HARMONIC DOMAIN ANALYSIS OF FIELD VARIABLES RELATED TO EDDY CURRENT AND HYSTERESIS LOSSES IN SATURATED LAMINATIONS

Nikola Rajaković* Adam Semlyen

Department of Electrical Engineering
University of Toronto
Toronto, Ontario, Canada

*On leave from University of Belgrade, Belgrade, Yugoslavia

Keywords: Harmonics, Laminations, Saturation, Transformers, Reactors, Hysteresis, Eddy Currents, Iron Losses.

Abstract - The paper gives a detailed analysis of the space and time distribution of the field variables B , H , and E inside saturating laminations, using the harmonic domain modeling method of Ref.[1]. A simple way has been devised for the representation of hysteresis. It is shown that saturation distorts the linear space distribution of the eddy currents but the assumption of linear distribution still leads to a very simple method for eddy current loss calculation which is remarkably accurate at 60 Hz.

The method of the paper is useful for performing accurate calculations in the presence of saturation and harmonics and for assessing simplified approaches.

INTRODUCTION

The analysis of the phenomena in a laminated iron core can be performed by computer simulation in the time domain with appropriate discretization of the laminations. In steady state however a phasor approach becomes of special interest being best fitted to the periodic nature of the phenomenon. Direct time domain simulation is for steady state problems a brute force approach requiring an excessive amount of computer time to reach steady state and possibly entailing significant error accumulation over the long time interval needed for the simulation.

The method of analysis used in this paper is a direct harmonic domain approach. It employs the methodology of our first paper on the topic of lamination modeling [1], but it analyzes the influence of all significant factors in more detail and it includes the representation of hysteresis. Previous work on the analysis of phenomena in laminations by different methods includes references [2],[3], and [4], while references [5] and [6] use the same basic method as the present paper but for other applications.

The significance of an accurate analysis of the behavior of a laminated iron core, taking into account both eddy currents and saturation with hysteresis, is related to the need for improved modeling of resonance and ferroresonance phenomena. A subsequent study will even address the problem of the quasi steady state phenomenon of inrush currents at the energization of a transformer. But gaining insight into what happens in the iron core, as a function of saturation level and type of excitation, is equally important.

The procedure of the paper is the following: the nonlinear magnetization characteristic is first approximated by a polynomial (this is needed for the direct harmonic domain analysis of systems with nonlinear components); then the laminations are discretized (into "sublaminations") and the related Maxwell equations are written in discretized form; these equations are then linearized, expressed in terms of incremental harmonic variables and solved by Newton's method.

88 SM 548-0 A paper recommended and approved by the IEEE Transformers Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1988 Summer Meeting, Portland, Oregon, July 24 - 29, 1988. Manuscript submitted January 26, 1988; made available for printing May 18, 1988.

The results of the paper are presented in two main sections. The first gives the space and time variation of the field variables and the effect on modeling of such factors as the applied input variable, truncation of harmonics, discretization of the laminations, and saturation level. The second section presents the exact loss calculation in the lamination and a simplified approach for the calculation of eddy current losses.

REVIEW OF THEORY

The general theory of harmonic domain analysis has been expounded in [1] and related mathematical concepts are presented in references [7] and [8]. Here we give only a short review of the main ideas in order to facilitate the understanding of the paper.

The harmonic domain analysis is based on the representation of a (time-) periodic function or variable

$$x = \sum_{h=-\infty}^{+\infty} x_h e^{jh\omega t} \quad (1)$$

by the harmonic vector

$$\mathbf{x} = [\dots x_h \dots]^T \quad (2)$$

Then the following remarks can be made:

- The equivalent of the multiplication xy of two time domain variables is, in the frequency domain, the convolution $\sum_k x_k y_{h-k}$ which we write simply as $\mathbf{x}\mathbf{y}$.
- The equivalent of time domain squaring x^2 is the self-convolution $\mathbf{x}\mathbf{x} = \mathbf{x}^2$.
- The equivalent of a power x^q is the repeated convolution $\mathbf{x}\mathbf{x} \dots \mathbf{x} = \mathbf{x}^q$.
- A nonlinear function of x , if expressed as a polynomial of x , has the harmonic domain equivalent also a polynomial with terms of the form $a_q \mathbf{x}^q$.
- The linearized form of a polynomial function $y=f(x)$ is $\Delta y = \mathbf{T}\Delta \mathbf{x}$, where $\mathbf{T}$ is the Toeplitz matrix of the elements of y' ; see [1] and [7].
- The equivalent of a time domain derivative of x is, in the harmonic domain, the product $\text{diag}(jh\omega)\mathbf{x}$.

