

2.7.4 Changes in $\chi$ vs. changes in a firm’s marginal revenue

The results above show how a rise in foreclosure costs, represented by a rise in $\chi$, is magnified via interactions between the cost of borrowing for firms and labor market frictions. Would we see similar
effects from a more general change in the marginal revenue of the firm? Such a change may be thought of as an exogenous fall in aggregate demand, or as a productivity shock. In the model environment, a change in marginal revenue is a change in average output. Suppose the change is symmetric, i.e. both the minimum and maximum possible output drop by the same amount. In order to make the results comparable to that of a change in $\chi$, I look at all changes relative to the calibrated steady state. For example, for an $X$ percent rise in the mortgage default rate, relative to the calibrated steady state, by what percent did $\chi$ rise, or $\bar{y}$ fall? How did the other relevant moments change? In what follows, I compare the impacts of a change in $\bar{y}$ only to the baseline version of the model.

Table 3 summarizes the differences in the behavior of selected moments in the cases of varying $\chi$ and $\bar{y}$, holding constant the percent change in mortgage default relative to the calibrated steady state. Columns 2 and 3 list the corresponding percent changes in $\chi$ and $\bar{y}$, respectively. The remaining columns list the percent change in the specified moment, relative to the calibrated steady state.

The following features are noteworthy. When $\chi$ ($\bar{y}$) rises (falls) just by a little bit, so that mortgage default rises by 2.5%, then there are few differences in the corresponding changes in the selected moments. That is, a small fall in average output works in a very similar way to a small rise in $\chi$. This is likely because both effects bring about similar falls in the value of a vacancy. The only exception is the change in the salvage rate, which falls under a rise in $\chi$, but rises slightly when there is a fall in $\bar{y}$. Since $\chi$ directly impacts the value of a foreclosed house, the former effect is unsurprising. The small rise in the mortgage default rate, is not enough to affect housing market liquidity significantly in either case. In the case of a fall in $\bar{y}$, even though default rises, less is loaned out, and thus easier to recover.

However, suppose default rises by a bit more. Then, the mechanisms which initiate the rise in default vary depending on if $\chi$ has risen or $\bar{y}$ has fallen. Specifically, given a high mortgage default rate, unemployment plays a much larger role in the case of an increase in $\chi$. The mechanism is as described previously, and works through the interaction of a fall in $E$, combined with labor market frictions. In the case of a fall in $\bar{y}$, while unemployment still rises significantly, the magnitude is smaller. The reason for this stems from changes in housing market liquidity. When $\chi$ does not rise, then there is less impact on the bank’s value of a foreclosed house. In the baseline model, the rise in $\chi$ instigates the fall in the size of a mortgage; when $\bar{y}$ falls, it is the rise in unemployment that instigates the fall in the size of a mortgage. Higher unemployment increases the mortgage default rate, and reduces the measure of eligible buyers in the housing market. However, because $\chi$ does not rise as well, there is a limit as to how much the bank limits the size of the mortgage. Thus, more is lent out relative to the case of a rise in $\chi$, so that the downpayment ratio and housing prices falls by less as well. Even though default is the same in both scenarios, the fact that the bank has a higher value a foreclosed house in the case of a fall
in \( \bar{y} \) makes the higher default rate more tolerable without having further feedback in the housing and labor markets.

In summary, although a fall in marginal revenue and a rise in \( \chi \) look qualitatively similar, comparing the two cases illustrates the role that an illiquid housing market can play in the magnitude of the changes both in the housing market and in the labor market. The magnitude of the impact of a fall in firm marginal revenue on the housing market is limited. Although mortgage lending standards tighten, the average losses to the bank remain largely unaffected. Furthermore, although the value of a vacancy to the firm falls, and, for a given amount borrowed, they are more likely to default, the risk of doing so is less than in the case in which \( \chi \) affects firm borrowing costs. This limits the impact on unemployment. However, in the baseline model, \( \chi \) further impacts lending standards and also the value of \( E \), which magnify the impacts on housing market liquidity and unemployment.

### 2.8 Conclusion

I have developed a theoretical model that builds on the housing market of HSZ and incorporates both firm borrowing frictions and a frictional labor market. The purpose of this has been to analyze how unemployment affects housing market liquidity in the presence of rising foreclosure costs. The reason for this is twofold. First, the magnitude of both the rise in unemployment and in the amount of default during the Great Recession suggest a strong link between unemployment and the housing market. Second, the model can shed light on contradictory empirical findings (Gerardi et al. 2015) on the extent to which default is strategic (e.g. due to the fall in value of a house) or necessary, due to an inability to repay.

In the model, default is never strategic — a fall in the value of a house is never large enough to outweigh the utility of home ownership. Default occurs only for “foresellers,” who default when faced with both financial distress and unemployment, and for highly indebted re-locaters, who no longer value housing. The measure of default rises when banks experience larger foreclosure costs. This occurs because a fall in the bank’s ability to foreclose also affects the cost of borrowing for a firm. The value of a job falls for the firm, and they create less vacancies. It now becomes harder for a household — foresellers in particular — to find a job. Faced with financial distress and no way to cope with the additional expenses, they are forced to default. The rise in default is further exacerbated by the increased difficulty in selling the house. Firstly, banks cut back on the size of mortgages due to rising foreclosure costs and increased risk of default. This makes it more difficult for potential home buyers to afford a house, and so they search longer for a less inexpensive house. Furthermore, since more residents are forced to sell their house, due to unemployment, the measure of houses for sale rises.
A key feature of the model is the impact of rising foreclosure costs on the cost of borrowing for a firm. This feature is realistic: the ability of a bank to foreclose on a house and on the funds available to lend to firms should be positively correlated. Furthermore, firm lending during the Great Recession fell to its lowest post-World War 2 level, and although difficult to disentangle supply and demand factors, both seem to have played a role. However, if this feature is absent, then the model predicts counterfactual falls in both housing default and unemployment. The reason for this is that, when banks decide to reduce the size of mortgages in response to rising foreclosure costs, households have less debt. This reduces the periodic mortgage payment they make, and thus reduces their need to search for high-wage jobs. With no contemporaneous fall in the ability to find a job, unemployment actually falls, and so does default.

A competing explanation for rising unemployment during the Great Recession is a fall in aggregate demand. I examine the more general case of falling marginal revenue for a firm, which would occur following an aggregate demand shock. I find that a relatively larger fall in marginal revenue is needed to match given changes in default and unemployment. Moreover, there is no fall in the salvage rate of foreclosed houses to banks — a feature that was noticeably apparent during the Great Recession.

The exogenous nature of a shock to $\chi$ on the cost of borrowing for a firm helps to see how unemployment can exacerbate housing market stress. However, endogenizing the relationship between general banking stress and firms’ ability to borrow would allow for a deeper analysis, especially to explain the persistence of unemployment over the Great Recession. I leave this task to future work.
Appendix

2.A Household Value Functions

Subperiod 1

The value functions for residents at the opening of the labor market are presented below.

A perpetual renter chooses a submarket \((\theta, w)\) to solve

\[
U^p_t = \max_{\theta, w} \gamma(\theta) W^p_t(w) + (1 - \gamma(\theta)) W^p_t(w^u) \text{ subject to } \theta = \Gamma(w).
\] (2.A.1)

Consider a buyer with no foreclosure flag. She is only eligible to buy a house if employed with a wage, \(w\), high enough to afford the downpayment. If she finds a match, then she searches to buy a house in the next subperiod, the value of which is \(V^b_t(w)\). Otherwise, she is employed, and continues to rent, the value of which is \(W^b_t(w^u)\) in subperiod 3:

\[
U^b_t = \max_{\theta, w} \gamma(\theta) V^b_t(w) + (1 - \gamma(\theta)) W^b_t(w^u) \text{ subject to } \theta = \Gamma(w).
\] (2.A.2)

Consider a buyer with a foreclosure flag. She cannot participate in the housing market in the next subperiod. If she succeeds in finding a job, she gets value \(W^f_t(s, a, w)\) in subperiod 3, where \(s\) denotes her financial distress status, and \(a\) her intra-temporal assets. Otherwise, she is unemployed, does not participate in the housing market, and gets value \(W^f_t(s, a, w^u)\):

\[
U^f_t(s) = \max_{\theta, w} \gamma(\theta) W^f_t(s, 0, w) + (1 - \gamma(\theta)) W^f_t(s, 0, w^u) \text{ subject to } \theta = \Gamma(w).
\] (2.A.3)

Let us now consider resident owners. Recall that \((m_0, n) = (0, 0)\) for owners without debt, and \(s\) indicates financial distress status, with
For any resident owner, the value function takes the form

$$U_t(s, m_0, n) = \max_{\theta, w} \gamma(\theta) V_t(s, m_0, n, w) + (1 - \gamma(\theta)) V_t(s, m_0, n, w^n) \text{ subject to } \theta = \Gamma(w). \quad (2.A.4)$$

If a resident owner with housing status \((s, m_0, n)\) finds a job, she enters the housing market with value \(V_t(s, m_0, n, w)\). Otherwise, she remains unemployed for the duration of the period, and enters the housing market with value \(V_t(s, m_0, n, w^n)\).

### Subperiod 3

Below, I describe the values at the beginning of subperiod 3 of period \(t\) for the households.

The value to a perpetual renter is

$$W^p_t(w) = u(w - RR_t) + \beta \left[ \frac{\pi_p L + (1 - \pi_p) E U^p_{t+1}}{\pi_h} \right] \quad (2.A.5)$$

Consumption in the current period is income net of rent. With probability \(\pi_p\), the perpetual renter is hit by the moving shock, leaves the city immediately, and receives the continuation value \(L\). Otherwise, she moves onto the next period as a renter.

A buyer with the foreclosure flag on her credit record has no access to credit or the housing market, and will remain a renter until either she relocates, or the foreclosure flag is lifted from her record. She enters subperiod 3 with intra-period asset holdings \(a\), which is positive when the outstanding debt on which she defaulted is less than the value of the foreclosed house to the mortgage company. We have

$$W^f_t(a, s, w) = u(w + a - RR_t - s\lambda) + \beta \left[ \frac{\pi_h L + (1 - \pi_h) \left( \pi_f E_t U^f_{t+1} + (1 - \pi_f) E_t U^b_{t+1} \right)}{1 - \pi_f} \right] \quad (2.A.6)$$

Here, \(s = 1\) if agent is financially distressed in the current period, and zero otherwise. Conditional on staying in the city, the foreclosure flag remains on the household’s record with probability \(\pi_f\), in which case she moves into the following period with value \(E_t U^f_{t+1}\). With probability \(1 - \pi_f\), the flag is lifted, and she will enter the labor market of the following period as a buyer with value \(E_t U^b_{t+1}\).

A buyer without the foreclosure flag is either a resident owner who successfully sold his house, or a
buyer who did not find a house earlier in the period. The former enters subperiod 3 with strictly positive assets $a$, from his successful sale, net of any outstanding mortgage debt. In the latter case, assets are zero. She will move on as a buyer in the next period with value $U_{t+1}^b$ if she remains in the city. The value of such a buyer is

$$W_t^b(a, w) = u(w + a - RR_t) + \beta \left( \pi_h L + (1 - \pi_h)E_t U_{t+1}^b \right)$$ (2.A.7)

In subperiod 3 of period $t$, a resident owner with a mortgage has the principle balance $d(m_0, n)$, where $n = 0, 1, \ldots, T - 1$. The owner makes the periodic mortgage payment, pays the maintenance cost, and financial distress costs (if applicable), and consumes the remaining income. Let $W_t^s(m_0, n, w)$ denote the value of such an owner. It follows that for $n = 0, 1, \ldots, T - 2$,

$$W_t^s(s, m_0, n, w) = u(w - x(m_0) - m - s\lambda) + z^H + \beta \left[ \pi_h \beta E_t V_{t+1}^R(m_0, n + 1) + (1 - \pi_h)E_t U_{t+1}^s(0, m_0, n + 1) \right]$$ (2.A.8)

If the owner is hit by the moving shock, then she will leave the city immediately and continue on with value $V_{t+1}^R(m_0, n + 1)$. She is still responsible for any mortgage debt. Conditional on not moving in the next period, the owner gets hit by an adverse shock with probability $\pi_d$. In this case, she will continue on into next period’s labor market as a distressed resident owner, with value $E_t U_{t+1}^s(1, m_0, n + 1)$. Otherwise, she enters the labor market in the next period as a non-distressed owner with value $E_t U_{t+1}^s(0, m_0, n + 1)$.

