On Bandwidth-Efficient Multiuser-Space-Time Signal Design and Detection

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Abstract—Signals designed for transmission over multiple transmit antennas are capable of achieving significant capacity gain. Traditional approaches aim at improving the single-user link with a centralized control over the set of transmit antennas. In this paper, by considering a set of independent and synchronized users communicating with the base station on the up-link, the joint signal can be viewed as space-time coded signal without a centralized control. Co-channel/inter-antenna interference presents a major impairment that limits the capacity. We propose a novel multiuser signal structure called interference-resistant modulation (IRM) to improve performance without coding nor bandwidth expansion. IRM can also be combined with fading-resistant modulation or space-time coding to yield additional gain when each user employs multiple transmit antennas. We prove that, both analytically and by simulations, the IRM with maximum-likelihood (ML) detection achieves the single-user performance asymptotically. Furthermore, to reduce the prohibitive complexity posed by ML detection, we propose a simple minimum-mean-square-error based precombining group detector and an interference cancellation scheme. It is shown that the proposed detector combined with IRM provides significant improvement over previous approaches.

Index Terms—Group detection, multiuser detection, multiuser signal design, multiple transmit antennas.

I. INTRODUCTION

RECENTLY, space-time signal design for multiple number of transmit/receive antennas has been attracting serious attention, partly due to the promise of remarkable spectral efficiency in theory and partly due to the development of unique signal structures [1], [2], which achieve high capacity in practice. This remarkable gain is owed to the transmitter's centralized control, which allows the elegant signal structure being built in a cooperative fashion over all transmit antennas. While there has been much success in this point-to-point single-user link, the issue of multiuser signal design has not gained as much popularity. Traditional approaches are mainly based on the use of orthogonal signaling, such as time division multiple access (TDMA) and code division multiple access (CDMA). In this paper, we design space-time signals for a group of uncoordinated/independent but synchronized users on the reverse link. The group of users' signals can be perceived as space-time coded signals on a single-user link without a centralized control; that is, the signal codeword from individual user is confined

to its own transmitting antennas. We first explore the possibilities of transmitting signals from all users without any bandwidth expansion nor coding. Hence, without orthogonal signaling, the co-channel interference presents a major impairment to the quality of the received signal. To combat this undesired effect, a novel signal structure called interference-resistant modulation (IRM) is proposed. IRM is a rotated form of multi-dimensional modulation, and signals from different users are distinguished by having unique angles of rotation associated with their signal constellations. The idea of rotated constellation has been addressed by several authors in the past [1], [10], yet mainly for the single-user link. A more recent approach suggested in [3] involves the design of rotated constellations for the multiuser system when the channel is impaired only by additive white Gaussian noise (AWGN) and, therefore, it is not suitable for fading channels. In our approach, the signal design is targeted for fading channels, which is a more realistic assumption, especially when antenna arrays are used at the transmitter/receiver. Furthermore, when each user deploys multiple transmit antennas, the conventional space-time code can be combined with IRM to enhance individual user's link performance while reducing the effect of co-channel interference. We also propose a joint rotation approach in which the IRM is combined with the previously proposed fading-resistant scheme [1] to provide interference-and-fading-resilient communication.

Next, to address the prohibitive complexity of maximum-likelihood (ML) detection and IRM, we propose a simple minimum mean square error (MMSE) precombining group detector. The principle behind precombining group detection is that a linear filter first suppresses the interference from users which are not included in the group of desired users. ML detection is then performed over the remaining target group of users. Yet, the group detection methods studied in [8] and [9] are based on linear decorrelator, which inevitably leads to noise enhancement. We will show that the proposed signal structure combined with the MMSE-based precombining group detector outperforms its decorrelator-based counterpart. Furthermore, an interference cancellation technique is introduced with the proposed group detector to yield additional gain.

For performance analysis, a thorough study on multiuser approach to narrow-band cellular communications can be found in [8]. That paper has shown that asymptotically, there is no loss in receive diversity gain in the presence of interfering users if ML multiuser detection is considered. The main contribution in this paper is the proposed IRM and ML detection, where we prove that the asymptotic performance (in terms of P_b , average probability of bit error) of any user is identical to that when there are no interferers. This result applies to any number of

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Fig. 1. Multiuser channel model.

transmit antennas each user employs, for both coded and uncoded system.

This paper is organized as follows. A brief system model of the multi-antenna system with multiple users is discussed in Section II. In Section III, the principle behind the IRM is explained and the design criteria are proposed. This section considers only one transmit antenna per user for introductory purpose. Then in Section IV, the IRM and space-time code are combined for signal transmission across multiple transmit antennas per user. In Section V, the interference-fading resistant modulation (IFRM) scheme is proposed to mitigate both effects, using IRM and previously derived results from fading-resistant modulation. Section VI contains various examples to illustrate the constructions of rotated constellation. The MMSE-based precombining group detector is introduced in Section VII. In Section VIII, the asymptotic performance of IRM is discussed in detail. Sections IX and X contain simulation results and concluding remarks, respectively.

II. SYSTEM MODEL

The channel model is depicted in Fig. 1. Each user is transmitting with M_T antennas while the receiver has M_R antenna elements. The channel is subject to frequency nonselective fading and the fading paths between each pair of transmit and receive antennas are independent. Let h_{nm}^i be the channel gain between transmit antenna n of user i and receive antenna m . It is modeled as zero-mean complex Gaussian random variable with variance 0.5 per dimension. The channel is quasistatic so that the channel gains remain constant for the transmission of a block of signals and the receiver has perfect knowledge of all the channel gains. At the receiver, the decision statistics are produced by the demodulators matched to the signal pulse waveform at symbol rate $1/T_s$. Assuming the signals from different

users are symbol-synchronous, the received signal at antenna m during time t is given by

$$r_t^m = \sum_{i=1}^N \sum_{n=1}^{M_T} h_{nm}^i c_t^{i,n} \sqrt{E_s} + \eta_t \quad (1)$$

where N is the number of users, $c_t^{i,n}$ is the symbol transmitted by antenna n of user i at time t , and the noise η_t is modeled as zero-mean AWGN with variance $N_0/2$ per dimension. The average signal power, E_s , is the same for all users. In this paper, no bandwidth expansion (such as spread-spectrum, additional time, or frequency channel) is considered for multiuser transmission. We consider only BPSK signals but as will be shown, the IRM will transform the BPSK signals into multilevel pulse amplitude modulated (PAM) signals.

