Modal analysis and experimental research on a planetary reducer with small tooth number difference

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Modal analysis and experimental research on a planetary reducer with small tooth number difference

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Abstract

In order to comprehensively analyze the modal characteristics of planetary reducer with small tooth number difference, on the basis of the coupling of the shaft–bearing–gear–shell, firstly a finite element modal analysis model was established in the ABAQUS, the teeth meshing sites were constrained by binding, bearings were simulated by spring elements, then the natural frequencies and corresponding vibration modes of the reducer were obtained by applying the Lanczos method. Afterwards the hammering modal experiment on the reducer was carried out utilizing the LMS Test. Lab, and the modal data were analyzed by employing the method of modal identification named PolyMAX, the modal frequencies and damping ratios were achieved, also the experimental modal parameters were validated according to the modal assurance criterion named MAC. The research results indicate that the lowest-order natural frequency of the reducer is 148.53Hz, which is much higher than the rotation frequency of eccentric shaft, double gear and output gear, also the two-stage gear mesh frequencies are away from the natural frequencies, therefore the reducer under normal operating conditions will not cause coupling resonance. This research work has provided a theoretical basis and experimental reference for the dynamic structure optimization of planetary reducer.

Key words

Planetary reducer, Small tooth number difference, Natural frequency, PolyMAX, Modal experiment
1 Introduction

The planetary reducer with small tooth number difference is compact, small in size, light in weight, large in transmission ratio, large in carrying capacity, high in efficiency and long in service life. It has been widely used in the mechanical equipment of transportation, mining, metallurgy, chemical, textile, construction, instrumentation and other industrial sectors (Feng 1982). Therefore, it has received high attention at home and abroad. The structural dynamic performance of the gear transmission directly reflects the stability and reliability of the transmission system. Over the years, domestic and foreign scholars have conducted extensive and in-depth research on the modal characteristics of planetary reducers based on theoretical modes and finite element methods.

Bu et al. (2011, 2012) developed a dynamic model of herringbone planetary gear trains considering the asymmetric stiffness and coupling characteristics of sliding bearings. Four stiffness coefficients were used to describe the support oil film stiffness of planet wheel sliding bearing. And then, the free vibration characteristic of the system was analyzed. For the gear power coupling transmission system of the wind power transmission box load-split two-stage planetary gear train, Wang et al. (2013, 2014) constructed the dynamic model of the multi-stage coupling transmission system using the centralized parameter method. With the comprehensive consideration of gear tooth meshing error, damping, stiffness, interstage coupling stiffness, moment of inertia of component and other factors, the inherent characteristics of the system were studied, it was concluded that there were eight typical vibration modes in the system. However, each component in the model was regarded as a rigid body, the torsional rigidity of each support and the friction of the system were not considered, and the mean value of time-varying meshing
stiffness was taken as the meshing stiffness of each gear pair. Zhang et al. (2015) divided the star-wheel reducer with small tooth number difference into subsystems such as output shaft, input shaft, star wheel axle and translational star wheel based on the substructure synthesis, and constructed the motion differential equations of each subsystem through the Newton's mechanics method. With the comprehensive consideration of the radial bearing deformation of each bearing, the gear pair deformation, the indexing error and eccentricity error of the eccentric sleeve on the input shaft and the star wheel axle, the deformation coordination conditions of each link in the system were constructed, and the elastic dynamic equation of the star wheel reducer was constructed by combining the differential equations of the subsystems. The intrinsic properties were obtained by solving the eigenvalue problem of the equation. For a two-stage closed planetary gear set, Zhang et al. (2016, 2017) established a translational-rotation dynamics model considering rotation and translational displacement, and studied its dynamic response and avoiding resonance. The natural frequency and mode shape were obtained by converting the kinetic equation into a matrix form. The results showed that the vibration of the device can be divided into several types of modes: rigid body, rotation and translation.

Parker RG and its collaborators combined the analysis method with the finite element method to systematically study the dynamics of the planetary gear transmission system, significant results have been achieved in the vibration mode analysis (Wu 2010; Guo and Parker 2010b; Parker and Wu 2010), dynamics modeling and analysis (Guo and Parker 2010a), modal characteristics (Cooley and Parker 2013a, 2013b; Ericson and Parker 2013, 2014), etc.