For $n = T - 1$, we have

$$W_t^s(s, m_0, T - 1, w) = u(w - x(m_0) - m - s\lambda) + z^H + \beta \left[ \pi_h \beta E_t V_{t+1}^R(0, 0) + (1 - \pi_h)E_t U_{t+1}^s(0, 0) \right]$$ (2.A.9)

In this case, the current period is the last time that the owner makes a payment, and is debt-free for the remainder of her time in the city. Thus, she will continue on with value $E_t V_{t+1}^R(0, 0)$ if hit by the moving shock, and with value $E_t U_{t+1}^s(0, 0)$ otherwise.

A new owner who has just purchased a house in the preceding subperiod makes a downpayment on the house (the difference between the purchase price and the total debt $m_0$). The periodic mortgage payment starts in the next period. Thus, the amount of debt going into the next period is $d(m_0, 0)$. Let $W_t^o(p, m_0, w)$ denote the value of a new owner:
The zero profit condition of the mortgage arm of the bank is given by 2.4.17. Recall that 

\[ W_t^0(p, m_0, w) = u(w - (p_t - m_0) - m) + z^H + \beta \left( \pi_h \mathbb{E}_t V_{t+1}^R(m_0, 0) + (1 - \pi_h) \left( \pi_d \mathbb{E}_t U_{t+1}^0(1, m_0, 0) + (1 - \pi_d) \mathbb{E}_t U_{t+1}^0(0, m_0, 0) \right) \right) \]  

(2.A.10)

Finally, owners without mortgage debt do not suffer from financial distress shocks. They stay in the city until hit by a moving shock. The value of this owner is

\[ W_t^s(0, 0, 0, w) = u(w - m) + z^H + \beta \left( \pi_h \mathbb{E}_t V_{t+1}^R + (1 - \pi_h) \mathbb{E}_t U_{t+1}^s(0, 0, 0) \right) \]  

(2.A.11)

### 2.B Derivation of \( P_t^\delta(m_0, n) \)

The zero profit condition of the mortgage arm of the bank is given by 2.4.17. Recall that \( \delta = (m_0; r_m) \) is the mortgage held by a resident owner and \( P_t^\delta(m_0, n) \) the mortgage’s present value at the end of subperiod 2 of period \( t \) after \( n \geq 0 \) payments have been made up to period \( t \). Similarly, \( P_t^{\delta R}(m_0, n) \) is the present value of the mortgage held by a relocated owner. In this case, \( n \geq 1 \) because one repayment has been made by the beginning of the first subperiod 2 after relocation. Because the expression is quite cumbersome, it is helpful to break it down into parts:

- Suppose that, next period, the resident owner of type \( j \in \{s, sd\} \) experiences no moving shock. The probability that she sells her house is \( \tau(\theta_{jt}^*(m_0, n + 1, w^j)) \), in which case the bank gets the minimum of the selling price and the outstanding debt: \( \min \{ (\hat{p}_t^*(w), d(m_0, n + 1)) \} \). If the resident owner does not sell the house, then she chooses whether or not to default. If she defaults, the bank receives the minimum of the outstanding debt and the value of adding the house to its REO inventory in the next period, \( \mathbb{E}_t V_{t+2}^{REO} \). Otherwise, the resident owner makes periodic payment \( x(m_0) \), and the value of the mortgage to the bank next period is \( P_t^\delta(m_0, n + 1) \). We weight the value to the mortgage company in this scenario by the employment status of the resident owner, and discount to the current period:

\[
P_{jt}(n) = \frac{1}{1 + r + \phi} \left[ \gamma(\theta_{jt}^*(m_0, n)) \left[ \tau(\theta_{jt}^*(m_0, n, w^j)) \right] \min \left\{ p_t^*(w^j), d(m_0, n) \right\} + (1 - \tau(\theta_{jt}^*(m_0, n, w^j))) \times \right.

\left. \{ D_t^*(m_0, n, w^j) \} \min \left\{ \mathbb{E}_t V_{t+1}^{REO}, d(m_0, n) \right\} + (1 - D_t^*(m_0, n, w^j)) (\mathbb{E}_t P_{t+1}^\delta(m_0, n + 1) + x(m_0)) \right] 

+ (1 - \gamma(\theta_{jt}^*(m_0, n))) \left[ \tau(\theta_{ht}^*(m_0, n, w^u)) \right] \min \left\{ p_t^*(w^u), d(m_0, n) \right\} + (1 - \tau(\theta_{ht}^*(m_0, n, w^u))) \times \right.

\left. \{ D_t^*(m_0, n, w^u) \} \min \left\{ \mathbb{E}_t V_{t+1}^{REO}, d(m_0, n) \right\} + (1 - D_t^*(m_0, n, w^u)) (\mathbb{E}_t P_{t+1}^\delta(m_0, n + 1) + x(m_0)) \right] 
\]
Suppose the resident owner of type \( j \in \{ s, sd \} \) receives a moving shock. 

\[
P_{h,t}(n) = \frac{1}{1 + i + \phi} \left[ \tau(\theta_{h,t}^{R}(m_0, n)) \min \{ p_{t}^{R}(d(m_0, n)) \} + (1 - \tau(\theta_{h,t}^{R}(m_0, n))) \times \right.
\]
\[
\left. \left[ D_{t}^{R}(m_0, n) \min \{ \mathbb{E}_{t}V_{t+1}^{REO}(d(m_0, n)) \} + (1 - D_{t}^{R}(m_0, n))(x(m_0) + \mathbb{E}_{t}P_{t+1}(m_0, n+1)) \right] \right]
\]

It follows then, for all \( n \in \{0, 1, \ldots, T - 1\} \):

\[
P_{t}^{S}(m_0, n) = \mathbb{E}_{t} \left[ (1 - \pi_{h}) \left[ (1 - \pi_{d}) P_{s,t+1}(n+1) + \pi_{d}P_{sd,t+1}(n+1) \right] + \pi_{h}P_{h,t+1}(n+1) \right] \quad (2.B.1)
\]

And, for all \( n - 1 \in \{1, \ldots, T - 1\} \):

\[
P_{t}^{SR}(m_0, n) = \mathbb{E}_{t} \left[ \tau(\theta_{t+1}^{R})(p_{t+1}(m_0, n+1), d(m_0, n+1)) \right. + \left. (1 - \tau(\theta_{t+1}^{R})) \left[ D_{t+1}^{R}(m_0, n+1) \min \{ \beta_{V_{t+2}}^{REO}(d(m_0, n+1)) \} \right. \right. \]
\[
\left. \left. + (1 - D_{t+1}^{R}(m_0, n+1))(P_{t+1}^{SR}(m_0, n+1) + x(m_0)) \right] \right] \quad (2.B.2)
\]

### 2.C Laws of Motion

I now proceed to describe the evolution of the stock of households, all measured at the opening of the housing market in period \( t \). The stock variables are normalized by the total population \( Q_{t} \). In per capita values, the stock variables are the (relative) measures of

- perpetual renters, \( P_{t} \),
- buyers without the foreclosure flag, \( B_{t} \),
- buyers with the foreclosure flag, \( B_{t}^{F} \),
- resident owners, \( S_{t}(m_0, n) \),
- relocated owners, \( S_{t}^{R}(m_0, n) \),
- the construction firm’s inventory, \( S_{t}^{F} \), and
- the foreclosed housing inventory, \( S_{t}^{REO} \).
Note that since jobs last for only one period, there is no evolution of the stock of employed/unemployed households.

Period-\(t\) perpetual renters consist of perpetual renters from period \(t - 1\) who survive the moving shock, and of newly-entering perpetual renters. The measure \(P_t\) thus evolves according to

\[
(1 + \mu)P_t = (1 - \pi_f)P_{t-1} + (1 - \psi)G(\epsilon_t^c)\mu
\]  
(2.C.1)

Potential buyers with the foreclosure flag at the beginning of period \(t\) consist of the following households who were not hit by the moving shock at the end of period \(t - 1\): (i) buyers whose foreclosure flags are still on record in period \(t\), and (ii) resident owners who defaulted in period \(t - 1\). Resident owners are summed over the financial distress status \(s \in \{0, 1\}\).\(^{10}\) Moreover, within each category, we have both employed and unemployed households, and so I sum over the wage.\(^{11}\) Thus we have:

\[
(1 + \mu)B^F_t = (1 - \pi_h)\left\{\pi_f B^F_{t-1} + \sum_{s \in \{0, 1\}} \sum_{n=0}^{T-1} \left[p(s) \left(1 - \tau(\theta^s_{c, t-1}(s, m_{0, t-1-n}, n), w_t)) \times D^s_{t-1}(s, m_{0, t-1-n}, n, w_t)\right)\right] \right\}. \]  
(2.C.2)

Here, \(S_{t-1}(s, m_{0, t-1-n}, n, w_t)\) is the measure of indebted owners who have made \(n\) periodic payments by the beginning of \(t - 1\) on a mortgage with volume \(m_{0, t-1-n}\) at origination, with income \(w_t\) at the closing of the labor market in period \(t\).

Consider now the measure of buyers without the foreclosure flag. It is made up of the employed fraction of following groups: (i) newly entering buyers; (ii) previously flagged buyers who have their flag lifted; (iii) non-relocating buyers from the previous period who participated in the housing market in period \(t - 1\) and did not get to trade; and (iv) successful sellers in the previous period who remain residents. We can combine the latter two groups into one; period \(t - 1\) resident sellers are buyers in period \(t\), and so the net change in buyers between periods is represented by period \(t - 1\) buyers who successfully traded with non-resident owners:

\[
(1 + \mu)B_t = \gamma(\theta^b_{c, t}) \left[\psi G(\epsilon_t^c)\mu + (1 - \pi_f)B^F_{t-1} + (1 - \pi_h)\left\{B_{t-1} - \tau(\theta^R_{h, t-1}(s_{t-1} - \tau(\theta^R_{REO} S^R_{t-1} - \sum_{n=0}^{T-1} \tau(\theta^R_{h, t-1}(m_{0, t-1-n}, n)) S^R_{t-1}(m_{0, t-1-n}, n)) \right\} \right]\right] \]  
(2.C.3)

\(^{10}\)Note that owners without mortgage debt are not subject to the default shock.

\(^{11}\)This is equivalent to employment status, and takes on only two values for each type of household: the wage and the unemployment benefit.
Let \( S_t(m_{0,t}, n) \) be the measure of indebted owners who have made \( n \) periodic payments by the beginning of period \( t \) on a mortgage with volume \( m_{0,t-n} \) at origination. For \( n > 0 \), this measure evolves according to:

\[
(1 + \mu)S_t(m_0, n) = (1 - \pi_h) \left[ \sum_{s \in \{0, 1\}} \sum_wp(s)(1 - \tau(\theta_{h,t-1}^s(s, m_{0,t-n}, n-1, w))) \times (1 - D^s_{t-1}(s, m_{0,t-n,n-1}, w)) S_{t-1}(s, m_{0,t-n}, n-1, w) \right]. \tag{2.C.4}
\]

That is, the current period owners with an ongoing mortgage are those indebted owners from the previous period who were not hit by the moving shock, and did not sell their house, either because they did not want to or could not, and did not default. These owners are summed over employment and financial distress status. For \( n = 0 \), \( S_t(m_0, 0) \) represents the measure of resident owners who just purchased a house in the last period (and were not hit by the moving shock). Note that the measure of new owners can be accounted by the measure of successful sellers in the previous period. Thus, we have

\[
(1 + \mu)S_t(m_0, 0) = (1 - \pi_h) \left\{ \sum_{s \in \{0, 1\}} \sum_{w=0}^{T-1} \sum_{n=0}^{T-1} p(s)\tau(\theta_{h,t-1}^s(s, m_{0,t-1-n}, n, w)) S_{t-1}(s, m_{0,t-1-n}, n, w) + \tau^R_{h,t-1}(m_{0,t-1-n}) S_{t-1}^R(m_{0,t-1-n}) + \sum_w \tau(\theta_{h,t-1}^{sg})(0, 0, 0, w) + \tau(\theta_{h,t-1}^c) S_{t-1}^C + \tau(\theta_{h,t-1}^{REO}) S_{t-1}^{REO} \right\}. \tag{2.C.5}
\]

