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Quasi-Grammian solutions of the generalized Heisenberg magnet model

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Abstract

In this paper we use standard binary Darboux transformation to obtain the quasi-Grammian multisoliton solutions of generalized Heisenberg magnet model in two dimensions. We also discuss the model based on the Lie group $SU(n)$ and obtain explicit solutions of the model for $SU(2)$ case.

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1 Introduction

In the study on the dynamics of nonlinear waves in various branches of physics and fluids, solitons, rogue waves and lumps have been solved and investigated for many nonlinear differential equations \cite{1}-\cite{7}, \cite{31}-\cite{34}. During the past few decades, there has been an increasing interest in the field of classical and quantum integrability of Heisenberg magnet (HM) model \cite{9}-\cite{18}. The interest in GHM model is due to its wide range of applicability in magnetism. Due to the versatility of GHM model, it is known to be the elementary object for the study of theory of magnetism \cite{8}. The HM model based upon Hermitian symmetric spaces has been investigated in \cite{15}-\cite{18}. The Darboux transformation of the generalized Heisenberg magnet GHM model and its soliton solutions in terms of quasideterminants has been presented in \cite{19}. In this paper we study the standard binary Darboux transformation of the GHM model and obtain the exact solutions in terms of quasi-Grammians. We employ the technique which is introduced in \cite{20} and calculate

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the standard binary Darboux transformation by introducing the Darboux matrices for the GHM model for direct and adjoint Lax pairs we then construct the binary Darboux matrix by composing the two Darboux matrices. Further, we calculate the quasi-Grammian multisoliton solutions by using iterated binary Darboux transformations. Finally in the last section, we take the system based upon Lie group $SU(N)$ and derive the $SU(2)$ based explicit solutions.

The Hamiltonian of the GHM model is given by [11]

$$H = \frac{1}{2} \text{Tr}((\partial_x U)^T(\partial_x U)), \quad (1.1)$$

where ‘$T$’ is the transpose and $U(x, t)$ is a matrix-valued function which takes values in the Lie algebra $g1(n)$ of general linear group $GL(n)$. The corresponding equation of motion can be defined as

$$\partial_t U = \{H, \partial_x U\}. \quad (1.2)$$

Equation (1.2) can be written as

$$\partial_t U = [U, \partial^2_x U]. \quad (1.3)$$

The generalized Heisenberg magnet model (1.3) can be written as the compatibility condition of the following Lax pair

$$\partial_x \Psi(x, t; \lambda) = \frac{1}{1-\lambda} U(x, t) \Psi(x, t; \lambda), \quad (1.4)$$

$$\partial_t \Psi(x, t; \lambda) = \left(\frac{c^2}{(1-\lambda)^2} U + \frac{1}{1-\lambda} [U, U_x]\right) \Psi(x, t; \lambda), \quad (1.5)$$

where $\lambda$ can be a real or complex parameter and $\Psi$ is an invertible $n \times n$ matrix valued function belonging to $GL(n)$. The compatibility condition of the above system (1.4), (1.5) is the zero curvature condition.
The above equation (1.6) is equivalent to the equation of motion (1.2). In the next section we will retrace the steps of elementary Darboux transformation for the direct and adjoint spaces and then we will calculate the standard binary Darboux transformation by combing the two elementary Darboux transformations of the GHM model.

2 Darboux transformation on direct and adjoint Lax pairs

In this section we discuss the elementary Darboux transformation on the direct and adjoint Lax pairs [21]-[30]. The one-fold Darboux transformation on matrix solution to Lax pair (1.4),(1.5) is defined as

\[ \tilde{\Psi}(x,t;\lambda) = D(x,t,\lambda)\Psi(x,t;\lambda), \] (2.1)

where \( D(x,t,\lambda) \) is the Darboux matrix. We can make the following ansatz for the Darboux matrix \( D(x,t,\lambda) \)

\[ D(x,t,\lambda) = \lambda I - M(x,t), \] (2.2)

where \( M(x,t) \) is an \( n \times n \) matrix function and \( I \) is an \( n \times n \) identity matrix. The Darboux matrix transforms the matrix solution \( \Psi \) in space \( \nu \) to \( \tilde{\Psi} \) in space \( \tilde{\nu} \).

\[ D(\lambda) : \nu \longrightarrow \tilde{\nu}. \] (2.3)

The new solution \( \tilde{\Psi} \) satisfies the following Lax pair, i.e,

\[ \partial_x \tilde{\Psi}(x,t;\lambda) = \frac{1}{1-\lambda} U \tilde{\Psi}(x,t;\lambda), \] (2.4)
\[
\partial_t \tilde{\Psi}(x, t; \lambda) = \left( \frac{c^2}{(1 - \lambda)^2} \tilde{U} + \frac{1}{1 - \lambda} [\tilde{U}, \tilde{U}_x] \right) \tilde{\Psi}(x, t; \lambda),
\]  

(2.5)

where \( \tilde{U} \) satisfies the equation of motion (1.3). We can check the covariance of Lax pair (1.4), (1.5) under Darboux transformation by substituting equation (2.1) in equation (2.4), (2.5). It implies the following Darboux transformation on the matrix field \( U \) is

\[
\tilde{U} = U + M_x.
\]  

(2.6)

The matrix \( M \) satisfies the following conditions

\[
M_x(I - M) = [U, M],
\]  

(2.7)

\[
\]  

(2.8)

The matrix \( M \) can be written as

\[
M = \Theta \Lambda \Theta^{-1},
\]  

(2.9)

where \( \Theta \) is the particular matrix solution of the Lax pair defined as

\[
\Theta = \Psi(\lambda_1) \mid e_1 >, ..., \Psi(\lambda_n) \mid e_n > = \mid \theta_1 >, ..., \mid \theta_n >.
\]

Each column \( \mid \theta_i > = \Psi(\lambda_i) \mid e_i > \) in \( \Theta \) is a column solution of the Lax pair (1.4),(1.5) when \( \lambda = \lambda_i \), i.e

\[
\partial_x \theta_i = \frac{1}{1 - \lambda_i} U \theta_i,
\]  