Qiu et al. (2018) proposed a rotational-translational-axial dynamic model of the planetary gear under pitching base motion. Dynamic responses of the planetary gear were obtained by numerical
integration, and spectrum analysis was conducted using the Fourier transform. Spectra of the rotational and translational vibrations of the central components showed completely different characteristics because of the structure symmetry of the planetary gear. Mbarek et al. (2018) presented a comparison between an Experimental Modal Analysis test, an Operational Modal Analysis test and an Order Based Modal Analysis applied on a recirculating energy planetary gear, the back to back planetary gears modal parameters were identified. Furthermore, they investigated a modal analysis of planetary gear transmission for different loading conditions and under mesh stiffness fluctuation (Mbarek et al. 2019). Tatar et al. (2019) developed a six degrees of freedom dynamic model of a planetary geared rotor system with equally spaced planets by considering gyroscopic effects. The dynamic model was created using a lumped parameter model of the planetary gearbox and a finite element model of the rotating shafts using Timoshenko beams. The modal analysis results showed that torsional and axial vibrations on the shafts were coupled in the helical gearing configuration due to the gear helix angle whereas these vibrations became uncoupled for spur gearing.

Although the study of the modal characteristics of the internal meshing planetary reducer is common, some are separate analysis of the gear transmission system and the box. However, for the NN type planetary reducer with small tooth number difference, considering the coupling effect of the shaft-bearing-gear-housing, combining with finite element method and experimental modal method, the comprehensive modal characteristics analysis is rare.

In this paper, the NN type planetary reducer with small tooth number difference is studied. Firstly, the finite element free modal analysis is carried out to determine its natural frequency and mode shape. Then, LMS Test.Lab is used to perform the hammer modal experiment to obtain the
modal frequency and damping ratio. The finite element analysis values are basically consistent
with the test values. It provides reference for optimizing the structure and transmission parameters
of the planetary reducer, avoiding resonance, reducing noise and improving working performance.

2 Structure and principle of planetary reducer

The assembly structure of the planetary reducer with small tooth number difference (Wang et al.
2007) is shown in Fig.1. When the motor drives the eccentric shaft to rotate around its geometric
center o-o (2000 rpm), the eccentric force forces the slewing bearing to push the double gear away
from the center of the frame in the radial direction, while the ball transmits the rotation of the
eccentric shaft to the double gear through rolling, so that it meshes with the fixed gear to achieve
planetary motion (both revolutionary and self-rotating). When the left double gear meshes with the
fixed gear according to the above motion, the first-stage transmission is realized, and when the
right double gear meshes with the output gear, the second-stage transmission is realized. The gear
parameters are shown in Table 1.

Fig.1 Assembly diagram of the planetary reducer

Table 1 Gear parameters of the planetary reducer

3 Natural characteristics analysis of planetary reducer

3.1 Natural characteristics theory

The vibration differential equation (Kelly 2000) of the reducer system is
\[ M\ddot{x} + C\dot{x} + Kx = f(t) \]  \hspace{1cm} (1)

where \( M \), \( C \), and \( K \) are the equivalent mass, damping coefficient, and stiffness matrix of the system, respectively; \( \ddot{x}, \dot{x}, x \) are vibration acceleration, velocity, and displacement, respectively; \( f(t) \) is the load vector of external excitation.

Since the natural characteristics of the system are independent of the external load, the influence of the external load can be neglected; and when the natural characteristics are analyzed, the damping has little effect on it, the damping term can be neglected, and the undamped free vibration equation of the reducer is established:

\[ M\ddot{x} + Kx = 0 \]  \hspace{1cm} (2)

Let the \( i \)-th order vibration frequency of harmonic motion be \( \omega_i \), the vibration mode be \( \phi \), and substitute the main vibration \( x = \phi \sin(\omega_i t + \phi) \) into equation (2), and obtain the corresponding characteristic equation:

\[ (K - \omega_i^2 M)\phi = 0 \]  \hspace{1cm} (3)

equation (3) is an \( n \)-th equation for \( \omega_i^2 \), sharing \( n \) pairs of roots. These \( n \) pairs of roots are called \( n \) feature pairs, i.e. \( n \) feature vectors \( \phi \) and \( n \) eigenvalues \( \omega_i^2 \). The eigenvectors reflect the spatial form of the gear system when it vibrates according to the natural frequency \( \omega_i \), so it is also called the mode vector or modal.