The above statements describe the basics of the harmonic domain algebra. They are applied to the differential equations of a lamination

$$\frac{\partial H}{\partial x} = \sigma E \quad (3a)$$

$$\frac{\partial E}{\partial x} = \dot{B} \quad (3b)$$

which can be combined into

$$\frac{\partial^2 H}{\partial x^2} = \sigma \dot{B} \quad (4)$$

In these equations H is related to B by the nonlinear (polynomial) expression

$$H = a_1 B + a_p B^p + a_q B^q \quad (5)$$

We discretize the laminations in $2n$ layers which we call sublaminations. This defines the discrete points $0, 1, \dots, n$, point 0 being in the middle and point n at the surface of the lamination. Sublamination k is between points $k-1$ and k . For input and output, the variables of interest are E_n and H_n , at the surface of the lamination. They may be specified individually or by a linear combination of the two, as it will be discussed later.

The analysis is performed by using the discretized form of equations (3) and (4); for details see Ref.[1].

MODELING OF HYSTERESIS

In equation (5) H and B are periodic functions of time and the coefficients a_1 , a_p , and a_q are real. In the harmonic domain, H and B consist of complex phasors of harmonic frequencies and a complex coefficient a_1 could well take care of a phase shift between the components of B and H . This, in essence, can be the solution of hysteresis modeling since it produces the hysteresis loop at flux densities below saturation and independently of the magnitude of the base frequency, which is a fundamental characteristic of the hysteresis phenomenon. Thus, a complex a_1 produces a frequency independent phase shift, built into the magnetizing curve. The resultant losses will be proportional to $B_{\max}^2$. If a different exponent is desired (say, 1.6) one may have to experiment with the other two coefficients of the polynomial (5) also taken as complex.

The implementation of the idea of a complex coefficient a_1 has to satisfy the condition that the elements of the vector H are complex conjugate if B satisfies this requirement. For this it is necessary that a_1 be taken as the complex conjugate pair $a_1 = a'_1 \pm j a''_1$ the first value multiplying the positive first harmonic B_1 and the second value multiplying the negative first harmonic B_{-1} . All other harmonics will be multiplied by the real coefficient a'_1 . This assures that the hysteresis curve will be smooth, unaffected by the harmonics of the input B .

We note that the first value of a_1 must be in the first quadrant of the complex plane for having the components of H shifted forward with respect to those of B as needed for the correct reproduction of the hysteresis curve. The angle of a_1 can be estimated easily by examining the hysteresis curve: a'_1 is roughly equal to the slope H/B in the origin and a''_1 is the intercept on the H axis of a hysteresis loop with $B_{\max}=1$. The exact final values can be obtained by further experimentation.

Fig.1 illustrates the generation of hysteresis curves by the method described. The particular form of eqn.(5) being used in the harmonic domain, with H and B in S.I. units (see Ref.[1]), is

$$H = 51.8025B + 0.218B^{17} + 0.135B^{21} \quad (6a)$$

with

$$a_1 = 51.8025 \pm j30 \quad (6b)$$

for the first harmonic. To obtain this figure, the phasors H_h are calculated directly in the harmonic domain from eqns.(6) for B of different magnitudes. Then both B and H are expressed as in equation (1) and numerical values calculated for different values of time.

Fig.1 Magnetizing curves with hysteresis obtained by the harmonic domain calculations with eqn.(6)

FIELD VARIABLES

General Overview

Harmonic domain calculations are very powerful but their accuracy and the required computational expenditure depends on the retained number of harmonics and the discretization level (number of sublaminations). Insight can be gained into the field distribution inside the laminations by using the new approach. Of particular interest are the effect of input type (specified E_n or H_n or a combination of the two, at the lamination surface), and of the saturation level on the field distribution and on the iron losses. These topics will be examined in the following sections.