III. DESIGN CRITERIA OF IRM

As an introductory section for IRM, the number of transmit antennas per user, M_T , is fixed to one. Let the row vector $\mathbf{c}^i = [c_1^i, \dots, c_l^i]$ denote the transmitted codeword of user i , where l is the block-length which is assumed to be greater than M_T . Note that in an uncoded system, a codeword represents a block of data symbols. Next, we may combine codewords from different users to form a *joint codeword*, a $N \times l$ matrix $\mathbf{C}$, in which the rows are $\mathbf{c}^i$, $1 \leq i \leq N$. The joint ML detection is considered and we proceed by computing the pairwise error probability. With ideal state information at the receiver, the probability of erroneously detecting the transmitted codeword $\mathbf{C}$ as codeword $\hat{\mathbf{C}}$ is approximated by [2]

$$P(\mathbf{C} \rightarrow \hat{\mathbf{C}} | h_{nm}^i \forall i, n, m) \leq \exp(-d^2(\mathbf{C}, \hat{\mathbf{C}})E_s/4N_0) \quad (2)$$

where

$$d^2(\mathbf{C}, \hat{\mathbf{C}}) = \sum_{m=1}^{M_R} \sum_{t=1}^l \left| \sum_{i=1}^N h_{nm}^i (c_t^i - \hat{c}_t^i) \right|^2. \quad (3)$$

As in [2], we define the error matrix as $\mathbf{B}(\mathbf{C}, \hat{\mathbf{C}}) = \hat{\mathbf{C}} - \mathbf{C}$. The problem we tackle here is the same as that of space-time coding, in which multiple antennas are used for the single-user link, except that now each transmit antenna is not coordinated. The design criteria for original space-time codewords are summarized as follows [2].

- *The Rank Criterion:* Maximize the minimum rank of $\mathbf{B}(\mathbf{C}, \hat{\mathbf{C}})$ over all pairs of distinct codewords.
- *The Determinant Criterion:* Maximize the minimum coding advantage, which is given by, $d_H(\mathbf{C}, \hat{\mathbf{C}}) = \prod_{i=1}^r \lambda_i$, the product of eigenvalues of matrix $\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}}) = \mathbf{B}(\mathbf{C}, \hat{\mathbf{C}})\mathbf{B}(\mathbf{C}, \hat{\mathbf{C}})^H$, taken over all pairs of distinct codewords, for a given target transmit diversity gain r

where $(\cdot)^H$ denotes the conjugate transpose. Due to the restriction that each user's own codeword is limited to its transmitting antenna, the above design criteria are not sufficient to generate good signal design. To understand this, we first focus on the two-users scenario. Let $\mathbf{x} = (\hat{c}_1^1 - c_1^1, \hat{c}_2^1 - c_2^1, \dots, \hat{c}_l^1 - c_l^1)$ and

$\mathbf{y} = (\hat{c}_1^2 - c_1^2, \hat{c}_2^2 - c_2^2, \dots, \hat{c}_l^2 - c_l^2)$ denote the pairwise difference of codewords transmitted by user 1 and 2, respectively. We may express $\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}})$ as follows:

$$\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}}) = \begin{bmatrix} \mathbf{xx}^H & \mathbf{xy}^H \\ \mathbf{y}^H \mathbf{x} & \mathbf{yy}^H \end{bmatrix} \quad (4)$$

The term $\mathbf{xy}^H$ and its complex conjugate approximately quantify the effect of inter-antenna interference (i.e., co-channel interference) on the pairwise error probability. The traditional way to eliminate interference is by introducing orthogonal signaling, such as TDMA or CDMA. To see this, assume that the transmission rate is 0.5 b/s/Hz for each user (i.e., the sum-rate is 1 b/s/Hz). For the first $l/2$ time slots, each user transmits with full-rate 1 b/s/Hz (BPSK). And for the second $l/2$ time slots, user 1 repeats transmitting the previous signals, while user 2 transmits the negative replica of its previous signals. Hence, the matrix $\mathbf{B}(\mathbf{C}, \hat{\mathbf{C}})$ and $\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}})$ can be expressed as follows:

$$\mathbf{B}(\mathbf{C}, \hat{\mathbf{C}}) = \begin{bmatrix} \mathbf{x} & \mathbf{x} \\ \mathbf{y} & -\mathbf{y} \end{bmatrix}, \quad \mathbf{A}(\mathbf{C}, \hat{\mathbf{C}}) = \begin{bmatrix} 2\mathbf{xx}^H & 0 \\ 0 & 2\mathbf{yy}^H \end{bmatrix}. \quad (5)$$

Hence, the co-channel interference is totally eliminated. In fact, the above formulation is equivalent to modulating the signals with orthogonal sequence as in CDMA and, therefore, can be generalized to any number of users. Nevertheless, the requirement is the expansion of bandwidth in proportion to the number of users.

In this paper, our goal is to minimize the co-channel interference without bandwidth expansion. Suppose both users are transmitting at 1 b/s/Hz continuously without any coding nor orthogonal signaling. The detrimental effect is that the matrix $\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}})$ is no longer diagonal, and some pairs of joint codewords reduce the rank of $\mathbf{A}$ to one (e.g., $\mathbf{x} = \mathbf{y}$). To mitigate this effect introduced by co-channel interference, we propose the use of IRM. Its construction involves the multiplication of a rotation matrix to the original codeword and is analogous to the idea of fading-resistant modulation [1] and signal-space diversity [10]. Specifically, given a codeword $\mathbf{c}$, we right-multiply it by an orthogonal rotation matrix $\mathbf{R}$. Each user is assigned a unique rotation matrix, which applies to all codewords belonging to the same user, and each joint codeword takes the following form:

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}^1 \mathbf{R}_1 \\ \mathbf{c}^2 \mathbf{R}_2 \\ \vdots \\ \mathbf{c}^N \mathbf{R}_N \end{bmatrix} \quad (6)$$

where $\mathbf{R}_i, i = 1, \dots, N$ are the corresponding rotation matrices for users 1 to N . It is easy to see that the single-user performance is invariant to such rotational transform. For two-users scenario, we only need to apply rotation to one user. We replace $\mathbf{y}$ by $\mathbf{yR}_2$ and note the results as follows:

$$\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}}) = \begin{bmatrix} \mathbf{xx}^H & \mathbf{xR}_2^H \mathbf{y}^H \\ \mathbf{yR}_2 \mathbf{x}^H & \mathbf{yy}^H \end{bmatrix}. \quad (7)$$

We now investigate the effect of the rotation matrix on the joint codeword with multiple users. In general, certain joint codeword pairs, with errors belonging only to one user (e.g., $\mathbf{x} = \mathbf{0}, \mathbf{y} \neq \mathbf{0}$ in above example), experience a transmit diversity gain of one (rank $\mathbf{B} = 1$), regardless of any rotation

Fig. 2. Transmitter structure for a multiuser system with IRM.

applied to the codewords. Rotation also has no effect on the coding gain associated with such codeword pairs. From the traditional space-time coding principle, which aims at minimizing the maximum pairwise error probability, we know that these lowest-rank-and-coding-gain codeword pairs, which limit the system performance at high signal-to-noise ratio (SNR), are the only optimization factors considered. Yet, the diversity gain of a number of codeword pairs, with errors on different users (e.g., $\mathbf{x} \neq \mathbf{0}, \mathbf{y} \neq \mathbf{0}$), can be increased by applying proper rotations. The maximum transmit diversity gain achievable by a joint codeword pair is equal to $\min(l, z)$, where z is the number of nonzero rows in $\mathbf{B}$. As we will show, by increasing the rank associated with these codeword pairs, the co-channel interference is suppressed at high SNR. Hence, the new rank criterion appropriate in this multiuser setting is that it should handle different codeword pairs according to $\min(l, z)$. For the new determinant criterion, we take a more heuristic approach: one may consider codeword pairs with possibilities of achieving a diversity gain of $\min(l, z) = 2$, as they present the greatest pairwise error sources out of all those which can be attacked through proper rotations. Although the significant of coding gains associated with these codeword pairs diminishes at high SNR, it is worth maximizing them from a performance optimization point of view. As a result, given N users, we choose $\mathbf{R}_i, 1 \leq i \leq N$ according to the following criteria.