### 3.2 Finite element analysis of natural characteristics

#### 3.2.1 Analysis model

The finite element modal analysis model of the reducer is established by ABAQUS, and the unconstrained free modal analysis is carried out. After the free meshing, the model contains 467054 units and 125493 nodes.
In view of the free modal analysis, there is no need to impose external constraints on the reducer. The gear teeth meshing portion adopts a binding constraint. In order to avoid local stress concentration caused by the simulated spring unit between the single nodes of the contact surface, the spring unit is added to simulate the bearing according to the actual contact surface of the bearing. The reducer has four deep groove ball bearings and one needle bearing. The deep groove ball bearing adopts four symmetrically distributed spring simulations: in the working coordinate system, two node sets are established, and the two node sets are respectively coupled with the two actual contact faces, and a spring simulated bearing is added between the two node sets as shown in Fig.2 (A and B are the node sets of the eccentric shaft and the double gear, respectively, a spring is connected between the two points, the spring defines the stiffness value in the working coordinate system). The needle bearing uses a symmetrical two-row spring, each row consists of four springs distributed symmetrically.

Fig.2 Simulating bearings by using springs

3.2.2 Analysis results

The free mode is solved by the Lanczos method which is suitable for extracting multi-order modes of complex models. According to the free modal analysis theory, the first 6 order frequency is all zero, and the first 20 order natural frequencies and vibration modes extracted are shown in Table 2. Due to space limitations, only the mode shapes of some orders are shown in Fig.3. Since the bearing housing and the fixed gear are fully constrained, the low order is basically no deformation except for the high-order micro-vibration deformation, they have been hidden for easy observation.
Table 2 The first 20 orders natural modes of the planetary reducer

(R-Rotation around; S-Swing around; T-Translation along)

Fig.3 The several orders mode shapes of the planetary reducer

The modal analysis results show that the lowest natural frequency of the reducer is 148.53 Hz, which is much higher than the rotation frequency of the eccentric shaft 33.33 Hz (2000/60), the rotation frequency of the double gear 37.5 Hz (2250/60) and the rotation frequency of the output gear 33.75 Hz (2025/60). In the first 20 natural frequencies, there is no close frequency to the meshing frequency of the double gear and the fixed gear 1500 Hz (37.5 Hz × 40 teeth), and the meshing frequency of the double gear and the output gear 1687.5 Hz (37.5 Hz × number of teeth 45). Therefore, under normal operating conditions, there is no phenomenon that the rotational frequency or the meshing frequency coincides with the natural frequency to resonate.

4 Experimental modal analysis of planetary reducer

4.1 Experimental modal theory

Experimental modal analysis (EMA), also known as the experimental process of modal analysis, is based on the combination of theoretical analysis and experimental techniques. By fitting the curve of the excitation and response (such as displacement, velocity, acceleration, etc.) transfer function of the structure, the parameter identification method is used to obtain the modal parameters of the structure, namely the natural frequency, the mode shape, and the damping ratio.

For the planetary reducer, the Fourier transform with zero initial conditions for both ends of equation (1) is obtained (Verboven 2002):
\[ [M(j\omega)^2+C(j\omega)+K] \cdot X(\omega) = F(\omega) \quad (4) \]

Let \( H(\omega) = [M(j\omega)^2+C(j\omega)+K]^{-1} \), then equation (4) is

\[ X(\omega) = H(\omega) \cdot F(\omega) \quad (5) \]

where \( H(\omega) \) is the frequency response function matrix of the system, i.e., the transfer function matrix; \( F(\omega) \) is the Fourier transform of the external excitation load; \( X(\omega) \) is the Fourier transform of the vibration displacement.

Let \( m_r, k_r, c_r, \varphi_r, \omega_r, \zeta_r \) be the \( r \)th modal mass, stiffness, damping, mode shape, natural frequency, damping ratio, then between the \( i \)-th response point and the \( j \)-th excitation point the frequency response function (Yam et al. 1996) is:

\[
H_y(\omega) = \sum_{i=1}^{n} \frac{\varphi_i \varphi_{jr}}{k_r - \omega^2 m_r + j \omega c_r} = \sum_{i=1}^{n} \frac{\varphi_i \varphi_{jr}}{k_r (1 - \lambda_r^2 + 2j \zeta_r \lambda_r)} \quad (6)
\]

where \( \lambda_r = \omega/\omega_r, \quad \omega_r = \sqrt{k_r/m_r}, \quad \zeta_r = c_r/(2m_r \omega_r) \)

In order to determine all modal parameter information, it is only necessary to measure any column of the frequency response function matrix (single-point excitation multi-output SIMO, fix hammer and pick up each point) or any line (multi-point excitation single output MISO, move hammer and pick up single point).