Finally, the measure of resident owners without a mortgage, \( S_t(0, 0) \), is given by

\[
(1 + \mu)S_t(0, 0) = (1 - \pi_h) \left\{ \sum_{s \in \{0, 1\}} \sum_w \left\{ p(s)(1 - \tau(\theta_{h,t-1}^s(s, m_{0,t-T}, T-1, w))) \times (1 - D^s_{t-1}(s, m_{0,t-T}, T-1, w)) S_{t-1}(s, m_{0,t-T}, T-1, w) \right. \right. \\
\left. + (1 - \tau(\theta_{h,t-1}^{sg}(w))) S_{t-1}(0, 0, 0, w) \right\} \right\}. \tag{2.C.6}
\]

Similarly, \( S^R_t(m_0, n) \) is the measure of relocated owners who have made \( n \) periodic payments at the beginning of period \( t \). Again, the loan volume at origination is \( m_{0,t-n} \). Moreover, \( S^R_{t-1}(0, 0) \) represents the measure of debt-free relocated owners. It follows that

\[
(1 + \mu)S^R_t(m_0, n) = (1 - \tau(\theta_{h,t-1}^R(m_{0,t-n}, n-1)))(1 - D^R_{t-1}(m_{0,t-n}, n-1)) S^R_{t-1}(m_0, n-1) + \pi_h(1 - \pi_d) \times
\]
The stock of houses in the REO inventory at the beginning of period 
that are made available in period \( t \). This stock includes the inventory that did not get sold in the previous period, and the newly built houses

\[
\sum_w (1 - \tau^s(\theta^s_{h,t-1}(0, m_0, t-n, n-1, w))) (1 - D^s_{t-1}(0, m_0, t-n, n-1, w)) S_{t-1}(0, m_0, n-1, w).
\]

\[ (1 + \mu)S^R_t(m_0, 0) = \pi_h \left\{ \sum_{s \in \{0, 1\}} \sum_w \sum_{n=0}^{T-1} p(s) \tau(\theta^s_{h,t-1}(s, m_0, t-1-n, n, w)) S_{t-1}(s, m_0, t-1-n, n, w) + \right. \\
\left. \sum_{n=0}^{T-1} \tau(\theta^R_{h,t-1}(m_0, t-1-n, n)) S^R_{t-1}(m_0, t-1-n, n) + \sum_w \tau(\theta^{sg}_{h,t-1}) S_{t-1}(0, 0, 0, w) + \tau(\theta^g_{h,t-1}) S^R_{t-1}(0, 0) + \tau(\theta^{REO}_{h,t-1}) S^{REO}_{t-1} \right\}. 
\]

\[ (1 + \mu)S^R_t(0, 0) = \pi_h \sum_w \left\{ (1 - \pi_d)(1 - \tau(\theta^s_{h,t-1}(0, m_0, t-T, T-1, w))) \\
\times (1 - D^s_{t-1}(0, m_0, t-T, T-1, w)) S_{t-1}(0, m_0, t-T, T-1, w) + (1 - \tau(\theta^{sg}_{h,t-1}(w))) S_{t-1}(0, 0, 0, w) + \right. \\
\left. (1 - \tau(\theta^R_{h,t-1}(m_0, t-T, T-1))) (1 - D^R_{t-1}(m_0, t-T, T-1)) S^R_{t-1}(m_0, t-T, T-1) + \right. \\
\left. (1 - \tau(\theta^{REO}_{h,t-1})) S^{REO}_{t-1}(0, 0) \right\}. 
\]

Let \( S^c_t \) be the stock of houses in the representative construction firm’s inventory at the start of period \( t \). This stock includes the inventory that did not get sold in the previous period, and the newly built houses that are made available in period \( t \), the latter of which is measured by \( N_t \). Thus \( S^c_t \) evolves according to

\[ (1 + \mu)S^c_t = (1 - \tau(\theta^c_{h,t-1})) S^c_{t-1} + N_t. \]

The stock of houses in the REO inventory at the beginning of period \( t \), \( S^{REO}_t \), evolves according to

\[ (1 + \mu)S^{REO}_t = (1 - \tau(\theta^{REO}_{h,t-1})) S^{REO}_{t-1} + \]

\[ \sum_{s \in \{0, 1\}} \sum_w \sum_{n=0}^{T-1} p(s) (1 - \tau(\theta^s_{h,t-1}(s, m_0, t-n-1, n, w))) D^s_{t-1}(s, m_0, t-n-1, n, w) S_{t-1}(s, m_0, t-n-1, n, w) + \]

\[ \sum_{n=1}^{T-1} (1 - \tau(\theta^R_{h,t-1}(m_0, t-n-1, n))) D^R_{t-1}(m_0, t-n-1, n) S^R_{t-1}(m_0, t-n-1, n) \]
Since any depreciation is offset by maintenance, the total housing stock in the city evolves as

\[ H_{t+1} = H_t + N_t. \]  

(2.C.12)

Finally, for accounting purposes, the total measure of buyers searching to trade in the housing market, \( B_t^{\text{sum}} \), can be derived as

\[
B_t^{\text{sum}} = \sum_{s \in \{0, 1\}} \sum_{w} \sum_{n=0}^{T-1} \theta_{h,t-1}^{s}(s,m_{0,t-n},n,w)S_t(s,m_{0,t-n},n,w) + \sum_{n=0}^{T-1} \theta_{h,t-1}^{R}(m_{0,t-n},n)S_t^{R}(m_{0,t-n},n) \\
+ \sum_{w} \theta_{h,t-1}^{s}(0,0,0,w)S_t(0,0,0,w) + \theta_{h,t-1}^{c}(0,0) + \theta_{h,t-1}^{REO}S_t^{REO}. 
\]  

(2.C.13)

In particular, the measure of buyers in an active submarket is the product of the corresponding market tightness and the number of sellers in this specific submarket.
## 2.D Tables

Table 1: Parameters in common with HSZ

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
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</thead>
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<tr>
<td>$\beta$</td>
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<td>Annual interest rate</td>
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<tr>
<td>$\pi_p$</td>
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<td>Annual mobility of renters</td>
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<tr>
<td>$\pi_h$</td>
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<td>Annual mobility of owners</td>
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<td>$\xi$</td>
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<td>Median price elasticity of demand</td>
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<td>$i$</td>
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<td>International bond annual yield</td>
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<td>$T$</td>
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<td>Fixed-rate mortgage maturity (years)</td>
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<tr>
<td>$\mu$</td>
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<td>Annual population growth rate</td>
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<td>$\pi_f$</td>
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<tr>
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<td>Average land-price-to-income ratio</td>
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<td>Residential housing gross depreciation rate</td>
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<tr>
<td>$\zeta$</td>
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<td>Median price elasticity of new construction</td>
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<td>Average rent-to-price ratio</td>
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Parameters determined jointly

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<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\chi}$</td>
<td>0.52805</td>
<td>Loss severity rate</td>
<td>46%</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.0220</td>
<td>Average downpayment ratio</td>
<td>20%</td>
</tr>
<tr>
<td>$\varrho$</td>
<td>0.010</td>
<td>Average annual FRM-yield</td>
<td>7.20%</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.98</td>
<td>Fraction of households that rent</td>
<td>33.3%</td>
</tr>
<tr>
<td>$\pi_d$</td>
<td>0.0680</td>
<td>Annual foreclosure rate</td>
<td>1.6%</td>
</tr>
<tr>
<td>$z^H$</td>
<td>0.4000</td>
<td>Average loan-to-income ratio at origination</td>
<td>2.72</td>
</tr>
<tr>
<td>$\omega_h$</td>
<td>0.41</td>
<td>Average fraction of delinquent loans repossessed</td>
<td>33.5%</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.1370</td>
<td>Average housing price relative to annual income</td>
<td>3.2%</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>6.200</td>
<td>Relative volatility of population growth</td>
<td>0.17%</td>
</tr>
</tbody>
</table>
### Chapter 2. Housing Liquidity and Unemployment: The Role of Firm Financial Frictions

Table 2: Remaining Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.65</td>
<td>See discussion</td>
<td>N/A</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>1.0609</td>
<td>Normalization</td>
<td>4.0%</td>
</tr>
<tr>
<td>$w_b$</td>
<td>0.8784</td>
<td>See discussion</td>
<td>N/A</td>
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</table>

**Parameters determined jointly**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_h$</td>
<td>0.1798</td>
<td>Probability of successful sale of new houses</td>
<td>15%</td>
</tr>
<tr>
<td>$\nu$</td>
<td>6</td>
<td>Elasticity of firm lending with respect to house prices</td>
<td>0.7%</td>
</tr>
<tr>
<td>$j_E$</td>
<td>0.02</td>
<td>No risk premium of lending to firms in steady state</td>
<td>0%</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>1.7919</td>
<td>Average wage-to-output ratio</td>
<td>0.72%</td>
</tr>
<tr>
<td>$j^c$</td>
<td>0.0295</td>
<td>Firm default rate</td>
<td>6%</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.4229</td>
<td>Unemployment rate</td>
<td>5%</td>
</tr>
<tr>
<td>$\eta_l$</td>
<td>0.6595</td>
<td>Job finding probability at the minimum wage</td>
<td>Normalized to 99.9%</td>
</tr>
<tr>
<td>$\alpha_c$</td>
<td>0.4678</td>
<td>Firm risk premium</td>
<td>2%</td>
</tr>
</tbody>
</table>

Table 3: Percent changes in $\chi$ vs. changes in $\bar{y}$, selected moments

<table>
<thead>
<tr>
<th>Default rate</th>
<th>$\chi$</th>
<th>$\bar{y}$</th>
<th>$\Delta$PTI</th>
<th>DP ratio</th>
<th>Salvage rate</th>
<th>Unemployment rate</th>
<th>Mean wage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\chi$</td>
<td>$\bar{y}$</td>
<td>$\chi$</td>
<td>$\bar{y}$</td>
<td>$\chi$</td>
<td>$\bar{y}$</td>
<td>$\chi$</td>
</tr>
<tr>
<td>2.5</td>
<td>0.81</td>
<td>-0.21</td>
<td>-1.16</td>
<td>-0.97</td>
<td>1.40</td>
<td>0.96</td>
<td>-0.58</td>
</tr>
<tr>
<td>6.8</td>
<td>3.00</td>
<td>-0.58</td>
<td>-5.57</td>
<td>-2.43</td>
<td>4.30</td>
<td>2.15</td>
<td>-2.32</td>
</tr>
<tr>
<td>9.0</td>
<td>3.43</td>
<td>-0.81</td>
<td>-6.63</td>
<td>-2.89</td>
<td>5.41</td>
<td>2.35</td>
<td>-2.51</td>
</tr>
<tr>
<td>20.0</td>
<td>5.40</td>
<td>-1.61</td>
<td>-10.73</td>
<td>-6.65</td>
<td>9.58</td>
<td>4.56</td>
<td>-3.98</td>
</tr>
</tbody>
</table>
Figure 1: Steady-state distributions of mortgage status respectively among resident owners by employment status (upper panel) and household sellers (lower panel)
Figure 2: Leverage and seller behavior. The top panel shows the choices of selling probability by foresellers. The bottom panel shows the corresponding choices of selling price.