(2.10)

\[
\partial_t \theta_i = \left( \frac{c^2}{(1 - \lambda_i)^2} U + \frac{1}{1 - \lambda_i} [U, U_x] \right) \theta_i.
\]  

(2.11)
where \( i = 1, 2, \ldots, n \). Let take \( n \times n \) invertible diagonal matrix having entries \( \lambda_i \) are the eigenvalues corresponding to the eigenvectors \( \theta_i \) i.e \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n) \), so the equation (2.10), (2.11) can be written as

\[
\partial_x \Theta = U \Theta (I - \Lambda)^{-1},
\]

\[
\partial_t \Theta = c^2 U \Theta (i - \Lambda)^{-2} + [U, U_x] \Theta (I - \Lambda)^{-1}.
\]

The Darboux transformation of the generalized Heisenberg magnet model in terms of particular matrix solution \( \Theta \) with particular eigenvalue matrix \( \Lambda \) is given by

\[
\tilde{\Psi} = (\lambda I - \Theta \Lambda \Theta^{-1}) \Psi,
\]

\[
\tilde{U} = (\lambda I - \Theta \Lambda \Theta^{-1}) U (\lambda I - \Theta \Lambda \Theta^{-1})^{-1}.
\]

In terms of quasideterminants we can write

\[
\tilde{\Psi} = \begin{vmatrix} \Theta & \Psi \\ \Theta \Lambda^{-1} & \lambda^{-1} \Psi \end{vmatrix}.
\]

\[
\tilde{U} = \begin{vmatrix} \Theta_1 & I \\ \Theta_1 (I - \Lambda) & 0 \end{vmatrix} U \begin{vmatrix} \Theta_1 & I \\ \Theta_1 (I - \Lambda) & 0 \end{vmatrix}^{-1}.
\]

The result can be generalized to construct \( N \)-fold Darboux transformation on matrix solution \( \Psi \) as (for detail see [19])

\[
\Psi[N] = \begin{vmatrix} \Theta_1 & \cdots & \Theta_N & \Psi \\ \Theta_1 \Lambda_1 & \cdots & \Theta_N \Lambda_N & \lambda \Psi \\ \vdots & \cdots & \vdots & \vdots \\ \Theta_1 \Lambda_1^N & \cdots & \Theta_N \Lambda_N^N & \lambda^N \Psi \end{vmatrix}.
\]
Similarly the expression for $U[N]$ is given as

$$
U[N] = \begin{vmatrix}
\Theta_1 & \cdots & \Theta_N & I \\
\Theta_1(I - \Lambda_1) & \cdots & \Theta_N(I - \Lambda_N) & O \\
\vdots & \ddots & \vdots & \vdots \\
\Theta_1(I - \Lambda_1)^N & \cdots & \Theta_N(I - \Lambda_N)^N & O
\end{vmatrix}
\times U \times \begin{vmatrix}
\Theta_1 & \cdots & \Theta_N & I \\
\Theta_1(I - \Lambda_1) & \cdots & \Theta_N(I - \Lambda_N) & O \\
\vdots & \ddots & \vdots & \vdots \\
\Theta_1(I - \Lambda_1)^N & \cdots & \Theta_N(I - \Lambda_N)^N & O
\end{vmatrix}^{-1}
$$

The $N$-fold Darboux transformation on the matrix solution $\Psi$ can also be expressed in terms of projection operator is

$$
\Psi[N] = \prod_{k=0}^{N-1} \left( I - \frac{\mu_{N-k} - \overline{\mu}_{N-k}}{\lambda - \overline{\mu}_{N-k}} P[N - k] \right) \Psi,
$$

where the Hermitian projector is defined as

$$
P[k] = \sum_{i=1}^{n} \frac{\left| \theta_i[k] \right| > \theta_i[k]}{\left| \theta_i[k] \right|} \left( 1 - \frac{\mu_{N-k} - \overline{\mu}_{N-k}}{\lambda - \overline{\mu}_{N-k}} \right),
$$

such that $P^*[K] = P[K]$ and $P^2[K] = P[K]$. For the case of Darboux transformation in adjoint space, we may rewrite equations (1.4) and (1.5) by taking adjoint as

$$
\partial_x \varphi(x; t; \eta) = -\frac{1}{1 - \eta} U^\dagger \varphi(x; t; \eta),
$$

$$
\partial_t \varphi(x; t; \eta) = \left( -\frac{c^2}{(1 - \eta)^2} U^\dagger + \frac{1}{1 - \eta} [U^\dagger, U^\dagger_x] \right) \varphi(x; t; \eta).
$$

In the above equation $\eta$ is a real (or complex) parameter and $\varphi$ is an invertible $n \times n$ matrix in $\nu^\dagger = \{ \varphi \}$ space. The Darboux matrix $D(\eta)$ transforms the matrix solution $\varphi$ in space $\nu^\dagger$ to a new matrix solution $\tilde{\varphi}$ in $\tilde{\nu}^\dagger$, i.e.
\[ D(\eta) : \psi \to \tilde{\psi}. \]  
\hspace{10cm} (2.17)

The one-fold Darboux transformation for the matrix solution \( \varphi \) is given as

\[
\tilde{\varphi} \equiv D(\eta)\varphi = -(\eta I - \Xi\Omega^{-1})\varphi,
\]

where \( \Xi = \text{diag}(\eta_1, \ldots, \eta_n) \) is the eigenvalue matrix. The matrix function \( \Omega \) is an invertible non-degenerate \( n \times n \) matrix and is given by

\[
\Omega = (\varphi(\eta_1) \mid 1 >, \ldots, \varphi(\eta_n) \mid n>) = (|\rho_1>, \ldots, |\rho_n>).
\]

The \( N \)-fold Darboux transformation on matrix solution \( \varphi \) can be expressed as

\[
\varphi[N] = \begin{vmatrix}
\Omega_1 & \cdots & \Omega_N & \varphi \\
\Omega_1\Xi_1 & \cdots & \Omega_N\Xi_N & \eta\varphi \\
\vdots & \ddots & \vdots & \vdots \\
\Omega_1\Xi_1^N & \cdots & \Omega_N\Xi_N^N & \eta^N\varphi
\end{vmatrix},
\]  
\hspace{10cm} (2.18)