The frequency response function of the system can be obtained from the power spectral density during the experiment. The formula is obtained by equation (5):

\[
H(\omega) = \frac{X(\omega) \cdot X(\omega)}{F(\omega) \cdot X(\omega)} = \frac{S_{XX}(\omega)}{S_{EF}(\omega)} \quad (7)
\]

where \( S_{XX}(\omega) \) is the self-power spectral density of the response; \( S_{EF}(\omega) \) is the mutual power spectral density. To evaluate the linear correlation between response and excitation, a coherence function (Heylen et al. 1998) is introduced:
\[
\gamma_{FX}^2(\omega) = \frac{|S_{FX}(\omega)|^2}{S_{FF}(\omega) \cdot S_{XX}(\omega)}
\]  

(8)

where \(S_{FF}(\omega)\) is the self-power spectral density of the excitation, and the coherence function \(\gamma^2\) takes a value between zero and one. When the value is close to one, it indicates that there is a good linear relationship between the response signal and the excitation signal after averaging. The experimental data is correct.

4.2 Experimental program

The LMS Test.Lab is used to perform a free-mode experiment on the planetary reducer by hammering (pulse excitation). The reducer is axially knocked by pulsing hammer with the force sensor, the response signal was collected through the three-axis acceleration sensor. The excitation signal \(f(t)\) and the vibration response signal \(x(t)\) are input to the LMS SCADAS data acquisition front end, and the frequency domain signals \(F(\omega)\) and \(X(\omega)\) after the Fourier transform are processed by the dynamic signal analysis, then, the frequency response function \(H(\omega)\) of the system can be calculated through the equation (7). Using the frequency response function, the modal parameters of the system are obtained by the PolyMAX modal parameter identification method.

In view of the small size, light weight and compact structure of the reducer, a three-axis acceleration sensor with wide dynamic range, wide working frequency range, strong anti-interference ability, high sensitivity, light weight is selected so as to minimize the influence of the additional mass of the sensor on the structural characteristics of the tested reducer. The main equipment used is shown in Table 3.

| Table 3 Main test equipment and analysis instrument |

https://mc06.manuscriptcentral.com/tcsme-pubs
The elastic suspension of the reducer is used to approximately simulate the free-free boundary condition in the free modal experiment. The modal test site is shown in Fig.4: the reducer is suspended on a rigid test bench by a rubber band with a small stiffness coefficient and ensures that the eccentric shaft is in a horizontal position, and the acceleration sensor is adhered to the distal end surface of the fixed gear of the reducer in the axial direction.

Fig.4 Modal experiment on the planetary reducer

In the experiment, the vibration frequency response function was obtained by single-point tapping, single-point picking, and repeated excitation. In order to improve the recognition accuracy of the modal parameters, it is necessary to properly arrange the positions of the excitation point and the pickup point to minimize the modal omission. In order to effectively stimulate the multi-order modal frequency of the reducer, the points are evenly distributed along the circumference on the distal end surface of the fixed gear of the reducer (the same side as the sensor), and are sequentially struck axially. Then, the same points are placed on the opposite side of the reducer, and the taps are sequentially performed in the axial direction.

It is difficult to ensure that each pulse excitation is the same at the time of sampling, so multiple tests are used to average. In order to improve the signal-to-noise ratio and eliminate some noise and interference factors, the experiment should be struck at multiple intervals (the magnitude of each tap should be equivalent), and the interval should be long enough to avoid aliasing of the response of two taps.

4.3 Experimental results
The LMS Test. Lab Modal Analysis module is used for data selection and post-processing analysis in the experiment.

According to the established experimental scheme, the excitation points are hammered in turn, and the self-power spectral density, mutual power spectral density, frequency response function and coherence function of each excitation point and response point along the X, Y and Z directions can be obtained as shown in Fig.5-8. There is a compact structure and a small volume for the reducer, and the effective transmission time of the excitation signal is very short, and the attenuation is fast, so the spectral density function, the frequency response function, and the coherence function curve are relatively flat, and there is no obvious continuous peak and valley. For the picked three-way signal, there is the largest amplitude in X-direction (end face normal), which is the main factor involved in modal analysis identification.

Fig.5 Self-power spectral density

Fig.6 Cross power spectral density

Fig.7 Frequency response function

Fig.8 Coherence function

From the coherence function map shown in Fig.8, the X-direction is considered. The coherence coefficients of most of the natural frequency segments are above 0.9, indicating that the response signal and the excitation signal have a good linear relationship after averaging, and the collected
data is highly reliable.