Figure 3: Financial distress and job search behavior. The top panel shows the choices of job-finding probability by distressed sellers. The bottom panel shows the corresponding wage.
Chapter 2. Housing Liquidity and Unemployment: The Role of Firm Financial Frictions

Figure 4: Steady state average loan-to-income ratio (left panel) and downpayment ratio (right panel)

Figure 5: Steady state average rate of mortgage default (left panel) and unemployment (right panel)
Chapter 2. Housing Liquidity and Unemployment: The Role of Firm Financial Frictions

Figure 6: Steady state average price-to-income ratio (left panel) and mortgage salvage rate (right panel)

Figure 7: Steady state average loan-to-income ratio (left panel) and downpayment ratio (right panel) by economic environment
Figure 8: Steady state average rate of mortgage default (left panel) and unemployment (right panel) by economic environment

Figure 9: Steady state average price-to-income ratio (left panel) and mortgage salvage rate (right panel) by economic environment
Figure 10: Difference in steady state average wage between variable and fixed $E$ by labor market type
Chapter 3

Housing Market Distress and Unemployment: A Dynamic Analysis

3.1 Introduction

The Great Recession of 2007-2009 is largely believed to have been instigated by the collapse in housing prices and the resulting financial crisis. Unemployment rose sharply from 4.7% just prior to the recession, to its peak of 10%, and recovered at its slowest pace in the post-World War 2 period. This has led to the hypothesis that the housing crisis played an important role in the sharp and persistent rise in unemployment. There are two prominent hypotheses linking the collapse in housing markets to unemployment. The first is that home equity losses and the need for homeowners to deleverage reduced aggregate demand, and thus the need for firms to hire. The second hypothesis is that banks and lending institutions suffered large losses due to the collapse in the mortgage-backed securities market and the rise in foreclosures, which restricted their ability to lend to firms, which reduced these firms’ ability and desire to hire. In particular, the bank loan growth rate declined for 10 consecutive quarters following the start of the Great Recession, with total borrowing falling by 5 percent of GDP, more than in any other post-World War 2 recession.

This paper analyzes the strength of the credit supply hypothesis. In particular, the objective is to analyze how, and to what extent, an initial negative shock of 5 percent to housing market liquidity can
propagate through the aggregate economy, with a special focus on unemployment. The shock is captured by a temporary, but persistent, fall in a financial intermediary’s ability to sell a foreclosed house, which raises the costs associated with mortgage default. The shock is meant to capture, in a very simple way, the collapse of the secondary mortgage market. When mortgage-backed securities were liquid, i.e. could be sold easily at a given price, then mortgage originators were able to offload the risk associated with mortgage default. The collapse of this market made this more difficult, and increased the bank’s overall cost to foreclosing on a house.

The mechanism works as follows. The initial shock increases the cost of mortgage default on existing mortgages. All else equal, this reduces the resources available to lend to firms, or more generally, increases the cost of renting capital from the bank. This reduces labor demand, raises unemployment and lowers wages.

Consider the effect on house sellers. It is more difficult for indebted owners, particularly those with high leverage, to make payments on their house. Selling their house may be difficult however. Firstly, the initial shock raises default premiums, making it more difficult for potential buyers to obtain a large mortgage. This reduces housing demand, which is further reduced by the fall in overall income. Secondly, highly leveraged sellers are limited by how much they can lower their selling price. Selling is only optimal if the funds obtained from doing so are sufficient to pay off remaining debt. Overall, housing liquidity falls further. Unsuccessful sellers may then find default unavoidable. The rise in mortgage default further raises the mortgage costs of the bank, potential reducing the resources available to firms, increasing their cost of credit, and propagating the initial shock.

I find that the shock to the bank’s ability to sell a foreclosed house propagates through the rest of the economy when pass-through to firms is strong. Suppose that firms are charged an initial premium beyond the typical rental rate of capital, and that this premium is linked to housing prices in the economy. This can be thought of as capturing, in a simple manner, the observation that bank credit supply tends to be higher to all clients when housing markets are strong (Flannery and Lin, 2015), that is when housing prices are high, and vice versa. In this case, the drop in housing prices caused by the initial shock affects a firm’s cost of borrowing to a larger extent, causing them to cut back on employment. I find that a shock that persists for one year raises the unemployment rate by 1.1 percentage points.

An important feature of the mechanism at work in the model is the presence of liquidity spirals, as described by Hedlund (2016b). A liquidity spiral arises from the interaction of illiquid housing markets, as captured by market search frictions, and endogenous credit constraints. When it is difficult to sell a house, a potential seller must lower their price if they want to increase the likelihood of selling. Highly leveraged homeowners however, have a lower bound as to what price they can accept, which means it
will take longer to sell a house. Some owners may be forced into foreclosure. Higher foreclosure rates are then priced into new mortgages in the form of a higher default premium. This affects potential buyers, who are faced with smaller mortgages, and lowers housing demand and thus, housing liquidity. This creates a spiral of falling housing prices, difficulty selling, increased foreclosures, and tighter credit.

I incorporate an additional endogenous step to the liquidity spiral by introducing a frictional labor market and endogenous unemployment and income variation. The increase in mortgage costs arising from the initial shock raises the cost of capital for firms and reduces labor demand. Labor income falls endogenously, both at the intensive and extensive margins. This makes it more difficult for households to make payments, or obtain mortgages, making defaults more likely and tightening household credit constraints.

This paper uses a heterogeneous agent macroeconomic model that features (i) directed search in both the housing and labor markets; (ii) endogenous supply of (long-term) mortgage credit (iii) and equilibrium mortgage default. In the housing market, buyers choose the size and price of house for which they search, and sellers choose the price at which to try and sell a house. Since buyers and sellers are heterogeneous, this gives rise to an endogenous distribution of submarket tightnesses and corresponding matching probabilities. On the labor side, ex-ante identical firms direct their search by posting a wage in order to attract heterogeneous workers. This gives rise to an endogenous distribution of wages and matching probabilities. A matched firm combines a worker with capital in order to produce. Capital is rented from perfectly competitive financial intermediaries.

Financial intermediaries trade bonds and mortgages with households, sell their stock of foreclosed housing, and accumulate capital to rent to firms.

At first glance, the rich heterogeneity in the model makes it seem intractable. Buyers and sellers differ endogenously by income, employment, wage, housing and financial assets. However, I utilize an approach by Hedlund (2016b) that satisfies a variation of the notion of block recursivity, developed by Menzio and Shi (2010), to make the model tractable. The model includes passive real estate brokers that intermediate trades in the housing market. They sell houses to potential buyers, which they buy from sellers. Buyers and sellers thus trade with identical agents, rather than each other, and this makes the level of heterogeneity manageable. Household decisions do not depend on the entire distribution of households, but only on the value of the shadow price of housing. When it comes to the labor market, ex-ante identical firms and one-period duration jobs allow me to manage heterogeneity and maintain the tractability of the model.

This paper expands upon the results in Chapter 2. Using a model with similar features, notably that of an illiquid housing market, I look at the impact of a rise in foreclosure costs on unemployment that
arises through firm financial frictions. While there are some differences in the model, the mechanism of interest is the same. There are two important differences in the analysis, however. The first difference is that, in Tewfik (2016), the link between foreclosure costs and a financial intermediary’s ability to lend to firms is exogenously imposed. That is, when foreclosure costs are higher for an intermediary, this is directly passed on to firms in the form of a higher lending premium. The current paper partially endogenizes the link; while the existence of the premium is exogenously imposed, the premium depends on the shadow price of housing, which is determined in equilibrium, and allows me to more accurately measure the magnitude of the effect on unemployment.

The second important difference is that Tewfik (2016) conducts a primarily stationary equilibrium analysis. That is, the macroeconomic changes for different levels of foreclosure costs is analyzed. The analysis is instrumental in highlighting the potential magnitude of the mechanism at work in this model even absent any dynamic or propagation effects. However, in a stationary equilibrium analysis, mortgages adjust instantly, which limits any propagation effects resulting from a bank having relatively large mortgages on their balance sheet when a negative shock occurs. A dynamic analysis allows us to examine the role that liquidity spirals play in magnifying over time the initial shock to housing liquidity. That is, the magnitude of the liquidity spiral that is generated determines the strength of the propagation of the shock from the housing market to unemployment, and back to the housing market. This analysis is possible specifically due to the tractability of the model that arises when passive real estate agents are introduced.

The remainder of this paper is organized as follows. Section 3.2 reviews the relevant literature. Section 3.3 outlines the model. Section 3.4 briefly describes the equilibrium in the economy. Section 3.5 outlines the calibration strategy. Section 3.6 outlines the experiment and results, and Section 3.7 concludes.

3.2 Related Literature

This paper contributes to several strands of the literature on the housing crisis of the Great Recession. There is a substantial literature relating the housing crisis to high and persistent levels of unemployment. While not exhaustive, two common explanations have arisen. The first is that massive defaults resulted in debt deleveraging and correspondingly low aggregate demand, thus reducing the incentive of firms to hire (e.g. Mian et al. 2013 and Mian and Sufi, 2014). The second story is that banks saw their balance sheets take a hit from widespread mortgage default and losses in the value of asset-backed securities, which forced them to cut back lending to firms, inhibiting employment. It is the second explanation
that I focus on in this paper.

An ample body of empirical evidence suggests that banks that were more affected by the fall in housing prices cut back on lending more, not only to households, but also to their commercial clients as well (e.g. Ivashina and Scharfstein (2010), Helbling and Terrones (2003), and Huang and Stephens (2015)). The cutback to lending resulted in affected firms reducing employment (e.g. Haltenhoff et al., 2014). Duygan-Bump et al. (2010) find that businesses that relied on external finance experienced the largest increases in unemployment. Greenstone and Mas (2014) show that the fall in lending resulted in a 1.4 percentage fall in employment for small firms, and a 0.8 percentage fall in total unemployment. Benotolila et al. (2015) find much more substantial effects for the case of Spain, where the clients of banks that were bailed out had the largest employment losses. This paper uses the empirical evidence as motivation and provides a theoretical model by which to study the strength of the mechanism described above, notably in how much persistence in unemployment can be generated by a housing crisis that affects firm credit supply.

This paper is also related to the literature on the financial accelerator framework (e.g. Kiyotaki and Moore, 1997 and Bernanke et al., 1999). In these models, land or real estate functions as collateral for firms, so when a shock leads to a fall in the value of collateral, firms cut back on investment and production, which propagates the original shock. A more recent strand of literature focuses on an accelerator effect on the household side, notably how household credit constraints propagate income shocks through house price movements (e.g. Ortalo-Magne and Lady, 2006 and Liu et al., 2016). The present paper takes elements of each. While firms do not hold collateral, they are affected by falls in credit supply that stem from housing price falls, forcing them to cut back on employment and production. This leads to mortgage default and further house price falls, thus amplifying the original shock. In this case, the main source of income shocks stems from the increased likelihood of unemployment, brought on by shocks that occur in the housing market. The rise in mortgage default not only tightens household credit constraints, but also negatively affects bank balance sheets, thus further propagating the shock.

This paper is perhaps most related to the literature on search models of housing, specifically those papers that highlight the role that housing illiquidity plays in explaining both housing dynamics and the propagation of those dynamics throughout the rest of the macroeconomy. This is evident in papers by Head, Lloyd-Ellis, and Sun (2014), Garriga and Hedlund (2016), Head, Sun, and Zhou (2016), Hedlund (2016a,b), and Tewfik (2016), to name a few. Doing so allows for the role that time to sell plays in the propagation of housing market outcomes throughout the rest of the economy. Empirically, time to sell varies over the business cycle and rose sharply during the Great Recession. Even if households have positive home equity, if they have trouble paying off their mortgage, they may still be forced into default
if they cannot sell fast enough. The importance of time to sell is stronger when households have a large loan-to-value ratio as well, as this limits how flexible they can be in lowering the house price in order to sell more quickly. This plays an important role in generating the liquidity spirals described by Hedlund (2016a), which are responsible for magnifying income shocks.

The above papers also emphasize the role that directed search plays in housing liquidity. They are able to make such housing search models tractable by utilizing the notion of block recursivity developed by Menzio and Shi (2010). This allows for the inclusion of rich heterogeneity of households into a full production economy, while make the dynamics computationally tractable. I extend this by incorporating a frictional labor market that allows for heterogeneity in unemployment outcomes, as well as linking realizations in unemployment to housing market activity.

3.3 Model

3.3.1 Households

Each period, households search for employment in submarkets characterized by a wage and tightness, or the ratio of firms to potential workers. Successful workers are paid a wage $w$ per unit of labor efficiency. Labor efficiency is stochastic, and composed of a persistent component, $s \in S$, which follows a finite Markov chain, and a transitory component $e$, with distribution function $F(e)$. A household initially draws $s$ from the invariant distribution $\Pi(s'|s)$. Households who are unsuccessful in finding a job receive an unemployment benefit $w^u$.

A household’s periodic utility is given by

$$U_t = u(c, c_h),$$

where $c$ denotes the composite consumption good and $c_h$ denotes housing services. Households enjoy a utility premium by consuming housing services from houses that they own. That is, an owner with a house of size $h \in \{h_1, h_2, h_3\}$ receives utility $c_h = h$, while a renter consumes at most $c_h \leq h$. A renter obtains housing services each period in a competitive spot market, while a house may be bought and sold in a frictional housing market, which will be described later.