- **Rank Criterion:** For any joint codeword pair, increase the rank of matrix $\mathbf{B}$ to r , where $r = \min(l, z)$, and z is the number of nonzero rows in matrix $\mathbf{B}$.
- **Determinant Criterion:** Over all joint codeword pairs with $\min(l, z) = 2$, maximize the minimum coding gain, $\prod_{i=1}^2 \lambda_i$, where $\lambda_i, i = 1, 2$ are the eigenvalues of the corresponding matrix $\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}})$.

Fig. 2 depicts the block diagram for construction of multiuser signals using IRM.

The complexity in performing ML detection of IRM signals depends on the number of users and the code structure. For uncoded system, the number of codewords (i.e., blocks of data symbols) increases exponentially with the block-length. Hence, the complexity is $\mathcal{O}(2^{lN})$. For coded system, we may take convolutional code as an example. As rotation introduces memory

into the code symbols, the number of states in the trellis increases exponentially with the size of the block that is under rotation. Since the block-length of a codeword is usually very large, we may alternatively divide the entire codeword into equal number of codeword segments, each of which has length $l_s < l$. Rotation (with size $N \times l_s$) can be readily applied over each segment such that the resulting codewords satisfy the same criteria discussed above. To further simplify the process, one may choose to rotate as few number of segments as possible while the rank criterion is satisfied (only the rank criterion is required for achieving single-user performance, see Section VIII). Now, we assume that the code has rate k/n and $l_s = n$ so that the memory's length remains unchanged compared with non-IRM. Using similar argument as in [4], we find that the complexity is $\mathcal{O}(2^{N\nu})$ where ν is the code constraint length.

IV. IRM WITH SPACE-TIME CODES

In this section, we demonstrate that the IRM can be employed in the general case when each user employs space-time code with arbitrary number of transmit antennas. The number of interfering signals is increased to $N \times M_T - 1$, including the inter-antenna self-interference. As we will see in Section VIII, with IRM, the asymptotic performance is not affected by such enhanced impairment as the single-user performance (a user with M_T transmit antennas) remains achievable.

Again, we assume that the code block-length, $l > M_T$. The construction of multiuser signals consists of two steps: 1) To maintain bandwidth efficiency, each user employs the identical space-time coding scheme appropriate for the number of transmit antennas it has without any bandwidth expansion. These single-user space-time coding schemes can be of any type developed in the past [2], [6]. 2) IRM is then constructed by applying the same rotation matrix $\mathbf{R}_i$ to the signals from each transmit antenna of a particular user [same as (6), except that each codeword vector is replaced by the corresponding $M_T \times l$ matrix]. The rank criterion and determinant criterion considered in choosing different $\mathbf{R}_i$ for different users are basically the same as those when $M_T = 1$. However, now, the joint codeword pairs considered for maximizing the minimum coding gain are those with $\min(l, z) = 2 \cdot M_T$, provided that each individual user's code is able to achieve full transmit diversity gain M_T .

It then remains to show that the application of IRM has no effect on the space-time coding scheme employed. The following proposition is required.

Proposition 1 [5]: Unitary transformation u is an isometry with respect to d_H , i.e., $d_H(u(\mathbf{C}), u(\hat{\mathbf{C}})) = d_H(\mathbf{C}, \hat{\mathbf{C}})$ where $d_H(\mathbf{C}, \hat{\mathbf{C}}) = \prod_{i=1}^r \lambda_i$, i.e., the product of eigenvalues of matrix $\mathbf{A}(\mathbf{C}, \hat{\mathbf{C}})$.

Since the interference-resistant rotation is unitary, the d_H remains unchanged. The performance of the signaling scheme is, therefore, invariant to the rotation of IRM.

V. INTERFERENCE-FADING RESISTANT MODULATION (IFRM)

Besides co-channel interference, fading is another major impairment that limits the capacity of wireless systems. The method in [1] is an effective fading-resistant scheme

which can exploit full transmit diversity gain from multiple antennas. As this fading-resistant scheme and the proposed method both require rotations of the original constellation, it is natural to study the effect of joint rotations and construct the IFRM. For the fading-resistant modulation, each user deploys $M_T > 1$ transmit antennas without any coding, while orthogonal signaling (e.g., TDMA or CDMA) is used to eliminate inter-antenna interference from the same user. Let $\mathbf{d}_i = [d_1^i \ d_2^i \ \dots \ d_l^i]$ be the block of l data symbols belonging to user i . If $M_T = l$, the mapping from the data stream to different antennas/time-slots is as follows:

$$\mathbf{d}_i = [d_1^i \ d_2^i \ \dots \ d_l^i] \rightarrow \mathbf{C}_i = \text{diag}(d_1^i, d_2^i, \dots, d_l^i) \cdot \mathbf{\Gamma} \quad (8)$$

where $\mathbf{C}_i$ is the transmitted signal codeword of the i th user and $\mathbf{\Gamma}$ is an orthogonal matrix with which the inter-antenna interference is eliminated (i.e., TDMA or CDMA when $\mathbf{\Gamma}$ is an identity or Hadamard matrix, respectively); from now on, unless otherwise stated, TDMA is assumed. It is clear that at the receiver, the decision statistic for user i , say, during time t is the sum of independently faded d_t^i plus noise, where $1 \leq t \leq l$. The fading-resistant constellation is constructed by multiplying each signal vector by a rotation matrix, i.e., $\mathbf{d}_i \mathbf{R}_i$, where $\mathbf{R}_i$ is chosen based on the following criterion:

- *Symbol Product Distance:* Maximize the minimum symbol product distance given by $d_P = \prod_{j=1}^{M_T} (d_j - \hat{d}_j)^2$ over all pairs of $\mathbf{d}$ and $\hat{\mathbf{d}}$.