The PolyMAX (Peeters et al. 2004) modal identification method in LMS Test. Lab belongs to the multi-degree-of-freedom time domain identification method, also known as the Polyreference least squares complex frequency domain method. It is the multi-input form of the least squares complex frequency domain method (LSCF), and is a multi-degree-of-freedom method for overall estimating the pole and modal participation factors. The steady state diagram generated by PolyMAX can identify highly dense modes and has high recognition accuracy for each mode frequency, damping and mode shape. It is an ideal modal analysis method based on transfer function. Set the analysis bandwidth to 4096 Hz, select the required frequency response data, and perform a lumped average on the transfer function. The steady state diagram of the modal parameter identification using the PolyMAX method is shown in Fig.9.

Fig.9 Steady state diagram of the modal parameters identification

In the steady state diagram, the absissa is the frequency, the left ordinate is the real amplitude of the frequency response function, and the right ordinate is the assumed number of poles. f, d, v, s represent the frequency, damping and frequency, vector, frequency and damping of the hypothetical pole and the steady state of the vector within the tolerance range (Heylen et al. 1998). When the modal order is picked up, the frequency corresponding to s in the longitudinal comparison set is taken as the first order. The first 17 orders modal frequencies and damping ratios of the reducer are thus picked up, as shown in Table 4, and compared with the previous theoretical analysis values. The analytical value is in good agreement with the experimental value, and the relative error is small. The error of 12th order is the largest, probably because the tapping point is
just the node of a certain mode shape of the reducer, resulting in distortion of the excitation signal.

Table 4 The first 20 orders natural frequencies and damping ratios of the planetary reducer

It is known from Table 4 that the order of theoretical analysis modal is more than that of experimental modal, and the experimental modal frequency is mainly concentrated below 2500 Hz. With the experimental method of the overall free mode, it is only possible to arrange measuring points on the outer surface of the reducer, so detailed vibration information inside the reducer cannot be obtained. In addition, the free-mode hammer excitation is not pure white noise, due to the limitations of experimental conditions and methods, the higher-order modes of the reducer cannot be excited. For vibration analysis, it is generally concerned with the low-order modes of the structure, so there is certain engineering application value in the experiment.

4.4 Modal verification

The accuracy of the experimental modal analysis results can be directly determined by modal verification. The modal assurance criterion (MAC) is a commonly used modal accuracy determination method, which is used to represent the plausibility of modality. The MAC (Carne and Dohrmann 1994) values of the two-mode shape vectors $\varphi_r$ and $\varphi_s$ are defined as:

$$\text{MAC}_{rs} = \frac{|\varphi_r^T \varphi_s|^2}{(\varphi_r^T \varphi_r) \cdot (\varphi_s^T \varphi_s)}$$  \hspace{1cm} (9)

If there is a linear correlation between the complex vectors $\varphi_r$ and $\varphi_s$, the MAC value is close to one; if the two are linearly independent, the MAC value is close to zero. Therefore, the false and overlapping modes can be effectively eliminated by the modal confidence criterion. Figure 10 is a histogram of the first 17 orders experimental modal data MAC matrix of the reducer. The MAC
value analysis shows that the MAC value between the same vector on the diagonal is one. The MAC values of the remaining modal vectors are all less than one, that is, independent of each other, indicating that the modal parameters extracted by the experiment are more accurate.

![Fig.10 Column diagram of the MAC matrix](https://mc06.manuscriptcentral.com/tcsme-pubs)

### 5 Conclusions

(1) Establish the finite element model of planetary gear reducer with small tooth number difference, solve its eigenvalue by Lanczos method, and extract the first 20 natural frequencies and vibration modes. The lowest order natural frequency is 148.53 Hz, which is much higher than that of the eccentric shaft, double gear and output gear. In the first 20 natural frequencies, there is no close frequency to the meshing frequency of the double gear and the fixed gear, and the meshing frequency of the double gear and the output gear, so the coupling resonance will not be induced under normal working conditions.

(2) Based on EMA theory, the modal test of the reducer is carried out by hammering method. The modal data is obtained by PolyMAX modal identification method, and the natural frequency and damping ratio are obtained. The experimental modal data is more accurate by MAC test.