Households may smooth consumption by buying one-period bonds at price $q_b \in (0, 1)$, which are bought from perfectly competitive financial intermediaries. A homeowner may also smooth consumption by borrowing against the value of their house and taking on long-term mortgage debt. A mortgage contract will be described in Section 3.3.6.
3.3.2 Consumption good sector and the labor market

Ex-ante identical consumption good firms operate a production function that requires one unit of labor to operate. Firms produce according to

$$Y_c = z_c F(k).$$

Here, $k$ denotes capital per match. Firms are free to enter the sector. In order to find a worker, firms search in a frictional labor market. Search is directed, as in Shi (2009). That is, the labor market is divided into submarkets denoted by $(w, \theta_r)$, where $w$ is the wage, and $\theta_r$ is the tightness, or the ratio of firms to workers. Given the characteristics of the submarket, firms and workers choose which one to visit. Within each submarket, they are randomly matched according to a matching function $T(U, V)$, which is increasing in workers $(U)$ and firms $(V)$, and exhibits constant returns to scale. The matching probability of a worker, $q(\theta_r)$ is increasing in $\theta_r$, while the matching probability of a firm, $p(\theta_r)$ falls with $\theta_r$.

A match lasts for the current period. A matched firm rents capital from financial intermediaries at rental rate $r_k + r(p_h)$. Here, $r(p_h)$ is a premium that decreases with the price of new housing. The introduction of such a premium is meant to capture, in a simple way, the phenomenon that banks are more willing to lend cheaply to all clients when the housing market is booming, as observed for example, in Flannery and Lin (2015). However, the value of the premium retains an element of endogeneity, as $p_h$ is determined endogenously.

Firms choose the amount of capital to solve

$$\max_k zF(k) - \{r_k + r(p_h)\} k - w.$$ 

Let $A$ denote the aggregate state of the economy. The optimal amount of capital satisfies

$$r_k(A) + r(p_h(A)) = zF_k(k^* (A)).$$

(3.3.1)

All firms choose the same amount of capital. The value of a match is then

$$J(w) = zF(k^*) - \{r_k + r(p_h)\} k^* - w.$$ 

A firm must incur a vacancy cost $j_v$ to search for a worker. This implies that the value of posting a vacancy in submarket $(w, \theta_r)$ is

$$J_v(w, \theta_r) = -j_v + p(\theta_r)J(w).$$
Firms create vacancies as long \( J_v(\cdot, \cdot) > 0 \). Additionally, since firms are ex-ante identical, they must be indifferent between entering any active submarket. Together, the value of a vacancy is driven down to zero, which yields a trade-off between the wage and level of tightness:

\[
J_v = 0. \tag{3.3.2}
\]

Output can be consumed, added to the capital stock, or used to build new housing and rental units.

### 3.3.3 Housing services for renters

Landlords use a linear, reversible technology to produce housing services at rate \( A_h \) from the consumption good. Housing service are sold on a competitive spot market at price \( r_h \). Profit maximization implies

\[
r_h = \frac{1}{A_h}. \tag{3.3.3}
\]

### 3.3.4 Construction sector

Construction firms use land \( L \), and structures \( S_h \) to produce new housing according to a constant returns to scale technology described by

\[
Y_h = z_h F_h(L, S_h).
\]

The government supplies a fixed amount \( \bar{L} > 0 \) of new land/permits each period, and all revenues go to unproductive government spending. Construction firms sell new houses directly to real estate firms at price \( p_h \). They choose land and structures to maximize profit:

\[
\max_{L,S_h} p_h z_h F_h(L, S_h) - p_L L - S_h.
\]

A house is subject to stochastic depreciation at rate \( \delta_h \).\(^1\) The housing stock then accumulates according to

\[
H' = (1 - \delta_h) H + Y_h'.
\]

### 3.3.5 Real estate sector

The housing market is characterized by search frictions. There exists a continuum of real estate firms, whose sole purpose is to match bilaterally with house sellers and buyers. Sellers attempt to sell their

\(^1\)Complete depreciation avoids the need to deal with situations where mortgaged homeowners find themselves underwater because a portion of their house depreciates. I assume complete mortgage forgiveness in the low probability event that a house depreciates.
house to real estate firms, before buyers have the opportunity to attempt to buy from a real estate firm.

Decentralized house selling

Sellers direct their search to real estate agents by choosing a submarket that is characterized by \((x_s, h) \in \mathbb{R}_+ \times H\), where \(x_s \geq 0\) is the trading price, and \(h \in H\) is the size of the seller’s house. A seller incurs a utility cost \(\xi\) if they fail to match. A real estate firm chooses a measure \(\Omega_s(x_s, h)\) of agents to visit submarket \((x_s, h)\), incurring cost \(\kappa_s h\) per agent to do so. Agents and sellers are then matched randomly according to a matching process defined by the function \(M_s(\Omega_s, S)\), which is increasing in both arguments, and exhibits constant returns to scale. The matching probability for the seller is given by

\[
p_s(\theta_s(x_s, h)) = \frac{M_s(\Omega_s, S)}{S} = M_s(\theta_s(x_s, h), 1),
\]

where \(\theta_s(x_s, h)\) denotes the submarket tightness. The probability of a match for a real estate agent is

\[
\alpha_s(\theta_s(x_s, h)) = \frac{p_s(\theta_s(x_s, h))}{\theta_s(x_s, h)}.\]

A seller can only match with one real estate agent, whereas an agent can match with multiple sellers. Agents cannot hold housing inventories. A successful seller immediately moves out of his house, while those who are unsuccessful make a decision to either stay in their house, or if applicable, default on their mortgage. This decision will be described shortly.

Decentralized house buying

Buyers direct their search to real estate agents by choosing a submarket that is characterized by \((x_b, h)\), where \(x_b > 0\) is the trading price. A real estate firm chooses a measure \(\Omega_b(x_b, h)\) of real estate agents to visit submarket \((x_b, h)\), incurring cost \(\kappa_b h\) per agent to do so. Agents and buyers are then matched randomly according to a matching process defined by the function \(M_b(\Omega_b, B)\), which is increasing in both arguments, and exhibits constant returns to scale. The matching probability for a buyer is given by

\[
p_b(\theta_b(x_b, h)) = \frac{M_b(\Omega_b, B)}{B} = M_b(\theta_b(x_b, h), 1),
\]

where \(\theta_b(x_b, h)\) denotes the submarket tightness. The probability of a match for a real estate agent is

\[
\alpha_b(\theta_b(x_b, h)) = \frac{p_b(\theta_b(x_b, h))}{\theta_b(x_b, h)}.\]

A successful buyer can move into the house immediately, while those who are unsuccessful rent for the remainder of the period.
Market tightness

Real estate firms make three decisions. They choose how much new housing, \( Y_h \), to purchase from construction firms, and hire agents \( \Omega_s \) and \( \Omega_b \) to intermediate trades between buyers and sellers. They do so to solve

\[
\max_{Y_h \geq 0, \Omega_s(x_s, h) \geq 0, \Omega_b(x_b, h) \geq 0} \int \left[-\kappa_s h + \alpha_s(\theta_s(x_s, h; A))(x_s)\right] \Omega_s(dx_s, dh) - p_h(A)Y_h + \\
\int \left[-\kappa_b h + \alpha_b(\theta_b(x_b, h; A))x_b\right] \Omega_b(dx_b, dh),
\]

subject to

\[
Y_h + \int h\alpha_s(\theta_s(x_s, h; A))\Omega_s(dx_s, dh) \geq \int h\alpha_b(\theta_b(x_b, h; A))\Omega_b(dx_b, dh)
\]

The constraint holds with equality in equilibrium so that the housing market clears in expectation. Let \( \mu \) denote the multiplier on the constraint. Solving the problem yields the following:

\[
\mu = p_h
\]

\[
\kappa_b h \geq \alpha_b(\theta_b(x_b, h; A))(x_b - p_h(A)h) \text{ and } \theta_b(x_b, h; A) \geq \text{ with comp. slackness}
\]

\[
\kappa_s h \geq \alpha_s(\theta_s(x_s, h; A))(p_h(A)h - x_s) \text{ and } \theta_s(x_s, h; A) \geq \text{ with comp. slackness}
\]

3.3.6 Financial sector

Perfectly competitive financial intermediaries sell bonds \( b' \in B > 0 \) and issue mortgages \( m' \in M > 0 \) to households, accumulate capital to rent to firms, and attempt to sell houses within their real-estate owned (REO) inventory. Capital accumulates according to

\[
K' = (1 - \delta_k)K + I.
\]

Here, \( \delta_k \) denotes the depreciation rate of capital, and \( I \) denotes investment. Intermediaries have access to international bond financing. They are charged the risk-free rate \( i \). However, as in Hedlund (2016b), I concentrate on the case of a closed economy.
Mortgages

A borrower with mortgage $m'$ receives $q_0^0 m'$ at origination, where $q_0^0 \in (0, 1)$ is the mortgage price. Since the financial sector is perfectly competitive, each mortgage depends only on the loan size and the borrower’s characteristics, so that an intermediary earns zero expected profit on each individual loan.\(^2\)

Borrowers choose how much of their loan to repay each period. They incur interest $r_m$ on the remainder of their loan. If a borrower wishes to take out a larger mortgage, they must first repay their existing one.

Intermediaries incur proportional origination and servicing costs, denoted by $\zeta$ and $\phi$, respectively, over the life of each mortgage. There are two ways in which mortgages may not be repaid. First, a borrower’s house may depreciate, in which case the remainder of the mortgage is entirely forgiven. Secondly, a borrower always has the option to default, in which case the intermediary will foreclose on the defaulter’s house.

The origination costs and default risk are both priced into the mortgage price, $q_0^0$, while the servicing cost and depreciation risk are priced into the interest rate $r_m$. In this way, the mortgage state can be fully described by the remaining loan balance.

If an indebted homeowner defaults, the remaining debt is erased, and a foreclosure flag ($f = 1$) is placed on their record. This flag remains on the record at the beginning of the next period with probability $\gamma_f \in (0, 1)$, with the defaulter losing access to mortgage markets as long as the flag remains. The house is immediately seized by the intermediary and placed in the REO inventory. The intermediary attempts to sell it in the current period. If there is a profit from a successful sale, the balance is returned to the defaulter. The intermediary has reduced search efficiency $\lambda \in (0, 1)$ and loses a fraction of $\chi$ of the selling price.

**Profit maximization**

Each period, intermediaries choose next period’s capital $K'$, issue bonds $B'$ to households, and originate $n_m(m', b', h)$ mortgages of each type $(m', b', h)$. Period-$t$ cash flows are discounted at the international bond interest rate. The intermediary prices long-duration assets to market. These assets are composed of existing, or vintage, mortgages and REO inventories. That is, intermediaries sell their long-term assets at the beginning of the period, distribute ex post losses or gains to households, and then repurchase these long-term assets at the end of the period.

\(^2\)As in Hedlund (2016a), the government evenly divides all ex-post profits or losses among households through a proportional wealth tax. In this way, there is no need to specify the owners of the intermediaries.
Profit maximization implies

\[
q_b(A) = \frac{1}{1 + \delta b(A)} = \frac{1}{1 - \delta_k + \mathbb{E}\{r_k(A') + r(p_h(A'))\}}
\]

\[
q_m(A) = \frac{1}{1 + r_m(A)} = \frac{1 - \delta_b}{1 + \phi} q_b(A),
\]

with next period’s capital equaling the sum of end-of-period bond issuances minus the value of unsold REO inventories and new and vintage mortgages.