Similarly the \( N \)-fold Darboux transformation on \( U^\dagger \) gives

\[
U^\dagger[N] = \begin{vmatrix}
\Omega_1 & \cdots & \Omega_N & I \\
\Omega_1(I - \Xi_1) & \cdots & \Omega_N(I - \Xi_N) & O \\
\vdots & \ddots & \vdots & \vdots \\
\Omega_1(I - \Xi_1)^N & \cdots & \Omega_N(I - \Xi_N)^N & [O]
\end{vmatrix}^{-1} \times U \times \begin{vmatrix}
\Omega_1 & \cdots & \Omega_N & I \\
\Omega_1(I - \Xi_1) & \cdots & \Omega_N(I - \Xi_N) & O \\
\vdots & \ddots & \vdots & \vdots \\
\Omega_1(I - \Xi_1)^N & \cdots & \Omega_N(I - \Xi_N)^N & [O]
\end{vmatrix}.
\]  
\hspace{10cm} (2.19)

In terms of Hermitian projector we may rewrite equation (2.18) as
\[ \varphi[N] = \prod_{k=0}^{N-1} \left( I - \frac{v_{N-k} - \eta_{N-k}}{\eta - \eta_{N-k}} P[N-k] \right) \varphi, \]

where the Hermitian projector in this case is given by

\[ P[k] = \sum_{i=1}^{n} \langle \rho_i[k] | \rho_i[k] \rangle \quad \text{for} \quad k = 1, 2, ..., K. \quad (2.20) \]

By using the equation (1.4), (1.5) and (2.15), (2.16) for the column solutions \( |\theta_i \rangle \) and the row solutions \( \langle \rho_i | \) of direct and adjoint Lax pairs respectively, it can be easily shown that the expressions (2.14) and (2.20) are same.

### 3 Standard binary Darboux transformation

In order to construct the standard binary Darboux transformation we follow the approach given in [30]. Consider a new space \( \hat{u} \) which is copy of the direct space \( u \) such that if \( \varphi \in u \) then \( \hat{\varphi} \in \hat{u} \). The equation of motion and the compatibility condition will have the similar form as for the direct space equations (1.3) and (1.6). That is,

\[ \partial_x \hat{\Psi}(x,t;\lambda) = \frac{1}{1-\lambda} \hat{U}(x,t) \hat{\Psi}(x,t;\lambda), \quad (3.1) \]

\[ \partial_t \hat{\Psi}(x,t;\lambda) = \left( \frac{c^2}{(1-\lambda)^2} \hat{U} + \frac{1}{1-\lambda} [\hat{U}, \hat{U}_x] \right) \hat{\Psi}(x,t;\lambda). \quad (3.2) \]

The specific solutions for the direct and adjoint spaces are \( \Theta, \Omega \) respectively. So the corresponding solutions for the space \( \hat{u} \) are \( \hat{\Theta} \in \hat{u} \) and \( \hat{\varphi} \in \hat{\varphi} \). Let us now assume

\[ i(\hat{\Theta})e^{\hat{\varphi}}, \quad (3.3) \]

then from equations (2.3) and (2.17) we can write the transformation as

\[ D^{(1)}(\lambda) : v^\dagger \rightarrow \hat{v}^\dagger, \quad (3.4) \]
since $\varphi \in v^\dagger$, we have
\[
i(\hat{\Theta}) = D^{(-1)\dagger}(\lambda)\varphi.
\] (3.5)

Also form
\[
D^\dagger(\lambda)(i(\Theta)) = 0,
\] (3.6)
we obtain
\[
i(\Theta) = \Theta^{(-1)\dagger},
\] (3.7)
similarly
\[
i(\hat{\Theta}) = \hat{\Theta}^{(-1)\dagger}.
\] (3.8)

Substituting equation (3.8) in equation (3.5), we get
\[
\hat{\Theta}^{(-1)\dagger} = D^{(-1)\dagger}(\lambda)\varphi,
\] (3.9)

By using equation (2.2) and (2.9) in above expression (3.9), we get
\[
\hat{\Theta} = ((\lambda I - \Theta\Lambda\Theta^{-1})^{(-1)}\varphi)^{(-1)\dagger} = (\lambda I - \Theta\Lambda\Theta^{-1})\varphi^{(-1)\dagger} = \Theta\Delta^{-1},
\] (3.10)
where the potential $\Delta$ is given by
\[
\Delta(\Theta, \varphi) = (\varphi^\dagger\Theta)(\lambda I - \Lambda)^{-1}.
\] (3.11)

Similarly for the adjoint space
\[
\hat{\Omega} = \Omega\Delta^{(-1)\dagger},
\] (3.12)
where
\[
\Delta(\Psi, \Omega) = -((\lambda I - \Xi^\dagger)^{-1}(\Omega^\dagger\Psi).
\] (3.13)

By writing equations (3.11) and (3.13) in matrix form for the solutions $\Theta$ and $\Omega$, we obtain the condition on the potential $\Delta$ as
\[
\Xi^\dagger\Delta(\Theta, \Omega) - \Delta(\Theta, \Omega)\Lambda = \Omega^\dagger\Theta,
\] (3.14)
where the matrix $\Delta$ is given by
\[
\Delta(\Theta, \Omega)_{ij} = \frac{(\Omega^\dagger\Theta)_{ij}}{\eta_i - \lambda_j}.
\] (3.15)
We can now define Darboux matrix in hat space as
\[
\hat{D}(\lambda) \equiv (\lambda I - \hat{S}) = (\lambda I - \hat{\Theta}\Xi\hat{\Theta}^{-1}),
\]
(3.16)
where
\[
\hat{D}(\lambda)\hat{\Psi} = \tilde{\Psi}.
\]
(3.17)

The above formalism may be summarized as
\[
D(\lambda) : \nu \rightarrow \tilde{\nu}, \\
\hat{D}(\lambda) : \hat{\nu} \rightarrow \tilde{\nu}, \\
D(\eta) : \nu^\dagger \rightarrow \tilde{\nu}^\dagger.
\]
(3.18)