(3) The comparison analysis shows that the finite element analysis results are in good agreement with the experimental data, and the relative error between the two natural frequencies is small, which provides a reference for optimizing the structure and transmission parameters of the planetary reducer and reducing the vibration noise.
Acknowledgment

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References


Wu, X.H. 2010. Vibration of planetary gears having an elastic continuum ring gear. The Ohio State University, Columbus.


Table 1 Gear parameters of the planetary reducer

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<th>Double gear</th>
<th>Output gear</th>
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<tr>
<td>Module $m$ / mm</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Tooth number $Z$</td>
<td>-45</td>
<td>40 / 45</td>
<td>-50</td>
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<tr>
<td>Pressure angle $\alpha$ / °</td>
<td>20</td>
<td>20</td>
<td>20</td>
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<tr>
<td>Addendum coefficient $h_a$</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
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<tr>
<td>Tip clearance coefficient $c$</td>
<td>0.25</td>
<td>0.25</td>
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<tr>
<td>Diameter of dedendum circle $d_f$ / mm</td>
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<td>38.4 / 43.4</td>
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<td>Tooth width $b$ / mm</td>
<td>6</td>
<td>5 / 5.5</td>
<td>6</td>
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<tr>
<td>Rotation speed $n$ / (r/min)</td>
<td>/</td>
<td>-250</td>
<td>-25</td>
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Table 2 The first 20 orders natural modes of the planetary reducer

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<th>No. of order</th>
<th>Natural frequency (Hz)</th>
<th>Mode shape of vibration</th>
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<tr>
<td>7</td>
<td>148.53</td>
<td>R(Y+)</td>
</tr>
<tr>
<td>8</td>
<td>452.66</td>
<td>S(X+)</td>
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<td>9</td>
<td>610.58</td>
<td>T(Y+)</td>
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<tr>
<td>10</td>
<td>779.80</td>
<td>T(Y+), S(Z+)</td>
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<td>11</td>
<td>1175.6</td>
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<tr>
<td>12</td>
<td>1232.1</td>
<td>S(X-)</td>
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<td>13</td>
<td>1572.2</td>
<td>R(Y+), T(Z+)</td>
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<td>14</td>
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<td>20</td>
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<td>S(Z-)</td>
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(R-Rotation around; S-Swing around; T-Translation along)
Table 3 Main test equipment and analysis instrument

<table>
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<tr>
<th>Name</th>
<th>Type</th>
<th>Performance parameter</th>
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<tr>
<td>Dell mobile workstation</td>
<td>M4600</td>
<td>17-2620M/4GB/500GB</td>
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<tr>
<td>Portable dynamic signal analyzer</td>
<td>LMS SCADAS Mobile, SCM01</td>
<td>8-channel, Sampling rate, 102.4kHz, resolution, 24, SNR 105dB</td>
<td>Resonance frequency 75kHz, linearity ±1%FS</td>
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<tr>
<td>Impact force hammer</td>
<td>Dytran 5800B4T</td>
<td>500LbF, 10mV/LbF</td>
<td></td>
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<tr>
<td>Acceleration sensor</td>
<td>PCB 356A01, SN LW 117394</td>
<td>Sensitivity 5.28mV/g, bias 9.8V</td>
<td>XYZ triaxial</td>
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</table>

Table 4 The first 20 orders natural frequencies and damping ratios of the planetary reducer

<table>
<thead>
<tr>
<th>No. of order</th>
<th>Analysis value</th>
<th>Experimental value</th>
<th>Damping ratio /%</th>
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Table 1 Gear parameters of the planetary reducer

Table 2 The first 20 orders natural modes of the planetary reducer

Table 3 Main test equipment and analysis instrument

Table 4 The first 20 orders natural frequencies and damping ratios of the planetary reducer

Fig.1 Assembly diagram of the planetary reducer

Fig.2 Simulating bearings by using springs

Fig.3 The several orders mode shapes of the planetary reducer

Fig.4 Modal experiment on the planetary reducer

Fig.5 Self-power spectral density

Fig.6 Cross power spectral density

Fig.7 Frequency response function

Fig.8 Coherence function

Fig.9 Steady state diagram of the modal parameters identification

Fig.10 Column diagram of the MAC matrix
Needle cage
Double gear
Rotary arm bearing
Eccentric shaft
Input bearing

Fixed gear
Output gear
Phillips screw
Bearing housing
Output bearing

Fig. 1 Assembly diagram of the planetary reducer
Fig. 2 Simulating bearings by using springs
8th-order mode shape

10th-order mode shape
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12th-order mode shape

14th-order mode shape
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