Mortgage prices satisfy the recursive relationship:

\[
q^0(m', b', w, h, s; A) = q_m(A) \frac{1}{1 + \zeta} \mathbb{E}\{p_s(\theta_s(x'_s, h; A'))m' + [1 - p_s(\theta_s(x'_s, h; A'))]\}
\times \left[ d'' \min m', J_{REO}'(h; A') + (1 - d'') \right] 
\times \left( m' - (1 + \phi)q_m(A')m'' \mathbb{I}_{m'' \leq m'} + \Pi(m'', b'', h, s'; A') \mathbb{I}_{m'' \leq m'} \right) \}
\]

\[
\mathcal{A}' = G(\mathcal{A}, \lambda')
\]

Here, \(x''_s\) is the homeowner’s next period choice of selling price, \(m''\) is the new mortgage balance, \(b''\) is his current bond buying choice, and \(d''\) is his expected default choice. Additionally, \(J_{REO}\) denotes the intermediary’s value for repossessing the borrower’s house, and \(\Pi\) is the continuation value of the mortgage,

\[
\Pi(m'', b'', h; A') = q^0(m'', b'', h; A')(1 + \zeta)(1 + \phi)m''.
\]

Finally, \(G\) is the aggregate law of motion, where \(\lambda'\) is the search efficiency in period \(t + 1\) of the intermediary for selling a foreclosed house.

**Foreclosure sales**

Intermediaries can sell their REO inventories like any other seller, but at a reduced search efficiency. The value to intermediaries of a repossessed house satisfies

\[
J_{REO}(h; A) = R_{REO}(h; A) + (1 - \delta_h)q_b(A)\mathbb{E}J_{REO}'(h; A')
\]

\[
R_{REO}(h; A) = \max \left\{0, \max_{x_s \geq 0} \lambda p_s(\theta_s(x_s, h; A))(1 - \chi)x_s - (1 - \delta_h)q_b(A)\mathbb{E}J_{REO}'(h; A') \right\}.
\]
\[ A' = G(A, \lambda') \] (3.3.11)

The exact description of the intermediary’s balance sheet is provided in Appendix A.

### 3.3.7 Household problem

There are four subperiods within each period. A household can be described by its individual state. The first component is last period’s choice of bonds, \( b \). A household also learns its stochastic and persistence labor efficiency shocks, \( e \) and \( s \), respectively. Households with a previous foreclosure flag learn whether or not it is removed in the current period. Additionally, homeowners are characterized by their house size \( h \) and mortgage balance \( m \geq 0 \). Households start by searching for a job in the labor market.

The aggregate state \( A = \{ \lambda, \Phi, K, \{ H_{REO}(h) \}_h \} \) consists of the intermediary’s selling efficiency, the distribution \( \Phi \) of homeowners and renters over individual states, the capital stock \( K \), and the REO housing stock \( \{ H_{REO}(h) \}_h \). I describe the household value functions by starting in subperiod 4, and working back to subperiod 1.

#### Subperiod 4: Consumption and saving

A household’s income can be described as \( y = (1 - \tau)w \cdot e \cdot s + b \), which is the after-tax sum of its wage or unemployment benefit, \( w \), and bond holdings, \( b \). For a homeowner, expenditures consist of the consumption good, new bond purchases, and mortgage payments. Expenditures cannot be greater than income:

\[ c + q_b(A)b' + m - \tilde{q}_m(m', b', w, h, s; A) \leq y, \] (3.3.12)

where \( \tilde{q}_m(\cdot; A) = q_m(A) \) if owners choose \( m' \leq m \) and \( \tilde{q}_m(\cdot; A) = q_m^0(\cdot; A) \) otherwise.

Let us first consider homeowners. The value function of a homeowner with good credit can be described as

\[
V_{own}(y, m, h, s, 0; A) = \max_{c > 0, m', b' \geq 0} u(c, h) + \beta \mathbb{E}[(1 - \delta_h)W_{own}^{L}(m', b', h, s', 0; A') + \delta_h W_{rent}^{L}(b', s', 0; A')] \\
\text{subject to} \ (3.3.13)
\]

Equation (3.3.12),

\[ \tilde{q}_m^0(m', b', w, h, s; A)m' \mathbb{I}_{m' \geq m} \leq p_h(A), \]

\[ A = G(A, \lambda') \]
Here, $W_{own}^L(\cdot, \cdot, \cdot, \cdot)$ denotes the value of the homeowner at the start of the next period, before the labor market opens. He remains an owner as long as his house does not depreciate. If so, he starts next period as a renter, with value $W_{rent}^L(\cdot, \cdot, \cdot)$.

The value of being a homeowner with bad credit is described as

$$V_{own}(y, 0, h, s, 1; \mathcal{A}) = \max_{c > 0, b'} u(c, h) + \beta \mathbb{E}[(1 - \delta_h)W_{own}^L(m', b', h, s', f'; \mathcal{A}') + \delta_h W_{rent}^L(b', s', f'; \mathcal{A}')]$$

subject to

$$c + q_b(\mathcal{A})b' \leq y,$$
$$\mathcal{A} = G(\mathcal{A}, \lambda')$$

Here, the expectation also takes into account the idiosyncratic probability that the foreclosure flag is removed.

Let us now consider renters, who purchase housing services on a period-by-period basis. Their budget constraint is

$$c + r_h c_h + q_b(\mathcal{A})b' \leq y.$$  \hspace{1cm} (3.3.15)

Renters with good credit are described by the following value function:

$$V_{rent}(y, s, 0; \mathcal{A}) = \max_{b', c \geq 0, c_h \in [0, h]} u(c, c_h) + \beta \mathbb{E} W_{rent}^L(s', 0, \mathcal{A}')$$

subject to

Equation (3.3.15),
$$\mathcal{A} = G(\mathcal{A}, \lambda')$$

Analogously, renters with bad credit have the following value function:

$$V_{rent}(y, s, 1; \mathcal{A}) = \max_{b', c \geq 0, c_h \in [0, h]} u(c, c_h) + \beta \mathbb{E} W_{rent}^L(s', f', \mathcal{A}')$$

subject to

Equation (3.3.15),
$$\mathcal{A} = G(\mathcal{A}, \lambda')$$

As before, the expectation also takes into account the idiosyncratic probability that the foreclosure flag is removed.
Subperiod 3: House buying

Renters, as well as successful home sellers from the previous subperiod, can search to buy a house. They do so by directing their search to a submarket \((x_b, h)\). If a renter has bad credit, he cannot take out a mortgage and is bound by his own income: \(x_b \leq y\). A renter with good credit can buy a house greater than his income: \(x_b \leq y + y(w, h; \mathcal{A})\), where \(y > 0\) reflects the ability of new buyers to take out a mortgage in the next subperiod. The option value of searching for a house is

\[
R_{\text{buy}}(y, s, 0; \mathcal{A}) = \max\{0, \max_{h \in H, x_b \leq y + y} p_b(\theta_b(x_b, h; \mathcal{A})) \times [V_{\text{own}}(y - x_b, 0, h, 0; \mathcal{A}) - V_{\text{rent}}(y, s, 0; \mathcal{A})]\},
\]

(3.3.18)

\[
R_{\text{buy}}(y, s, 1; \mathcal{A}) = \max\{0, \max_{h \in H, x_b \leq y} p_b(\theta_b(x_b, h; \mathcal{A})) \times [V_{\text{own}}(y - x_b, 0, h, 1; \mathcal{A}) - V_{\text{rent}}(y, s, 1; \mathcal{A})]\},
\]

(3.3.19)

Subperiod 2b: Mortgage default

Subperiod 2 can be subdivided into the house selling period, and the consequences of the outcome of attempting to sell a house. An indebted homeowner may either default, or choose to remain in his house and continue to make payments. The value function for a homeowner deciding whether to default is

\[
W_{\text{own}}^h(y, m, h, s, 0; \mathcal{A}) = \max\{(V_{\text{rent}} + R_{\text{buy}})(y + \max\{0, J_{\text{REO}}(h; \mathcal{A}) - m\}, 1; \mathcal{A}), V_{\text{own}}(y, m, h, s, 0; \mathcal{A})\}.
\]

(3.3.20)

If he defaults, he becomes a renter with a foreclosure flag on his record. He has the option to buy in subperiod 3. Furthermore, his income may be adjusted if there is some positive amount left after the intermediary sells the value of his house. Otherwise, he continues onto subperiod 3 as an owner.

For a homeowner with no debt who fails to sell, there is no default decision:

\[
W_{\text{own}}^h(y, m, h, s, 0; \mathcal{A}) = V_{\text{own}}(y, m, h, s, 0; \mathcal{A}).
\]

(3.3.21)
Subperiod 2a: House selling

Homeowners in subperiod 2 decide whether to try to sell their house. An owner with house size \( h \) who sells chooses a submarket \((x_s, h)\). The option value of selling can be described as follows.

\[
R_{sell}(y, m, h, s, 0, A) = \max \{0, \max_{y+x_s \geq m} \ p_s(\theta_s(x_s, h); A) \left[ (V_{rent} + R_{buy})(y + x_s - m, s, 0; A) - \right. \right.
\]
\[
W_{own}(y, m, h, s, 0; A) - (1 - p_s(\theta_s(x_s, h); A)) \xi \left. \right\} \tag{3.3.22}
\]

\[
R_{sell}(y, 0, h, s, 1, A) = \max \{0, \max_{x_s} p_s(\theta_s(x_s, h); A) \left[ (V_{rent} + R_{buy})(y + x_s, s, 1; A) - \right. \right.
\]
\[
V_{own}(y, 0, h, s, 1; A) - (1 - p_s(\theta_s(x_s, h); A)) \xi \left. \right\} \tag{3.3.23}
\]

Recall that \( \xi \) is the cost of attempting, but failing, to sell.

Subperiod 1: Job search

Households at the beginning of the period must search for a job. They choose a desired wage \( w \), and direct their search to submarket \((w, \theta_l(s))\). The value of searching, respectively, for a homeowner is

\[
R_{Lown}^L(b, m, h, s, f; A) = \max_w p_l(\theta_l(w, s; A)) \left[ (W_{own} + R_{sell})(1 - \tau)(w \cdot e \cdot s + b), m, h, s, f; A \right] - \right.
\]
\[
(W_{own} + R_{sell})(w^u + b - \tau, m, h, s, f; A) \left. \right\} \tag{3.3.26}
\]

For a renter, we have

\[
R_{Lrent}^L(b, s, f; A) = \max_w p_l(\theta_l(w, s; A)) \left[ (V_{rent} + R_{buy})(1 - \tau)(w \cdot e \cdot s + b), s, f; A \right] - \right.
\]
\[
(V_{rent} + R_{buy})(w^u + b - \tau, s, f; A) \left. \right\} \tag{3.3.29}
\]
3.4 Equilibrium

3.4.1 Block recursivity in the housing and labor markets

As in Hedlund (2016b), this model utilizes a more general application of block recursivity developed by Menzio and Shi (2010). Let us first consider the housing market. Technically, the submarket tightnesses $\theta_s(x_s, h; A)$ and $\theta_b(x_b, h; A)$ depend on the aggregate state $A$, which includes the infinite dimensional distribution of households. However, as shown in the real estate firm’s problem, these tightnesses manifest their dependency on the aggregate state only through $p_h$.

$$
\theta_b(x_b, h; p_h(A)) = \alpha_b^{-1}\left( \frac{\kappa_b h}{x_b - p_h(A)h} \right),
$$

(3.4.1)

$$
\theta_s(x_s, h; p_h(A)) = \alpha_s^{-1}\left( \frac{\kappa_s h}{p_h(A)h - x_s} \right).
$$

(3.4.2)

More specifically, the shadow price of housing is a sufficient statistic for $A$ when calculating submarket tightness, which makes the model significantly more tractable. Block recursivity arises for two reasons. The first is that housing search is directed. The second is due to the existence of real estate agents, who are free to enter any submarket. Together, these imply that the distribution of households across submarkets matters only through its impact on $p_h$.

Let us now consider the labor market. Here, too, the endogenous distribution of households does not influence the tightness in any submarket. Search is also directed, as the wage is posted in advance of matching. Additionally, firms start the period off ex-ante identical, and thus free entry into any submarket yields a trade-off between wage and submarket tightness. Together with jobs lasting for one period, the heterogeneous households separate into different submarkets depending on their individual state.