All computations have been performed for laminations of thickness $2d=0.33 \times 10^{-3}$ m, conductivity $\sigma=6 \times 10^5$ S/m and density $\rho=7.8 \times 10^3$ kg/m³. We have assumed $2n = 8$ sublaminations and harmonics up to order 15 for the base case (other values were also used). The base frequency is 60 Hz. D.c. offset and even harmonics have not been considered in this study.

Effect of Input Type

Three types of input have been employed:

- Ideal sinusoidal voltage source (E_n specified). This case is important because it approximates the operating conditions of transformers and rotating machines.
- Ideal sinusoidal current source (H_n specified).
- General source (a linear relation between E_n and H_n is specified).

We examine next the general case since it is appropriate for the analysis of all situations.

Equation (36) of reference [1] gives a linear relation between the surface variables of the form

$$\alpha H_n + \beta E_n = \gamma \quad (7)$$

On the other hand, the voltage-current relation of the external system can be given in the form of a Thevenin (or Norton) equivalent

$$v + Z_T i = v_T \quad (8)$$

Here

$$v = 2E_n w N n \quad (9a)$$

$$i = H_n h / n \quad (9b)$$

where

n – number of turns

N – number of laminations

w – width of laminations

h – height of the winding

Substitution of (9) into (8) yields an equation similar to (7), needed to have a completely defined problem.

Figures 2 a, b, and c represent the same six quantities over one period of time: B_4 and H_4 (at the surface of the lamination), B_0 and H_0 (at the center), and E_4 (at the surface) and E_2 (at the middle of the half-lamination). The figures differ by the type of input: in Fig.2a the input is a sinusoidal E_4 such that the peak flux density without eddy currents (B_{eff}) would be 1.65 T; in Fig.2b the input is a sinusoidal H_4 to produce the same B_{eff} ; and in Fig.2c the input is a general excitation, as described previously, with E_4 very close to the value in Fig.2a.

Fig.2a will be considered as reference for subsequent studies in which some of the parameters will be varied.

In Fig.2a we can see the penetration delay of $B(t)$ at the zero crossing, but this disappears at flux densities beyond the saturation level. Consequently, B peaks in all sublaminations at the same time and therefore E has the zero crossing in all layers simultaneously. An interesting fact is the non-smooth curve obtained for E_2 (in the middle of the half-lamination). This is not due to truncation of harmonics or discretization effects (as we have ascertained by careful testing). It can be explained by the character of the magnetizing curve with a fast varying slope beyond the saturation knee. Because of this, B has the strong variations of slope around the saturation point, as seen in the figure. The dips and peaks in E simply reflect the changes of slope of B .

Fig.2b indicates that the type of excitation affects dramatically the nature of all variables. In particular, B is now flat over a significant time interval and almost the same in all layers. No propagation effects can be discerned in any of the variables in the region of saturation. The truncation of harmonics is however visible. The effect can be alleviated at the cost of using more harmonics. This is however not necessary once we know the nature of this effect. As expected, the curves for E show sharp peaks since B varies fast around its zero crossing.

(a) Ideal sinusoidal voltage source (E_4)

Fig.2c pertaining to a general type of input shows much similarity to Fig.2a because of the relatively small value of the external impedance. E_4 is of course not perfectly sinusoidal. This situation is relevant to devices supplied from practical power sources. The frequency dependent impedance filters out the higher harmonics in the current so that the 15th harmonic in H_4 is in this case 6 times smaller than in the case of Fig.2a.

Fig.2
(c) General source

(b) Ideal sinusoidal current source (H_4)

Fig.2 Time variation of representative field variables

Effect of Truncation of Harmonics

Truncation of harmonics has the strongest effect in the situation of Fig.2b where the input is H_4 . Figure 3, obtained with 55 retained harmonics, demonstrates that the curve of B_4 can be made arbitrarily smooth.

It is clear that, in general, the deeper the saturation, the larger the number of harmonics needed for accurate modeling. However, since most practical cases will be closer to the situation of Fig.2a, which corresponds to a voltage source, the highest harmonic needed for modeling will most likely not exceed 15.