We require that applying \(\hat{D}(\lambda)\) given by equation (3.16) on equation (3.1) and (3.2), the Lax pair remains invariant
\[
\partial_x \tilde{\hat{\Psi}}(x,t;\lambda) = \frac{1}{1-\lambda} \tilde{U}(x,t)\tilde{\Psi}(x,t;\lambda),
\]
\[
\partial_t \tilde{\hat{\Psi}}(x,t;\lambda) = \left(\frac{c^2}{(1-\lambda)^2} \tilde{U} + \frac{1}{1-\lambda} [\tilde{U}, \tilde{U}_x]\right) \tilde{\Psi}(x,t;\lambda),
\]
where the Darboux transformation on matrix field \(\hat{\Psi}\) and \(\hat{U}\) in hat space \(\hat{\nu}\) given by
\[
\tilde{\hat{\Psi}} = (\lambda I - \hat{\Theta}\Xi\hat{\Theta}^{-1})\hat{\Psi},
\]
(3.19)
\[
\tilde{\hat{U}} = (I - \hat{\Theta}\Xi\hat{\Theta}^{-1})\hat{U}(I - \hat{\Theta}\Xi\hat{\Theta}^{-1})^{-1}.
\]
(3.20)

From equation (3.18), we have
\[
\hat{D}(\lambda)\hat{\Psi} = D(\lambda)\Psi,
\]
which implies that
\[
\hat{\Psi} = \hat{D}^{-1}(\lambda)D(\lambda)\Psi.
\]
(3.21)

This transformation is known as the standard binary Darboux transformation because it relates the two solution \(\hat{\Psi}\) and \(\Psi\). By substituting the values of \(\hat{D}^{-1}(\lambda)\) and \(D(\lambda)\) from equation (3.16) and equation (2.2) we get
\[
\hat{\Psi} = (\lambda I - \hat{\Theta}\Xi\hat{\Theta}^{-1})^{-1}(\lambda I - \Theta\Lambda\Theta^{-1})\Psi,
\]
\[
= \hat{\Theta}(\lambda I - \Xi\hat{\Theta}^{-1}\hat{\Theta}^{-1}\Theta(\lambda I - \Lambda))\Theta^{-1}\Psi.
\]
(3.22)
By using the expression (3.10) in above equation (3.22), we get

\[ \hat{\Psi} = \Theta \Delta(\Theta, \Omega)^{-1}(\lambda I - \Xi)^{-1}\Delta(\Theta, \Omega)\Theta^{-1}\Theta(\lambda I - \Lambda)\Theta^{-1}\Psi \]

\[ = \Theta \Delta(\Theta, \Omega)^{-1}(\lambda I - \Xi)^{-1}(\lambda \Delta(\Theta, \Omega) - \Delta(\Theta, \Omega)\Lambda)\Theta^{-1}\Psi. \]  

(3.23)

Now by substituting the value of \( \Delta(\Theta, \Omega)\Lambda \) from equation (3.14) in the above equation (3.23), we get

\[ \hat{\Psi} = \Theta \Delta(\Theta, \Omega)^{-1}(\lambda I - \Xi)^{-1}\left(\lambda I - \Xi\right)^{-1}\left(I + \Theta \Delta(\Theta, \Omega)^{-1}\Xi\right)\Psi, \]

\[ = \Psi + \Theta \Delta(\Theta, \Omega)^{-1}(\lambda I - \Xi)^{-1}\Xi\Psi, \]  

(3.24)

By using equation (3.13) in above equation (3.24) we obtain

\[ \hat{\Psi} = \Psi - \Theta \Delta(\Theta, \Omega)^{-1}\Delta(\Psi, \Omega). \]  

(3.25)

In terms of quasideterminant the above equation (3.25) can be written as

\[ \hat{\Psi} = \begin{vmatrix} \Delta(\Theta, \Omega) & \Delta(\Psi, \Omega) \\ \Theta & \Psi \end{vmatrix}. \]

This is known as the quasi-Grammian solution of the GHM. Similarly for the adjoint space \( \hat{\varphi} \in \hat{\upsilon}^\dagger \) we obtain,

\[ \hat{\varphi} = \varphi - \Xi \Delta(\Theta, \Theta)^{-1}\Delta(\Theta, \varphi) = \begin{vmatrix} \Delta(\Theta, \Omega)^\dagger & \Delta(\Theta, \varphi)^\dagger \\ \Omega & \varphi \end{vmatrix}. \]  

(3.26)

Now the standard binary Darboux transformation on matrix field \( U \) as given in equation (3.20) is

\[ \hat{U} = (I - \Theta \Xi \Theta^{-1})^{-1}(I - \Theta \Lambda \Theta^{-1})U(I - \Theta \Lambda \Theta^{-1})^{-1}(I - \Theta \Xi \Theta^{-1}). \]  

(3.27)

By using equation (3.10), we get

\[ \hat{U} = (I - \Theta \Delta(\Theta, \Omega)^{-1}\Xi \Delta(\Theta, \Theta)\Theta^{-1})^{-1}(I - \Theta \Lambda \Theta^{-1})U(I - \Theta \Lambda \Theta^{-1})^{-1}(I - \Theta \Delta(\Theta, \Omega)^{-1}\Xi \Delta(\Theta, \Theta)\Theta^{-1}). \]  

(3.28)
Again by substituting equation (3.14) in above expression, we get

\[
\hat{U} = \left( I - \Theta \Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1} \right)^{-1}(I - \Theta \Lambda \Theta^{-1}) \\
\times U(I - \Theta \Lambda \Theta^{-1})^{-1}(I - \Theta \Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1}), \\
= \left( I - \Theta \Lambda \Theta^{-1} - \Theta \Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1} \right) \times U(I - \Theta \Lambda \Theta^{-1} - \Theta \Delta(\Theta, \Omega)^{-1}\Theta^\dagger), \\
= \Theta(I - \Lambda - \Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1})^{-1}(I - \Theta \Lambda \Theta^{-1} - \Theta \Delta(\Theta, \Omega)^{-1}\Theta^\dagger), \\
= \Theta \left( I - \Lambda \left( I - \Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1} \right) \right)^{-1} \times U(I - \Theta \Lambda \Theta^{-1} - \Theta \Delta(\Theta, \Omega)^{-1}\Theta^\dagger), \\
= \Theta \left( I - \frac{\Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1}}{\left( I - \Lambda \right)} \right)^{-1} \times U \left( I - \frac{\Delta(\Theta, \Omega)^{-1}(\Delta(\Theta, \Omega) - \Theta^\dagger)\Theta^{-1}}{\left( I - \Lambda \right)} \right)^{-1}, \\
= \left( I - \Theta \Delta(\Theta, \Omega)^{-1}\Theta^\dagger \right)^{-1} U \left( I - \Theta \Delta(\Theta, \Omega)^{-1}\Theta^\dagger \right), \\
= \left( I - \Theta \Delta(\Theta, \Omega)^{-1}(I - \Lambda)^{-1}\Theta^\dagger \right)^{-1} U \left( I - \Theta \Delta(\Theta, \Omega)^{-1}(I - \Lambda)^{-1}\Theta^\dagger \right). \tag{3.29}
\]