3.4.2 Determining the shadow housing price

The shadow housing price $p_h$ adjusts so that expected housing supply is equal to expected housing demand, where the expectation takes into account the likelihood of trade. Housing supply $S_h(p_h; A)$ is composed of new housing and existing houses sold by both homeowners and intermediaries:

$$
S_h(p_h; A) = Y_h(p_h; A) + S_{REO}(p_h; A) + \sum_h \int hp_s(\theta_s(x_s^*, h; p_h))\Phi_{own}(dy, dm, dh, ds, df).
$$

(3.4.3)
Housing demand \( D_h(p_h; \mathcal{A}) \) is the housing purchased by matched buyers:

\[
D_h(p_h; \mathcal{A}) = \int h^* p_b(\theta_b(x_b^*, h^*); p_h) \Phi_{\text{rent}}(dy, ds, df). 
\] (3.4.4)

The definition of a recursive equilibrium is given in Appendix B. In order to solve for the equilibrium, as in Krusell and Smith (1998), I assume that households form expectations over an approximate aggregate state space. Households can forecast future prices when they can forecast capital, \( K \), and the shadow housing price, \( p_h \). Together with the exogenous shock \( \lambda \) to the intermediary’s ability to sell a foreclosed house, the approximate state space is formed.

### 3.5 Calibration

I calibrate the steady state of the economy to match the same selected facts of the U.S. economy as in Hedlund (2016b). Additionally, since one of the contributions of this paper is the addition of a more involved labor market, I calibrate the additional parameters to target selected labor market facts. Table 1 lists the parameters, their values, and the targets.

#### 3.5.1 Households

Utility is constant elasticity of substitution within-period and constant relative risk aversion across time:

\[
U(c, c_h) = \begin{cases} \frac{\omega c^{\frac{1}{\nu}} + (1 - \omega)c_h^{\frac{1}{\nu}}}{\frac{\nu}{1 - \sigma}} & \sigma \neq 1 \\ \frac{1}{1 - \sigma} & \sigma = 1 \end{cases} 
\] (3.5.1)

The value of the intertemporal elasticity of substitution is taken from the literature (see Flavin and Nakagawa, 2008), and is set at \( \nu = 0.13 \). The remaining parameters, in addition to the discount factor, \( \beta \), are determined jointly. Three targets are most relevant to the values of these parameters. The first is the ratio of (non-residential) capital to GDP, which takes a value of 1.64. The next is the ratio of average value of housing to earnings, which data from the 1998 Survey of Consumer Finance suggests is 3.62. Finally, these parameters are used to match the ratio of mortgage debt to earnings, which the same survey suggests stands at around 2.03.

#### 3.5.2 Production sectors

In the consumption sector, given capital \( K_c \), one unit of labor can produce output \( Y_c \) according to
\[ Y_e = z_e K^{\alpha_K} \]

The share of capital is determined independently and set to \( \alpha_K = 0.26 \). Total factor productivity is calibrated jointly with other parameters; however, the most relevant target it is set to match is the mean quarterly labor earnings. Annual earnings are normalized to 1, so that the quarterly analog would be 0.25.

Production in the housing sector follows

\[ Y_h = z_h L^{\alpha_L} S_h^{1-\alpha_L} \]

The share of land is set independently at \( \alpha_L = 0.33 \), as suggested according to data from the Lincoln Institute of Land Policy. Recall that housing is subject to stochastic depreciation. The data suggests a 2.5\% yearly rate of depreciation, so that \( \delta_h = 0.00625 \). The amount of land, \( \bar{L} \), is fixed and set to match the ratio of housing investment to output. The productivity of housing production, \( z_h \), is set to match the ratio of housing to output, which, following Hedlund (2016b), is targeted to be 1.04.

### 3.5.3 Labor sector

The labor efficiency process, \( \ln(e \cdot s) \), follows

\[
\begin{align*}
\ln(s') &= \rho_s \ln(s) + \epsilon', \\
\epsilon' &\sim \mathcal{N}(0, \sigma^2_s), \\
\ln(e) &\sim (0, \sigma^2_e).
\end{align*}
\]

The calibration procedure follows that outlined in the Appendix of Hedlund (2016b). The stochastic component, \( \ln(e) \) has compact support, and the persistent component, \( \ln(s) \), is approximated with a 3-state Markov chain.

The labor matching function has a Cobb Douglas specification:

\[
\mathcal{T}(U, V) = A_F U^{\sigma_F} V^{1-\sigma_F} \quad (3.5.2)
\]

Both \( A_F \) and \( \sigma_F \) are calibrated jointly in order to match the U.S. mean unemployment rate of 5\%, and a mean wage-to-output ratio of 0.7, which is consistent with data from the Bureau of Economic Analysis for the period prior to 2007.
Additional parameters relating to the labor market are the firm vacancy cost, $j_c$, and the unemployment benefit, $w_u$, which are calibrated jointly. They are set to target the ratio of unemployment benefits to earnings, which was around 0.50 pre-2000 (O’Leary and Rubin, 1997), and the probability of mortgage default given unemployment, which Gerardi et al. (2015) suggest is around 10%.

3.5.4 Real estate firms

Matching functions in the real estate section have a Cobb Douglas specification.

$$M_j(B, S) = A_j B^{\gamma_j} S^{1-\gamma_j}, j \in \{b, s\}$$

Here, real estate agents can function either as buyers ($B$) or sellers ($S$). The parameters $A_j$, $\gamma_j$, entry costs $\kappa_j$, and utility cost for household sellers, $\xi$, are calibrated jointly. Here, $\gamma_b, \gamma_s$ target the average buyer and seller duration in weeks, respectively, the values for which are taken from a combination of the Census Bureau, the National Delinquency Survey, and the National Association of Realtors. The parameters related to a real estate agent’s buying behavior, $A_b$ and $\kappa_b$, are set to match the minimum and maximum buying premiums of 0.5% and 2.5%, respectively, which are consistent with Gruber and Martin (2003). Similarly, $A_s$ is set to target a housing turnover rate of 2%, and $\kappa_s$ is set to match a maximum selling discount of 20%. The entry cost is set to match an annual foreclosure rate of 1.4%, which is consistent with data from the National Delinquency Survey by the Mortgage Bankers Association. Additionally, the minimum house size $\underline{h}$ is set to match an average home ownership rate is 64%, which is the rate reported by the Census Bureau for the households whose head is between the ages of 35 and 44.

3.5.5 Financial sector

Following data on closing costs from the Federal Housing Finance Board, I set mortgage origination costs, $\zeta$, to be 3%. The mortgage servicing cost is set at $\phi = 0.0000415$ to target a spread of 2.65% between mortgage interest rates and bond yields. Capital depreciates at $\delta_K = 0.025$ to match an annual depreciation rate of 10%.

The firm interest rate premium, $r(p_h)$ has a linear specification:

$$r(p_h) = \beta_0 + \beta_1 p_h, \ \beta_1 < 0$$

The following two choices pin down ($\beta_0, \beta_1$). First, I normalize $r(p_h) = 0$ in the stationary equilib-
rium. Second, I use the observation that, at the trough of the Great Recession, the Federal Reserve reduced interest rates by 125 basis points, which corresponded with a 30% fall in house prices. Since commercial interest rates may not have been so much higher absent the intervention, I conservatively set a 30% fall in house prices to be associated with a 63 basis point rise in \( r(p_h) \).

### 3.5.6 Foreclosure environment

As is standard in the literature (e.g. Hedlund, 2016b and Head et al. 2017), a foreclosure flag stays on a defaulter's record with probability \( \gamma_f = 0.95 \) in order to match a waiting period of 5 years before a household has access to mortgage markets. Pennington and Cross (2006) find an average foreclosure selling discount of 22%, which is used to target REO foreclosure costs, \( \chi \). The REO search efficiency, \( \lambda \), is set to match the length of time a foreclosed house remains on the market. Again, Pennington and Cross (2006) find that this is about 1 year. The remaining parameters related to foreclosure, REO foreclosure costs, \( \chi \), and search efficiency, \( \lambda \), are determined jointly.

### 3.6 Experiment and Results

The main experiment I perform is as follows. I look at what happens when there is a fall in the search efficiency of a bank when selling a house in their REO inventory. The option value of selling a house for the bank is given by equation (3.3.10). Thus, the shock reduces this option value by lowering \( \gamma \). All else equal, the shock reduces the recovery rate of selling a foreclosed house. This is meant to capture, in a simple way, the increased difficulty that banks and other financial institutions faced once the mortgage-backed securities market dried up, thus increasing costs associated with mortgage default.

Before proceeding with the numerical results, let us consider, for a moment, the impact of a temporary shock to \( \lambda \) that lasts for only one period. This raises the costs associated with mortgage default in the current period. Since it is a purely temporary shock, all else equal, there would be no impact on mortgage lending behavior today. No vintage or new mortgages would have an increased risk of default. The impact on the overall house price level will be limited; the bank may reduce the price at which it sells a house, but since the shock dissipates in the following period, the bank may also choose to wait. The minimal fall in house prices, combined with a predetermined capital stock, implies that neither changes in the interest rate premium nor changes in the capital available to firms, would impact current hiring behavior. In theory, if the shock is large enough to significantly raise default costs in the current period, there may be less capital available for firms to rent tomorrow, which would raise rental rates tomorrow. However, bond and mortgage prices would adjust so that the impact on the change in capital
would be limited. Thus, any propagation would dissipate within the following period. Thus, for there to be significant propagation effects, the initial shock must be persistent. In this way, house prices would fall over time, the effects of which would make their way through the economy.

I allow the shock to persist for a total of one year before it returns to its steady state value. I choose the size of the shock to be 5 percent, such that prices would need to fall on impact by 5 percent in order for the probability of an REO sale to remain at its steady state value.\(^3\) I analyze the results under two scenarios. The first scenario is the baseline case in which firms face an interest rate premium associated with \(p_h\). In the second scenario, I omit this premium, so that \(r(p_h) = 0\) for all \(p_h\). The results indicate that for the shock to propagate beyond the housing market, the interest rate premium must be present. Without the premium, the shock directly raises the costs associated with mortgage default, but does not affect the amount of resources, or capital, available to firms. Thus, there is no effect on their hiring behavior or unemployment. The remainder of this section explains this in greater detail.

3.6.1 Results

Figure 1 illustrates the impulse response functions generated from the experiment. The relevant variables are the shadow housing price, \(p_h\), the firm borrowing premium, \(r(p_h)\), the unemployment rate, the average time on the market (TOM), the default rate, and the non-residential investment rate.

In the baseline model, the shadow price of housing, \(p_h\), drops on impact, and eventually falls 10 percent below its steady state level. There is a corresponding rise in the interest rate premium charged to firms. Over the duration of the shock, it rises by an annualized rate a fraction of 1 percentage point. The overall increase in the cost of capital lowers the value of a job vacancy to the firm. The fall in the measure of job vacancies makes it more difficult for households to find a job, and so the unemployment rate rises. Panel C of Figure 1 shows the response in unemployment. At its peak, the unemployment rate rises by 1.1 percentage points. For employed workers, average wages fall as well.

The fall in income, both from the rise in unemployment and the reduction in wages, makes it more difficult for indebted homeowners to make payments on their mortgages. The level of difficulty depends on the size of the mortgage. If a homeowner with a high loan-to-value (LTV) ratio has trouble making a payment or refinancing, he will be forced to sell his house. However, the price at which the house can be sold must be sufficiently high so as to cover mortgage repayment. After the shock, selling at such a high price becomes more difficult. Banks lower the size of mortgage issuances, so that potential buyers must either search for cheaper housing, or make a larger downpayment. Regardless, buyers are

\(^3\)From peak to trough, house prices in the U.S. fell on average 30 percent during the Great Recession. I choose 5 percent so that any further fall in house prices remains endogenous.
unwilling to pay as much for houses than they were prior to the shock. This increases the risk of default for high-LTV homeowners. Panel E illustrates the IRF of the total default rate for homeowners, and for homeowners with LTV ratios greater than 95%. The latter account for more than half of the total measure of defaults.

Together, the falls in house prices and mortgage issuance bring about a fall in housing market liquidity, as evidenced by the increasing time on the market (TOM) for a house (Panel D). The average TOM rises by 2 months. This is because the resulting rise in mortgage defaults increases the riskiness of any mortgage, which puts further pressure on banks to reduce mortgage issuances. Thus, the shock causes a liquidity spiral, as observed in Hedlund (2016b). In Hedlund (2016b), shocks to productivity bring about a fall in income, which increases mortgage default risk. The increase in default risk results tightens credit constraints faced by households in the form of the default premium, or equivalently, a fall in the size of a mortgage issued. This makes it even more difficult for buyers to buy a relatively more expensive house, and puts further downward pressure on prices, thus continuing the spiral. The same mechanism is at work here, as well, with two key differences. The first difference is that it is a shock that occurs in the housing market that instigates the liquidity spiral. The second difference is that there is one additional step in the liquidity spiral through the impact on firms. The continuing fall in the house price level raises the premium on renting capital, increasing the cost of capital, so that firms cut back on employment even more, and further increasing mortgage default risk.