Fig.3 Time variation of B_4 and H_4 calculated with harmonics up to 55

Effect of Discretization

Fig.4 represents B and H at the surface and in the center of a lamination discretized into $2n=2$ layers with E at the surface as (sinusoidal) input. We note that, compared with the case of Fig.2a, there is a pronounced difference in B_0 , the flux density at the center, noticeable especially in the hump corresponding to the saturation knee. It is interesting to note that the two-layer discretization yields the same magnitude for the magnetic field at the center as at the surface which contradicts our expectation for discernible skin effect in the unsaturated case. For an analytical proof of this fact, see the Appendix. It is thus clear that a two-layer discretization is completely inadequate. We have found, for example, 40% error in the calculation of eddy current losses in comparison with the base case. On the other hand, $2n=16$ yields almost identical results as the base case with $2n=8$ sublaminations. For $2n=6$ we have found (for the same sinusoidal input E) a very small ($\approx 10^{-3}\%$) difference in the magnitude of the magnetic field variables, compared to the case with $2n=8$. However, there is a small phase shift difference in H at the surface and, since the active power losses are very sensitive to the phase shift between H_n and E_n , the error in the eddy current losses is $\approx 0.7\%$ if $2n=6$. Consequently $2n=8$ appears to be an appropriate upper limit for discretization at 60 Hz.

Fig.4 Time variation of B_n , B_0 , H_n , and H_0 calculated with only $2n=2$ sublaminations

Fig.5 Time variation of the same field variables as in Fig.2a but without saturation

Effect of Saturation

Fig.5 presents the ideal linear case because it serves to emphasize special features due to saturation. The same six variables are represented as in Fig.2a. It can be seen that skin effect is less pronounced with saturation, both in magnitude and in the phase shift related to the propagation of the field toward the center of the lamination, because saturation has the effect of producing almost identical curve shapes in the saturated region. This has produced the same zero crossing of E in all layers and is not true any more in the non-saturated case. E is of course smooth inside the lamination, in contrast to the case shown in Fig.2a.

Profiles of B and E

Figures 6a and 6b give three dimensional representations of B as a function of time and of depth inside the lamination for both the saturated and the unsaturated case and an ideal sinusoidal voltage source as input. The saturated case corresponds to $B_{eff} = 1.65$ T while the unsaturated case has been simulated for the perfectly linear magnetization characteristic corresponding to the first term of eqn.(5). The saddle point in Fig. 6a can be observed more clearly on the modified scale insert. Skin effect is manifest by the change of both magnitude and phase shift towards the center of the lamination. These effects gradually disappear with saturation. Clearly, B is not sinusoidal.

(a) Unsaturated case

(b) Saturated case

Fig.6 Time and space variation of B

A derivative of the representations of B are the three dimensional plots in Fig.7 of E and the corresponding profiles with respect to depth, for different values of time in Fig.8. Fig.7a corresponds to the unsaturated case, while Fig.7b corresponds to the saturated case of Fig.6b. From the latter one may note that a linear variation of E is expected where B is large and therefore constant. This appears around the zero crossing of E . Elsewhere saturation produces undulations and, correspondingly nonlinear profiles for E . It can be noted from figures 8a and 8b that the nonlinearity of the profiles corresponds to a lagging effect in the middle of the half-laminations. Since the eddy current losses are directly related to the space and time distribution of E , Fig.7b can be used to visualize the effect of saturation on the eddy current losses. It appears plausible that the undulations may affect the losses only in a small degree due to effects of cancellation. This problem will be examined more closely in the next section.

We note that hysteresis has no discernible effect on any of the profiles examined, except at low saturation levels.

Fig.7 Time and space variation of E

Fig.8 Profiles of E with sinusoidal time variation at the surface

CALCULATION OF IRON LOSSES

The losses in a lamination can be calculated with high accuracy using the Poynting vector at the surface of the lamination. The possible errors have been examined in previous sections and can be reduced arbitrarily by appropriate discretization and by keeping a sufficient number of harmonics. Therefore, this approach can serve as a basis for assessing any approximate calculation method.