It was shown in [1] that rotated constellation brings about a significant improvement. Now, we generalize the above scenario to multiuser transmission without bandwidth expansion. We assume that the block-length l considered is equal to $M_T \cdot N$ and, therefore, the number of data symbols transmitted through the same antenna within a block is $l/M_T = N$. These co-antenna data symbols form a subblock $\mathbf{c}_b^i = [c_{(b-1)N+1}^{i_b} \ \dots \ c_{bN}^{i_b}]$, with the superscript i_b denoting the antenna b of user i , where the subblock b is transmitted. The number of such subblocks, each undergoing independent fading, is $l/N = M_T$. In this case, the single-user performance is determined by the Product-Euclidean distance (d_{PE})

$$d_{PE}(\mathbf{C}_i, \hat{\mathbf{C}}_i) = \prod_{b=1}^{M_T} |\mathbf{c}_b^i - \hat{\mathbf{c}}_b^i|^2. \quad (9)$$

The joint codeword is simply formed by stacking the individual users' codewords, $\mathbf{C} = [\mathbf{C}_1 \ \mathbf{C}_2 \ \dots \ \mathbf{C}_N]^T$. For example, when $N = 2$, $M_T = 2$, $l = 4$ and $\mathbf{\Gamma} = \mathbf{I}$, it follows that:

$$\mathbf{C} = \begin{bmatrix} d_1^1 & d_2^1 & 0 & 0 \\ 0 & 0 & d_3^1 & d_4^1 \\ d_1^2 & d_2^2 & 0 & 0 \\ 0 & 0 & d_3^2 & d_4^2 \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{c}_1^1 & 0 \\ 0 & \mathbf{c}_2^1 \\ \mathbf{c}_1^2 & 0 \\ 0 & \mathbf{c}_2^2 \end{bmatrix}. \quad (10)$$

Next, if we assume that each user only employs *one* antenna, another equivalent channel model, called block-wise fast fading, can be introduced. In this channel, the block of signals is partitioned into subblocks, and each subblock undergoes independent fading while the symbols within each subblock undergo static flat fading. Clearly, it is identical to the aforementioned

model where multiple transmit antennas and orthogonal signaling are used. The following analysis applies to both models.

The pairwise error probability between two joint codewords can be expressed as in (2) by

$$d^2(\mathbf{C}, \hat{\mathbf{C}}) = \sum_{m=1}^{M_R} \sum_{b=1}^{M_T} \left| \sum_{i=1}^N h_{bm}^i (\mathbf{c}_b^i - \hat{\mathbf{c}}_b^i) \right|^2 \quad (11)$$

where b is the index for subblock. We have explicitly shown the role of subblocks in the above expression. At high SNR, the pairwise error probability can be shown to be

$$P(\mathbf{C} \rightarrow \hat{\mathbf{C}}) \leq \prod_{b=1}^{M_T} \left[\left(\prod_{k=1}^{r_b} \lambda_{b,k} \right)^{-M_R} (E_s/4N_o)^{-r_b M_R} \right] \quad (12)$$

where $\lambda_{b,k}, k = 1, 2, \dots, r_b$ are the eigenvalues of matrix $\mathbf{A}(b) = \mathbf{B}(b)\mathbf{B}(b)^H$ having rank r_b , which consists of only co-channel subblocks b from each user, i.e., $\mathbf{B}(b) = \mathbf{C}(b) - \hat{\mathbf{C}}(b)$, where $\mathbf{C}(b) = [(\mathbf{c}_b^1} \ (\mathbf{c}_b^2} \ \dots \ (\mathbf{c}_b^N)]^T$. It follows that the rank of $\mathbf{B} (= \mathbf{C} - \hat{\mathbf{C}})$ is equal to the sum of ranks $\sum_{b=1}^{M_T} r_b$. The signal design criterion shall aim at achieving full transmit diversity gain (M_T) associated with each user. This is done by maximizing the minimum Product-Euclidean distance over all distinct codeword pairs belonging to each individual user. To minimize the interference effect using IRM, the criteria are the same as those of IRM with space-time codes. Due to orthogonal signaling, the determinant criterion reduces to the following:

- *Product-Determinant Criterion:* Maximize the minimum coding gain given by $\prod_{b=1}^{M_T} d_H(\mathbf{C}(b), \hat{\mathbf{C}}(b))$ for joint codeword pairs with $\min(l, z) = 2M_T$.

Since we are considering joint rotations with large block-length l , the degrees of freedom associated with rotations grow significantly. To reduce complexity, we devise the following suboptimal algorithm in choosing good rotation angles for the IFRM.

- Step 1: Note that each subblock is of equal length. These subblocks are divided and re-grouped: The k th group is formed by taking the k th bit from each subblock, e.g., user 1's k th group is $g_k = [c_k^1, c_{N+k}^1, \dots, c_{N(M_T-1)+k}^1]$. An identical fading-resistant rotation matrix is applied to each group ($1 \leq k \leq M_T$) and for all users.
- Step 2: Construct IRM independently for signals in each $\mathbf{C}(b)$, $1 \leq b \leq M_T$.

Thus, the joint problem is decomposed into two individual problems which were considered previously. Furthermore, due to the fact that the coding gain can be expressed as the product of determinants, the dimension for IRM rotation is reduced to that of the subblock. Step 1 basically performs the fading-resistant rotation to combat fading as if there were no interferers. To justify the validity of the above algorithm, it is required to prove that the IRM applied in Step 2 does not destroy the fading-resistant property achieved from Step 1. Proposition 1 is sufficient for such a proof, since the single-user performance metric d_{PE} (9), which is equal to d_H (when orthogonal signaling is used among transmit antennas), is unchanged after rotation. The main function of IRM is, therefore, to increase the rank of the error matrix

$\mathbf{B}$ of the joint-codeword pairs without affecting the single-user d_{PE} . Likewise, in block-wise fast fading channels, rotation can be applied across several signal subblocks and the IFRM will work equally well.

VI. CONSTRUCTION OF ROTATED CONSTELLATIONS

As l is the time span of one codeword, each codeword may then represent a constellation point in a l -dimensional space. To satisfy the criteria listed in previous sections, constellations for different users are rotated uniquely with distinct angles. One method of obtaining the best set of angles is to vary $\binom{l}{2}$ rotation angles for each user according to a numerical optimization or search algorithm. In the following, we demonstrate several examples of IRM for different number of transmit antennas per user. Recall that N and M_T is the number of users, and the number of transmit antennas per user, respectively.

A. IRM

Case 1) $N = 2, l = 2, M_T = 1$. The rotation matrix can be expressed as

$$\mathbf{R} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}. \quad (13)$$

Due to symmetry, only the second user is required to rotate its constellation. Running the search using a discretization interval of 0.1° , we obtain an optimal angle of 63.4° . With this angle, the rank criterion is satisfied (i.e., error matrix, $\mathbf{B}$, with rank 1 and nonzero errors scattered among both users, is transformed to a matrix of rank 2) and the minimum determinant over all possible rank-2 matrices $\mathbf{A}$ s is found to be 3.188.