In terms of quasideterminants, we have

\[
\hat{U} = \begin{vmatrix}
(I - \Lambda)\Delta(\Theta, \Omega) & \Omega^\dagger & \Omega^\dagger \\
\Theta & I & I
\end{vmatrix}^{-1} U \begin{vmatrix}
(I - \Lambda)\Delta(\Theta, \Omega) & \Omega^\dagger \\
\Theta & I
\end{vmatrix}. \tag{3.30}
\]

For the next iteration of binary Darboux transformation, let \( \Theta_1, \Theta_2 \) be the two particular solutions of Lax pair (1.4), (1.5) having \( \Lambda = \Lambda_1 \) and \( \Lambda = \Lambda_2 \) respectively. Similarly for the adjoint space let \( \Omega_1, \Omega_2 \) be the two particular solutions having \( \Xi = \Xi_1 \) and \( \Xi = \Xi_2 \). By using the notation \( \Psi[1] = \Psi, \ U[1] = U \) and \( \Psi[2] = \Psi, \ U[2] = \hat{U} \), we write the two fold binary Darboux transformation on \( \Psi \) as

\[
\Psi[3] = \Psi[2] - \Theta[2] \Delta(\Theta[2], \Omega[2])^{-1} \Delta(\Psi[2], \Omega[2]), \tag{3.31}
\]

where \( \Theta[1] = \Theta_1, \ \Omega[1] = \Omega_1, \ \Theta[2] = \Psi[2] \mid_{\Psi \to \Theta_2}, \Omega[2] = \varphi[2] \mid_{\varphi \to \Omega_2} \). Also note that by using the definition of potential \( \Delta \) and equation (3.15), we have

\[
\Delta(\Psi[2], \varphi[2]) = \Delta(\Psi_1, \varphi_1) - \Delta(\Theta_1, \varphi_1)\Delta(\Theta_1, \Omega_1)^{-1}\Delta(\Psi_1, \Omega_1),
\]

\[
\Delta(\Psi_1, \varphi_1) = \begin{vmatrix}
\Delta(\Theta_1, \Omega_1) & \Delta(\Psi_1, \Omega_1) \\
\Delta(\Theta_1, \varphi_1) & \Delta(\Psi_1, \varphi_1)
\end{vmatrix}. \tag{3.32}
\]
Similarly as from the above equation
\[
\Delta(\Theta[2], \Omega[2]) = \Delta(\Theta_2, \Omega_2) - \Delta(\Theta_1, \Omega_2)\Delta(\Theta_1, \Omega_1)^{-1}\Delta(\Theta_2, \Omega_1),
\]
\[
= \begin{bmatrix}
\Delta(\Theta_1, \Omega_1) & \Delta(\Theta_2, \Omega_1) \\
\Delta(\Theta_1, \Omega_2) & \Delta(\Theta_2, \Omega_2)
\end{bmatrix}.
\]  
(3.33)

By using the expression (3.32), (3.33) in above equation (3.31), we get
\[
\Psi[3] = \begin{bmatrix}
\Delta(\Theta_1, \Omega_1) & \Delta(\Psi, \Omega_1) \\
\Theta_1 & \Theta_1
\end{bmatrix} - \begin{bmatrix}
\Delta(\Theta_1, \Omega_1) & \Delta(\Theta_2, \Omega_1) \\
\Delta(\Theta_1, \Omega_2) & \Delta(\Theta_2, \Omega_2)
\end{bmatrix}^{-1} \begin{bmatrix}
\Delta(\Theta_1, \Omega_1) & \Delta(\Theta_2, \Omega_1) \\
\Delta(\Psi, \Omega_1) & \Delta(\Psi, \Omega_2)
\end{bmatrix},
\]
\[
= \begin{bmatrix}
\Delta(\Theta_1, \Omega_1) & \Delta(\Theta_2, \Omega_1) & \Delta(\Psi, \Omega_1) \\
\Delta(\Theta_1, \Omega_2) & \Delta(\Theta_2, \Omega_2) & \Delta(\Psi, \Omega_2)
\end{bmatrix},
\]  
(3.34)

where we have used the noncommutative Jacobi identity for calculating (3.34). The Nth iteration of binary Darboux transformation is
\[
\Psi[N] = \Psi[K] - \Theta[K]\Delta(\Theta[K], \Omega[K])^{-1}\Delta(\Psi[K], \Omega[K]),
\]
\[
= \begin{bmatrix}
\Delta(\Theta[K], \Omega[K]) & \Delta(\Psi[K], \Omega[K]) \\
\Theta[K] & \Theta[K]
\end{bmatrix},
\]
\[
= \begin{bmatrix}
\Delta(\Theta_1, \Omega_1) & \cdots & \Delta(\Theta_K, \Omega_1) & \Delta(\Psi, \Omega_1) \\
\vdots & \vdots & \vdots & \vdots \\
\Delta(\Theta_1, \Omega_K) & \cdots & \Delta(\Theta_K, \Omega_K) & \Delta(\Psi, \Omega_K)
\end{bmatrix},
\]  
(3.35)