While there is evidence of a liquidity spiral, the magnitude of the effect is limited by the response in non-residential investment. The capital stock is determined by the difference between bonds purchased by households and total mortgage costs (default costs, vintage and new origination costs, net of payments). Default costs rise while costs associated with vintage and new mortgages fall, due to rising default premiums. The net change in the capital stock, or investment, depends on the response of bond purchases. The response depends on a household’s housing status and current debt portfolio. Households with a high LTV ratio, who are at the greatest risk of default following a negative income shock (i.e. lower wage or unemployment benefit), do not have large bond purchases, so the fall is minimal. The fall in bond purchases is larger, on the other hand, for those with mid-level LTV ratios. For these households, if they sell quickly, they can pay off their debt. They can sell quickly because they can accept a lower price for their house. However, if they fail to sell, they can rely on previous bond purchases to pay their bills. Thus, they reduce bond purchases following the shock. Households with low LTV ratios face a trade-off when purchasing bonds. Purchases may fall due to lower income. However, the incentive to save is larger. Overall, they decrease asset purchases, but only slightly. Overall, bond purchases fall. The capital stock, or equivalently, investment, falls marginally. Thus, the liquidity spirals stem primarily
from the feature that firms face an higher cost of capital due to the presence of a premium that is linked to housing prices, rather than a rise in the rental rate on its own independent of the premium.

Once the shock has dissipated, recovery is relatively quick. Recovery hinges on the rise of house prices. The shock caused a direct drop in house prices, which continued to fall due to higher default risk, which stemmed from an increase in unemployment and a fall in income. However, the fall in house prices is limited by debt overhang. That is, households with high LTV ratios are limited by the degree to which they can drop their selling price – prices exhibit downward rigidity. Once the shock dissipates, the direct cost of mortgage default falls, as well as the increased risk of default stemming from a fall in income. House prices are not upwardly rigid and can rise quickly. The interest rate premium charged to firms falls, and hiring recovers. Moreover, since non-residential investment fell only minimally over the duration of the shock, there is no need for investment to recover. Within three months, house prices and unemployment return to their steady state values.

From the discussion above, it is clear that much of the response to the initial shock, especially in unemployment, stems from the presence of the rental rate premium for firms, \( r(p_h) \). If the premium is absent, that is \( r(p_h) = 0 \), then unemployment remains unchanged. The rise in mortgage default costs associated with the shock is almost completely offset by a fall in the size of new mortgage originations. There is no change in the supply of capital available to firms, and thus there is nothing that alters the measure of job vacancies they choose to create. The corresponding IRFs are illustrated in Figure 2.

### 3.6.2 Discussion

The results of the paper suggest that a shock that alters the ability of the bank to sell a foreclosed house is passed on throughout the economy only when the bank's ability to lend to firms is affected as well. In the model, this occurs when the interest rate charged to firms depends not only on the marginal productivity of capital and market clearing, but also on the shadow price of housing. This formulation is meant to capture, in a simple way, the complexity in which house prices, or the health of the housing market in general, affects banks' lending decisions in reality. For example, Flannery and Lin (2015) find that a 1 percent increase in real estate prices increases small business lending by 0.93 percent. Under this formulation, we see that the shock propagates throughout the rest of the economy, but that the duration of the propagation does not extend far beyond the duration of the shock. One important reason for this is the magnitude of the liquidity spiral that occurs. In Hedlund (2016b), liquidity spirals are an important magnification mechanism by which a productivity shock is passed throughout the rest of the economy. The productivity shock reduces housing liquidity by reducing household income, which makes
Chapter 3. Housing Market Distress and Unemployment: A Dynamic Analysis

it harder to buy, and thus to sell, a house. The mortgage default premium rises, further reducing housing liquidity, and so starts the spiral. The mechanism at work here is similar, in that the increase in the rental rate of capital to firms brings about an increase in unemployment and thus a reduction in income. However, the magnitude of the shock and the rise in unemployment implies that the magnification effect of the liquidity spiral is smaller.\(^4\)

This suggests that the credit supply story can provide a partial explanation for the link between the housing crisis and unemployment observed during the Great Recession. However, it alone cannot account for such a high and persistent unemployment rate. It is clear that there are some components missing in the model that may produce a larger effect. If an intermediary were subject to a reserve constraint, then it may reduce lending to all clients following a large and unexpected rise in the cost of default. Moreover, informational frictions may also account for a stronger impact on the bank’s balance sheet that may reduce firm lending to a larger extent. For example, if a bank must incur a cost to observe the creditworthiness of a home buyer, then it may choose to pay the cost less frequently if it expects the default premiums costs to remain low for the foreseeable future. In this case, following a shock to the bank’s ability to sell a foreclosed house, it would incur a larger cost associated with default due to the presence of "sub-prime" borrowers. The addition of such complexities remains the subject of future work.

3.7 Conclusion

I have developed a theoretical model that incorporates both frictional housing and labor markets in order to study the quantitative important of the credit supply story linking the housing crisis to unemployment during the U.S. Great Recession. The model features directed search in both markets, as well as a notion of block recursivity, that allows me to analyze the impact of a shock to a financial intermediary’s ability to lend to sell foreclosed houses. The shock is meant to capture, in a simply way, the difficulty in offloading mortgage default risk that occurred when secondary mortgage markets dried up. I find that, when the shock is passed onto firms in the form of an endogenous credit premium, unemployment subsequently rises, and raising default costs and increasing the default premium on new mortgages. This causes a further fall in housing market liquidity and exacerbates the initial shock. Recovery is quick, however, due to a strong recovery in housing prices. This suggests that the credit supply story provides a partial, but incomplete, explanation for the high and persistent unemployment rate observed during the Great Recession.

\(^4\)The effect is also affected by the initial size of the shock. Due to the long computation time, analyzing the impact of a larger shock will remain the subject of future work.
Appendix

3.A The Financial Intermediary’s Balance Sheet

The period-\(t\) cash flows of an intermediary is given below. The following expression is calculated after any aggregate shocks occur, but before any market opens. Given wage/benefit income \(w\), denote the period-\(t\) return on an existing mortgage \(m\) on a non-depreciated house as

\[
\gamma_{1,t}(w) = p_s(\theta_{s,t}(x^*_{s}(w), h))m_t + [1 - p_s(\theta_{s,t}(x^*_{s}(w), h))] \times \\
\left[(1 - d^*(w))\left[ m_t - \frac{(1 + \phi)m_{t+1}^*}{1 + r_{m,t}} + \Pi_t(m_{t+1}^*, b_{t+1}^*, h, s_t) + d^*(w)J_{REO,t}(h) \right] \right]
\]

Then, the period-\(t\) cash flows of an intermediary can be expressed as

\[
\pi_t = (1 - \delta_c + r_t)K_t - K_{t+1} - B_t + q_{b,t}B_{t+1} + \\
\sum_{n_{m,t}} \sum_{s_t} (1 - \delta_h) \left\{ p(\theta_t(w^*))\gamma_{1,t}(w^*) + [1 - p(\theta_t(w^*))] \gamma_{1,t}(w^u) \right\} \pi(s_t|s_{t-1})n_{m,t} + \\
\sum_{h \in H} J_{REO,t}(h)H_{REO,t}(h) - \sum_{n_{m,t}} \sum_{s_t} [p(\theta_t(w^*))[1 - p_s(\theta_{s,t}(x^*_{s}(w), h))]d^*(w^*) + (1 - p(\theta_t(w^*)))][1 - p_s(\theta_{s,t}(x^*_{s}(w^u), h))]d^*(w^u)] \times \\
(1 - \delta_h) \left\{ J_{REO,t}(h) - \lambda p_s(\theta_{s,t}(x^{REO*}_{s,t}, h))(1 - \chi)x^{REO*}_{s,t}(h) \right\} \pi(s_t|s_{t-1})n_{m,t} + \\
\sum_{h \in H} J_{REO,t}(h) - \lambda p_s(\theta_{s,t}(x^{REO*}_{s,t}, h))(1 - \chi)x^{REO*}_{s,t}(h)H_{REO,t}(h)
\]

value of new REOs prior to selling

cost of purchasing unsold new REOs

cost of purchasing unsold REO inventories

125
\[
- \sum_{i_{m,t+1}}^n q^0_{m,t}(m_{t+1}, b_{t+1}, h_t)(1 + \zeta)(1 + \phi)m_{t+1}n_{m,t+1}
\]

cost of new and vintage mortgages

Here, an asterisk on a variable denotes an optimal decision.

3.B Definition of an Equilibrium

A recursive equilibrium consists of:

- household value and policy functions;
- firm functions \( k(A), L(A), S_h(A) \);
- intermediary functions \( J_{REO}(h; A), R_{REO}(h; A), x^{REO}_s(h; A) \), and \( K(A) \);
- prices \( r_h, q_b(A), r_m(A), r_k(A), p_h(A), p_h(A) \), and \( q^0_{m}(m', b', w, h, s; A) \);
- housing market tightnesses \( \theta_b(x_b, h; p_h(A)) \) and \( \theta_s(x_s, h; p_h(A)) \);
- labor market tightnesses \( \theta_l(w, s; A) \);
- an aggregate law of motion \( \dot{A} = G(A, \lambda) \);

such that the following statements hold:

1. **Household optimality**: The value/policy functions solve Equations (3.13) to (3.29).
2. **Firm optimality**: Equation (3.1) is satisfied.
3. **Labor market tightness**: The variable \( \theta_l \) satisfies Equation (3.2).
4. **Intermediary optimality**: Conditions (3.6) to (3.8) are satisfied.
5. **Housing market tightnesses**: The variables \( \theta_s \) and \( \theta_b \) satisfy Equation (3.5).
6. **Shadow housing price**: \( p_h(A) \) satisfies \( D_h(p_h(A); A) = S_h(p_h(A); A) \).
7. **Land/permits**: We have \( L(A) = \bar{L} \).

---

In equilibrium, the rental rate of capital adjusts to clear the market: \( k \times e = K \), where \( e \) is the employment rate. Technically, this depends on the number of matched firms. Adding employment complicates the computation, so in order to simplify, I make the following assumption. Firms that remain unmatched may operate the production technology themselves without hiring a worker; however, they must consume what they produce themselves (i.e. it cannot be sold to households). This can be thought of as either home production or some sort of training to maintain the technology. Self-operating the technology requires a fixed amount of effort. Thus, firms will always choose to hire a worker first. They will always choose to operate the technology as long as doing so is greater than the effort exerted. Once operated, they will choose the same level of capital as the other firms. Thus, all firms choose \( k^* = K/1 \).
8. Capital market clears: We have $K_c(A) = K(A)$.

9. Resource constraint: Total use of the consumption good equals total production, i.e.

- Total production:

$$Y = zF(K_c, 1) \left[ \int M(s, \theta(t))\Phi_{rent}(db, ds, df) + \int M(s, \theta(t))\Phi_{own}(db, dm, dh, ds, df) \right]$$

$$= zF(K_c, 1)\eta \left[ \int \theta(1-\eta)\Phi_{rent}(db, ds, df) + \int \theta(1-\eta)\Phi_{own}(db, dm, dh, ds, df) \right]$$

- Total use of the consumption good, i.e.

$$\int c_{rent}(dy, ds, df) + \int c_{own}(dy, dm, dh, ds, df) + \int S_h + S_r$$

10. Aggregate law of motion: The law of motion is consistent with the Markov process induced by exogenous processes $\pi_s, \pi_\chi$, and all relevant policy functions.
### Table 1: Model calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<th>Target</th>
<th>Model</th>
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<td>η</td>
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<td>Intertemporal elasticity of substitution</td>
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<td><strong>Parameters determined jointly</strong></td>
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<td>Home ownership rate</td>
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<td>0.01625</td>
<td>Unemployed default rate</td>
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Figure 1: Impulse response functions following a 1-year shock to $\lambda$
Figure 2: Comparison of impulse response functions following a 1-year shock to $\lambda$ in the baseline case (solid line) and when $r(p_h) = 0$ for all $p_h$ (dashed line)
Bibliography