Case 2) $N = 2, l = 4, M_T = 1$. For four-dimensional constellation, there are six degrees of freedom. It takes too long to do an exhaustive search and we turn to an iterative construction of the Hadamard matrix as used in [10]

$$\mathbf{R} = \begin{bmatrix} \cos \theta_1 & \sin \theta_1 & -\cos \theta_2 & -\sin \theta_2 \\ -\sin \theta_1 & \cos \theta_1 & \sin \theta_2 & -\cos \theta_2 \\ \cos \theta_2 & \sin \theta_2 & \cos \theta_1 & \sin \theta_1 \\ -\sin \theta_2 & \cos \theta_2 & -\sin \theta_1 & \cos \theta_1 \end{bmatrix}. \quad (14)$$

Due to the unitary and orthogonality constraint, $\theta_2 = 180^\circ + \theta_1$. We run the search using intervals of 0.1° and obtain $\theta_1 = 26^\circ, \theta_2 = 206^\circ$. The rank criterion is satisfied and the minimum determinant is found to be 6.3312.

Case 3) $N = 3, l = 2, M_T = 1$. In this case, the first user's constellation remains unchanged while two unique rotation matrices are generated, one for each remaining user. Note that the maximum rank produced by an error matrix involving more than one user is two. Running the search using a discretization interval of 1° , we obtain an optimal angle of 30° and 60° , respectively, for each rotation matrix. With these angles, the rank criterion is satisfied and the minimum determinant over all possible rank-two error matrices is found to be 2.14.

Case 4) $N = 2, l = 4, M_T = 2$ and space-time block code (STBC). In this example with two users, each user employs two transmit antennas and STBC is considered. As $l = N \cdot M_T = 4$, a block with length of four is constructed by concatenating two 2×2 STBC [6], which is originally proposed for two transmit antennas over two symbol periods. That is, given $\mathbf{d}$, the following mapping is applied:

$$[d_1 d_2 d_3 d_4] \rightarrow \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \\ -d_2 & d_1 & -d_3 & d_4 \end{bmatrix} \quad (15)$$

where the entry of row i and column j represents the symbol transmitted from antenna i during time j . While both users are using the same STBC, rotation is applied to one user only. Since $l = 4$, the degree of freedom for rotation increases to six and an exhaustive search becomes infeasible. Using a procedure similar to [1], a random number of points are selected and deepest gradient search is performed. A local maximum (which is, therefore, suboptimal) is found as $(45^\circ, 45^\circ, 45^\circ, 45^\circ, 45^\circ, 45^\circ)$. The rank criterion is satisfied and the minimum determinant over all possible rank-four error matrices is found to be one.

B. IFRM

Example: $N = 2, l = 4$, block-wise fast fading with fading period = 2. In this example, each independently faded subblock is of length 2. Using the procedure proposed in the last section, the first step is the multiplication by a fading-resistant rotation matrix. The data block $\mathbf{d} = [d_1 d_2 d_3 d_4]$ is first divided into two groups: $g_1 = [d_1 d_3]$ and $g_2 = [d_2 d_4]$. The rotation matrix applied to each group is in the same form as (13). Using the result from [1], the angle is found as 31.7° . Note that this step is repeated for all users. Next, for the second user, the IRM is applied for its two groups: $[d_1 d_2], [d_3 d_4]$. The rotation matrix is also in the same form as (13). Using a discretization interval of 1° , the two angles found for these two groups are 16° and 36° , respectively. The rank criterion is satisfied and the minimum determinant is found to be 0.01.

VII. GROUP MMSE DETECTOR

To reduce the prohibitive complexity of ML joint detection, we propose a groupwise MMSE detector which provides a balance tradeoff between complexity and performance. Given N users, the IRM is constructed independently over an individual group of G users, where G is chosen according to the complexity and performance requirement. The basic principle of detection is to perform ML detection over the target group and treat the remaining groups as interference. A novel feature which makes the proposed detector distinct from previous ones [9] is that it is based on MMSE detection, rather than the decorrelator.

In this section, we assume that each user only employs one transmit antenna for simplicity, and it is straightforward to generalize to $M_T > 1$. Assume that G is equal to l and number of groups is equal to $N_g = N/G$. Let the code vector for the i th group be $\mathbf{d}_i = [c_1^{i1}, c_1^{i2}, \dots, c_1^{iG}, c_2^{i1}, \dots, c_l^{iG}]^T$, where the superscript ik represents the k th user in group i . Without loss of

generality, assume that group 1 is our desired group. Then, by stacking the sampled outputs with temporal ordering, we denote the received vector as $\mathbf{r} = [r_1^1, r_1^2, \dots, r_1^{M_R}, r_2^1, \dots, r_l^{M_R}]^T$, which can be expressed as follows:

$$\mathbf{r} = \mathbf{H}_1 \mathbf{d}_1 + \sum_{i=2}^{N_g} \mathbf{H}_i \mathbf{d}_i + \eta \quad (16)$$

where $\mathbf{H}_i$ is the i th group's channel matrix which is block-diagonal and defined as

$$\mathbf{H}_i = \text{diag}\{\mathbf{H}_i(1), \mathbf{H}_i(2), \dots, \mathbf{H}_i(l)\} \quad (17)$$

and

$$\mathbf{H}_i(\mathbf{t}) = \begin{bmatrix} h_{11}^{i1} & h_{11}^{i2} & \dots & h_{11}^{iG} \\ h_{12}^{i1} & h_{12}^{i2} & \dots & h_{12}^{iG} \\ \dots & \dots & \dots & \dots \\ h_{lM_R}^{i1} & h_{lM_R}^{i2} & \dots & h_{lM_R}^{iG} \end{bmatrix}, \quad 1 \leq t \leq l. \quad (18)$$

Thus, $\mathbf{H}_i$ contains the quasistatic fading path gains from users in group i to receive antennas.

The MMSE-based detector is to satisfy the following criterion:

$$\min \|\mathbf{W}\mathbf{r} - \mathbf{d}_1\|^2 = \min \text{tr}[(\mathbf{W}\mathbf{r} - \mathbf{d}_1)(\mathbf{W}\mathbf{r} - \mathbf{d}_1)^H] \quad (19)$$

where $\mathbf{W}$ is a $lG \times lG$ matrix. The resulting vector $\mathbf{y} = \mathbf{W}\mathbf{r}$ contains the decision variables for the desired signal group. Using standard minimization techniques, the optimal $\mathbf{W}_{\text{opt}}$ is found as

$$\mathbf{W}_{\text{opt},1} = \mathbf{H}_1^H \mathbf{R}_{\text{corr}}^{-1} \quad (20)$$

where

$$\mathbf{R}_{\text{corr}} = \sum_{i=1}^{N_g} \mathbf{H}_i \mathbf{H}_i^H + N_0^2 \mathbf{I}. \quad (21)$$

The next step is to perform joint ML detection by the following rule:

$$\hat{\mathbf{d}}_1 = \arg \min_{\mathbf{d}} (\mathbf{r} - \mathbf{W}_{\text{opt},1} \mathbf{H}_1 \mathbf{d}_1)^H \times \mathbf{Q}_1^{-1} (\mathbf{r} - \mathbf{W}_{\text{opt},1} \mathbf{H}_1 \mathbf{d}_1) \quad (22)$$

where $\mathbf{Q}_1 = \mathbf{W}_{\text{opt},1} (\sum_{i=2}^{N_g} \mathbf{H}_i \mathbf{H}_i^H + N_0^2 \mathbf{I}) \mathbf{W}_{\text{opt},1}^H$.