where \(K = N - 1\). Similarly the Nth iteration for the adjoint binary Darboux transformation is given as
\[
\varphi[N] = \varphi[K] - \Omega[K]\Delta(\Theta[K], \Omega[K])^{-1}\Delta(\Theta[K], \varphi[K])^\dagger,
\]
\[
= \begin{bmatrix}
\Delta(\Theta[K], \Omega[K])^\dagger & \Delta(\Theta[K], \varphi[K])^\dagger \\
\Omega[K] & \Omega[K]
\end{bmatrix},
\]
\[
= \begin{bmatrix}
\Delta(\Theta_1, \Omega_1)^\dagger & \cdots & \Delta(\Theta_K, \Omega_1)^\dagger & \Delta(\Theta_1, \varphi)^\dagger \\
\vdots & \vdots & \vdots & \vdots \\
\Delta(\Theta_1, \Omega_K)^\dagger & \cdots & \Delta(\Theta_K, \Omega_K)^\dagger & \Delta(\Theta_K, \varphi)^\dagger
\end{bmatrix},
\]  
(3.36)
Similarly the multisoliton solution \( U[N] \) can be written as

\[
U[N] = \left| \begin{array}{cccc}
(I - \Lambda_1)\Delta(\Theta_1, \Omega_1) & \cdots & (I - \Lambda_N)\Delta(\Theta_N, \Omega_1) & \Omega_1^\dagger \\
\vdots & \ddots & \vdots & \vdots \\
(I - \Lambda_1)\Delta(\Theta_1, \Omega_N) & \cdots & (I - \Lambda_N)\Delta(\Theta_N, \Omega_N) & \Omega_N^\dagger \\
\Theta_1 & \cdots & \Theta_N & \mathbb{1}
\end{array} \right|^{-1} 
\times U \times 
\left| \begin{array}{cccc}
(I - \Lambda_1)\Delta(\Theta_1, \Omega_1) & \cdots & (I - \Lambda_N)\Delta(\Theta_N, \Omega_1) & \Omega_1^\dagger \\
\vdots & \ddots & \vdots & \vdots \\
(I - \Lambda_1)\Delta(\Theta_1, \Omega_N) & \cdots & (I - \Lambda_N)\Delta(\Theta_N, \Omega_N) & \Omega_N^\dagger \\
\Theta_1 & \cdots & \Theta_N & \mathbb{1}
\end{array} \right|.
\]

(3.37)

Similar expression can be obtained for the \( N \)th iteration of \( U^\dagger \). Therefore we see that with the use of standard binary Darboux transformation we can calculate the gramian type solutions for the GHM model and also the potential can be presented in terms of quasideterminants. Thus by constructing the bilinear Darboux transformation in the form of spectral parameters, we can get the expression of the matrix solutions in terms of quasi-grammians, which have the different form from the solutions which we have calculated by using elementary Darboux transformation. Along with the solutions of linear system we are able to calculate explicit quasideterminant expression of the potential \( \Delta \) in the form of particular solutions of linear system. It is important to note that the spectral parameters remains same in binary Darboux transformation. We have considered the eigenfunctions (solutions of direct Lax pair) and adjoint eigenfunctions (solutions of adjoint Lax pair) and the bilinear potential \( \Delta \) is linked to each pair of (direct and adjoint) solutions.

4 Explicit solutions for the \( SU(2) \) system

In this section, we calculate explicit expression of soliton solution. We first discuss the GHM model based on \( SU(n) \). In this case the spin function \( U \) takes the values in the Lie algebra \( su(n) \). So that one can decompose spin function into components i.e., \( U = U^a T^a \) where the generators \( T^a \) \((a = 1, 2, \ldots, n^2)\) are anti-hermitian \( n \times n \) matrices with normalization \( \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \). These generators satisfy the following algebra

\[
[T^a, T^b] = f^{abc} T^c,
\]

(4.1)
where $f^{abc}$ are the structure constant of Lie algebra $su(n)$. For any $X \in su(n)$, we write $X = X^a T^a$ and $U^a = -2\text{Tr}(U^a)$. The matrix field $U$ belongs to the Lie algebra $su(n)$ of the Lie group $SU(n)$, therefore

$$U^\dagger = -U, \quad \text{Tr}(U) = 0. \quad (4.2)$$

Now for the case $SU(2)$, we define the matrix field $U$ as

$$U = i \begin{pmatrix} U_1 & U_2 \\ U_3 & U_4 \end{pmatrix}. \quad (4.3)$$

We now take the seed solution which is given as

$$U_0 \equiv U = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \quad (4.4)$$

So the solution of the linear system (1.4) and (1.5) can be written as

$$\Psi(x, t; \lambda) = \begin{pmatrix} e^{i \left( \frac{1}{1 - \lambda} x + \frac{4}{(1 - \lambda)^2} t \right)} & 0 \\ 0 & e^{-i \left( \frac{1}{1 - \lambda} x + \frac{4}{(1 - \lambda)^2} t \right)} \end{pmatrix}. \quad (4.5)$$

The particular matrix solution $\Theta$ of the direct Lax pair can be written by using the above equation (4.5) as

$$\Phi = (\theta_1, \theta_2) = \begin{pmatrix} e^{i \left( \frac{1}{1 - \lambda} x + \frac{4}{(1 - \lambda)^2} t \right)} & e^{i \left( \frac{4}{(1 - \lambda)^2} t \right)} \\ -e^{i \left( \frac{1}{1 - \lambda} x + \frac{4}{(1 - \lambda)^2} t \right)} & e^{-i \left( \frac{1}{1 - \lambda} x + \frac{4}{(1 - \lambda)^2} t \right)} \end{pmatrix}. \quad (4.6)$$

By substituting $\lambda_2 = \lambda_1$, we can write the above expression (4.6)

$$\Theta = \begin{pmatrix} e^{i \left( \frac{4}{(1 - \lambda_1)^2} t \right)} & e^{i \left( \frac{4}{(1 + \lambda_1)^2} t \right)} \\ -e^{-i \left( \frac{4}{(1 - \lambda_1)^2} t \right)} & e^{-i \left( \frac{4}{(1 + \lambda_1)^2} t \right)} \end{pmatrix}. \quad (4.7)$$