A simple approach for the calculation of the losses due to eddy currents only is based on the observation, discussed previously, that the profile of E is linear, or almost linear on the average. With this assumption,

$$p(t) = 2 \int_0^d \sigma E(x,t)^2 dx = \frac{2\sigma d}{3} E(t)^2_n \quad (10a)$$

and the average or active power for a single lamination of 1 m^2 surface area results

$$P = \frac{2\sigma d}{3} E_{n,RMS}^2 \quad (10b)$$

where $E_{n,RMS}$ represents the RMS value of E at the surface, calculated as the root mean square of the peak values of the harmonics.

The simplified approach gives almost perfect results for the unsaturated case at 60 Hz with ideal sinusoidal voltage source (error less than 1%), and small errors in the saturated case (less than 3%). At 300 Hz the errors become of the order of 20%.

In the case of an impressed sinusoidal H the application of the simplified approach of equation (10b) requires the following steps. First, B has to be calculated, including all its harmonics, by solving (5) in the harmonic domain. We assume in this case that both H and B are uniformly distributed over the cross-section of the lamination. Since this is not true in reality, especially in the case of the unsaturated case, an error will exist in the value of the flux and in the corresponding E at the surface. Second, we calculate $E_{n,RMS}$, as defined above, and use it for the calculation of the eddy current losses. The resultant error at 60 Hz is up to about 20% in the unsaturated case but less than 1% in the saturated case when the flux distribution is in fact practically uniform. The discrepancies become of course larger at 300 Hz. There, the error in the saturated case is close to 10% and completely beyond any acceptable value in the unsaturated case.

Fig.9 gives the eddy current losses as a function of B_{eff} (this has been defined as the value of a uniform flux density corresponding to a flux equal to the actual one and thus being a measure for the flux), for a 60 Hz sinusoidal E_n or H_n as input. In the case of E_n being the input, the curve is quadratic, as expected, because E_n and B_{eff} are proportional and the losses are proportional to E_n^2 . In the case of H_n as the input, the losses are up to 6 times higher because saturation distorts the time variation of B in such a way that its zero crossing becomes very steep, with associated large E , as shown in Fig.2b. Of course, with decreasing saturation, the two curves get closer. We note that in practical conditions, pure H_n excitation is unlikely to be encountered. The figure illustrates however the significance of the type of excitation for the magnitude of eddy current losses.

Fig.9 Eddy current losses versus effective flux density B_{eff}

We did not represent the hysteresis losses in combination with the eddy current losses because they appear independently and because their modeling by equation (6b) is based on the knowledge of these losses as input. Therefore no new results can be expected. Fig.10 illustrates the mechanism of hysteresis losses for a slow sinusoidal voltage input to the lamination of Fig.1.

Fig.10 Variation of E (slow, sinusoidal, $B_{\max} = 1.4$ T), H , and p_{hys} for a lamination with hysteresis as in Fig.1

The calculated losses are in good agreement with data published in different sources, e.g. [9] and [10]. With our lamination data we have obtained for the base case (specified sinusoidal E_n at 60 Hz and 1.65 T) $P_{eddy} = 1.415$ W/kg and, with the commonly accepted ratio of hysteresis/eddy current losses of 2/3, the total losses are $P = 2.358$ W/kg. The corresponding value, for the same conditions, is in Ref.[10], page 95, $P = 2.4$ W/kg. Reference [9] gives also very close values. In order to obtain hysteresis losses as 2/3 of the eddy current losses, the imaginary part of the coefficient a_1 in eqn.(6b) should be 14.2.

CONCLUSIONS

The paper gives detailed and comprehensive insight into the space and time distribution of the field variables, B , H , and E , which describe the phenomena in a saturating laminated iron core. Such accurate and complete representation was previously not available because of the lack of an adequate method of analysis in the harmonic domain.

The harmonic domain analysis method of the paper permits a very easy representation of hysteresis. It was found that hysteresis affects to a small degree the variation of H and B but has no noticeable influence on E and thus, as expected, does not interact with the corresponding eddy current losses.

It appears that even though saturation renders B practically constant, the variation of E is less close to linear than in the unsaturated case. Still, the assumption of linear distribution of eddy currents leads to a very accurate and simple approach for the calculation of eddy current losses at 60 Hz.