The complexity of the group MMSE detection method is dominated by the calculation of the MMSE filter's coefficients $\mathbf{W}_{\text{opt}}$ and the ML detection performed over the target group. The calculation of $\mathbf{W}_{\text{opt}}$ involves the matrix inversion $\mathbf{R}_{\text{corr}}^{-1}$, which can be computed based on the recursive method

$$\Psi^{(0)} = N_0^{-2} \mathbf{I} \quad (23)$$

$$\mathbf{u} = \begin{bmatrix} h_{11}^{ij} & h_{12}^{ij} & \dots & h_{lM_R}^{ij} \end{bmatrix}^T \quad i = 1, 2, \dots, N_g, \quad j = 1, 2, \dots, G \quad (24)$$

$$\Psi^{(k)} = \Psi^{(k-1)} - \frac{\Psi^{(k-1)} \mathbf{u} \mathbf{u}^H \Psi^{(k-1)}}{1 + \mathbf{u}^H \Psi^{(k-1)} \mathbf{u}}. \quad (25)$$

Using this method, the dominant computation is the $N \cdot M_R^3$ multiplications. To calculate $\mathbf{Q}^{-1}$, we may use the similar recursive method while $\mathbf{u}$ represents the columns in $\mathbf{W}_{\text{opt}} \mathbf{H}$. The

computation involves $N \cdot G^3$ multiplications. Combined with the computations required in the ML detection, the total complexity is $\mathcal{O}(2^{lG} + N(M_R^3 + G^3))$.

A. Group Detector With Interference Cancellation (IC)

The performance of the group MMSE detector can be further improved when combined with IC. First, similar to [7], the *reliability* of a group j is defined as follows:

$$\Delta_j = \min_{\mathbf{d}} (\mathbf{r} - \mathbf{W}_{\text{opt},j} \mathbf{H}_j \mathbf{d}_j)^H \mathbf{Q}_j^{-1} (\mathbf{r} - \mathbf{W}_{\text{opt},j} \mathbf{H}_j \mathbf{d}_j) \quad (26)$$

where

$$\mathbf{W}_{\text{opt},j} = \mathbf{H}_j^H \mathbf{R}_{\text{corr}}^{-1}, \quad \text{and} \quad \mathbf{R}_{\text{corr}}^{-1} = \sum_{i \in U} \mathbf{H}_i \mathbf{H}_i^H + N_0^2 \mathbf{I} \quad (27)$$

$$\mathbf{Q}_i = \mathbf{W}_{\text{opt},j} \left(\sum_{i \neq j, i \in U} \mathbf{H}_i \mathbf{H}_i^H + N_0^2 \mathbf{I} \right) \mathbf{W}_{\text{opt},j}^H \quad (28)$$

and, we denote U as the set of groups which have not been subtracted during the iterative IC procedure. The reliability indicates the accuracy of detecting group j . The iterative IC algorithm works as follows. During each stage, the group with the best reliability is chosen and hard decisions are made for symbols within that group. These decisions are fed back to the detector and subtracted from the incoming signal of that stage. The remaining signals are processed repeatedly in the next stage and the process terminates when all groups have been detected. Our algorithm can now be summarized as follows.

- 1) Rank the Δ_j , where $j \in U$ according to (26).
- 2) $\hat{\mathbf{d}}_k$ is obtained using (22), where $\Delta_k < \Delta_j$, $k \neq j, \forall j \in U$.
- 3) $\mathbf{r} = \mathbf{r} - \mathbf{H}_k \hat{\mathbf{d}}_k$, $U = U - k$
- 4) Repeat 1. Stop when $U = \emptyset$.

Finally, the complexity of the IC algorithm can be estimated as follows. The calculation of the reliability also involves two matrix inversions $\mathbf{R}_{\text{corr}}^{-1}$ and $\mathbf{Q}_j^{-1}$, for which the complexities are discussed above. The computations involved in sorting the reliabilities is $\mathcal{O}(|U| \log |U|)$. To simplify the analysis, we may upper bound the complexity by assuming that $|U| = N_g$ in all iterations, while the number of iterations is equal to N_g . Hence, the total complexity of the IC algorithm is bounded by $\mathcal{O}(N_g^2 \log N_g + N_g N M_R^3 + N_g^2 N G^3 + N_g^2 2^{lG})$. It is clear that when the size of group is small and N is large, significant reduction in complexity can be achieved at the expense of performance degradation.

VIII. ASYMPTOTIC PERFORMANCE OF IRM

In this section, we will present one of our main results regarding the asymptotic performance (i.e., as $E/N_0 \rightarrow \infty$) of ML joint detection when users are transmitting with IRM. The results are valid for both coded/uncoded system. A data block of finite length is considered. The codeword length l is always greater than M_T . In the analysis, the encoder used can be of arbitrary type, either space-time or time (for $M_T = 1$), and it is assumed that it provides full transmit diversity gain M_T .

For uncoded system, encoder simply means identical mapping. Comparison between the proposed and ordinary modulation is based on the asymptotic performance of average bit-error probability (BEP) or P_b , relative to that of the reference user with the same number of transmit antennas but no interferers ($P_{b,\infty}$)

$$\lim_{N_0 \rightarrow 0} \frac{P_b}{P_{b,\infty}}. \quad (29)$$

First, assume that IRM is not yet applied. Without loss of generality, we compute the BEP of the k th bit of the data block for the first (reference) user, P_b^k . Let $\mathbf{C}$ be the joint codeword initially transmitted and $\mathcal{C}^k$ be the set of codewords which results from encoding a data block which has a different bit in the k th position. Denote the event of erroneously deciding *any* codeword of this type as $\mathbf{C} \rightarrow \mathcal{C}^k$, which is the union of events, $\bigcup_{\mathbf{C} \in \mathcal{C}^k} \mathbf{C} \rightarrow \hat{\mathbf{C}}$. Note that $P[\mathbf{C} \rightarrow \mathcal{C}^k | \mathbf{C}]$ denotes the probability of such event *without* considering the decision regions of other good codewords (denoted as $\mathcal{C}^{\bar{k}}$) which result from encoding a data block having the same k th bit. To account for the effect of these good codewords which reduce the BEP, we define $\mathbf{C} \rightarrow \mathcal{C}^k / \mathcal{C}^{\bar{k}}$ as the event of deciding in favor of good codewords if their decision regions are taken into account, otherwise, we decide in favor of codewords in $\mathcal{C}^k$. Also, let $E_{\mathbf{C}}$ denote the expectation with respect to $\mathbf{C}$. It then follows that:

$$P_b^k = E_{\mathbf{C}}(P[\mathbf{C} \rightarrow \mathcal{C}^k | \mathbf{C}] - P[\mathbf{C} \rightarrow \mathcal{C}^k / \mathcal{C}^{\bar{k}} | \mathbf{C}]). \quad (30)$$