By using the definition given in equation (2.9), and substituting equation (4.7), we get

$$M = \begin{pmatrix} \lambda_1 \tanh u & -\lambda_1 e^w \sec hu \\ -\lambda_1 e^{-w} \sec hu & -\lambda_1 \tanh u \end{pmatrix}. \quad (4.8)$$

where

$$u(x, t) = i \left( \frac{1}{1 - \lambda_1} - \frac{1}{1 + \lambda_1} \right)x + 4i \left( \frac{1}{1 - \lambda_1} - \frac{1}{1 + \lambda_1} \right)t, \quad w(x, t) = i \left( \frac{1}{1 - \lambda_1} + \frac{1}{1 + \lambda_1} \right)x + 4i \left( \frac{1}{1 - \lambda_1} + \frac{1}{1 + \lambda_1} \right)t. \quad (4.9)$$

Also we have

$$I - M = \begin{pmatrix} I - \lambda_1 \tanh u & \lambda_1 e^w \sec hu \\ \lambda_1 e^{-w} \sec hu & I + \lambda_1 \tanh u \end{pmatrix} = \begin{pmatrix} U_1 & U_2 \\ U_3 & U_4 \end{pmatrix}. \quad (4.10)$$
The iterated solution $U[1]$ can be written as

$$U[1] = (\lambda I - M)(\lambda I - M)^{-1}.$$  

(4.11)

By using equations (4.5) and (4.10), we may write $\Psi[1]$ as

$$\Psi[1] = D(\lambda) \Psi = \begin{pmatrix} (\lambda - \lambda_1 \tanh u) e^{A} & (\lambda_1 e^u \sec \hbar u) e^{-A} \\ (\lambda_1 e^{-u} \sec \hbar u) e^{A} & (\lambda + \lambda_1 \tanh u) e^{-A} \end{pmatrix},$$

(4.12)

where we have

$$A(x, t) = \frac{i}{1 - \lambda} x + \frac{4i}{(1 - \lambda)^2} t.$$  

Again by analogy of direct case, we can write solution for the adjoint case as

$$\Omega = \begin{pmatrix} e^{i \frac{1}{\xi_1} x + \frac{4}{1 - \xi_1^2} t} & e^{i \frac{1}{\xi_1} x + \frac{4}{1 - \xi_1^2} t} \\ -e^{-i \frac{1}{\xi_1} x + \frac{4}{1 - \xi_1^2} t} & e^{-i \frac{1}{\xi_1} x + \frac{4}{1 - \xi_1^2} t} \end{pmatrix},$$

(4.13)

In order to obtain the expression for $\tilde{U}$, we start from the definition of $\Delta(\Theta, \Omega)$ given in (3.15). Also we take $\xi = -\xi$ and we use equations (4.7) and (4.13) to obtain

$$\Delta(\Theta, \Omega) = \begin{pmatrix} 2 \cosh p & 2 \sinh \kappa \\ \xi_1 - \lambda_1 & \xi_1 + \lambda_1 \end{pmatrix}$$

(4.14)

where

$$p(x, t) = i \left( \frac{1}{1 - \lambda_1} - \frac{1}{1 - \xi_1} \right) x + 4i \left( \frac{1}{(1 - \lambda_1)^2} - \frac{1}{(1 - \xi_1)^2} \right) t,$$

$$q(x, t) = i \left( \frac{1}{1 - \lambda_1} - \frac{1}{1 + \xi_1} \right) x + 4i \left( \frac{1}{(1 - \lambda_1)^2} - \frac{1}{(1 + \xi_1)^2} \right) t,$$

$$k(x, t) = i \left( \frac{1}{1 + \lambda_1} - \frac{1}{1 - \xi_1} \right) x + 4i \left( \frac{1}{(1 + \lambda_1)^2} - \frac{1}{(1 - \xi_1)^2} \right) t,$$

$$l(x, t) = i \left( \frac{1}{1 + \lambda_1} - \frac{1}{1 + \xi_1} \right) x + 4i \left( \frac{1}{(1 + \lambda_1)^2} - \frac{1}{(1 + \xi_1)^2} \right) t.$$  

(4.15)

Now we consider

$$\hat{M} = \Theta \Delta(\Theta, \Omega)^{-1}(I - \Lambda)^{-1} \hat{\Omega}^\dagger = \begin{pmatrix} \hat{M}_{11} & \hat{M}_{12} \\ \hat{M}_{21} & \hat{M}_{22} \end{pmatrix},$$

(4.16)

where

$$\hat{M} = \frac{2}{K(1 - \lambda_1^2)} \begin{pmatrix} (1 + \lambda_1) e^p \cosh l & (1 + \lambda_1) e^p \sinh q \\ (1 + \lambda_1) e^p \sinh q & (1 + \lambda_1) e^q \cosh p \end{pmatrix}.$$  

(4.17)
where

\[ K = \frac{4 \sinh k \sinh q}{(\xi_1 + \lambda_1)^2} - \frac{4 \cosh p \cosh l}{(\xi_1 - \lambda_1)^2}. \]  \hspace{1cm} (4.17)

Also

\[
\begin{align*}
\hat{p}(x, t) &= i \left( \frac{1}{1 - \lambda_1} + \frac{1}{1 - \xi_1} \right) x + 4i \left( \frac{1}{(1 - \lambda_1)^2} + \frac{1}{(1 - \xi_1)^2} \right) t, \\
\hat{q}(x, t) &= i \left( \frac{1}{1 - \lambda_1} + \frac{1}{1 + \xi_1} \right) x + 4i \left( \frac{1}{(1 - \lambda_1)^2} + \frac{1}{(1 + \xi_1)^2} \right) t, \\
\hat{k}(x, t) &= i \left( \frac{1}{1 - \lambda_1} + \frac{1}{1 - \xi_1} \right) x + 4i \left( \frac{1}{(1 - \lambda_1)^2} + \frac{1}{(1 - \xi_1)^2} \right) t, \\
\hat{l}(x, t) &= i \left( \frac{1}{1 + \lambda_1} + \frac{1}{1 + \xi_1} \right) x + 4i \left( \frac{1}{(1 + \lambda_1)^2} + \frac{1}{(1 + \xi_1)^2} \right) t, \hspace{1cm} (4.18)
\end{align*}
\]

where the matrix field in hat space is given by

\[
\hat{U} = (I - \hat{M})^{-1} U (I - \hat{M}), \hspace{1cm} (4.19)
\]