Discretization into sublaminations is essential for the satisfactory analysis of the phenomena in laminations and calculation of losses. The method of the paper gives accurate and reliable values for losses at any frequency and saturation level, regardless of the type of input, and is thus useful for the calculation of some periodic phenomena as for instance ferroresonance.

ACKNOWLEDGEMENTS

Financial support from the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged. The first author wishes to express his appreciation to the Faculty of Electrical Engineering of the University of Belgrade for his study leave at the University of Toronto.

REFERENCES

- [1] A. Semlyen and N. Rajaković, "Harmonic Domain Modeling of Laminated Iron Core", IEEE Paper No. 88 WM 047-3.
- [2] J. Avila-Rosales and A. Semlyen, "Iron Core Modeling for Electrical Transients", IEEE. Trans. on Power Apparatus and Systems, Vol. PAS-104, No. 11, November 1985, pp. 3189-94.
- [3] A.E. Emanuel, "The Effect of Nonsinusoidal Excitation on Eddy Current Losses in Saturated Iron", IEEE Paper No. 87 WM 127-4.
- [4] I.D. Mayergoyz, F.M. Abdel-Kader, and F.P. Emad, "On Penetration of Electromagnetic Fields into Nonlinear Conducting Media", Journal of Applied Physics, Vol.55, No.3, 1 February 1984, pp. 618-629.
- [5] A. Semlyen, "Steady-State Analysis of Nonlinear Dynamic Systems with Periodic Excitation Based on Linearization in Harmonic Space", Canadian Electrical Engineering Journal, Vol.11 No.3, 1986, pp. 114-117.
- [6] A. Semlyen, E. Acha, and J. Arrillaga, "Newton-Type Algorithms for the Harmonic Phasor Analysis of Non Linear Power Circuits in Periodical Steady State with Special Reference to Magnetic Non-Linearities", IEEE Paper No. 87 WM 170-4.
- [7] G.H. Golub and C.F. Van Loan, "Matrix Computations", The John Hopkins University Press, 1983.
- [8] R. Vichnevetsky, "Computer Methods for Partial Differential Equations - Volume I", Prentice Hall, 1981.
- [9] "Electrical Engineering Handbook", Siemens AG, Heyden & Son Limited, 1976.
- [10] S. Austen Stigant and A.C. Franklin, "The J&P Transformer Book", 10th Edition, Newnes-Butterworths, 1976.

APPENDIX

Discretization of Equations (3) for Two Layers

With subscript 0 for the center and 1 for the surface of the lamination, trapezoidal discretization of (3) yields for harmonic h , taking into account that $E_0 = 0$,

$$H_1 - H_0 = \frac{d\sigma E_1}{2} \quad (\text{A-1a})$$

$$B_1 + B_0 = \frac{2E_1}{j\omega d} \quad (\text{A-1b})$$

Assuming that the input E_1 is pure imaginary (corresponding to a sine function), equations (A-1) yield

$$\text{Re}(H_1) = \text{Re}(H_0) \quad (\text{A-2a})$$

or

$$\text{Re}(B_1) = \text{Re}(B_0) \quad (\text{A-2b})$$

and

$$\text{Im}(B_1) = -\text{Im}(B_0) \quad (\text{A-2c})$$

Consequently, $B_1 = B_0^*$, and therefore $|B_1| = |B_0|$. This contradicts the fact that, especially in the unsaturated case, the flux density is not uniform. The resulting errors are significant, as shown in the main text.

Nikola Rajaković was born in Yugoslavia in 1952. He graduated from the University of Belgrade where he also received his M.Sc. and Ph.D. degrees. He is now an assistant professor in Power Systems at the same university. He has published over twenty papers and one textbook, and has worked in numerous power system projects. His present research interest is in Harmonic Modeling and Optimization of Power Systems.

Adam Semlyen (Fellow IEEE) was born and educated in Rumania where he obtained a Dipl. Ing. degree and his Ph.D. He started his career with an electric power utility and held an academic position at the Polytechnic Institute of Timisoara, Rumania. In 1969 he joined the University of Toronto where he is a professor in the Department of Electrical Engineering. His research interests include the Steady State and Dynamic Analysis of Power Systems, Electromagnetic Transients, and Power System Optimization.