Our aim is to express P_b^k in terms of the interference-free $P_{b,\infty}$. To do so, we introduce the union bound and partition the events defined above as follows. Let $\mathcal{C}_1^k$, being the subset of $\mathcal{C}^k$, denote the set of codewords with which the error matrix, $\mathbf{B}$, contains zero rows from row $M_T + 1$ to $N \cdot M_T$ (first M_T rows belong to the reference user). For completeness, $\mathcal{C}_1^k$ denotes the set, a subset of $\mathcal{C}^k$, with which the error matrix, $\mathbf{B}$, contains at least $M_T + 1$ nonzero rows. Likewise, $\mathbf{C} \rightarrow \mathcal{C}^k / \mathcal{C}^{\bar{k}}$ is partitioned into four events: $\mathbf{C} \rightarrow \mathcal{C}_1^k / \mathcal{C}_1^{\bar{k}}$, $\mathbf{C} \rightarrow \mathcal{C}_1^k / \mathcal{C}_1^{\bar{k}}$, $\mathbf{C} \rightarrow \mathcal{C}_1^k / \mathcal{C}_1^{\bar{k}}$, $\mathbf{C} \rightarrow \mathcal{C}_1^k / \mathcal{C}_1^{\bar{k}}$. For notation purposes, the above events are labeled as J_1, J_2, J_3, J_4 , respectively. It follows that:

$$P_b^k \leq E_{\mathbf{C}} \left(P[\mathbf{C} \rightarrow \mathcal{C}_1^k | \mathbf{C}] + P[\mathbf{C} \rightarrow \mathcal{C}_1^k | \mathbf{C}] - P[J_1 \bigcup J_2 | \mathbf{C}] - P[J_3 \bigcup J_4 | \mathbf{C}] \right) \quad (31)$$

$$\leq E_{\mathbf{C}} (P[\mathbf{C} \rightarrow \mathcal{C}_1^k | \mathbf{C}] + P[\mathbf{C} \rightarrow \mathcal{C}_1^k | \mathbf{C}] - P[J_1 | \mathbf{C}]) \quad (32)$$

where the first inequality is due to the union bound and the fact that $P[(\mathbf{C} \rightarrow \mathcal{C}_1^k) \cap (\mathbf{C} \rightarrow \mathcal{C}_1^{\bar{k}}) | \mathbf{C}] \geq P[(J_1 \cup J_2) \cap (J_3 \cup J_4) | \mathbf{C}]$. The second inequality is due to ignoring events J_2, J_3, J_4 . Next, we examine the case with no interference. The BEP can be expressed (using similar notations as above for user 1, codeword $\mathbf{C}_1$) as

$$P_{b,\infty} = E_{\mathbf{C}_1} \left(P[\mathbf{C}_1 \rightarrow \mathcal{C}_1^k | \mathbf{C}_1] - P[\mathbf{C}_1 \rightarrow \mathcal{C}_1^k / \mathcal{C}_1^{\bar{k}} | \mathbf{C}_1] \right). \quad (33)$$

Note the following claim.

Claim: $P_{b,\infty} = E_{\mathbf{C}}(P[\mathbf{C} \rightarrow \mathcal{C}_1^k | \mathbf{C}] - P[J_1 | \mathbf{C}])$.

Proof: When the joint codeword $\mathbf{C}$ contains $\mathbf{C}_1$, it is clear that

$$P[\mathbf{C}_1 \rightarrow \mathbf{c}_1^k | \mathbf{C}_1] - P[\mathbf{C}_1 \rightarrow \mathbf{c}_1^k / \mathbf{c}_1^k | \mathbf{C}_1] = P[\mathbf{C} \rightarrow \mathbf{c}_1^k | \mathbf{C}] - P[J_1 | \mathbf{C}] \quad (34)$$

since the calculation of probability of above events on both sides involve the identical set of error matrices. Note that all $\mathbf{C}$ s which include $\mathbf{C}_1$ give the same probability of above events. Since each $\mathbf{C}_1$ is included in the same number of joint codewords $\mathbf{C}$, the expectation over $\mathbf{C}_1$ is the same as that over $\mathbf{C}$. $\square$

With this claim, it is valid to substitute $P_{b,\infty}$ into (32). And since $P_{b,\infty} \leq P_b^k$ [8], it follows that:

$$P_{b,\infty} \leq P_b^k \leq P_{b,\infty} + E_C P[\mathbf{C} \rightarrow \mathbf{c}_1^k | \mathbf{C}]. \quad (35)$$

Now, we present the following theorem.

Theorem: Given a system with N users and each equipped with M_T transmit antennas, if IRM is used such that the IRMs rank criterion is satisfied, the asymptotic performance of ML joint detection is as follows:

$$\lim_{N_0 \rightarrow 0} \frac{P_b}{P_{b,\infty}} = 1. \quad (36)$$

That is, the single-user performance with M_T transmit antennas is achieved.

Proof: First, it is required to prove that the single-user performance of the signaling scheme employed by each user remains unchanged with the application of IRM. The proposition 1 already provides such a proof. Hence, it is valid to use the same single-user $P_{b,\infty}$ in the analysis of the multiuser case.