On comparison of equation (4.16) with (4.19), we get

\[
\begin{align*}
\hat{U}_1 &= 1 - \hat{M}_{11}, \\
&= 1 - \frac{2}{K(1 - \lambda_1^2)} \left( \frac{(1 + \lambda_1)e^p \cosh l}{\lambda_1 - \xi_1} + \frac{(\lambda_1 - 1)e^q \sinh k}{\xi_1 + \lambda_1} + \frac{(1 + \lambda_1)e^k \sinh q}{\xi_1 + \lambda_1} + \frac{(1 - \lambda_1)e^l \cosh p}{\xi_1 - \lambda_1} \right), \\
\hat{U}_2 &= -\hat{M}_{12}, \\
&= -\frac{2}{K(1 - \lambda_1^2)} \left( \frac{(1 + \lambda_1)e^p \cosh l}{\xi_1 - \lambda_1} + \frac{(\lambda_1 - 1)e^q \sinh k}{\xi_1 + \lambda_1} - \frac{(1 + \lambda_1)e^k \sinh q}{\xi_1 + \lambda_1} + \frac{(1 - \lambda_1)e^l \cosh p}{\xi_1 - \lambda_1} \right), \\
\hat{U}_3 &= -\hat{M}_{21}, \\
&= -\frac{2}{K(1 - \lambda_1^2)} \left( \frac{(1 + \lambda_1)e^{-p} \cosh l}{\xi_1 - \lambda_1} + \frac{(1 - \lambda_1)e^{-q} \sinh k}{\xi_1 + \lambda_1} + \frac{(1 + \lambda_1)e^{-k} \sinh q}{\xi_1 + \lambda_1} + \frac{(1 - \lambda_1)e^{-l} \cosh p}{\xi_1 - \lambda_1} \right), \\
\hat{U}_4 &= 1 - \hat{M}_{22}, \\
&= 1 - \frac{2}{K(1 - \lambda_1^2)} \left( \frac{(1 + \lambda_1)e^{-p} \cosh l}{\lambda_1 - \xi_1} + \frac{(1 - \lambda_1)e^{-q} \sinh k}{\xi_1 + \lambda_1} - \frac{(1 + \lambda_1)e^{-k} \sinh q}{\xi_1 + \lambda_1} + \frac{(1 - \lambda_1)e^{-l} \cosh p}{\xi_1 - \lambda_1} \right). \hspace{1cm} (4.20)
\end{align*}
\]
Now we consider the special case (reduction) when $\lambda = -\xi$ which gives $q = 0 = k$, $p(x, t) = u(x, t)$ and $l(x, t) = -u(x, t)$. Also $\hat{p}(x, t) = \hat{l}(x, t) = w(x, t)$, the expressions (4.20) becomes

$$
\begin{align*}
U_1 &= 1 - \lambda_1 \tanh p, \\
U_2 &= \lambda_1 e^{\hat{p}} \text{sech} p, \\
U_3 &= \lambda_1 e^{-\hat{p}} \text{sech} p, \\
U_4 &= 1 + \lambda_1 \tanh p.
\end{align*}
$$

Note that the expressions (4.21) are similar with that we obtained for the elementary Darboux transformation (4.10). When we compare equation (4.10) with equation (4.20), we clearly see that the solutions obtained by the elementary Darboux transformation are different from the solutions obtained by standard binary Darboux transformation which contains the contribution of both the direct and adjoint system. But if we take $\lambda = -\xi$, the solutions obtained from both the techniques become same as shown in expression (4.21). Consider the eigenvalue $\lambda_1 = e^{i\theta}$, also using expression (4.4), we may write $U[1]$ as

$$
U[1] = \begin{pmatrix} iU' & U_+ \\ -U_- & -iU'' \end{pmatrix},
$$

(4.22)

![Figure 1: Bright ($U'$) and Dark ($U_+$) soliton](https://mc06.manuscriptcentral.com/cjp-pubs)
where

\[
U' = 1 - (1 + \cos \theta) \tanh^2 u, \\
U_+ \equiv \bar{U}_- = -ie^v[(1 + \cos \theta) \tanh u + i \sin \theta] \sec hu.
\] (4.23)

From (4.23), we see that \(U^\dagger[1] = -U[1]\) and \(\Tr(U[1]) = 0\). Therefore, we can say that the above expression (4.23) is an explicit equation based upon \(SU(2)\) one-soliton solution of the GHM model presented in Fig. 1. The dark and bright type two-soliton solutions can be calculated by using the two sets of particular matrix solutions \(\Psi(x, t; \lambda_1)\) having \(\Lambda_1 = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \bar{\lambda}_1 \end{pmatrix}\) and \(\Psi(x, t; \lambda_2)\) with \(\Lambda_2 = \begin{pmatrix} \lambda_2 & 0 \\ 0 & \bar{\lambda}_2 \end{pmatrix}\), which are presented in Figs. 2 and 3. The interaction of dark and bright type two soliton are presented in Fig. 4.

![Figure 2: Dark type two-soliton: for the numerical values \(\lambda_1 = 1 + 0.09i, \lambda_2 = 1+0.003i\).](image)

![Figure 3: Bright type two-soliton: for the numerical values \(\lambda_1 = 1 + 0.09i, \lambda_2 = 1+0.003i\).](image)
So, we can obtain solutions in the form of direct and adjoint space parameters by using the standard binary Darboux transformation which cannot be obtained with the help of elementary Darboux transformation, so this is the advantage of the standard binary Darboux transformation. Similarly one can obtain explicit expression for multisoliton solution of the model.

5 Conclusion

In this paper, we have composed the elementary Darboux transformation of the generalized Heisenberg magnet model and calculated the standard binary Darboux transformation of the model. By iterating the standard binary Darboux transformation we have obtained the multisolitons of the model. We have also calculated the potential in the form of quasideterminant. We have also considered the case of generalized Heisenberg magnet model based on the Lie group $SU(2)$, and calculated the explicit expressions of Grammian solutions of the system. We can further apply this method on the supersymmetric version of generalized Heisenberg magnet model and quasi-grammian supersolitons can be obtained by using this technique. We can also compare it with the dressing method. Another outlook of this work is the bilinearization of the model.
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References