Prior to the application of IRM, the event $\mathbf{C} \rightarrow \mathbf{c}_1^k$ can be partitioned into two types of event. As subsets of $\mathbf{C}_1^k$, let $\mathbf{C}_{1,r=M_T}^k$ and $\mathbf{C}_{1,r>M_T}^k$ denote the set of codewords with which the error matrix $\mathbf{B}$ has rank M_T and greater than M_T , respectively, (note that there is no $r < M_T$). Using the union bound, the right-hand side of (35) can be expressed as

$$P_b^k \leq P_{b,\infty} + E_C \left[\sum_{\mathbf{C} \in \mathbf{C}_{1,r=M_T}^k} P(\mathbf{C} \rightarrow \hat{\mathbf{C}} | \mathbf{C}) \right] + E_C \left[\sum_{\mathbf{C} \in \mathbf{C}_{1,r>M_T}^k} P(\mathbf{C} \rightarrow \hat{\mathbf{C}} | \mathbf{C}) \right]. \quad (37)$$

At low N_0 , each pairwise error probability can be expressed in the form $K \cdot (E/N_0)^{-rM_R}$, where K is a constant and r is the rank of $\mathbf{B}$. By denoting $\hat{K}_r$ as the maximum K out of all pairs having the same rank r , (37) can be further simplified and bounded by dropping the expectation

$$P_b^k \leq P_{b,\infty} + D_{M_T} \hat{K}_{M_T} (E/N_0)^{-M_T M_R} + \sum_{i=M_T+1}^{NM_T} D_i \hat{K}_i (E/N_0)^{-iM_R} \quad (38)$$

where D_i is a constant which is equal to the weighted sum of different pairs with error matrix of rank i over all $\mathbf{C}$. Since $P_{b,\infty}$

Fig. 3. Performance of ML detection with/without IRM; two users, uncoded.

can also be expressed as $K \cdot (E/N_0)^{-M_T}$ at low N_0 [11], where K is another constant, it follows that at low N_0 :

$$\frac{P_b^k}{P_{b,\infty}} \leq 1 + \frac{D_{M_T} \hat{K}_{M_T}}{K} + \sum_{i=M_T+1}^{M_T N} \frac{D_i \hat{K}_i (E/N_0)^{(-i+M_T)M_R}}{K}. \quad (39)$$

Thus, the asymptotic performance can now be shown as

$$1 \leq \lim_{N_0 \rightarrow 0} \frac{P_b^k}{P_{b,\infty}} \leq 1 + \frac{D_{M_T} \hat{K}_{M_T}}{K}. \quad (40)$$

When rotation is applied to all users such that the IRMs rank criterion is satisfied, D_{M_T} (the event $\mathbf{C} \rightarrow \mathbf{c}_{1,r=M_T}^k$) is reduced to zero and the right-hand side of (40) becomes one. The proof is then complete. Note that the IRM criterion is a more-than-sufficient condition for the right-hand side of (40) to become one, since only the set of rank- M_T error matrices needs to be eliminated. $\square$

In comparison with the results obtained [8] for the uncoded transmission and $M_T = 1$, where it is shown that P_b has the same asymptotic order as $P_{b,\infty}$ only (i.e., both achieve the same diversity gain), we proceed even further by showing that P_b approaches the exact interference-free performance using IRM.

IX. SIMULATION RESULTS

First, we investigate the effect of using IRM upon the system performance. All users transmit with the same average power. The examples of IRM studied in Section VI are examined here. The number of receive antenna is always one, unless stated otherwise. The first two cases both involve one reference user and one interferer but with block-length equal to two and four, respectively. Fig. 3 shows the BEP versus the SNR (E/N_0). For comparison, the single-user (interference-free) BEP performance curve, along with that of two-users with normal BPSK, are shown. Notice that at low SNR, the improvement of IRM over non-IRM BPSK is not obvious while at high SNR, the gap in performance increases. Moreover, the performance of IRM approaches the exact performance of the interference-free case,

Fig. 4. The effect of good rotations in IRM; optimal angle versus nonoptimal angle.

Fig. 5. Performance of ML detection with/without IRM; three users, uncoded.

in agreement with the conclusion reached in Section VIII. As shown in Fig. 4, the threshold where the two curves merge is moved to a lower SNR by increasing the coding gain with the optimal angle of rotation. This is because the influence of error matrices with high rank diminishes at high SNR, and by maximizing the coding gain associated with these matrices, their effects disappear at a relatively lower SNR. On the other hand, when the block length is increased to four, there is no additional improvement over that of the length of two, as shown in Fig. 3. Hence, there is no reason to apply a rotation with dimension greater than the number of users in this scenario. This is because the maximum rank achievable is nevertheless limited by the total number of users.

When the number of co-channel users increases to three, similar observations are noted in Fig. 5. The performance gap between non-IRM and single-user is larger than that for two users, due to stronger interference. By using IRM, the gap is closed at high SNR, and therefore, the improvement is more significant (4 dB) as this stronger interference is mitigated.

Fig. 6. Performance of IRM with STBC; two users, two transmitters each.

Fig. 7. Performance of IFRM; two users, two transmitters each.

Next, we examine the performance of IRM with space-time code when two transmit antennas are used for each user. In Fig. 6, first we note that the single-user performance is superior to that of using one transmit antenna due to an additional diversity gain. With the application of IRM, the gap between the single-user and the two-users closes at high SNR. Thus, the IRM remains resistive to interference while retaining the diversity gain associated with each user.

Fig. 7 illustrates the performance of IFRM in a block-wise fast fading channel. As mentioned previously, it is also applicable for multiple transmit antennas with orthogonal signaling. The single-user performance curve indicates the result of applying fading-resistant modulation which exploits the transmit diversity gain. With IRM, the two-users performance again approaches that of single-user. In comparing this result to that of using IRM with STBC, it becomes apparent that the latter outperforms the former, although both achieve the same order of diversity gain. Nevertheless, the IFRM becomes useful in the block-wise fast fading channel, in which the STBC with IRM is not applicable. Also, the design of space-time codes, such as

Fig. 8. Performance of linear group detectors; $M_R = 3$, four users, two groups.

STBC, becomes too complicated for large M_T (or even impossible for some M_T). On the other hand, the IFRM can be easily generalized to large M_T .

Next, in Fig. 8, the performance of proposed MMSE-based group detector is shown. The number of users is four, and they are divided into two groups of two. Number of receive antennas M_R is set to three. Note that even without IRM, MMSE-based group detector outperforms its decorrelator counterpart. With IRM, the performance is further improved, indicating that the MMSE-based group detector is able to retain the interference-resistant ability of IRM while having low complexity. Fig. 8 also illustrates the performance of MMSE-based group detector with interference cancellation. As expected, its performance with IRM provides the most pronounced improvement, which is about 1 dB over the same scheme without IRM.

X. CONCLUSION

We propose a novel multiuser signal structure called IRM. In a synchronized multiuser system, where each user transmits independently and multiple antennas are deployed at receiver, the joint signal design problem bears close resemblance to that of traditional single-user space-time codes except that centralized/coordinated design now becomes impossible. New design criteria are, therefore, suggested and the proposed scheme, IRM, is shown to be an effective way to satisfy them without any coding nor bandwidth expansion. When each user employs multiple transmit antennas, IRM can be combined with space-time codes or previously proposed fading-resistant modulation to remedy the impairment due to both fading and interference. We prove, both analytically and by simulations, that single-user performance is achieved asymptotically without any bandwidth expansion, for both uncoded and coded system with arbitrary number of transmit antennas. In order to reduce the high complexity of ML detection and IRM, we propose the MMSE-based group precombining detector. By partitioning large number of users into smaller groups and applying IRM

independently to these groups, this low-complexity receiver provides significant improvement over the previous approaches.

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